

## IDENTIFICATION IN DIFFERENTIATED PRODUCTS MARKETS USING MARKET LEVEL DATA

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We present new identification results for nonparametric models of differentiated products markets, using only market level observables. We specify a nonparametric random utility discrete choice model of demand allowing rich preference heterogeneity, product/market unobservables, and endogenous prices. Our supply model posits nonparametric cost functions, allows latent cost shocks, and nests a range of standard oligopoly models. We consider identification of demand, identification of changes in aggregate consumer welfare, identification of marginal costs, identification of firms' marginal cost functions, and discrimination between alternative models of firm conduct. We explore two complementary approaches. The first demonstrates identification under the same nonparametric instrumental variables conditions required for identification of regression models. The second treats demand and supply in a system of nonparametric simultaneous equations, leading to constructive proofs exploiting exogenous variation in demand shifters and cost shifters. We also derive testable restrictions that provide the first general formalization of Bresnahan's (1982) intuition for empirically distinguishing between alternative models of oligopoly competition. From a practical perspective, our results clarify the types of instrumental variables needed with market level data, including tradeoffs between functional form and exclusion restrictions.

**KEYWORDS:** Nonparametric identification, discrete choice models, differentiated products oligopoly, simultaneous equations models.

### 1. INTRODUCTION

MODELS OF DISCRETE CHOICE BETWEEN DIFFERENTIATED PRODUCTS PLAY a central role in the modern industrial organization (IO) literature and are used in a wide range of other applied fields of economics.<sup>2</sup> Often, the discrete

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<sup>2</sup>Applications include studies of the sources of market power (e.g., Berry, Levinsohn, and Pakes (1995), Nevo (2001)), mergers (e.g., Nevo (2000), Capps, Dranove, and Satterthwaite (2003)), welfare gains from new goods/technologies (e.g., Petrin (2002), Eizenberg (2011)), network effects (e.g., Rysman (2004), Nair, Chintagunta, and Dube (2004)), product promotions (e.g., Chintagunta and Honoré (1996), Allenby and Rossi (1999)), environmental policy (e.g., Goldberg (1998)), vertical contracting (e.g., Villas-Boas (2007), Ho (2009)), equilibrium product quality (e.g., Fan (2013)), media bias (e.g., Gentzkow and Shapiro (2010)), asymmetric information and insurance (e.g., Cardon and Hendel (2001), Bundorf, Levin, and Mahoney (2012), Lustig (2010)), trade policy (e.g., Goldberg (1995), Berry, Levinsohn, and Pakes (1999), Goldberg and Verboven (2001)), residential sorting (e.g., Bayer, Ferreira, and McMillan (2007)), voting (e.g., Gordon and Hartmann (2010)), and school choice (e.g., Hastings, Kane, and Staiger (2010), Neilson (2013)).

choice demand model is combined with an oligopoly model of supply in order to estimate markups, predict equilibrium responses to policy, or test hypotheses about firm behavior. Typically, these models are estimated using econometric specifications incorporating functional form restrictions and parametric distributional assumptions. Such restrictions may be desirable in practice: estimation in finite samples always requires approximations and, since the early work of McFadden (1974), an extensive literature has developed providing flexible discrete choice models well suited to estimation and inference. Furthermore, parametric structure is necessary for the extrapolation involved in many out-of-sample predictions. However, an important question is whether these parametric functional form and distributional assumptions play a more fundamental role in determining what is learned from the data. In particular, are such assumptions essential for identification?

Here, we examine the nonparametric identifiability of models in the spirit of Berry, Levinsohn, and Pakes (1995) (henceforth, "BLP") and a large applied literature that has followed. We focus on the common situation in which only market level data are available, as in BLP. In such a setting, one observes market shares, market characteristics, product prices and characteristics, and product/market level cost shifters, but not individual choices or firm costs. We consider identification of demand, identification of changes in aggregate consumer welfare, identification of marginal costs, identification of firms' marginal cost functions, and discrimination between alternative models of firm conduct. We also provide guidance for applied work by focusing attention on the essential role of instrumental variables, clarifying the types of instruments needed in this setting, and pointing out tradeoffs between functional form and exclusion restrictions. Our primary motivation is to develop a nonparametric foundation for a class of empirical models used widely in practice. Nonetheless, our analysis may also suggest new estimation and/or testing approaches (parametric, semiparametric, or nonparametric).

On the demand side, the models motivating our work incorporate two essential features. One is rich heterogeneity in preferences, which allows flexibility in demand substitution patterns.<sup>3</sup> The second is the presence of product/market level unobservables. Because these unobservables are known by firms and consumers, they give rise to endogeneity of prices. Only by explicitly modeling these unobservables can one account simultaneously for endogeneity and heterogeneity in preferences for product characteristics (see Section 2). Both features are essential to reliable estimation of demand elasticities in differentiated products markets. Surprisingly, this combination of features has not been treated in the prior literature on identification. Indeed, although there is a

<sup>3</sup>See, for example, the discussions in Domencich and McFadden (1975), Hausman and Wise (1978), and Berry, Levinsohn, and Pakes (1995). Early models of discrete choice with heterogeneous tastes for characteristics include those in Quandt (1966, 1968).

large literature on identification of discrete choice models, there are no nonparametric or semiparametric identification results for even the linear random coefficients random utility model widely used in the applied literature that motivates us.

On the supply side, the empirical literature on differentiated products employs equilibrium oligopoly models, building on early insights of Rosse (1970) and Bresnahan (1981). By combining the model of oligopoly competition with estimates of demand, one can infer marginal costs and examine a range of market counterfactuals. Following BLP, recent work typically allows for latent cost shocks and unobserved heterogeneity in cost functions, but employs a parametric specification of costs.

Our results show that the primary requirement for identification is the availability of instrumental variables. It is not surprising that instruments are needed. Less obvious is what types of exclusion restrictions suffice in this setting. Focusing on the demand side, it is intuitive that identification requires instruments generating exogenous variation in choice sets, including changes in prices. For example, BLP exploited a combination of exogenous own-product characteristics, characteristics of alternative products, and additional shifters of markups and/or costs. Following Bresnahan (1981), BLP made an intuitive argument that changes in the exogenous characteristics of competing products should help to identify substitution patterns. Ignoring the other shifters of markups and costs, characteristics of competing products are sometimes referred to as “BLP instruments.” However, there has been no general formal statement about the role of “BLP instruments”—why they would aid identification and whether they are sufficient alone to identify demand. One difficult question has been: how could product characteristics that are not excluded from the demand system help to identify demand?

We show that the “BLP instruments” are in fact useful. For our formal results, they are necessary but not sufficient: we require additional instruments such as cost shifters or proxies for costs (e.g., prices in other markets). However, we also discuss tradeoffs between functional form and exclusion restrictions, including cases in which cost shifters alone or the “BLP instruments” alone could suffice. In Berry and Haile (2010), we considered identification of demand when one has consumer level choice data. There, we obtained results requiring fewer instruments, in some cases allowing identification when only exogenous product characteristics (the “BLP instruments”) are available.

In the following, we begin with demand, positing a nonparametric random utility discrete choice model. We require one important index restriction on how product/market-specific unobservables enter preferences, but the model is otherwise very general. Our first result shows that standard nonparametric instrumental variables conditions (completeness conditions discussed in, e.g., Newey and Powell (2003) or Andrews (2011)) suffice for identification of demand. As usual, adding a requirement of quasilinear preferences can then allow identification of changes in aggregate consumer welfare.

We then move to the supply side of the model. Given identification of demand, specifying a model of oligopoly competition allows identification of marginal costs through firms' first-order conditions. Identification of firms' marginal cost *functions* then obtains with the addition of standard nonparametric instrumental variables conditions.

Next, we drop the nonparametric instrumental variables conditions and consider an alternative approach, combining the demand model and supply model in a single system of nonparametric simultaneous equations. This approach requires some additional structure and stronger exclusion conditions, but enables us to offer constructive proofs using more transparent variation in excluded demand shifters and cost shifters. These results therefore complement those obtained using more abstract completeness conditions.

Finally, we consider discrimination between alternative models of firm "conduct," that is, alternative models of oligopoly competition. The nature of oligopoly competition is itself a fundamental question of modern industrial organization, and, in practice, the choice of supply model can have important implications for estimates, counterfactual simulations, and policy implications. We offer the first general formalization of Bresnahan's (1981, 1982) early intuition for empirically discriminating between alternative "oligopoly solution concepts." Our results do not require "rotations" of demand. And, unlike prior formal results (Lau (1982)), ours allow product differentiation, heterogeneous firms, latent shocks to demand and costs, and oligopoly models outside the problematic "conjectural variations" framework.

Together, these results provide a positive message regarding the faith we may have in a large and growing body of applied work on differentiated products markets. This message is not without qualification: in addition to the index restrictions, our results require instruments with sufficient variation on both the demand and cost side. However, while "adequate exogenous variation" is a strong requirement for any nonparametric model, the requirement here is no stronger than for regression models. Put differently, the functional form and distributional assumptions typically used in practice play their usual roles: approximation in finite samples and compensation for the gap between the exogenous variation available in practice and that required to discriminate between all nonparametric models.

To our knowledge, we provide the first and only results on the nonparametric identification of market level differentiated products models of the sort found in BLP and related applications in IO. However, our work is connected to several theoretical and applied literatures. In the following section, we briefly place our work in the context of the prior literature. We then set up the model in Section 3. Our analysis based on nonparametric instrumental variable conditions is presented in Section 4. Section 5 develops our alternative simultaneous equations approach. We take up discrimination between oligopoly models in Section 6. Section 7 provides a discussion focused on tradeoffs between functional form and exclusion restrictions. We conclude in Section 8.

## 2. RELATED LITERATURE

There is a large body of work on the identification of discrete choice models.<sup>4</sup> Much of that literature considers models allowing for heterogeneous preferences through a random coefficients random utility specification, but ruling out endogeneity. Ichimura and Thompson (1998), for example, studied a linear random coefficients binary choice model. Briesch, Chintagunta, and Matzkin (2010) considered multinomial choice, allowing some generalization of a linear random coefficients model. In contrast to this literature, our formulation of the underlying random utility model specifies only random utilities, not random parameters that interact with observables to generate random utilities. This allows us to substantially relax functional form and distributional assumptions relied on in earlier work.

Also essential to our demand model is the endogeneity of prices. Several papers address the identification of discrete choice models with endogeneity. Examples include Lewbel (2000, 2006), Honoré and Lewbel (2002), Hong and Tamer (2003), Blundell and Powell (2004), and Magnac and Maurin (2007). These considered linear semiparametric models, allowing for a single additive scalar shock (analogous to the extreme value or normal shock in logit and probit models) that may be correlated with some observables. Among these, Lewbel (2000, 2006) considered multinomial choice. Extensions to nonadditive shocks were considered in Matzkin (2007a, 2007b). Compared to these papers, we relax some functional form restrictions and, more fundamentally, add the important distinction between market/choice-specific unobservables and individual heterogeneity in preferences. This distinction allows the model to define comparative statics that account for both heteroskedasticity (heterogeneity in tastes for product characteristics) and endogeneity.<sup>5</sup> For example, to define a demand elasticity, one must quantify the changes in market shares resulting from an exogenous change in price. Accounting for heterogeneity in consumers' marginal rates of substitution between income and other characteristics requires allowing the price change to affect the covariance matrix (and other moments) of utilities. On the other hand, controlling for endogeneity requires holding fixed the market/choice-specific unobservables. Meeting both requirements is impossible in models with a single composite error for each product.

Blundell and Powell (2004), Matzkin (2004), and Hoderlein (2009) have considered binary choice with endogeneity in semiparametric triangular models, leading to the applicability of control function methods or the related idea

<sup>4</sup>Important early work includes Manski (1985, 1988) and Matzkin (1992, 1993), which examined semiparametric models with exogenous regressors.

<sup>5</sup>Matzkin (2004) (Section 5.1) made a distinction between choice-specific unobservables and an additive preference shock, but in a model without random coefficients or other sources of heteroskedasticity/heterogeneous tastes for product characteristics. See also Matzkin (2007a, 2007b).

of “unobserved instruments” (see also Petrin and Train (2010), Altonji and Matzkin (2005), and Gautier and Kitamura (2013)).<sup>6</sup> However, standard models of oligopoly pricing in differentiated products markets imply that each equilibrium price depends, in general, on the entire vector of demand shocks (and typically, the vector of cost shocks as well). This rules out a triangular structure and, therefore, a control function approach except under restrictive functional form restrictions (see Blundell and Matzkin (2010)). Nonetheless, some of our results use a related strategy of inverting a multiproduct supply and demand system to recover the entire vector of shocks to costs and demand. This could be interpreted as a generalization of the control function approach. It is also related to the small literature on nonparametric simultaneous equations. Indeed, we show that our demand and supply model can be transformed into a system of simultaneous equations with a general form first explored by Matzkin (2007a, 2008) (see also the discussion in Section 6). Here, we provide both a new result and an application of a result in Berry and Haile (2013).

In the literature on oligopoly supply, Rosse (1970) introduced the idea of using first-order conditions for imperfectly competitive firms to infer marginal costs from prices and demand parameters. Although Rosse considered the case of monopoly, subsequent work extended the idea to models of simultaneous price-setting, quantity-setting, and to “conjectural variations” models. In a simple case, this literature might consider an IV regression of “price minus markup” on a parametric form for marginal cost, including an additive unobserved cost shock. Bresnahan (1981, 1987) estimated the supply model using the reduced form for prices implied by the demand parameters and oligopoly pricing first-order conditions. BLP incorporated structural errors (demand shocks and cost shocks) and explicitly solved the multiproduct oligopoly pricing first-order conditions for the implied levels of marginal cost. Their approach is a parametric version of the nonparametric strategy we take. However, rather than focusing on a single model of supply, we exploit the “connected substitutes” property of demand (Berry, Gandhi, and Haile (2013)) in order to provide identification results applicable to a wide range of oligopoly supply models.

Our insights regarding discrimination between alternative oligopoly models are related to ideas from the early empirical IO literature on inferring firm conduct from market outcomes. Bresnahan (1982), in particular, provided influential intuition for how “rotations of demand” could distinguish between alternative oligopoly models. While Bresnahan’s intuition suggested wide applicability, formal results (Lau (1982)) have been limited to deterministic homogeneous goods conjectural variations models (with linear “conjectures”), and have required exogenous variation that alters both the level and slope of aggregate demand (a point emphasized by Nevo (1998)).<sup>7</sup> Our results avoid

<sup>6</sup>See also Chesher (2003) and Imbens and Newey (2009).

<sup>7</sup>Bresnahan’s (1982) intuitive insight concerns discrimination between pairs of oligopoly models. Lau (1982) focused on the particular implementation recommended by Bresnahan (1982,

these restrictions and point to a broader class of cross-market or cross-product variation leading to testable restrictions that can discriminate between alternative models.

Turning to concurrent unpublished papers, Berry and Haile (2009) explored related ideas in the context of a “generalized regression model” (Han (1987)). Berry and Haile (2010) considered identification of discrete choice models in the case of “micro data,” where one observes the choices of individual consumers as well as characteristics specific to each consumer and product. The distinction between “market data” and “micro data” has been emphasized in the recent industrial organization literature (e.g., Berry, Levinsohn, and Pakes (2004)), but not the econometrics literature. A key insight in Berry and Haile (2010) is that, within a market, the market/choice-specific unobservables are held fixed. One can therefore learn key features of a random utility model by exploiting within-market variation—variation that is not confounded by variation in the market/choice-specific unobservables. Such a strategy cannot be applied to market level data, but was exploited throughout Berry and Haile (2010). There, we show that this additional variation can reduce the need for exclusion restrictions, can make additional instruments available, and can allow one to drop the index structure relied on throughout the present paper. Matzkin (2010) considered estimation of simultaneous equations models of the form studied by Matzkin (2008) and included a pair of new identification results that could be extended to that form. Fox and Gandhi (2009) explored the identifiability of a discrete choice model in which consumer types are multinomial and conditional indirect utility functions are analytic.<sup>8</sup> Fox and Gandhi (2011) explored an extension in which the dimension of the product/market level unobservables exceeds that of the choice set, showing that *ex ante* average features of demand can still be identified. Chiappori and Komunjer (2009) combined a simultaneous equations approach with completeness conditions to consider identification using “micro data” (cf. Berry and Haile (2010)) in a random utility discrete choice demand model with an additive structure.

### 3. DEMAND MODEL

#### 3.1. *Random Utility Discrete Choice*

Each consumer  $i$  in market  $t$  chooses a good  $j$  from a set  $\mathcal{J}_t = \{0, 1, \dots, J_t\}$ . We use the terms “good” and “product” interchangeably. A “market” consists

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1989), which involves estimation of conjectural variations parameters (often referred to as “conduct parameters”). Corts (1999) and Reiss and Wolak (2007) discussed fundamental problems with the conduct parameter approach.

<sup>8</sup>Bajari, Fox, Kim, and Ryan (2012) considered identification in a linear random coefficients model without endogeneity, assuming that the distribution of an additive i.i.d. preference shock is known. Gandhi, Kim, and Petrin (2011) considered identification and estimation of a particular parametric variation on the standard BLP model.

of a continuum of consumers (with total measure  $M_t$ ) in the same choice environment. In practice, markets are often defined geographically and/or temporally; however, other notions are permitted. For example, residents of the same city with different incomes, races, or family sizes might be split into distinct markets. Formally, a market  $t$  is defined by  $(\mathcal{J}_t, \chi_t)$ , where

$$\chi_t = (x_t, p_t, \xi_t)$$

(with support  $\mathcal{X}$ ) represents the characteristics of products and/or markets. Observed exogenous characteristics of the products or market are represented by  $x_t = (x_{1t}, \dots, x_{J_t t})$ , with each  $x_{jt} \in \mathbb{R}^K$ . The vector  $\xi_t = (\xi_{1t}, \dots, \xi_{J_t t})$ , with each  $\xi_{jt} \in \mathbb{R}$ , represents unobservables at the level of the product and/or market. These may reflect unobserved product characteristics and/or unobserved variation in tastes across markets. Although a scalar unobservable for each product  $j > 0$  is standard in the literature, it is an important restriction that will constrain the ways in which the distribution of utilities can vary across markets. On the other hand, a need to have no more than one structural error per observed choice probability may not be surprising.

Finally,  $p_t = (p_{1t}, \dots, p_{J_t t})$ , with each  $p_{jt} \in \mathbb{R}$ , represents endogenous observable characteristics, that is, those correlated with the structural errors  $\xi_t$ . The restriction to one scalar endogenous observable for each good/market reflects standard practice but is not essential.<sup>9</sup> We refer to  $p_{jt}$  as “price,” reflecting the leading case. Without loss of generality, we henceforth condition on  $\mathcal{J}_t = \mathcal{J} = \{0, 1, \dots, J\}$ .<sup>10</sup>

Consumer preferences are represented with a random utility model.<sup>11</sup> Consumer  $i$  in market  $t$  has conditional indirect utilities

$$v_{i0t}, v_{i1t}, \dots, v_{iJ_t t}$$

for the goods. For simplicity, we will refer to these as “utilities.” Without loss, we normalize utilities relative to that of good 0 for each consumer, implying

$$v_{i0t} = 0 \quad \forall i, t.$$

Any observed characteristics of good 0 in market  $t$  are then treated as market-specific characteristics common to all products  $j > 0$  in that market. In applications, it is common for good 0 to represent the composite “outside good,”

<sup>9</sup>The modifications required to allow higher dimensional  $p_{jt}$  are straightforward, although the usual challenge of finding adequate instruments would remain. See the discussion in Appendix C, which also demonstrates that, in some special cases, additional instruments will not be necessary. For an empirical analysis with multiple endogenous characteristics per product, see Fan (2013).

<sup>10</sup>We do not impose any restriction linking distributions of preferences across markets with different values of  $J_t$ . An interesting question is whether imposing such restrictions would allow weakening of our identification conditions.

<sup>11</sup>See, for example, Block and Marschak (1960), Marschak (1960), McFadden (1974), and Manski (1977) for pioneering work.

that is, the decision to purchase none of the goods explicitly studied, and this outside good may have no observable characteristics.<sup>12</sup> Our formulation allows this as well.

Conditional on  $\chi_t$ , the utilities  $(v_{i1t}, \dots, v_{iJt})$  are independent and identically distributed (i.i.d.) across consumers and markets, with joint distribution

$$F_v(v_{i1t}, \dots, v_{iJt} | \chi_t).$$

We assume  $\arg \max_{j \in \mathcal{J}} v_{ijt}$  is unique with probability 1. Choice probabilities (market shares) are then given by

$$(1) \quad s_{jt} = \sigma_j(\chi_t) = \Pr\left(\arg \max_{k \in \mathcal{J}} v_{ikt} = j | \chi_t\right), \quad j = 0, \dots, J.$$

Of course,  $\sigma_0(\chi_t) = 1 - \sum_{j=1}^J \sigma_j(\chi_t)$ . We assume that, for all  $\chi_t \in \mathcal{X}$ ,  $\sigma_j(\chi_t) > 0$  for all  $j \in \mathcal{J}$ .<sup>13</sup> Let  $s_t = (s_{1t}, \dots, s_{Jt})$  and  $\sigma(\chi_t) = (\sigma_1(\chi_t), \dots, \sigma_J(\chi_t))$ .

### 3.2. An Index Restriction

So far, the only restriction placed on the random utility model is the restriction to a scalar product/market unobservable  $\xi_{jt}$  for each  $t$  and  $j = 1, \dots, J$ . We now add an important index restriction. Partition  $x_{jt}$  as  $(x_{jt}^{(1)}, x_{jt}^{(2)})$ , with  $x_{jt}^{(1)} \in \mathbb{R}$ . Let  $x_t^{(1)} = (x_{1t}^{(1)}, \dots, x_{Jt}^{(1)})$  and  $x_t^{(2)} = (x_{1t}^{(2)}, \dots, x_{Jt}^{(2)})$ . Define the linear indices

$$(2) \quad \delta_{jt} = x_{jt}^{(1)} \beta_j + \xi_{jt}, \quad j = 1, \dots, J$$

and let  $\delta_t = (\delta_{1t}, \dots, \delta_{Jt})$ .

ASSUMPTION 1:  $F_v(\cdot | \chi_t) = F_v(\cdot | \delta_t, x_t^{(2)}, p_t)$ .

Assumption 1 requires that  $\xi_t$  and  $x_t^{(1)}$  affect the distribution of utilities only through the indices  $\delta_t$ . With this assumption, variation in  $p_{jt}$  and  $x_{jt}^{(2)}$  can have arbitrary effects on the way variation in  $\xi_{jt}$  changes the distribution of  $v_{ijt}$ , but  $x_{jt}^{(1)}$  and  $\xi_{jt}$  are perfect substitutes. Put differently, the marginal rate of substitution between a unit of the characteristic measured by  $x_{jt}$  and that measured by  $\xi_{jt}$  must be constant. This linear structure is actually stronger than necessary,

<sup>12</sup>We have implicitly assumed that any unobservable specific to good 0 is held fixed. In the special case in which the unobservables  $\xi_{jt}$  enter utilities linearly, a nonconstant unobservable  $\xi_{0t}$  could be accommodated simply by redefining the structural errors as  $\tilde{\xi}_{jt} = \xi_{jt} - \xi_{0t} \forall j > 0$ . In Berry, Gandhi, and Haile (2013) we showed that, outside the case of single discrete choice, one can use an artificial notion of “good zero” that allows stochastic unobservables for every real good. This yields a demand system with the same structure studied here, but where “good zero” is only a technical device rather than one of the available options.

<sup>13</sup>It should be clear that at most a bound could be obtained on the distribution of utility for a good with zero market share.

and we show in Appendix B that identification with a nonparametric IV approach can also be obtained with nonseparable indices  $\delta_{jt} = \delta_j(x_{jt}^{(1)}, \xi_{jt})$ . The essential requirement is that  $x_{jt}^{(1)}$  and  $\xi_{jt}$  enter through an index that is strictly monotonic in  $\xi_{jt}$ .

EXAMPLE 1: In applied work, it is common to generate the conditional joint distributions  $F_v(\cdot|\chi_t)$  from an analytical form for random utility functions

$$v_{ijt} = v_j(x_t, \xi_t, p_t; \theta_{it}),$$

where  $\theta_{it}$  is a finite-dimensional parameter. The most common form is a linear random coefficients model, for example,

$$(3) \quad v_{ijt} = x_{jt}\beta_{it} - \alpha_{it}p_{jt} + \xi_{jt} + \epsilon_{ijt},$$

with  $\theta_{it} = (\alpha_{it}, \beta_{it}, \epsilon_{i1t}, \dots, \epsilon_{ijt})$  independent of  $\chi_t$  and i.i.d. across consumers and markets. Endogeneity of price is reflected by correlation between  $p_{jt}$  and  $\xi_{jt}$  conditional on  $x_{jt}$ . The specification (3) generalizes the BLP model by dropping its parametric distributional assumptions and allowing correlation among the components of  $\theta_{it}$ .<sup>14</sup> In this model, a sufficient condition for Assumption 1 is that one component of  $x_{jt}$  have a degenerate coefficient, an assumption made in most applications. Compared to this example, our model relaxes several restrictions. For example, we do not require the linearity of (3), finite-dimensional  $\theta_{it}$ , or even the exclusion of  $(x_{kt}, \xi_{kt}, p_{kt})$  from  $v_j(\cdot)$  for  $j \neq k$ .

Because the unobservables  $\{\xi_{1t}, \dots, \xi_{Jt}\}$  have no natural location or scale, we must normalize them in order to have a unique representation of preferences.<sup>15</sup> We normalize the scale by setting  $\beta_j = 1 \forall j$ , yielding

$$\delta_{jt} = x_{jt}^{(1)} + \xi_{jt}.$$

This leaves a location normalization on each  $\xi_{jt}$  to be made later.

Henceforth, we condition on an arbitrary value of  $x_t^{(2)}$  and suppress it. Although this provides considerable simplification of notation, we emphasize that the distribution of utilities can vary in a quite general way with  $x_t^{(2)}$ . For simplicity, we then let  $x_{jt}$  represent  $x_{jt}^{(1)}$  and let  $\chi_t$  (with support  $\mathcal{X}$ ) now represent  $(x_t^{(1)}, p_t, \xi_t)$ . Given Assumption 1, we will often abuse notation and write

$$\sigma(\delta_t, p_t)$$

instead of  $\sigma(\chi_t)$  for the sake of clarity.

<sup>14</sup>In the BLP model, price enters through a nonlinear interaction with random coefficients. Since the distinction has no substantive implication for our purposes, we ignore this and refer to the model with linear interactions as the BLP model.

<sup>15</sup>For example, let  $\tilde{\xi}_t = \alpha + \xi_t\beta$  and  $\tilde{F}_v(\cdot|x_t, p_t, \tilde{\xi}_t) = F_v(\cdot|x_t, p_t, \xi_t) \forall (x_t, p_t, \xi_t)$ . Then, for every  $(\alpha, \beta) \in \mathbb{R}^J \times \mathbb{R}_+^J$ , we have a different representation of the same preferences.

3.3. *Connected Substitutes*

Standard discrete choice models inherently involve choice among weak gross substitutes.<sup>16</sup> For example, if  $v_{ijt}$  is strictly decreasing in its price  $p_{jt}$ , a fall in  $p_{jt}$  will (all else equal) raise the market share of good  $j$  and (weakly) lower market shares of all other goods. With the following “connected substitutes” assumption (Berry, Gandhi, and Haile (2013)), we will strengthen this notion of choice among substitute goods in two ways. First, we require that the index  $\delta_{jt}$  also “act like (minus) price” in the sense that a rise in  $\delta_{jt}$  will (all else equal) weakly lower the market shares of all goods  $k \neq j$ . Second, we will require a minimal degree of *strict* substitution among the goods.

DEFINITION 1: Let  $\lambda_t$  denote either  $-p_t$  or  $\delta_t$ . Goods  $(0, 1, \dots, J)$  are *connected substitutes in  $\lambda_t$*  if both

- (i)  $\sigma_k(\delta_t, p_t)$  is nonincreasing in  $\lambda_{jt}$  for all  $j > 0, k \neq j, (\delta_t, p_t) \in \mathbb{R}^{2J}$ ; and
- (ii) at each  $(\delta_t, p_t) \in \text{supp}(\delta_t, p_t)$ , for any nonempty  $\mathcal{K} \subseteq \{1, \dots, J\}$  there exist  $k \in \mathcal{K}$  and  $\ell \notin \mathcal{K}$  such that  $\sigma_\ell(\delta_t, p_t)$  is strictly decreasing in  $\lambda_{kt}$ .

ASSUMPTION 2: *Goods  $(0, 1, \dots, J)$  are connected substitutes in  $-p_t$  and in  $\delta_t$ .*

Following the definition, Assumption 2 has two parts. Part (i) requires the goods to be weak gross substitutes in  $p_t$  and in  $\delta_t$ . Part (ii) requires some strict substitution as well—loosely speaking, enough to justify treating  $\mathcal{J}$  as the relevant choice set. In particular, if part (ii) failed, there would be some strict subset  $\mathcal{K}$  of goods that substitute only to other goods in  $\mathcal{K}$ . A sufficient condition for Assumption 2 is that each  $v_{ijt}$  be strictly increasing in  $\delta_{jt}$  and  $-p_{jt}$ , but unaffected by  $(\delta_{kt}, p_{kt})$  for  $k \neq j$ . Given the index restriction, this would be standard. But this is stronger than necessary; for example, while Assumption 2 implies that the aggregate choice probability  $\sigma_j(\delta_t, p_t)$  is strictly increasing in  $\delta_{jt}$  (see Berry, Gandhi, and Haile (2013)), it still allows an increase in  $\delta_{jt}$  to lower  $v_{ijt}$  for a positive measure of consumers. Thus,  $x_{it}$  and  $\xi_{jt}$  need not be vertical characteristics.<sup>17</sup>

Berry, Gandhi, and Haile (2013) provided additional discussion of the connected substitutes property and demonstrated that it captures a feature common among standard models of differentiated products demand. This will be a key condition allowing us to “invert” our supply and demand models.

<sup>16</sup>Complements can be accommodated by defining the “products” as bundles of the individual goods (e.g., Gentzkow (2007)).

<sup>17</sup>For example,  $x_{jt}^{(1)}$  and  $\xi_{jt}$  could be observed and unobserved factors contributing to a “horizontal” characteristic—say, the acceleration capacity of a car—which consumers as a whole like, but which some consumers dislike. This is an additional way in which our setup generalizes the standard linear random coefficients model of Example 1.

### 3.4. Observables

Let  $z_t$  denote a vector of instruments excluded from the demand model (we discuss exclusion restrictions below). In the market data environment, the observables include  $(M_t, x_t, p_t, s_t, z_t)$ . When we add a model of supply, we will sometimes add to this list a vector of exogenous cost shifters  $w_t$  and instruments  $y_t$  excluded from firms' marginal cost functions. Allowable overlap between elements of  $(M_t, w_t, x_t, z_t, y_t)$  will be characterized below.

## 4. IDENTIFICATION UNDER NONPARAMETRIC IV CONDITIONS

### 4.1. Identification of Demand

We first consider identification of the demand system, that is, of the mapping

$$\sigma(\cdot) = (\sigma_1(\cdot), \dots, \sigma_J(\cdot)).$$

A key step in our identification argument is the inversion of the system of share equations

$$(4) \quad s_{jt} = \sigma_j(\delta_t, p_t), \quad j = 0, \dots, J.$$

The following result, shown in Berry, Gandhi, and Haile (2013), generalizes a well-known invertibility result for linear discrete choice models in Berry (1994).

**LEMMA 1:** *Consider any price vector  $p$  and any market share vector  $s = (s_1, \dots, s_J)$  such that  $s_j > 0$  for all  $j$  and  $\sum_{j=1}^J s_j < 1$ . Under Assumptions 1 and 2, there is at most one vector  $\delta$  such that  $\sigma_j(\delta, p) = s_j \forall j$ .*

With this result, for any  $(s_t, p_t)$  in their support we can write

$$(5) \quad \delta_{jt} = \sigma_j^{-1}(s_t, p_t), \quad j = 1, \dots, J,$$

or

$$(6) \quad x_{jt} + \xi_{jt} = \sigma_j^{-1}(s_t, p_t), \quad j = 1, \dots, J.$$

This equation provides a reminder of the need to normalize the location of each  $\xi_{jt}$ . In this section, we choose the normalization

$$E[\xi_{jt}] = 0, \quad j = 1, \dots, J.$$

Equation (6) has a form similar to, but different from, a standard nonparametric regression model with endogenous regressors and a linear structural error. The standard model takes the form  $\tilde{y}_t^{(1)} - \phi(\tilde{y}_t^{(2)}) = \tilde{u}_t$ , where  $(\tilde{y}_t^{(1)}, \tilde{y}_t^{(2)})$

are endogenous and  $\phi(\cdot)$  is unknown. Newey and Powell (2003) showed identification of  $\phi(\cdot)$  using excluded instrumental variables with dimension at least equal to that of  $\tilde{y}_t^{(2)}$ . In similar notation, our inverted demand model (6) takes the form  $\phi_j(\tilde{y}_{jt}^{(1)}, \tilde{y}_{jt}^{(2)}) - \tilde{x}_{jt} = \tilde{u}_{jt}$  for each  $j$ . Here, all endogenous variables appear inside the unknown transformation  $\phi_j$ , and an essential instrument,  $\tilde{x}_{jt}$ , is not excluded. This precludes direct application of the Newey–Powell result. However, we will see that following their identification *proof* will allow us to demonstrate identification in our setting, using as instruments the exogenous  $\tilde{x}_t \in \mathbb{R}^J$  in combination with excluded instruments of dimension at least equal to that of  $\tilde{y}_t^{(2)}$ .

Before stating the instrumental variables conditions, recall that, in addition to the exogenous product characteristics  $x_t$ , the vector  $z_t$  represents instruments for  $p_t$  excluded from the determinants of  $(v_{it}, \dots, v_{jt})$ . Standard excluded instruments include cost shifters (e.g., input prices) or proxies for cost shifters such as prices of the same good in other markets (e.g., Hausman (1996), Nevo (2001)). In some demand applications, characteristics of consumers in “nearby” markets will be appropriate instruments (e.g., Gentzkow and Shapiro (2010), Fan (2013)). We make the following exclusion and completeness assumptions.

ASSUMPTION 3: For all  $j = 1, \dots, J$ ,  $E[\xi_{jt}|z_t, x_t] = 0$  almost surely.

ASSUMPTION 4: For all functions  $B(s_t, p_t)$  with finite expectation, if  $E[B(s_t, p_t)|z_t, x_t] = 0$  almost surely, then  $B(s_t, p_t) = 0$  almost surely.

Assumption 3 is a standard exclusion restriction, requiring mean independence between the instruments and the structural errors  $\xi_{jt}$ .<sup>18</sup> Assumption 4 requires completeness of the joint distribution of  $(z_t, x_t, s_t, p_t)$  with respect to  $(s_t, p_t)$ .<sup>19</sup> This is a nonparametric analog of the standard rank condition for linear models. In particular, it requires that the instruments move the endogenous variables  $(s_t, p_t)$  sufficiently to ensure that any function of these variables can be distinguished from others through the exogenous variation in the instruments. See Newey and Powell (2003), Severini and Tripathi (2006), Andrews (2011), and references therein for helpful discussion and examples. We emphasize that we require both the excluded instruments  $z_t$  and the exogenous

<sup>18</sup>Observe that we do not require any restriction on the joint distribution of  $x_t^{(2)}$  and  $\xi_t$ . Thus our description of  $(x_t^{(1)}, x_t^{(2)})$  as “observed exogenous characteristics,” while consistent with standard practice, suggests less flexibility than what is actually permitted. For example, if  $\xi_{jt}$  and  $x_{jt}^{(2)}$  are jointly determined, this presents no problem for the identification of price elasticities as long as  $x_{jt}^{(1)}$  is independent of  $\xi_t$  conditional on  $x_t^{(2)}$ . We provide an example in Appendix C.

<sup>19</sup>Identification with weaker forms of completeness follow from the same argument used below.  $L^2$ -completeness (Andrews (2011)) would suffice under the mild restriction that the sum  $x_{jt} + \xi_{jt}$  has finite variance for all  $j$ . If  $\xi_{jt}$  and  $x_{jt}$  were assumed to have bounded support for all  $j$ , bounded completeness would suffice.

demand shifters  $x_t$ . Neither would suffice alone for Assumption 4:  $x_t$  has dimension of only  $J$ , while  $z_t$  can affect market shares only through prices.

Newey and Powell (2003) used analogs of Assumptions 3 and 4 to show the identifiability of a separable nonparametric regression model. The following result shows that the same argument can be applied to show identification of demand in our setting.

**THEOREM 1:** *Under Assumptions 1–4, for all  $j = 1, \dots, J$ , (i)  $\xi_{jt}$  is identified with probability 1 for all  $t$ , and (ii) the function  $\sigma_j(\chi_t)$  is identified on  $\mathcal{X}$ .*

**PROOF:** For any  $j$ , rewriting (6) and taking expectations conditional on  $z_t, x_t$ , we obtain

$$E[\xi_{jt}|z_t, x_t] = E[\sigma_j^{-1}(s_t, p_t)|z_t, x_t] - x_{jt},$$

so that by Assumption 3,

$$E[\sigma_j^{-1}(s_t, p_t)|z_t, x_t] - x_{jt} = 0 \quad \text{a.s.}$$

Suppose there is another function  $\tilde{\sigma}_j^{-1}$  satisfying

$$E[\tilde{\sigma}_j^{-1}(s_t, p_t)|z_t, x_t] - x_{jt} = 0 \quad \text{a.s.}$$

Letting  $B(s_t, p_t) = \sigma_j^{-1}(s_t, p_t) - \tilde{\sigma}_j^{-1}(s_t, p_t)$ , this implies

$$E[B(s_t, p_t)|z_t, x_t] = 0 \quad \text{a.s.}$$

But by Assumption 4, this requires  $\tilde{\sigma}_j^{-1} = \sigma_j^{-1}$  almost surely, implying that  $\sigma_j^{-1}$  is identified. Repeating for all  $j$ , each  $\xi_{jt}$  is then uniquely determined with probability 1 by (6), proving part (i). Because choice probabilities are observed and all arguments of the demand functions  $\sigma_j(\chi_t)$  are now known, part (ii) follows immediately. *Q.E.D.*

In Appendix B, we show that if we strengthen the two instrumental variables assumptions as in Chernozhukov and Hansen (2005), we can generalize the model to allow

$$\delta_{jt} = \delta_j(x_{jt}, \xi_{jt}),$$

where each  $\delta_j$  is an unknown function that is strictly increasing in its second argument. This avoids the linearity of the index, the requirement that  $x_{jt}$  and  $\xi_{jt}$  be perfect substitutes, and the monotonicity of  $\sigma_j(\chi_t)$  in  $x_{jt}$ . Thus, the essential requirement is strict monotonicity of the index  $\delta_{jt}$  in the latent demand shock  $\xi_{jt}$ .

4.2. Identification of Changes in Consumer Welfare

The demand system  $\sigma(\chi_t)$  is the only primitive from the consumer model required for most purposes motivating estimation in differentiated products markets. The important exception is evaluation of changes in consumer welfare. It should be clear that, in general, changes in individual consumer welfare are not identified from market level data.<sup>20</sup> However, as usual, an assumption of quasilinear preferences will allow one to define a valid notion of aggregate consumer welfare (consumer surplus) in terms of aggregate demand (see, e.g., McFadden (1981), Small and Rosen (1981)).

ASSUMPTION 5:  $v_{ijt} = \mu_{ijt} - p_{jt}$ , where the random variables  $(\mu_{i1t}, \dots, \mu_{iJt})$  are independent of  $p_t$  conditional on  $(x_t, \xi_t)$ .

We use this assumption only in this section of the paper. Note that the restriction  $v_{ijt} = \mu_{ijt} - p_{jt}$  would have no content alone: because  $p_t \in \chi_t$ , any joint distribution  $F_v(\cdot|\chi_t)$  could be replicated by appropriate specification of the distribution  $F_\mu(\cdot|\chi_t)$  of  $(\mu_{i1t}, \dots, \mu_{iJt})$  conditional on  $\chi_t$ . The conditional independence adds the requirement that, fixing  $(x_t, \xi_t)$ , prices affect the joint distribution of utilities only through the linear terms. Thus, Assumption 5 is a natural notion of quasilinearity for a random utility setting.<sup>21</sup> If one specified  $v_{ijt} = v_j(x_t, \xi_t; \theta_{it}) - p_{jt}$ , where  $\theta_{it}$  is a possibly infinite-dimensional random parameter (Example 1 is a special case), Assumption 5 holds when  $\theta_{it} \perp\!\!\!\perp p_t$  conditional on  $(x_t, \xi_t)$ ; that assumption is standard in the literature.<sup>22</sup> We emphasize that the conditional independence assumption does not imply an absence of endogeneity: arbitrary dependence between  $p_t$  and  $(x_t, \xi_t)$  is still permitted. Note that, under Assumptions 1 and 5, we can replace  $F_\mu(\cdot|\chi_t)$  with  $F_\mu(\cdot|\delta_t)$ .

The most common welfare questions in applications concern the effects of price changes, for example, those resulting from a merger, a tax, or a change in

<sup>20</sup>Knowledge of the conditional distributions  $F_v(\cdot|\chi_t)$  would not imply identification of such welfare changes, since a given change in the distribution does not reveal the change in any individual's welfare. Many parametric models enable identification of changes in individual welfare (e.g., compensating/equivalent variation), even with market level data, by specifying each consumer (or each set of identical consumers) as associated with a particular realization of random parameters that completely determine her utilities (see Example 1).

<sup>21</sup>Under the seemingly more general specification  $v_{ijt} = \mu_{ijt} - \alpha_{it} p_{jt}$ , with the random coefficient  $\alpha_{it} > 0$  a.s., one can normalize the scale of each consumer's utilities without loss by setting  $\alpha_{it} = 1$ .

<sup>22</sup>In industrial organization, see, for example, Berry, Levinsohn, and Pakes (1995) and the extensive applied literature that follows. In the econometrics literature, see, for example, Ichimura and Thompson (1998), Briesch, Chintagunta, and Matzkin (2010), or Gautier and Kitamura (2013). Exceptions are models in which no distinction is made between latent heterogeneity in preferences (here,  $\theta_{it}$ ) and unobserved characteristics of goods (here,  $\xi_t$ ). As discussed in Section 2, such models have severe limitations in applications to demand.

the form of competition. If  $F_\mu(\cdot|\delta_t)$  admits a joint density  $f_\mu(\cdot|\delta_t)$ , the change in consumer surplus when prices change from  $p^0$  to  $p^1$  is given, as usual, by a line integral  $\int_a^b \sigma(\delta_t, \alpha(\tau)) \cdot \alpha'(\tau) d\tau$ , where  $\alpha$  is a piecewise smooth path from  $\alpha(a) = p^0$  to  $\alpha(b) = p^1$ . Thus, identification of demand on  $\mathcal{X}$  immediately implies identification of the aggregate welfare effects associated with any pair of price vectors  $(p^0, p^1)$  lying in a path-connected subset of  $\text{supp } p_t|\delta_t$ .

Whether the welfare effects of other types of changes in the environment are identified depends on the support of prices. Just as in classical consumer theory, evaluating the welfare effects of a change in the characteristics of goods requires sufficient exogenous price variation to allow integration under the entire left tail of demand (see, e.g., Small and Rosen (1981)).

We can also see this by observing that, under Assumption 5,  $F_v(\cdot|\chi_t)$  determines the distribution of consumers' money-metric indirect utility functions and can be used to directly calculate aggregate consumer surplus. Under Assumptions 1 and 5,  $F_v(\cdot|\chi_t)$  is completely determined by  $F_\mu(\cdot|\delta_t)$  and the given level of prices. Thus, while not necessary for identification of all changes in consumer surplus, identification of  $F_\mu(\cdot|\delta_t)$  for all  $\delta_t$  is sufficient.

Under Assumption 5, analysis of point identification of  $F_\mu(\cdot|\delta_t)$  is standard. With the notational convention  $p_{0t} = 0$ , let  $\dot{p}_{jt}$  denote the vector of price differences  $(p_{kt} - p_{jt})_{k \in \mathcal{J} \setminus j}$  and let  $\Gamma_j$  denote the matrix such that  $\Gamma_j p_t = \dot{p}_{jt}$  (see, e.g., Thompson (1989)). Let  $\mathcal{F}$  denote the set of all cumulative distribution functions on  $\mathbb{R}^J$ . Under Assumptions 1 and 5,  $F_\mu(\cdot|\delta_t)$  is an element of  $\mathcal{F}$  satisfying

$$(7) \quad \sigma_0(\delta_t, p_t) = F_\mu(p_t|\delta_t), \quad \text{and}$$

$$(8) \quad \sigma_j(\delta_t, p_t) = \int 1\{\Gamma_j m \leq \Gamma_j p_t\} dF_\mu(m|\delta_t), \quad j = 1, \dots, J,$$

on the support of  $p_t|\delta_t$ . In general, given identification of  $\sigma(\chi_t)$  on  $\mathcal{X}$ , (7) and (8) deliver partial identification of  $F_\mu(\cdot|\delta_t)$  for each  $\delta_t$ . However, under the large support condition

$$\text{supp } p_t|\delta_t \supset \text{supp } \mu_t|\delta_t,$$

equation (7) directly provides point identification. Furthermore, in this case the restrictions in (8) are redundant to (7) (see, e.g., Thompson (1989)).

With more limited support, what is learned about  $F_\mu(\cdot|\delta_t)$  combines information obtained from each function  $\sigma_j(\delta_t, p_t)$ . Figure 1 illustrates for  $J = 2$  (cf. Thompson (1989)). Given a price vector  $p_t$ , panel (a) divides the space of  $\mu_{it}$  into regions leading to each possible consumer choice. Panel (b) illustrates how the boundaries between regions adjust with prices. For example, a fall in the price of good 1 leads consumers with  $\mu_{it}$  in the horizontally hatched region to switch to good 1. With each  $\sigma_j(\delta_t, p_t)$  identified, the measure of consumers in each region is known. This provides not only point identification of  $F_\mu(\cdot|\delta_t)$

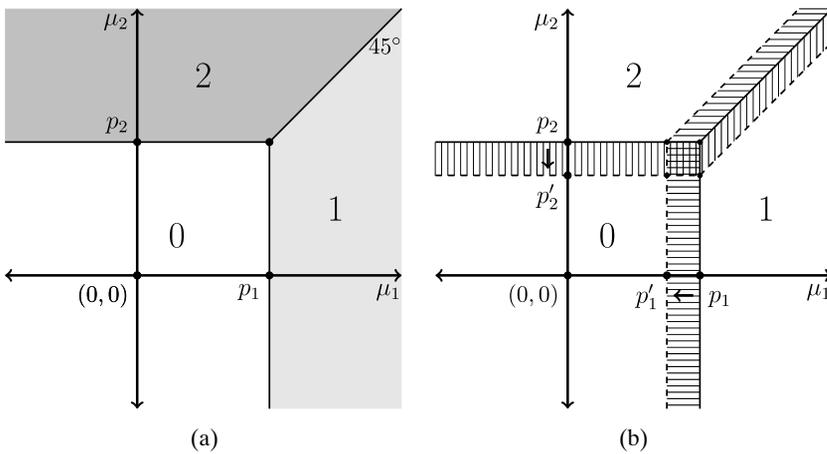


FIGURE 1.—(a) Choice regions with  $J = 2$ . (b) Price changes move choice regions.

on the support of  $p_i|\delta_i$  (from (7)), but also restrictions on the probability measure associated with strips of  $\mathbb{R}^2$  extending outside the support. For example, if  $F_\mu(\cdot|\delta_i)$  admits a density and prices have support on an open set containing both  $p_1$  and  $p_2$ , (7) and (8) imply  $\int_{-\infty}^{p_2} f_\mu(p_1, \mu_2|\delta_i) d\mu_2 = \partial\sigma_0(\delta_i, p)/\partial p_1$ ,  $\int_{-\infty}^{p_1} f_\mu(\mu_1, p_2|\delta_i) d\mu_1 = \partial\sigma_0(\delta_i, p)/\partial p_2$ ,  $\int_{p_2}^{\infty} f_\mu(\mu_2 - p_2 + p_1, \mu_2|\delta_i) d\mu_2 = \partial\sigma_2(\delta_i, p)/\partial p_1$ , and  $f_\mu(p_1, p_2|\delta_i) = \partial^2\sigma_0(\delta_i, p)/\partial p_1 \partial p_2$ . These expressions demonstrate identification of the integral of  $f_\mu(\cdot|\delta_i)$  over regions corresponding to the limits of those illustrated in panel (b) (in the last case, the cross-hatched region) as the price changes shrink to zero.

### 4.3. Identification of Marginal Costs

In the applied literature, adding a supply side to the model typically proceeds by specifying a functional form for firm costs and an extensive form for the competition between firms. A set of first-order conditions characterizing equilibrium prices and quantities in terms of firms' costs and features of demand (e.g., own- and cross-price elasticities) then allow recovery of firms' equilibrium marginal costs and, under additional exclusion restrictions, their marginal cost functions. Given identification of demand, it is straightforward to show that a similar approach can be applied to demonstrate identification in a nonparametric setting.<sup>23</sup>

<sup>23</sup>Although the results below will reference the particular demand model and sufficient conditions for identification of demand developed above, this is not essential. What is required in this section is that the demand elasticities be known in every market and that the goods be connected substitutes in prices. The same is true of the results in Appendix A and the result on testability

An identification approach based on first-order conditions requires differentiability of firms' profit functions with respect to their choice variables. This will be assured if the market share functions  $\sigma_j(\chi_t)$  are continuously differentiable with respect to prices, and we will assume this directly. We also slightly strengthen part of the connected substitutes assumption (Assumption 2) by requiring  $\partial\sigma_k(\chi_t)/\partial p_j > 0$  whenever  $\sigma_k(\chi_t)$  is strictly increasing in  $p_j$ .

ASSUMPTION 6: (i)  $\sigma_j(\chi_t)$  is continuously differentiable with respect to  $p_k \forall j, k \in \mathcal{J}$ ; (ii) at each  $\chi_t \in \mathcal{X}$ , for any nonempty  $\mathcal{K} \subseteq \{1, \dots, J\}$ , there exist  $k \in \mathcal{K}$  and  $\ell \notin \mathcal{K}$  such that  $\partial\sigma_\ell(\chi_t)/\partial p_{kt} > 0$ .

Let  $mc_{jt}$  denote the marginal cost of production of good  $j$  in market  $t$  given the observed levels of output  $(q_{1t}, \dots, q_{Jt})$ . For now, we place no restriction on the structure of firm costs. However, we follow the parametric literature in assuming one has committed to a model of supply that implies a system of first-order conditions defining firm behavior. In the typical model, each first-order condition can be solved to express  $mc_{jt}$  as a known function of equilibrium prices, equilibrium quantities (determined by  $s_t$  and  $M_t$ ), and the first-order derivatives of the demand system  $\sigma$ .<sup>24</sup> We let  $D(\chi_t)$  denote the  $J \times J$  matrix of partial derivatives  $[\frac{\partial\sigma_k(\chi_t)}{\partial p_\ell}]_{k,\ell}$ .

ASSUMPTION 7a: For each  $j = 1, \dots, J$ , there exists a known function  $\psi_j$  such that, for all  $(M_t, \chi_t, s_t)$  in their support,

$$(9) \quad mc_{jt} = \psi_j(s_t, M_t, D(\chi_t), p_t).$$

Although Assumption 7a is a high-level condition, the following remark (proved in Appendix A) demonstrates that, given connected substitutes in prices and Assumption 6, Assumption 7a follows from the first-order conditions characterizing equilibrium in the standard oligopoly supply models typically considered in the empirical literature, where  $\psi_j$  might be interpreted as a "generalized residual marginal revenue function" for product  $j$ . Relying on this high-level assumption thus allows us to provide results for a variety of supply models at once.

REMARK 1: Suppose that the goods  $\{0, 1, \dots, J\}$  are connected substitutes in  $-p_t$  and that supply is characterized by any of the following models, each allowing single-product or multi-product firms:

- (i) marginal cost pricing;

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of the supply model in Theorem 9 below. See Berry, Gandhi, and Haile (2013) for examples of other demand structures satisfying the connected substitutes condition.

<sup>24</sup>This does not require that the first-order conditions have a unique solution for prices or quantities.

- (ii) monopoly pricing (or joint profit maximization);
- (iii) Nash equilibrium in a complete information simultaneous price-setting game;
- (iv) Nash equilibrium in a complete information simultaneous quantity-setting game.

Then under Assumption 6, Assumption 7a holds.

This remark demonstrates that Assumption 7a describes a feature common to standard models of oligopoly supply. This feature allows identification of marginal costs. In particular, the following result follows immediately from (9) and Theorem 1, since the latter implies identification of  $D(\chi_t)$ .

**THEOREM 2:** *Suppose Assumptions 1–4, 6, and 7a hold. Then  $mc_{jt}$  is identified for all  $t, j = 1, \dots, J$ .*

Theorem 2 shows that marginal costs (and, therefore, markups) are identified without any additional exclusion condition or any restriction on firms’ cost functions. Given any of the standard models of oligopoly supply described above, only the differentiability assumption (Assumption 6) has been added to the conditions we required for identification of demand.

Marginal costs and markups are sometimes the only objects of interest on the supply side. However, unless one assumes constant marginal costs, these will not be sufficient for counterfactual questions involving changes in equilibrium quantities. We therefore move now to consider identification of firms’ marginal cost functions.

Let  $\mathcal{J}_j$  denote the set of goods produced by the firm producing good  $j$ . Let  $q_{jt} = M_t s_{jt}$  denote the quantity of good  $j$  produced, and let  $Q_{jt}$  denote the vector of quantities of all goods  $k \in \mathcal{J}_j$ . We allow each marginal cost  $mc_{jt}$  to depend on all quantities  $Q_{jt}$ , on observable cost shifters  $w_{jt}$  (which may include or consist entirely of demand shifters), and on an unobserved cost shifter  $\omega_{jt} \in \mathbb{R}$ . Thus, we let

$$(10) \quad mc_{jt} = \tilde{c}_j(Q_{jt}, w_{jt}, \omega_{jt}),$$

where the unknown function  $\tilde{c}_j$  may differ arbitrarily across goods.

By Theorem 2, each  $mc_{jt}$  can be treated as known, so (10) takes the form of a standard nonparametric regression equation. We have a standard endogeneity problem: firm output  $Q_{jt}$  is correlated with the marginal cost shock  $\omega_{jt}$ . However, typically there are many available instruments. Any demand shifter excluded from the cost shifters  $w_{jt}$  is a candidate instrument. These can include observable demand shifters  $x_{kt}$  for  $k \in \mathcal{J}_j$ . Furthermore, because standard oligopoly models imply that demand shifters for *all* goods affect every market share through the consumer choice problem and equilibrium, demand shifters  $x_{kt}$  for  $k \notin \mathcal{J}_j$  are also available as instruments. Other possible

instruments include observables like population that vary only at the market level and may or may not directly affect market *shares*, but do affect quantities. Thus, there will often be a large number of instruments for  $Q_{jt}$ .<sup>25</sup> Indeed, the large number of instruments may lead to testable overidentifying restrictions, something we discuss in Section 6.

With adequate instruments, identification of the marginal cost function is straightforward. One way to obtain a formal result is to assume monotonicity of  $\bar{c}_j$  in the unobserved cost shifter  $\omega_{jt}$  and apply the identification result of Chernozhukov and Hansen (2005) for nonparametric nonseparable instrumental variables regression models. Alternatively, one could consider the separable nonparametric specification

$$(11) \quad mc_{jt} = \bar{c}_j(Q_{jt}, w_{jt}) + \omega_{jt},$$

in which case identification of the unknown function  $\bar{c}_j$  follows by direct application of Newey and Powell's (2003) identification result for separable nonparametric regression models.<sup>26</sup> We give a formal statement only for the latter case. Let  $y_{jt}$  denote instruments excluded from the own-product cost shifters, as discussed above, and make the following exclusion and completeness assumptions.

ASSUMPTION 8:  $E[\omega_{jt}|w_{jt}, y_{jt}] = 0$  almost surely for all  $j = 1, \dots, J$ .

ASSUMPTION 9: For all  $j = 1, \dots, J$  and all functions  $B(Q_{jt}, w_{jt})$  with finite expectation, if  $E[B(Q_{jt}, w_{jt})|w_{jt}, y_{jt}] = 0$  almost surely, then  $B(Q_{jt}, w_{jt}) = 0$  almost surely.

THEOREM 3: Suppose marginal costs take the form in (11) and that Assumptions 1–4, 6, 7a, 8, and 9 hold. Then for all  $j = 1, \dots, J$ , (i) the marginal cost functions  $\bar{c}_j(Q_{jt}, w_{jt})$  are identified, and (ii)  $\omega_{jt}$  is identified with probability 1 for all  $t$ .

PROOF: Immediate from Theorem 2 and Newey and Powell (2003). *Q.E.D.*

#### 4.4. Identifying Cost Shocks Without a Supply Model

We conclude our exploration of nonparametric instrumental variables approaches to identification by providing conditions under which the latent cost shocks  $\omega_{jt}$  can be identified without specifying a particular oligopoly

<sup>25</sup>In practice, it is usually assumed that only  $q_{jt}$ , not  $Q_{jt}$ , enters the cost of good  $j$ . In that case, only a single excluded instrument would be required.

<sup>26</sup>Unlike our prior use of a Newey–Powell inspired argument to identify demand, there is nothing nonstandard about equation (11): the left-hand-side variable is endogenous and all instruments are excluded from the equation.

model. In this section, we consider identification of these shocks alone, not of marginal costs themselves. This will prove useful below because it will enable us to obtain our strongest results regarding discrimination between alternative oligopoly models in Section 6.<sup>27</sup> In addition, some of the structure introduced here will prove useful to our exploration of identification using nonparametric simultaneous equations in Section 5.

We begin by replacing the assumption of a known supply model (Assumption 7a) with the following less restrictive condition.

ASSUMPTION 7b: *For each  $j = 1, \dots, J$ , there exists a (possibly unknown) function  $\psi_j$  such that, for all  $(M_t, \chi_t, s_t)$  in their support,*

$$(12) \quad mc_{jt} = \psi_j(s_t, M_t, D(\chi_t), p_t).$$

Assumption 7b is identical to Assumption 7a except that the functions  $\psi_j$  now need not be known. We require only that a first-order condition of this form exist. As already noted, Appendix A shows that this exists in standard static oligopoly models, including those typically considered in empirical work.<sup>28</sup> Violations of this condition could arise in ill-defined models (e.g., models with no equilibrium) or in models with behavior characterized by corner solutions.

With this relaxation of Assumption 7a, we will require additional structure on the marginal cost function. Partition the cost shifters  $w_{jt}$  as  $(w_{jt}^{(1)}, w_{jt}^{(2)})$ , with  $w_{jt}^{(1)} \in \mathbb{R}$ , and define the “cost index”

$$\kappa_{jt} = w_{jt}^{(1)} \gamma_j + \omega_{jt}.$$

Parallel to our index restriction on the demand model, we now require that  $w_{jt}^{(1)}$  and  $\omega_{jt}$  affect marginal cost only through the index  $\kappa_{jt}$ .

ASSUMPTION 10: *For all  $j = 1, \dots, J$ ,  $mc_{jt} = c_j(Q_{jt}, \kappa_{jt}, w_{jt}^{(2)})$ , where  $c_j$  is strictly increasing in  $\kappa_{jt}$ .*

This is an important restriction, but one that is satisfied in many standard models.<sup>29</sup> As with our model of demand, we focus on the case of a linear index, although only monotonicity of the index in  $\omega_{jt}$  is essential.

<sup>27</sup>As discussed below (see, e.g., footnote 33), knowledge of the structural errors  $(\xi_t, \omega_t)$  can allow identification of some supply counterfactuals without fully specifying the model of supply.

<sup>28</sup>Without changing any argument, we could generalize Assumption 7b (likewise Assumption 7a) by allowing  $\psi_j$  to depend arbitrarily on the demand system  $\sigma(\cdot)$ . We use the simpler condition since, in the examples we have considered, the matrix of derivatives  $D(\chi_t)$  suffices.

<sup>29</sup>Note that  $w_{jt}^{(1)}$  and  $\omega_{jt}$  could be any known transformations of observed and unobserved cost shifters.

We normalize  $\gamma_j = 1$  for each  $j$  without loss. Continuing to condition on (and suppress)  $x_t^{(2)}$ , we now also condition on a value of  $w_t^{(2)} = (w_{1t}^{(2)}, \dots, w_{Jt}^{(2)})$  and suppress it in the notation by letting  $w_{jt}$  now denote  $w_{jt}^{(1)}$ . We let  $w_t = (w_{1t}, \dots, w_{Jt})$ .

The following lemma provides a supply-side analog to our previous result on the invertibility of demand. It shows that, under Assumptions 7b and 10, there is a unique vector of cost indices  $(\kappa_{1t}, \dots, \kappa_{Jt})$  consistent with any vector of equilibrium prices and market shares.

LEMMA 2: *Given any  $(M_t, s_t, p_t)$ , there is exactly one  $(\kappa_{1t}, \dots, \kappa_{Jt}) \in \mathbb{R}^J$  consistent with Assumptions 1, 2, 7b, and 10.*

PROOF: By Lemma 1,  $\sigma_k(\chi_t) = \sigma_k(\delta_t, p_t) = \sigma_k(\sigma^{-1}(s_t, p_t), p_t)$ , so that we may write

$$D(\chi_t) = \tilde{D}(s_t, p_t)$$

for some function  $\tilde{D}$ . Assumption 7b then yields

$$(13) \quad c_j(Q_{jt}, \kappa_{jt}) = \psi_j(s_t, M_t, \tilde{D}(s_t, p_t), p_t).$$

By Assumption 10, the function  $c_j$  can be inverted, yielding

$$(14) \quad \kappa_{jt} = c_j^{-1}(\psi_j(s_t, M_t, \tilde{D}(s_t, p_t), p_t); Q_{jt}).$$

Since each  $q_{jt} = M_t s_{jt}$ , the right-hand side is a function of  $s_t, p_t$ , and  $M_t$ . *Q.E.D.*

Henceforth we condition on a value of  $M_t$  and suppress it in the notation. We then rewrite (14) as

$$(15) \quad w_{jt} + \omega_{jt} = \pi_j^{-1}(s_t, p_t), \quad j = 1, \dots, J.$$

Each function  $\pi_j^{-1}$  involves the composition of several functions, including  $c_j^{-1}$  and  $\psi_j$ ; although we have not yet defined a function  $\pi_j$ , we use the notation  $\pi_j^{-1}$  as a reminder that this represents an “inversion” of supply-side equilibrium conditions. Let  $\pi^{-1}(s_t, p_t) = (\pi_1^{-1}(s_t, p_t), \dots, \pi_J^{-1}(s_t, p_t))$ .

Equation (15) takes the same form as (6), and we can use it in the same way, exploiting the exogenous cost shifters  $w_t$  and demand shifters  $x_t$  as instruments.

ASSUMPTION 11:  $E[\omega_{jt}|w_t, x_t] = 0$  almost surely for all  $j = 1, \dots, J$ .

ASSUMPTION 12: *For all functions  $B(s_t, p_t)$  with finite expectation, if  $E[B(s_t, p_t)|w_t, x_t] = 0$  almost surely, then  $B(s_t, p_t) = 0$  almost surely.*

**THEOREM 4:** *Suppose Assumptions 1–4, 6, 7b, and 10–12 hold. Then for all  $j = 1, \dots, J$ ,  $\omega_{jt}$  is identified with probability 1 for all  $t$ .*

**PROOF:** This follows by observing that the argument used in the proof of Theorem 1 can be applied to (15) with trivial modification to recover the inverse pricing relations  $\pi_j^{-1}$  and the cost shocks  $\omega_{jt}$ . *Q.E.D.*

Since we have held fixed both  $x_t^{(2)}$  and  $w_t^{(2)}$ , Assumption 12 implies

$$x_t^{(1)} \notin (w_t^{(1)}, w_t^{(2)}),$$

$$w_t^{(1)} \notin (x_t^{(1)}, x_t^{(2)}).$$

These restrictions were not required previously, and in practice they may rule out some demand shifters for the role of  $x_t^{(1)}$ . However, several common types of observable product characteristics will remain good candidates. For example, in BLP (see also Petrin (2002)), a measure of a car's fuel cost—miles per dollar—enters its demand but is excluded from its marginal cost, the latter depending on the physical measure miles per gallon. This is an example of a class of instruments involving interactions between product characteristics (which may affect marginal cost) and market characteristics (which may not). The latter might include market demographics, local climate, local distribution or transportation networks, or local prices/availability of complementary goods. Other types of candidate instruments include product characteristics produced with market-specific fixed costs, such as product-specific advertising or other marketing measures (e.g., Nevo (2001), Goeree (2008)), firm locations (e.g., Capps, Dranove, and Satterthwaite (2003)), or a firm's local offerings of complementary products (e.g., Berry and Jia (2010)). In some applications, product characteristics themselves may vary without affecting firm costs due to technological constraints (e.g., satellite television reception in Goolsbee and Petrin (2004)) or other exogenous market-specific factors (climate, topography, transportation network). As these examples suggest, appropriate instruments can be found in a large share of the most prominent applications of BLP-type models. Nonetheless, the required exclusion restriction does require careful attention in practice.

## 5. A SIMULTANEOUS EQUATIONS APPROACH

In this section, we consider identification of the entire demand and supply model in a system of nonparametric simultaneous equations. This alternative approach requires some additional structure, but complements the nonparametric IV approach by demonstrating that one can replace the abstract completeness conditions with more transparent conditions on excluded demand shifters and cost shifters. This leads to constructive proofs with connections to

classic identification arguments for models of demand and supply in perfectly competitive markets.

Throughout this section, we maintain the demand index and connected substitutes assumptions (Assumptions 1 and 2). We also maintain the cost index restriction of Assumption 10 and assume supply is characterized by a (possibly unknown) set of first-order conditions (Assumption 7b). Exploiting Lemmas 1 and 2, demand and supply are then characterized by the system of  $2J$  equations in (6) and (15):

$$\begin{aligned}x_{jt} + \xi_{jt} &= \sigma_j^{-1}(s_t, p_t), & j = 1, \dots, J, \\w_{jt} + \omega_{jt} &= \pi_j^{-1}(s_t, p_t), & j = 1, \dots, J.\end{aligned}$$

These equations take a form considered previously in Section 4.2 of Matzkin (2008), which provides an identification result that could be applied directly to our system of inverted equations. The result exploits a large support assumption and restrictions on the density of the unobservables. While it has both support and density conditions, Matzkin's (2008) result allows for indexes that are nonlinear in  $x_j$  (and  $w_j$ ), a significant advantage relative to the linear index we feature in our results (see also Appendix B). Matzkin (2010) provided additional results for the linear index model that do not require full support, but which rely on a different set of restrictions on the joint density of the structural errors. Unlike the density restriction used in Matzkin (2008), these alternative density restrictions rule out the multivariate Normal distribution, among others (see Berry and Haile (2013)).

In this section, we add two alternative results. The first drops Matzkin's (2008) restriction on the joint density of the structural errors, demonstrating that the large support condition suffices alone. The second result provides an alternative density restriction that allows for identification with limited support for  $(x_t, w_t)$ . This density restriction holds for many common densities, including the multivariate Normal.

Here, a different location normalization of  $\xi_t$  and  $\omega_t$  will be convenient. For each  $j$ , take arbitrary  $(x^0, w^0)$  and arbitrary  $(s^0, p^0)$  in the support of  $(s_t, p_t)|(x^0, z^0)$  and let

$$(16) \quad \begin{aligned}\sigma_j^{-1}(s^0, p^0) - x_j^0 &= 0, & j = 1, \dots, J, \\ \pi_j^{-1}(s^0, p^0) - w_j^0 &= 0, & j = 1, \dots, J.\end{aligned}$$

To facilitate our use of a change of variables, we will make the following assumption.

**ASSUMPTION 13:** *There is a unique vector of equilibrium prices associated with any  $(\delta_t, \kappa_t)$ .*

A strong sufficient condition would be that, at the true marginal cost and demand functions, the equilibrium first-order conditions have a unique solution (for prices) given any  $(\delta_t, \kappa_t)$ . However, this is often difficult to verify in models of product differentiation (see, e.g., Caplin and Nalebuff (1991)), and it is not hard to construct examples admitting multiple equilibria. Importantly, then, Assumption 13 does not require that the model have a unique equilibrium. If there are multiple equilibria, the assumption does require an equilibrium selection rule such that the same prices  $p_t$  arise whenever  $(\delta_t, \kappa_t)$  is the same.<sup>30</sup> This rules out random equilibrium selection or equilibrium selection based on  $x_{jt}$  or  $\xi_{jt}$  instead of their sum  $\delta_{jt}$  (and similarly for  $\kappa_{jt}$ ). This does not seem particularly strong, but is not something we required previously. With this assumption, we can write

$$(17) \quad p_t = \pi(\delta_t, \kappa_t)$$

for some unknown function  $\pi: \mathbb{R}^{2J} \rightarrow \mathbb{R}^J$ .

The change-of-variables argument also requires regularity conditions enabling one to relate the joint density of the latent structural errors  $(\xi_{1t}, \dots, \xi_{Jt}, \omega_{1t}, \dots, \omega_{Jt})$  to the joint density of the observables  $(s_t, p_t)$ .

ASSUMPTION 14:  $(\xi_t, \omega_t)$  have a positive joint density  $f_{\xi, \omega}$  on  $\mathbb{R}^{2J}$ .

ASSUMPTION 15: The function  $\pi(\delta_t, \kappa_t)$  is differentiable, and the functions  $\sigma^{-1}(s_t, p_t)$  and  $\pi^{-1}(s_t, p_t)$  are continuously differentiable.

Finally, we will require exclusion and support conditions on the demand and cost shifters. Assumption 16 requires full independence from the structural errors, and Assumption 17 ensures that the instruments have sufficient variation to trace out the inverse demand and supply functions. As noted already, below we illustrate that the full support assumption is not necessary if we instead impose a shape restriction on the distribution of the unobservables.

ASSUMPTION 16:  $(x_t, w_t) \perp\!\!\!\perp (\xi_t, \omega_t)$ .

ASSUMPTION 17:  $\text{supp}(x_t, w_t) = \mathbb{R}^{2J}$ .

We first show identification of demand, which does not require us to specify the form of oligopoly competition.

<sup>30</sup>The recent literature on the estimation of dynamic oligopoly models makes use of similar assumptions on equilibrium selection, although typically requiring that the equilibrium selection depend only on observables. A key point in both cases is that the selection rule need not be estimated; the identification argument only requires that such a rule exist.

**THEOREM 5:** *Suppose Assumptions 1, 2, 6, 7b, 10, and 13–17 hold. Then, for all  $j = 1, \dots, J$ , (i)  $\xi_{jt}$  is identified for all  $t$ , and (ii) the function  $\sigma_j(\delta_t, p_t)$  is identified on  $\mathcal{X}$ .*

**PROOF:** Recalling (17), let  $\tilde{\sigma}(\delta_t, \kappa_t) = \sigma(\delta_t, \pi(\delta_t, \kappa_t))$ . Since  $\sigma$  and  $\pi$  are differentiable,  $\tilde{\sigma}$  is differentiable. Further, the vector function  $(\sigma^{-1}(s_t, p_t), \pi^{-1}(s_t, p_t))$  is the inverse of  $(\tilde{\sigma}(\delta_t, \kappa_t), \pi(\delta_t, \kappa_t))$ . Thus, by the chain rule,  $(\sigma^{-1}(s_t, p_t), \pi^{-1}(s_t, p_t))$  has nonsingular Jacobian matrix  $\mathbb{J}(s_t, p_t)$ .<sup>31</sup> We observe the joint density of market shares and prices conditional on  $(x_t, z_t)$ . By a standard change of variables, under Assumption 16 this joint density is related to that of  $(\xi_t, \omega_t)$  by

$$(18) \quad f_{s,p}(s_t, p_t | x_t, w_t) = f_{\xi,\omega}(\sigma_1^{-1}(s_t, p_t) - x_{1t}, \dots, \pi_J^{-1}(s_t, p_t) - w_{Jt}) |\mathbb{J}(s_t, p_t)|.$$

Since the joint density  $f_{\xi,\omega}(\sigma_1^{-1}(s_t, p_t) - x_{1t}, \dots, \pi_J^{-1}(s_t, p_t) - w_{Jt})$  must integrate to 1, we learn the value of the Jacobian determinant from

$$\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f_{s,p}(s_t, p_t | x_t, w_t) dx_t dw_t = |\mathbb{J}(s_t, p_t)|.$$

Combined with (18), this implies that the value of

$$f_{\xi,\omega}(\sigma_1^{-1}(s_t, p_t) - x_{1t}, \dots, \pi_J^{-1}(s_t, p_t) - w_{Jt})$$

is identified for all  $\forall s_t, p_t, x_t, w_t$ . Then, since

$$(19) \quad \int_{\tilde{w}, \tilde{x}_-j, \tilde{x}_j \geq x_j} f_{\xi,\omega}(\sigma_1^{-1}(s, p) - \tilde{x}_1, \dots, \pi_J^{-1}(s, p) - \tilde{w}_J) d\tilde{x} d\tilde{w} = F_{\xi_j}(\sigma_j^{-1}(s, p) - x_j),$$

$F_{\xi_j}(\sigma_j^{-1}(s_t, p_t) - x_{jt})$  is identified for all  $s_t, p_t, x_{jt}$ . From the normalization (16), we know  $F_{\xi_j}(\sigma_j^{-1}(s^0, p^0) - x_j^0) = F_{\xi_j}(0)$ . So, for any  $(s_t, p_t)$ , we can find  $x_j^*$  (unique by Assumption 14) such that  $F_{\xi_j}(\sigma_j^{-1}(s_t, p_t) - x_j^*) = F_{\xi_j}(0)$ . This reveals  $\sigma_j^{-1}(s_t, p_t) = x_j^*$ , so the function  $\sigma_j^{-1}$  is identified. With equation (6), this proves part (i). Repeating for all  $j$  proves part (i). Part (ii) then follows as in the proof of Theorem 1. *Q.E.D.*

If we are willing to assume a particular model of supply, that is, to strengthen Assumption 7b to Assumption 7a, identification of marginal costs follows directly from the identification of demand (Theorem 5) and equation (9). We omit a formal statement of this result.

<sup>31</sup>Let  $\tau = (s_t, p_t)$ ,  $\phi = (\tilde{\sigma}, \pi)$ ,  $\phi^{-1} = (\sigma^{-1}, \pi^{-1})$ , and then differentiate the identity  $\phi(\phi^{-1}(\tau)) = \tau$ .

Alternatively, without strengthening Assumption 7b, we can apply the argument used in the proof of Theorem 5 (part (i)) to show identification of the latent cost shocks  $\omega_{jt}$  and the reduced-form equilibrium pricing function  $\pi(\delta_t, \kappa_t)$ .

**THEOREM 6:** *Suppose the hypotheses of Theorem 5 hold. Then (i)  $\omega_{jt}$  is identified for all  $t$  and  $j = 1, \dots, J$ ; (ii)  $\pi(\delta_t, \kappa_t)$  is identified.*

**PROOF:** The argument used in the proof of Theorem 5 can be repeated with trivial modification to recover the inverse supply relations  $\pi_j^{-1}$  and the cost shocks  $\omega_{jt}$ .<sup>32</sup> Given Theorem 5, identification of the mapping  $\pi(\delta_t, \kappa_t)$  is then immediate. *Q.E.D.*

Part (i) of this result will be useful for discriminating between oligopoly models (see Section 6). Part (ii) allows identification of many supply-side counterfactuals despite the fact that we have not required the form of oligopoly competition to be known.<sup>33</sup> Of course, when the stronger Assumption 7a holds, part (i) would permit identification of firms’ marginal cost functions: all variables in the equations

$$mc_{jt} = c_j(Q_{jt}, \kappa_{jt})$$

would be known, identifying the functions  $c_j$ . We omit a formal statement of this result.

Theorems 5 and 6 demonstrate the identifiability of demand and marginal costs using an argument with close links to classical arguments for supply and demand models: excluded cost and demand shifters with sufficient support can trace out the entire demand and supply structure. The results exploit the large support Assumption 17. This type of condition is familiar from the prior literature on discrete choice models, simultaneous equations models, and even classic supply and demand models. However, a large support assumption is not essential. The following result offers one alternative.

**THEOREM 7:** *Let  $Y_t = (\xi_{1t}, \dots, \xi_{Jt}, \omega_{1t}, \dots, \omega_{Jt})$  and suppose Assumptions 1, 2, 6, 7b, 10, and 13–16 hold. In addition, suppose (a) the support of  $(x_t, w_t)$  has nonempty interior, (b)  $f_{\xi, \omega}$  is twice differentiable, and (c) for almost all  $(s_t, p_t)$ , there exists  $(x^*(s_t, p_t), w^*(s_t, p_t))$  such that the matrix*

$$\frac{\partial^2 \ln f_{\xi, \omega}(\sigma_1^{-1}(s_t, p_t) - x_1^*(s_t, p_t), \dots, \pi_J^{-1}(s_t, p_t) - w_J^*(s_t, p_t))}{\partial Y_t \partial Y_t'}$$

<sup>32</sup>At equation (19), integrate instead over  $\{\tilde{x}, \tilde{z}_{-j}, \tilde{z}_j \geq z_j\}$  and then use the normalization  $\pi_j^{-1}(s^0, p^0) - w_j^0 = 0$ .

<sup>33</sup>Given Assumption 13, identification of the function  $\pi(\delta_t, \kappa_t)$  follows immediately once the structural errors  $\xi_t$  and  $\omega_t$  have been identified. Theorems 1 and 4 together, therefore, provide an alternative set of sufficient conditions.

is nonsingular. Then, for all  $j = 1, \dots, J$ , (i)  $\xi_{jt}$  and  $\omega_{jt}$  are identified for all  $t$ , (ii) the function  $\sigma_j(\chi_t)$  is identified on  $\mathcal{X}$ , and (iii) the function  $\pi_j(\delta_t, \kappa_t)$  is identified on  $\text{supp}(\delta_t, \kappa_t)$ .

PROOF: Nonsingularity of the Jacobian matrix  $\mathbb{J}(s_t, p_t)$  follows as in the proof of Theorem 6, again leading to equation (18). Identification of the functions  $\sigma_j^{-1}$  and  $\pi_j^{-1}$  for  $j = 1, \dots, J$  then follows from Theorem 3 in Berry and Haile (2013). Since  $p_t$  and  $s_t$  are observed, the results follow immediately. *Q.E.D.*

This alternative result illustrates an available tradeoff between support requirements on demand/cost shifters and shape restrictions on the distribution of the structural errors.<sup>34</sup> For example, if the second-derivative matrix referenced in Theorem 7 is nonsingular almost everywhere (the multivariate Normal is one example), the support of  $(x_t, w_t)$  can be arbitrarily small.

## 6. DISCRIMINATING BETWEEN OLIGOPOLY MODELS

A central question in the empirical industrial organization literature is the nature of firm “conduct,” that is, the form of competition between firms.<sup>35</sup> Firm conduct has important implications for markups, profits, efficiency, and counterfactual predictions. Bresnahan (1982), citing formal results in Lau (1982), offered an influential insight for how “rotations” of demand could be used to discriminate between alternative models of oligopoly competition based on market level data (see also Bresnahan (1989)). The class of models considered by Lau was very limited, including only deterministic conjectural variations models, with linear conjectures, homogeneous goods, and symmetric firms. However, Bresnahan’s original intuition suggested much broader applicability.

Here, we show that a generalization of Bresnahan’s ideas can indeed be given a formal foundation in a much more general setting. Our approach utilizes the equilibrium relation (12), in which the marginal cost of a good  $j$  is equated to the value of a function  $\psi_j$  determined by the true oligopoly model. This leads to testable restrictions that can be used to discriminate between alternative models of firm conduct, each implying a different function  $\psi_j$ . In contrast to the earlier focus on rotations of demand, we show that there is a much broader range of cross-market and cross-firm variation that can be exploited.

<sup>34</sup>Our condition on the second derivative matrix of  $f_{\xi, \omega}$  in Theorem 7 is similar to a condition Chiappori and Komunjer (2009) used in combination with a completeness condition to show identifiability of a discrete choice demand model with individual level data.

<sup>35</sup>Following the literature, we consider the testability of specific models of firm conduct. This analysis can alternatively be interpreted as developing restrictions that can provide point identification or partial identification of the true model of conduct within a specified class of models. Open questions include what restrictions on the class of models would ensure point identification, and whether the restrictions we consider are exhaustive.

We consider two cases. We obtain our strongest results in “case 1,” where we suppose we have identified the cost indices without specifying the oligopoly model, as in Theorem 4 or Theorem 6. This case, therefore, requires both the cost index structure and exclusion of the demand shifters  $x_t$  (i.e.,  $x_t^{(1)}$ ) from firms’ marginal costs. “Case 2” provides a weaker set of testable restrictions without these requirements.

6.1. Case 1

We begin with a simple example motivated by Bresnahan (1982). Consider a market with a single producer and suppose the null hypothesis is that the firm prices at marginal cost. The alternative is that the firm is a profit-maximizing monopolist. Panel (a) of Figure 2 shows the market demand curve  $DD_t$ . Under the monopoly hypothesis, the function  $\psi_j$  in Assumption 7b is the marginal revenue curve  $MR_t$ . We label this curve  $\psi_{jt}^1$ , indicating the alternative hypothesis. Under the null of marginal cost pricing, however, it is the demand curve that is the function  $\psi_j$ . We label this  $\psi_{jt}^0$ . The observed equilibrium outcome  $E_t$  in market  $t$  maps to two possible values of marginal cost at the quantity  $q_t$ , depending on the model.

Suppose that the monopoly model is the true model. Hold the cost index fixed and consider a change in market conditions that “rotates” the curve  $\psi_{jt}^1$  around the point  $(q_t, mc_t^1)$ —that is, such that  $\psi_{jt}^1$  still intersects this point. This is illustrated in panel (b) with the curve  $\psi_{jt'}^1$ . Associated with this new marginal revenue curve is a market demand curve  $\psi_{jt'}^0$ . Since the true model is monopoly, the new observed equilibrium outcome is  $E_{t'}$ . Under the alternative, the implied marginal cost at quantity  $q_t$  is still  $mc_t^1$ . Under the null, the implied marginal cost is  $mc_{t'}^0$ , which is different from  $mc_t^0$ . But since neither the cost index nor the quantity changed, this is impossible. This contradiction rules out the false null.

This is a particularly simple example, designed to be similar to that in Bresnahan (1982) and to use a similar type of exogenous variation (although rotating marginal revenue, not demand). But it points toward a more general strategy for discriminating between alternative models of supply based on the following result.

**THEOREM 8:** *Suppose the hypotheses of either Theorem 4 or Theorem 6 hold. Then the oligopoly supply model is testable. In particular, whenever  $c_j(\cdot) = c_{j'}(\cdot)$  and  $(Q_{jt}, \kappa_{jt}) = (Q_{jt'}, \kappa_{jt'})$ , with  $t' \neq t$  and/or  $j' \neq j$ , the testable restriction*

$$(20) \quad \psi_j(s_t, \tilde{D}(s_t, p_t), p_t) = \psi_{j'}(s_{t'}, \tilde{D}(s_{t'}, p_{t'}), p_{t'})$$

*must hold.*

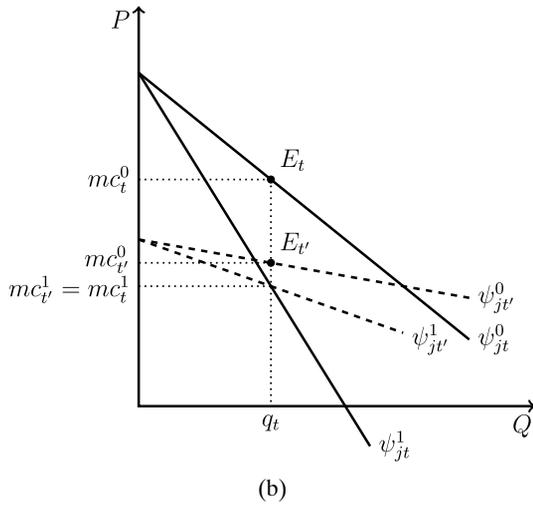
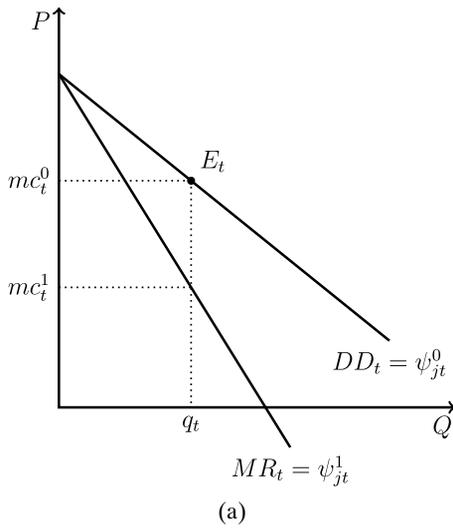


FIGURE 2.—(a) Possible marginal costs under two models. (b) Rotation of  $\psi_j^1$  rules out the false model.

PROOF: By Lemma 1,  $D(\chi_t) = \tilde{D}(s_t, p_t)$  (see the proof of Lemma 2), so by Assumptions 7b and 10 we have

$$(21) \quad c_j(Q_{jt}, \kappa_{jt}) = \psi_j(s_t, \tilde{D}(s_t, p_t), p_t) \quad \forall j, t.$$

Since the right-hand side of (21) cannot change unless the left-hand side does, (20) must hold. To see that this restriction is testable, first observe that when the hypotheses of Theorem 4 or Theorem 6 hold, the matrix  $\tilde{D}(s_t, p_t)$  and

vector  $(\kappa_{1t}, \dots, \kappa_{jt})$  are identified for all  $t$ . Further,  $Q_{jt}, s_t,$  and  $p_t$  are directly observed for all  $j$  and  $t$ . Thus, given any candidate functions  $(\psi_1, \dots, \psi_j)$ , both sides of (20) are known. Q.E.D.

In the example above, we had  $(Q_{jt}, \kappa_{jt}) = (Q_{jt'}, \kappa_{jt'})$  for the single firm  $j$  and examined the restriction  $\psi_j(s_t, \tilde{D}(s_t, p_t), p_t) = \psi_j(s_{t'}, \tilde{D}(s_{t'}, p_{t'}), p_{t'})$ . With more than one good, if we assume  $c_j(\cdot) = c_{j'}(\cdot)$  for some  $j' \neq j$  (a common assumption is that  $c_j(\cdot) = c(\cdot) \forall j > 0$ ), then Theorem 8 requires  $\psi_j(s_t, \tilde{D}(s_t, p_t), p_t) = \psi_{j'}(s_t, \tilde{D}(s_t, p_t), p_t)$  whenever  $(Q_{jt'}, \kappa_{jt'}) = (Q_{jt}, \kappa_{jt})$ . Thus, unlike the example above, the restriction can be applicable within a single market and does not require any changes in market demand. Further, there is no a priori restriction on the class of models that can be considered. We require only variation between two markets  $t$  and  $t'$  or products  $j$  and  $j'$  that (i) leaves the relation between quantities and marginal cost unchanged (i.e.,  $c_j(\cdot, \kappa_{jt}) = c_{j'}(\cdot, \kappa_{jt'})$ ), (ii) leaves equilibrium quantities unchanged for the relevant firm(s) under the true model (i.e.,  $Q_{jt} = Q_{jt'}$ ), but (iii) would imply a change in at least one of these equilibrium quantities under the false model.

Conditions providing such variation will depend on the models considered, but include a much broader class than the rotations of market demand considered by Lau (1982) and Bresnahan (1982, 1989). For example, even if preferences and market size are identical in markets  $t$  and  $t'$  (i.e., there is no change in demand),  $\psi_{jt}$  and  $\psi_{jt'}$  can differ under the false null due to variation in the number of competing firms, the set of competing goods, characteristics of competing products ( $x_{-jt}^{(1)}$  and/or  $\xi_{-jt}$ ), or costs of competing firms ( $w_{-jt}^{(1)}$  and/or  $\kappa_{-jt}$ ).<sup>36</sup> Since we have shown how to identify the demand shocks  $\xi_{jt}$  without knowing the oligopoly model, the variation exploited here need not be exogenous. Further, although we have conditioned on  $x_t^{(2)}, w_{-jt}^{(2)}$ , and  $M_t$ , variation in any of these will typically alter  $\psi_j$  and/or  $\tilde{D}(s_t, p_t)$ , and may also be exploited.

The following example illustrates some of this broader applicability in another simple but less trivial model. While the logit model in the example is still simple enough to give analytical results (a feature not required to allow application of Theorem 8), it is closer to those used to study differentiated products markets in practice.

6.1.1. *Logit Example*

Consider a cross section of markets, each with two inside goods and an outside good. Demand is known to the researcher and characterized by a multinomial logit model, where  $\sigma_j(\delta, p) = \frac{e^{\delta_j - p_j}}{1 + e^{\delta_1 - p_1} + e^{\delta_2 - p_2}}$ . For simplicity, let each firm have constant marginal cost  $mc_j = c_j(\kappa_j)$ . This is not essential and we do not

<sup>36</sup>Recall that different time periods in the same geographic area typically are treated as distinct “markets.”

assume that the researcher knows that marginal costs are constant.<sup>37</sup> Supply is characterized by Nash equilibrium in a complete information simultaneous move duopoly price-setting game. Equilibrium prices are then characterized implicitly by the first-order conditions<sup>38</sup>

$$(22) \quad mc_j = p_j - \frac{1}{1 - \sigma_j(\delta, p)}, \quad j = 1, 2.$$

Let  $(s, p)$  denote one observed equilibrium outcome (market shares and prices). Now consider another outcome  $(s', p')$  realized when  $mc_1$  and  $\delta$  are held fixed,  $mc_2$  falls to  $mc'_2$ , and the market size exogenously grows from  $M$  to  $M'$ . Because prices are strategic complements, both equilibrium prices fall. So by (22),  $s'_1 < s_1$ . However, let the increase in market size leave

$$q_1 = Ms_1 = M's'_1 = q'_1.$$

Since both prices decline, the market share of the outside good falls, that is,  $s'_0 < s_0$ . This further implies  $s'_2 > s_2$ . We will show that the two data points  $(p_1, p_2, s_1, s_2, M)$  and  $(p'_1, p'_2, s'_1, s'_2, q'_1, M')$  are sufficient to rule out the most commonly considered alternative models of supply.

*Marginal Cost Pricing.* Let  $\tilde{mc}_1$  denote the value of firm 1's marginal cost at quantity  $q_1$  inferred under the (incorrect) hypothesis of marginal cost pricing. Since  $q_1 = q'_1$ , Theorem 8 requires that  $\tilde{mc}_1$  rationalize both data points, that is,

$$p_1 = \tilde{mc}_1 = p'_1.$$

Since  $p'_1 < p_1$ , this is a contradiction.

*Quantity Competition.* With logit demand, inverse demand for good  $j$  is given by

$$(23) \quad p_j = \delta_j + \ln(s_0) - \ln(s_j).$$

Since  $s_0 = 1 - s_1 - s_2$ , we have  $\frac{\partial p_1}{\partial s_1} = -\frac{1}{s_0} - \frac{1}{s_1}$  and  $\frac{\partial p_1}{\partial s_2} = -\frac{1}{s_0}$ . If supply is characterized by Nash equilibrium in a complete information simultaneous move quantity-setting game, firm 1 chooses  $s_1$  to maximize  $Ms_1p_1 - C_1(Ms_1)$ , where

<sup>37</sup>If it were known that each  $c_j$  is constant in  $Q_{jt}$ , the requirement  $Q_{jt} = Q_{j't}$  in Theorem 8 could be dropped.

<sup>38</sup>Recall that in the logit model,  $\frac{\partial \sigma_j(\delta, p)}{\partial p_j} = -\sigma_j(\delta, p)(1 - \sigma_j(\delta, p))$ , while  $\frac{\partial \sigma_j(\delta, p)}{\partial p_k} = \sigma_j(\delta, p)\sigma_k(\delta, p)$ .

$C_1(\cdot)$  is the total cost function of firm 1. This yields the first-order condition (after substituting for  $\frac{\partial p_1}{\partial s_1}$ )

$$(24) \quad p_1 - mc_1 - \frac{s_1}{s_0} - 1 = 0.$$

As above, because  $q_1 = q'_1$ , if (24) is the correct first-order condition, one marginal cost must rationalize both data points, that is, there must be some  $\widetilde{mc}_1$  such that

$$(25) \quad \widetilde{mc}_1 = p_1 - \frac{s_1}{s_0} - 1 = p'_1 - \frac{s'_1}{s'_0} - 1.$$

With logit demand,  $\frac{s_1}{s_0} = e^{\delta_1 - p_1}$ , so (25) requires

$$p_1 - p'_1 = e^{\delta_1 - p_1} - e^{\delta_1 - p'_1},$$

which is impossible because  $p_1 > p'_1$  implies  $e^{\delta_1 - p_1} < e^{\delta_1 - p'_1}$ .

*Joint Profit Maximization.* Finally, consider the alternative model in which prices (or quantities) are set to maximize the joint profits from the two inside goods,

$$p_1 M s_1 - C_1(M s_1) + p_2 M s_2 - C_2(M s_2).$$

Prices would be determined by the first-order conditions

$$1 - (1 - s_1)(p_1 - mc_1) + s_2(p_2 - mc_2) = 0,$$

$$1 - (1 - s_2)(p_2 - mc_2) + s_1(p_1 - mc_1) = 0.$$

Straightforward algebra yields

$$\frac{1}{s_0} - (p_1 - mc_1) = 0.$$

Following the logic above, if this is the correct model, we must have

$$\widetilde{mc}_1 = p_1 - \frac{1}{s_0} = p'_1 - \frac{1}{s'_0}.$$

But this cannot hold, since  $p'_1 < p_1$  and  $s'_0 < s_0$ .

### 6.2. Case 2

In Case 1, we used the fact that if the function  $\psi_j$  is correctly specified, there must be a single cost function  $c_j$  such that equation (21) always holds. We now

consider a version of the same idea that is applicable without the additional index and exclusion restrictions that enable identification of each  $\kappa_{jt}$  without specifying  $\psi_j$ . In this case, the restriction (21) no longer has bite when comparing pairs of observations: variation in the right-hand side can always be attributed to latent variation in  $\kappa_{jt}$ . However, one can obtain a testable restriction by taking expectations conditional on exogenous variables.

To demonstrate this, we return to the separable specification of the marginal cost function (11)

$$(26) \quad mc_{jt} = \bar{c}_j(Q_{jt}, w_{jt}) + \omega_{jt}$$

and maintain the exclusion restrictions of Assumption 8.<sup>39</sup> Consider a candidate specification of  $\psi_j$ . Taking expectations of (21) using (26) and Assumption 8, we have

$$(27) \quad E[\bar{c}_j(Q_{jt}, w_{jt})|w_{jt}, y_{jt}] = E[\psi_j(s_t, M_t, \tilde{D}(s_t, p_t), p_t)|w_{jt}, y_{jt}] \quad \text{a.s.}$$

Thus, a testable restriction of the model is that there exist some (possibly counterfactual) marginal cost function  $\tilde{c}_j(Q_{jt}, w_{jt})$  such that

$$(28) \quad E[\tilde{c}_j(Q_{jt}, w_{jt})|w_{jt}, y_{jt}] = E[\psi_j(s_t, M_t, \tilde{D}(s_t, p_t), p_t)|w_{jt}, y_{jt}] \quad \text{a.s.}$$

**THEOREM 9:** *Suppose the hypotheses of either Theorem 1 or Theorem 5 hold, Assumption 8 is satisfied, and marginal costs take the separable form (26). Then the oligopoly supply model is testable. In particular, for each  $j > 0$ , there must exist a function  $\tilde{c}_j$  consistent with (28).*

This can be seen as a weaker version of the restriction (21) considered in Case 1—weaker because the restriction applies only to conditional expectations rather than to each value of  $\psi_j(s_t, M_t, \tilde{D}(s_t, p_t), p_t)$ .<sup>40</sup>

Note that we did not assume the completeness condition (Assumption 9) used in Theorem 3. This is because it is possible that variation in the instruments  $w_{jt}, y_{jt}$  can rule out a false supply model even when there is no subset of these instruments that would point identify the marginal cost function when this is the true model. Nonetheless, a natural special case is that in which the instruments  $w_{jt}, y_{jt}$  lead to overidentification. For example, suppose there are two distinct subsets of the instruments that would allow point identification of a marginal cost function  $\tilde{c}_j(Q_{jt}, w_{jt})$  consistent with (28) using Theorem 3 and a candidate function  $\psi_j$ . Let  $\omega_{jt}^*$  denote the cost shocks implied by the first set

<sup>39</sup>A similar testable restriction could be derived with a nonseparable cost function and full independence of the instruments.

<sup>40</sup>The power of the restriction may be increased if one imposes shape restrictions on the marginal cost function, for example, weak monotonicity in output.

of identifying instruments, and let  $\omega_{jt}^{**}$  denote those implied by the second. A testable restriction of the candidate oligopoly specification is that  $\omega_{jt}^* = \omega_{jt}^{**}$  for all  $j, t$ .

7. DISCUSSION: FUNCTIONAL FORM VERSUS INSTRUMENTS

Our results demonstrate that strong functional form restrictions are not necessary for identification of demand or firm costs, or for discrimination between alternative models of oligopoly competition. Rather, the essential requirement is the availability of instruments. A need for instruments to identify demand and supply is not surprising, and it should be comforting that this is essentially all that is required. From a practical perspective, our results clarify the types of exclusion restrictions needed. Intuition from elementary supply and demand models might have suggested that identification of demand would require only cost shifters or other excluded instruments for prices. However, we required instruments for both prices and “quantities” (market shares), even for identification of demand alone. This need arises from the fact that, in a multi-good setting, demand is defined by a system of equations, each depending on the  $J$ -dimensional endogenous price vector  $p_t$  as well as the  $J$ -dimensional latent demand shocks  $\xi_t$ . One can obtain a system of equations with one unobservable per equation when the demand system is inverted (Lemma 1) to give

$$(29) \quad \xi_{jt} = \sigma_j^{-1}(s_t, p_t) - x_{jt}, \quad j = 1, \dots, J.$$

But now each equation depends on the  $2J$  endogenous variables  $(s_t, p_t)$ . As we have shown, because  $x_{jt}$  is exogenous and enters these equations with a known (normalized) coefficient, it is available as one such instrument. Thus, the demand shifters  $x_t$  provide  $J$  of the  $2J$  instruments required for identification, leaving a need for  $J$  additional instruments such as product-specific cost shifters.

We have seen that once demand is identified, identification of supply features requires either no new instruments (to identify marginal costs) or few instruments relative to those likely to be available (to identify marginal cost functions). Focusing then on the IV requirements for identification of demand, it is useful to compare the inverse demand equations (29) to those in more familiar parametric models. In a multinomial logit model with linear random utilities, inverse demand takes the form (see, e.g., Berry (1994))

$$(30) \quad \xi_{jt} = \ln\left(\frac{s_{jt}}{s_{0t}}\right) + \alpha p_{jt} - x_{jt}\beta,$$

or, equivalently,

$$(31) \quad \tilde{\xi}_{jt} = \frac{1}{\beta} \ln\left(\frac{s_{jt}}{s_{0t}}\right) + \frac{\alpha}{\beta} p_{jt} - x_{jt},$$

where  $\tilde{\xi}_{jt} = \frac{1}{\beta} \xi_{jt}$ , giving a different scale normalization of the latent demand shocks. It is clear from either equation that identification of the unknown parameters  $(\alpha, \beta)$  requires, in addition to the exogenous  $x_{jt}$ , an excluded instrument for price. Valid instruments could include cost shifters or the “BLP instruments”  $x_{-jt}$ . Observe that in (31), as in (29),  $x_{jt}$  enters with a known coefficient; however, it provides exogenous variation in the function of market shares,  $\ln(\frac{s_{jt}}{s_{0t}})$ , which enters with unknown coefficient  $\frac{1}{\beta}$ .

In the nested logit model, inverse demand takes the form (again see Berry (1994))

$$\xi_{jt} = [\ln(s_{jt}) - \ln(s_{0t}) - \theta_g \ln(s_{j/g,t})] + \alpha p_{jt} - x_{jt} \beta$$

or

$$(32) \quad \tilde{\xi}_{jt} = \frac{1}{\beta} [\ln(s_{jt}) - \ln(s_{0t}) - \theta_g \ln(s_{j/g,t})] + \frac{\alpha}{\beta} p_{jt} - x_{jt}.$$

Here,  $\theta_g$  is a parameter governing substitution within/between subgroups of products (“nests”) and  $s_{j/g,t}$  is the within-group share of product  $j$ . Relative to the more restrictive multinomial logit, one needs an additional instrument due to the presence of the endogenous variable  $\ln(s_{j/g,t})$ , which enters with unknown coefficient  $\theta_g$ . The exogenous demand shifters  $x_{-jt}$  are available for this role, in which case  $x_{jt}$  may again be viewed as providing the needed exogenous variation in the market shares appearing in (32).

Finally, in the BLP model, inverse demand takes the richer parametric form

$$(33) \quad \xi_{jt} = \tilde{\delta}_j(s_t, x_t, p_t, \Sigma) - \alpha p_{jt} - x_{jt} \beta$$

(or, equivalently,  $\tilde{\xi}_{jt} = \frac{1}{\beta} \tilde{\delta}_j(s_t, x_t, p_t, \Sigma) - \frac{\alpha}{\beta} p_{jt} - x_{jt}$ ), where the function  $\tilde{\delta}_j$  maps market shares to “mean utilities”  $x_{jt} \beta - \alpha p_{jt} + \xi_{jt}$ . This function is known up to the parameters  $\Sigma$ , which govern the distribution of the random coefficients. Since  $(s_t, p_t)$  are endogenous, identification requires not only an excluded instrument for  $p_{jt}$ , but also a sufficient number of instruments (“BLP instruments” and/or others) to identify all parameters  $(\alpha, \beta, \Sigma)$ . This need for additional instruments arises from the presence of unknown parameters interacting with the endogenous variables  $(s_t, p_t)$ .

We can see that these examples are special cases of a general parametric model

$$(34) \quad \xi_{jt} = \sigma_j^{-1}(s_t, p_t; \theta) - x_{jt},$$

where  $\theta$  is a finite-dimensional parameter. Further, as additional flexibility in substitution patterns is allowed, the inverse demand function  $\sigma_j^{-1}(s_t, p_t; \theta)$  takes a form that depends in a richer way on the vectors of market shares and

prices, with the need for exclusion restrictions growing accordingly. Our equation (29) is the natural nonparametric extension of these examples. And as a result, our instrumental variables requirements are an extension of those in these familiar parametric models.

One message from this discussion is that there are tradeoffs between a priori functional form restrictions and the need for exclusion restrictions. It also suggests a greater range of alternatives that might be explored in future work by focusing directly on the form of the inverse demand system. For example, suppose (as in many empirical applications) price enters only through the demand index, so that  $\delta_{jt} = x_{jt} - \alpha p_{jt} + \xi_{jt}$ , with  $\alpha$  an unknown parameter. Equation (29) then takes the semiparametric form

$$\xi_{jt} = \sigma_j^{-1}(s_t) + \alpha p_{jt} - x_{jt}.$$

Identification of demand would require only a single instrument (e.g., a market level cost shifter) beyond the exogenous product characteristics  $x_t$ . In fact, the same reduction in the need for excluded instruments can be obtained without linearity of the index. Suppose instead that

$$\delta_{jt} = \delta_j(x_{jt}, p_{jt}, \xi_{jt}),$$

where  $\delta_j$  is an unknown function required only to be strictly increasing in  $\xi_{jt}$ . This is a trivial variation on the model considered in Appendix B, and applying the argument given there yields

$$s_{jt} = h_j(s_{-jt}, x_{jt}, p_{jt}, \xi_{jt}),$$

where  $h_j$  is an unknown function required only to be strictly increasing in  $\xi_{jt}$ . This equation takes the form of a nonseparable nonparametric regression model (e.g., Chernozhukov and Hansen (2005)), and identification of  $h_j$  (and therefore of each  $\xi_{jt}$ , yielding identification of demand) requires only a single excluded instrument beyond the exogenous product characteristics  $x_t$ .<sup>41</sup>

Finally, it is instructive to consider the case in which prices are exogenous (independent of  $\xi_t$ ) and can therefore be fixed (as we did with  $x_t^{(2)}$ ) and dropped from the notation to consider identification. One might speculate that identification holds without any exclusion restrictions. However, this is incorrect, again due to the determination of each quantity as a function of the entire vector  $\xi_t$  of latent demand shocks. In this case, (29) takes the form

$$\xi_{jt} = \sigma_j^{-1}(s_t) - x_{jt}.$$

<sup>41</sup>Another possible restriction is the symmetry/exchangeability assumption  $\sigma_j(\delta_t, p_t) = \sigma(\delta_{jt}, p_{jt}, \delta_{-jt}, p_{-jt})$  for all  $j$ . This would allow within-market (cross-product) variation to contribute to identification and would also allow a weaker version of the usual completeness condition.

The presence of the  $J$ -dimensional vector of endogenous variables  $s_t$  in the unknown function  $\sigma_j$  requires at least  $J$  instruments.<sup>42</sup> As before, the entire  $J$ -vector  $x_t$  is available. However, no alternative instruments are available; in particular, exogenous shifters of costs (or markups) do not help since these enter only through their effect on the (fixed) prices. Thus, identification is possible here only because the index restriction leaves  $x_t$  available as instruments. This suggests not only the necessity of the “BLP instruments,” but the crucial role of our index restriction for making them available.

This example clarifies several other things as well. One is the fundamental difference between models like ours and those often considered in the econometrics literature on discrete choice, which lack the structural errors  $\xi_t$ . In those models, an absence of endogeneity would make identification of demand trivial: observed conditional choice probabilities would *be* demand. Another is the essential role of the assumptions we do make on the random utility model. The connected substitutes assumption allows us to invert the demand system, yielding equations taking a form nearly identical to that of nonparametric regression models. Monotonicity of the index  $\delta_{jt}$  in the structural error  $\xi_{jt}$  ensures that each “regression” equation is monotonic in  $\xi_{jt}$  as required for existing identification results for regression models. And, as already noted, the fact that  $x_{jt}$  enters only through the index  $\delta_{jt}$  yields the essential exclusion of  $x_{-jt}$  from the inverse demand function  $\sigma_j^{-1}$ .

## 8. CONCLUSION

We have considered nonparametric identification in a class of differentiated products models used in a large and growing body of empirical work in IO and other fields of economics. Our results demonstrate that the nonparametric foundation for these models is strong, the primary requirement being the availability of instruments providing adequate exogenous variation in the endogenous prices and quantities. Thus, functional form and/or distributional assumptions relied on in practice can be viewed as playing only their usual roles: smoothing, extrapolation, and compensation for the gap between the exogenous variation necessary to discriminate between all functions and that available in practice. Our results provide guidance on the key sources of variation one should look for in applications, and point to tradeoffs between a priori functional form assumptions and the need for exclusion restrictions.

Although we motivated our analysis of demand with a widely used class of discrete choice models, most of our results hold in other types of demand models. For example, our results on the identification of demand extend immediately to any environment in which one can invert the demand system to obtain

<sup>42</sup>Note the contrast to the literature on semiparametric “transformation” models, for example, Horowitz (1996), which take a similar form but have only one endogenous variable entering the unknown transformation.

the inverse demand system (5). Berry, Gandhi, and Haile (2013) provided a range of examples. Many of our results on identification and falsifiability of the supply model also extend immediately whenever identification of demand holds.

Finally, while we have focused exclusively on identification, our analysis may suggest new estimation or testing approaches. A critical aspect of our identification strategy is to work directly with the economic primitives of interest on the demand side of the model, in particular the demand functions  $\sigma_j$ . This approach contrasts with the usual strategy of building up a random utility model from random parameters, using this to derive demand. Our approach allowed us to obtain positive results while also substantially relaxing functional form and distributional assumptions relied on previously. This approach may prove useful for developing new estimators as well, whether parametric, semiparametric, or nonparametric.<sup>43</sup>

#### APPENDIX A: INVERTIBILITY OF OLIGOPOLY FIRST-ORDER CONDITIONS

In the text, we relied on a high-level assumption ensuring that oligopoly first-order conditions can be inverted to solve for each marginal cost as a function of prices, market shares, and features of the demand system (Assumption 7a or the weaker Assumption 7b). Here, we show that, as claimed in Remark 1, our high-level assumption is satisfied in standard models of differentiated products oligopoly under supply-side assumptions already maintained. These models include complete information games with simultaneous choice of prices or quantities, with single- or multi-product firms. Joint profit maximization (perfect collusion) is a special case. Thus, given any one of these models, Assumption 7a could be viewed as a lemma (proved in this appendix) rather than an assumption.

As emphasized in the text, the strategy of solving first-order conditions for marginal costs has a long history in the IO literature (e.g., Rosse (1970), Bresnahan (1981, 1982, 1987), and Berry, Levinsohn, and Pakes (1995)). The innovation in this appendix is the demonstration, under general nonparametric conditions, of the invertibility of particular substitution matrices. A key condition is that the goods be connected substitutes in prices.

Throughout this appendix, we will fix the market size  $M_t$  and suppress it. Below, we will make use of the following result, which follows immediately from Theorem 2 in Berry, Gandhi, and Haile (2013).

**LEMMA 3:** *Suppose goods  $\{0, 1, \dots, J\}$  are connected substitutes in  $-p_t$  and let Assumption 6 hold. Then, for all  $\chi_t \in \mathcal{X}$ , every principal submatrix of  $D(\chi_t)$  is nonsingular.*

<sup>43</sup>A recent working paper, Gandhi and Nevo (2011), pursued a semiparametric approach. Souza-Rodrigues (2011) considered a nonparametric approach for the closely related “generalized regression model.”

### A.1. Marginal Cost Pricing

With marginal cost pricing, the result is immediate: the function  $\psi_j(s_t, D(\chi_t), p_t)$  is simply  $p_{jt}$ .

### A.2. Price Setting

The most common model of supply for empirical work on differentiated products markets is Nash equilibrium in a complete information simultaneous price-setting game. Recall that  $\mathcal{J}_j$  denotes the set of products produced by the firm that produces good  $j$ , for example, the singleton  $\{j\}$  in the special case of single-product firms or the full choice set  $\mathcal{J}$  in the case of joint profit-maximization (perfect collusion).

Letting  $\sigma_{jt} = \sigma_j(\chi_t)$  as shorthand, the first-order condition for the price of good  $j$  is

$$\sigma_{jt} + \sum_{k \in \mathcal{J}_j} (p_{kt} - mc_{kt}) \frac{\partial \sigma_{kt}}{\partial p_{jt}} = 0.$$

Following Bresnahan (1981, 1987), the first-order conditions for all firms can then be written in matrix form as

$$(A.1) \quad \sigma_t + \Delta_t(p_t - mc_t) = 0,$$

where  $\sigma_t = (\sigma_{1t}, \dots, \sigma_{Jt})'$  and the  $(j, k)$  element of the square matrix  $\Delta_t$  is equal to  $\partial \sigma_{kt} / \partial p_{jt}$  if products  $k$  and  $j$  are produced by the same firm, and zero otherwise. Following BLP, the supply-side “inversion” for marginal cost takes the form

$$mc_t = p_t + \Delta_t^{-1} \sigma_t.$$

Thus, Assumption 7a will be satisfied if the matrix  $\Delta_t$  is invertible. Observe that by simultaneous permutation of rows and columns (i.e., reordering of the goods to match the ownership structure), one can obtain from  $\Delta_t$  a matrix that is block diagonal, with each block being a principal submatrix of  $D(\chi_t)'$ . Invertibility of this new matrix, and therefore of  $\Delta_t$ , follows from Lemma 3.

### A.3. Quantity Setting

Consider the alternative model of simultaneous quantity-setting with complete information, again with Nash equilibrium as the solution concept. A fundamental requirement of this model is existence of an inverse demand function. Theorem 1 of Berry, Gandhi, and Haile (2013) ensures that when goods are connected substitutes in prices, given any  $(x_t, \xi_t)$  in their support, there is a unique price vector  $p_t$  that solves  $s_t = \sigma(\chi_t)$ . This implies existence

of an inverse demand function on  $\sigma(\mathcal{X})$ , which we write in vector form as  $p_t = \rho(s_t, x_t, \xi_t)$ .

If the inverse demand function is differentiable, equilibrium quantities are characterized by  $J$  first-order conditions of the form

$$mc_{jt} = p_{jt} + \sum_{k \in \mathcal{J}_j} \frac{\partial \rho_k}{\partial s_{jt}} s_{kt}.$$

Thus, Assumption 7a holds as long as the derivatives  $\frac{\partial \rho_k}{\partial s_{jt}}$  exist. Lemma 3 ensures that when goods are connected substitutes in prices, the matrix  $D(\chi_t)$  is invertible. So, given Assumption 6, the inverse function theorem ensures that derivatives of the inverse demand function exist and are equal to the elements of  $D(\chi_t)^{-1}$ , that is,

$$\frac{\partial \rho_k}{\partial s_{jt}} = [D(\chi_t)^{-1}]_{kj}.$$

APPENDIX B: IDENTIFICATION WITH A NONLINEAR INDEX

Here, we discuss conditions under which the linearity of the indices  $\delta_{jt}$  and  $\kappa_{jt}$  can be relaxed. One possible relaxation is to allow the nonlinear specification

$$\begin{aligned} \delta_{jt} &= g_j(x_{jt}) + \xi_{jt}, \\ \kappa_{jt} &= h_j(w_{jt}) + \omega_{jt}, \end{aligned}$$

where  $g_j$  and  $h_j$  are unknown strictly increasing functions. This partial relaxation drops the requirement of linearity in the demand and cost shifters, while maintaining separability and monotonicity. Our identification results using simultaneous equations (Theorems 5–7) can be extended to this setting by combining the arguments used in the results above with the result in Section 4.2 of Matzkin (2008).<sup>44</sup> Because these extensions are straightforward, we omit a formal statement.

For the remainder of this appendix, we consider an extension of the non-parametric IV approach, where the restrictions on the index can be relaxed further. We focus on the demand index and henceforth redefine

$$\delta_{jt} = \delta_j(x_{jt}, \xi_{jt}),$$

with  $\delta_j$  strictly increasing in its second argument. Relative to (2), this specification drops the assumptions of linearity, separability, and monotonicity in  $x_{jt}$ , retaining only the key requirement of strict monotonicity in  $\xi_{jt}$ .

<sup>44</sup>See also Theorem 1 in Berry and Haile (2013).

Two lemmas will be useful. The first is taken from Berry, Gandhi, and Haile (2013) (their Lemma 3).

LEMMA 4: *Under Assumptions 1 and 2, for any  $(\delta, p)$  and  $(\delta', p)$  in the support of  $(\delta_t, p_t)$ , if  $\delta$  and  $\delta'$  are such that  $I^+ \equiv \{j : \delta'_j > \delta_j\}$  is nonempty, we have  $\sum_{j \in I^+} \sigma_j(\delta', p) > \sum_{j \in I^+} \sigma_j(\delta, p)$ .*

LEMMA 5: *Suppose Assumptions 1 and 2 hold. Then for all  $j = 1, \dots, J$  and any  $s$  such that  $\sum_{j=0}^1 s_j = 1$  and  $s_j > 0$ ,  $\sigma_j^{-1}(s, p)$  is strictly increasing in  $s_j$ .*

PROOF: Arguing by contradiction, take  $j = 1$  without loss and suppose

$$(B.1) \quad s'_1 > s_1, \quad s'_j = s_j \quad \forall j > 1$$

but

$$\delta'_1 = \sigma_1^{-1}(s', p) \leq \sigma_1^{-1}(s, p) = \delta_1.$$

Because probabilities sum to 1, we must have  $s'_0 < s_0$ , that is,

$$(B.2) \quad \sigma_0(\delta', p) < \sigma_0(\delta, p).$$

If the set  $I^+ \equiv \{j : \delta'_j > \delta_j\}$  is nonempty, by Lemma 4 we have

$$\sum_{j \in I^+} \sigma_j(\delta', p) > \sum_{j \in I^+} \sigma_j(\delta, p).$$

Since  $1 \notin I^+$  and  $0 \notin I^+$ , this contradicts the hypothesis that  $s'_j = s_j$  for all  $j > 1$ . Thus,  $I^+$  must be empty, that is,

$$\delta'_j \leq \delta_j \quad \forall j > 0.$$

But in that case, Assumption 2 requires  $\sigma_0(\delta', p) \geq \sigma_0(\delta, p)$ , contradicting (B.2). *Q.E.D.*

Lemma 1 (which did not rely on linearity of the index) guarantees that, for all  $j$  and  $t$ ,

$$\delta_j(x_{jt}, \xi_{jt}) = \sigma_j^{-1}(s_t, p_t)$$

for some functions  $\sigma_j^{-1}$ ,  $j = 1, \dots, J$ . Since  $\delta_j(x_{jt}, \xi_{jt})$  is strictly increasing in  $\xi_{jt}$ , we can then write

$$\begin{aligned} \xi_{jt} &= \delta_j^{-1}(\sigma_j^{-1}(s_t, p_t); x_{jt}) \\ &\equiv g_j(s_t, p_t, x_{jt}) \end{aligned}$$

for some unknown function  $g_j$ . Moreover, since  $\delta_j(\cdot)$  is strictly increasing in  $\xi_{jt}$ ,  $\delta_j^{-1}(\cdot; x_{jt})$  is strictly increasing. So, by Lemma 5,  $g_j$  must be strictly increasing in  $s_{jt}$ . Inverting  $g_j(\cdot)$ , we obtain

$$(B.3) \quad \begin{aligned} s_{jt} &= g_j^{-1}(\xi_{jt}; s_{-jt}, p_t, x_{jt}) \\ &\equiv h_j(s_{-jt}, p_t, x_{jt}, \xi_{jt}), \end{aligned}$$

where  $s_{-jt}$  denotes  $(s_{kt})_{k \neq j}$  and  $h_j$  is an unknown function that is strictly increasing in  $\xi_{jt}$ . The shares  $s_{jt}$  and  $s_{-jt}$  are bounded by definition, and we may assume without loss that  $p_t$  has been transformed to be bounded as well.

For arbitrary  $j > 0$ , consider the identification of the functions  $h_j$  in equation (B.3), which has the form of a nonseparable nonparametric regression function. Because  $x_{jt}$  is exogenous, we can condition on it and drop it from the notation, rewriting (B.3) as

$$(B.4) \quad s_{jt} = h_j(s_{-jt}, p_t, \xi_{jt}).$$

We will assume that  $\xi_{jt}$  has an atomless marginal distribution. Then, without loss, we can normalize  $\xi_{jt}$  to have a standard uniform marginal distribution. We will also assume that  $(s_t, p_t)$  are continuously distributed conditional on  $(z_t, x_{-jt})$  and let  $f_{jsp}(s_{-jt}, p_t | z_t, x_{-jt})$  denote the conditional density of  $s_{-jt}, p_t$ . Let  $f_{js}(s_{jt} | s_{-jt}, p_t, z_t, x_{-jt})$  denote the conditional density of  $s_{jt}$ .

Let  $\epsilon_1$  and  $\epsilon_2$  be some small positive constants. Let  $\alpha_j(s_{-jt}, p_t)$  denote the set

$$\{s \in (0, 1) : f_{js}(s | s_{-jt}, p_t, z_t, x_{-jt}) \geq \epsilon_1 \ \forall (z_t, x_{-jt}) \text{ such that } f_{jsp}(s_{-jt}, p_t | z_t, x_{-jt}) > 0\}.$$

For each  $j$  and  $\tau \in (0, 1)$ , define  $\mathcal{L}_j(\tau)$  as the convex hull of functions  $m_j(\cdot, \cdot, \tau)$  that satisfy (a) for all  $(z_t, x_{-jt})$ ,  $\Pr(s_{jt} \leq m_j(s_{-jt}, p_t, \tau) | z_t, x_{-jt}) \in [\tau - \epsilon_2, \tau + \epsilon_2]$ ; and (b) for all  $(s_{-jt}, p_t)$ ,  $m_j(s_{-jt}, p_t, \tau) \in \alpha_j(s_{-jt}, p_t)$ . Consider the following instrumental variables conditions, from Chernozhukov and Hansen (2005, Appendix C).

ASSUMPTION 18:  $\xi_{jt} \perp\!\!\!\perp (z_t, x_{-jt}) \ \forall t$ .

ASSUMPTION 19: For all  $j = 1, \dots, J$  and  $\tau \in (0, 1)$ :

(i) let  $\varepsilon_{jt} \equiv s_{jt} - h_j(s_{-jt}, p_t, \tau)$ . For any bounded function  $B_j(s_{-jt}, p_t, \tau) = m_j(s_{-jt}, p_t, \tau) - h_j(s_{-jt}, p_t, \tau)$  with  $m_j(\cdot, \cdot, \tau) \in \mathcal{L}_j(\tau)$  and

$$\begin{aligned} \zeta_j(s_{-jt}, p_t, z_t, x_{-jt}, \tau) &\equiv \int_0^1 f_{\varepsilon_j}(bB_j(s_{-jt}, p_t, \tau) | s_{-jt}, p_t, z_t, x_{-jt}) db \\ &> 0, \end{aligned}$$

$E[B_j(s_{-jt}, p_t, \tau)\zeta_j(s_{-jt}, p_t, z_t, x_{-jt}, \tau)|z_t, x_{-jt}] = 0$  a.s. only if  $B_j(s_{-jt}, p_t, \tau) = 0$  a.s.;

(ii) the conditional density  $f_{\epsilon_j}(e|s_{-jt}, p_t, z_t, x_{-jt})$  of  $\epsilon_{jt}$  is continuous and bounded in  $e$  over  $\mathbb{R}$  a.s.;

(iii)  $h_j(s_{-jt}, p_t, \tau) \in \alpha_j(s_{-jt}, p_t)$  for all  $(s_{-jt}, p_t)$ .

Assumption 18 strengthens the exclusion restriction of Assumption 3 to require full independence. Assumption 19 is a type of “bounded completeness” condition that replaces Assumption 4 in the text. It was used previously by Chernozhukov and Hansen (2005) to demonstrate nonparametric identification of quantile treatment effects.

**THEOREM 10:** *Suppose Assumptions 1, 2, 18, and 19 hold. Then (i)  $\xi_{jt}$  is identified with probability 1 for all  $t$ , and (ii) the function  $\sigma_j(\cdot)$  is identified on  $\mathcal{X}$ .*

**PROOF:** Identification of  $h_j(\cdot, \tau)$  for each  $\tau \in (0, 1)$  follows from Theorem 4 of Chernozhukov and Hansen (2005) after noting that, for each value of  $\xi_{jt} \in (0, 1)$ , the model (B.4) is equivalent to a special case of the model they considered. Parts (i) and (ii) then follow immediately, as in the proof of Theorem 1. *Q.E.D.*

Since  $j$  was arbitrary, this shows that the analog of Theorem 1 (i.e., identification of demand) can be obtained with the relaxed index structure.

#### APPENDIX C: AN EXAMPLE: MULTIPLE ENDOGENOUS PRODUCT CHARACTERISTICS

As discussed in the text, extension of our results to the case of more than one endogenous product characteristic is straightforward provided that sufficient instruments are available. For example, if we let each  $p_{jt}$  have two components, Theorem 1 still holds as stated, although the completeness condition of Assumption 4 would require  $z_t$  to have at least  $2J$  components instead of  $J$ . In what follows, we point out that in some special cases, additional instruments may not be necessary, at least for identification of key features of demand such as own- and cross-price elasticities.

The important observation is that the exclusion conditions relied on in the text required independence (or mean independence) between instrument vector  $x_t^{(1)}$  and the structural errors  $\xi_t$  conditional on  $x_t^{(2)}$  (or, in some cases, between  $(x_t^{(1)}, w_t^{(1)})$  and  $(\xi_t, \omega_t)$  conditional on both  $x_t^{(2)}$  and  $w_t^{(2)}$ ). This type of conditional independence is possible even when  $(x_{jt}^{(1)}, x_{jt}^{(2)}, \xi_{jt})$  are all jointly determined.

Consider a simple model of automobile product design. Suppose  $x_{jt}^{(2)}$  consists of indicators for automobile class—for example, luxury sedan, pickup truck, etc.,  $x_{jt}^{(1)}$  is a measure of size, and  $\xi_{jt}$  is an index representing unmeasured

quality—e.g., leather seats, electronic features, and safety features.<sup>45</sup> Suppose further that

$$\begin{aligned}x_{jt}^{(1)} &= A(x_{jt}^{(2)}) + \epsilon_{jt}^{(1)}, \\ \xi_{jt} &= B(x_{jt}^{(2)}) + \epsilon_{jt}^{(2)},\end{aligned}$$

where  $\epsilon_{jt}^{(1)}$ ,  $\epsilon_{jt}^{(2)}$  are independent of  $x_{jt}^{(2)}$ . An interpretation is that  $A(x_{jt}^{(2)})$  and  $B(x_{jt}^{(2)})$  are the “standard” size and unmeasured quality for an automobile in class  $x_{jt}^{(2)}$ , with  $\epsilon_{jt}^{(1)}$  and  $\epsilon_{jt}^{(2)}$  representing idiosyncratic deviations from the standard in each dimension arising through the design process. Here, size of the car  $x_{jt}^{(1)}$ , its class  $x_{jt}^{(2)}$ , and its unobserved quality  $\xi_{jt}$  are all correlated. However, conditional on  $x_{jt}^{(2)}$ ,  $x_{jt}^{(1)}$  is independent of  $\xi_{jt}$ .

Thus, in this example, identification of demand (conditional on  $x_t^{(2)}$ ) follows from Theorem 1 without modification. Note, however, that the effects of an exogenous change in  $x_t^{(2)}$  on demand would not be identified without additional structure such as that provided by the example.

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<sup>45</sup>It should be clear that additional observables that are exogenous conditional on  $x_t^{(2)}$  are easily accommodated.

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