

Roll the DICE Again: Economic Models of Global Warming

Chapter 2

William D. Nordhaus and Joseph Boyer

Yale University
October 25, 1999

Note: This is the “manuscript edition” of the book by the same title to be published by MIT Press. The manuscript edition is for the personal use of readers and may not be sold or used without the written permission of MIT Press.

Version is DICE v.101599

Chapter 2. The Structure And Derivation of RICE-99

1. Introduction

In this chapter, we present an overview of RICE-99. This section describes the structure of the model verbally, while subsequent sections present the equations of the model. The following chapter then discusses the calibration of the major components of the models.

In considering climate-change policies, the fundamental tradeoff that society faces is between consumption today and consumption in the future. By taking steps to slow emissions of greenhouse gases today, the economy reduces the amount of output that can be devoted to consumption and productive investment. The return for this “climate investment” is lower damages and therefore higher consumption in the future. The climate investments involve reducing fossil-fuel consumption or moving to low-carbon fuels; in return for this investment, the impacts on agriculture, coastlines, and ecosystems as well as the potential for catastrophic climate change will be reduced.

But the lags between emissions reductions and climatic impacts are extraordinarily long and uncertain, and this fact makes the economic and scientific questions treacherous. Nations must decide whether they will take climate investments now in order to slow

climate change over the coming centuries. Few societal decisions, and no personal ones except those involving Pascal's wager, have comparable time horizons, and this encourages political decision makers to temporize on costly steps.

The major challenge in RICE-99 has been to develop a model of the world economy that captures the major properties of medium- and long-run economic growth of the major countries and regions over the next century. Outside of the rarified and highly stylized models used in the climate-change integrated-assessment models, there are essentially no models of the world economy upon which to draw. Useful ingredients can be obtained for the population projections from demographers, who do in fact prepare long-term projections. But for other important variables, ones determining capital formation and technological change, particularly for countries outside the United States and Western Europe, it has been necessary to develop long-term projections *de novo*.

The model operates in periods of 10 years. All flow variables in the empirical model are reported as flows per year, while the convention is that stocks are measured at the beginning of the period.

2. Verbal Description

A. Economic sectors

The approach taken here is to view climate change in the framework of economic growth theory. This approach was developed by Frank Ramsey in the 1920s (see Ramsey [1928]), made rigorous by Tjalling Koopmans and others in the 1960s (see especially Koopmans [1967]), and is summarized by Robert Solow in his masterful exposition of economic-growth theory [1970]. In the neoclassical growth model, society invests in tangible capital goods, thereby abstaining from consumption today, in order to increase consumption in the future.

The DICE/RICE models are the extension of the Ramsey model to include climate investments in the environment. Emissions reductions in the extended model are analogous to investment in the mainstream model. That is, we can view concentrations of GHGs as “negative capital,” and emissions reductions as lowering the quantity of negative capital. Sacrifices of consumption that lower emissions prevent economically harmful climate change and thereby increase consumption possibilities in the future.

In the description that follows, we will focus on the fully regionalized model, the RICE model. Most of the statements apply equally well to the globally aggregated DICE model, which is discussed in chapter 5.

The world is composed of several regions. Some regions consist of a single sovereign country (such as the U.S. or China) while other regions (like OECD Europe or the Low Income region) contain many countries. Each region is assumed to have a well-defined set of preferences, represented by a “social welfare function,” which determines choices about the path for consumption and investment. The social welfare function is increasing in the per capita consumption of each generation, with diminishing marginal utility of consumption. The importance of a generation’s per capita consumption depends on its relative size. The relative importance of different generations is affected by a pure rate of time preference; because a positive time preference is assumed, current generations are favored over future generations.

Regions are assumed to maximize the social welfare function subject to a number of economic and geophysical constraints. The decision variables that are available to the economy are consumption, the rate of investment in tangible capital, and the climate investments, primarily reductions of GHG emissions.

The model contains both a traditional economic sector found in many economic models and a novel climate sector designed for climate-change modeling. We first describe the traditional sector of the economy — the economy without any considerations of climate change.

Each region is assumed to produce a single commodity which can be used for either consumption or investment. In the model, all changes in welfare, including those due to climate change, are included in our definition of consumption of this single commodity. Thus, we will sometimes refer to consumption of this all-inclusive commodity as “generalized consumption.”

There is no international trade in goods or capital except in exchange for carbon emissions permits. That is, we allow regions to trade only for the sake of paying other regions to lower their emissions or to receive payment for lowering emissions.

Each region is endowed with an initial stock of capital and labor and an initial and region-specific level of technology. Population growth and technological change are exogenous while capital accumulation is determined by optimizing the flow of consumption over time.

The major methodological change in the economic sector is a respecification of the production relations in RICE from earlier vintages. (DICE retains the simple “reduced-form” production structure from earlier vintages.) RICE-99 defines a new input into production called “carbon-energy.” Carbon-energy can be thought of as the energy services derived from fossil fuel consumption. Fossil fuel consumption in the model is equal to the carbon content of all fossil-fuel consumption. In other words, energy use is

lumped into a single aggregate where the different fuels are aggregated using carbon weights. Thus we refer to the marginal product or cost of, supply of, and allocation across regions of carbon-energy rather than coal, petroleum, and natural gas.

Output is produced with a Cobb-Douglas production function in capital, labor, and carbon-energy inputs. Technological change takes two forms: economy-wide technological change and carbon-saving technological change. Economy-wide technological change is Hicks-neutral, while carbon-saving technological change is modeled as reducing the ratio of CO₂ emissions to carbon-energy inputs. For convenience, both carbon-energy and industrial emissions are measured in carbon units. The procedure is quite intuitive if one thinks of carbon-energy as coal.

We calibrate the energy-related parameters using data on energy use, energy prices, and energy-use price elasticities. These allow an empirically based carbon reduction curve, whereas most current integrated assessment models make “reasonable” but not data-based specifications of demand. On the supply side, the earlier DICE and RICE models assumed that carbon fuels are superabundant at a fixed supply price. In RICE-99, a supply curve for carbon-energy is introduced. The supply curve allows for limited (albeit huge) long-run supplies at rising costs. Because of the optimal-growth framework, carbon-energy is efficiently allocated across time, which implies that low-cost carbon resources have scarcity prices (called “Hotelling rents”) and that carbon-

energy prices rise over time.

B. Climate-related sectors

The "non-traditional" part of the model contains a number of geophysical relationships that link together the different forces affecting climate change. This part contains a carbon cycle, a radiative forcing equation, climate-change equations, and a climate-damage relationship.

In the earlier DICE/RICE models, endogenous emissions included CO₂ and CFCs. In RICE-99 and DICE-99, endogenous emissions are limited to industrial CO₂. This reflects projections by the IPCC and within the DICE/RICE models that indicate that the radiative forcings from uncontrolled CO₂ concentrations are likely to be nearly five times larger than those from the combined effect of non-CO₂ GHGs and aerosols (see Table 3-9, which is discussed in Chapter 3). The major change here is that the chlorofluorocarbons (CFCs) are now outside the climate-change control strategy; this specification reflects the fact that CFCs are strictly controlled outside the framework of the climate-change agreements under different protocols. Other anthropogenic contributions to climate change are also taken as exogenous. These include CO₂

emissions from land-use changes, non-CO₂ GHGs, and sulfate aerosols.¹ Although it would be more complete to endogenize other GHGs and aerosols (and five other gases are in principle included in the Kyoto Protocol), these are extremely complex and poorly understood.

The original DICE and RICE models used an empirical approach to estimating the carbon flows, estimating the parameters of the emissions-concentrations equation from data on emissions and concentrations. A number of commentators noted that this approach may understate the long-run atmospheric retention of carbon because it assumes an infinite sink of carbon in the deep oceans. DICE-99 and RICE-99 replace the earlier treatment with a structural approach that uses a three-reservoir model calibrated to existing carbon-cycle models. The basic idea is that the deep oceans provide a finite sink for carbon in the long run. In the new specification, we assume that there are three reservoirs for carbon — the atmosphere, a quickly mixing reservoir in the upper oceans and the short-term biosphere, and the deep oceans. Carbon flows in both directions between adjacent reservoirs. The mixing between the biosphere/shallow ocean reservoir and the deep oceans is extremely slow. The RICE/DICE-99 approach matches the original DICE model and other calculations in the early periods but has better long-run properties. A full discussion of the new approach is contained in chapter 4.

Climate change is represented by global mean surface temperature, and the

relationship uses the consensus of climate modelers and a lag suggested by coupled ocean-atmospheric models. The climate module is unchanged from the original DICE and RICE models.

Understanding the economic impacts of climate change continues to be the thorniest issue in climate-change economics. Estimates of climate-change impacts in most integrated assessment modeling rely on a wide variety of estimates of the damage from climate change in different sectors for different regions. Starting with Nordhaus [1989, 1991a], assessments tended to organize impacts of climate change in the framework of national economic accounts, with additions to reflect non-market activity. The present study follows first-generation approaches by analyzing impacts on a sectoral basis. There are two major differences from many earlier studies. First, the approach focuses on deriving estimates for all regions rather than for the U.S. alone. This focus is obviously necessary both because global warming is a global problem and because the impacts are likely to be significantly larger in poorer countries. Second, this study focuses more heavily on the non-market aspects of climate change with particular importance given to the potential for catastrophic risk; this approach is taken because of the finding of the first-generation studies that the impacts on market sectors are likely to be relatively limited. The major results are that impacts are likely to differ sharply by region. We estimate that Russia and other high-income countries (principally Canada) will benefit slightly from a modest global warming . At the other extreme, low income regions —

particularly Africa and India — and Western Europe appear to be quite vulnerable to climate change. The United States appears to be relatively less vulnerable to climate change than many countries. The results are discussed in detail in chapter 4.

3. Derivation of the Equations of RICE-99

We now discuss in detail the equations of RICE-99. The relationships are divided into three groups: the objective function, the economic relationships, and the geophysical relationships. Although the economic sectors are conventional in their approach, modifying them for the climate-change problem requires careful attention, and the major issues are considered in the first two subsections. The major issues of the climate sector and the interaction of economy and climate are analyzed in subsection C.

A. Objective function

A central organizing framework of the DICE/RICE models is that the purpose of economic and environmental policies is to improve the living standards or consumption of people now and in the future. The relevant economic variable is “generalized consumption,” which denotes a broad concept that includes not only traditional market purchases of goods and services like food and shelter but also non-market items such as leisure, cultural amenities, and enjoyment of the environment.

The fundamental assumption we adopt is that policies should be designed to optimize the flow of generalized consumption over time. This approach rests on the view that more consumption is preferred to less. Moreover, increments of consumption become less valuable as consumption levels increase. In technical terms, we model these assumptions by assuming that regions maximize a social welfare function that is the discounted sum of the population-weighted utility of per capita consumption. This social welfare function is a mathematical representation of three basic value judgments: (i) higher levels of consumption have higher worth; (ii) there is diminishing marginal valuation of consumption as consumption increases; and (iii) the marginal social utility of consumption is higher for the current generation than for a future generation of the same size with the same per capita consumption.

RICE adds a significant level of complexity to the original DICE model by incorporating the simultaneous growth paths of different regions. The exact objective function, or criterion to be maximized, for region J is:

$$(2.1) \quad W_J = \sum_t U[c_J(t), L_J(t)]R(t)$$

where W_J is the objective function of region J, $U[c_J(t), L_J(t)]$ is the utility of consumption for region J, $c_J(t)$ is the flow of consumption per capita during period t, $L_J(t)$ is the population at time t, and $R(t)$ is the pure time preference discount factor. The exact form of the utility function will be described shortly.

Utility is discounted by a factor that represents social time preference among different generations. The pure rate of time preference $\rho(t)$, which underlies the time preference discount factor $R(t)$, becomes an important parameter in this approach; the parameter $\rho(t)$ is assumed to decline over time, and the pure time preference discount factor is then given by:

$$(2.2) \quad R(t) = \prod_{v=0}^t [1 + \rho(v)]^{-10}$$

The pure rate of time preference is a choice parameter that is implicit in many societal decisions, such as fiscal and monetary policies. In conjunction with other parameters, it is closely connected with the market rate of interest (or marginal productivity of capital) and with the savings rate. The original RICE and DICE models used a constant pure rate of time preference of $\rho(t) = 3$ percent per year. The constant rate of 3 percent per year was considered to be consistent with historical savings data and interest rates. In DICE-99 and RICE-99, the pure rate of time preference is assumed to decline over time because of the assumption of declining impatience. The rate of time preference starts at 3 percent per year in 1995 and declines to 2.3 percent per year in 2100 and 1.8 percent per year in 2200.²

B. Economic constraints

The next set of equations represents the different regions. The first equation is the definition of utility, which was described and motivated in the previous subsection. Utility represents the current value of economic well-being and is assumed equal to the size of population $[L_j(t)]$ times the utility of per capita consumption $u[c_j(t)]$. Equation (2.3) uses the general case of a power function to represent the form of the utility function:³

$$(2.3) \quad U [c_j(t), L_j(t)] = L_j(t) \{c_j(t)^{1-\alpha} - 1\} / (1-\alpha)$$

In this equation, the parameter α is a measure of the social valuation of different levels of consumption, which has several interpretations. It represents the curvature of the utility function, the elasticity of the marginal utility of consumption, or the rate of inequality aversion. Operationally, it measures the extent to which a region is willing to reduce the welfare of high-consumption generations to improve the welfare of low-consumption generations. In the RICE and DICE models, we take (the limit of) $\alpha = 1$, which yields the logarithmic or Bernoullian utility function:

$$(2.3') \quad U [c_j(t), L_j(t)] = L_j(t) \{ \log [c_j(t)] \}$$

For most regions, the growth of population is assumed to follow an exponential path, and the basic projection method is as follows: Population growth in the initial period is taken from U.N. data, as discussed below. We then assume that the growth rate declines over time at a geometrically declining rate. More precisely, let $g^{\text{pop}}_j(t)$ be the population growth rate in region J and period t and δ^{pop}_j be its constant rate of decline. We then have the growth rate of population in time t as:

$$(2.4) \quad g^{\text{pop}}_j(t) = g^{\text{pop}}_j(0) \exp(-\delta^{\text{pop}}_j t)$$

It is easily verified that this assumption leads to a stable population. Its advantage is that the population trajectory can be represented by two parameters and can be easily

fit to different projections. The parameters chosen for RICE-99 produce a global population growth rate of 1.5 percent per year for the initial decade and a rate of decline in the global population growth rate of about 20 percent per decade. The global asymptotic maximum population is 11.5 billion people.

Production is represented by a modification of a standard neoclassical production function. For region J, output or GDP [$Q_J(t)$] is given by a constant-returns-to-scale Cobb-Douglas production function in capital [$K_J(t)$], labor [$L_J(t)$], and carbon-energy $ES_J(t)$. Carbon-energy represents “energy services.” Carbon emissions is related to energy services by an efficiency index function; this function changes over time to reflect carbon-saving technological change.

$$(2.5a) \quad Q_J(t) = \Omega_J(t) \{ A_J(t) K_J(t)^\gamma L_J(t)^{1-\beta_J-\gamma} ES_J(t)^{\beta_J} - c^E_J(t) ES_J(t) \}$$

$$(2.5b) \quad ES_J(t) = \varsigma_J(t) E_J(t)$$

In equation (2.5a), γ is the elasticity of output with respect to capital and is assumed to be 0.3. β_J is the elasticity of output with respect to energy services (discussed further below), and the term $(1 - \beta_J - \gamma)$ is the output elasticity with respect to labor. $A_J(t)$ represents the

level of Hick-neutral technological change. The term $\Omega_j(t)$ is a damage coefficient that relates to the impact of climate change on output and is described below. Labor inputs are equal to population; this is identical to assuming they are proportional to population and adjusting $A_j(t)$ by a constant factor. Capital accumulation is described below, and the carbon-energy aggregate is discussed next. The term $[c_j^E(t) ES_j(t)]$ in (2.5a) subtracts from gross output the costs of producing carbon-energy.

Equation (2.5b) then shows the relationship between carbon-energy inputs and energy services. Technological change in the energy sector is “carbon-augmenting,” where $\zeta_j(t)$ is the level of carbon-augmenting technology. Because of carbon-augmenting technological change, society is able to squeeze more energy services per unit of carbon-energy.

A major uncertainty in the model involves projecting the growth of $A_j(t)$, or total factor productivity (TFP), into the future. TFP growth is assumed to slow gradually over the next three centuries until eventually stopping. The exact technique for deriving estimates is described in Chapter 3, section 3, subsection A. The technical formula within the DICE and RICE models for projecting TFP is similar to that introduced above for population growth. Let $g_j^A(t)$ be the TFP growth rate in period t and δ_j^A be its constant rate of decline. Productivity growth at time t is then:

$$(2.6) \quad g^A_J(t) = g^A_J(0)\exp(-\delta^A_J t)$$

where δ^A_J is chosen so that $A_J(t)$ tends asymptotically to A_J^* , where A_J^* is the assumed asymptotic level of total factor productivity for region j .

In a one-sector closed economy $Q_J(t)$ equals $C_J(t) + I_J(t)$, where $C_J(t)$ is consumption and $I_J(t)$ is investment. In RICE-99, regions can trade carbon emissions permits for goods. With trade, the constraint on regional expenditures becomes

$$(2.7) \quad Q_J(t) + \tau_J(t)[\Pi_J(t) - E_J(t)] = C_J(t) + I_J(t)$$

where $\Pi_J(t)$ is the number of carbon emissions allowances allocated to region J and $\tau_J(t)$ is the price of each emissions permit. The second term on the left-hand side of (2.7) measures the net revenues a region receives from its purchase and sale of permits. If its emissions exceed its allocation of permits, it has to buy more permits than it sells, and its net revenue is negative. We will refer to $\tau_J(t)$ below as the “carbon tax,” because it functions just like a tax on carbon, but it can also be interpreted as the market price of emissions permits. The allocation of emissions permits is determined by agreement among the parties. Each region also takes the carbon tax to be exogenous.

A central research and policy issue is the number and composition of emissions

trading blocs. A trading bloc B is a set of regions for which the carbon tax (or permit price) is equalized and for which total emissions cannot exceed the total allocation of permits. Grouping regions into trading blocs makes it easy to analyze the impacts of policies such as the Kyoto Protocol that call for emissions trading. Equation set (2.7') gives the mathematical conditions for the permit allocations and carbon taxes in a trading bloc:

(2.7') $t_J(t) = t_b(t)$ for all J who are members of B

$$\sum_{J \in B} H_J(t) \geq \sum_{J \in B} E_J(t)$$

$$\sum_{J \in B} H_J(t) = \sum_{J \in B} E_J(t) \text{ if } t_b > 0$$

$$t_b(t) \geq 0$$

Each region is in exactly one trading bloc. The most frequent number of trading blocs in the cases considered in the book is one – the entire world.

The next equation is the definition of per capita consumption:

$$(2.8) \quad c_j(t) = C_j(t)/L_j(t)$$

The evolution of the capital stock is given by

$$(2.9) \quad K_j(t) = K_j(t-1)(1-\delta_k)^{10} + 10 \times I_j(t-1)$$

where δ_k is the annual rate of depreciation of the capital stock. We assume that capital depreciates at 10 percent per annum. The coefficient of 10 on $I_j(t-1)$ in equation (2.9) reflects the convention that investment is measured at annual rates while the period in the model is 10 years.

The next set of relations involves the supply side of the energy market. The cost of carbon-energy is:

$$(2.10) \quad c_j^E(t) = q(t) + \text{Markup}_j^E(t)$$

where $c_j^E(t)$ is the cost per unit of carbon-energy in region J, $q(t)$ is the wholesale price of carbon-energy exclusive of the Hotelling rent, and $\text{Markup}_j^E(t)$ is a markup on energy costs. The wholesale price, $q(t)$, is assumed to be equalized in different regions. The markup captures regional differences in transportation, distribution costs, and national energy taxes and is assumed to be constant over time. We interpret energy taxes as

Pigovian taxes that reflect the external costs of energy production and consumption.

Note that the cost of carbon-energy in (2.10) does not depend on $\zeta_j(t)$, the ratio of carbon-energy to carbon services. We have modeled carbon-saving technical change so that it has no output-enhancing effect. In RICE-99, total factor productivity, $A_j(t)$, increases aggregate productivity, but the role of decarbonization, $\zeta_j(t)$, is to reduce the ratio of carbon emissions to carbon-energy and carbon emissions.

The next equation defines cumulative use of carbon-energy:

$$(2.11) \quad \text{CumC}(t) = \text{CumC}(t-1) + 10 \times E(t)$$

where $\text{CumC}(t)$ is the cumulative consumption of carbon-energy at the end of period t and $E(t)$ is world use of carbon-energy in period t . $E(t)$ is the sum of carbon-energy use across regions.

The next equation represents the supply curve of carbon energy.

$$(2.12) \quad q(t) = \xi_1 + \xi_2[\text{CumC}(t)/\text{CumC}^*]^{\xi_3}$$

In equation (2.12), $q(t)$ is the wholesale (supply) price of carbon-energy while ξ_1 , ξ_2 , and ξ_3 are parameters.⁴ $CumC^*$ is a parameter which represents the inflection point beyond which the marginal cost of carbon-energy begins to rise sharply.

C. Concentrations, climate-change, and damage equations

The next set of relationships has proven a major challenge because there are no well-established empirical regularities and very little history that can be drawn upon to represent the linkage between economic activity and climate change. As with the economic relationships, it is desirable to use a parsimonious specification so that the theoretical model is transparent and so that the optimization model is empirically tractable. The methodology is drawn from macroeconomics, in which economic behavior is represented by equations that capture the behavior of broad aggregates (such as consumers or investors). The challenge in modeling climate-change economics is that aggregate relationships are needed for optimization approaches like the DICE and RICE models.

The first link is between economic activity and greenhouse-gas emissions. In the DICE/RICE-99 models, greenhouse gases affect climate through their radiative forcing. Of the suite of GHGs, only industrial CO_2 is endogenous in the model. The other GHGs (including CO_2 arising from land-use changes) are exogenous and projected on the basis

of current analysis by the IPCC, IIASA, and other scientific groups. Nearly 80 percent of the radiative forcing in 2100 comes from CO₂ in RICE-99, and more than 90 percent of cumulative CO₂ emissions come from industrial sources, so we devote most of our attention to industrial CO₂.

In the original DICE model, the accumulation and transportation of emissions were assumed to follow a simple process in which CO₂ decayed in the atmosphere at a constant rate. This has been revised in light of inconsistencies with established carbon-cycle modeling.

The new treatment uses a structural approach with a three-reservoir model calibrated to existing carbon-cycle models. The basic idea is that the deep oceans provide a limited, albeit vast, sink for carbon in the long run. In the new specification, we assume that there are three reservoirs for carbon — the atmosphere, a quickly mixing reservoir in the upper oceans and the biosphere, and the deep oceans. Each of the three reservoirs is assumed to be well-mixed in the short run, while the mixing between the upper reservoirs and the deep oceans is assumed to be extremely slow. We assume that CO₂ accumulation and transportation can be represented as the following linear three-reservoir model.

$$(2.13a) \quad M_{AT}(t) = 10 \times ET(t) + \phi_{11} M_{AT}(t-1) - \phi_{12} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$$

$$(2.13b) \quad M_{UP}(t) = \phi_{22} M_{UP}(t-1) + \phi_{12} M_{AT}(t-1) - \phi_{21} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1) - \phi_{23} M_{UP}(t-1)$$

$$(2.13c) \quad M_{LO}(t) = \phi_{33} M_{LO}(t-1) - \phi_{32} M_{LO}(t-1) + \phi_{23} M_{UP}(t-1)$$

where $M_{AT}(t)$ is the end-of-period mass of carbon in the atmosphere, $M_{UP}(t)$ is the mass of carbon in the upper reservoir (biosphere, and upper oceans), $ET(t)$ is global CO_2 emissions including those arising from land-use changes, and $M_{LO}(t)$ is the mass of carbon in the lower oceans. The coefficient ϕ_{ij} is the transfer rate from reservoir i to reservoir j (per period), where i and $j = AT, UP,$ and LO . The calibration of equations (2.13a), (2.13b), and (2.13c) is described in chapter three.

The next step concerns the relationship between the accumulation of GHGs and climate change. This sector uses the same specification as in the original DICE/RICE models because there have been no major developments that would lead to a revision of the underlying approach. Climate modelers have developed a wide variety of approaches for estimating the impact of rising GHGs on climatic variables. On the whole, existing models are much too complex to be included in economic models, particularly ones that are used for optimization. Instead, we employ a small structural model that captures the

basic relationship between GHG concentrations, radiative forcings, and the dynamics of climate change.

Accumulations of GHGs lead to global warming through increasing the warming at the surface by increased radiation. The relationship between GHG accumulations and increased radiative forcing, $F(t)$, is derived from empirical measurements and climate models. We characterize the relationship as follows:

$$(2.14) \quad F(t) = \eta \{ \log[M_{AT}(t)/M_{AT}^{PI}] / \log(2) \} + O(t)$$

where $M_{AT}(t)$ is the atmospheric concentration of CO_2 in billion metric tons of carbon (GtC) and $F(t)$ is the increase in radiative forcing since 1900 in watts per square meter (W/m^2), which is the standard measure of radiative forcing. $O(t)$ represents the forcings of other GHGs (CFCs, CH_4 , N_2O , and ozone) and aerosols. These other gases represent a small fraction of the total warming potential, their sources are poorly understood, and techniques for preventing their buildup are sketchy today; they are therefore taken as exogenous. The term M_{AT}^{PI} is the pre-industrial level of atmospheric concentrations of CO_2 (taken to be about 280 parts per million).

The list of exogenous components of forcing included in $O(t)$ represents a departure from previous versions of the RICE/DICE models, which considered CFCs to

be endogenous and did not include the effects of aerosols. The forcings from non-CO₂ GHGs and aerosols are much lower in the current version, reflecting lower anticipated effects of CFCs and the cooling effect of aerosols. These offset slightly higher projections of forcing from methane, nitrous oxide, and tropospheric ozone. All these issues are discussed in detail in the next chapter.

The parameterization of radiative forcing from CO₂ in (2.14) is not controversial. It relies upon a variety of data on atmospheric concentrations and combines those into a series on radiative forcing as described in the most recent comprehensive IPCC report (IPCC [1996a]). The major assumption for the present modeling is the finding that a doubling of CO₂ concentrations would lead to an increase in radiative forcing by 4.1 W/m².

The next set of equations provides the link between radiative forcing and climate change. Here again, the specification is identical to the original DICE/RICE models. Higher radiative forcings warm the atmospheric layer, which then warms the upper ocean, gradually warming the deep oceans. The lags in the system are primarily due to the thermal inertia of the different layers. We can write the model as follows:

$$(2.15a) \quad T(t) = T(t-1) + \sigma_1 \{F(t) - \lambda T(t-1) - \sigma_2 [T(t-1) - T_{LO}(t-1)]\}$$

$$(2.15b) \quad T_{LO}(t) = T_{LO}(t-1) + \sigma_3[T(t-1) - T_{LO}(t-1)]$$

where $T(t)$ is the increase in the globally and seasonally averaged temperature in the atmosphere and the upper level of the ocean since 1900. $T_{LO}(t)$ is the increase of temperature in the deep oceans. $F(t)$ is the increase in radiative forcing in the atmosphere, λ is a feedback parameter, and the σ_i are transfer coefficients reflecting the rates of flow and the thermal capacities of the different sinks.

Equations (2.15a) and (2.15b) can be understood in terms of a simple example of the impact of a warming source on a pool of water. Suppose that a heating lamp is turned on [this being the increase in $F(t)$ or radiative forcings]. The top part of the pool along with the air at the top are gradually warmed, and the lower part of the pool be gradually warmed as the heat diffuses to the bottom. The lags in the warming of the surface in this simple example are determined by the size of the pool (that is, by its thermal inertia) and by the rate of mixing of the different levels of the pool. This set of equations was fully described for the original DICE model in Nordhaus [1994b].

The next link in the chain is the economic impact of climate change on human and natural systems. Estimating the damages from greenhouse warming has proven extremely elusive. For the purpose of this study, it is assumed that there is a relationship between

the damage from greenhouse warming and the extent of warming. More specifically, the relationship between global-temperature increase and income loss is given by:

$$(2.16) \quad D_j(t) = \theta_{1,j} T(t) + \theta_{2,j} T(t)^2$$

where $D_j(t)$ is the damage from climate change for a region as a fraction of its output net of climate damages and relates the damage to the change in global mean temperature. The damage function is a quadratic function, and the damage relationships are described in Chapter 4.

Finally, we can include the damage function into the production function in (2.5) using the Ω coefficient as follows:

$$(2.17) \quad \Omega_j(t) = 1/[1+D_j(t)]$$

Equations (2.1) through (2.17) form the RICE-99 model that is analyzed in the subsequent chapters. Appendix A lists the equations of RICE-99 in a single place. The major variables are summarized in Appendix C. The GAMS computer code for the RICE-99 model is listed as Appendix D.

4. Equilibrium in the Market for Carbon-Energy

In a competitive equilibrium of the model sketched above, the demand for carbon energy satisfies the following condition:

$$(2.18a) \quad \beta_J \Lambda_J(t) E S_J(t)^{\beta_J - 1} = c_J^E(t) + h(t)/\zeta_J(t) + \tau_J(t)/\zeta_J(t)$$

where $\Lambda_J(t)$ is a scaling factor which equals $\Omega_J(t) A_J(t) K_J(t)^\gamma \Lambda_J(t)^{1-\gamma-\beta_J}$.

Rewriting, we obtain:

$$\begin{aligned}
 (2.18b) \quad E_J(t) &= [1/\zeta_J(t)] \{ [c_J^E(t) + h(t)/\zeta_J(t) + \tau_J(t)/\zeta_J(t)] \\
 &\quad / \beta_J \Lambda_J(t) \}^{1/(\beta_J-1)} \\
 &= \zeta_J^t [t_b(t)]
 \end{aligned}$$

The market price includes three terms: the cost of production of carbon-energy, the “Hotelling” rent representing the effect of current extraction of carbon fuels on future extraction costs, and the carbon tax. Both the carbon tax and the Hotelling rent are applied only to the carbon content of carbon-energy; they are therefore adjusted by the ratio of carbon to carbon-energy in (2.18a). Subtracting the regional markup from the market price yields the wholesale price of carbon-energy.

Summing (2.18b) across regions in a trading bloc and substituting in (2.7'), we get the equilibrium condition in the market for industrial emissions permits:

$$(2.19) \quad \sum_{J \in B} H_J(t) \geq \sum_{J \in B} z_J^t(t_b(t))$$

with the inequality becoming an equality if the carbon tax is greater than zero. $\zeta_J^t [t_b(t)]$ is the right-hand side of (2.18b). This equation says that total demand for emissions in a trading bloc cannot exceed the supply.

5. Policy in RICE-99

Policymakers (or modelers analyzing policy) can use either carbon taxes or emissions permits as the instrument of policy in RICE-99. In practice, there are many ways to accomplish these indirectly or in combination.

Equation (2.19) says that a policymaker can view either the carbon tax in each trading bloc or the total emissions permits allocated to each trading bloc as a policy variable. If the policymaker specifies the total permits for a trading bloc, then the carbon tax is determined by the necessity to equate demand and supply. If the policymaker specifies the carbon tax, then the total permit allocation of a trading bloc is determined, although the policymaker can choose how to split up the permits among the members of the trading bloc. The user can always satisfy (2.19) for any schedule of carbon taxes by simply granting each region permits equal to its emissions from (2.18b) at the market or equilibrium carbon tax.

Setting the carbon tax to zero in all regions will produce the “reference” or “baseline” case of the model, a projection of what will happen if no government action is taken to slow global warming. In the baseline case, emissions are determined by an unregulated market.

A Pareto-optimal policy – designed as a policy which induces the economically-efficient level of emissions – can be achieved by setting the carbon tax in each region equal to the global environmental shadow price of carbon. The environmental shadow price of carbon is the impact through environmental channels of a unit of emissions today on the present value of consumption in all regions in all future periods.

As we will see in later simulations, policies to slow global warming will have quite different costs and benefits in different regions. Some regions are likely to be more affected by climate change, and the costs of an efficient policy are also likely to be quite asymmetric. The allocation of carbon permits within a trading bloc is a way of influencing the distribution of gains and losses from climate-change policy. Granting a region emissions permits in excess of its emissions will transfer to that region permit revenues which are collected from other regions.⁵

Granting each region a number of permits equal to its emissions will ensure that no transfers occur via permit purchases and sales. A distribution of emissions permits which leads to no redistribution of income among nations is called a *revenue-neutral permit allocation*; this is equivalent to a regime in which countries set harmonized carbon taxes with no transfers among countries.

While we have interpreted the policy choice of the user as a permit-trading

arrangement, any combination of taxes and allocated permits that satisfies the constraints above could also be interpreted as a fiscal regime with a given carbon tax and tax revenues. The usual way in which a uniform carbon-tax plan is assumed to work, where regions harmonize their carbon taxes and each redistributes its revenues to its own citizens in a lump sum fashion, could be implemented in our model by setting carbon taxes equal in all regions and allocating permits in a revenue-neutral fashion.

Endnotes:

1. Although total carbon emissions include both industrial and land emissions, often we will refer to the endogenous component, industrial emissions, as simply “emissions” or “carbon emissions.”

2. A comprehensive review of issues involved in discounting the distant future is contained in the essays contained in Portney and Weyant [1999]. A full discussion of the discount rate question in the context of the DICE and RICE models is contained in Nordhaus [1994b] and Nordhaus [1998a].

3. This formulation subtracts one from the power function in the numerator of (3.1) so that the limit of the expression is $L_j(t)[\log(c_j(t))]$ as α tends to 1.

4. In earlier versions of the revised RICE model, a backstop technology was introduced at a cost of around \$500 per ton of carbon. The current RICE-99 and DICE-99 do not include backstop technologies. Omitting a backstop technology implies that the price of carbon-energy can rise to extremely high levels in the future; that also implies that the current Hotelling rent will be high relative to the with-backstop model and that emissions in the RICE-99 model are therefore somewhat lower than in a model with a backstop technology. Experiments indicate that the effect of adding a backstop technology is relatively small over the next century and not worth the additional complexity.

5. This assumes the carbon tax is not zero.