Documentation for DICE-2006, November 2006 round

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This documentation provides the modeling details on the DICE model runs. The basic DICE model is similar to the one used in William Nordhaus and Joseph Boyer, *Warming the World: Economic Modeling of Global Warming*, MIT Press, Cambridge, Mass, 2000. We have updated the major economic and environmental variables so that the first step is the decade centered on 2005 from 1995.

Updated parameter estimates are provided in Appendix A.

The baseline model with no emissions reductions is provided in Appendix B. This model uses the GAMS modeling system.

To get the Runs in the *Stern Review* paper:

1. Run 1 is the baseline model with the emissions reduction parameter set optimally.
2. Run 2 is run 1 with the social rate of time preference set at 0.001 per year.
3. Run 3 is run 1 with the social rate of time preference set at 0.001 per year and then the elasticity of the marginal utility of consumption calibrated so that the savings rates and interest rate is the same as Run 1. For this, the MUC is equal to 2.25.

Appendix A. Parameter adjustments to DICE-99

	Variable	New data	Adjustment
1.	Atmospheric CO2, 2000, GtC	787	Atmospheric concentration from CDIAC for 2000 x 2.13
2.	Global surface temperature, 2000 increase from 1900, degrees C	0.71	Take average 1880 - 1920 and 1995-2005 from Hadley center.
3.	Deep ocean temperature, 2000 increase from 1900, degrees C	0.3	From Stocker and Schnitker model run (http://earth.usc.edu/~geol150/variability/deepocean.html)
4.	World GDP, 2005 (billions, 2000 US$)	46,289	Corderi estimates
5.	Reflator from 2000$ to 2006$	1.16168	From www.bea.gov; 2006Q2 and Q3 were 115.905 116.431.
6.	World GDP, 2005 (billions, 2006 US$)	53,773	Product of last two lines
7.	World population, millions, 2005	6409	Corderi estimates
8.	Global CO2 emissions, 2005, GtC	7.518	CDIAC and Corderi
9.	Land CO2 releases, 2005	0	New estimates on deforestation suggest zero or negative; in any case, likely to be small.
	Non-CO2 forcings:		
10.	From 1880 to 2000 (W/m2)	1.2	GISS estimates, http://www.giss.nasa.gov/data/simodel/ghgases/ (downloaded November 2006)
11.	From 2000 to 2100 (W/m2)	0.5	GISS estimates, http://www.giss.nasa.gov/data/simodel/ghgases/ (downloaded November 2006)
12.	Cumulative carbon consumption (GtC)	6000	This module had been turned off in the last round and has been reactivated.

Appendix B. DICE-2006 used for the Stern Review calculations

***** Comments for users of this version of the DICE model:

** D_111606b_alt_base.gms
** New base DICE-2006 as of November 16, 2006

***** This is the DICE model recalibrated to data for November 2006.
** This is the run for the optimal case with baseline discounting.
** To obtain the Stern runs, the social rate of time preference must be changed to .001
** To obtain a baseline, the MIU.fx must be set to .01 by removing the * from the
** front of the statement (about two-thirds of the way down the program).

***** Major changes include:
** 1. Respecifying the utility function as constant elasticity
** 2. Resetting the initial condisions for output, capital, population
** 3. Setting the initial conditions for the carbon cycle and temperatures
** 4. The source of the economic and emissions data are generally Corderi
** 5. Recalibrated from DICE-98 as of November 24, 1998

***** The program

```
SETS  T             Time periods                /1*100/
     TFIRST(T)       First period
     TLAST(T)        Last period
     tearly(T)       First 20 periods
     TLATE(T)        Second 20 periods;

SCALARS

 ELASMU   Elasticity of marginal utility of consumption   /1.00001 /
 SRTP     Initial rate of social time preference per year /.03     /
 DR       Decline rate of social time preference per year /.0025719/
 GL0      Growth rate of population per decade           /.080    /
 DLAB     Decline rate of pop growth per decade          /.300    /
 A0       Initial level of total factor productivity     /.0276   /
 GA0      Initial growth rate for technology per decade  /.150    /
 DELA     Decline rate of technol. change per decade     /.005    /
 SIG0     CO2-equivalent emissions-GNP ratio             /.1416   /
 GSIGMA   Growth of sigma per decade                     /-.150   /
 desig    Decline rate of decarbonization                /.0065   /
 desig2   Quadratic term in decarbonization             /-.00035 /
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```
DK      Depreciation rate on capital per year          /.100    /
GAMA    Capital elasticity in production function      /.300    /
MAT1990 Concentration in atmosphere 2000   (GtC)       /787     /
MU1990  Concentration in upper strata 2000 (GtC)       /900     /
ML1990  Concentration in lower strata 2000 (GtC)       /19230   /
b11     Carbon cycle transition matrix                 /0.66616 /
b12     Carbon cycle transition matrix                 /0.33384 /
b21     Carbon cycle transition matrix                 /0.27607 /
b22     Carbon cycle transition matrix                 /0.60897 /
b23     Carbon cycle transition matrix                 /0.11496 /
b32     Carbon cycle transition matrix                 /0.00422 /
b33     Carbon cycle transition matrix                 /0.99578 /
TL0     2000 lower strat. temp change (C) from 1900    /.300    /
T0      2000 atmospheric temp change (C)from 1900      /.710    /
Q0      2005 world gross output trill 2006 US dollars  /47.1    /
LL0     2005 world population millions                 /6409    /
K0      2005 value capital trill 2006 US dollars       /120     /
C1      Climate-equation coefficient for upper level   /.226    /
LAM     Climate feedback factor                        /1.41    /
C3      Transfer coeffic. upper to lower stratum       /.440    /
C4      Transfer coeffic for lower level               /.020    /
A1      Damage coeff linear term                       /-.0045  /
A2      Damage coeff quadratic term                    /.0035   /
COST10  Intercept control cost function                /.030    /
COST2   Exponent of control cost function              /2.15    /
ET0     C Emiss from land (GtC per decade)             /0.00    /
Dmiufunc Decline in abatement-cost function (per decade) /-.08  /
Decmiu  Change in decline of cost function             /.500    /
Coefopt1 Scaling coefficient in the objective function /81.1    /
Coefopt2 Scaling coefficient in the objective function /23292   /
Tlimit  Limit on temperature increase                  /10      /
Exfc2000 Estimate of 2000 forcings of non-CO2 GHG      / 0.35   /
Exfc2100 Estimate of 2000 forcings of non-CO2 GHG      / 0.70   /    ;


PARAMETERS
 L(T)         Level of population and labor
 AL(T)        Level of total factor productivity
 SIGMA(T)     CO2-equivalent-emissions output ratio
 R(T)         Instantaeous rate of social time preference
 RR(T)        Average utility social discount rate
 GA(T)        Growth rate of productivity from 0 to T
 FORCOTH(T)   Exogenous forcing for other greenhouse gases
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```
  GL(T)        Growth rate of labor 0 to T
  gcost1       Growth of cost factor
  GSIG(T)      Cumulative improvement of energy efficiency
  ETREE(T)     Emissions from deforestation
  cost1(t)     Cost function for abatement ;

TFIRST(T) = YES$(ORD(T) EQ 1);
TLAST(T)  = YES$(ORD(T) EQ CARD(T));
TEARLY(T) = YES$(ORD(T) LE 20);
TLATE(T)  = YES$(ORD(T) GE 21);
DISPLAY TFIRST, TLAST;

GL(T) = (GL0/DLAB)*(1-exp(-DLAB*(ord(t)-1)));
L(T)=LL0*exp(GL(t));


ga(T)=ga0*EXP(-dela*10*(ORD(T)-1));
al("1") = a0;
LOOP(T,
al(T+1)=al(T)/((1-ga(T)));
);


gsig(T)=gsigma*EXP (  -desig*10*(ORD(T)-1) - desig2*10* ((ord(t)-1)**2)  )   ;
sigma("1")=sig0;
LOOP(T,
sigma(T+1)=(sigma(T)/((1-gsig(T+1))));
);

gcost1(T)=dmiufunc*EXP(-decmiu*10*(ORD(T)-1));
cost1("1")=cost10;
LOOP(T,
cost1(T+1)=cost1(T)/((1+gcost1(T+1)));
);


ETREE(T) = ET0*(1-0.1)**(ord(T)-1);

R(T)=srtp*EXP(-DR*10*(ORD(T)-1));
RR("1")=1;
LOOP(T,
RR(T+1)=RR(T)/((1+R(T))**10);
);
```

```
FORCOTH(T)= Exfc2000+ .1*(Exfc2100-Exfc2000)*(ORD(T)-1)$
    (ORD(T) LT 12)+ 0.35$(ORD(T) GE 12);


VARIABLES
 MIU(T)   Emission control rate GHGs
 FORC(T)  Radiative forcing in watts per m2
 TE(T)    Temperature of atmosphere in C
 TL(T)    Temperatureof lower oceans degrees C
 MAT(T)   Carbon concentration in atmosphere (b.t.c.)
 MU(T)    Carbon concentration in shallow oceans (b.t.c.)
 ML(T)    Carbon concentration in lower oceans (b.t.c.)
 E(T)     CO2-equivalent emissions billions t
 C(T)     Consumption trillions US dollars
 K(T)     Capital stock trillions US dollars
 CPC(T)   Per capita consumption thousands US dol
 PCY(t)   Per capita income thousands US dol
 I(T)     Investment trillions US dollars
 S(T)     Savings rate as fraction of gross world product
 RI(T)    Real interest rate per annum
 Y(T)     Output or gross world product
 CCA(T)   Cumulative industrial carbon emissions (GTC)
 FEL(t)   Instantaneous utility function

 UTILITY;

POSITIVE VARIABLES MIU, TE, E, Mat, mu, ml, Y, C, K, I, CCA ;

EQUATIONS

*new
      CCTFIRST(T)     First period cumulative carbon
      CCACCA(T)       Cumulative carbon emissions

 UTIL      Objective function
 YY(T)     Output equation
 CC(T)     Consumption equation
 KK(T)     Capital balance equation
 KK0(T)    Initial condition for capital
 KC(T)     Terminal condition for capital
 CPCE(t)   Per capita consumption definition
 PCYE(T)   Per capita income definition
 EE(T)     Emissions process
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```
SEQ(T)    Savings rate equation
RIEQ(T)   Interest rate equation
FORCE(T)  Radiative forcing equation
MMAT0(T)  Starting atmospheric concentration
MMAT(T)   Atmospheric concentration equation
MMU0(T)   Initial shallow ocean concentration
MMU(T)    Shallow ocean concentration
MML0(T)   Initial lower ocean concentration
MML(T)    Lower ocean concentration
TTE(T)    Temperature-climate equation for atmosphere
TTE0(T)   Initial condition for atmospheric temperature
TLE(T)    Temperature-climate equation for lower oceans
TLE0(T)   Initial condition for lower ocean
tlim(t)   Limit temperature
FELEQ(t)    Instantaneous utility function equation
;

** Equations of the model

*new
CCTFIRST(TFIRST).. CCA(TFIRST)=E=0;
CCACCA(T+1)..      CCA(T+1)=E=CCA(T)+ E(T);

tlim(t)..          te(t) =l= Tlimit;
KK(T)..         K(T+1) =L= (1-DK)**10 *K(T)+10*I(T);
KK0(TFIRST)..   K(TFIRST) =E= K0;
KC(TLAST)..     .02*K(TLAST) =L= I(TLAST);

EE(T)..         E(T)=G=10*SIGMA(T)*(1-MIU(T))*AL(T)*L(T)**(1-
GAMA)*K(T)**GAMA + ETREE(T);
FORCE(T)..      FORC(T) =E=  4.1*((log(Mat(T)/596.4)/log(2)))+FORCOTH(T);

MMAT0(TFIRST).. MAT(TFIRST) =E= MAT1990;
MMU0(TFIRST)..  MU(TFIRST) =E= MU1990;
MML0(TFIRST)..  ML(TFIRST) =E= ML1990;
MMAT(T+1)..     MAT(T+1) =E= MAT(T)*b11+E(T)+MU(T)*b21;
MML(T+1)..      ML(T+1) =E= ML(T)*b33+b23*MU(T);
MMU(T+1)..      MU(T+1) =E= MAT(T)*b12+MU(T)*b22+ML(T)*b32;

TTE0(TFIRST)..  TE(TFIRST) =E= T0;
TTE(T+1)..      TE(T+1) =E= TE(t)+C1*(FORC(t)-LAM*TE(t)-C3*(TE(t)-TL(t)));
TLE0(TFIRST)..  TL(TFIRST) =E= TL0;
TLE(T+1)..      TL(T+1) =E= TL(T)+C4*(TE(T)-TL(T));
```

```
YY(T)..        Y(T) =E= AL(T)*L(T)**(1-GAMA)*K(T)**GAMA*(1-
cost1(t)*(MIU(T)**cost2))
            /(1+a1*TE(T)+ a2*TE(T)**2);

SEQ(T)..       S(T) =e= I(T)/(.001+Y(T));
RIEQ(T)..      RI(T) =E= GAMA*Y(T)/K(T)- (1-(1-DK)**10)/10  ;
CC(T)..        C(T) =E= Y(T)-I(T);
CPCE(T)..      CPC(T) =e= C(T)*1000/L(T);
PCYE(T)..      PCY(T) =e= Y(T)*1000/L(T);
*FELEQ(T)..     FEL(T) =e=  LOG(C(T)/L(T));
FELEQ(T)..     FEL(T) =e=   ((C(T)/L(T))**(1-ELASMU)-1)/(1-ELASMU);

UTIL..         UTILITY =E= SUM(T, 10 *RR(T)*L(T)*(FEL(T))/coefopt1)+ coefopt2 ;


**  Upper and Lower Bounds: General conditions imposed for stability

MIU.up(T)      = 1.0;
MIU.lo(T)      = 0.000001;
K.lo(T)        = 1;
MAT.lo(T)      = 10;
MU.lo(t)       = 100;
ML.lo(t)       = 1000;
C.lo(T)        = 2;

** Fix savings assumption if operational

*s.fx(t)=.22;

** Emissions control policy. Current setting is for optimal policy.
** Reinstate equation "Miu.fx(t) = .0" for no-control run.

** Constraint for emissions; = .01 for base; removed for optimal

*MIU.fx(T)=.01;

** Upper limit global mean temperature. Set at 12 for convergence except in limit
cases

TE.up(t)       = 12;

** Cumulative limits on carbon use at 6000 GtC
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CCA.up(T) = 6000;

** Solution options

option iterlim = 99900;
option reslim = 99999;
option solprint = on;
option limrow = 0;
option limcol = 0;

model CO2 /all/;

solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;
solve CO2 maximizing UTILITY using nlp ;

** Display of results

display y.l, c.l, s.l, k.l, miu.l, e.l, te.l, forc.l, ri.l;
display cc.m, ee.m, kk.m, tte.m, cpc.l, tl.l, pcy.l, i.l;
display sigma, rr, l, al, forcoth, etree;
display mat.l,mu.l,ml.l;

** Note to users
** From here on, these are outputs to get the results.

Parameters
Year(t)        Date
Indem(t)       Industrial emissions (b.t.c. per year)
Wem(t)         Total emissions (b.t.c. per year);

Year(t)        = 2005 +10*(ord(t)-1);
Indem(t)       = e.l(t)-etree(t);
Wem(t)  = e.l(t);

Parameters

Tax(t)         Carbon tax ($ per ton)
damtax(t)      Concentration tax ($ per ton)
dam(t)         Damages
cost(t)        Abatement costs;
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```
tax(t)    = -1*ee.m(t)*1000/(kk.m(t));
damtax(t) = -1*mmat.m(t)*1000/kk.m(t);
dam(t)    = y.l(t)*(1-1/(1+a1*te.l(t)+ a2*te.l(t)**2));
cost(t)   = y.l(t)*(cost1(t)*(miu.l(t)**cost2));

** Note to USERS. These statements must be modified for your local machine.
** Please see the GAMS manual for instructions on modification.

File d98oute;
D98oute.pc=5;
D98oute.pw=250;
Put d98oute;
Put / "base (no control) run";
Put / "year";
Loop (tearly, put year(tearly)::0);
Put / "output";
Loop (tearly, put y.l(tearly)::3);
Put / "pccon";
Loop (tearly, put cpc.l(tearly)::3);
Put / "savrate";
Loop (tearly, put s.l(tearly)::4);
Put / "indem";
Loop (tearly, put indem(tearly)::4);
Put / "sigma";
Loop (tearly, put sigma(tearly)::4);
Put / "temp";
Loop (tearly, put te.l(tearly)::3);
Put / "conc";
Loop (tearly, put mat.l(tearly)::3);
Put / "ctax";
Loop (tearly, put tax(tearly)::2);
Put / "intrate";
Loop (tearly, put ri.l(tearly)::3);
Put / "discrate";
Loop (tearly, put rr(tearly)::5);
Put / "prod";
Loop (tearly, put al(tearly)::5);
Put / "exogforc";
Loop (tearly, put forcoth(tearly)::3);
Put / "pop";
Loop (tearly, put l(tearly)::3);
Put / "etree";
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```
Loop (tearly, put etree(tearly)::4);
Put / "margy";
Loop (tearly, put yy.m(tearly)::3);
Put / "margc";
Loop (tearly, put cc.m(tearly)::3);
Put / "miu";
Loop (tearly, put miu.l(tearly)::3);
Put / "total emissions";
Loop (tearly, put wem(tearly)::3);
Put / "interest rate";
Loop (tearly, put ri.l(tearly)::4);
Put / "damages";
Loop (tearly, put dam(tearly)::5);
Put / "abatement cost";
Loop (tearly, put cost(tearly)::5);
Put /"objective function";
Put utility.l::3;

File d98outL;
D98outL.pc=5;
D98outL.pw=250;
Put d98outL;
Put / "base (no control) run";
Put / "year";
Loop (tlate, put year(tlate)::0);
Put / "output";
Loop (tlate, put y.l(tlate)::3);
Put / "pccon";
Loop (tlate, put cpc.l(tlate)::3);
Put / "savrate";
Loop (tlate, put s.l(tlate)::4);
Put / "indem";
Loop (tlate, put indem(tlate)::6);
Put / "sigma";
Loop (tlate, put sigma(tlate)::6);
Put / "temp";
Loop (tlate, put te.l(tlate)::3);
Put / "conc";
Loop (tlate, put mat.l(tlate)::6);
Put / "ctax";
Loop (tlate, put tax(tlate)::6);
Put / "intrate";
Loop (tlate, put ri.l(tlate)::3);
```

```
Put / "discrate";
Loop (tlate, put rr(tlate)::5);
Put / "prod";
Loop (tlate, put al(tlate)::6);
Put / "exogforc";
Loop (tlate, put forcoth(tlate)::3);
Put / "pop";
Loop (tlate, put l(tlate)::3);
Put / "etree";
Loop (tlate, put etree(tlate)::4);
Put / "margy";
Loop (tlate, put yy.m(tlate)::3);
Put / "margc";
Loop (tlate, put cc.m(tlate)::3);
Put / "miu";
Loop (tlate, put miu.l(tlate)::3);
Put / "total emissions";
Loop (tlate, put wem(tlate)::3);
Put / "interest rate";
Loop (tlate, put ri.l(tlate)::4);
Put / "damages";
Loop (tlate, put dam(tlate)::5);
Put / "abatement cost";
Loop (tlate, put cost(tlate)::5);

display gsig, sigma;
display ga, al, cost1, gcost1, tax;
display cca.l;
```