Roll the DICE Again: Economic Models of Global Warming

Appendix B

William D. Nordhaus and Joseph Boyer

Yale University
October 25, 1999

Version is DICE v.101599

Appendix B. Equations of DICE-99

(B.1) $$W = \sum_{t} U[c(t),L(t)]R(t)$$

(B.2) $$R(t) = \prod_{v=0}^{t} [1+ \rho(v)]^{-10}$$

$$\rho(t) = \rho(0)\exp(-g^{\rho}t)$$

(B.3) $$U[c(t),L(t)] = L(t)\{\log[c(t)]\}$$

(B.4) $$g^{pop}(t) = g^{pop}(0)\exp(-\delta^{pop}t)$$

$$L(t) = L(0)\exp(\int_{0}^{t} g^{pop}(t))$$

(B.5) $$Q(t) = \Omega(t)(1-b_1(t)\mu(t)^{b2})\, A(t)\, K(t)^{\gamma}\, L(t)^{1-\gamma}$$

(B.6) $$g^{A}(t) = g^{A}(0)\exp(-\delta^{A}t)$$

$$A(t) = A(0)\exp(\int_{0}^{t} g^{A}(t))$$

(B.7) $$\Omega(t) = 1/[1+D(t)]$$

(B.8) $$D(t) = \theta_1\, T(t) + \theta_2\, T(t)^2$$

(B.9) $$g^{b}(t) = g^{b}(0)\exp(-\delta^{b}t)$$

$$b_1(t) = b_1(t-1)/(1+g^{b}(t))$$

$$b_1(0) = b_1^*$$

(B.10) $$E(t) = (1-\mu(t))\sigma(t)A(t)\, K(t)^{\gamma}\, L(t)^{1-\gamma}$$

(B.11) $$g^{\sigma}(t) = g^{\sigma}(0)\exp(-\delta^{\sigma}_1 t - \delta^{\sigma}_2 t^2)$$

$$\sigma(t) = \sigma(t-1)/(1+ g^{\sigma}(t))$$

$$\sigma(0) = \sigma^*$$

(B.12) $$Q(t) + \tau(t)[\Pi(t) - E(t)] = C(t) + I(t)$$

(B.13) $$II(t) = E(t)$$

(B.14) $$c(t)=C(t)/L(t)$$

(B.15) $$K(t) = K(t-1)(1-\delta_K)^{10} + 10 \times I(t-1)$$

$$K(0) = K^*$$

(B.16) $$LU(t) = LU(0)(1-\delta_l)^t$$

$$ET(t) = E(t) + LU(t)$$

(B.17a) $$M_{AT}(t) = 10 \times ET(t) + \phi_{11} M_{AT}(t-1) - \phi_{12} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$$

$$M_{AT}(0) = M_{AT}^*$$

(B.17b) $$M_{UP}(t) = \phi_{22} M_{UP}(t-1) + \phi_{12} M_{AT}(t-1) - \phi_{21} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) - \phi_{23} M_{UP}(t-1)$$

$$M_{UP}(0) = M_{UP}^*$$

(B.17c) $$M_{LO}(t) = \phi_{33} M_{LO}(t-1) - \phi_{32} M_{LO}(t-1) + \phi_{23} M_{UP}(t-1)$$

$$M_{LO}(0) = M_{LO}^*$$

(B.18) $$F(t) = \eta\{\log[M_{AT}(t)/M_{AT}^{PI}]/\log(2)\} + O(t)$$

$$O(t) = -0.1965 + 0.13465t \qquad t < 11$$
$$= 1.15 \qquad t > 10$$

(B.19) $$T(t) = T(t-1) +\sigma_1\{F(t) - \lambda T(t-1) - \sigma_2[T(t-1) - T_{LO}(t-1)]\}$$

$$T(0) = T^*$$

(B.20) $$T_{LO}(t) = T_{LO}(t-1) + \sigma_3[T(t-1) - T_{LO}(t-1)]$$

$$T_{LO} = T_{LO}^*$$