Roll the DICE Again: Economic Models of Global Warming

Appendix A

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Appendix A. Equations of RICE-99

Sets:	Time periods	t (1995=0, 2005=1, etc)
	Regions	J
	Trading blocs	b

(A.1) $$W_J = \sum_t U[c_J(t), L_J(t)]R(t)$$

(A.2) $$R(t) = \prod_{v=0}^{t} [1+\rho(v)]^{-10}$$

$$\rho(t) = \rho(0)\exp(-g^{\rho}t)$$

(A.3) $$U[c_J(t), L_J(t)] = L_J(t)\ \{\log[c_J(t)]\}$$

(A.4) $$g^{pop}{}_J(t) = g^{pop}{}_J(0)\exp(-\delta^{pop}{}_J t)$$

$$L_J(t) = L_J(0)\exp(\int_0^t g_J^{pop}(t))$$

(A.5a) $$Q_J(t) = \Omega_J(t)\{\ A_J(t)\ K_J(t)^{\gamma}\ L_J(t)^{1-\beta_J-\gamma} ES_J(t)^{\beta_J} - c^E{}_J(t)\ ES_J(t)\ \}$$

(A.5b) $$ES_J(t) = \varsigma_J(t)E_J(t)$$

$$g^Z{}_J(t) = g^Z{}_J(0)\exp(-\delta^Z{}_J t)$$

$$V_J(t) = V_J(0)\exp(\int_0^t g_J^Z(t))$$

(A.6) $$g^A{}_J(t) = g^A{}_J(0)\exp(-\delta^A{}_J t)$$

$$A_J(t) = A_J(0)\exp(\int_0^t g_J^A(t))$$

(A.7) $Q_J(t) + \tau_J(t)[\Pi_J(t) - E_J(t)] = C_J(t) + I_J(t)$

$$(A.7') \quad t_J(t) = t_b(t)\ \forall J \in b$$

$$\sum_{J\in b} II_J(t) \geq \sum_{J\in b} E_J(t)$$

$$\sum_{J\in b} II_J(t) = \sum_{J\in b} E_J(t) \quad if t_b > 0$$

$$t_b(t) \geq 0$$

(A.8) $c_J(t) = C_J(t)/L_J(t)$

(A.9) $K_J(t) = K_J(t-1)(1-\delta_K)^{10} + 10 \times I_J(t-1)$

$K_J(0) = K_J*$

(A.10) $c^E{}_J(t) = \Lambda[\varsigma_J(t)]q(t) + markup^E{}_J$

$\Lambda[\varsigma_J(t)] = 1$

(A.11) $CumC(t) = CumC(t-1) + 10 \times E(t)$

$$E(t) = \sum_J E_J(t)$$

(A.12) $q(t) = \xi_1 + \xi_2[CumC(t)/CumC*]^{\xi 3}$

(A.13) $M_{AT}(t) = 10 \times ET(t) + \phi_{11} M_{AT}(t-1) - \phi_{12} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$

$LU_J(t) = LU_J(0)(1-\delta_l)^t$

$ET(t) = \sum_J (E_J(t) + LU_J(t))$

$M_{AT}(0) = M_{AT}*$

(A.13b) $M_{UP}(t) = \phi_{22} M_{UP}(t-1) + \phi_{12} M_{AT}(t-1) - \phi_{21} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) - \phi_{23} M_{UP}(t-1)$

$M_{UP}(0) = M_{UP}*$

(A.13c) $M_{LO}(t) = \phi_{33} M_{LO}(t-1) - \phi_{32} M_{LO}(t-1) + \phi_{23} M_{UP}(t-1)$

$M_{LO}(0) = M_{LO}*$

(A.14) $F(t) = \eta\{\log[M_{AT}(t)/M_{AT}^{PI}]/\log(2)\} + O(t)$

$O(t) = -0.1965 + 0.13465t \qquad t < 11$
$\quad = 1.15 \qquad t > 10$

(A.15a) $T(t) = T(t-1) + \sigma_1\{F(t) - \lambda T(t-1) - \sigma_2[T(t-1) - T_{LO}(t-1)]\}$

$T(0) = T*$

(A.15b) $T_{LO}(t) = T_{LO}(t-1) + \sigma_3[T(t-1) - T_{LO}(t-1)]$

$T_{LO} = T_{LO}*$

(A.16) $D_J(t) = \theta_{1,J}\, T(t) + \theta_{2,J}\, T(t)^2$

(A.17) $\Omega_J(t) = 1/[1+D_J(t)]$