

Roll the DICE Again: Economic Models of Global Warming

Appendix C

William D. Nordhaus and Joseph Boyer

Yale University
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Appendix C. Variable List

Variables are listed in the order they appear in Appendices A and B.

1. Exogenous variables and parameters

<i>Variable</i>	<i>Description</i>	<i>Units</i>
$L(t)$	Population	Millions
$R(t)$	Social time preference discount factor	Pure number
$\rho(t)$	Social time preference discount rate	Rate per year
g^{ρ}	Growth rate of $\rho(t)$	Rate per decade
$\rho(0)$	Initial social time preference discount rate	Rate per year
$g^{\text{pop}}(t)$	Growth rate of population	Rate per decade
$g^{\text{pop}}(0)$	Initial population growth rate	Rate per decade
δ^{pop}	Rate of decline of $g^{\text{pop}}(t)$	Rate per decade
$L(0)$	Initial population	Millions
$A(t)$	Total factor productivity	Determined by units of inputs in production function
γ	Elasticity of output with respect to capital	Pure number
β	Elasticity of output with respect to carbon-energy	Pure number
$\zeta(t)$	Ratio of carbon-energy to industrial carbon emissions	Pure number
$g^{\zeta}(t)$	Growth rate of $\zeta(t)$	Rate per decade
$g^{\zeta}(0)$	Initial growth rate of $\zeta(t)$	Rate per decade
δ^{ζ}	Rate of decline of $g^{\zeta}(t)$	Rate per decade
$\zeta(0)$	Initial ratio of carbon-energy to industrial carbon emissions	Pure number
$g^A(t)$	Growth rate of $A(t)$	Rate per decade
$g^A(0)$	Initial productivity growth rate	Rate per decade
δ^A	Rate of decline of $g^A(t)$	Rate per decade
$A(0)$	Initial total factor productivity	Determined by units

		of inputs in production function
$\Pi(t)$	Industrial carbon emissions permits	GtC per year
δ_K	Rate of depreciation of capital	Rate per year
K^*	Initial capital stock	Trillions of 1990 dollars
markup ^E	Regional energy services markup	\$1000 / ton carbon
ξ_1, ξ_2, ξ_3	Parameters of long-run industrial emissions supply curve	Varies by parameter
CumC*	Point of diminishing returns in carbon extraction	GtC
$\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \phi_{23}, \phi_{32}, \phi_{33}$	Parameters of carbon transition matrix	Pure number
LU(t)	Land-use carbon emissions	GtC per year
LU(0)	Initial land-use carbon emissions	GtC per year
δ_1	Rate of decline of land-use emissions	Rate per decade
M_{AT}^*	Initial atmospheric concentration of CO ₂	GtC
M_{UP}^*	Initial concentration of CO ₂ in upper oceans/biosphere	GtC
M_{LO}^*	Initial concentration of CO ₂ in deep oceans	GtC
η	Increase in radiative forcing due to doubling of CO ₂ concentrations from preindustrial levels.	W/m ²
M_{AT}^{PI}	Preindustrial CO ₂ concentration	GtC
O(t)	Increase in radiative forcing over preindustrial levels due to exogenous anthropogenic causes	W/m ²
$\sigma_1, \sigma_2, \sigma_3, \lambda$	Temperature dynamics parameters η/λ is the climate sensitivity, or equilibrium increase in temperature due to CO ₂ concentration doubling	Varies by parameter
T*	Initial atmospheric temperature	°C over 1900 level
T_{LO}^*	Initial ocean temperature	°C over 1900 level
θ_1, θ_2	Parameters of damage function	Varies by parameter

Specific to DICE-99

$b_1(t)$	Coefficient on control rate in abatement cost function in DICE-99	Pure number
b_2	Exponent on control rate in abatement cost function in DICE-99	Pure number
$g^b(t)$	Rate of decline of $b_1(t)$	Rate per decade
$g^b(0)$	Initial rate of decline of $b_1(t)$	Rate per decade
δ^b	Rate of decline of $g^b(t)$	Rate per decade
b_1^*	Initial value of $b_1(t)$	Pure number
$\sigma(t)$	Base-case ratio of industrial emissions to output in DICE-99	Tons / \$ 1000
$g^\sigma(t)$	Rate of decline of $\sigma(t)$	Rate per decade
$g^\sigma(0)$	Initial rate of decline of $\sigma(t)$	Rate per decade
$\delta_1^\sigma, \delta_2^\sigma$	Parameters that determine rate of decline of $g^\sigma(t)$	Pure numbers
σ^*	Initial value of $\sigma(t)$	Tons / \$ 1000

2. Endogenous variables– RICE-99

W	Welfare	Utils
U(t)	Utility during period t	Utils
c(t)	Per capita consumption	\$ Million / Person per year
Q(t)	Output	Trillions of 1990 dollars per year
$\Omega(t)$	Climate-change damage factor on gross output	Pure number
K(t)	Capital stock	Trillions of 1990 dollars
ES(t)	Carbon-energy; energy services from carbon fuels	GtC per year
$c^E(t)$	Cost of carbon energy	\$1000 per ton
E(t)	Industrial CO ₂ emissions	GtC per year
$\tau(t)$	Industrial emission permit price	\$1000 per ton
	Note: Could also treat permit price as exogenous, in which case allocation of permits are constrained by (2.7') and (A-2.13) (See Chapter 2, part 4)	
C(t)	Consumption	Trillions of 1990 dollars per year
I(t)	Investment	Trillions of 1990 dollars per year

$\tau_b(t)$	Industrial emissions permit price in trading bloc b	\$1000 per ton
$q(t)$	Cost of extraction of industrial emissions	\$1000 per ton carbon
CumC(t)	Cumulative industrial carbon emissions	GtC
E(t)	World industrial carbon emissions	GtC per year
$M_{AT}(t)$	Atmospheric CO ₂ concentration	GtC
ET(t)	World total CO ₂ emissions	GtC per year
$M_{UP}(t)$	Upper oceans/biosphere CO ₂ concentration	GtC
$M_{LO}(t)$	Lower oceans CO ₂ concentration	GtC
F(t)	Radiative forcing, increase over preindustrial level	W/m ²
T(t)	Atmospheric temperature, increase over 1900 level	°C
$T_{LO}(t)$	Lower ocean temperature, increase over 1900 level	°C
D(t)	Climate damage as fraction of net output	pure number

Specific to DICE-99

$\mu(t)$	Industrial emission control rate In DICE	pure number
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