Notes on Graduate International Trade[1]

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Abstract

These notes are prepared for the first part of the part of (720) Graduate International Trade at Yale.

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Chapter 1

An introduction into deductive reasoning

1.1 Inductive and deductive reasoning

Inductive or empirical reasoning is the type of reasoning that moves from specific observations to broader generalizations and theories. **Deductive** reasoning is the type of reasoning that moves from axioms to theorems and then applies the predictions of the theory to the specific observations.

Inductive reasoning has failed in several occasions in economics. According to Prescott (see Prescott (1998)) "The reason that these inductive attempts have failed ... is that the existence of policy invariant laws governing the evolution of an economic system is inconsistent with dynamic economic theory. This point is made forcefully in Lucas' famous critique of econometric policy evaluation."

Theories developed using deductive reasoning must give assertions that can be falsified by an observation or a physical experiment. The consensus is that if one cannot potentially find an observation that can falsify a theory then that theory is not scientific (Popper).

A general methodology of approaching a question using deductive reasoning is the following:

1) Observe a set of empirical (stylized) facts that your theory has to address and/or are relevant to the questions that you want to tackle,

2) Build a theory,

3) Test the theory with the data and then use your theory to answer the relevant questions,

4) Refine the theory, going through step 1

1.2 Employing and testing a model

A vague definition of two methodologies: calibration and estimation:

Calibration is the process of picking the parameters of the model to obtain a match between the observed distributions of independent variables of the model and some key dimensions of the data. More formally, calibration is the process of establishing the relationship between a measuring device and the units of measure. In other words, if you think about the model as a "measuring device" calibrating it means to parameterize it to deliver sensible quantitative predictions.

Estimation is the process of picking the parameters of the model to minimize a function of the errors of the predictions of the model compared to some pre-specified targets. It is the approximate determination of the parameters of the model according to some pre-specified metric of differences between the model and the data to be explained.

It is generally considered a good practice to stick to the following principles (see Prescott (1998) and the discussion in Kydland and Prescott (1994)) when constructing quantitative models:

1. When modifying a standard model to address a question, the modification continues to display the key facts that the standard model was capturing.

2. The introduction of additional features in the model is supported by other evidence for these particular additional features.

3. The model is essentially a measurement instrument. Thus, simply estimating the magnitude of that instrument rather than calibrating the model can influence the ability of the model to be used as a measuring instrument. In addition the model's selection (or in particular, parametric specification) has to depend on the specific question to be addressed, rather than the answer we would like to derive. For example, "if the question is of the type, how much of fact X can be accounted for by Y, then choosing the parameter values in such a way as to make the amount accounted for as large as possible according to some metric makes no sense."

4. Researchers can challenge existing results by introducing new quantitatively relevant features in the model, that alter the predictions of the model in key dimensions.

1.3 International Trade: The Macro Facts

Chapter 2 of Eaton and Kortum (2011) manuscript

Chapter 2

An introduction to modeling

2.1 The Heckscher-Ohlin model

The Heckscher-Ohlin (H-O) model of international trade is a general equilibrium model that predicts that patterns of trade and production are based on the relative factor endowments of trading partners. It is a perfect competition model. In its benchmark version it assumes two countries with identical homothetic preferences and constant return to scale technologies (identical across countries) for two goods but different endowments for the two factors of production. The model's main prediction is that countries will export the good that uses intensively their relatively abundant factor and import the good that does not. We will present a very simple version of this model. Country i's representative consumer's problem is

$$\max a_1 \log c_1^i + a_2 \log c_2^i$$
$$s.t.\ p_1 c_1^i + p_2 c_2^i \leq r^i \bar{k}^i + w^i \bar{l}^i$$

The production technologies of good ω in the two countries are identical and given by

$$y_\omega^i = z_\omega \left(k_\omega^i\right)^{b_\omega} \left(l_\omega^i\right)^{1-b_\omega}, i, \omega = 1, 2$$

and where $0 < b_2 < b_1 < 1$. This implies that good 1 is more capital intensive than good 2. Assume for simplicity that $\bar{k}^1/\bar{l}^1 > \bar{k}^2/\bar{l}^2$. This implies that country 1 is capital abundant relative to country 2. Finally, goods, labor, and capital markets clear. One of the common assumptions for the H-O model is that there is no factor intensity reversal which in our example is always the case given the Cobb-Douglas production function (one good is always more capital intensive than the other, with the capital intensity given by b_ω).

2.1.1 Autarky equilibrium

We first solve for the autarky equilibrium for country i. This is easy especially if we consider the social planner problem, but we will compute the competitive equilibrium instead. The Inada conditions for the consumer's utility function imply that both goods will be produced in equilibrium. Thus, we just have to take FOC for the consumer and look at cost minimization for the firm. For the consumer we have

$$\max a_1 \log c_1^i + a_2 \log c_2^i$$
$$s.t.\ p_1^i c_1^i + p_2^i c_2^i \leq r^i \bar{k}^i + w^i \bar{l}^i$$

which implies

$$a_1 = \lambda^i p_1^i c_1^i \quad (2.1)$$
$$a_2 = \lambda^i p_2^i c_2^i \quad (2.2)$$
$$p_1^i c_1^i + p_2^i c_2^i = r^i \bar{k}^i + w^i \bar{l}^i \quad (2.3)$$

This gives

$$p_2^i c_2^i = \frac{a_2}{a_1} p_1^i c_1^i \ . \quad (2.4)$$

The firm's cost minimization problem

$$\min r^i k_\omega^i + w^i l_\omega^i$$
$$\text{s.t. } y_\omega^i \leq z_\omega \left(k_\omega^i\right)^{b_\omega} \left(l_\omega^i\right)^{1-b_\omega}$$

implies the following equation, under the assumption that both countries produce both goods,

$$\frac{b_\omega}{(1-b_\omega)} l_\omega^i w^i = r^i k_\omega^i \ .$$

We can also use the goods market clearing to get

$$\begin{aligned} c^i_\omega &= z_\omega \left(k^i_\omega\right)^{b_\omega} \left(l^i_\omega\right)^{1-b_\omega} \implies \\ c^i_\omega &= z_\omega l^i_\omega \left(\frac{b_\omega}{1-b_\omega}\frac{w^i}{r^i}\right)^{b_\omega}. \end{aligned}$$

Thus,

$$\begin{aligned} b_\omega p^i_\omega z_\omega \left(k^i_\omega\right)^{b_\omega} \left(l^i_\omega\right)^{1-b_\omega} &= r^i k^i_\omega \\ b_\omega p^i_\omega c^i_\omega &= r^i k^i_\omega \end{aligned}$$

similarly

$$\begin{aligned} \left(1-b_\omega\right) p^i_\omega z_\omega \left(k^i_\omega\right)^{b_\omega} \left(l^i_\omega\right)^{1-b_\omega} &= w^i l^i_\omega \\ \left(1-b_\omega\right) p^i_\omega c^i_\omega &= w^i l^i_\omega \end{aligned}$$

Zero profits in equilibrium give us

$$\begin{aligned} p^i_\omega &= \frac{r^i k^i_\omega}{b_\omega z_\omega \frac{r^i k^i_\omega}{w^i}\frac{(1-b_\omega)}{b_\omega}\left(\frac{b_\omega}{(1-b_\omega)}\frac{w^i}{r^i}\right)^{b_\omega}} \\ &= \frac{\left(w^i\right)^{1-b_\omega}\left(r^i\right)^{b_\omega}}{z_\omega \left(1-b_\omega\right)^{1-b_\omega}\left(b_\omega\right)^{b_\omega}} \end{aligned}$$

We can also derive the labor used in each sector. From the consumer's FOCs, together with the expressions for p^i_ω and c^i_ω derived above, we obtain:

$$a_\omega = \lambda^i p^i_\omega c^i_\omega \implies$$

$$\left(1-b_\omega\right)\frac{a_\omega}{\lambda^i} = w^i l^i_\omega$$

this implies that

$$l^i_1 \frac{\left(1-b_2\right) a_2}{\left(1-b_1\right) a_1} = l^i_2$$

We can use the labor market clearing condition and get

$$\begin{aligned} l_2^i + l_1^i &= \bar{l} \implies \\ l_1^i \frac{(1-b_2)\, a_2}{(1-b_1)\, a_1} + l_1^i &= \bar{l} \implies \\ l_1^i &= \frac{\bar{l}}{\left(\frac{(1-b_2)a_2}{(1-b_1)a_1} + 1\right)} . \end{aligned}$$

The results are similar for capital and thus,

$$k_1^i = \frac{\bar{k}}{\left(\frac{b_2 a_2}{b_1 a_1} + 1\right)} \quad ,$$

This implies

$$\frac{\bar{l}^i}{\bar{k}^i} = \frac{r^i}{w^i} \frac{\sum_\omega (1-b_\omega)\, a_\omega}{\sum_\omega b_\omega a_\omega} \tag{2.5}$$

Thus, in a labor abundant country capital is relatively more expensive as we would expect. We can finally use the goods' market clearing to get the values for c_ω^i's.

2.1.2 Free Trade Equilibrium

In the two country example, free trade implies that the price of each good is the same in both countries. Therefore, we will denote free trade prices without a country superscript. In the two country case it is important to distinguish among three conceptually different cases: in the first case both countries produce both goods, in the second case one country produces both goods and the other produces only one good, and in the last case each country produces only one good.

We first define the free trade equilibrium. A free trade equilibrium is allocations for consumers $\left(\hat{c}_\omega^i,\ i, \omega = 1, 2\right)$, allocations for the firm $\left(\hat{k}_\omega^i, \hat{l}_\omega^i,\ i, \omega = 1, 2\right)$, and prices $\left(\hat{w}_\omega^i, \hat{r}_\omega, \hat{p}_\omega,\ i, \omega = 1, 2\right)$ such that

1. Given prices consumer's allocation maximizes her utility for $i = 1, 2$
2. Given prices the allocations of the firms solve the cost minimization problem in $i = 1, 2$,

$$\begin{aligned} b_\omega p_\omega z_\omega \left(k_\omega^i\right)^{b_\omega - 1} \left(l_\omega^i\right)^{1-b_\omega} &\leq r^i \ , \text{ with equality if } y_\omega^i > 0 \\ (1-b_\omega)\, p_\omega z_\omega \left(k_\omega^i\right)^{b_\omega} \left(l_\omega^i\right)^{-b_\omega} &\leq w^i \ , \text{ with equality if } y_\omega^i > 0 \end{aligned}$$

3. Markets clear

$$
\begin{aligned}
\sum_i \hat{c}^i_\omega &= \sum_i \hat{y}^i_\omega, \;\; \omega = 1,2 \\
\sum_\omega \sum_i \hat{k}^i_\omega &= \sum_i \bar{k}^i \\
\sum_\omega \sum_i \hat{l}^i_\omega &= \sum_i \bar{l}^i.
\end{aligned}
$$

2.1.3 No specialization

We analyze the three cases separately. First, let's think of the case in which both countries produce both goods.

$$
\max a_1 \log c^i_1 + a_2 \log c^i_2
$$
$$
s.t. \; p_1 c^i_1 + p_2 c^i_2 \leq r^i \bar{k}^i + w^i \bar{l}^i
$$

$$
\begin{aligned}
a_1 &= \lambda^i p_1 c^i_1 && (2.6) \\
a_2 &= \lambda^i p_2 c^i_2 && (2.7) \\
p_1 c^i_1 + p_2 c^i_2 &= r^i \bar{k}^i + w^i \bar{l}^i && (2.8)
\end{aligned}
$$

This implies again that

$$
p_2 c^i_2 = \frac{a_2}{a_1} p_1 c^i_1 \tag{2.9}
$$

When both countries produce both goods the firms cost minimization problem implies the following two equalities,

$$
\begin{aligned}
b_\omega p_\omega z_\omega \left(k^i_\omega\right)^{b_\omega - 1} \left(l^i_\omega\right)^{1-b_\omega} &= r^i \;, \\
\left(1 - b_\omega\right) p_\omega z_\omega \left(k^i_\omega\right)^{b_\omega} \left(l^i_\omega\right)^{-b_\omega} &= w^i \;,
\end{aligned}
$$

which in turn imply

$$
\frac{b_\omega \left(l^i_\omega\right) w^i}{\left(1 - b_\omega\right) r^i} = k^i_\omega \;. \tag{2.10}
$$

Additionally, from zero profits,

$$p_\omega = \frac{\left(r^i\right)^{b_\omega}\left(w^i\right)^{1-b_\omega}}{z_\omega\left(b_\omega\right)^{b_\omega}\left(1-b_\omega\right)^{1-b_\omega}} \tag{2.11}$$

and, of course, technologies (by assumption) and prices (due to free trade) are the same in the two countries. Notice that the equality (2.11) is true for $i = 1,2$ this implies that

$$\begin{aligned}
\left(r^1\right)^{b_\omega}\left(w^1\right)^{1-b_\omega} &= \left(r^2\right)^{b_\omega}\left(w^2\right)^{1-b_\omega} \quad \omega = 1,2 \\
\left(\frac{r^1}{r^2}\right)^{b_\omega} &= \left(\frac{w^2}{w^1}\right)^{1-b_\omega} \quad \omega = 1,2
\end{aligned}$$

Noticing that the above expression holds for $\omega = 1,2$ and replacing these two equations in one another we have

$$\begin{aligned}
\left(\frac{w^2}{w^1}\right)^{\frac{(1-b_2)b_1}{b_2}-1+b_1} &= 1 \Longrightarrow \\
w^2 &= w^1
\end{aligned}$$

and of course

$$r^2 = r^1 \ .$$

This shows that we have factor price equalization (FPE) in the free trade equilibrium.

>From the cost minimization of the firm and the market clearing, $c_\omega^i = y_\omega^i$, we have

$$\begin{aligned}
b_\omega p_\omega^i z_\omega \left(k_\omega^i\right)^{b_\omega}\left(l_\omega^i\right)^{1-b_\omega} &= r^i k_\omega^i \Longrightarrow \\
b_\omega p_\omega y_\omega^i &= r^i k_\omega^i \Longrightarrow \\
p_\omega y_\omega^i &= \frac{r^i k_\omega^i}{b_\omega} \ .
\end{aligned}$$

Summing up over i and using FPE we have

$$p_\omega\left(\sum_i y_\omega^i\right) = \frac{r}{b_\omega}\left(\sum_i k_\omega^i\right) \ . \tag{2.12}$$

The equations (2.6) and (2.7) imply

$$\frac{a_\omega}{\lambda^1} + \frac{a_\omega}{\lambda^2} = p_\omega\left(c_\omega^1 + c_\omega^2\right) \tag{2.13}$$

Using goods market clearing we have

$$\begin{aligned}
\sum_i \frac{a_\omega}{\lambda^i} &= p_\omega \left(c_\omega^1 + c_\omega^2\right) = p_\omega \sum_i y_\omega^i = \frac{r}{b_\omega} \sum_i k_\omega^i \implies \\
b_\omega a_\omega \sum_i \frac{1}{\lambda^i} &= r \sum_i k_\omega^i \implies
\end{aligned}$$

$$\begin{aligned}
\left(\sum_i \frac{1}{\lambda^i}\right) \sum_\omega b_\omega a_\omega &= r \sum_\omega \sum_i k_\omega^i \implies \\
\sum_i \frac{1}{\lambda^i} &= \frac{r\left(\bar{k}^1 + \bar{k}^2\right)}{\sum_\omega b_\omega a_\omega} \qquad (2.14)
\end{aligned}$$

and in a similar manner

$$\sum_i \frac{1}{\lambda^i} = \frac{w\left(\bar{l}^1 + \bar{l}^2\right)}{\sum_\omega (1-b_\omega)\, a_\omega} \ . \qquad (2.15)$$

Using (2.14) and (2.15) we can determine the w/r ratio

$$\frac{\bar{l}^1 + \bar{l}^2}{\bar{k}^1 + \bar{k}^2} = \frac{r}{w} \frac{\sum_\omega (1-b_\omega)\, a_\omega}{\sum_\omega b_\omega a_\omega} \ . \qquad (2.16)$$

Assuming that one country is more capital abundant than the other (say $\bar{k}^1/\bar{l}^1 > \bar{k}^2/\bar{l}^2$), the equilibrium factor price ratio r/w under free trade lies in between the autarky factor prices of the two countries (determined in equation 2.5).

Using the relationships for the capital labor ratio (2.10) together with the above expression and factor market clearing conditions we can derive the equilibrium labor used from each country in each sector. Using the capital labor ratios for the second good and for both countries we get:

$$\begin{aligned}
\frac{w}{r}\left[\left(\bar{l}^i - l_2^i\right)\frac{b_1}{(1-b_1)} + \left(l_2^i\right)\frac{b_2}{(1-b_2)}\right] &= \bar{k}^i \\
\frac{l_2^i}{\bar{l}^i} &= \frac{(1-b_2)(1-b_1)}{b_2 - b_1}\left(\frac{r}{w}\frac{\bar{k}^i}{\bar{l}^i} - \frac{b_1}{(1-b_1)}\right) \\
\frac{l_2^i}{\bar{l}^i} &= \frac{(1-b_2)(1-b_1)}{b_1 - b_2}\left(\frac{b_1}{(1-b_1)} - \frac{\sum_\omega b_\omega a_\omega}{\sum_\omega (1-b_\omega)\, a_\omega}\frac{\bar{l}^1 + \bar{l}^2}{\bar{k}^1 + \bar{k}^2}\frac{\bar{k}^i}{\bar{l}^i}\right)
\end{aligned}$$

You may notice two things in this expression. First, if initial endowments of the two countries are inside a relative range, there is diversification since $l_j^i > 0$. If the endowments of a country for a given good are not in this range, then a country specializes in the other good (this range of endowments that implies diversification in production is commonly referred to as the cone of diversification). Second, conditional on diversification labor abundant countries use relatively more labor in the labor intensive sector.

What is the share of consumption for each country? We can use the FOC from the consumer's problem to obtain

$$\begin{aligned} p_1 c_1^i \left(1+\frac{a_2}{a_1}\right) &= r\bar{k}^i + w\bar{l}^i \implies \\ c_1^i &= \frac{r\bar{k}^i + w\bar{l}^i}{p_1\left(1+\frac{a_2}{a_1}\right)} \implies \\ c_1^i &= \frac{1}{(1-b_\omega)} \frac{w\bar{l}^i}{p_1\left(1+\frac{a_2}{a_1}\right)} \end{aligned} \tag{2.17}$$

where in the last equivalence we used equation (2.10). Obtaining the rest of the allocations and prices is straightforward. In fact, you can show that if the production function exhibits CRS and the capital-labor ratio for both countries is fixed (in a given sector), total production can be represented by[1]

$$y_\omega = z_\omega \left(\sum_i k_\omega^i\right)^{b_\omega} \left(\sum_i l_\omega^i\right)^{1-b_\omega} .$$

We can determine $\sum_i k_\omega^i$, $\sum_i l_\omega^i$ by combining expression (2.14) with (2.12), (2.13) and using the market clearing condition. This gives

$$\frac{b_\omega a_\omega}{\sum_\omega b_\omega a_\omega} = \frac{\sum_i k_\omega^i}{\sum_i \bar{k}^i} , \tag{2.18}$$

[1] Assume that $k^1/l^1 = k^2/l^2$. We only have to prove that

$$A\left(k^1+k^2\right)^b \left(l^1+l^2\right)^{1-b} = A\left(k^1\right)^b \left(l^1\right)^{1-b} + A\left(k^2\right)^b \left(l^2\right)^{1-b} .$$

We have that

$$\begin{aligned} A\left(\frac{k^1+k^2}{l^1+l^2}\right)^b &= A\left(\frac{k^1}{l^1}\right)^b \frac{l^1}{l^1+l^2} + A\left(\frac{k^2}{l^2}\right)^b \frac{l^2}{l^1+l^2} \implies \\ \left(\frac{k^1+k^2}{l^1+l^2}\right)^b &= \left(\frac{k^1}{l^1}\right)^b \implies \\ \left(\frac{\left(k^2 l^1\right)/l^2 + k^2}{l^1+l^2}\right) &= \left(\frac{k^1}{l^1}\right) \implies \\ \frac{k^2}{l^2} &= \frac{k^1}{l^1} \end{aligned}$$

and similarly for labor.

2.1.4 Specialization

[HW]

2.1.5 The 4 big theorems.

In this final section for the H-O model we will state main theorems that hold in the benchmark model with two countries and two goods. Variants of these theorems hold under less or more restrictive assumptions. Our approach will still be as parsimonious as possible.[2]

Theorem 1 (Factor Price Equalization) *Assume countries engage in free trade, there is* ***no*** *specialization (thus there is diversification) in equilibrium and there is no factor intensity-reversal, then factor prices equalize across countries.*

Proof. *See main text* ■

Theorem 2 (Rybczynski Theorem) *Assume that the economies remain always incompletely specialized. An increase in the relative endowment of a factor will increase the ratio of production of the good that uses the factor intensively.*[3]

Theorem 3 (Stolper-Samuelson) *Assume that the economies remain always incompletely specialized. An increase in the relative price of a good increases the real return to the factor used intensively in the production of that good and reduces the real return to the other factor.*

Theorem 4 (Heckscher-Ohlin) *Each country will produce the good which uses its abundant factor of production more intensively.*

2.2 Armington model

- The Armington (1969) model is based on the (ad-hoc) assumption that consumers have a certain desire for foreign goods.

[2]For a detailed treament you can look at the books of Feenstra (2003) and Bhagwati, Panagariya, and Srinivasan (1998).

[3]If prices were fixed a stronger version of the theorem can be proved.

- Though primitive, it can generate relationships that are key to understanding the next generation of models. The main relationships that arise in the model appear in many of the models of the next generation. This turns out to be great given that empirically the gravity approach had some success (gravity is a good statistical representation of trade flows).

- The gravity equation that we will get will serve as a great benchmark for thinking about bilateral trade flows.

2.2.1 The model

- N countries.

- Production: Assume no production, but each country is endowed with $\bar{y}_i$ of its own good.

- Consumption: Consumers' welfare in country j is

$$U_j = \left[\sum_{\upsilon=1}^{N} \alpha_{\upsilon j}^{1/\sigma} x_{\upsilon j}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

- Assume the "iceberg" transportation cost τ_{ij} - the total amount of a good sent from country i in order for 1 unit of the good to reach country j.

- Thus, total output is $\bar{y}_i = \sum_{\upsilon=1}^{N} \tau_{i\upsilon} x_{i\upsilon}$.

- To derive the demand we look at the first order conditions of the consumer in country j for goods originating from countries υ, i, which imply

$$\frac{\alpha_{ij}^{1/\sigma} x_{ij}^{(\sigma-1)/\sigma-1}}{\alpha_{\upsilon j}^{1/\sigma} x_{\upsilon j}^{(\sigma-1)/\sigma-1}} = \frac{p_{ij}}{p_{\upsilon j}}$$

given i,j, we raise the last expression to the power of $-\sigma$, rearrange $\alpha_{\upsilon j}, \alpha_{ij}$'s and we sum up over all goods $\upsilon = 1, .., N$.

$$\begin{aligned} \frac{p_{ij}}{p_{\upsilon j}} \frac{x_{ij}}{x_{\upsilon j}} &= \frac{\alpha_{ij}}{\alpha_{\upsilon j}} \left(\frac{p_{ij}}{p_{\upsilon j}} \right)^{1-\sigma} \implies \\ \sum_{\upsilon} \alpha_{\upsilon j} \left(p_{\upsilon j}^{1-\sigma} \right) &= \frac{1}{p_{ij} x_{ij}} \alpha_{ij} \left(p_{ij} \right)^{1-\sigma} \sum_{\upsilon} p_{\upsilon j} x_{\upsilon j} \end{aligned}$$

$$x_{ij}p_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} a_{ij} \sum_{\upsilon} p_{\upsilon j} x_{\upsilon j}$$

where

$$P_j = \left[\sum_{\upsilon=1}^{N} \alpha_{\upsilon j} \left(p_{\upsilon j}\right)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{2.19}$$

and

$$X_j = \left[\sum_{\upsilon=1}^{N} \alpha_{\upsilon j}^{1/\sigma} x_{\upsilon j}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} .$$

For CES demand we have that $X_j P_j$ equals total spending (simply replace the demand function in the expression $\sum_{\upsilon} p_{\upsilon j} x_{\upsilon j}$) and thus $X_j P_j = \sum_{\upsilon} p_{\upsilon j} x_{\upsilon j}$. Thus, a change in the price of a good sold in country j changes total sales in j with an elasticity of $1-\sigma$ (ignoring the General Equilibrium effect in P_j). Furthermore, from balanced trade, income defined as y_i equals spending and thus

$$p_{ij}x_{ij} = \alpha_{ij}\left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} y_j. \tag{2.20}$$

2.2.2 Gravity

Now we will derive an expression that is closer to the gravity equation. First, notice that the trade share of country i in country j is given by

$$\begin{aligned}
\lambda_{ij} &= \frac{x_{ij}p_{ij}}{\sum_{\upsilon} x_{\upsilon j} p_{\upsilon j}} \\
&= \frac{a_{ij} p_{ij}^{1-\sigma}}{\sum_{\upsilon} a_{\upsilon j} p_{\upsilon j}^{1-\sigma}} \\
&= \frac{a_{ij} p_{ii}^{1-\sigma} \tau_{ij}^{1-\sigma}}{\sum_{\upsilon} a_{\upsilon j} p_{\upsilon\upsilon}^{1-\sigma} \tau_{\upsilon j}^{1-\sigma}} \qquad (2.21)
\end{aligned}$$

From trade balance we also have

$$y_i = p_{ii}\bar{y}_i = p_{ii}\sum_j \tau_{ij} x_{ij} = \sum_j p_{ij} x_{ij}.$$

Let $p_{ij} = p_{ii}\tau_{ij}$ and sum across j's.

$$y_i = \sum_j p_{ij}x_{ij} = p_{ii}^{1-\sigma} \sum_{\upsilon} \alpha_{i\upsilon} \left(\frac{\tau_{i\upsilon}}{P_{\upsilon}}\right)^{1-\sigma} y_{\upsilon} \implies \tag{2.22}$$

$$p_{ii}^{1-\sigma} = \frac{y_i}{\sum_v \alpha_{iv} \left(\frac{\tau_{iv}}{P_v}\right)^{1-\sigma} y_v} \tag{2.23}$$

Replacing in 2.20 we get

$$\begin{aligned} X_{ij} &= p_{ij}x_{ij} = \alpha_{ij}\left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} y_j \implies \\ X_{ij} &= \alpha_{ij}p_{ii}^{1-\sigma}\left(\frac{\tau_{ij}}{P_j}\right)^{1-\sigma} y_j \\ &= y_i y_j \frac{\alpha_{ij}}{\sum_v \alpha_{iv}\left(\frac{\tau_{iv}}{P_v}\right)^{1-\sigma} y_v}\left(\frac{\tau_{ij}}{P_j}\right)^{1-\sigma} \end{aligned} \tag{2.24}$$

which shows that the bilateral trade spending is related to the product of the GDPs of the two countries (gravity!!), the distance/tradecost and a GE component.

The price index P_j is a measure of bilateral trade resistance. If a country is very isolated from the rest of the world, the domestic prices will be high on average and, thus, the domestic price index will also be high. Given that large countries "trade a lot with themselves" the changes in trade costs affect their price index only by a little. The opposite is true for small countries. Additionally, $\sum_j \left(\frac{\tau_{ij}}{P_j}\right)^{1-\sigma} y_j$ is a measure of how much selling opportunities i has overall.

2.2.3 Welfare

We will now show that welfare in relationship to trade is given by a simple relationship involving the trade to GDP ratio and parameters of the model (but no other equilibrium variables). We will be revisiting this relationship multiple times in this course. Using expression (2.21) we have

$$\lambda_{ij} = \frac{a_{ij}p_{ij}^{1-\sigma}}{P_j^{1-\sigma}} \ .$$

Thus

$$P_j = \left(\frac{a_{jj}p_{jj}^{1-\sigma}}{\lambda_{jj}}\right)^{1/(1-\sigma)}$$

Since $p_{ij} = \tau_{ij}p_{ii}$ then welfare can be written as

$$W_j = \frac{y_j}{P_j} = \frac{p_{jj}\bar{y}_j}{P_j} = \frac{p_{jj}}{a_{jj}^{1-\sigma}p_{jj}}\lambda_{jj}^{-1/(\sigma-1)}\bar{y}_j = \frac{\lambda_{jj}^{-1/(\sigma-1)}}{a_{jj}^{1-\sigma}}\bar{y}_j \ .$$

Notice that the welfare dependents only on changes in the trade to GDP ratio, λ_{jj}, with an elasticity of $-1/(\sigma-1)$ which is the inverse of the trade elasticity (see expression 2.21 the coefficient on the trade costs)

2.3 Monopolistic Competition with Homogeneous Firms

The formulation that we will develop in this section is based on the monopolistic competition framework of Krugman (1980) and the subsequent analysis of Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008). We assume that there are $i = 1, ..., N$ countries and there is a measure L_i of consumers in each country i. Let $\omega \in \Omega$ be a potentially differentiated variety, where Ω is the set of all potential varieties. In order for a firm from country i to produce a differentiated variety it has to incur a fixed cost of entry in terms of domestic labor, f_i^e. We define as X_j the total spending of country i. Since labor is the only factor of production and in equilibrium all profits would be accrued to labor the total spending equals labor income, $X_j = w_j L_j$, where w_j is the wage and L_j the labor force.

2.3.1 Consumer's problem

The problem of the representative consumer from country j is

$$\max \left(\sum_{i=1}^{N} \int_{\Omega_i} x_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

$$\text{s.t.} \sum_{i=1}^{N} \int_{\Omega_i} p_{ij}(\omega)\, x_{ij}(\omega)\, d\omega = w_j,$$

where $x_{ij}(\omega)$ is the quantity demanded of good ω, $p_{ij}(\omega)$ is the price of that good, and $\sigma > 1$ is the elasticity of substitution. We assume (consistently hereafter) inelastically supplies his unit of labor endowment

The above implies the following CES demand for the consumers in country j

$$x_{ij}(\omega) = \left(\frac{p_{ij}(\omega)}{P_j} \right)^{-\sigma} \frac{w_j L_j}{P_j},$$

$$P_j = \left(\int_{\Omega} p_{ij}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}.$$

2.3.2 Firm's problem

Each firm is a monopolist of a variety. All firms in country i have a common productivity, ϕ_i, and produce one unit of the good using $\frac{1}{\phi_i}$ units of labor. Firms have to pay a fixed cost of production in terms of domestic labor, f_i. They also incur an iceberg transportation cost, τ_{ij}, to ship the good from country i to country j.[4] Profit maximization implies that optimal pricing for a firm selling from country i to country j is

$$p_{ij}\left(\phi_i\right) = \frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi_i}.$$

We will make the notation a bit cumbersome by carrying around the ϕ_i's in order to allow for direct comparison of our results with the heterogeneous firms example that will be studied later on. Given the Dixit-Stiglitz demand, the variable profits of the firm from operating in country j, revenues in country j minus labor cost of production for that country, is simply revenues divided by the elasticity of substitution σ,

$$\pi_{ij}^V\left(\phi_i\right) = \frac{p_{ij}\left(\phi_i\right)^{1-\sigma}}{P_j^{1-\sigma}}\frac{w_jL_j}{\sigma}\ .$$

Based on variable profits, the firm will have to decide whether paying the fixed entry cost is profitable to which we turn next.

2.3.3 Equilibrium

In order to determine the equilibrium of the model, we have to consider the free entry condition and the labor market clearing condition. The free entry condition implies that in a given country entry occurs until the point where expected profits are equal to zero for the firms of that country: the sum of the revenue in each destination minus its total labor cost of production minus the fixed cost of entry

[4]Notice that here we haven't introduced fixed costs of exporting. Introducing these costs will change the analysis in that we may have countries for which all the firms chose not to export depending on values of the fixed costs and other variables. More extreme predictions can be delivered if the production cost f_i is only a cost to produce domestically and independent of the exporting cost. However, in order to create a true extensive margin of firms (i.e. more firms exporting when trade costs decrease) requires heterogeneity either in the productivities of firms (as we will do later on in the notes) or in the fixed costs of selling to a market (see Romer (1994)).

equals to zero for each firm. This implies that

$$\begin{aligned}
\sum_j \frac{p_{ij}\left(\phi_i\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{w_j L_j}{\sigma} - w_i f_i^e &= 0 \implies \\
\sum_j \frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij} w_i}{\phi_i}\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{w_j L_j}{\sigma} &= w_i f_i^e \implies \\
\frac{\sigma}{\sigma-1}\frac{1}{\sigma}\frac{w_i}{\phi_i}\sum_j \frac{\tau_{ij}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij} w_i}{\phi_i}\right)^{-\sigma}}{P_j^{1-\sigma}} w_j L_j &= w_i f_i^e \implies \\
\frac{1}{\sigma-1}\frac{w_i}{\phi_i} x_i &= w_i f_i^e \implies \\
x_i &= f_i^e \phi_i \left(\sigma - 1\right),
\end{aligned}$$

where by slightly abusing the notation we define $x_i \equiv \sum_j \frac{\tau_{ij}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij} w_i}{\phi_i}\right)^{-\sigma}}{P_j^{1-\sigma}} w_j L_j$.

The labor market clearing condition implies that (where J_i is the mass of operating domestic firms)

$$\begin{aligned}
J_i\left[\sum_{\upsilon=1}^{N} \frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{i\upsilon} w_i}{\phi_i}\right)^{-\sigma}}{P_\upsilon^{1-\sigma}} w_\upsilon L_\upsilon \frac{\tau_{i\upsilon}}{\phi_i} + f_i^e\right] &= L_i \implies \\
J_i\left[\sum_{\upsilon=1}^{N} \frac{\tau_{i\upsilon}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{i\upsilon} w_i}{\phi_i}\right)^{-\sigma}}{P_\upsilon^{1-\sigma}} \frac{w_\upsilon L_\upsilon}{\phi_i} + f_i^e\right] &= L_i \implies \\
J_i\left[\frac{x_i}{\phi_i} + f_i^e\right] &= L_i \implies \\
J_i\left[f_i\left(\sigma-1\right) + f_i^e\right] &= L_i \implies
\end{aligned}$$

$$J_i = \frac{L_i}{\sigma f_i^e}. \tag{2.25}$$

Notice that the equilibrium measure of entrants is independent of variable trade costs.

2.3.4 Gravity

Now we compute the fraction of total income in country j spent on goods from country i, λ_{ij},

$$\begin{aligned}\lambda_{ij} &= \frac{\frac{J_i\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi_i}\right)^{1-\sigma}}{P_j^{1-\sigma}}w_jL_j}{\sum_{\upsilon=1}^{N}\frac{J_\upsilon\left(\frac{\sigma}{\sigma-1}\frac{\tau_{\upsilon j}w_\upsilon}{\phi_\upsilon}\right)^{1-\sigma}}{P_j^{1-\sigma}}w_jL_j} \\ &= \frac{J_i\left(\frac{\tau_{ij}w_i}{\phi_i}\right)^{1-\sigma}}{\sum_{i=1}^{N}J_\upsilon\left(\frac{\tau_{\upsilon j}w_\upsilon}{\phi_\upsilon}\right)^{1-\sigma}}\end{aligned}$$

and using equation (2.25), we have

$$\lambda_{ij} = \frac{\frac{L_i}{f_i^e}\left(\tau_{ij}w_i\right)^{1-\sigma}\left(\phi_i\right)^{\sigma-1}}{\sum_{\upsilon=1}^{N}\frac{L_\upsilon}{f_\upsilon^e}\left(\tau_{\upsilon j}w_\upsilon\right)^{1-\sigma}\left(\phi_\upsilon\right)^{\sigma-1}}. \tag{2.26}$$

Notice that in order to compute wages across countries, we can use the following condition implied by balanced trade:

$$w_iL_i = \sum_{\upsilon=1}^{N}\lambda_{i\upsilon}L_\upsilon w_\upsilon.$$

Given that there is free entry of firms and payments for entry costs are accrued to labor, total labor income w_iL_i is the total income in each country i.

2.3.5 Welfare

Denote bilateral sales of country i to country j as X_{ij}. Related to the above, we also have

$$\begin{aligned}\lambda_{ij} &= \frac{X_{ij}}{X_j} = \\ &= \frac{\frac{J_i\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi_i}\right)^{1-\sigma}}{P_j^{1-\sigma}}w_jL_j}{w_jL_j},\end{aligned}$$

which implies that

$$P_j = \left(J_i\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi_i}\left(\lambda_{ij}\right)^{\frac{1}{\sigma-1}}.$$

Looking at the domestic market share of j, we have

$$P_j = (J_j)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \frac{w_j}{\phi_j} (\lambda_{jj})^{\frac{1}{\sigma-1}} . \tag{2.27}$$

Finally, welfare is given by

$$\frac{w_j}{P_j} = \frac{\sigma - 1}{\sigma} \phi_j \frac{(J_j)^{\frac{1}{\sigma-1}}}{(\lambda_{jj})^{\frac{1}{\sigma-1}}},$$

with J_j given by equation (2.25).

If we interpret the effect of trade liberalization as a change in trade to GDP ratio, λ_{jj}, then the change in welfare before and after a trade liberalization is given by

$$\frac{\frac{w'_j}{P'_j}}{\frac{w_j}{P_j}} = \left(\frac{\lambda'_{jj}}{\lambda_{jj}} \right)^{-\frac{1}{\sigma-1}} . \tag{2.28}$$

2.4 Ricardian model

The Ricardian model is a model of perfect competition where countries produce the same goods using different technologies. The Ricardian model predicts that countries may specialize in the production of certain ranges of goods.

2.4.1 The two goods case

We consider the simple version of the model with two countries and two goods. In order to get as much intuition as possible we will first consider the case where both countries specialize in the production of one good.

The production technologies in the two countries $i = 1, 2$ are different for the two goods $\omega = 1, 2$ and given by

$$y^i_\omega = z^i_\omega \left(l^i_\omega \right), i, \omega = 1, 2 \ .$$

Assume that country 1 has absolute advantage in the production of both goods

$$\begin{aligned} z^2_1 &< z^1_1 \ , \\ z^2_2 &< z^1_2 \ . \end{aligned}$$

Assume that country 1 has comparative advantage in the production of good 1 and country 2 in good

2

$$\frac{z_1^2}{z_2^1} < \frac{z_1^1}{z_2^1} \ .$$

Assume Cobb-Douglas preferences. The consumer's problem is

$$\max a_1 \log c_1^i + a_2 \log c_2^i$$
$$s.t. \ p_1 c_1^i + p_2 c_2^i \leq w^i \bar{l}^i.$$

Consumer optimization implies that

$$p_2^i c_2^i = \frac{a_2}{a_1} p_1^i c_1^i \quad (2.29)$$
$$p_1^i c_1^i + p_2^i c_2^i = w^i \bar{l}^i \quad (2.30)$$

Autarky

Using firms cost minimization and the Inada conditions (that ensure that the consumer actually wants to consume both goods) from the consumer problem we directly obtain that

$$p_1^i z_1^i = w = p_2^i z_2^i.$$

Using the goods market clearing

$$c_\omega^i = y_\omega^i \text{ for } \omega = 1, 2 \ ,$$

together with labor market clearing

$$l_\omega^i = a_\omega \bar{l}^i \ ,$$

we get labor allocated to each good. Using the production function and goods market clearing we can obtain the rest of the allocations.

Free trade

Under free trade international prices equalize. Relative productivity patterns will determine specialization. There can be three possible specialization patterns, two where one country specializes and the other diversifies and one where both countries specialize.

Proposition 5 (Specialization) *Under the assumptions we stated in the beginning, at least one country specializes in the free trade equilibrium.*

Proof. If not then the firm's cost minimization together with the consumer FOCs would imply

$$\frac{z_1^1}{z_2^1} = \frac{z_1^2}{z_2^2}$$

a contradiction. ■

Notice that using equation (2.29), the goods market clearing, and the production function we can get the relative price in the complete specialization cases being

$$\frac{p_1}{p_2} = \frac{a_1}{a_2} \frac{z_2^2 \bar{l}^2}{z_1^1 \bar{l}^1} \, .$$

With this at hand we can determine the restriction that would deliver specialization. In fact, when a country produces both goods and using cost minimization it has to be the case that

$$\frac{z_1^i}{z_2^i} = \frac{p_1}{p_2} \, .$$

2.4.2 The model with a continuum of goods

The model of Dornbusch, Fischer, and Samuelson (1977) is based on the Ricardian model where trade and specialization patterns are determined by different productivities.[5] There is absolute advantage due to higher productivity in producing certain goods, but also comparative advantage due to lower opportunity cost of producing some goods. The main drawback of the simple Ricardian model, similar to that of the Heckscher-Ohlin model, is in the complexity of solving for the patterns of specialization for a large number of industries.

[5] The notes in this chapter are partially based on Eaton and Kortum (2011).

Breakthrough: Dornbusch, Fischer, and Samuelson (1977) used a continuum of sectors. The characterization of the equilibrium ended up being very easy.

- Perfect competition
- 2 countries (H, F)
- Continuum of goods $\omega \in [0, 1]$
- CRS technology (labor only)
- Cobb-Douglas Preferences with equal share in each good
- Ideberg trade costs τ_{HF}, τ_{FH}

Normalize the domestic wage to 1. Basically we want to compare the price of the good as offered by the domestic country to the price of the good offered by the foreign country in order to find the set of goods produced by the foreign economy in equilibrium (re-ordered in a decreasing order of productivity in $[\underline{\omega}, 1]$), where $\underline{\omega}$ is the lowest productivity still produced in the foreign country

$$\frac{w_F}{z_F(\omega)} < \frac{\tau_{HF}}{z_H(\omega)} \implies A(\omega) \equiv \frac{z_F(\omega)}{z_H(\omega)} > \frac{w_F}{\tau_{HF}}$$

and we can define

$$A(\underline{\omega}) = \frac{w_F}{\tau_{HF}}. \tag{2.31}$$

Out of these goods some will be exported to the home country. To find the set of such goods we have to find the set of goods produced by the domestic country. This is simply finding the ω that satisfies

$$A(\bar{\omega}) = \tau_{FH} w_F \tag{2.32}$$

and $[0, \bar{\omega}]$ is the set of goods produced by the home country. In order to get sensible relationships from the model, DFS parametrize $\frac{z_F(\omega)}{z_H(\omega)}$ by using a monotonic function.

In this last case we can invert A and get the exact range of goods produced by each country. Labor market clearing implies that

$$L_H = \underline{\omega} w_F L_F + \bar{\omega} L_H$$

inverting A we can explicitly solve for wages using this equation.

2.4.3 Where DFS stop and EK start

Eaton and Kortum (2002) (henceforth EK) treat productivities $z_i(\omega)$ as an independent realization of a random variable Z_i independently distributed according to the same distribution F_i for each good ω in country i. Given the continuum of goods (using a LLN argument) we can determine with certainty the fraction of goods produced by each country. This way EK are able to overcome the complications faced by the standard Ricardian framework and go much further in developing an analytical quantitative trade framework.

Assume that the random variable Z_i follows the Frechet distribution[6]:

$$\Pr(Z_i \leq z) = \exp\left[-A_i z^{-\theta}\right] .$$

The parameter $A_i > 0$ governs country's overall level of efficiency (absolute advantage) (with more productive countries having higher A_i's). The parameter $\theta > 1$ governs variation in productivity across different goods (comparative advantage) (higher θ less dispersed).

Now we will split the $[0,1]$ interval by thinking of $\bar{\omega}$ as the probability that the relative productivity of F to H is less than $\tilde{A}$, where $\tilde{A}$ can either be defined by (2.31) or by (2.32). Therefore, in order to determine $\bar{\omega}$ which is defined as the share of goods that the domestic country produces we simply compute the probability that the domestic country is the cheapest provider of the good across all the range of productivities. For example using (2.32) for the definition of $\tilde{A}$ we can derive

$$\begin{aligned}
\bar{\omega} &= \lambda_{HH} \\
&= \Pr\left[\frac{z_F}{z_H} \leq \tilde{A}\right] \\
&= \Pr\left[z_F \leq \tilde{A} z_H\right] \\
&= \int_0^{+\infty} \underbrace{\exp\left[-A_F\left(\tilde{A}\right)^{-\theta}\right]}_{\Pr\left(z(\omega)\leq \tilde{A}_{zH}(\omega)\right)} \underbrace{dF_H(z)}_{\text{density of } z_H(\omega)} \\
&= \int_0^{+\infty} \exp\left[-A_F\left(\tilde{A}z\right)^{-\theta}\right] \theta A_H(z)^{-\theta-1} \exp\left[-A_H(z)^{-\theta}\right] dz \\
&= \frac{A_H}{A_H + A_F \tilde{A}^{-\theta}}
\end{aligned}$$

[6] See the appendix for the properties of the Frechet distribution and the next chapter for a derivation from first principles.

Country H is spending $(1-\bar{\omega})\, w_H L_H$ on imports (given Cobb-Douglas) which implies

$$X_{FH} = \frac{A_F \left(w_F \tau_{HF}\right)^{-\theta}}{A_H + A_F \left(w_F \tau_{HF}\right)^{-\theta}} w_H L_H$$

Notice that this relationship is similar to the relationship (2.21) derived with the assumption of the Armington aggregator but with an exponent $-\theta$. A lower value of θ generates more heterogeneity. This means that the comparative advantage exerts a stronger force for trade against resistance impose by the geographic barrier τ_{in}. In other words with low θ there are many outliers that overcome differences in geographic barriers (and prices overall) so that changes in w's and τ's are not so important for determining trade.

Chapter 3

Modeling the production

The purpose of this chapter is to develop a general model for production in which different assumptions on technology and competition will give us different workhorse frameworks important for the quantitative analysis of trade. Our analysis is based on the exposition of (Eaton and Kortum (2011)) and earlier results of (Kortum (1997)) and (Eaton and Kortum (2002)).

3.1 A theory of technology starting from first principles

We start with a very general framework under the following assumptions

- Time is continuous and there is a continuum of goods with measure $\mu(\Omega)$.
- Ideas for good ω (ways to produce the same good with different efficiency) arrive at location i at date t at a Poisson rate with intensity

$$\bar{a} R_i(\omega, t)$$

  where we think of $\bar{a}$ as research productivity and R as research effort.

- The quality of ideas is a realization from a random variable Q drawn independently from a Pareto distribution with $\theta > 1$, so that

$$\Pr[Q > q] = \left(q/\underline{q}\right)^{-\theta}, \ q \geq \underline{q}$$

where $\underline{q}$ is a lower bound of productivities. Note that the probability of an idea being bigger than q conditional on ideas being bigger than a threshold, is also Pareto (see appendix for the properties of the Pareto distribution).

The above assumptions together imply that the arrival rate of an idea of efficiency $Q \geq q$ is

$$\bar{a}R_i(\omega, t)\left(q/\underline{q}\right)^{-\theta}.$$

(normalize this with $\underline{q} \rightarrow 0$, $\bar{a} \rightarrow +\infty$ such that $\bar{a}\underline{q}^{-\theta} \rightarrow 1$ in order to consider all the ideas in $(0, +\infty)$).

Important assumption: There is no forgetting of ideas. Thus, we can summarize the history of ideas for good ω by

$$A_i(\omega, t) = \int_{-\infty}^{t} R_i(\omega, \tau)\, d\tau.$$

The number of ideas with efficiency $Q > q'$ is therefore distributed Poisson with a parameter $A(\omega, t)(q')^{-\theta}$ (using the previous normalization).

The unit cost for a location i of producing good ω with an efficiency of q is $c = w_i/q$. Given all the above, the expected number of techniques providing unit cost less than c is distributed Poisson with parameter

$$\Phi_i(\omega, t)\, c^{\theta}$$

where

$$\Phi_i(\omega, t) = A_i(\omega, t)\, w_i^{-\theta}.$$

But notice that this delivers back unit costs that are conditionally Pareto distributed

$$\Pr\left[C \leq c' | C \leq c\right] = \Pr\left[Q \geq \frac{w}{c'} = q' | Q \geq \frac{w}{c} = q\right] = \left(\frac{c'}{c}\right)^{\theta} = \left(\frac{q}{q'}\right)^{\theta}.$$

In what follows set

$$\Phi = \Phi_i(\omega, t).$$

Definition 6 $C^{(k)}$ *is the k'th lowest unit cost technology for producing a particular good. Given this definition we have the main theorem for the joint distribution of the order statistics* $C^{(k)}$

Theorem 7 *The joint density* $C^{(k)}, C^{(k+1)}$ *is*

$$\begin{aligned} g\left(C^{(k)}=c_k, C^{(k+1)}=c_{k+1}\right) &\equiv g_{k,k+1}\left(c_k, c_{k+1}\right) \\ &= \frac{\theta^2}{(k-1)!}\Phi^{k+1}c_k^{\theta k-1}c_{k+1}^{\theta k-1}\exp\left(-\Phi c_{k+1}^{\theta}\right) \end{aligned}$$

for $0 < c_k \leq c_{k+1} < \infty$ *while the marginal density of* $C^{(k)}$ *is:*

$$g_k\left(c_k\right) = \frac{\theta}{(k-1)!}\Phi^k c_k^{\theta k-1}\exp\left(-\Phi c_k^{\theta}\right)$$

for $0 < c_k < +\infty$

Proof. The distribution of a cost C conditional on $C \leq \bar{c}$ is:

$$\begin{aligned} F\left(c|\bar{c}\right) &= \left(\frac{c}{\bar{c}}\right)^{\theta} \quad c \leq \bar{c} \\ F\left(c|\bar{c}\right) &= 1 \qquad\quad c > \bar{c} \end{aligned}$$

The probability that a cost is less than c_k is $F\left(c_k|\bar{c}\right)$. Thus, if we have n techniques with unit cost less than $\bar{c}$, where $c_k \leq c_{k+1} \leq \bar{c}$, the probability that k are less than c_k while the remaining are greater than c_{k+1} is given by the multinomial:

$$\left(\begin{array}{c} n \\ k \end{array}\right) F\left(c_k|\bar{c}\right)^k\left(1-F\left(c_{k+1}|\bar{c}\right)\right)^{n-k}$$

Taking the cross derivative of this expression wrt to c_k, c_{k+1} gives

$$g_{k,k+1}\left(c_k, c_{k+1}|\bar{c}, n\right) = \frac{n!F\left(c_k|\bar{c}\right)^{k-1}\left[1-F\left(c_{k+1}|\bar{c}\right)\right]^{n-k-1}F'\left(c_k|\bar{c}\right)F'\left(c_{k+1}|\bar{c}\right)}{(k-1)!\,(n-k-1)!}$$

for $c_{k+1} \geq c_k$ and $n \geq k+1$. For $n < k+1$ we can define $g_{k,k+1}\left(c_k, c_{k+1}|\bar{c}, n\right) = 0$. We also know that n is drawn from a Poisson distribution with parameter $\Phi\bar{c}^{\theta}$, the expectation of this joint distribution unconditional on n is:

$$\begin{aligned}
g_{k,k+1}\left(c_k, c_{k+1}|\bar{c}\right) &= \sum_{n=0}^{\infty} \underbrace{\frac{\exp\left(-\Phi\bar{c}^{\theta}\right)\left(\Phi\bar{c}^{\theta}\right)^n}{n!}}_{\substack{\text{prob } n \text{ ideas arrived for} \\ \text{a particular good}}} \underbrace{g_{k,k+1}\left(c_k, c_{k+1}|\bar{c}, n\right)}_{\substack{\text{conditional on } n \text{ prob } C^{(k)}=c_k, \\ C^{(k+1)}=c_{k+1}}} = \\
&= \sum_{n=k+1}^{\infty} \frac{\exp\left(-\Phi\bar{c}^{\theta}\right)\left(\Phi\bar{c}^{\theta}\right)^n}{n!} \frac{n! F\left(c_k|\bar{c}\right)^{k-1}\left[1 - F\left(c_{k+1}|\bar{c}\right)\right]^{n-k-1} F'\left(c_k|\bar{c}\right) F'\left(c_{k+1}|\bar{c}\right)}{(k-1)!\,(n-k-1)!} \\
&= \frac{\left(\Phi\bar{c}^{\theta}\right)^{k+1} \exp\left(-\Phi\bar{c}^{\theta} F\left(c_{k+1}|\bar{c}\right)\right) F\left(c_k|\bar{c}\right)^{k-1} F'\left(c_k|\bar{c}\right) F'\left(c_{k+1}|\bar{c}\right)}{(k-1)!} \\
&\quad \sum_{m=0}^{\infty} \exp\left(-\Phi\bar{c}^{\theta}\right)\left(\Phi\bar{c}^{\theta}\right)^m \frac{\exp\left(-\Phi\bar{c}^{\theta} F\left(c_{k+1}|\bar{c}\right)\right)\left[1 - F\left(c_{k+}|\bar{c}\right)\right]^m}{m!} \\
&= \frac{\left(\Phi\bar{c}^{\theta}\right)^{k+1} \exp\left(-\Phi\bar{c}^{\theta} F\left(c_{k+1}|\bar{c}\right)\right) F\left(c_k|\bar{c}\right)^{k-1} F'\left(c_k|\bar{c}\right) F'\left(c_{k+1}|\bar{c}\right)}{(k-1)!} 1
\end{aligned}$$

Making use of the derivation of $F\left(c|\bar{c}\right)$ we have that

$$g_{k,k+1}\left(c_k, c_{k+1}|\bar{c}\right) = \frac{\theta^2}{(k-1)!} \Phi^{k+1} c_k^{\theta k-1} c_{k+1}^{\theta-1} \exp\left(-\Phi c_{k+1}^{\theta}\right)$$

We can derive the marginal density by making use of the above expression. We have that

$$\begin{aligned}
g_k(c_k) &= \int_{c_k}^{\infty} g_{k,k+1}(c_k, c_{k+1}) dc_{k+1} \\
&= \frac{\theta^2}{(k-1)!} \Phi^{k+1} c_k^{\theta k-1} \int_{c_k}^{\infty} c_{k+1}^{\theta-1} e^{-\Phi c_{k+1}^{\theta}} dc_{k+1}.
\end{aligned}$$

Now by making the substitution $u = c_{k+1}^{\theta}$,

$$\begin{aligned}
\int_{c_k}^{\infty} c_{k+1}^{\theta-1} e^{-\Phi c_{k+1}^{\theta}} dc_{k+1} &= \theta^{-1} \int_{c_k^{\theta}}^{\infty} e^{-\Phi u} du \\
&= \theta^{-1} \Phi^{-1} e^{-\Phi c_k^{\theta}}.
\end{aligned}$$

Therefore

$$\begin{aligned}
g_k(c_k) &= \frac{\theta^2}{(k-1)!} \Phi^{k+1} c_k^{\theta k-1} \left(\theta^{-1} \Phi^{-1} e^{-\Phi c_k^{\theta}}\right) \\
&= \frac{\theta}{(k-1)!} \Phi^k c_k^{\theta k-1} e^{-\Phi c_k^{\theta}}\ , \qquad (3.1)
\end{aligned}$$

as asserted. ■

This result will be the base for a series of lemmas to be discussed later on. First, by noticing that $F_k'(c_k) = g_k(c_k)$ we can directly compute the probability $\Pr\left[C^{(k)} \leq \tilde{c}_k\right]$:

Lemma 8 *The distribution of the k'th lowest cost $C^{(k)}$ is:*

$$\Pr\left[C^{(k)} \leq \tilde{c}_k\right] = F_k(\tilde{c}_k) = 1 - \sum_{\upsilon=0}^{k-1} \frac{\left(\Phi \tilde{c}_k^{\theta}\right)^{\upsilon}}{\upsilon!} e^{-\Phi \tilde{c}_k^{\theta}} \tag{3.2}$$

This gives us that the distribution of the lowest cost ($k = 1$) is the Frechet distribution

$$F_1(\tilde{c}_1) = 1 - \exp\left(-\Phi \tilde{c}_1^{\theta}\right)$$

Now in this context we will assume that ideas are randomly assigned to goods across the continuum. Given that there is a large number of goods (say of measure $\mu(\Omega)$) in the continuum we can drop the ω notation by simply denoting $A_i(\omega, t) = A_i(t)/\mu(\Omega)$ to be the total amount of ideas available for a good, in location i at time t. Given the above, the measure of goods with cost less than c is $\Phi_i(t)/\mu(\Omega)\, c^{\theta}$ and the distribution of the lowest cost $C^{(1)}$ (the frontier idea) is

$$F_1(c_1) = 1 - \exp\left(-\left(\Phi_i(t)/\mu(\Omega)\right) \tilde{c}_1^{\theta}\right)$$

Thus a set of $\mu(\Omega)\, F_1(c_1)$ ideas can be produced at a cost less than c_1. We will proceed under this convention in the rest of this chapter.

Using the general technology framework we developed above and different assumptions on the competition structure we will be able to derive main quantitative models that are widely used in the recent international trade literature.

3.2 Application I: Perfect competition (EK)

- Assume perfect competition
- Assume that there is a unit continuum of available goods and CES preferences. The utility

function of the representative consumer is

$$U_j = \left[\int_0^1 x_j(\omega)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$$

where the elasticity of substitution is $\sigma > 0$. If the spending in country j is X_j and the demand function for good ω is

$$x_j(\omega) = \frac{(p_j(\omega))^{-\sigma}}{(P_j)^{1-\sigma}} X_j.$$

- Assume iceberg transportation costs of trade
- In perfect competition only the lowest cost producer of a good will supply that particular good.

Thus, we want to derive the distribution of the minimum price over a set of prices offered by producers in different countries

$$p_j = \min\{p_{1j}, ..., p_{Nj}\}$$

In order to find this distribution we take advantage of the properties of extreme value distributions. Everything turns out to work beautifully! Let c_i be the unit cost and labor be the only factor of production. The probability that country i can sell in j with a price less than p is simply given by

$$G_{ij}(p) = 1 - \exp\left(-\Phi_{ij} p^{\theta}\right)$$

where $\Phi_{ij} = A_i (c_i \tau_{ij})^{-\theta}$.[1] Taking in account all the source countries (including the home country) we have that the distribution of prices in country j is given by

$$\begin{aligned} G_j(p) &= \Pr(P_j \leq p) \\ &= 1 - \Pr(P_j \geq p) \\ &= 1 - \prod_{i=1}^{N} \Pr(P_{ij} \geq p) \\ &= 1 - \prod_{i=1}^{N} [1 - G_{ij}(p)] \\ &= 1 - e^{-\Phi_j p^\theta} \end{aligned}$$

where $\Phi_j \equiv \sum_{i=1}^{N} \Phi_{ij}$.[2]

Given the above technology framework and the assumption of perfect competition we have that the market share of country i to country j (which is the same as the probability of supplying a good given that the distribution of prices is source independent) is:

$$\lambda_{ij} = \int_0^{+\infty} \underset{s \neq i}{\Pi} [1 - G_{sj}(q)] \, dG_{ij}(q) = \frac{A_i (c_i \tau_{ij})^{-\theta}}{\sum_{\upsilon=1}^{N} A_\upsilon (c_\upsilon \tau_{\upsilon j})^{-\theta}} = \frac{\Phi_{ij}}{\Phi_j} \tag{3.3}$$

[1] Starting from the Frechet disribution of productivities we can derive the distribution of prices offered by country i to n :

$$\begin{aligned} G_{ij}(p) &= \Pr(p_{ij} \leq p) \\ &= \Pr\left(\frac{c_i}{Z_i} \tau_{ij} \leq p\right) \\ &= \Pr\left(\frac{c_i}{p} \tau_{ij} \leq Z_i\right) \\ &= 1 - F_i\left(\frac{c_i}{p} \tau_{ij}\right) \\ &= 1 - \exp\left(-T_i \left(\frac{c_i}{p} \tau_{ij}\right)^{-\theta}\right) \\ &= 1 - \exp\left(-\psi_{ij} p^\theta\right) \end{aligned}$$

where $\psi_{ij} = \Phi_i (c_i \tau_{ij})^{-\theta}$.

[2] We can show that the distribution of prices of goods that country n actually buys from i is independent of i. This result can be obtained by showing that

$$\tilde{G}_{ij}(p) = \frac{\int_0^p \prod_{k \neq i} [1 - G_{kj}(q)] \, dG_{ij}(q)}{\lambda_{ij}} .$$

This also implies that the fraction of goods for coutnry i to country n is the same as the fraction of sales of country i to country n.

Using the following proposition and the assumption of the CES demand we can directly derive the price index

Proposition 9 *For each order* k*, the* b*'th moment* $(b > -\theta k)$ *is*

$$E\left[\left(C^{(k)}\right)^b\right] = \left(\Phi^{-1/\theta}\right)^b \frac{\Gamma\left[\left(\theta k + b\right)/\theta\right]}{(k-1)!},$$

where $\Gamma(\alpha) = \int_0^{+\infty} y^{\alpha-1} e^{-y} dy$.

Proof. First consider $k = 1$, where suppressing notation we denote by the marginal density of $C^{(k)}$,

$$g_k(c) = \frac{\theta}{(k-1)!} \Phi^k c_k^{\theta k-1} \exp\left[-\Phi c_k^\theta\right]$$

$$\begin{aligned} E\left[\left(C^{(1)}\right)^b\right] &= \int_0^{+\infty} c^b g_1(c)\, dc \\ &= \int_0^{+\infty} \Phi\theta c^{\theta+b-1} \exp\left[-\Phi c^\theta\right] dc \end{aligned}$$

changing the variable of integration to $\upsilon = \Phi c^\theta$ and applying the definition of the gamma function, we get

$$\begin{aligned} E\left[\left(C^{(1)}\right)^b\right] &= \int_0^{+\infty} (\upsilon/\Phi)^{b/\theta} \exp\left[-\upsilon\right] d\upsilon \\ &= (\Phi)^{-b/\theta}\, \Gamma\left[\frac{\theta+b}{\theta}\right] \end{aligned}$$

well defined for $\theta + b > 0$. For general k we have

$$\begin{aligned} E\left[\left(C^{(k)}\right)^b\right] &= \int_0^{+\infty} c^b g_k(c)\, dc \\ &= \int_0^{+\infty} c^b \frac{\theta}{(k-1)!} \Phi^k c^{\theta k-1} \exp\left[-\Phi c^\theta\right] dc \\ &= \frac{\Phi^{k-1}}{(k-1)!} \int_0^{+\infty} c^{b+\theta k-\theta} \theta \Phi c^{\theta-1} \exp\left[-\Phi c^\theta\right] dc \\ &= \frac{\Phi^{k-1}}{(k-1)!} E\left[\left(C^{(1)}\right)^{b+\theta(k-1)}\right] \end{aligned} \tag{3.4}$$

■

The above proposition shows that the price index for a country i is

$$P_i = \gamma^{PC} \Phi_i^{-1/\theta}$$

$$\gamma^{PC} = \Gamma \left[\frac{\theta + 1 - \sigma}{\theta} \right]^{1/(1-\sigma)}$$

where Γ is the gamma function.

Finally, using a methodology similar to the one outlined above for the Krugman model we can compute the welfare index which is given by

$$\frac{w_j}{P_j} = \frac{1}{\gamma^{PC}} \left(\frac{A_j}{\lambda_{jj}} \right)^{\frac{1}{\theta}}. \tag{3.5}$$

3.2.1 Key contributions of EK

- Models heterogeneous sectors in a Ricardian GE model of trade.
- Estimates of θ using alternative empirical specifications. Estimation of gravity models will be a topic of section 7.
- Implement a model for counterfactual experiments.
- Most importantly: Laid the foundation for the quantitative analysis of international trade at the firm level. It was followed by models such as those of Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) that model other forms of competition (Bertrand and monopolistic respectively) and allowed subsequent work to bring models of trade closer to the trade data in many dimensions.

3.3 Application II: Bertrand competition (Bernand, Jensen & EK)

- Consider the case where different producers have access to different technologies. If we assume Bertrand competition, the cost distribution will be given by the frontier producer ($k = 1$ in the expression 3.2) but prices are related to the distribution of the second lowest cost ($k = 2$). Since the lowest cost supplier is the one that will sell the good, the probability that a good is supplied from i to j is

$$\frac{A_i (w_i \tau_{ij})^{-\theta}}{\sum_{v=1}^{N} A_v (w_v \tau_{vj})^{-\theta}}. \tag{3.6}$$

The price of a good ω in market j is:

$$p_j(\omega) = \min\{C_{2j}(\omega), \bar{m}C_{1j}(\omega)\}$$

where we will define $C_{ij}(\omega)$ to be the cost of the i'th minimum cost producer of good ω in country j, and $\bar{m} = \sigma/(\sigma-1)$ is the optimal markup that a monopolist firm would charge (assuming CES preferences with an elasticity σ). Thus, assuming heterogeneity among technology costs for firms we will derive the distribution of unit costs in each given country.

Define again $C^{(k)}$ as the k'th lowest unit cost technology for producing a particular good. We have the following Lemma

Lemma 10 *The distribution of $C^{(k+1)}$ conditional on $C^{(k)} = c_k$ is:*

$$\Pr\left[C^{(k+1)} \leq c_{k+1} | C^{(k)} = c_k\right] = 1 - \exp\left[-\Phi\left(c_{k+1}^{\theta} - c_k^{\theta}\right)\right] \;, c_{k+1}^{\theta} \geq c_k^{\theta} \geq 0$$

Proof. We have

$$\begin{aligned} \Pr\left[C^{(k+1)} \leq c_{k+1} | C^{(k)} = c_k\right] &= \int_{c_k}^{c_{k+1}} \frac{g_{k,k+1}(c_k, c)}{g_k(c_k)} dc \\ &= \int_{c_k}^{c_{k+1}} \theta\Phi c^{\theta-1} \exp\left[-\Phi c^{\theta} + \Phi c_k^{\theta}\right] dc \\ &= 1 - \exp\left[-\Phi\left(c_{k+1}^{\theta} - c_k^{\theta}\right)\right]. \end{aligned}$$

■

Thus, the distribution of $C^{(2)}$ is stochastically increasing in c_1 and hence decreasing in $z_1 = w/c_1$. Thus, high productivity producers are more likely to charge a lower price. The above relationship also implies that (defining $m = c_{k+1}/c_k$)

$$\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m | C^{(k)} = c_k\right] = 1 - \exp\left[-\Phi c_k^{\theta}\left(m^{\theta} - 1\right)\right]$$

The distribution of the ratio $M = C^{(2)}/C^{(1)}$ given $C^{(1)} = c_1$ is:

$$\Pr\left[M \leq m | C^{(1)} = c_1\right] = 1 - \exp\left[-\Phi c_1^{\theta}\left(m^{\theta} - 1\right)\right] \;.$$

We have that the lower c_1, the more likely a high markup. Thus, in this context low-cost producers are more likely to charge a high markup.

Notice that the markup with Bertrand competition that BEJK consider is

$$M(\omega) = \min\left\{\frac{C^{(2)}(\omega)}{C^{(1)}(\omega)}, \bar{m}\right\}$$

We can figure out the distribution of the markup for a given country. Notice that for any $m \leq \bar{m}$, conditional on $C_i^{(2)} = c_2$, we have

$$\begin{aligned}
\Pr\left[M \leq m | C_i^{(2)} = c_2\right] &= \Pr\left[c_2/m \leq C_i^{(1)} \leq c_2 | C_i^{(2)} = c_2\right] \\
&= \frac{\int_{c_2/m}^{c_2} g_i(c_1, c_2)\, dc_1}{\int_0^{c_2} g_i(c_1, c_2)\, dc_1} \\
&= \frac{c_2^\theta - \left(c_2^\theta/m\right)^\theta}{c_2^\theta} \\
&= 1 - m^{-\theta}.
\end{aligned}$$

This derivation implies that the distribution of markups does not depend on c_2.[3] Thus, we have proved the following proposition:

Proposition 11 *Under Bertrand competition the distribution of the markup M is:*

$$\Pr[M \leq m] = F_M(m) = 1 - m^{-\theta}$$

for $m \leq \bar{m}$. With probability $\bar{m}^{-\theta}$ the markup is $\bar{m}$. The distribution of the markup is independent of $C^{(2)}$.

[3] To compute the unconditional distribution of productivities for $m \leq \bar{m}$ we have that

$$\begin{aligned}
\Pr[M \leq m] &= \int_0^{+\infty} \left(1 - \exp\left[-\Phi c_1^\theta \left(m^\theta - 1\right)\right]\right) \theta c_1^{\theta-1} \exp\left(-\Phi c_1^\theta\right) dc_1 \\
&= \exp\left(-\Phi c_1^\theta\right) - \left.\frac{\exp\left[-\Phi c_1^\theta m^\theta\right]}{m^\theta}\right|_0^{+\infty} \\
&= 1 - 1/m^\theta
\end{aligned}$$

and with probability $\bar{m}^{-\theta}$ the markup is $\bar{m}$.

Using the above results we have

$$\begin{aligned}
P_i^{1-\sigma} &= \int_1^{\infty} E\left[p_i^{1-\sigma}|M=m\right]\theta m^{-\theta-1}dm \\
&= \int_1^{\bar{m}} E\left[\left(C_i^{(2)}\right)^{1-\sigma}\right]\theta m^{-(\theta+1)}dm + \int_{\bar{m}}^{+\infty} E\left[\left(\bar{m}C_i^{(2)}/m\right)^{1-\sigma}\right]\theta m^{-(\theta+1)}dm \\
&= E\left[\left(C_i^{(2)}\right)^{1-\sigma}\right]\left[\left(1-\bar{m}^{-\theta}\right)+\bar{m}^{-\theta}\frac{\theta}{1+\theta-\sigma}\right]
\end{aligned}$$

where in the second equality we used the fact that the distribution of markups is independent of the second lowest cost. We have already calculated $E\left[\left(C_i^{(2)}\right)^{1-\sigma}\right]$ in equation (3.4). Thus, the price index under Bertrand competition is

$$P_i = \gamma^{BC}\Phi_i^{-1/\theta}$$

$$\gamma^{BC} = \left[\left(1-\bar{m}^{-\theta}\right)+\bar{m}^{-\theta}\frac{\theta}{1+\theta-\sigma}\right]^{1/(1-\sigma)}\Gamma\left(\frac{2\theta+1-\sigma}{\theta}\right)^{1/(1-\sigma)}$$

Finally, using again the results of the theorem we have

$$\begin{aligned}
\Pr\left[C^{(k)} \leq c_k|C^{(k+1)}=c_{k+1}\right] &= \int_0^{c_k}\frac{g_{k,k+1}\left(c,c_{k+1}\right)}{g_{k+1}\left(c_{k+1}\right)}dc \\
&= \int_0^{c_k}\frac{\frac{\theta^2}{(k-1)!}\Phi^{k+1}c^{\theta k-1}c_{k+1}^{\theta-1}\exp\left(-\Phi c_{k+1}^{\theta}\right)}{\frac{\theta}{(k)!}\Phi^{k+1}c_{k+1}^{\theta(k+1)-1}e^{-\Phi c_{k+1}^{\theta}}}dc \\
&= \int_0^{c_k}\theta k\frac{c^{\theta k-1}}{c_{k+1}^{\theta k}}dc \\
&= \left(\frac{c_k}{c_{k+1}}\right)^{\theta k} \qquad (3.7)
\end{aligned}$$

and simply replacing for $c_k = \frac{c_{k+1}}{m}$ in expression (3.7) we can get

$$\begin{aligned}
\Pr\left[\frac{C^{(k+1)}}{C^{(k)}} \leq m|C^{(k+1)}=c_{k+1}\right] &= \Pr\left[C^{(k)} \geq \frac{c_{k+1}}{m}|C^{(k+1)}=c_{k+1}\right] \\
&= 1-\Pr\left[C^{(k)} \leq \frac{c_{k+1}}{m}|C^{(k+1)}=c_{k+1}\right] \\
&= 1-m^{-\theta k}
\end{aligned}$$

Given that under Bertrand competition this is the markup (for $k=1$) in the case that the lowest cost firm does not charge monopoly pricing this distribution gives us an idea of the distribution of the markups, which is independent of the second lowest cost. In fact, lengthy derivations (see the appendix of BEJK) can show with brute force that the distribution of offered prices is source country independent (see their online appendix). This result implies that the probability that a good is supplied from i to j, given by equation (3.6), is also the market share of i to j, λ_{ij}.

3.3.1 Key contributions of BEJK

- Develops a firm level model and explicitly tests its predictions with firm-level data.
- Models a framework where firms markups are variable and depending on competition. Alternative models of variable markups can be developed in monopolistic competition by allowing for a preference structure that departs from the CES aggregator (see section 4.4).
- Develops a methodology of simulating an artificial economy with heterogeneous firms and finding the parameters of this economy that brings the predictions of the model closer to the data.
- Acknowledges the fact that "measured productivity" when measured as nominal output over employment is constant in models with constant markups. It developes a model that can deliver variable markups.

3.4 Application III: Monopolistic competition (Chaney-Melitz)

- Utility function
$$U_j = \left[\int_{\Omega} x_j\left(\omega\right)^{(\sigma-1)/\sigma} d\omega\right]^{\sigma/(\sigma-1)}$$
where the measure of Ω is $\mu\left(\Omega\right)$ and $x_j(\omega)$ is a quantity of a variety available to consumers in j.
- Consumers derive income from labor the ownership of domestic firms.
- $\mu\left(\Omega\right) \subset [0, +\infty)$ is the set of available varieties. Let I_i the measure of ideas that fall randomly into goods. In some sense $I_i/\mu\left(\Omega\right)$ ideas correspond to each good.
- In the probabilistic context we described above, the monopolistic competition model arises in a very natural way. Let the distribution of the lowest cost for a good to be Frechet such that
$$F_1\left(c_1\right) = 1 - \exp\left(-\frac{I_i}{\mu\left(\Omega\right)} c_1^{\theta}\right) .$$

The measure of firms with unit cost less than $C^{(1)} \leq c_1$, is $\mu(\Omega) F_1(c_1)$. Taking the limit of this expression for the number of potential varieties $\mu(\Omega) \to +\infty$ we can show that the distribution of the best producer's cost of a variety is Pareto. The Pareto distribution in terms of productivities is defined as $\Pr(\Phi \leq \phi) = 1 - \frac{A_i}{\phi^\theta}$, where $\phi \in [A_i^{1/\theta}, +\infty)$. More details are given in appendix (9.1).

- In particular, given the fact that the Dixit-Stiglitz preferences always imply an interior solution, we need to introduce a cost of entry that is "sizable" compared to the marginal profit from entering. Melitz, following Krugman, uses a simple uniform fixed cost.

Firms decide to enter each market by maximizing their profit

$$\pi_{ij}(\phi) = \max_p \frac{p^{1-\sigma}}{P_j^{1-\sigma}} X_j - \frac{\tau_{ij}}{\phi} \frac{p^{-\sigma}}{P_j^{1-\sigma}} w_i X_j - w_j F_j$$

where X_j is the expenditure in market j and fixed costs are paid in terms of labor in country j. Prices are

$$p_{ij}(\phi) = \frac{\sigma}{\sigma-1} \frac{\tau_{ij}}{\phi} w_i$$

which implies that variable profits are

$$\frac{\left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij}}{\phi} w_i\right)^{1-\sigma}}{P_j^{1-\sigma}} \frac{X_j}{\sigma}$$

and thus only the firms that have

$$\phi \geq \phi_{ij}^*$$

where

$$\left(\phi_{ij}^*\right)^{\sigma-1} = \frac{\sigma w_j F_j P_j^{1-\sigma}}{\left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i\right)^{1-\sigma} X_j} \tag{3.8}$$

will enter. Replacing this definition, sales can be rewritten in the simple form:

$$p_{ij}(\phi) q_{ij}(\phi) = \sigma w_j F_j \left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}.$$

3.4.1 Gravity-Aggregation

Given the Pareto distribution of productivities, we can compute trade flows from a country to another by performing the simple integration

$$\begin{aligned} X_{ij} &= \underbrace{J_i \Pr\left(\Phi \geq \phi_{ij}^*\right)}_{\#\text{ exporters from } i \text{ to } j} \underbrace{\int_{\phi_{ij}^*} \sigma w_j F_j \left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1} \frac{\theta \frac{(A_i)^\theta}{\phi^{\theta+1}}}{\Pr\left(\Phi \geq \phi_{ij}^*\right)} d\phi}_{\text{av. sales per exporter}} \\ &= J_i \int_{\phi_{ij}^*} \sigma w_j F_j \left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1} \theta \frac{(A_i)^\theta}{\phi^{\theta+1}} d\phi \\ &= J_i \left(\frac{A_i}{\phi_{ij}^*}\right)^\theta \frac{\theta\sigma}{\theta-\sigma+1} w_j F_j \end{aligned}$$

Derivations reveal that the share of profits in this model is a fixed proportion of total income $\eta = (\sigma-1)/(\theta\sigma)$. Using that result, the measure of entrants can be written as a function of trade flows,

$$M_{ij} = \frac{X_{ij}}{\frac{\sigma w_j F_j}{1-1/\tilde{\theta}}} \ ,$$

where

$$\tilde{\theta} = \theta / (\sigma - 1) \ .$$

Finally, trade shares are given by

$$\lambda_{ij} = \frac{A_i (\tau_{ij})^{-\theta} J_i w_i^{-\theta}}{\sum_v A_v (\tau_{vj})^{-\theta} J_v (w_v)^{-\theta}} \tag{3.9}$$

3.4.2 Welfare

Using equation (3.9) and the price index

$$P_j^{1-\sigma} = \sum_v M_{ij} \int_{\phi_{ij}^*} p_{ij}(\phi)^{1-\sigma} \mu_{ij}(\phi) \, d\phi$$

–where $\mu_{ij}(\phi)$ is again the conditional productivity density of firms from i that sell to j– as well as (3.8) we can write welfare as[4]

$$\frac{w_j}{P_j} = (\lambda_{jj})^{-1/\theta} \left(\frac{c_j}{L_j^{1-\theta/(\sigma-1)}} \frac{\theta}{\theta - \sigma + 1} \right)^{1/\theta} \tag{3.10}$$

where

$$c_j = (F_j)^{1-\theta/(\sigma-1)} A_j \sigma^{1-\theta/(\sigma-1)} \left(\frac{\sigma}{\sigma - 1} \right)^{-\theta} (1-\eta)^{1-\theta/(\sigma-1)} \tag{3.11}$$

3.4.3 Key contributions of Melitz

- Established a macroeconomic framework where the concept of the firm had a meaning while the model was tractable and amenable to a variety of exercises. In this framework it is possible to think about trade liberalization and firms in GE.
- In addition the number of varieties offered to the consumer is potentially changing when the fundamentals change (e.g. trade liberalization).
- Explicitly modeled the importance of reallocation of production through the death of the least productive firms.
- Extensions by Helpman, Melitz, and Yeaple (2004), Chaney (2008), Helpman, Melitz, and Rubinstein (2008) and EKK10 made the framework applicable to actual quantitative exercises.
- Paper by EKK10 shows how to go from the one form of competition to the other.

[4] The original Melitz setup assumed a free entry condition and that firms learn their productivity after incurring a fixed cost of entry f^e. The implications of such an assumption will be studied later on.

3.5 Application IV: Modeling Vertical Specialization

In this section we will present some straightforward ways of introducing intermediate inputs into the heterogeneous firms models. We will comment on the different ways in which the production theory developed above can be used.

3.5.1 Each good is both final and intermediate

In their heterogeneous sectors framework Eaton and Kortum (2002) have used the intermediate inputs structure initially proposed by Krugman and Venables (1995). The idea is that the production of each good requires labor and intermediate inputs, with labor having a constant share ι. Intermediates comprise the full set of goods that are also used as finals and they are combined according to the same CES aggregator. Therefore, the overall price index in country i, P_i (derived in previous sections), becomes the appropriate index of intermediate goods prices in this case. The cost of an input bundle in country i is thus

$$c_i = w_i^{\iota} P_i^{1-\iota}$$

The overall changes in the predictions of the model are small, but the main effect is that trade shares are now affected by ι and thus

$$\lambda_{ij} = \frac{A_i \tau_{ij}^{-\theta} \left(w_i^{\iota} P_i^{1-\iota} \right)^{-\theta}}{\sum_{\upsilon=1}^{N} A_{\upsilon} \tau_{\upsilon j}^{-\theta} \left(w_{\upsilon}^{\iota} P_{\upsilon}^{1-\iota} \right)^{-\theta}} .$$

3.5.2 Each good has a single specialized intermediate input

Yi (2003) develops a model where endogenous vertical specialization into different stages of production is allowed. The output $y^2(\omega)$ for a final good $\omega \in \Omega$ is produced using input from a uniquely specialized intermediate good $y^1(\omega)$. The corresponding production functions are

$$\begin{aligned} y_i^2(\omega) &= z_i^2(\omega)\, l_i^2(\omega)^{\iota}\, y_i^1(\omega)^{1-\iota} \ , i = 1, 2 \\ y_i^1(\omega) &= z_i^1(\omega)\, l_i^1(\omega) \ , i = 1, 2 \end{aligned}$$

where the output of each one of the stages can be produced by either countries and $1 - \iota$ is the share of intermediates into production. The model is essentially a two stages Dornbusch, Fischer, and

Samuelson (1977) model with Perfect Competition in all the markets. The interesting feature of the Yi (2003) model is that the degree of specialization in either stage of production for a given country is endogenous and depends on trade barriers and the comparative advantage of the two countries.[5] When for a given good both stages of the production are performed abroad, trade of that good is more sensitive to trade cost changes. Yi (2003) uses this feature of the model to offer an explanation of the rapid growth of world trade during the past decades.

The main drawback of his approach is that calibration is constrained by the usage of the Dornbusch, Fischer, and Samuelson (1977) framework. Thus, Yi (2003) can use general monotonic functions for the relative productivity of one of the stages of production between the two countries but not of both. Of course, this setup is very difficult to be generalized in more than two countries.

3.5.3 Each good uses a continuum of inputs

Arkolakis and Ramanarayanan (2008) propose a different intermediate inputs structure by merging and generalizing the two approaches described above. Goods are produced in two stages with the second stage of production (production of "final goods") using goods produced in the first stage ("intermediate goods"). Production is vertically specialized to the extent that one country uses imported intermediate goods to produce output that is exported. There is a continuum of measure one of goods in the first stage of production, and in the second stage of production. We index both intermediate and final goods by ω, although they are distinct commodities.

Each first-stage intermediate input ω can be produced with a CRS labor only technology given by

$$y_i^1(\omega) = z_i^1(\omega)\, l_i^1(\omega) \;, \tag{3.12}$$

with efficiency denoted by $z_i^1(\omega)$. The technology for producing output of final good ω is:[6]

$$y_2^i(\omega) = z_i^2(\omega)\left(l_i^2\right)^{\iota}\left(\int m_i\left(\omega,\omega'\right)^{\frac{\sigma-1}{\sigma}} d\omega'\right)^{\frac{(1-\iota)\sigma}{\sigma-1}} \;, \tag{3.13}$$

[5] While in the Eaton and Kortum (2002) intermediates good framework there are two possible production patterns for the good that is sold in a given market (either home or foreign is the producer of the sold good) in the model of Yi (2003) there are 4 for the two stages of a given variety. These are (HH) Home (country) produces stages 1 and 2, (FF) Foreign produces stages 1 and 2, (HF) Home produces stage 1, Foreign produces stage 2 and (FH) Foreign produces stage 1, Home produces stage 2.

[6] Unless otherwise noted, integration is over the entire set of goods in the relevant stage of production.

where $m^i(\omega, \omega')$ is the use of intermediate good ω' in the production of final good ω. The parameter σ is the elasticity of substitution between different intermediate inputs.

We use the probabilistic representation of Eaton and Kortum (2002) for good-specific efficiencies. For each country i and stage s, z_s^i in (3.12) and (3.13) is drawn from a Fréchet distribution characterized by the cumulative distribution function

$$F_i^s(z) = e^{-A_i^s z^{-\theta}} ,$$

for $s = 1, 2$ and $i = 1, 2$, where $A_i^s > 0$ and $\theta > 1$. Efficiency draws are independent across goods, stages, and countries. The probability that a particular stage-s good ω can be produced in country i with efficiency less than or equal to z_i^s is given by $F_i^s(z_i^s)$. Since draws are independent across the continuum of goods, $F_i^s(z_i^s)$ also denotes the fraction of stage-s goods that country i is able to produce with efficiency at most z_i^s.

Following Eaton and Kortum (2002), it is straightforward to show that the distribution of prices of stage-1 goods that country i offers to country j equals

$$G_{ij}^s(p) = 1 - e^{-A_s^i \left(q_s^{ij}\right)^{-\theta} p^\theta} ,$$

where q_s^{ij} is the unit cost of producing and shipping the good. This means that the overall distribution of prices of stage-s goods available in country j is

$$G_s^j(p) = 1 - e^{-\Phi_s^j p^\theta} , \tag{3.14}$$

where

$$\Phi_s^j \equiv \sum_v A_s^v \left(q_s^{vj}\right)^{-\theta} . \tag{3.15}$$

The probability that country j buys a certain good from country i, as Eaton and Kortum (2002) show, equals

$$\lambda_s^{ij} = \frac{A_s^i \left(q_s^{ij}\right)^{-\theta}}{\Phi_s^j} . \tag{3.16}$$

As in Eaton and Kortum (2002), it is also true that, because the distribution of stage-s goods actually purchased by country j from country i is equal to the overall price distribution G_s^j, the fraction λ_s^{ij} of goods purchased from country i also equals to the fraction of country j's total expenditures on stage-s goods that it spends on goods from country i.

The interesting feature of this intermediate inputs structure is that the specialization patterns introduced by Yi (2003) still hold. However, the model is much easier to calibrate given that the function that determined comparative advantage can be easily linked to observable trade shares for each stage of production.

Chapter 4

Modeling the demand side

4.1 The CES demand structure

We can consider the generalized CES structure

$$\left(\int_{\Omega} \left(\sum_{k=1}^{N} x_k \left(\omega \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\frac{\sigma-1}{\sigma}}{\frac{\varepsilon-1}{\varepsilon}}} d\omega \right)^{\frac{1}{\frac{\sigma-1}{\sigma}}}$$

that delivers the demand

$$x_i \left(\omega \right) = \left(\frac{p_i \left(\omega \right)}{P \left(\omega \right)} \right)^{-\varepsilon} \left(\frac{P \left(\omega \right)}{P} \right)^{-\sigma} X \ ,$$

with

$$\begin{aligned} P \left(\omega \right) &= \left[\sum_{\upsilon=1}^{N} p_{\upsilon} \left(\omega \right)^{1-\varepsilon} d\omega \right]^{1/(1-\varepsilon)} \ , \\ P &= \left[\int_0^J P \left(\omega \right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \ . \end{aligned}$$

and X being the overall spending.

Serving a market incurs an entry cost

a) $\varepsilon \to \infty, F = 0$ PC EK02

b) $\varepsilon \to \infty, F = 0$ Bertrand BEJK

c) $\varepsilon = \sigma$, $F > 0$ monopolistic competition Melitz-Chaney

d) $\varepsilon > \sigma$, $F \geq 0$ (with either $F > 0$ or $\varepsilon \to \infty$) and Cournot, Atkeson and Burstein.

By using the CES structure and assuming $\varepsilon = \sigma$ together with increasing cost to sell more in a market it results in the context of Arkolakis (2010) that we will introduce below.

4.2 Extension I: Market penetration costs

The CES benchmark proved extremely useful for many applications. Its main weakness is in predicting the behavior of small firms-goods as Eaton, Kortum, and Kramarz (2010). These firms-goods tend to be a very large part of trade in a trade liberalization and as time evolves. To address this fact, a simple extension presented in Arkolakis (2010) does the job by modeling the fixed entry costs as cost of reaching individual consumers into individual destinations.

Each good is produced by at most a single firm and firms differ ex-ante only in their productivities ϕ and their country of origin $i = 1, ..., N$. We denote the destination country by j. The preferences for consumer l are given by the standard symmetric constant elasticity of substitution (CES) objective function:

$$U^l = \left(\int_{\omega \in \Omega^l} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma-1}{\sigma}} ,$$

where $\sigma \in (1, +\infty)$ is the elasticity of substitution. When a good produced with a productivity ϕ from country i is included in the choice set of consumer l, Ω^l, the demand of this consumer is given by,

$$x_{ij}(\phi) = y_j \frac{p_{ij}(\phi)^{-\sigma}}{P_j^{1-\sigma}} , \tag{4.1}$$

where $p_{ij}(\phi)$ is the price charged in country j, y_j the income per capita of the consumer, and P_j a price aggregate of the goods in the choice set of the consumer. An unrealistic assumption of the CES framework introduced by Dixit and Stiglitz is that all the consumers have access to the same set of goods Ω^l. This formulation departs from the standard formulation of trade models with CES preferences by proposing a formulation where Ω^l can be different for different consumers. In order to be able to fully characterize the general equilibrium of the model, we assume that consumers are reached independently by different firms and that each firm pays a cost to reach a fraction n of the consumers. In equilibrium, all firms ϕ from country i will reach the same fraction of consumers in country j and thus their 'effective' sales will be:[1]

$$t_{ij}(\phi) = \underbrace{n_{ij}(\phi) L_j}_{\substack{\text{consumers} \\ \text{reached in } j}} \underbrace{y_j \frac{p_{ij}(\phi)^{1-\sigma}}{P_j^{1-\sigma}}}_{\text{sales per-consumer}}$$

where L_j is the measure of the population of country j. In order to give foundations to the market penetration cost function as an explicit function of $n_{ij}(\phi)$ we depart from the standard formulation where there is a uniform fixed cost to enter the market and sell to all the consumers there. Instead, we consider an alternative formulation that intends to broadly capture the marketing costs incurred by the firm in order to increase their sales in a particular market. The marketing costs are modeled as increasing access costs that the firms pay in order to access an increasing number of customers in each given country. Due to market saturation, reaching additional consumers becomes increasingly difficult once a relatively large fraction of them has already been reached. Based on a derivation of a marketing technology from first principles the cost function of reaching a fraction n of a population of L consumers in Arkolakis (2010) is derived to be

$$f(n) = \begin{cases} \frac{L^{\alpha}}{\psi} \frac{1-(1-n)^{1-\beta}}{1-\beta} & \text{if } \beta \in [0,1) \cup (1,+\infty) \\ \frac{L^{\alpha}}{\psi} \log(1-n) & \text{if } \beta = 1 \end{cases} .$$

$1/\psi$ denotes the productivity of search effort and $a \in [0,1]$ regulates returns to scale of marketing costs with respect to the population size of the destination country. The parameter β determines how steeply the cost to reach additional consumers is rising. However, for any parametrization of β the marginal cost to reach the very first consumers in a given market j is always positive (the derivative is always bigger than zero). Thus, only firms with productivity above some threshold ϕ_{ij}^* will have high enough revenues from the very first consumers to find it profitable to enter the market.[2] The case where $\beta = 1$ corresponds to the benchmark random search case of Butters (1977) and Grossman and Shapiro (1984). If $\beta = 0$ the function implies a linear cost to reach additional consumers, which in turn is isomorphic to the case of Melitz (2003)-Chaney (2008) given that firms reach either all the consumers in a market or none.

[1] Given the existence of a continuum of firms and consumers I am making use of the Law of Large Numbers. This implies that $n_{ij}(\phi)$ from a probability becomes a fraction. The application of the Law of Large Numbers also implies that P_j is now a function of $n_{ij}(\phi)$'s and has a given value for all consumers.

[2] With no additional heterogeneity across firms this implies a hierarchy of exporting destinations

The production side of the firm is standard. Labor is the only factor of production. The firm ϕ uses a production function that exhibits constant returns to scale and productivity ϕ. It incurs an iceberg transportation cost τ_{ij} to ship a good from country i to country j. This implies that the optimal price of the firm is a constant markup $\sigma/(\sigma-1)$ over the unit cost of producing and shipping the good, $w_j\tau_{ij}/\phi$. The equilibrium of the model retains many of the desirable properties of the benchmark quantitative framework for considering bilateral trade flows develop by Eaton and Kortum (2002) and particularly the gravity structure. It also allows for endogenous decision of exporting and non-exporting of firms as in Melitz (2003).

How can this additional feature of endogenous market penetration costs help the model to address facts on exporters? The following version of the proposition proved in Arkolakis (2010) computes the responses of firm's sales in a trade liberalization episode:

Proposition 12 (Elasticity of trade flows and firm size)
The partial elasticity of a firm's sales in market j with respect to variable trade costs, $\varepsilon_{ij}(\phi) = |\partial \ln t_{ij}(\phi)/\partial \ln \tau_{ij}|$, is decreasing with firm productivity, ϕ, i.e. $d\varepsilon_{ij}(\phi)/d\phi < 0$ for all $\phi \geq \phi^*_{ij}$.

Proof. Compute the partial elasticity of trade flows $t_{ij}(\phi)$ with respect to a change in τ_{ij}, namely $|\partial \ln t_{ij}(\phi)/\partial \ln \tau_{ij}| = |\zeta(\phi)| \times \left|\partial \ln \phi^*_{ij}/\partial \ln \tau_{ij}\right|$, where

$$\zeta(\phi) = \underbrace{(\sigma-1)}_{\substack{\text{intensive margin}\\ \text{of per-consumer}\\ \text{sales elasticity}}} + \underbrace{\frac{\sigma-1}{\beta}\left[\left(\frac{\phi}{\phi^*_{ij}}\right)^{(\sigma-1)/\beta} - 1\right]^{-1}}_{\substack{\text{extensive margin of}\\ \text{consumers elasticity}}}.$$

Notice that $\zeta(\phi) \geq 0$ for $\phi \geq \phi^*_{ij}$. $\zeta(\phi)$ is also decreasing in ϕ and thus decreasing in initial export sales. In fact, as $\beta \to 0$ then $\zeta(\phi) \to (\sigma-1)$ for all $\phi \geq \phi^*_{ij}$. ■

The proposition implies that trade liberalization benefits relatively more the smaller exporters in a market. The parameter β governs both the heterogeneity of exporters cross-sectional sales and also the heterogeneity of the growth rates of sales after a trade liberalization.

4.3 Extension II: Multiproduct firms

We now turn to an extension of the basic CES setup that can accomodate multiproduct firms. This extension is suggested by Arkolakis and Muendler (2010) and is modeling the idea of "core-competency" within the standard heterogeneous firms setup of Melitz (2003).

A conventional "variety" offered by a firm ω from source country i to destination j is the product composite

$$x_{ij}(\omega) \equiv \left(\sum_{g=1}^{G_{ij}(\omega)} x_{ijg}(\omega)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $G_{ij}(\omega)$ is the number of products that firm ω sells in country d and $x_{ijg}(\omega)$ is the quantity of product g that consumers consume. The consumer's utility at destination j is a CES aggregation over these bundles

$$U_j = \left(\sum_{i=1}^{N} \int_{\omega \in \mathbf{\Omega}_{ij}} x_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \quad \text{for } \sigma > 1, \tag{4.2}$$

where $\mathbf{\Omega}_{ij}$ is the set of firms that ship from source country i to destination j. For simplicity we assume that the elasticity of substitution across a firm's products is the same as the elasticity of substitution between varieties of different firms.[3]

The consumer's first-order conditions of utility maximization imply a product demand

$$x_{ijg}(\omega) = \frac{(p_{ijg}(\omega))^{-\sigma}}{P_j^{1-\sigma}} X_j, \tag{4.3}$$

where p_{ijg} is the price of variety ω product g in market j and we denote by X_j the total spending of consumers in country j. The corresponding price index is defined as

$$P_j \equiv \left[\sum_{\upsilon=1}^{N} \int_{\omega \in \mathbf{\Omega}_{\upsilon j}} \sum_{g=1}^{G_{\upsilon j}(\omega)} p_{\upsilon jg}(\omega)^{-(\sigma-1)} \, d\omega \right]^{-\frac{1}{\sigma-1}}. \tag{4.4}$$

A firm of type ϕ chooses the number of products $G_{ij}(\phi)$ to sell to a given market j. The firm makes each product $g \in \{1, 2, \ldots, G_{ij}(\phi)\}$ with a linear production technology, employing local labor with efficiency ϕ_g. When exported, a product incurs a standard iceberg trade cost so that $\tau_{ij} > 1$ units must be shipped from i for one unit to arrive at destination j. We normalize $\tau_{ii} = 1$ for domestic sales. Note that this iceberg trade cost is common to all firms and to all firm-products shipping from i to j.

[3] Arkolakis and Muendler (2010) generalize the model to consumer preferences with two nests. The inner nest contains the products of a firm, which are substitutes with an elasticity of ε. The outer nest aggregates those firm-level product lines over firms and source countries, where the product lines are substitutes with a different elasticity $\sigma \neq \varepsilon$. The general case of $\varepsilon \neq \sigma$ generates similar predictions at the firm-level and at the aggregate bilateral country level.

Without loss of generality we order each firm's products in terms of their efficiency so that $\phi_1 \geq \phi_2 \geq \ldots \geq \phi_{G_{ij}}$. A firm will enter export market j with the most efficient product first and then expand its scope moving up the marginal-cost ladder product by product. Under this convention we write the efficiency of the g-th product of a firm ϕ as

$$\phi_g \equiv \frac{\phi}{h(g)} \qquad \text{with} \quad h'(g) > 0. \tag{4.5}$$

We normalize $h(1) = 1$ so that $\phi_1 = \phi$. We think of the function $h(g) : [0, +\infty) \to [1, +\infty)$ as a continuous and differentiable function but we will consider its values at discrete points $g = 1, 2, \ldots, G_{ij}$ as appropriate.

Related to the marginal-cost schedule $h(g)$ we define firm ϕ's product efficiency index as

$$H(G_{ij}) \equiv \left(\sum_{g=1}^{G_{ij}} h(g)^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}. \tag{4.6}$$

This efficiency index will play an important role in the firm's optimality conditions for scope choice.

As the firm widens its exporter scope, it also faces a product-destination specific incremental local entry cost $f_{ij}(g)$ that is zero at zero scope and strictly positive otherwise:

$$f_{ij}(0) = 0 \quad \text{and} \quad f_{ij}(g) > 0 \quad \text{for all } g = 1, 2, \ldots, G_{ij}, \tag{4.7}$$

where $f_{ij}(g)$ is a continuous function in $[1, +\infty)$.

The incremental local entry cost $f_{ij}(g)$ accommodates fixed costs of production (e.g. with $0 < f_{ii}(g) < f_{ij}(g)$). In a market, the incremental local entry costs $f_{ij}(g)$ may increase or decrease with exporter scope. But a firm's local entry costs

$$F_{ij}(G_{ij}) = \sum_{g=1}^{G_{ij}} f_{ij}(g)$$

necessarily increase with exporter scope G_{ij} in country j because $f_{ij}(g) > 0$. We assume that the incremental local entry costs $f_{ij}(g)$ are paid in terms of importer (destination country) wages so that $F_{ij}(G_{ij})$ is homogeneous of degree one in w_j. Combined with the preceding varying firm-product efficiencies, this local entry cost structure allows us to endogenize the exporter scope choice at each destination j.

A firm with a productivity ϕ from country i faces the following optimization problem for selling to destination market j

$$\pi_{ij}(\phi) = \max_{G_{ij}, p_{ijg}} \sum_{g=1}^{G_{ij}} \left(p_{ijg} - \tau_{ij} \frac{w_i}{\phi / h(g)} \right) \frac{(p_{ijg})^{-\sigma}}{P_j^{1-\sigma}} X_j - F_{ij}(G_{ij}).$$

The firm's first-order conditions with respect to individual prices p_{ijg} imply product prices

$$p_{ijg}(\phi) = \bar{m}\, \tau_{ij}\, w_i\, h(g)/\phi \tag{4.8}$$

with an identical markup over marginal cost $\bar{m} \equiv \sigma/(\sigma-1) > 1$ for $\sigma > 1$. A firm's choice of optimal prices implies optimal product sales for product g

$$p_{ijg}(\phi) x_{ijg}(\phi) = \left(\frac{P_j}{\bar{m}\, \tau_{ij}\, w_i} \frac{\phi}{h(g)} \right)^{\sigma-1} X_j. \tag{4.9}$$

Summing (4.9) over the firm's products at destination j, firm ϕ's optimal total exports to destination j are

$$t_{ij}(\phi) = \sum_{g=1}^{G_{ij}(\phi)} p_{ijg}(\phi)\, x_{ijg}(\phi) = \left(\frac{P_j}{\bar{m}\, \tau_{ij}\, w_i} \phi \right)^{\sigma-1} X_j\, H(G_{ij}(\phi))^{-(\sigma-1)}, \tag{4.10}$$

where $H(G_{ij})$ is a firm's product efficiency index from (4.6). The term $H(G_{ij}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{ij}(\phi)$.

Given constant markups over marginal cost, profits at a destination j for a firm ϕ selling G_{ij} are

$$\pi_{ij}(\phi) = \left(\frac{P_j}{\bar{m}\, \tau_{ij}\, w_i} \phi \right)^{\sigma-1} \frac{X_j}{\sigma} H\left(G_{ij}\right)^{-(\sigma-1)} - F_{ij}\left(G_{ij}\right).$$

The following assumption is required for the firm optimization to be well defined:

$$z'_{ij}(G) > 0 \tag{4.11}$$

where $z_{ij}(G) \equiv f_{ij}(G)\, h(G)^{\sigma-1}$

Under this assumption, the optimal choice for $G_{ij}(\phi)$ is the largest $G \in \{0, 1, \ldots\}$ such that operating profits from that product equal (or still exceed) the incremental local entry costs:

$$\begin{aligned}\left(\frac{P_j}{\bar{m}}\,\tau_{ij}\,w_i\,\frac{\phi}{h(G)}\right)^{\sigma-1}\frac{X_j}{\sigma} &\geq f_{ij}(G) \quad \Longleftrightarrow \\ \pi_{ij}^{g=1}(\phi)\equiv\left(\frac{P_j\,\phi}{\bar{m}\,\tau_{ij}\,w_i}\right)^{\sigma-1}\frac{X_j}{\sigma} &\geq f_{ij}(G)\,h(G)^{\sigma-1}\equiv z_{ij}(G). \end{aligned}\tag{4.12}$$

Operating profits from the core product are $\pi_{ij}^{g=1}(\phi)$, and operating profits from each additional product g are $\pi_{ij}^{g=1}(\phi)/h(g)^{\sigma-1}$.

Assumption 4.11 is comparable to a second-order condition (for perfectly divisible scope in the continuum version of the model, Assumption 4.11 is equivalent to the second order condition). When Assumption 4.11 holds we will say that a firm faces *overall diseconomies of scope*.

We can express the condition for optimal scope more intuitively and evaluate the optimal scope of different firms. Firm ϕ exports from i to j iff $\pi_{ij}(\phi) \geq 0$. At the break-even point $\pi_{ij}(\phi) = 0$, the firm is indifferent between selling its first product to market j and remaining absent. Equivalently, reformulating the break-even condition and using the above expression for minimum profitable scope, the productivity threshold ϕ_{ij}^* for exporting from i to j is given by

$$\left(\phi_{ij}^*\right)^{\sigma-1}\equiv\frac{\sigma f_{ij}(1)}{X_j}\left(\frac{\bar{m}\,\tau_{ij}\,w_i}{P_j}\right)^{\sigma-1}. \tag{4.13}$$

In general, using (4.13), we can define the productivity threshold $\phi_{ij}^{*,G}$ such that firms with $\phi \geq \phi_{ij}^{*,G}$ sell at least G_{ij} products as

$$\left(\phi_{ij}^{*,G}\right)^{\sigma-1}=\frac{z_{ij}(G)}{f_{ij}(1)}\,\left(\phi_{ij}^*\right)^{\sigma-1}, \tag{4.14}$$

under the convention that $\phi_{ij}^* \equiv \phi_{ij}^{*,1}$. Note that if Assumption 4.11 holds then $\phi_{ij}^* < \phi_{ij}^{*,2} < \phi_{ij}^{*,3} < \ldots$ so that more productive firms introduce more products in a given market. So $G_{ij}(\phi)$ is a step-function that weakly increases in ϕ.

Using the above definitions, we can rewrite individual product sales (4.9) and total sales (4.10) as

$$\begin{aligned}p_{ijg}(\phi)x_{ijg}(\phi) &= \sigma\,f_{ij}(1)\left(\frac{\phi}{\phi_{ij}^*}\right)^{\sigma-1}h(g)^{-(\sigma-1)} \\ &= \sigma\,z_{ij}\left[G_{ij}(\phi)\right]\left(\frac{\phi}{\phi_{ij}^{*,G}}\right)^{\sigma-1}h(g)^{-(\sigma-1)} \end{aligned}\tag{4.15}$$

and

$$t_{ij}(\phi) = \sigma\, f_{ij}(1) \left(\frac{\phi}{\phi^*_{ij}}\right)^{\sigma-1} H\left[G_{ij}(\phi)\right]^{-(\sigma-1)}. \tag{4.16}$$

The following proposition summarizes the findings.

Proposition 13 *If Assumption 4.11 holds, then for all $i, j \in \{1, \dots, N\}$*

- *exporter scope $G_{ij}(\phi)$ is positive and weakly increases in ϕ for $\phi \geq \phi^*_{ij}$;*
- *total firm exports $t_{ij}(\phi)$ are positive and strictly increase in ϕ for $\phi \geq \phi^*_{ij}$.*

Proof. The first statement follows directly from the discussion above. The second statement follows because $H(G_{ij}(\phi))^{-(\sigma-1)}$ strictly increases in $G_{ij}(\phi)$ and $G_{ij}(\phi)$ weakly increases in ϕ so that $t_{ij}(\phi)$ strictly increases in ϕ by (4.16). ■

There are two key differences to the Melitz (2003) setup. The first is the term $H(G_{ij}(\phi))^{-(\sigma-1)}$ that reflects multi-product choice within the firm. Adding new products make this term higher, but with core-competency these new products are less and less important for overall sales. The second difference with the Melitz setup is the fixed cost term $F_{ij}\left(G_{ij}\right)$ that jointly with H determines the products optimization. These two features properly estimated from the data can be used to evaluate the prediction of this setup for a number of facts on multi-product exporters. We will come back to this point when we talk about the estimation of firm-level models.

4.3.1 Gravity and Welfare

The market shares in this model are given by

$$\lambda_{ij} = \frac{J_i A_i (w_i \tau_{ij})^{-\theta} f_{ij}(1)^{-\tilde{\theta}} \bar{F}_{ij}}{\sum_v J_v A_v (w_v \tau_{vj})^{-\theta} f_{vj}(1)^{-\tilde{\theta}} \bar{F}_{vj}}$$

where $f_{ij}(1)^{-\tilde{\theta}} \bar{F}_{ij} = \sum_{G=1}^{\infty} f_{ij}(G)^{-(\tilde{\theta}-1)} h(G)^{-\theta}$ and $\tilde{\theta} = \frac{\theta}{\sigma-1}$. The key new insight is that changes in the entry cost will have a different effect on overall trade than in the Melitz (2003) setup insofar they affect the entry costs for different products differently. Conditional on overall trade flows though, the welfare gains from trade are given by an expression that is similar to the Melitz (2003) setup. Thus, the difference is in the counterfactual predictions with respect to changes in trade costs.

4.4 Non-Homothetic demand structures

4.4.1 Melitz-Otavianno

In this part, we solve a version of the monopolistic competition model with heterogeneous firms and linear demand (Melitz and Ottaviano (2008)). We extend the model to a multi-country setting, and allow for income effects by dispensing with the outside sector. The main results of Melitz-Ottaviano still hold; in particular, firm mark-ups depend on firm size. However, we highlight a number of new results for the model. First, the distribution of sales of small firms is more skewed than the simple Pareto distribution, which is what the constant elasticity of substitution (CES) model would imply. This result is similar to Arkolakis (2010). Second, when compared to the CES framework, the model lacks an additional degree of freedom in order to match both bilateral aggregate trade across countries and the distribution of sales. Finally, the equilibrium number of entrants is a function of the population and other constants, but not of bilateral trade costs. This result is the same as the one found by Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008) for the CES model.

Solving for the FOCs

Assume a measure L of identical consumers, where each one of them is endowed with 1 unit of labor and does not value leisure. Preferences of a representative consumer in country j over a continuum of products $\omega \in \Omega$ are given by

$$U_J = \alpha \int_{\Omega_j} x_j^c(\omega)\, d\omega - \frac{1}{2}\gamma \int_{\Omega_j} \left(x_j^c(\omega)\right)^2 d\omega - \frac{1}{2}\eta \left(\int_{\Omega_j} x_j^c(\omega)\, d\omega\right)^2$$

where α, η, γ are all positive and $x^c(\omega)$ is the quantity consumed. The consumer maximizes this utility function subject to the budget constraint

$$\int_{\Omega_j} x_j^c(\omega) p_j(\omega) d\omega = w_j,$$

where w is the unit wage and $p(\omega)$ is the price of good ω. The FOCs of the above problem yield $(\forall x^c(\omega) > 0)$:

$$\mu_j p_j(\omega) = \alpha - \gamma x_j^c(\omega) - \eta \int_{\Omega_j} x_j^c(\omega)\, d\omega. \tag{4.17}$$

where μ_j is the Lagrangian multiplier of the consumer in country j. Also, we can derive:

$$x_j^c(\omega) = \frac{1}{\gamma}\left(\alpha - \mu_j p_j(\omega) - \eta \int_{\Omega_j} x_j^c(\omega)\, d\omega\right). \tag{4.18}$$

Let $\Omega_j^* \subset \Omega_j$ represent consumed varieties, and let M be the measure of this set. Defining:

$$\bar{q}_j^c := \frac{1}{M_j}\int_{\Omega_j^*} x_j^c(\omega) d\omega, \qquad \bar{p}_j := \frac{1}{M_j}\int_{\Omega_j^*} p_j(\omega) d\omega,$$

and integrating (4.18) over all $\omega \in \Omega^*$ yields:

$$\begin{aligned} \mu_j \bar{p}_j &= \alpha - \gamma \bar{q}_j^c - \eta M_j \bar{q}_j^c \implies \\ \bar{q}_j^c &= \frac{\alpha - \mu_j \bar{p}_j}{\gamma + \eta M_j}. \end{aligned}$$

Using (4.18), demand for variety ω for a country with a continuum of consumers of measure L_j is:

$$\frac{L_j}{\gamma}\left(\alpha - \mu_j p_j(\omega) - \eta \int_{\Omega_j} x_j^c(\omega)\, d\omega\right).$$

We will consider a symmetric equilibrium where all the firms from source country i with productivity ϕ choose the same equilibrium variables. It follows that $q_{ij}(\phi) = 0$ exactly when

$$\mu_j p_{ij}(\phi_{ij}^*) = \mu_j p_{ij}^* := \alpha + \eta M_j \mu_j \frac{\bar{p}_j - \alpha}{\gamma + \eta M_j} = \frac{\gamma\alpha + \eta M_j \bar{p}_j \mu_j}{\gamma + \eta M_j}. \tag{4.19}$$

Firm problem: The profit maximization problem of the firm with productivity draw ϕ is

$$\begin{aligned} \pi_{ij}(\phi) &= \max_{q,p} pq - \tau_{ij}\frac{w_i}{\phi} q \\ &= \max_{p} p\left(\frac{\alpha L_j}{\gamma + \eta M_j} - \frac{L_j}{\gamma}\mu_j p + \frac{\eta L_j}{\gamma}\frac{M_j \bar{p}_j \mu_j}{\gamma + \eta M_j}\right), \end{aligned}$$

where we have replaced for the iceberg transportation costs and the production function for a firm ϕ

in the cost function of producing and shipping the good abroad. The above problem implies the FOC

$$\frac{L_j}{\gamma}\left(\mu_j p_{ij}\left(\phi\right) - \mu_j \tau_{ij}\frac{w_i}{\phi}\right) = q_{ij}\left(\phi\right). \tag{4.20}$$

The FOCs also imply that for the marginal firm with $\phi = \phi_{ij}^*$, $q_{ij}\left(\phi\right) = 0$:

$$p_{ij}\left(\phi^*\right) = \tau_{ij}\frac{w_i}{\phi_{ij}^*}$$

By further manipulating the FOC we can get that

$$p_{ij}\left(\phi\right) = \frac{1}{2}\left(p_{ij}^* + \tau_{ij}\frac{w_i}{\phi}\right) \tag{4.21}$$

and therefore using (4.21) the quantity is given by

$$q_{ij}\left(\phi\right) = \frac{L_j}{\gamma}\left(\frac{1}{2}\mu_j p_{ij}^* - \frac{1}{2}\mu_j \tau_{ij}\frac{w_i}{\phi}\right) \tag{4.22}$$

The sales of the firm using the above two equations are

$$\begin{aligned} p_{ij}\left(\phi\right) q_{ij}\left(\phi\right) &= \left(\tau_{ij}\frac{w_i}{\phi_{ij}^*} + \tau_{ij}\frac{w_i}{\phi}\right)\frac{1}{2}\mu_j\frac{L_j}{\gamma}\frac{1}{2}\left(\tau_{ij}\frac{w_i}{\phi_{ij}^*} - \tau_{ij}\frac{w_i}{\phi}\right) \implies \\ p_{ij}\left(\phi\right) q_{ij}\left(\phi\right) &= \mu_j\frac{L_j}{4\gamma}\left(\left(\tau_{ij}\frac{w_i}{\phi_{ij}^*}\right)^2 - \left(\tau_{ij}\frac{w_i}{\phi}\right)^2\right) \end{aligned} \tag{4.23}$$

Total sales

We have that

$$\mu_j p_{ij}(\phi_{ij}^*) = \alpha + \eta M_j \mu_j \frac{\bar{p}_j - \alpha}{\gamma + \eta N_j}$$

and looking at the case $\eta = 0$

$$\begin{aligned} \mu_j p_{ij}(\phi_{ij}^*) &= \alpha \implies \\ \mu_j w_i &= \frac{\alpha \phi_{ij}^*}{\tau_{ij}} \end{aligned} \tag{4.24}$$

and thus using equation (4.23) can be written as

$$p_{ij}(\phi)\,q_{ij}(\phi) = \frac{w_i L_j}{4\gamma}\alpha\tau_{ij}\left(\frac{1}{\phi_{ij}^*} - \frac{\phi_{ij}^*}{\phi^2}\right) \tag{4.25}$$

and thus the average sales are given by

$$\begin{aligned}\overline{p_{ij}q_{ij}} &= \int_{\phi_{ij}^*} \frac{w_i L_j}{4\gamma}\alpha\tau_{ij}\left(\frac{1}{\phi_{ij}^*} - \frac{\phi_{ij}^*}{\phi^2}\right)\theta\frac{\left(\phi_{ij}^*\right)^\theta}{\phi^{\theta+1}}d\phi \\ &= \theta\alpha^2\frac{L_j}{\mu_j 4\gamma}\left(\frac{2}{\theta}\frac{1}{(\theta+2)}\right) .\end{aligned}$$

Notice that the last line implies that average sales per firm are not source country specific. The number of firms from source i selling to country j is

$$M_{ij} = J_i\frac{A_i}{\left(\phi_{ij}^*\right)^\theta}$$

and therefore total sales are given

$$X_{ij} = J_i\frac{A_i}{\left(\phi_{ij}^*\right)^\theta}\theta\alpha^2\frac{L_j}{\mu_j 4\gamma}\left(\frac{2}{\theta}\frac{1}{(\theta+2)}\right)$$

where J_i can be determined using the free entry and labor market clearing conditions. In particular the budget constraint (which is equivalent to labor market clearing) implies that

$$\begin{aligned}w_i L_i &= \sum_j J_i\frac{A_i}{\left(\phi_{ij}^*\right)^\theta}\int_{\phi^*}^\infty p_{ij}(\phi)q_{ij}(\phi)\,dG_{\phi_{ij}^*}(\phi) \\ w_i L_i &= \sum_j J_i\frac{A_i}{\left(\phi_{ij}^*\right)^\theta}\frac{L_j\theta}{4\gamma}\mu_j\frac{w_i^2}{(\phi_{ij}^*)^2}\tau_{ij}^2\left(\frac{1}{\theta} - \frac{1}{\theta+2}\right)\end{aligned} \tag{4.26}$$

profits are given by

$$\begin{aligned}\sum_\upsilon\left(\frac{\phi_{ii}^*}{\phi_{i\upsilon}^*}\right)^\theta\int_{\phi_{i\upsilon}^*}^\infty \pi_{i\upsilon}(\phi)dG_{\phi^*}(\phi) &= \theta(\phi_{ii}^*)^\theta\sum_\upsilon \mu_\upsilon\frac{L_\upsilon}{4\gamma}\int_{\phi^*}^\infty\left[\left(\tau_{i\upsilon}\frac{w_i}{\phi_{i\upsilon}^*}\right) - \left(\tau_{i\upsilon}\frac{w_i}{\phi}\right)\right]^2\phi^{-\theta-1}d\phi \\ &= \theta w_i^2\sum_\upsilon\frac{\mu_\upsilon}{\left(\phi_{i\upsilon}^*\right)^2}\frac{L_\upsilon}{4\gamma}\left(\frac{\phi_{ii}^*}{\phi_{i\upsilon}^*}\right)^\theta(\tau_{i\upsilon})^2\left[\frac{1}{\theta} + \frac{1}{\theta+2} - \frac{2}{\theta+1}\right]\end{aligned}$$

Therefore the free entry condition implies

$$\frac{A_i}{(\phi_{ii}^*)^\theta}\sum_v\left(\frac{\phi_{ii}^*}{\phi_{iv}^*}\right)^\theta\frac{L_v}{2\gamma}\mu_v\frac{w_i}{(\phi_{iv}^*)^2}(\tau_{iv})^2\left[\frac{1}{(\theta+1)(\theta+2)}\right]=f^e$$

now replacing the above equation inside (4.26) we obtain

$$w_iL_i=\sum_v J_i\frac{A_i}{(\phi_{iv}^*)^\theta}\frac{L_v\theta}{4\gamma}\mu_v\frac{w_i^2}{(\phi_{iv}^*)^2}\tau_{iv}^2\left(\frac{1}{\theta}-\frac{1}{\theta+2}\right)$$

$$J_i=\frac{L_i}{(\theta+1)\,f^e} \tag{4.27}$$

Thus, the number of entrants is independent of tariffs and trade in general. This is the same as in the CES model as we will show later on when we will introduce the free entry condition.

Discussions of the results

Regarding the distribution of sales we can use expressions (4.25), (4.24) and noticing that from the Pareto distribution we have that

$$\frac{\phi_{ij}^*}{\phi}=(1-\mathrm{Pr})^{1/\theta}$$

where Pr is the percentile of sales of the firm from i in country j. Sales can be written as

$$p_{ij}(\phi)\,q_{ij}(\phi)=\mu_j\left(\frac{w_i}{\phi_{ij}^*}\right)^2\frac{L_j}{4\gamma}\left(1-(1-\mathrm{Pr})^{2/\theta}\right)$$

and when we use the average sales to divide

$$\int_{\phi_{ij}^*}p_{ij}(\phi)\,q_{ij}(\phi)\,dG_{\phi_{ij}^*}(\phi)=\mu_j\frac{L_j}{4\gamma}\tau_{ij}\frac{w_i}{\phi_{ij}^*}\left(\frac{2}{\theta+2}\right)\ .$$

We finally obtain the percentile sales normalized by mean sales

$$\frac{y_{ij}(\mathrm{Pr})}{\bar{y}}=\frac{1-(1-\mathrm{Pr})^{2/\theta}}{\frac{2}{\theta+2}}\ .$$

which have the desirable property that they are independent of market size.

Finally, notice that trade shares can be easily derived and are given by,

$$\lambda_{ij} = \frac{J_i A_i w_i^{-\theta} \tau_{ij}^{-\theta}}{\sum_v J_v A_v w_v^{-\theta} \tau_{vj}^{-\theta}} \tag{4.28}$$

which is identical to what the CES models described above yield.

Using the Linear Demand for quantitative analysis

Notice that the only coefficient affecting the distribution of sales is the curvature of the Pareto, θ. This fact highlights one of the main drawbacks of using the simple linear demand model for quantitative analysis in trade: the same parameter will be the only one appearing in the gravity equations, not allowing enough degrees of freedom to both match the observed distribution of firm sales and the total trade among countries (see the appendix in the original Melitz-Ottaviano paper for the derivation of total trade flows as a function of trade costs and wages and also expression (4.28)). On the contrary, the model with CES demand has one additional degree of freedom since both θ and the elasticity of demand, σ, appear in the expression for the distribution of sales. Thus, a generalization of the CES framework such as the framework in Arkolakis (2010) allows both for a micro-foundation of the deviations from CES demand (so that there is a theoretical underpinning for a non-strictly CES demand structure) and a quantitatively successful framework for predicting trade flows across countries. Of course the model with linear demand may be extended in the future to other forms of demand that incorporate linear demand as a subcase. In addition, the linear demand framework is successful in terms of delivering variable markups, something that a model based on CES is not easy to get. Finally, average sales are regulated by the fixed cost in the endogenous cost model, while in the homothetic demand model they are a function of equilibrium variables. That feature makes it very difficult to finetune the model without fixed costs to get the fact that average sales increase with the size of the market where firms sell with a certain elasticity (see Eaton, Kortum, and Kramarz (2004)).

Chapter 5

Closing the model

In order to close the models we constructed above we need to determine wages for each country in the general equilibrium. The individual goods markets are already assumed to clear since we replaced the consumer demand directly in the sales of the firm for each good.

5.1 Equilibrium in the labor market

Total labor income equals the sum of the labor income originating from sales across each destination:

$$w_i L_i = (1 - \eta_i) \sum_j \lambda_{ij} y_j$$

where η_i is the share of profits out of total revenue for country i, λ_{ij} is the market share of country i in country j and y_j is the total spending of country j. If profits from domestic firms are accrued to domestic labor only, then this condition is equivalent to labor market clearing. In the case of the Pareto distribution of productivities, η_i, turns out to be a constant independent of i.

5.2 The free entry condition

Many papers assuming firm heterogeneity and monopolistic competition, including the original paper of Melitz (2003), assume that additional firms can freely choose to enter in the economy. These papers assume that by paying a fixed entry cost, f^e, in advance firms can enter the market and draw a productivity realization. If a firm gets a productivity draw that is below ϕ^*_{ii}, then it exits immediately without operating.[1] It turns out that in the simple monopolistic competition framework with Pareto distribution of productivities of firms the free entry condition is irrelevant (see Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008)): the model with free entry is simply isomorphic to one with a predetermined number of entrants (essentially the Chaney (2008) version of Melitz (2003)). The only difference between the two models is that all the profits are accrued to labor allocated for the production of the fixed cost of entry.

To show the point of Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008) first note that, in the equilibrium, because of free entry, the expected profits of a firm must be equal to entry costs.[2] Using the free entry condition and a Pareto distribution with shape parameter $\theta > \sigma - 1$, c.d.f. $G(\phi; A_i) = 1 - \frac{A_i}{\phi^\theta}$, and support $[A_i^{1/\theta}, +\infty)$ we have[3]

$$\sum_{\upsilon} \int_{\phi_{i\upsilon}^*} \frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{i\upsilon} w_i}{\phi}\right)^{1-\sigma}}{P_\upsilon^{1-\sigma}\sigma} w_\upsilon L_\upsilon \theta \frac{(\phi_{i\upsilon}^*)^\theta}{\phi^{\theta+1}} \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} d\phi - \sum_{\upsilon} \int_{\phi_{i\upsilon}^*} w_\upsilon f_{i\upsilon} \theta \frac{(\phi_{i\upsilon}^*)^\theta}{\phi^{\theta+1}} \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} d\phi = \frac{w_i f^e}{\frac{A_i}{(\phi_{ii}^*)^\theta}} \Leftrightarrow$$

$$\sum_{\upsilon} w_\upsilon f_{i\upsilon} \frac{\theta}{\theta - \sigma + 1} \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} - \sum_{\upsilon} w_\upsilon \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} f_{i\upsilon} = \frac{w_i f^e}{\frac{A_i}{(\phi_{ii}^*)^\theta}} \Leftrightarrow$$

$$\sum_{\upsilon} w_\upsilon f_{i\upsilon} \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} \left(\frac{\theta}{\theta - \sigma + 1} - 1\right) = \frac{w_i f^e}{\frac{A_i}{(\phi_{ii}^*)^\theta}} \Leftrightarrow$$

$$\sum_{\upsilon} \frac{w_\upsilon}{w_i} f_{i\upsilon} \frac{(\phi_{ii}^*)^\theta}{(\phi_{i\upsilon}^*)^\theta} \frac{\sigma - 1}{\theta - \sigma + 1} = \frac{f^e}{\frac{A_i}{(\phi_{ii}^*)^\theta}}. \tag{5.1}$$

[1] We assume that the parameters of the model are such that the lower productivity threshold $\phi_{ij}^* > \phi_{ii}^* > b_i$, $\forall i, j$, $i \neq j$.

[2] Essentially, we assume that there exists a perfect capital market, which requires firms to pay a fixed entry cost before drawing a productivity realization. Consequently, we multiply the LHS by $1 - G(\phi_{ii}^*, b_i)$, the probability of obtaining the average profit, since firms with profits below this average necessarily exit the market. Alternatively, we could have specified a more general case with intertemporal discounting, δ. In this case the expected profits from entry should equal the discounted entry cost in the equilibrium.

[3] An implication of free entry is that in the equilibrium all the profits are accrued to labor for the production of the entry cost.

5.2.1 Solving for Equilibrium

The equilibrium number of entrants in country i, J_i, is determined by the following labor market clearing condition:

$$\begin{aligned}
J_i\left(\sum_{\upsilon}\int_{\phi^*_{i\upsilon}}\frac{\left(\frac{\sigma}{\sigma-1}\frac{\tau_{i\upsilon}w_i}{\phi}\right)^{-\sigma}}{P_{\upsilon}^{1-\sigma}}\frac{\tau_{i\upsilon}}{\phi}w_{\upsilon}L_{\upsilon}\theta\frac{\left(\phi^*_{i\upsilon}\right)^{\theta}}{\phi^{\theta+1}}\frac{A_i}{\left(\phi^*_{i\upsilon}\right)^{\theta}}d\phi+f^e\right)+\sum_{\upsilon}N_{\upsilon}\frac{\left(\phi^*_{\upsilon\upsilon}\right)^{\theta}}{\left(\phi^*_{\upsilon i}\right)^{\theta}}f_{\upsilon i} &= L_i \Longrightarrow \\
N_i\left(\sum_{\upsilon}(\sigma-1)\frac{w_{\upsilon}}{w_i}f_{i\upsilon}\frac{A_i}{\left(\phi^*_{i\upsilon}\right)^{\theta}}\frac{\theta}{\theta-\sigma+1}+f^e\right)+\sum_{\upsilon}N_{\upsilon}\frac{\left(\phi^*_{\upsilon\upsilon}\right)^{\theta}}{\left(\phi^*_{\upsilon i}\right)^{\theta}}f_{\upsilon i} &= L_i. \qquad (5.2)
\end{aligned}$$

Substituting out equation (5.1), we obtain

$$J_i\left(\theta f^e+f^e\right)+\sum_{\upsilon}N_{\upsilon}\frac{\left(\phi^*_{\upsilon\upsilon}\right)^{\theta}}{\left(\phi^*_{\upsilon i}\right)^{\theta}}f_{\upsilon i}=L_i,$$

which, together with the price index, implies that

$$w_iL_i=\frac{\theta\sigma}{\theta-\sigma+1}\left(\sum_{\upsilon}N_{\upsilon}\frac{\left(\phi^*_{\upsilon\upsilon}\right)^{\theta}}{\left(\phi^*_{\upsilon i}\right)^{\theta}}w_if_{\upsilon i}\right),$$

and so

$$\begin{aligned}
J_if^e\left(\theta+1\right) &= L_i-L_i\frac{\theta-\sigma+1}{\theta\sigma} \Longrightarrow \\
J_i &= \frac{(\sigma-1)}{\theta\sigma f^e}L_i\ , \qquad (5.3)
\end{aligned}$$

which completes the derivation of the number of entrants.[4]

Notice that total export sales from country i to j are[5]

$$X_{ij}=\underbrace{J_i\frac{A_i}{\left(\phi^*_{ij}\right)^{\theta}}}_{\text{firms from } i \text{ in } j}\quad\underbrace{w_jf_{ij}\frac{\sigma\theta}{\theta-\sigma+1}}_{\text{average sales of operating firms}}. \qquad (5.4)$$

Define the fraction of total income of country j spent on goods from country i by λ_{ij}. Using the definition of total sales from i to j and equations (3.8) and (5.3), we have

[4] With a slightly altered proof the same results hold under the assumption that fixed costs are paid in terms of domestic labor.

[5] Average sales of firms from i conditional on operating in j are the same in the model with free entry and the one with a predetermined number of entrants.

$$\lambda_{ij} = \frac{X_{ij}}{\sum_{\upsilon} X_{\upsilon j}},$$

which gives that

$$\lambda_{ij} = \frac{L_i A_i \left(\tau_{ij} w_i\right)^{-\theta} f_{ij}^{1-\theta/(\sigma-1)}}{\sum_{\upsilon} L_{\upsilon} A_{\upsilon} \left(\tau_{\upsilon j} w_{\upsilon}\right)^{-\theta} f_{\upsilon j}^{1-\theta/(\sigma-1)}}. \tag{5.5}$$

All the key expressions derived in this context are the same as in Chaney (2008). In fact, the share of spending for fixed costs of entry is $(\sigma - 1) / (\theta\sigma)$. This is exactly the same as the profits share out of total income that we would get in the Chaney (2008) model if we assumed that domestic consumers own equal shares of domestic firms only.

Chapter 6

Some facts on disaggregated trade flows

6.1 Firm heterogeneity

- Firms appear to have huge differences in sales and measured productivities (Bernard, Eaton, Jensen, and Kortum (2003)–BEJK–)

- In fact, only a tiny fraction of firms export to at least one market and an even smaller fraction export to multiple destinations (only 16% of French firms sells to at least one destination other than France, 3.3% sell to at least 10 destinations and a mere .05% to 100 or more! See figure 10.1 drawn from Eaton, Kortum, and Kramarz (2010)). Moreover, exporters typically earn a small fraction of their total revenues from their exporting sales (BEJK).

- Exporters have a size advantage over non-exporters. In fact, exporters that sell to many countries sell more in total and in the domestic market than exporters that sell to few destinations or firms that sell only domestically (Eaton, Kortum, and Kramarz (2004), Eaton, Kortum, and Kramarz (2010) –EKK–). This fact is illustrated in Figure 10.1 given that the slope of the line in the plot is far less than 1 (around .35): including firms less successful in exporting means less than linear increase in total sales in France.

- The number of exporters entering a market, their average size and the total number of products sold increases with the size of the market, with an elasticity that is roughly constant. (Klenow and Rodríguez-Clare (1997), Hummels and Klenow (2005) EKK, Arkolakis and Muendler (2010) –AM–). The elasticity of entry for French exporters can be seen in Figure 10.2.

- The distribution of sales of firms in a country, conditional on selling to that country, is robust across countries. It features a Pareto tail when looking at the large firms, and large deviations from Pareto when looking at the small firms: there are too many "too" small guys selling to each destination. Figure 10.4 illustrates the distribution of size of firms in different destinations, grouping destinations in three categories depending on the overall sales of French firms there.

- Firms that sell more goods sell more per good (Bernard, Redding, and Schott (2010) and AM). This feature is true across destinations as Figure 10.3 indicates (AM). In fact the distribution of goods is also robust across destinations (AM).

- At a more disaggregated level, AM document that the most successful products of a firm (the metric being the rank of the product in the most popular market) are systematically more likely to be sold in other markets and conditional on being sold are systematically more likely to sell more than other less successful products. Table 10.5 summarizes the findings of AM.

The above facts suggests the existence of important trade barriers, that only relatively productive firms can overcome. In addition, the facts suggest that the costs of market penetration have similar characteristics across markets and that the same driving forces govern the behavior of firms.

6.2 Trade liberalization

- There is a substantial response of trade flows to price changes induced by changes in tariffs during trade liberalizations (see for example Romalis (2007)). This response is much larger than the response of trade flows to price changes over the business cycle frequency –2-3 years–. The elasticity to changes in tariffs has been estimated in the range of 8-10 while the one for short run adjustments around 1.5 to 2 (See Ruhl (2005) for a review).

- A large number of new firms engage in trade after trade liberalization (see discussion in Arkolakis (2010)). Also a large number of new products are traded after a trade liberalization (Kehoe and Ruhl (2003), Arkolakis (2010)). New goods typically come with very small sales (Arkolakis (2010)).

- Goods with little trade before a liberalization have higher growth rates of their trade flows after trade liberalization. (see figure 10.6 and Arkolakis (2010)).

- Trade liberalization forces the least productive firms to exit the market. (Bernard and Jensen (1999), Pavcnik (2002), Bernard, Jensen, and Schott (2003))

The above facts on trade liberalization suggest that firms respond to short run (e.g. exchange rate movements) changes differently than they respond to permanent changes (e.g tariff reductions). Their response to permanent changes depends also on their initial size. Whatever the explanation for this behavior, ultimately it should also be consistent with the previous facts on exporting behavior of heterogeneous firms.

6.3 Trade dynamics

- A large number of firms do not export continuously to a given destination (more than 40%). In addition a large number of new firms start exporting every year at a given destination. These new firms and the firms that die typically have tiny sales (Eaton, Eslava, Kugler, and Tybout (2008)).
- The growth rate of small exporters to a given destination is higher than the growth rate of exporters with large sales (Eaton, Eslava, Kugler, and Tybout (2008)).
- (Expected to be true: see Arkolakis (2009) and the facts presented by Sutton (2002)) The variance of the growth rate of small exporters to a given destination is larger than the variance of growth of large exporters.

Chapter 7

Estimating Models of Trade

Anderson and Van Wincoop (2003) developed a framework that was delivering structural relationships for trade among countries (or regions) based on the model analyzed in section (**??**). This model was useful to identify parameters related to the cost of distance and the border. As we showed in the previous chapters, and as elaborated in Anderson and Van Wincoop (2004) and Arkolakis, Costinot, and Rodríguez-Clare (2010), that basic setup has very similar properties in terms of bilateral aggregate trade and welfare to richer models of trade and heterogeneity. New, heterogeneous-firm models generate a number of predictions at the firm-level which can also be used to obtain key parameters of the model. In this chapter we will discuss the identification of key parameters of these models determining aggregated but also disaggregated trade. Alternative ways of estimating gravity equations are summarized in a survey by Anderson and Van Wincoop (2004)

7.0.1 The Anderson and van Wincoop procedure

Anderson and Van Wincoop (2003) develop a general equilibrium methodology to obtain estimates of the costs of trade in the model as a function of distance proxies.

Using the equation (2.22) we have

$$\frac{y_i}{y^W} = p_{ii}^{1-\sigma} \sum_j \alpha_{ij} \left(\frac{\tau_{ij}}{P_j} \right)^{1-\sigma} \frac{y_j}{y^W} \implies$$

and using

$$X_{ij} = p_{ij} x_{ij} = \alpha_{ij} p_{ii}^{1-\sigma} \left(\frac{\tau_{ij}}{P_j} \right)^{1-\sigma} y_j$$

we then have

$$X_{ij} = \frac{y_i y_j}{y^W}\alpha_{ij}\left(\frac{\tau_{ij}}{\left(\sum_{\upsilon}\alpha_{i\upsilon}\left(\frac{\tau_{i\upsilon}}{P_\upsilon}\right)^{1-\sigma}\frac{y_\upsilon}{y^W}\right)^{1/(1-\sigma)}P_j}\right)^{1-\sigma}$$

while by summing up over $\acute{\imath}$'s we can compute the price index,

$$\begin{aligned}\sum_{\upsilon'} X_{\upsilon' j} &= \sum_{\upsilon'}\frac{y_{\upsilon'} y_j}{y^W}\alpha_{\upsilon' j}\left(\frac{\tau_{\upsilon' j}}{\left(\sum_{\upsilon}\alpha_{i\upsilon}\left(\frac{\tau_{i\upsilon}}{P_\upsilon}\right)^{1-\sigma}\frac{y_\upsilon}{y^W}\right)^{1/(1-\sigma)}P_j}\right)^{1-\sigma} \implies \\ P_j &= \left[\sum_{\upsilon'}\frac{\alpha_{\upsilon' j}\left(\tau_{\upsilon' j}\right)^{1-\sigma}\frac{y_{\upsilon'}}{y^W}}{\sum_{\upsilon}\alpha_{i\upsilon}\left(\frac{\tau_{i\upsilon}}{P_\upsilon}\right)^{1-\sigma}\frac{y_\upsilon}{y^W}}\right]^{1/(1-\sigma)}\end{aligned}$$

where we used the fact that balanced trade implies $y_j = \sum_{\upsilon} X_{\upsilon j}$.

If we define

$$\Xi_\upsilon^{1-\sigma} = \sum_j \alpha_{\upsilon j}\left(\frac{\tau_{\upsilon j}}{P_j}\right)^{1-\sigma}\frac{y_j}{y^W}$$

(where y^W could represent the total income for all countries) then

$$P_j = \left[\sum_{\upsilon=1}\alpha_{\upsilon j}\frac{\left(\tau_{\upsilon j}\right)^{1-\sigma}}{\Xi_\upsilon^{1-\sigma}}\frac{y_\upsilon}{y^W}\right]^{1/(1-\sigma)}$$

under symmetric trade barriers, $\tau_{ij} = \tau_{ji}$, $\alpha_{ij} = \alpha_{ij}$, from the last equations it turns out that $\Xi_j = P_j$, so that

$$X_{ij} = \frac{y_i y_j}{y^W}\left(\frac{\tau_{ij}}{P_i P_j}\right)^{1-\sigma}$$

Anderson and Van Wincoop (2003) estimate the stochastic form of the equation

$$\ln\left(\frac{X_{ij}}{y_i y_j}\right) = k + a_1 \ln\tilde{\tau}_{ij} - a_2 D_{ij} - \ln P_i^{1-\sigma} - \ln P_j^{1-\sigma} + \varepsilon_{ij} \tag{7.1}$$

where D_{ij} is a dummy variable related to borders and $a_1 = (1-\sigma)\,\tilde{a}_1, \tilde{\tau}_{ij} = \tau_{ij}^{(1-\sigma)}$. The innovation of Anderson and vanWincoop was to perform this estimation expressing P_i, P_j as an explicit function of the model parameters, σ and $\tilde{a}_1$, a_2 as well as (observable) multilateral resistance terms. The authors cannot separately estimate σ since its effect on distance cannot be separately identified from $\tilde{a}_1$ with their methodology. Nevertheless, their method delivers much more sensible effects for the coefficient on borders. Estimation without considering P_i, P_j as a function of the parameters to be estimated overstates the effect of distance of trade. The intuition is that smaller countries are likely to have higher price indices since they impose trade barriers to larger countries.

7.0.2 The Head and Ries procedure

The Head and Ries (2001) procedure is another method of estimating the parameters on distance that dispenses of the need of computing the equilibrium of the model. If one looks at the relationship

$$\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}} = (\tau_{ij}\tau_{ji})^{1-\sigma} \tag{7.2}$$

then this relationship is an adjustment that takes care of the critique of Anderson and vanWincoop of neglecting the impact of parameters on general equilibrium variables. Parameters can be estimated through a linear regression.

7.0.3 The Eaton and Kortum procedure

Another approach that gives an unbiased estimate of parameter a_1 is to replace the inward and outward multilateral resistance indices and production variables, $y_i - \ln P_i^{1-\sigma}$ and $y_j - \ln P_j^{1-\sigma}$, with inward and outward region specific dummies. This approach is adopted by a series of papers (e.g. Eaton and Kortum (2002)).

Eaton and Kortum also provide a variety of different methods to estimate the parameter that governs the elasticity of trade. In the Eaton and Kortum (2002) model this is the parameter of the Frechet distribution that governs productivity heterogeneity, θ (whereas in the Armington model it is $\sigma - 1$). Using a relationship similar to (3.3) they can derive a relationship of the form

$$\ln \frac{X'_{ij}}{X'_{jj}} = -\theta \ln \tau_{ij} + S_i - S_j \tag{7.3}$$

where $S_i = A_i / (1-\iota) - \theta \ln w_i$, S_j are destination fixed effects and $X'_{ij} = X_{ij} - [(1-\iota)/\iota] \ln (X_{ii}/X_i)$ with $1-\iota$ the share of intermediates in manufacturing production.[1] They also use proxies for distance,

[1] Eaton and Kortum (2002) estimate

$$\ln \tau_{ij} = f + m_j + \delta_{ij}$$

where f includes distance and other geographic barrier fixed effects, m_j a destination fixed effect and δ_{ij} an error term.

border effects etc. for the first term in order to estimate $\theta \ln \tau_{ij}$ but while they can distinguish the effect of the components (proxies) of that term they cannot distinguish that effect from the effect of the multiplicative term θ. To address that problem and using their estimates from the previous stage for S_i they estimate

$$S_i = \frac{1}{1-\iota} \ln A_i - \theta \ln w_i$$

using technology and education fundamentals to be the proxies for A_i and data for wages adjusted for education. Using a 2SLS estimation they get $\theta = 3.6$.

The second alternative is to estimate the bilateral trade equation (7.3) using their proxy of $\ln(P_i d_{ij}/P_j)$, instead of the geography terms along with source and destination effects. The proxy for d_{ij} is constructed by looking at the (second) highest ratio of prices of homogeneous products across different destinations and the proxy for P_i/P_j as the average of these price ratios. Using a 2SLS and geography variables to instrument for the proxy of $\ln(P_i d_{ij}/P_j)$ their estimate for this procedure is a $\theta = 12.86$.

The favorite estimate of the Eaton and Kortum (2002) is the derivation of the θ using the trade shares equation in terms of prices

$$\frac{X_{ij}/X_j}{X_{ii}/X_i} = \left(\frac{P_i d_{ij}}{P_j}\right)^{-\theta} .$$

With simple method of moments, $-\theta$ is simply the ratio of the mean of $\ln \frac{X_{ij}/X_j}{X_{ii}/X_i}$ and their proxies of $\ln \frac{P_i d_{ij}}{P_j}$. Simonovska and Waugh (2009) propose an alternative estimation of the Eaton and Kortum (2002) by using the above equation and a simulated method of moments approach adapted from Eaton, Kortum, and Kramarz (2010).

7.0.4 Calibration of a firm-level model of trade

Parameters Determining Firm Sales Advantage We now turn to determine the parameters β and $\tilde{\theta} = \theta/(\sigma-1)$ of the Arkolakis (2010) by looking at the advantage of prolific exporters uncovered by equations the following structural relationships of the model, normalized average sales of firms from

To capture potential reciprocity in geographic barriers, they assume that the error term δ_{ij} consists of two components: $\delta_{ij} = \delta^1_{ij} + \delta^2_{ij}$. The country-pair specific component δ^2_{ij} (with variance σ^2_2) affects two-way trade, so that $\delta^2_{ij} = \delta^2_{ji}$, while δ^1_{ij} (with variance σ^2_1) affects one-way trade. This error structure implies that the variance-covariance matrix of δ diagonal elements $E(\delta_{ij}\delta_{ij}) = \sigma^1_1 + \sigma^2_2$ and certain nonzero off-diagonal elements $E(\delta_{ij}\delta_{ji}) = \sigma^2_2$.

France, F, conditional on selling to market j,

$$\frac{\bar{X}_{FF|j}}{\bar{X}_{FF}} = \frac{\frac{\left(\frac{M_{Fj}}{M_{FF}}\right)^{-1/\tilde{\theta}}}{1-1/\tilde{\theta}} - \frac{\left(\frac{M_{Fj}}{M_{FF}}\right)^{-1/\left(\tilde{\beta}\tilde{\theta}\right)}}{1-1/\left(\tilde{\theta}\tilde{\beta}\right)}}{\frac{1}{1-1/\tilde{\theta}} - \frac{1}{1-1/\left(\tilde{\theta}\tilde{\beta}\right)}} \tag{7.4}$$

and exporting intensity of firms in percentile $\Pr_{Fj}$ in market j,

$$\frac{t_{Fj}\left(\Pr_{Fj}\right)}{\bar{X}_{Fj}} / \frac{t_{FF}\left(\Pr_{FF}\right)}{\bar{X}_{FF}} = \frac{1-\left(1-\Pr_{Fj}\right)^{1/\left(\tilde{\theta}\beta\right)}}{\left(\frac{M_{Fj}}{M_{FF}}\right)^{-1/\tilde{\theta}} - \left(1-\Pr_{Fj}\right)^{1/\left(\tilde{\theta}\beta\right)}\left(\frac{M_{Fj}}{M_{FF}}\right)^{-1/\left(\tilde{\theta}\tilde{\beta}\right)}} \tag{7.5}$$

Notice that parameters θ and σ affect equations (7.4) and (7.5) only insofar they affect $\tilde{\theta}$. Higher θ implies less heterogeneity in firm productivities (and thus in firm sales), whereas higher σ translates the same heterogeneity in productivities to larger dispersion in sales.

For the calibration, Arkolakis (2010) uses a simple method of moments estimate. In particular, β and $\tilde{\theta}$ are picked so that the mean of the left-hand side is equal to the mean of the right-hand side for both equation (7.4) and equation (7.5) evaluated at the median percentile in each market j. The solution delivers $\beta = .915$ and $\tilde{\theta} = 1.65$. Notice that using equation (7.4), a method of moments estimate for the fixed model with $\beta = 0$ gives a $\tilde{\theta} = 1.49$.

To complete the calibration of the model, we need to assign magnitudes to σ and θ. Broda and Weinstein (2006) estimate the elasticity of substitution for disaggregated categories. The average and median elasticity for SITC 5-digit goods is 7.5 and 2.8, respectively (see their table IV). A value of $\sigma = 6$ falls in the range of estimates of Broda and Weinstein (2006) and yields a markup of around 1.2, which is consistent with those values reported in the data (see Martins, Scarpetta, and Pilat (1996)). In addition, $\tilde{\theta} = 1.65$ and $\sigma = 6$ imply that the marketing costs to GDP ratio in the model is around 6.6% within the range of marketing costs to GDP ratios reported in the data. Finally, this parameterization implies that $\theta = 8.25$ for the endogenous cost model which is very close to the main estimate of Eaton and Kortum (2002) (8.28) and within the range of estimates of Romalis (2007) $(6.2 - 10.9)$ and the ones reported in the review of Anderson and Van Wincoop (2004) $(5 - 10)$. Since the model retains the aggregate predictions of the Melitz-Chaney framework if θ is the same I will calibrate the two models to have $\theta = 8.25$. For the fixed cost model, given the calibrated $\tilde{\theta} = 1.49$, it implies a $\sigma = 6.57$.

Calibration for a multi-product firms model

Parametrizing a multi-products firm model requires to dig deeper into establishing predictions at the within-firm level. We will now briefly go over the calibration procedure of Arkolakis and Muendler (2010) for their model described in 4.3. Guided by various log-linear relationships observed in their data (see, for example Figure 10.3) they specify the following functional relationships

$$\begin{aligned} f_{ij}(g) &= f_{ij} \cdot g^{\delta} \quad \text{for } \delta \in (-\infty, +\infty), \\ h(g) &= g^{\alpha} \quad \quad \text{for } \alpha \in [0, +\infty). \end{aligned} \tag{7.6}$$

This specification gives product level sales for the g-th ranked product of the firm as

$$p_{ijg}(\phi)x_{ijg}(\phi) = \sigma f_{ij}(1)\, G_{ij}(\phi)^{\delta+\alpha(\sigma-1)} \left(\frac{\phi}{\phi_{ij}^{*,G}}\right)^{\sigma-1} g^{-\alpha(\sigma-1)} \, .$$

Using the logarithm of this structural relationship, a regression of the sales of the firm on a constant, a firm fixed effect and the number of the products of the firm obtains $\alpha\,(\sigma-1) = 2.66$ and $\delta \simeq -1.38$.

7.0.5 Estimation of a firm-level model

We present here the framework of Eaton, Kortum, and Kramarz (2010) that is the first work that estimates a multi-country firm-level model of trade making use of the firm-level data. The idea is to identify a set of micro facts on exporters and to develop a consistent modeling framework that would explain these micro observations using model relationships. Then the authors estimate the fundamental parameters of the model using the micro data. In this respect the paper of Eaton, Kortum, and Kramarz (2010) is parallel to the Eaton and Kortum (2002) framework.

The model

Sales of the firm are given by

$$t_{ij}\left(\omega\right) = a_j\left(\omega\right) n_{ij} \left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} ,$$

derived by asymmetric CES utility function with preference for each good affected by $a_j\left(\omega\right)$ (these could be interpreted as Armington type bias in a particular good). The term $a_j\left(\omega\right)$ reflects an exogenous demand shock specific to good ω in market j. The term P_j is the CES price index that will be analyzed in a moment.

Producers are heterogeneous and the unit cost for a producer from i in producing a good and shipping it to country j is

$$c_{ij}(\omega) = \frac{w_i \tau_{ij}}{z_i(\omega)}$$

where τ_{ij} is an iceberg cost. The measure of potential producers who can produce their good with efficiency at least z is

$$\mu_i(z) = A_i z^{-\theta} \ .$$

Given the unit cost this implies that the measure of goods that can be delivered to country j from anywhere in the world at unit cost c or less in j is

$$\begin{aligned} \mu_j(c) &= \sum_{\upsilon=1}^{N} \mu_{\upsilon j}(c) \\ &= \sum_{\upsilon=1}^{N} A_\upsilon (w_\upsilon \tau_{\upsilon j})^{-\theta} c^\theta \\ &\equiv \sum_{\upsilon=1}^{N} \Phi_{\upsilon j} c^\theta \\ &\equiv \Phi_j c^\theta \end{aligned}$$

Conditional on selling in a market the producer makes the profit from producer from i in j

$$\pi_{ij}(\omega) = \max_{p,n} \left(1 - \frac{c_j(\omega)}{p}\right) a_j(\omega) n \left(\frac{p}{P_j}\right)^{1-\sigma} X_j - \varepsilon_j(\omega) f_j \frac{1-(1-n)^{1-\beta}}{1-\beta} \ ,$$

where $c_j(\omega)$ is the unit production cost, $\varepsilon_j(\omega)$ an entry cost and $f_j > 0$. Producer charges a constant markup

$$p_{ij} = \bar{m} c_j(\omega) \ , \ \bar{m} = \sigma / (\sigma - 1)$$

Define

$$\eta_j(\omega) = \frac{a_j(\omega)}{\varepsilon_j(\omega)} \ .$$

Thus, we can describe seller's behavior in market j in terms of its cost draws $c_j(\omega) = c$, the demand shock $a_j(\omega) = a$, and the redefined entry shock $\eta_j(\omega) = \eta$. It can be shown using the results of section

4.2 combined with this framework that a firm will enter a market j iff its cost draw $c \geq \bar{c}_j(\omega)$

$$\bar{c}_j(\eta) = \left(\eta \frac{X_j}{\sigma f_j}\right)^{1/(\sigma-1)} \frac{P_j}{\bar{m}} \ . \tag{7.7}$$

Notice that the entry threshold depends on a only through η. For the firms with $c \geq \bar{c}_j(\omega)$ the fraction of buyers reached in a market will be (for $\beta > 0$)

$$n_{ij}(\eta, c) = 1 - \left(\frac{c}{\bar{c}_j(\eta)}\right)^{\frac{(\sigma-1)}{\beta}}$$

You can rewrite sales as

$$t_{ij}(\eta) = \varepsilon_j \left[1 - \left(\frac{c}{\bar{c}_j(\eta)}\right)^{\frac{(\sigma-1)}{\beta}}\right] \left(\frac{c}{\bar{c}_j(\eta)}\right)^{-(\sigma-1)} \sigma f_j$$

Notice that even though Eaton, Kortum, and Kramarz (2010) add these 3 levels of firm heterogeneity they can determine easily all the aggregate variables of the model. First, the price index is given by the following integration

$$\begin{aligned} P_j &= \left[\int\int \left(\int_0^{\bar{c}_j(\eta)} \alpha n_{ij}(\eta, c)\, \bar{m}^{1-\sigma} c^{1-\sigma} d\mu_j(c)\right) g(\alpha, \eta)\, d\alpha d\eta\right]^{-1/(\sigma-1)} \\ &= \bar{m}\left[\Phi_j \left(\frac{\theta}{\theta - \sigma + 1} - \frac{\theta}{\theta + (\sigma - 1)\frac{\beta - 1}{\beta}}\right) \int\int \alpha \bar{c}_j(\eta)^{\theta-(\sigma-1)} g(\alpha, \eta)\, d\alpha d\eta\right] \end{aligned}$$

which substituting for the entry hurdle (7.7) gives

$$P_j = \bar{m}\left(\kappa_1 \Phi_j\right)^{-1/\theta} \left(\frac{X_j}{\sigma f_j}\right)^{(1/\theta)-1/(\sigma-1)} ,$$

where

$$\kappa_1 = \left[\frac{\theta}{\theta - \sigma + 1} - \frac{\theta}{\theta + (\sigma - 1)\frac{\beta-1}{\beta}}\right] \int\int \alpha \eta^{\frac{\theta-(\sigma-1)}{\sigma-1}} g(\alpha, \eta)\, d\alpha d\eta \ ,$$

and $g(\alpha, \eta)$ is the joint density of the realizations of producer-specific costs. Second, from the model we can get a series of relationships directly related to observables. The measure of entrants in market

j is

$$\begin{aligned} M_j &= \int \bar{c}_j(\eta) g \\ &= \frac{\kappa_2}{\kappa_1} \frac{X_j}{\sigma f_j} \end{aligned}$$

where

$$\kappa_2 = \int \eta^{\theta/(\sigma-1)} g_2(\eta) \, d\eta$$

Number of firms selling from i to j

$$M_{ij} = \frac{\kappa_1}{\kappa_2} \frac{\lambda_{ij} X_j}{\sigma f_j},$$

where

$$\lambda_{ij} = \frac{\Phi_{ij}}{\Phi_j}$$

being the observed market share, which exactly the same as in the monopolistic competition model with productivity as the only source of variation. Finally, average sales are given by

$$\bar{X}_{ij} = \frac{\kappa_2}{\kappa_1} \sigma f_j$$

It also turns out that the distribution of sales in a market, and hence mean sales, is invariant to the location of the supplier.

Notice that all these relationships are derived independently of the actual distribution of demand and entry shocks. This separability allows for a very simple and generic solution of the model that retains the forces of the previous structure while allowing for additional levels of heterogeneity that brings the model closer to the data.

Estimation, simulated method of moments

There are particular steps in the estimation procedure proposed by the authors. They match 4 sets of moments (each set of moments is denoted as m)

a) The distribution of exporting sales in individual destinations by different percentiles in these destinations,

b) the sales of french firms in France of firms that sells in individual destinations by different percentiles in France,

c) normalized export intensity of firms by market by different percentiles in France,

d) the fraction of firms selling to each possible combination of the top seven exporting destinations.

These 4 set of moments contribute to the objective function

$$Q(m) = \sum_{k=1}^{\#m} w^k(m) \left(\hat{p}^k(m) - p^k(m)\right)^2$$

where $\hat{p}^k(m)$ are the simulated observations for each moment and $p^k(m)$ the ones related to the data. The authors use the following weights

$$w^k(m) = N/p^k(m)$$

where N is the number of firms in the data sample. With these weights each $Q(m)$ is a chi-square statistic with degrees of freedom given by the number of moments to be matched ($\#m$). Chi square is the limiting distribution of $Q(m)$ (for N large) under the null that the sampling error is the only source of error and, thus, observed sales follow a multinomial distribution with the actual probabilities as parameters. Hence, the means of the $Q(m)$'s equal their degrees of freedom and their variances twice their means.

The paper has a set of important contributions

- It identifies a set of statistics in the data that will be a rigorous test for all future trade theories.
- It develops a model that is consistent with these facts and can account for different levels of heterogeneity. In particular, it shows how the model can motivate research to interpret and "read" the data in a way consistent to the model.
- It develops an internally consistent methodology for estimating firm-level models.

Chapter 8

Trade Liberalization

8.1 Trade Liberalization and Firm Heterogeneity

There is a common perception that the gains from trade are larger than what quantitative general-equilibrium models of trade can explain. A recurring goal in the trade literature has been to find new channels through which such models can generate larger gains. Recently, authors such as Melitz (2003) have postulated additional gains from the "selection" effect compared to the extensive margin effect already postulated by Romer (1994). Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008) show that some of the key quantitative frameworks in international trade deliver (Krugman, Eaton and Kortum, the Chaney version of Melitz and Arkolakis) welfare expressions that are closely comparable. Arkolakis, Costinot, and Rodríguez-Clare (2010) show that for a wide class of perfect and monopolistic competition models of trade welfare gains from trade can be written as a function of two sufficient statistics: the share of spending that goes to domestic goods, λ_{jj}, and the elasticity of trade parameter, ε. Their result imply that changes in welfare can be written as

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon} \tag{8.1}$$

8.1.1 Trade Liberalization and Welfare gains (Arkolakis Costinot Rodriguez-Clare)

To understand the intuition for the main result of Arkolakis, Costinot, and Rodríguez-Clare (2010) we start the analysis from the simplest setup, the Armington model. The model is essentially identical to the model presented in section 2.2 assuming that the endowment is labor so that the price of the endowment is wage and that there are no preference shocks so that $\alpha_{ij} = 1$, $\forall i, j$. Changes in real income, $W_j = w_j/P_j$, in that model are given by

$$d\ln W_j = -\sum_{i=1}^{n} \lambda_{ij} \left(d\ln w_i + d\ln \tau_{ij} \right), \tag{8.2}$$

where we choose wage of country j as the numeraire. Changes in relative imports are such that

$$d\ln \lambda_{ij} - d\ln \lambda_{jj} = (1-\sigma) \left(d\ln w_i + d\ln \tau_{ij} \right). \tag{8.3}$$

Thus, both welfare and relative trade shares depend on terms of trade alone. Combining Equations (8.2) and (8.3), we obtain

$$d\ln W_j = \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d\ln \lambda_{jj} - d\ln \lambda_{ij} \right)}{1-\sigma} = \frac{d\ln \lambda_{jj}}{1-\sigma},$$

where the second equality derives from the fact that $\sum_{i=1}^{n} \lambda_{ij} = 1$. Integrating the previous expression between the initial equilibrium (before the shock) and the new equilibrium (after the shock), we finally get

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/(1-\sigma)}, \tag{8.4}$$

Three are the macro-level restrictions that are used to derive the result in a general perfect competition or monopilistically competitive setup. The first restriction is that the value of imports of goods must be equal to the value of exports of goods:

R1 *For any country j, $\sum_{i=1}^{n} X_{ij} = \sum_{i=1}^{n} X_{ji}$.*

In general,total income of the representative agent in country j may also depend on the wages paid to foreign workers by country j's firms as well as the wages paid by foreign firms to country j's workers. Thus, total expenditure in country j, $X_j \equiv \sum_{i=1}^{n} X_{ij}$, could be different from country j's total revenues, $Y_j \equiv \sum_{i=1}^{n} X_{ji}$. R1 rules out this possibility.

Aggregate profits are a constant share of revenues. Let Π_j denote country j's aggregate profits gross of entry costs (if any). The second macro-level restriction states that Π_j must be a constant share of country j's total revenues:

R2 *For any country j, Π_j/Y_j is constant.*

Under perfect competition, R2 trivially holds since aggregate profits are equal to zero. Under monopolistic competition with homogeneous firms, R2 also necessarily holds because of Dixit-Stiglitz preferences; see Krugman (1980). In more general environments, however, R2 is a non-trivial restriction.

The import demand system is CES. The last macro-level restriction is concerned with the partial equilibrium effects of variable trade costs on aggregate trade flows. Define the *import demand system* as the mapping from $(\boldsymbol{w}, \boldsymbol{J}, \boldsymbol{\tau})$ into $\boldsymbol{X} \equiv \{X_{ij}\}$, where $\boldsymbol{w} \equiv \{w_i\}$ is the vector of wages, $\boldsymbol{J} \equiv \{J_i\}$ is the vector of measures of goods that can be produced in each country, and $\boldsymbol{\tau} \equiv \{\tau_{ij}\}$ is the matrix of variable trade costs. This mapping is determined by utility and profit maximization given preferences, technological constraints, and market structure. It excludes, however, labor market clearing conditions as well as free entry conditions (if any) which determine the equilibrium values of $\boldsymbol{w}$ and $\boldsymbol{N}$. The third macro-level restriction imposes restrictions on the partial elasticities, $\varepsilon_j^{ii'} \equiv \partial \ln (X_{ij}/X_{jj}) / \partial \ln \tau_{i'j}$, of that system:

R3 *The import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$, and zero otherwise.*

Each elasticity $\varepsilon_j^{ii'}$ captures the percentage change in the relative imports from country i in country j associated with a change in the variable trade costs between country i' and j holding wages and the measure of goods that can be produced in each country fixed.

We will obtain the result for the case of monopolistic competition. Denoting α_{ij}^* the cutoff cost determining the entry of firms from country i in country j, i.e. $\pi_{ij}(\omega) > 0$ if and only if $\alpha_{ij}(\omega) < \alpha_{ij}^*$, the set of goods Ω_{ij} that country j buys from country i can be written as

$$\Omega_{ij} = \left\{ \omega \in \Omega | \alpha_{ij}(\omega) < \alpha_{ij}^* \equiv \sigma^{\frac{\sigma}{1-\sigma}} (\sigma - 1) \left(\frac{P_j}{c_{ij}} \right) \left(\frac{f_{ij} w_i^{\mu} w_j^{1-\mu}}{X_j} \right)^{\frac{1}{1-\sigma}} \right\}. \tag{8.5}$$

where $c_{ij} \equiv w_i \tau_{ij}$ and where we assume that the production of fixed costs f_{ij} is using a mix of domestic and foreign labor with respective shares μ, $1 - \mu$.

Combining this observation with Dixit-Stiglitz preferences, we get

$$X_{ij} = \frac{J_i \int_0^{\alpha_{ij}^*} [c_{ij}\alpha_i]^{1-\sigma} g_i(\alpha_i) \, d\alpha_i}{\sum_{i'=1}^n J_{i'} \int_0^{\alpha_{i'j}^*} [c_{i'j}\alpha_{i'}]^{1-\sigma} g_{i'}(\alpha_{i'}) \, d\alpha_{i'}} X_j, \tag{8.6}$$

where the density $g_i(\alpha_i)$ of goods with unit labor requirements α_i in Ω_{ij} is simply given by the marginal density of g. Noting that $\partial \ln \alpha^*_{ij} \big/ \partial \ln \tau_{ij} = \partial \ln \alpha^*_{jj} \big/ \partial \ln \tau_{ij} - 1$ and $\partial \ln \alpha^*_{ij} \big/ \partial \ln \tau_{i'j} = \partial \ln \alpha^*_{jj} \big/ \partial \ln \tau_{i'j}$ if $i' \neq i$, the import demand system now satisfies

$$\frac{\partial \ln (X_{ij}/X_{jj})}{\partial \ln \tau_{i'j}} = \varepsilon_j^{ii'} = \begin{cases} 1 - \sigma - \gamma_{ij} + \left(\gamma_{ij} - \gamma_{jj}\right)\left(\frac{\partial \ln \alpha^*_{jj}}{\partial \ln \tau_{ij}}\right) & \text{for} \quad i' = i \\ \left(\gamma_{ij} - \gamma_{jj}\right)\left(\frac{\partial \ln \alpha^*_{jj}}{\partial \ln \tau_{i'j}}\right) & \text{for} \quad i' \neq i \end{cases}, \tag{8.7}$$

where $\gamma_{ij} \equiv d \ln \int_0^{\alpha^*_{ij}} \alpha^{1-\sigma} g_i(\alpha)\, d\alpha \big/ d \ln \alpha^*_{ij}$.

Under free entry, labor market clearing and the representative agent's budget constraint still imply $d \ln Y_j = d \ln w_j = 0$, where the second equality derives from the choice of labor in country j as our numeraire. Changes in the consumer price index no longer satisfy $d \ln P_j = \sum_{i=1}^n \lambda_{ij} d \ln c_{ij}$, as in the case of Armington preferences (or perfect competition in general) reflecting the fact that, under monopolistic competition, consumers are not necessarily indifferent between the "cutoff" goods produced by different countries. Formally, small changes in real income are now given by

$$d \ln W_j = -d \ln P_j = -\sum_{i=1}^n \lambda_{ij} \left(d \ln c_{ij} + \frac{d \ln J_i + \gamma_{ij} d \ln \alpha^*_{ij}}{1 - \sigma} \right).$$

Using the definition of the cutoff α^*_{ij} this equation can be rearranged as

$$d \ln W_j = -\sum_{i=1}^n \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[\left(1 - \sigma - \gamma_{ij}\right) d \ln c_{ij} + \frac{\gamma_{ij}}{1 - \sigma} \left(d \ln \xi_{ij} + \mu d \ln w_i \right) + d \ln J_i \right],$$

where $\gamma_j \equiv \sum_i \lambda_{ij} \gamma_{ij}$. Similarly, changes in trade flows are now given by

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = \left(1 - \sigma - \gamma_{ij}\right) d \ln c_{ij} + \frac{\gamma_{ij}}{1 - \sigma} \left(d \ln \xi_{ij} + \mu d \ln w_i \right) + \left(\gamma_{ij} - \gamma_{jj}\right) d \ln \alpha^*_{jj} + d \ln J_i - d \ln J_j,$$

where we have used the fact that $d \ln \alpha^*_{ij} = d \ln \alpha^*_{jj} - d \ln c_{ij} + \left(d \ln \xi_{ij} + \mu d \ln w_i \right) / (1 - \sigma)$. Combining the two previous expressions reveals that

$$d \ln W_j = -\sum_{i=1}^n \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} - \left(\gamma_{ij} - \gamma_{jj}\right) d \ln \alpha^*_{jj} + d \ln J_j \right].$$

Since R3 implies $\gamma_{ij} = \gamma_{jj}$ and $1 - \sigma - \gamma_j = \varepsilon$ for all i, j, we obtain $d \ln W_j = \left(d \ln \lambda_{jj} - d \ln J_j \right) / \varepsilon$, in the exact same way as in the previous example. To conclude, we simply note that free entry implies

$\Pi_j = J_j F_j$. Since Π_j is proportional to Y_j by R1 and R2, we therefore have $d \ln J_j = d \ln Y_j = 0$. Combining the two previous observations and integrating, we finally obtain expression (8.1).

Going back to our various derivations in the previous chapters we can directly compare (2.28), (3.5), (3.10) and note that all the models deliver similar expressions for welfare gains from trade as a function of λ_{ii}, and thus the trade share of GDP. The expression for the Krugman and the Armington models is similar to the one derived in other models with heterogeneous firms such as the ones of Eaton and Kortum (2002), the Chaney (2008) version of Melitz (2003) and Arkolakis (2010). The only difference is that in the latter cases $\sigma - 1$ is replaced by the parameter that determines the heterogeneity of the productivities of the firms or productivities of sectors. In fact, the same thing holds for expression (2.28), which implies that the main quantitative models of trade with heterogeneous firms deliver exactly the same welfare predictions for the change of welfare in the case of a trade liberalization episode.

8.1.2 The Dekle Eaton Kortum Procedure for Counterfactuals

Dekle, Eaton, and Kortum (2008) have established a methodology for calculating counterfactual changes in the equilibrium variables with respect to changes in the iceberg costs or technology parameters. The merit of this approach is that it does not require prior information on the level of technology A_i and bilateral trade costs τ_{ij}, but rather only percentage changes in the magnitudes of these parameters. The idea is to use data for the endogenous variables λ_{ij}, y_j to calibrate the model in the initial equilibrium, and exploit the fact that the level of technology A_i and bilateral trade costs τ_{ij} are perfectly identified given the values for λ_{ij}, y_j.

The procedure can be applied to most of the frameworks above, and in fact delivers robust predictions for changes in trade and welfare as argued by Arkolakis, Costinot, and Rodríguez-Clare (2010), under the simple assumption that the elasticity of trade with respect to wages and trade costs is the same.

Denote the ratio of the variables in the new and the old equilibrium, e.g. $\hat{w}_j = w'_j / w_j$. We use labor in country j as our numeraire, $w_j = 1$. We will make crucial use of the fact that either profits are a constant fraction of income or that labor income is the only source of income in the models above so that we also obtain that $\hat{y}_i = \hat{w}_i$ for all $i = 1, ..., n$.

Under the assumption that the elasticity of trade with respect to wages and trade costs is the same, and equal to ε, the shares of expenditures on goods from country i in country j in the initial and new equilibrium, respectively, are given by

$$\lambda_{ij} = \frac{\chi_{ij} \cdot J_i \cdot (w_i \tau_{ij})^{\varepsilon}}{\sum_{i'=1}^{n} \chi_{i'j} \cdot J_{i'} \cdot \left(w_{i'} \tau_{i'j}\right)^{\varepsilon}}, \tag{8.8}$$

$$\lambda'_{ij} = \frac{\chi_{ij} \cdot J'_i \cdot \left(w'_i \tau'_{ij}\right)^{\varepsilon}}{\sum_{i'=1}^{n} \chi_{i'j} \cdot J'_{i'} \cdot \left(w'_{i'} \tau'_{i'j}\right)^{\varepsilon}}. \tag{8.9}$$

where χ_{ij} is some parameter of the model, other than τ_{ij} (e.g. bilateral fixed costs). Thus, for example, $\varepsilon = -\theta$ in the Eaton and Kortum (2002) model whereas $\varepsilon = -(\sigma - 1)$ in the Armington (1969) setup. Notice that an essential simplifying assumption is that J_i is a constant and does not depend on technology or bilateral trade costs.

Combining this observation with the above two equations we obtain

$$\hat{\lambda}_{ij} = \frac{(\hat{w}_i \hat{\tau}_{ij})^{\varepsilon}}{\sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}}. \tag{8.10}$$

>From the previous expression and the fact that $\hat{w}_j \hat{\tau}_{jj} = 1$ by our choice of numeraire we have that

$$\hat{\lambda}_{jj} = \frac{1}{\sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{\varepsilon}} .$$

For the models illustrated above, trade balance holds as argued by Arkolakis, Costinot, and Rodríguez-Clare (2010) so that in the new equilibrium:[1]

$$y'_i = \sum_{j'=1}^{n} \lambda'_{ij'} y'_{j'}. \tag{8.11}$$

Combining Equations (8.10) and (8.11) we obtain

$$y'_i = \sum_{j'=1}^{n} \frac{\left(\hat{w}_i \hat{\tau}_{ij'}\right)^{\varepsilon}}{\sum_{i'=1}^{n} \lambda_{i'j'} \left(\hat{w}_{i'} \hat{\tau}_{i'j'}\right)^{\varepsilon}} \lambda_{ij'} \hat{y}_{j'} y_{j'}.$$

[1] In fact, Arkolakis, Costinot, and Rodríguez-Clare (2010) show that formally, the trade balance condition and the condition that profits are a constant share of output, together with expression (8.10) are the conditions that are sufficient to deliver robust counterfactual predictions, as well as robust predictions for welfare gains from trade in a large class of models.

Using the property that $\hat{y}_i = \hat{w}_i$ we can rearrange the previous expression as

$$\hat{w}_i Y_i = \sum_{j'=1}^{n} \frac{\lambda_{ij'} \left(\hat{w}_i \hat{\tau}_{ij'}\right)^{\varepsilon}}{\sum_{i'=1}^{n} \lambda_{i'j'} \left(\hat{w}_{i'} \hat{\tau}_{i'j'}\right)^{\varepsilon}} \cdot \hat{w}_{j'} Y_{j'}. \tag{8.12}$$

The equilibrium changes in wages, w_i, and market shares, λ_{ij}, can be computed given expression (8.10) and (8.12), which completes the argument.

Chapter 9

Appendix

9.1 Distributions

This appendix explains the details of the two main distributions used in these notes[1].

9.1.1 The Fréchet Distribution

The type II extreme value distribution, also called the Fréchet distribution, is one of three distributions that can arise as the limiting distribution of the maximum of a sequence of independent random variables. The distribution function for the Fréchet distribution is

$$F(x) = \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\theta}\right\},$$

for $x > \mu$, where $\theta > 0$ is a shape parameter, $\sigma > 0$ is a scale parameter and $\mu \in \mathbb{R}$ is a location parameter. The density of the Fréchet distribution is

$$f(x) = \frac{\theta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{-\theta-1}\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\theta}\right\},$$

[1] Many thanks to Alex Torgovitsky for the preparation of this appendix

for $x > \mu$. If X is a Fréchet-distributed random variable then

$$
\begin{aligned}
E(X) &= \int_{\mu}^{\infty} x\frac{\theta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{-\theta-1} \exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\theta}\right\} dx \\
&= \sigma \int_{0}^{\infty} y^{-\frac{1}{\theta}} e^{-y} dy + \mu \int_{0}^{\infty} e^{-y} dy \\
&= \sigma \Gamma\left(\frac{\theta-1}{\theta}\right) + \mu,
\end{aligned}
$$

where $y := \left(\frac{x-\mu}{\sigma}\right)^{-\theta}$ and

$$
\Gamma(z) = \int_{0}^{\infty} t^{z-1} e^{-t} dt
$$

is the Gamma function. Now assume that $\mu = 0$, take $T := \sigma^{\theta}$ and rewrite the distribution function as

$$
F(x) = e^{-Ax^{-\theta}},
$$

so that the Fréchet distribution is now parameterized by θ, A. Notice that for any given θ and A is increasing in the scale parameter, σ. Figure 10.7, shows how θ and A affect the Fréchet distribution.

The Pareto Distribution

The Pareto distribution is parameterized by a shape parameter, $\theta > 0$, a scale parameter $m > 0$ and has support $[m, \infty)$ with distribution function

$$
F(x) = 1 - \left(\frac{m}{x}\right)^{\theta}.
$$

The density function is

$$
f(x) = \frac{\theta m^{\theta}}{x^{\theta+1}}.
$$

The n^{th} moment of a Pareto distributed random variable can easily be calculated as

$$
E(X^n) = \int_{m}^{\infty} x^n \theta m^{\theta} x^{-\theta-1} dx = \begin{cases} \frac{\theta m^n}{\theta-n}, & \text{if } \theta > n \\ +\infty, & \text{if } \theta \leq n \end{cases}
$$

which shows that the shape parameter controls the number of existent moments. Direct computation yields

$$E(X) = \frac{\theta m}{\theta - 1}, \quad \text{if } \theta > 1,$$
$$Var(X) = \frac{\theta m^2}{(\theta - 1)^2(\theta - 2)}, \quad \text{if } \theta > 2.$$

The Pareto distribution is an example of a power law distribution, which can be seen by observing that

$$\Pr[X \geq x] = \left(\frac{m}{x}\right)^{\theta}.$$

This implies that

$$\log(\Pr[X \geq x]) = \theta \log(m) - \theta \log(x),$$

so that the log of the mass of the upper tail past x is linear in $\log(x)$. For example, if the number of employees in a randomly sampled firm, X, is Pareto distributed, then the proportion of firms in the population that have more than x employees is linear with the number of employees on a log-log scale. This is related to a useful self-replicating feature of the Pareto distribution, which is that the distribution of X conditional on the event $[X \geq \bar{x}]$, where $\bar{x} \geq m$, is given by

$$\Pr[X \geq x | X \geq \bar{x}] = \frac{\Pr[X \geq x]}{\Pr[X \geq \bar{x}]} = \left(\frac{\bar{x}}{x}\right)^{\theta},$$

for $x \geq \bar{x}$. That is, truncating the Pareto distribution on the left produces another Pareto distribution with the same shape parameter! Figure 10.7, shows how θ and the initial point m affect the Pareto distribution.

Chapter 10

Figures and Tables

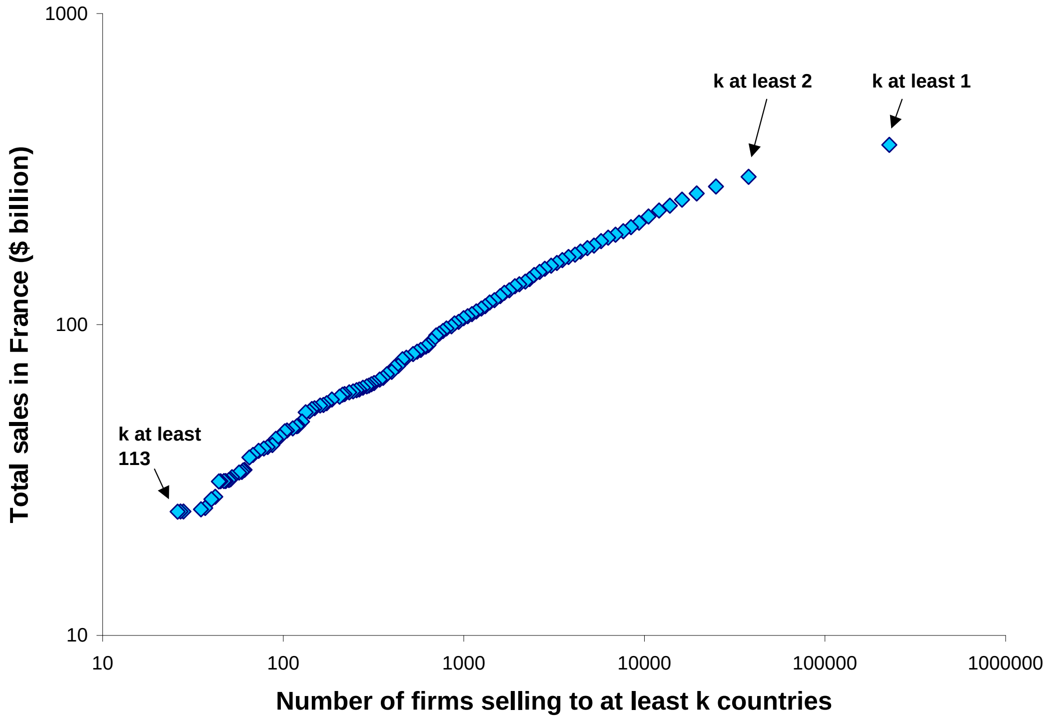

Figure 10.1: Sales in France from firms grouped in terms of the minimum number of destinations they sell to. Source: Eaton, Kortum, and Kramarz (2010).

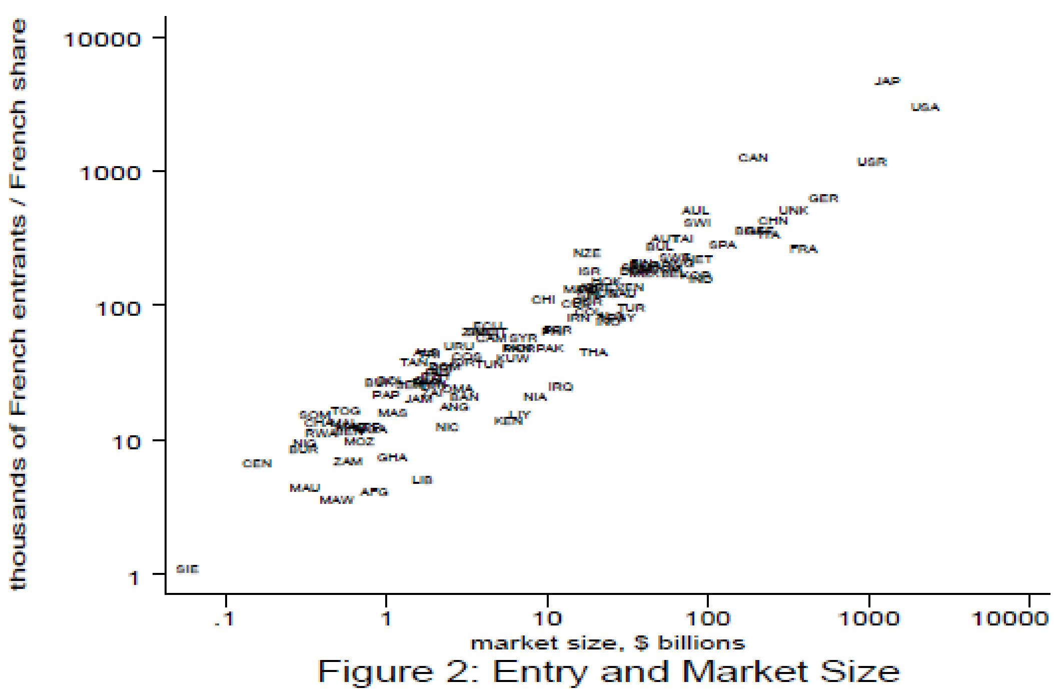

Figure 10.2: Market Size and French firm entry in 1986. *Source: Eaton, Kortum, and Kramarz (2004).*

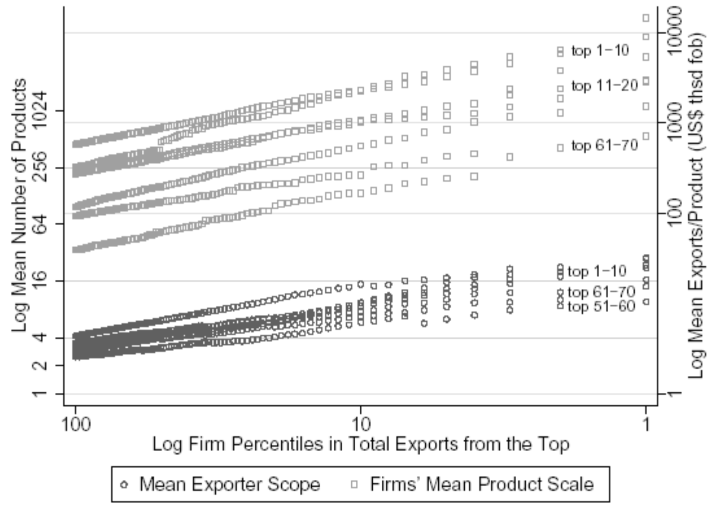

Figure 10.3: Distributions of average sales per good and average number of goods sold. Means taker over all firms larger or equal than the percentile considered in the graph. Source: Arkolakis and Muendler (2010). Products at the Harmonized-System 6-digit level. Destinations ranked by total exports.

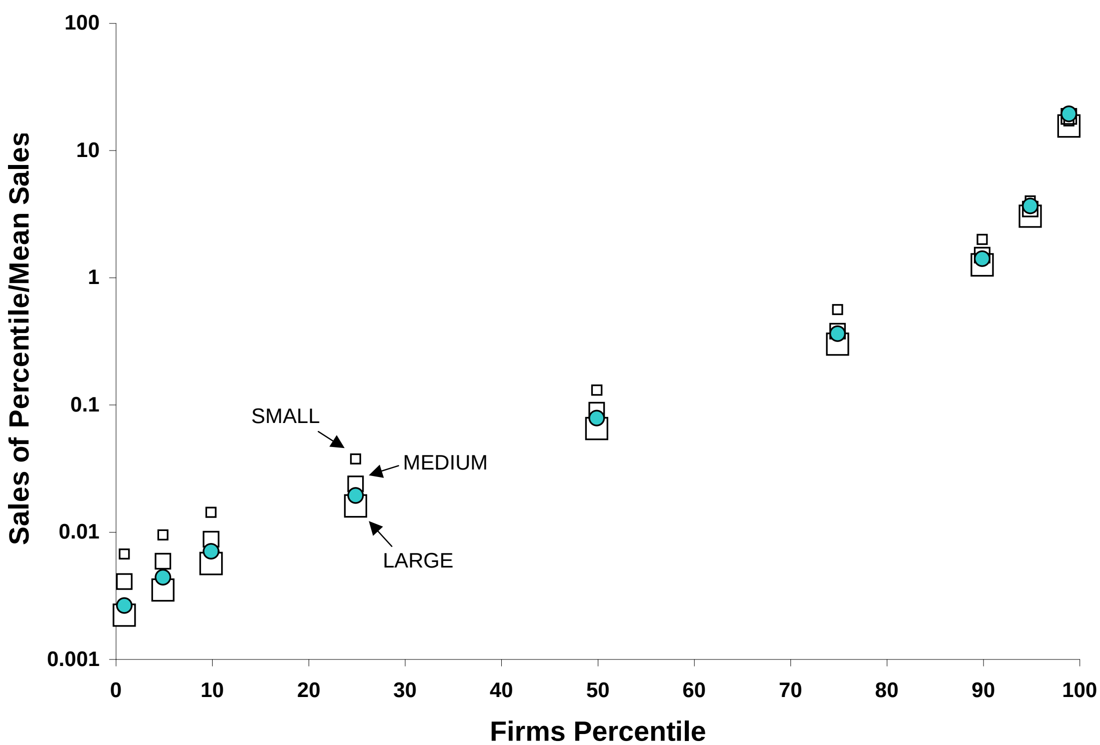

Figure 10.4: Distribution of sales for Portugal and means of other destinations group in terciles depending on total sales of French firms there. Each box is the mean over each size group for a given percentile and the solid dots are the sales distribution in Portugal. *Source Eaton, Kortum, and Kramarz (2010).*

Table 1: Overlaps between Reference Countries and Rest of World by Product Rank

Product rank in Ref. country	Reference country: USA				Reference country: Argentina			
	Overlap	Overlap top prd.	#Dest./ firm	#Firms	Overlap	Overlap top prd.	#Dest./ firm	#Firms
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	.83	.83	8.9	2,280	.77	.77	7.8	3,071
2	.54	.77	13.0	1,033	.54	.76	10.7	1,672
4	.36	.73	18.9	368	.38	.67	14.2	797
8	.34	.69	24.1	137	.30	.63	18.5	307
16	.26	.59	24.3	63	.24	.54	22.6	136
32	.24	.53	30.2	22	.22	.50	29.7	48
64	.15	.49	38.9	10	.29	.40	35.9	19
128	.13	.69	42.4	5	.11	.33	43.8	12

Source: *SECEX* 2000, manufacturing firms and their manufactured products.
Note: Destination counts in columns 3 and 7 are mean numbers of destinations to which firms with at least as many products as reported for a rank ship. Overlap in columns 1 and 5 is the proportion of destinations that a product of reported rank reaches relative to the overall destination counts (in columns 3 and 7). Overlap in columns 2 and 6 is the proportion of destinations that the top-selling product of firms with at least as many products as reported for a rank reaches relative to the overall destination counts (in columns 3 and 7). Products at the HS 6-digit level, ranked by decreasing export value within firm in reference country. Sample restricted to firm-products that ship to reference country and at least one other destination.

Figure 10.5: Product Rank, Product Entry and Product Sales for Brazilian Exporters

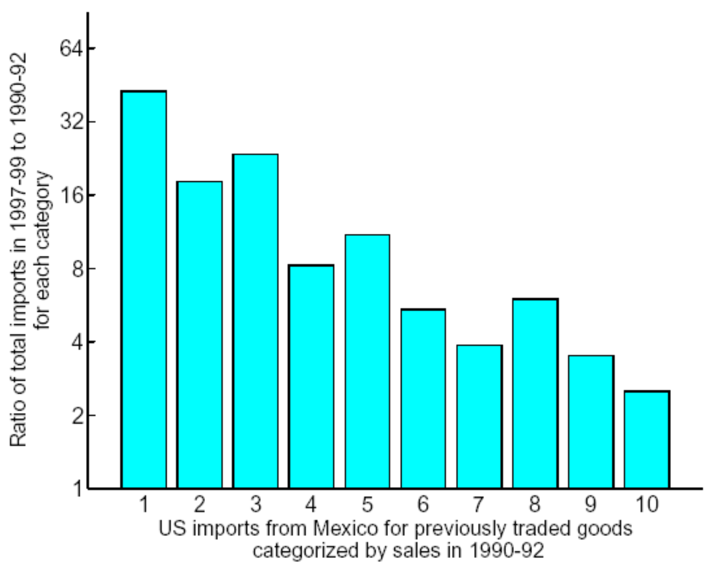

Figure 10.6: Increases in trade and initial trade. Source: Arkolakis (2009). Products at the Harmonized-System 6-digit level. Data are from www.sourceoecd.org

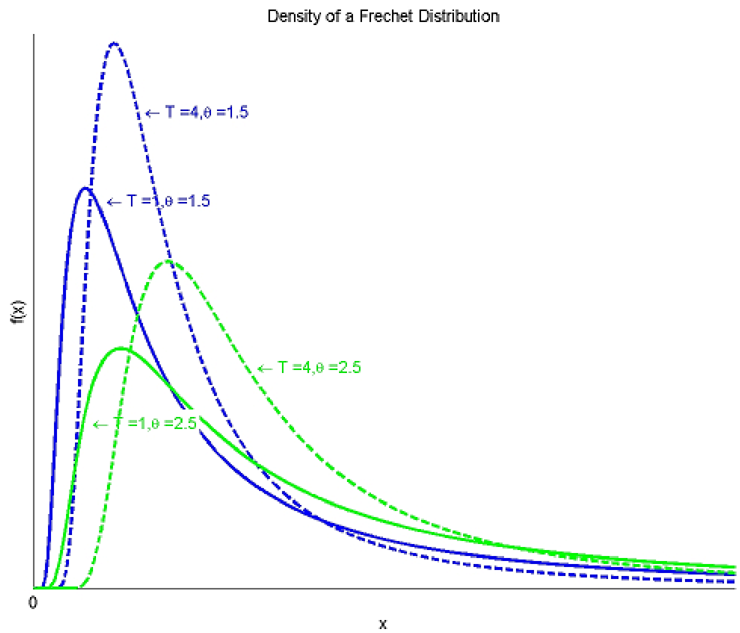

Figure 10.7: Frechet Distribution

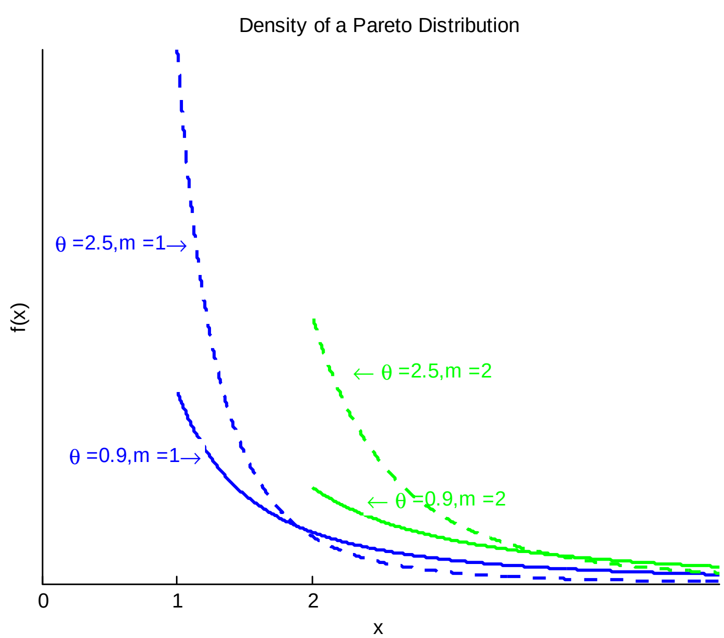

Figure 10.8: Pareto Distribution

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