# Idiosyncratic Risk Matters!*

Amit Goyal[†]        Pedro Santa-Clara [‡]

October 2001

## Abstract

This paper takes a new look at the tradeoff between risk and return in the stock market. We find a significant positive relation between average stock variance and the return on the market. There is, therefore, a tradeoff between risk and return in the stock market, except that risk is measured as total risk, *including idiosyncratic risk*, rather than only systematic risk. Further, we find that the variance of the market by itself has no forecasting power for the market return. These relations persist after we control for macroeconomic variables known to forecast the stock market. We show that idiosyncratic risk explains most of the variation of average stock risk through time and it is idiosyncratic risk that drives the forecastability of the stock market.

# Idiosyncratic Risk Matters!

**Abstract**

This paper takes a new look at the tradeoff between risk and return in the stock market. We find a significant positive relation between average stock variance and the return on the market. There is, therefore, a tradeoff between risk and return in the stock market, except that risk is measured as total risk, *including idiosyncratic risk*, rather than only systematic risk. Further, we find that the variance of the market by itself has no forecasting power for the market return. These relations persist after we control for macroeconomic variables known to forecast the stock market. We show that idiosyncratic risk explains most of the variation of average stock risk through time and it is idiosyncratic risk that drives the forecastability of the stock market.

# 1 Introduction

Merton's ICAPM suggests a positive relation between risk and return in the stock market.[1] Given the importance of this model, there is a long empirical literature that has tried to establish the existence of such a tradeoff between risk and return in the market.[2] Unfortunately, the results have been inconclusive. Often the relation between risk and return has been found insignificant, and sometimes even negative.

In this paper, we again investigate the relation between risk and return in the market. Our main innovation is to look at average stock risk in addition to market risk. We are motivated by the generalized lack of diversification of investors' portfolios. Recent papers by Barber and Odean (2000), Benartzi and Thaler (2001), and Falkenstein (1996) report that both individual investors' portfolios and mutual fund portfolios are surprisingly undiversified. The situation is even more extreme if we consider portfolios in a broader sense to include human capital and private equity. Heaton and Lucas (1997, 2000) and Vissing-Jørgensen and Moskowitz (2001) offer evidence that investors hold substantial amounts of idiosyncratic risk in the form of human capital and private equity. This lack of diversification suggests that the relevant measure of risk for many investors may be the total risk of individual stocks. To the extent that undiversified investors have an impact in the market, this should be reflected in the pricing of total stock risk, including idiosyncratic risk.

We measure the risk of the market in a given month by the variance of the daily market returns in that month. Similarly, we measure average stock risk in each month as the cross-sectional average of the variances of all the stocks traded in that month, where the variance of each stock is calculated with daily stock returns within the month. This is the same measure of average stock risk that was used by Campbell, Lettau, Malkiel, and Xu (2001) (hereafter CLMX). Our analysis is focussed on predictive regressions of market returns on these variance measures.

Consistent with some previous studies, we find that the lagged variance of the market has no forecasting power for the market return. This is evidence against a tradeoff between risk and return when risk is measured as systematic risk only. However, we do find a significant positive relation between lagged average stock variance and the return on the market. There does exist a tradeoff between risk and return in the stock market, except that risk is measured as total risk, *including idiosyncratic risk*, rather than only systematic risk.

To confirm these puzzling results, we run a battery of robustness checks. First, we implement a bootstrap experiment to check the small sample properties of our regres-

---

[1]However, Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993) offer models where a *negative* relation between return and volatility is consistent with equilibrium.

[2]See, for example, Baillie and DeGennaro (1990), Campbell (1987), Campbell and Hentschel (1992), Chan, Karolyi, and Stulz (1992), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993), Merton (1980), Nelson (1991), Pindyck (1984), Scruggs (1998), Turner, Startz, and Nelson (1989), and Whitelaw (1994).

sions. The distribution of the test statistics from the regressions of returns on variance measures can be distorted due to the high skewness and kurtosis of both the regressand and the regressors. Additionally, the regressions may suffer from the conjunction of the high persistence of regressors and their correlation with the dependent variable, which is the typical problem of predictive regressions studied by Stambaugh (1999). Our bootstrap experiment handles these issues simultaneously. The results from the experiment indicate that the relation between market return and average stock risk is not a small sample illusion.

Second, we extend the sample. Since our measure of average stock variance is only available since July of 1962, when CRSP started recording daily data, we use another measure recently proposed by Goyal and Santa-Clara (2001) (hereafter GS). That paper offers a measure of variance constructed using the cross-sectional average of the squared monthly returns of all stocks. GS find that this alternative measure has similar characteristics to the measure that uses daily data in the common sample (since 1962) but has the added advantage of being available since 1926, which is the beginning of the CRSP tapes. Using this alternative measure for the longer sample, we find that the coefficient on average variance remains significantly positive while the coefficient on market variance is still insignificant.

One potential explanation for our finding is that average stock variance might merely be proxying for business cycle fluctuations. Indeed, CLMX (2001) and GS (2001) have found that average stock risk is strongly pro-cyclical. There is also substantial evidence that macroeconomic variables related to the business cycle can forecast stock market returns.[3] Therefore, the effect of average stock variance on the stock market might only be proxying for variations in the business cycle. To test this "proxy" hypothesis, we examine the relation between stock market returns and average stock variance using macro variables known to forecast the stock market as controls for business cycle fluctuations. However, after controlling for the dividend-price ratio, the default and term spreads, and the relative riskfree interest rate, our results retain their significance.

To further establish the robustness of the predictor, we run regressions of the returns of a variety of portfolios other than the market on the variance measures. For portfolios sorted by size and book-to-market, and by industry classification, we find that the regressions yield significantly positive coefficients on average stock variance and insignificant coefficients on market variance. We conclude that the forecasting ability of average stock variance holds across a variety of portfolios.

To assess the economic significance of the predictability of stock returns, we construct a trading strategy in the spirit of Breen, Glosten, and Jagannathan (1989) based on out-of-sample forecasts of the market using the risk measures. For each month in the sample, we use the previous historical data on the conditioning variables to forecast the excess return on the market next period. If the predicted excess return is positive, the strategy invests fully in the stock market, otherwise the strategy invests in the riskfree asset. We

---

[3]See, for example, Campbell (1991), Campbell and Shiller (1988), Chen, Roll, and Ross (1986), Fama (1990), Fama and French (1988, 1989), and Keim and Stambaugh (1986).

find that the return of the trading strategy that conditions on average stock variance has a higher mean and a lower standard deviation (overall yielding a higher Sharpe ratio) than the return provided by buying and holding the market. Using the metric developed by Fleming, Kilby, and Ostdiek (2001), we find that a quadratic utility investor with relative risk aversion of 5 would be willing to pay 2.5 percent per year to a manager that was able to time the market with the average stock variance measure.

We study a decomposition of the measure of average stock risk using a factor model of returns. We show that average stock variance can be decomposed into a systematic component — driven by the factors' risk premia, the factors' variances, and the cross-sectional dispersion of the betas of the stocks on the factors — and an idiosyncratic component. We use two particular factor models — the market model and the Fama-French three-factor model — to estimate the two components of average stock variance. We find that on average throughout our sample, the idiosyncratic component represents 85 percent of the total average stock variance according to the market model, and 80 percent according to the Fama-French model. Additionally, it is the time variation of the idiosyncratic variance component that is responsible for most of the time variation of average stock variance. In the market model, the variation of the idiosyncratic component is 84 percent of the variation of the average stock variance. In the Fama-French model, this number is 70 percent. We use the estimated residuals from these factor models to compute an alternative measure of idiosyncratic variance. Regressions of the market return on the lagged measures of idiosyncratic variance corroborate the previous results. We conclude that it is the idiosyncratic component that drives the significance of average stock variance in explaining market returns.

Our results are complementary to the evidence of idiosyncratic risk being priced in the cross-section of stocks. In very early work, Douglas (1969) and Lintner (1965) found that the variance of the residuals from a market model was strongly significant in explaining the cross-sectional average returns of stocks. These results were largely disregarded after Miller and Scholes (1972), and Fama and Macbeth (1973) pointed the statistical pitfalls in these regressions. However, new evidence has surfaced about the pricing of idiosyncratic risk in the cross section. Lehmann (1990) reaffirms the results of Douglas (1969) after conducting a careful econometric analysis. Malkiel and Xu (1997, 2000) (hereafter MX) present comprehensive evidence of the importance of idiosyncratic risk in explaining the cross section of expected stock returns. MX run cross-sectional regressions of individual stock returns on their size, beta, and idiosyncratic volatility,[4] and find that idiosyncratic volatility is a significant regressor. Moreover, in multiple regression tests that include both idiosyncratic risk and size as explanatory variables, MX find that size becomes insignificant. MX also show that a factor portfolio constructed on the basis of idiosyncratic risk ("high minus low," à là Fama-French) helps explain the time variation of returns to different portfolios.

This paper is organized as follows. We introduce our risk measure and explore the main risk-return regressions in Section 2. Section 3 performs a host of robustness checks

---

[4]MX use either a market model or Fama-French factors to compute the variance of residuals from five years of monthly data prior to the cross-sectional regressions.

on the basic risk-return tradeoff. These include introducing a different measure of volatility, extending the sample, correcting for small sample inference problems, controlling for business cycle variables, and forecasting portfolios other than the market. We also assess the economic significance of our results in this section. Section 4 contains a discussion of our volatility measures. It includes additional checks using residuals from the market model and the Fama-French three factor model to construct alternative measures of idiosyncratic volatility. We conclude in Section 5 by positing some possible explanations for our findings.

# 2    Main Results

In this section, we first introduce our risk measures computed using daily data and discuss some salient features of those measures. Then we investigate the tradeoff between risk and return. We find that the lagged variance of the market has no forecasting power for the market return. However, we do find a significant positive relation between average stock variance and the return on the market.

## 2.1    Risk Measures

Each month we compute the variance of a portfolio $p$ using within-month daily return data:

$$V_{pt} = \sum_{d=1}^{D_t} r_{pd}^2 + 2 \sum_{d=2}^{D_t} r_{pd}\, r_{pd-1}, \tag{1}$$

where $D_t$ is the number of days in month $t$ and $r_{pd}$ is the portfolio's return on day $d$. The second term on the right hand side adjusts for the autocorrelation in daily returns using the approach proposed by French, Schwert, and Stambaugh (1987).[5] Similarly, we compute the average stock variance as the arithmetic average of the variance of each stock's daily returns:

$$V_t = \frac{1}{N_t} \sum_{i=1}^{N_t} \left[ \sum_{d=1}^{D_t} r_{id}^2 + 2 \sum_{d=2}^{D_t} r_{id}\, r_{id-1} \right], \tag{2}$$

where $r_{id}$ is the return on stock $i$ in day $d$, and $N_t$ is the number of stocks that exist in month $t$. Note that this is not strictly speaking a variance measure since we do not

---

[5]Note that if the autocorrelation of returns is less than -0.5, then the second term dominates and makes the variance estimate negative. Although this is never the case with portfolio volatility, it sometimes occurs (on average for 5 percent of the cases) for individual stocks. For these stocks, we ignore the second term and, instead, calculate the stock variance as the sum of squared returns only. Finally, stocks with less than 5 trading days in a month are excluded from computations for that month. The results that we present in this paper are robust to changes in these computational procedures.

demean returns before taking the expectation. However, for short holding periods, the impact of subtracting the means is not very important. For monthly excess returns of typical stocks, the expected squared return may overstate the variance by less than one percent.[6] The advantage of this approach, of course, is that it allows us to sidestep the issue of estimating conditional means for each stock. In the rest of the paper we ignore the distinction and refer to expected squared returns as variances.

We compute the risk measures using CRSP data from July of 1962 to December of 1999. This is the same sample period used by CLMX (2001) and corresponds to the availability of daily stock return data in CRSP. Each month, we use all the stocks which have a valid return for that month and a valid market capitalization at the end of the previous month to compute the measure of average stock variance.

Table 1 gives descriptive statistics on the volatility measures. In addition to the variance measures described above, we also report statistics on standard deviations. These are simply the square root of the corresponding variance measure ($SD_t = \sqrt{V_t}$, and $SD_{pt} = \sqrt{V_{pt}}$). Where there is no scope for confusion, we refer to standard deviation as 'volatility' in this paper. The first row (marked $r$) in Panel A of Table 1 gives the moments for a typical stock. These are computed as the average of the moments for all stocks.[7] Besides reporting the mean, median, standard deviation, minimum, and maximum, we also report the skewness and kurtosis to assess the normality of variables. Finally, we report the autoregressive root, the sum of the first twelve autoregressive coefficients, and the Augmented Dickey-Fuller test (with an intercept and twelve lags) to gauge the level of persistence in the time series of the variables.

Several features of the time series of the volatility measures are noteworthy from this table. First, the mean of average individual stock volatility (16.40 percent per month) is more than four times the mean volatility of the equal-weighted portfolio (3.56 percent per month). This indicates that idiosyncratic risk is likely to represent a large component of total stock risk.[8] Second, all the measures of volatility show substantial time variation. The 'standard deviation' of the volatility of the equal-weighted portfolio is 2.08 percent which is of the same order of magnitude as its mean of 3.56 percent. In contrast, the 'standard deviation' (4.19 percent per month) of the average individual stock volatility is only a fourth of its 'mean' (16.40 percent per month). This indicates that the average stock volatility is measured more precisely. Third, average stock volatility is very persistent with autoregressive root of 0.84. Moreover, the autocorrelation decays only slowly, as is evidenced by the large value of the sum of the first twelve autoregressive coefficients. The average stock volatility measure is substantially more persistent

---

[6]The computation is based on a typical stock mean return of 1.08 percent and a standard deviation of 16.53 percent (Table 1). The results of the regressions reported in the next subsection also do not change if we use an alternative measure of average stock variance adjusted for the mean. Using daily data, French, Schwert, and Stambaugh (1987) and Schwert (1989) also find the squared mean term is irrelevant to calculate variances. See section 4.1 for a detailed discussion of our risk measure.

[7]Since, the trading history is different for each stock, the averages are computed over different periods. This explains why the average return of a typical stock (1.04 percent) is different from the average return on the equal-weighted portfolio (1.29 percent).

[8]See section 4.1 for more on this issue.

than the volatility of the equal weighted portfolio, as measured by either the first-order autocorrelation or the first twelve autocorrelations.[9] This high level of persistence also points to the precision with which average stock risk is measured, since any estimation error would make the series faster mean reverting.

From Panel B of Table 1, we observe that the average stock volatility is not much correlated with the equal-weighted (0.324) and value-weighted (0.423) portfolio volatilities. Periods of high idiosyncratic risk are not necessarily the same as periods of high market risk. However, the equal-weighted and value-weighted portfolio volatilities are highly correlated (0.832) with each other. They are both good measures of market risk. The typical stock, as well as the equal- and value-weighted portfolios, are negatively contemporaneously correlated with the market volatility, in support of the leverage effect of Black (1976) and Christie (1982). In contrast, the average stock return and the market returns are *positively* contemporaneously correlated with average stock volatility. Duffee (1995) also finds a positive contemporaneous relation between individual stock returns and individual stock volatility. He suggests that the reason for the reversal of this relation at the aggregate level is the negative skewness of the market factor and the positive skewness of individual stock returns.

Figure 1 plots the time series of average stock volatility for our sample. The top panel of this figure plots the raw time series while the bottom panel plots their twelve-month moving average to smooth out high-frequency components (estimation noise). A remarkable feature of this graph (first noted by CLMX (2001)) is that average stock volatility shows a clear upward time trend during this sample period. The figure also plots the NBER recession months as shaded bars. It can be seen that average stock volatility tends to go up around recessions.[10] Figure 2 gives a graphical illustration of the time series of the equal-weighted and value-weighted portfolios' volatilities. These series do not show any significant trend. Although the market volatility was high in the 1970's and 1980's, it seems to have fallen back to average post-war levels in the 1990's. There is also evidence of procyclical behavior of the market volatility.

## 2.2  Risk and Return

We now explore the linkage between risk and return using the measures of volatility presented above.

The ICAPM predicts a positive relation between systematic risk and excess return

---

[9]CLMX (2001) also find that firm-level volatility is much more persistent than market-level volatility. Note also that the ADF statistic is unable to reject the presence of unit root for the average volatility. This, however, is not robust to the number of lags used, inclusion of a trend, test statistic (the Phillips-Peron test rejects the presence of a unit root strongly), or the sample period (see section 3.2 and Panel A of Table 4).

[10]Unreported regressions confirm the procyclical behavior of average volatility. Also see GS (2001) and Whitelaw (1994) for more extensive analysis of the predictability of average stock variance and average stock correlation using variables related to business cycle.

on the market. It is important to note that the predicted relation is between *ex-ante* expected returns and *ex-ante* forecasted volatility.[11] In our analysis, we regress realized excess returns on the lagged volatility measures. The fitted value of this regression gives the expected return conditional on the forecasted volatility. We use the current volatility as a proxy for the current expectation of next period's volatility, which can be justified by the high persistence of the volatility series.[12] Thus, the forecasting regression is:

$$r_{vwt+1} = \alpha + \beta\, X_t + \epsilon_{t+1}, \tag{3}$$

where $r_{vw}$ is the excess return on the market and $X$ includes different combinations of the market and average-stock risk measures.

Table 2 presents the results of regressions of the monthly value-weighted market return on the lagged measures of volatility. The first regression repeats the classic regression of market return on lagged market variance. The extant literature presents conflicting results on the sign of this coefficient. Campbell (1987), and Glosten, Jagannathan, and Runkle (1993) find a significantly negative relation, whereas Campbell and Hentschel (1992), French, Schwert, and Stambaugh (1987), and Scruggs (1998) find a significantly positive relation. For our data and sample period, we find a negative but insignificant coefficient.

However, the second regression shows that the lagged average stock variance is positively significant in explaining market returns. The coefficient has a $t$-statistic above two and the $\overline{R}^2$ of the regression is around one percent, which is high given the monthly frequency of the predictive regressions.[13] This significance actually increases when we include the market variance as a second regressor. Including both variance measures makes the coefficient on market variance significantly negative and the coefficient on average individual stock variance even more significantly positive.[14] The $\overline{R}^2$ of this regression is as high as two percent.

Although the formulation of the ICAPM suggests that risk should be measured by

---

[11]In contrast, the volatility feedback effect of Campbell and Hentschel (1992), and French, Schwert, and Stambaugh (1987) predicts a negative relation between *shocks* to volatility and *realized* returns. The leverage effect of Black (1976) and Christie (1982) also predicts a negative relation between *contemporaneous* returns and volatility. See Bekaert and Wu (2000) for a detailed analysis of both effects.

[12]The results are essentially the same if we replace volatilities by the fitted values from a ARMA model. See Schwert (1989, 1990b).

[13]To account for the heteroskedasticity and possible autocorrelation in returns, we report only Newey-West corrected $t$-statistics. The unadjusted $t$-statistic is 2.36.

[14]The cross-correlation between market variance and average stock variance is not high enough (0.413) to warrant issues of multicollinearity. We have also tested whether the importance of average stock volatility derives from it being a predictor of subsequent market volatility. However, the following regression shows that average stock variance has little predictive power for explaining future market variance:

$$V_{vw,t+1} = \underset{(3.82)}{0.001} + \underset{(1.92)}{0.161}\ V_{vw,t} + \underset{(0.65)}{0.005}\ V_t.$$

The sample period for this regression is 1962:07 to 1999:12 and Newey-West $t$-statistics are given in parenthesis.

variance, as a further check we also run regressions of market returns on lagged standard deviations and lagged log of variances. One potential problem with the regressions on the variance measures is the non-sphericity of residuals. As we pointed out in the discussion of Table 1, the times series of the variance measures display large kurtosis and skewness. This can potentially affect the distribution of standard errors and thereby impact the inference about parameter significance.[15] To the extent that transformations of the explanatory variables such as the square-root (in the case of the standard deviation) and log reduce the skewness and kurtosis, we should expect that the impact of these distortions is attenuated in the new regressions. Like Andersen, Bollerslev, Diebold, and Ebens (2001), we do find that the square root and log transformations of the variance measures are closer to normally distributed than the variances themselves. As reported in Table 1, the standard deviation measures have lower skewness and lower kurtosis than the variance measures. Similarly, the skewness coefficients of the logs of $V$ and $V_{vw}$ are 0.320 and 0.273 respectively, and the corresponding kurtosis coefficients are 2.741 and 3.838. These coefficients are actually quite close to those of a normally distributed random variable.

Lines 4 to 6 of Table 2 report the regressions on standard deviations and the last three lines correspond to the regression on logs of variances. The results with these transformed variables are even stronger than with variances as regressors![16] This is reassuring about the robustness of the relation between market returns and average stock risk. In contrast, the sign of the coefficient on market risk is not robust to the transformations. In regression 7, the sign on the log of the market variance is actually positive (although still insignificant). The coefficient on the measures of market risk is always insignificant except in regression 3.

Table 2 shows that there is a tradeoff between total risk — systematic *and* idiosyncratic — and return on the market. In the next section, we explore further the robustness of this result.

# 3    Robustness Checks

In this section, we confirm the puzzling relation between average stock variance and the return on the market through a battery of robustness checks. We run a bootstrap analysis of the significance of the relation between market return and average stock risk. We extend the sample using an alternative definition of average stock variance proposed by GS (2001) that does not require the use of daily data. We confirm the findings about the risk-return tradeoff after controlling for variables proxying for business cycle fluctuations. We also run regressions of the returns of a variety of portfolios other than the market on average stock variance. Finally we assess the economic significance of the

---

[15]The Newey-West correction to the $t$-statistics presented in the table partially addresses this concern. However it is possible that the adjustment is not sufficient.

[16]In the rest of the paper, we report only regressions using variances to measure risk, although using standard deviations or logs of variances would not change any of the results.

predictability of stock returns.

## 3.1   Correcting Small Sample Biases

We now investigate the small sample properties of the regressions of the previous section. There are two potential sources of problems that may affect the regressions reported in Table 2. First, both the regressand and the regressors display high levels of skewness and kurtosis, which can affect the distribution of the test statistics. Second, the regressors are highly persistent and contemporaneously correlated with the regressand. We study the impact of these problems on our inferences using a bootstrap experiment.[17]

The kurtosis of returns and variance measures may invalidate the use of the asymptotic distribution of the $t$-statistics in hypothesis tests since the standard deviation of the regression residuals understates their total variability. Additionally, as Miller and Scholes (1972) point out, the skewness of returns may create a spurious relation between ex-post realized returns and variance measures even when there is no relation ex-ante. Note that we are partially addressing these problems by using Newey-West corrected $t$-statistics. Also, the regressions of returns on the log and the square root of the lagged variance measures, which have lower skewness and kurtosis, are indicative that problems of non-spherical disturbances are unlikely to invalidate our inferences. Nevertheless, the problem deserves further attention.

The persistence of the variance measures coupled with their contemporaneous correlation with market returns may affect the results of regressions of market returns on lagged variances. As Stambaugh (1999) explains it:[18] "When a rate of return is regressed on a lagged stochastic regressor, [...] the regression disturbance is correlated with the regressor's innovation. The OLS estimator's finite sample properties can depart substantially from the standard regression setting." Are these biases responsible for a spuriously significant relation between the market return and average stock risk?

For the bootstrap experiment, we assume that both the average variance, $V$, and the market variance, $V_{vw}$, follow an AR(12) process that captures their strong serial dependence. Consistent with a prior of no predictability, we assume that market returns are iid. The assumed dynamics under the null hypothesis of our test are then:

$$
\begin{aligned}
r_{vwt+1} &= \mu + \epsilon_{t+1} \\
V_{t+1} &= \alpha + \sum_{i=0}^{11} \phi_i V_{t-i} + \xi_{t+1} \\
V_{vwt+1} &= \kappa + \sum_{i=0}^{11} \psi_i V_{vwt-i} + \eta_{t+1}.
\end{aligned}
\tag{4}
$$

---

[17]See Davison and Hinkley (1997), and Efron and Tibshirani (1993) for an exposition of bootstrap methods.

[18]See also Kendall and Stuart (1977).

9

We allow the innovations to the three variables, $\epsilon$, $\xi$, and $\eta$, to be contemporaneously correlated and do not impose any particular distributional assumption on them.

The parameter estimates for the VAR in (4) are given in Panel A of Table 3. They confirm that both the market variance and the average stock variance processes are highly persistent. This is especially true for $V$, which has an $\overline{R}^2$ of almost 60 percent in the regression on its own lags. The residuals of all three variables are highly skewed and leptokurtic. In particular, note the skewness and kurtosis coefficients of the innovations to the market variance process of 14 and 263, respectively. The last block of Panel A shows the correlation between the residuals from the VAR. While the correlation between the shocks to $V_{vw}$ and $r_{vw}$ is indeed substantial at -0.310, the correlation between residuals from $V$ and $r_{vw}$ is negligible at -0.045.[19]

The bootstrap experiment is carried out using the parameters from Panel A of Table 3. In each replication, we simulate paths of the variances, $V$ and $V_{vw}$, and the returns, $r_{vw}$, by sampling the estimated errors from the VAR with replacement and using the estimated dynamics from (4). Note that sampling from the fitted residuals ensures that the simulated paths will have the same skewness and kurtosis as the original time series. Also, we sample the innovations to the three variables concurrently to preserve their correlation structure. The simulated time series of the variables are then used in regressions of the form (3) that correspond to the first three lines of Table 2. For each regression, we compute the Newey-West corrected $t$-statistics for the coefficients. We generate an empirical distribution of these $t$-statistics from 10,000 replications of the bootstrap procedure.[20]

Panel B of Table 3 shows the results of the bootstrap experiment. For each regression of market returns on the lagged variance measures, we present the parameter estimates and the Newey-West $t$-statistics.[21] The third row of each regression shows the bootstrapped two-sided $p$-value of the $t$-statistic.

In regression 1, the $t$-statistic for the slope coefficient is -1.07, which is well within the 95 percent confidence interval from the bootstrap. This of course indicates that the variable is not significant in forecasting the market return. The $p$-value of the $t$-statistic from the bootstrap is 0.418, which is much higher than the asymptotic $p$-value (0.284

---

[19]These correlations are between the residuals of our VAR(12) specification. Stambaugh's (1999) specification involves an AR(1) process for the regressor and a one-period ahead predictive regression for the excess returns. Under that specification, the correlations are -0.289 for innovations between returns and market variance, and -0.022 for innovations between returns and average stock variance. Also, note that the results of Cavanagh, Elliott, and Stock (1995) and Elliott and Stock (1994) show that the asymptotic distribution for $t$-statistics is still valid in predictive regressions when regressors have a (near) unit root, provided that the correlation between the innovations is zero. This is the case of using average stock variance as a regressor.

[20]We bootstrap the $t$-statistics instead of the regression coefficients, as suggested by Efron and Tibshirani (1993), since the $t$-statistics do not depend on any nuisance parameters, such as the variance of the residuals.

[21]These numbers repeat the information in Table 2. The estimates are not exactly equal to that table because there is a slight difference in the sample. In order to estimate the VAR(12), we lose the first twelve observations of the sample.

for a $t$-statistic of -1.07). For this regression, there are indeed small-sample problems created by the large correlation between the innovations to the market return and its variance, as well as the high skewness and kurtosis of the market variance measure. In contrast, the bootstrapped $p$-value (0.017) for the $t$-statistic in equation 2 is much closer to its asymptotic value (0.010), confirming that the average stock variance measure is highly significant in explaining the return on the market. The coincidence between the bootstrapped and the asymptotic distribution of the $t$-statistic in this regression is likely due to the low correlation between the innovations to $V$ and the innovations to $r_{vw}$. Finally, in regression 3, both coefficients are still significant after correcting the small sample biases. Again, the bootstrapped confidence interval for the $t$-statistic on the coefficient of $V_{vw}$ is wider than its asymptotic counterpart while there is no such difference for the coefficient on $V$.

We conclude that the relation between market return and average stock risk is still significant after taking into account the small sample problems of our initial tests. However, the bootstrap results cast even more doubt about the significance of the relation between the market's return and its risk. In the rest of the paper, we report the bootstrapped $p$-values in addition to the sample $t$-statistics for all regressions involving variances.

## 3.2 Extending the Sample — Low-Frequency Measures of Variance

The sample used in the previous section starts only in July of 1962 because the daily returns on individual stocks required to construct the average stock variance measure are only available in the CRSP tapes since then. In order to assess the robustness of our results, we would like to use a longer time series. To do so, we compute an alternative measure of average stock variance first proposed by GS (2001). This measure is simply the average across stocks of squared monthly returns:

$$V_t^{lf} = \frac{1}{N_t} \sum_{i=1}^{N_t} r_{it}^2,$$ (5)

The superscript $lf$ stands for 'low frequency,' in contrast to the measure of equation (2) which uses high-frequency (daily) data. Since this measure requires only monthly return data on individual stocks, we are able to compute average stock variance since the beginning of the CRSP tapes in January of 1926. We refer the interested reader to GS (2001) for further details on this measure.

Table 4 presents descriptive statistics on the low-frequency measure. The descriptive statistics for the average stock volatility in the longer sample are similar to the statistics for the shorter sample, except for somewhat higher skewness and kurtosis coefficients. The average volatility is also slightly lower in the longer sample than it is in the shorter sample. Panel B directly compares the low-frequency and the high-frequency measures

11

in their common sample. The averages of both the low-frequency and the high-frequency measures of volatility are very close to each other. However, the 'standard deviation' of the low-frequency measure is higher than that of the high-frequency measure. Note that we are implicitly using a single monthly return squared to estimate each stock's variance. This is a very noisy measure of that stock's variance and although part of the noise in the estimator cancels out when we average across a large number of stocks, it is still present in the market variance. In contrast, the high-frequency measure of each stock's volatility (calculated using daily data) is much more precise and that precision carries to the measure of average stock volatility. Despite this, both high- and low-frequency measures of volatility are highly correlated, with a coefficient of 0.817, indicating that the low-frequency measure is able to capture largely the same information as the high-frequency measure. The top two panels of Figure 3 further illustrate the close correspondence between the two measures of volatility in their common sample. The bottom panels of this figure show the time series of the low-frequency measure since 1926. It can be seen that average volatility was very high in the pre-war period and that the upward trend in volatility mentioned in the previous section (and noted by CLMX (2001)) is a phenomenon only since the 1960's.

Table 5 analyzes the risk-return tradeoff using the low-frequency measure of variance. Panel A presents the regression results for the shorter sample period to facilitate comparison with the results of Table 2. The lagged average stock variance is again significantly positive in explaining the value-weighted market returns. The $t$-statistic (2.18) is somewhat lower than that in Table 3 (2.57) because the low-frequency measure is noisier, which creates an errors-in-variables problem. Similarly, in the common sample, the coefficient of the low-frequency measure (0.195) is lower than the coefficient of the high-frequency measure (0.336) due to the downward bias induced by errors-in-variables. The attenuation biases caused by measurement error in $V^{lf}$ are of the order of $1.72 = 0.336/0.195$ in the estimated coefficient and $1.18 = 2.57/2.18$ in the $t$-statistic. However, even with these biases, the bootstrap tests indicate that average variance is still significant at the 95 percent confidence level.

To construct a time series of market variance for the longer sample, we splice together two series. For the sample period of July of 1962 to December of 1999, we use CRSP daily value-weighted returns as before and for the earlier part of the sample we use daily data from Schwert (1990a). For the longer sample of 1927 to 1999 in Panel B of Table 5, we find that average stock variance is even more highly positively significant in forecasting the market's return. The estimated coefficient on average stock variance is 0.155 and the corresponding $t$-statistic is 2.83. The value-weighted market variance remains insignificant although with a coefficient that is now positive. This is evidence that the negative relation between stock return and market variance is not very robust and depends on the particular sample period or the definition of the market variance.[22] Note that, as mentioned before, the low-frequency measure of volatility is noisy leading to a downward bias in the coefficient and lower $t$-statistics, so the results in Table 5 are

---

[22]Campbell (1987) and Glosten, Jagannathan, and Runkle (1993) also find that it is difficult to sign the relation between the market's return and its variance.

12

that much more impressive. If we used the estimated attenuation biases reported in the previous paragraph to correct the coefficient and the $t$-statistic, we would obtain new estimates of $0.155 \times 1.72 = 0.267$ and $2.83 \times 1.18 = 3.34$ respectively. This is of course a "rough-and-ready" adjustment, but it helps make the coefficients and $t$-statistics of the regressions using $V$ and $V^{lf}$ comparable. Finally, the last regression in Panel B shows that the effect of average stock variance becomes even stronger after the inclusion of market variance as an additional regressor. These regressions confirm that our earlier results are not an artifact of a particular definition of average stock variance or of a particular sample period.

## 3.3 Controlling for the Business Cycle

One potential explanation for our finding is that average stock variance might merely be proxying for business cycle fluctuations. Indeed, CLMX (2001) and GS (2001) have found that average stock variance is strongly pro-cyclical. At the same time, there is substantial evidence that macroeconomic variables related to the business cycle can forecast stock market returns. A variety of papers including Campbell (1991), Campbell and Shiller (1988), Chen, Roll, and Ross (1986), Fama (1990), Fama and French (1988, 1989), and Keim and Stambaugh (1986), find evidence that the stock market can be predicted by variables such as the dividend-price ratio, the relative Treasury bill rate, the term spread, and the default spread. Therefore, the effect of average stock variance on the stock market might only be proxying for variations in the business cycle. To test this "proxy" hypothesis, we examine the relation between the stock market returns and the average stock variance using macro variables as controls for business cycle fluctuations.

The dividend-price ratio is calculated as the difference between the log of the last twelve month dividends and the log of the current price index of the CRSP value-weighted index. The three-month Treasury bill rate is obtained from Ibbotson Associates. The relative Treasury bill stochastically detrends the raw series by taking the difference between the Treasury bill rate and its twelve-month moving average. The term spread is calculated as the difference between the yield on long term government bonds and the Treasury bill rate, also obtained from Ibbotson Associates. The default spread is calculated as the difference between the yield on BAA- and AAA-rated corporate bonds, obtained from the FRED database. As a final variable, we include the lagged return on the market to control for the serial correlation in returns that might spuriously affect the predictability results.

Table 6 examines the forecasts of the market return controlling for the business cycle. The first regression uses just the control variables. In general, the predictability of the market by the control variables is in line with previous research. The dividend-price ratio is not significant for explaining returns, which is partly explained by our use of monthly (instead of annual) returns and previous research that shows that dividend-yield predictability is significant only at longer forecasting horizons.[23] The relative Treasury

---

[23]Additionally, Goyal and Welch (1999) show that this variable loses significance in the 1990's, which

bill rate is strongly significant with a negative coefficient in our sample. Both the term spread and the default spread are strongly significant in explaining returns.

In the regressions with control variables, we again use a bootstrap experiment to calculate $p$-values for the $t$-statistics. In contrast to the specification of equation (4), we impose a null of predictability from the business cycle variables alone. We still assume that both the average variance, $V$, and the market variance, $V_{vw}$, follow AR(12) processes. The assumed dynamics under the null hypothesis of our test are then:

$$
\begin{aligned}
r_{vwt+1} &= \beta\, Z_t + \epsilon_{t+1} \\
V_{t+1} &= \alpha + \sum_{i=0}^{11} \phi_i V_{t-i} + \xi_{t+1} \\
V_{vwt+1} &= \kappa + \sum_{i=0}^{11} \psi_i V_{vwt-i} + \eta_{t+1},
\end{aligned}
\tag{6}
$$

where $Z$ is a vector of conditioning variables (including an intercept). We again conduct the bootstrap experiment by sampling from the innovations $\epsilon$, $\xi$, and $\eta$. This allows the innovations to be contemporaneously correlated and does not impose any particular distributional assumption on them. The business cycle variables are not bootstrapped, and therefore we don't report $p$-values for the regression coefficients of these variables.

When we include the volatility measures along with the control variables in the regression, the findings of Table 2 are repeated. The coefficient on market variance is insignificantly negative and the coefficient on average stock variance is significantly positive. The last regression in this table uses all the variables to predict returns. The coefficients become even more significant than before! The magnitudes of the coefficients on the volatility measures in the regressions with the control variables are similar to the magnitudes reported in Table 2, which indicates that the effect of the variance measures is largely orthogonal to the control variables. Also, the $\overline{R}^2$ of the regressions goes up by as much as five percent when the volatility measures are added to the regression with control variables.

Table 7 repeats this exercise with the low-frequency measure of volatility. Panel A reports the results for the sample period of 1963 to 1999 and can be directly compared with the previous Table 6. The coefficient of the average stock variance is again positive (0.120), albeit with an insignificant $t$-statistic (1.25). This insignificance in undoubtedly explained by the errors-in-variables problem caused by the measurement noise in $V^{lf}$. The attenuation biases in the coefficient and the $t$-statistic are $0.363/0.120 = 3.03$ and $2.20/1.25 = 1.76$, respectively. For the full sample, the coefficient on $V^{lf}$ is 0.123, with a $t$-statistic of 1.71, which is again insignificant. However, if we use the "rough-and-ready" adjustment of the previous section to remedy the attenuation bias, the corrected coefficient would be $0.123 \times 3.03 = 0.373$ and the corrected $t$-statistic would be $1.71 \times 1.76 = 3.01$, which are again economically and statistically very significant. The market variance is negative in the shorter sample and positive in the longer sample but

---

are part of our sample.

insignificant in both samples (the bootstrapped $p$-value of the market variance in regression 4 of Panel A is more than 5 percent).[24] We conclude that the relation between market return and total stock risk is robust in subsamples.

## 3.4    Forecasting other Portfolios

In this section, we explore the forecasting power of average stock variance to predict returns on portfolios other than the market. This serves not only as a further robustness check on the previous results but also analyzes whether the predictability of the market is concentrated in a particular group of stocks.

In Table 8 we present results for the equal-weighted portfolio. We find that average stock variance is highly significant in explaining the equal-weighted portfolio's returns, with a higher coefficient, higher $t$-statistic, and higher $\overline{R}^2$ than in the corresponding regressions of value-weighted returns. Part of the increase in the explanatory power may be due to the portfolio's high autoregressive root (0.218, from Table 1). This serial correlation makes the equal-weighted portfolio easier to forecast than the value-weighted portfolio.

Next, we consider the predictability of portfolios formed on the basis of economically interesting characteristics. We form portfolios based on a double sort on book-to-market and size, and sorted by industry classification.[25] The results are presented in Table 9. To conserve space, we present results only for the following regression estimated with the full sample period of 1926 to 1999 (and the low-frequency volatility measure):

$$r_{pt+1} = a + b \, V_{vwt} + c \, V_t^{lf} + \epsilon_{t+1}, \tag{7}$$

where $r_p$ is the excess return on the various characteristic-sorted portfolios. We also suppress the bootstrapped $p$-values for these regressions for space considerations.

Panel A presents the results for 25 size and book-to-market sorted portfolios. For all the 25 portfolios, the coefficient on market volatility is insignificant. Except for the small market cap/low book-to-market, and the large market cap/high book-to-market portfolios, the remaining 23 size/book-to-market portfolios have significantly positive coefficients on the average stock variance. The average $t$-statistic on the coefficient $c$ across all portfolios is 2.94 and the average $\overline{R}^2$ is 3.19 percent, which compare favorably with the earlier regressions. It is also noteworthy that the coefficient $c$ declines with size across all book-to-market groups, implying that smaller stocks display higher predictability than larger stocks. There is no discernible trend in the $t$-statistics, however. This explains why the equal-weighted portfolio (which places a higher weight on smaller stocks) displays stronger predictability than the value-weighted portfolio. In Panel B,

---

[24]We can also carry out the bias adjustments in regressions involving both the average stock variance and the market variance measures. The corrected coefficient and $t$-statistic of the average stock variance in regression 4 of Panel B of Table 7 are 0.998 and 3.39, respectively.

[25]The return data for these portfolios were obtained from Ken French's web site http://web.mit.edu/kfrench/www/data_library.html.

we analyze seventeen industry portfolios and find that in twelve of these the coefficient on average stock variance is significantly positive. The coefficient on market volatility remains insignificant in all cases.

We conclude that the predictability of returns from the lagged average stock variance is quite general and holds across a wide variety of portfolios. Predictability is particularly strong for small stocks.

## 3.5  Assessing Economic Significance

In this subsection, we explore the economic significance of the regression results. We simulate the returns of a trading strategy based on out-of-sample forecasts of the market using the volatility measures. The trading strategy is similar to that of Breen, Glosten, and Jagannathan (1989). To conserve space, we focus only on forecasting with the time-series measure of volatility constructed using daily data.

For each month $T$ after July of 1967, we estimate the forecasting regression:[26]

$$r_{vwt+1} = \beta\, X_t + \epsilon_t, \quad t = 1, \ldots, T-1, \tag{8}$$

using all the data available upto time $T$. The regressors $X$ include different sets of the forecasting variables. The estimated $\beta$ coefficient is then used to forecast the market return at time $T+1$ as $\hat{r}_{vwT+1} = \hat{\beta}\, X_T$. At time $T$, the strategy invests all in the stock index if the forecasted excess return of stocks over the riskfree rate is greater than zero, otherwise it invests all in Treasury bills. At time $T+1$, the return of the portfolio is realized, a new regression is estimated, a return forecast computed, and new portfolio weights determined. In this way, we obtain a time series of return to the trading strategy.

The results from this trading strategy are given in Table 10. We choose four different sets of conditioning variables, $X_t$, for the exercise. The first three regressions correspond to the three rows of Table 2 and the fourth regression corresponds to regression 4 in Table 6. For all strategies, we present the annualized mean and the standard deviation of returns. We also report the number of months the trading strategy invests in the market portfolio.

The first row in Panel A gives the results of the market buy-and-hold strategy which invests all the funds in the market portfolio. For our sample period, the market portfolio had an excess return of 6.66 percent with the associated standard deviation of 15.72 percent annually. Use of the value-weighted market variance as a forecaster results in a slight deterioration of the mean (to 6.35 percent) but also in a marginal reduction in the standard deviation (to 14.76 percent). When we use the average stock variance, we not only get a higher mean (7.18 percent) but also a lower standard deviation (14.57

---

[26]Although the sample period starts in July of 1962, the forecasting exercise starts only in July of 1967, to allow for 60 months of data in the estimation of the first set of parameters. After that date, we use an expanding window to reflect the real-time nature of the problem, and to minimize the effect of spurious in-sample regression fit over the whole sample.

percent). The use of both the average stock variance and the market variance as signals results in only a slight improvement of the mean return over the trading strategy that uses market variance alone. When we use the macro variables, the market variance, and the average stock variance, we get much lower mean (5.62 percent) but also a substantial reduction in the standard deviation (10.31 percent).

To translate the gains into a measure of investors' welfare, we assume a quadratic utility investor with relative risk aversion of $\gamma$. The investor's utility at time $t + 1$ is given by:

$$U(W_{t+1}) = W_t \left( R_{st+1} - \frac{\gamma}{2(\gamma + 1)} R_{st+1}^2 \right),$$ (9)

where $R_s$ if the gross return on the trading strategy. To compare the expected utility of the trading strategy with simply buying and holding the market, we follow Fleming, Kilby, and Ostdiek (2001) and assume that the investor is willing to pay a constant fee $\Delta$ per period to a manager that implements the trading strategy. In other words, the fee is determined by the equation:

$$\sum_{t=1}^{T} \left[ R_{vw,t} - \frac{\gamma}{2(\gamma + 1)} R_{vw,t}^2 \right] = \sum_{t=1}^{T} \left[ R_{s,t} - \Delta - \frac{\gamma}{2(\gamma + 1)} (R_{s,t} - \Delta)^2 \right].$$ (10)

We report the fees in annualized terms in Panel B of Table 10 for three different values of $\gamma$. Even for modest levels of risk aversion, the investor is willing to pay a fee of 2.4 percent annually to invest in the strategy that forecasts the market based on the average stock variance measure alone. The fee increases with risk aversion and can be as much as 2.5 percent for more prudent investors. Also, note that although the in-sample $\overline{R}^2$ of regression 4 of Table 6 is the highest amongst all the combinations of conditioning variables that we consider in this section, the performance improvement from this set of variables is the worst amongst all the trading strategies. This reflects the poor out-of-sample performance of the macro variables to forecast stock returns.

We conclude that not only is the relation between market return and total risk robust out of sample, but also that conditioning on average stock risk can bring substantial economic benefits to investors. In the next section, we explore the determinants of average stock risk.

# 4  Discussion

In this section, we explore the constituents of our measure of average stock variance. We first decompose average stock variance into its systematic and idiosyncratic components. We show that not only does the idiosyncratic component represent a significant proportion of average stock variance but it also explains a high fraction of the time variation of average stock variance. We extract residuals from the market model and the Fama-French three factor model to isolate the effect of idiosyncratic risk. We show that

the measures of pure idiosyncratic risk are still successful in forecasting the market's return.

## 4.1   Interpreting the Volatility Measures

Both the high-frequency measure, $V_t$, and the low-frequency measure, $V_t^{lf}$, approximate the variance of a stock by its squared return. The cross-sectional average of squared returns is a measure of total risk, including both systematic and idiosyncratic components. To see this, assume that the returns to each stock $i$ is driven by a common factor $f$ and a firm-specific shock $\epsilon_i$. To simplify the exposition, assume that we use only one monthly return for each stock to estimate its variance (in line with our low-frequency measure of volatility), and that the factor loading for each stock is one.[27] The returns are generated by:

$$r_{it} = f_t + \epsilon_{it},$$

where $r_i$ is the return of stock $i$ in excess of the riskfree rate. Assume further that the idiosyncratic shocks and the factor shocks are uncorrelated. This implies that the cross-sectional average of the squared stock returns is:

$$V_t^{lf} = \frac{1}{N_t}\sum_{i=1}^{N_t} r_{it}^2 = f_t^2 + \frac{2}{N_t}f_t\sum_{i=1}^{N_t}\epsilon_{it} + \frac{1}{N_t}\sum_{i=1}^{N_t}\epsilon_{it}^2. \tag{11}$$

To compare average stock variance with market variance, consider the equal-weighted portfolio as a proxy for the market:[28]

$$r_{ewt} = \frac{1}{N_t}\sum_{i=1}^{N_t} r_{it} = f_t + \frac{1}{N_t}\sum_{i=1}^{N_t}\epsilon_{it}$$

$$V_{ewt}^{lf} = r_{ewt}^2 = f_t^2 + \frac{2}{N_t}f_t\sum_{i=1}^{N_t}\epsilon_{it} + \left(\frac{1}{N_t}\sum_{i=1}^{N_t}\epsilon_{it}\right)^2. \tag{12}$$

The first two terms of equation (12) are the same as those of equation (11). However, the contribution of the idiosyncratic component is divided further by the number of stocks in equation (12). For a large cross-section, the last term is negligible in equation (12) but not in equation (11).

We can explore further the components of total risk and the differences between total risk and systematic risk by assuming that $\epsilon_{it} \sim$ iid $N(0, \sigma_\epsilon^2)$, and $f_t \sim N(\mu_f, \sigma_f^2)$. To be more concrete assume (following the descriptive statistics in Table 1) that $\mu_f =$

---

[27]This discussion is only for illustration purposes. The results can easily be generalized to include many factors and different factor loadings for different stocks.

[28]We use the equal-weighted portfolio for simplicity, to avoid having to deal with market capitalization weights in the measures of volatility.

1 percent, $\sigma_f = 5$ percent, $\sigma_\epsilon = 15$ percent, and $N = 5,000$. Taking expectations of equations (11) and (12), we get:

$$\mathrm{E}[V_t^{lf}] \;=\; \mathrm{E}[f_t^2] + \frac{1}{N_t}\sum_{i=1}^{N_t}\mathrm{E}[\epsilon_{it}^2] = (\mu_f^2 + \sigma_f^2) + \sigma_\epsilon^2 \tag{13}$$

$$\mathrm{E}[V_{ewt}^{lf}] \;=\; \mathrm{E}[f_t^2] + \frac{1}{N_t^2}\sum_{i=1}^{N_t}\mathrm{E}[\epsilon_{it}^2] = (\mu_f^2 + \sigma_f^2) + \sigma_\epsilon^2/N_t. \tag{14}$$

Substituting the values of the parameters, we get:

$$\mathrm{E}[V_t^{lf}] \times 10^4 \;=\; 251 = \underbrace{\text{Systematic}}_{26} + \underbrace{\text{Idiosyncratic}}_{225}$$

$$\mathrm{E}[V_{ewt}^{lf}] \times 10^4 \;=\; 26 = \underbrace{\text{Systematic}}_{26} + \underbrace{\text{Idiosyncratic}}_{0}.$$

This decomposition illustrates that the effect of idiosyncratic risk is diversified away in the equal-weighted portfolio variance measure but not in the total stock variance measure. In fact, idiosyncratic risk constitutes almost 90 percent of total risk. Of course, these are just benchmark calculations. In the above, we did not account for the cross-sectional dispersion in betas, time variation in factor premia, and the changing cross-section of stocks. However, this analysis indicates that the variation in total risk is likely to be driven by idiosyncratic risk. In the next section, we construct measures of pure idiosyncratic risk using residuals from two popular asset pricing models and show that indeed most of the time variation in total risk is driven by time variation in residual risk.

It is also instructive to examine the measurement error in the two variance measures. We have:

$$\mathrm{Var}[V_t^{lf}] \;=\; \mathrm{Var}[f_t^2] + \frac{4}{N_t^2}\mathrm{E}[f_t^2]\sum_{i=1}^{N_t}\mathrm{E}[\epsilon_{it}^2] + \frac{1}{N_t^2}\sum_{i=1}^{N_t}\mathrm{Var}[\epsilon_{it}^2]$$

$$\;=\; 2\sigma_f^2(\mu_f^2 + \sigma_f^2) + 4(\mu_f^2 + \sigma_f^2)\sigma_\epsilon^2/N_t + 2\sigma_\epsilon^4/N_t \tag{15}$$

$$\mathrm{Var}[V_{ewt}^{lf}] \;=\; \mathrm{Var}[f_t^2] + \frac{4}{N_t^2}\mathrm{E}[f_t^2]\sum_{i=1}^{N_t}\mathrm{E}[\epsilon_{it}^2] + \frac{1}{N_t^3}\sum_{i=1}^{N_t}\mathrm{Var}[\epsilon_{it}^2]$$

$$\;=\; 2\sigma_f^2(\mu_f^2 + \sigma_f^2) + 4(\mu_f^2 + \sigma_f^2)\sigma_\epsilon^2/N_t + 2\sigma_\epsilon^4/N_t^2. \tag{16}$$

Substituting the values of the parameters, we get:

$$\mathrm{StdDev}[V_t^{lf}] \times 10^4 \;=\; 36.4$$

$$\mathrm{StdDev}[V_{ewt}^{lf}] \times 10^4 \;=\; 36.1,$$

so that the $t$-ratio (the mean dividend by the standard deviation) is 6.90 for the average stock variance measure and 0.72 for the market variance measure. Measurement error

in the risk measures leads to the errors-in-variables problem in the regressions alluded to earlier in this paper. However, $V^{lf}$ is measured more precisely in relation to its mean than is $V_{ew}^{lf}$. The source of this increased precision is the low measurement error in the pure idiosyncratic risk component. This makes the regressions reported in the next section (using a proxy for pure idiosyncratic risk) a clean robustness check for the effect of idiosyncratic risk on stock returns.

## 4.2   The Importance of Idiosyncratic Risk

In this subsection, we construct measures of pure idiosyncratic risk using residuals from two popular asset pricing models: the market model (MM) and the Fama-French three-factor model (FF). Using the market model, we construct a measure of idiosyncratic risk as follows. For each stock, we regress the complete history of that stock's monthly excess returns on the market excess return:[29]

$$r_{it} = \alpha + \beta_i MKT_t + \epsilon_{it}^{MM}. \tag{17}$$

The cross-sectional average of these residuals squared is used to construct the measure of average idiosyncratic risk. We define $V^{MM}$ as the measure of idiosyncratic risk computed from the market model's residuals as:

$$V_t^{MM} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \epsilon_{it}^{MM} \right)^2. \tag{18}$$

Similarly, we construct a measure of idiosyncratic risk from the Fama-French three factor model[30] by first regressing all stock's excess returns on the factors:

$$r_{it} = \alpha + \beta_{i1} SMB_t + \beta_{i2} HML_t + \beta_{i3} MKT_t + \epsilon_{it}^{FF} \tag{19}$$

$$V_t^{FF} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \epsilon_{it}^{FF} \right)^2. \tag{20}$$

Note that both $V^{MM}$ and $V^{FF}$ are *low-frequency* estimators of idiosyncratic variance.

From the time series of these alternative measures, we are able to decompose average total risk into its components. Over the sample period July of 1926 to December of 1999, $V^{MM}$ represents on average 85 percent of $V$, and $V^{FF}$ is on average 80 percent of $V$.[31] This provides evidence that idiosyncratic risk is indeed the major component of the total risk. Even more importantly, the idiosyncratic risk measures explain most of

---

[29]We use the notation $MKT$ instead of $r_{vw}$ for the excess return of the market in line with the common denomination of the Fama-French factors.

[30]See Fama and French (1993, 1996) for the discussion of the factors HML ('high minus low' book-to-market) and SMB ('small minus big' size). The data on the factors was obtained from Kenneth French's web site `http://web.mit.edu/kfrench/www/data_library.html`.

[31]This is in the ball park of our back-of-the-envelope calculations in the previous section.

the time variation in average risk. A contemporaneous regression of $V$ on $V^{MM}$ yields an $\overline{R}^2$ of 84 percent. For the Fama-French model, the $\overline{R}^2$ is 70 percent.

We use the idiosyncratic risk measures constructed above in regressions to predict stock returns. This is a robust way of investigating the importance of idiosyncratic risk in forecasting the market return. Since idiosyncratic risk is the largest component of total risk and explains most of its time variation, it is interesting to assess whether this component also drives the forecastability of the market. Additionally, and from a statistical perspective, the fact that the idiosyncratic component of total risk is measured with higher precision that the systematic component makes it likely to reduce the errors-in-variables problem in the forecasting regressions.

The results of regression of value-weighted return on $V^{MM}$ are presented in Table 11. For the sample period of 1963 to 1999, $V^{MM}$ has a $p$-value of 5.3 percent in univariate regression but is insignificant at the ten percent level when used in conjunction with business cycle variables. Panel B, however, shows that $V^{MM}$ is strongly significant over the whole sample. The error-in-variables problem mentioned earlier in the paper afflicts our results even here since we use just one observation for each stock to calculate the idiosyncratic variance of that stock for a particular month. Also, the varying perfor-mance of the idiosyncratic risk measures across different samples may reflect the quality of fit of the asset pricing model in those samples.

Table 12 presents results from using $V^{FF}$ as the measure of idiosyncratic risk. Using this alternative measure of risk does not alter the basic findings of earlier tables. $V^{FF}$ is even stronger than $V$ in predicting the next period's value-weighted returns. The $t$-statistic on $V^{FF}$ is 2.41 compared to 2.06 on $V$ (Table 2). The coefficient on the market variance remains negative as before. When we include both measures of risk, the results are unchanged. The results for the full sample of 1926 to 1999 are similar — the average idiosyncratic risk measure is significant only at ten percent level when used together with business cycle variables.

Sections 2 and 3 have shown that average stock risk is priced. In this section we have shown that idiosyncratic risk represents a large fraction of total risk and that it drives most of its variation through time. Idiosyncratic risk is also the portion of total risk that is measured with least noise. Therefore, when we run regressions of the market's return on a measure of pure idiosyncratic risk we generally get a positive and significant relation.

# 5   Conclusion

We have shown in this paper that there is a relation between market return and total stock risk, as opposed to systematic risk only. Our results are robust to small sample inference problems, in different sample periods, for alternative definitions of risk, for different portfolios, and persist even after controlling for business cycle variables known

to predict the stock market. This result is at odds with the theoretical literature which posits that only systematic risk should be priced in the market as investors can easily diversify idiosyncratic risk away. In fact, we show that it is the idiosyncratic component of total risk that drives the predictability of the market. The idiosyncratic component accounts for over 80 percent of total stock variance and over 70 percent of its variation through time. Moreover, when we regress market returns on pure idiosyncratic risk, we still retain significant predictive ability.

This does not mean that it is idiosyncratic risk *alone* that forecasts the market. Rather, we believe that it is total risk that matters to investors and is therefore priced in market returns. The significance of idiosyncratic risk as a forecaster of market returns derives only from its importance as a large component of total risk and the precision with which it is measured. Similarly, the general insignificance of systematic risk in forecasting the market is due to its low weight on total risk and the large error in its measurement.

One potential explanation for our finding would be that investors hold undiversified portfolios, so that their relevant measure of risk is indeed closely related to the average variance of individual stocks. Once some investors hold undiversified portfolios, all other investors must do the same.[32] When the representative investor is undiversified, an increase in idiosyncratic risk makes him want to lower the exposure to risky assets in general.

There is some support for investors holding undiversified portfolios. For instance, Barber and Odean (2000) find in their sample that the mean household's portfolio contains only 4.3 stocks (worth 47,334 dollars), and the median household invests in 2.61 stocks (worth 16,210 dollars). Benartzi and Thaler (2001) document that individuals hold a disproportionate amount of their pension plans in the stock of the company they work for. Falkenstein (1996) finds that even mutual funds hold substantial amounts of idiosyncratic risk.

There are rational and irrational justifications for limited diversification. Transaction costs and taxes restrict the portfolio holdings of investors. Employee compensation plans often give workers stock in their firms but restrict their capacity to sell their holdings, thereby leading to a concentrated exposure. Private information is another motive for holding large positions. Finally, Huberman (2001) surveys evidence from many sources to demonstrate that investors may just be more prone to investing in familiar stocks and often ignore the principles of portfolio diversification.

A related explanation, as suggested by Heaton and Lucas (1997, 2000), would be that the risk of human capital or the risk of (closely held) private companies is better proxied by the average variance of individual stocks rather than the variance of the stock market. If investors' holdings of human capital and private companies are substantial, then an increase in the riskiness of these assets (which would occur when average stock

---

[32]Malkiel and Xu (2000) show through an extension of CAPM that when some individuals do not hold the market, nobody can hold the market.

risk goes up) makes these investors less willing to hold other correlated risky assets, such as the stock market. Vissing-Jørgensen and Moskowitz (2001) report that private equity capital was worth more than public equity in the US until 1995 and is still of the same order of magnitude. Further, they find that over 45 percent of the net worth of investors with private businesses consists of private equity. Of this, more than 70 percent is concentrated in a single firm, implying that they hold very undiversified portfolios. Finally, these investors are likely to have impact in the market since they hold over twelve percent of the total public equity in the US. Vissing-Jørgensen and Moskowitz speculate that the lack of diversification of investors with private businesses may be explained by non-pecuniary benefits of control, their preference for skewness, and over confidence in their own success as entrepreneurs.

Our results have shown that idiosyncratic risk impacts market returns. They call for more research to understand the mechanism through which idiosyncratic risk is incorporated into asset prices.

# References

Abel, Andrew B., 1988, Stock Prices Under Time-Varying Dividend Risk: An Exact Solution in an Infinite-Horizon General Equilibrium Model, *Journal of Monetary Economics* 22, 375–393.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Heiko Ebens, 2001, The Distribution of Realized Stock Return Volatility, *Journal of Financial Economics* 61, 43–76.

Backus, David K., and Allan W. Gregory, 1993, Theoretical Relations Between Risk Premiums and Conditional Variance, *Journal of Business & Economic Statistics* 11, 177–185.

Baillie, Richard T., and Ramon P. DeGennaro, 1990, Stock Returns and Volatility, *Journal of Financial & Quantitative Analysis* 25, 203–214.

Barber, Brad M., and Terrance Odean, 2000, Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors, *Journal of Finance* 55, 773–806.

Bekaert, Geert, and Guojun Wu, 2000, Asymmetric Volatility and Risk in Equity Markets, *Review of Financial Studies* 13, 1–42.

Benartzi, Shlomo, and Richard H. Thaler, 2001, Naive Diversification Strategies in Defined Contribution Saving Plan, *American Economic Review* 91, 79–98.

Black, Fischer, 1976, Studies in Stock Price Volatility Changes, in *Proceedings of American Statistical Association, Business and Economic Statistics Section* pp. 177–181.

Breen, William, Lawrence R. Glosten, and Ravi Jagannathan, 1989, Economic Significance of Predictable Variations in Stock Index Returns, *Journal of Finance* 44, 1177–1189.

Campbell, John Y., 1987, Stock Returns and the Term Structure, *Journal of Financial Economics* 18, 373–399.

———— , 1991, A Variance Decomposition for Stock Returns, *Economic Journal* 101, 157–179.

———— , and Ludger Hentschel, 1992, No News Is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, *Journal of Financial Economics* 31, 281–318.

Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu, 2001, Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk, *Journal of Finance* 56, 1–43.

Campbell, John Y., and Robert J. Shiller, 1988, Stock Prices, Earnings, and Expected Dividends, *Journal of Finance* 43, 661–676.

Cavanagh, Christopher L., Graham Elliott, and James H. Stock, 1995, Inference in Models with Nearly Integrated Regressors, *Econometric Theory* 15, 1131–1147.

Chan, K. C., G. Andrew Karolyi, and Rene M. Stulz, 1992, Global Financial Markets and the Risk Premium on U.S. Equity, *Journal of Financial Economics* 32, 137–167.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, 1986, Economic Forces and the Stock Market, *Journal of Business* 59, 383–403.

Christie, Andrew A., 1982, The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects, *Journal of Financial Economics* 10, 407–432.

Davison, A.C., and D.V. Hinkley, 1997, *Bootstrap Methods and Their Application* . No. 1 in Cambridge Series in Statistical and Probabilistic Mathematics (Cambridge University Press: Cambridge).

Douglas, George W., 1969, Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency, *Yale Economic Essays* 9, 3–45.

Duffee, Gregory R., 1995, Stock Returns and Volatility: A Firm-Level Analysis, *Journal of Financial Economics* 37, 399–420.

Efron, Bradley, and Robert J. Tibshirani, 1993, *An Introduction to the Bootstrap* . No. 57 in Monographs on Statistics and Applied Probability (Chapman & Hall: New York).

Elliott, Graham, and James H. Stock, 1994, Inference in Time Series Regression When the Order of Integration of a Regressor Is Unknown, *Econometric Theory* 10, 672–700.

Falkenstein, Eric G., 1996, Preferences for Stock Characterstics as Revealed by Mutual Fund Portfolio Holdings, *Journal of Finance* 51, 111–135.

Fama, Eugene F., 1990, Stock Returns, Expected Returns, and Real Activity, *Journal of Finance* 45, 1089–1108.

———— , and Kenneth R. French, 1988, Dividend Yields and Expected Stock Returns, *Journal of Financial Economics* 22, 3–25.

———— , 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics* 25, 23–49.

———— , 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3–56.

———— , 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55–84.

Fama, Eugene F., and James D. Macbeth, 1973, Risk, Return and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607–636.

Fleming, Jeff, Chris Kilby, and Barbara Ostdiek, 2001, The Economic Value of Volatility Timing, *Journal of Finance* 56, 329–352.

French, Kenneth R., William Schwert, and Robert F. Stambaugh, 1987, Expected Stock Returns and Volatility, *Journal of Financial Economics* 19, 3–29.

Gennotte, Gerard, and Terry A. Marsh, 1993, Variations in Economic Uncertainty and Risk Premiums on Capital Assets, *European Economic Review* 37, 1021–1041.

Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle, 1993, On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *Journal of Finance* 48, 1779–1801.

Goyal, Amit, and Pedro Santa-Clara, 2001, The Volatility and Correlation of Common Stocks, Working paper, UCLA.

Goyal, Amit, and Ivo Welch, 1999, Predicting The Equity Premium, Working paper, UCLA.

Heaton, John, and Deborah Lucas, 1997, Market Frictions, Savings Behavior, and Portfolio Choice, *Journal of Macroeconomic Dynamics* 1, 76–101.

———, 2000, Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *Journal of Finance* 55, 1163–1198.

Huberman, Gur, 2001, Familiarity Breeds Investment, *Review of Financial Studies* 14, 659–680.

Keim, Donald B., and Robert F. Stambaugh, 1986, Predicting Returns in the Stock and Bond Markets, *Journal of Financial Economics* 17, 357–390.

Kendall, Sir Maurice, and Alan Stuart, 1977, *The Advanced Theory of Statistics* (McMillan: New York).

Lehmann, Bruce N., 1990, Residual Risk Revisited, *Journal of Econometrics* 45, 71–97.

Lintner, John, 1965, Security Prices and Risk: The Theory and Comparative Analysis of A.T.&T. and Leading Industrials, presented at the Conference on "The Economics of Regulated Public Utilities" at the University of Chicago Business School.

Malkiel, Burton G., and Yexiao Xu, 1997, Risk and Return Revisited, *Journal of Portfolio Management* 23, 9–14.

———, 2000, Idiosyncratic Risk and Security Returns, Working paper.

Merton, Robert C., 1980, On Estimating the Expected Return on the Market: An Exploratory Investigation, *Journal of Financial Economics* 8, 323–361.

Miller, Merton H., and Myron Scholes, 1972, Rates and Return in Relation to Risk: A Re-examination of Some Recent Findings, in Michael C. Jensen, ed.: *Studies in the Theory of Capital Markets* . pp. 47–78 (Praeger: New York).

Nelson, Daniel B., 1991, Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica* 59, 347–370.

Pindyck, Robert S., 1984, Risk, Inflation, and the Stock Market, *American Economic Review* 74, 335–351.

Schwert, William G., 1989, Why Does Stock Market Volatility Change Over Time?, *Journal of Finance* 44, 1115–1153.

―――― , 1990a, Indexes of United States Stock Prices from 1802 to 1987, *Journal of Business* 63, 399–431.

―――― , 1990b, Stock Volatility and the Crash of 87, *Review of Financial Studies* 3, 77–102.

Scruggs, John T., 1998, Resolving the Puzzling Intertemporal Relation Between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach, *Journal of Finance* 52, 575–603.

Stambaugh, Robert F., 1999, Predictive Regressions, *Journal of Financial Economics* 54, 375–421.

Turner, Christopher M., Richard Startz, and Charles R. Nelson, 1989, A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market, *Journal of Financial Economics* 25, 3–22.

Vissing-Jørgensen, Annette, and Tobias J. Moskowitz, 2001, The Private Equity Premium Puzzle, Working paper.

Whitelaw, Robert F., 1994, Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns, *Journal of Finance* 49, 515–541.

## Table 1: Descriptive Statistics of Returns and Volatility Measures

This table presents descriptive statistics on returns and measures of volatility. The sample period is 1962:07 to 2000:12 (450 monthly observations). $r$ is the typical stock return and $r_p$ is the portfolio return, where $ew$ and $vw$ stand for the equal- and value-weighted portfolios respectively. $V$ is the average stock variance, $SD$ is the average stock standard deviation calculated as the square root of $V$, $V_p$ and $SD_p$ are the portfolio variance and standard deviation respectively. $V_p$ and $V$ are calculated using daily data corrected for serial correlation, as shown in equations (1) and (2) respectively. 'Skew' is the skewness, 'Kurt' is the kurtosis, 'AR$_1$' is the first-order autocorrelation, and 'AR$_{1:12}$' is the sum of the first 12 autocorrelation coefficients. 'ADF' is the Augmented Dickey-Fuller statistic for presence of unit root calculated with an intercept and 12 lags. The critical values for rejection of unit root are -3.4475 and -2.8684 at 1 percent and 5 percent level. The first row in Panel A gives average statistics for all stocks.

### Panel A: Descriptives

|          | Mean   | Median  | StdDev | Min     | Max    | Skew    | Kurt     | AR$_1$  | AR$_{1:12}$ | ADF     |
|----------|--------|---------|--------|---------|--------|---------|----------|---------|-------------|---------|
| $r$      | 0.0104 | -0.0090 | 0.1646 | -0.3612 | 0.6928 | 0.9584  | 7.3933   | -0.0289 | -0.0862     | —       |
| $r_{ew}$ | 0.0129 | 0.0153  | 0.0556 | -0.2708 | 0.2992 | -0.2239 | 6.4965   | 0.2180  | 0.1480      | -5.6449 |
| $r_{vw}$ | 0.0111 | 0.0132  | 0.0436 | -0.2249 | 0.1656 | -0.4648 | 5.4971   | 0.0448  | -0.0528     | -5.7363 |
| $V$      | 0.0286 | 0.0245  | 0.0157 | 0.0099  | 0.1226 | 1.9232  | 8.7414   | 0.7999  | 7.4019      | -1.1031 |
| $SD$     | 0.1640 | 0.1564  | 0.0419 | 0.0995  | 0.3501 | 1.0031  | 4.4407   | 0.8426  | 8.3058      | -1.2680 |
| $V_{ew}$ | 0.0017 | 0.0009  | 0.0032 | 0.0001  | 0.0515 | 10.0219 | 140.2248 | 0.1449  | 0.6274      | -4.9511 |
| $SD_{ew}$| 0.0356 | 0.0305  | 0.0208 | 0.0105  | 0.2270 | 3.1770  | 22.4899  | 0.3591  | 2.1791      | -3.8937 |
| $V_{vw}$ | 0.0019 | 0.0012  | 0.0036 | 0.0001  | 0.0671 | 13.7457 | 242.8002 | 0.1707  | 0.8344      | -4.7519 |
| $SD_{vw}$| 0.0386 | 0.0345  | 0.0202 | 0.0089  | 0.2591 | 3.8199  | 35.3703  | 0.4479  | 3.0480      | -3.7424 |

### Panel B: Cross-Correlation

|           | $r$     | $r_{ew}$ | $r_{vw}$ | $V$    | $SD$   | $V_{ew}$ | $SD_{ew}$ | $V_{vw}$ |
|-----------|---------|----------|----------|--------|--------|----------|-----------|----------|
| $r$       | 1       |          |          |        |        |          |           |          |
| $r_{ew}$  | 0.3506  | 1        |          |        |        |          |           |          |
| $r_{vw}$  | 0.3012  | 0.8471   | 1        |        |        |          |           |          |
| $V$       | 0.0333  | 0.0693   | 0.0693   | 1      |        |          |           |          |
| $SD$      | 0.0369  | 0.0807   | 0.0830   | 0.9848 | 1      |          |           |          |
| $V_{ew}$  | -0.1103 | -0.2911  | -0.3148  | 0.3794 | 0.3241 | 1        |           |          |
| $SD_{ew}$ | -0.0903 | -0.2136  | -0.2515  | 0.3558 | 0.3244 | 0.8902   | 1         |          |
| $V_{vw}$  | -0.1331 | -0.3124  | -0.2817  | 0.4127 | 0.3606 | 0.8965   | 0.7303    | 1        |
| $SD_{vw}$ | -0.1201 | -0.2909  | -0.2244  | 0.4363 | 0.4225 | 0.7954   | 0.8317    | 0.8645   |

## Table 2: Forecasts of Value-Weighted Portfolio Return

This table presents the results of a 1-month ahead predictive regression of the excess value-weighted portfolio return on lagged explanatory variables. $V$ is the average stock variance, $V_{vw}$ is the value-weighted volatility. $V_{vw}$ and $V$ are calculated using daily data, as shown in equations (1) and (2) respectively. The sample period is 1962:07 to 1999:12 (449 monthly observations). The first row in each regression is the coefficient and the second row is the Newey-West adjusted $t$-statistic.

| | CNST | $V_{vw}$ | $V$ | $SD_{vw}$ | $SD$ | $\ln(V_{vw})$ | $\ln(V)$ | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|
| | | **Variance** | | | | | | |
| 1. | 0.007 | -0.582 | | | | | | 0.01% |
| | (3.36) | (-1.02) | | | | | | |
| 2. | -0.003 | | 0.313 | | | | | 1.01% |
| | (-0.73) | | (2.42) | | | | | |
| 3. | -0.004 | -1.391 | 0.448 | | | | | 1.88% |
| | (-1.08) | (-2.54) | (3.64) | | | | | |
| | | | | **Standard Deviation** | | | | |
| 4. | 0.006 | | | -0.002 | | | | -0.22% |
| | (1.27) | | | (-0.02) | | | | |
| 5. | -0.013 | | | | 0.118 | | | 1.04% |
| | (-1.78) | | | | (2.66) | | | |
| 6. | -0.013 | | | -0.130 | 0.145 | | | 1.11% |
| | (-1.75) | | | (-0.89) | (2.98) | | | |
| | | | | | | **ln Variance** | | |
| 7. | 0.009 | | | | | 0.001 | | -0.20% |
| | (0.77) | | | | | (0.31) | | |
| 8. | 0.042 | | | | | | 0.010 | 0.93% |
| | (3.03) | | | | | | (2.63) | |
| 9. | 0.039 | | | | | -0.002 | 0.011 | 0.81% |
| | (2.57) | | | | | (-0.61) | (2.69) | |

# Table 3: Bootstrap Results

This table presents results of the bootstrap experiment described in section 3.1. The bootstrap is carried out under the null of no predictability of excess value-weighted returns. Both $V$ and $V_{vw}$ are assumed to follow an AR(12) process, the parameters of which are reported in Panel A. Panel A also gives the correlations between the residuals of the AR processes. For each bootstrap replication, the errors are drawn simultaneously for the three variables to preserve their correlation structure. Panel B presents regression results for the sample 1963:08-1999:12 (slightly different from the sample of Table 2). The first row in each regression shows the estimated coefficient and the second row its the Newey-West $t$-statistic. The third row gives the 95 percent critical values of the $t$-statistic from the empirical distribution generated by 100,000 replications of the bootstrap.

## Panel A: Parameter Estimates for VAR

### AR(12) for $V$

| | CNST | $V_{t-1}$ | $V_{t-2}$ | $V_{t-3}$ | $V_{t-4}$ | $V_{t-5}$ | $V_{t-6}$ | $V_{t-7}$ | $V_{t-8}$ | $V_{t-9}$ | $V_{t-10}$ | $V_{t-11}$ | $V_{t-12}$ | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.001 | 0.514 | 0.207 | 0.038 | -0.082 | 0.079 | -0.005 | -0.023 | -0.002 | 0.043 | 0.019 | 0.005 | 0.170 | 68.11% |
| | (1.50) | (4.40) | (3.00) | (0.87) | (-1.61) | (1.54) | (-0.11) | (-0.35) | (-0.04) | (0.65) | (0.36) | (0.10) | (1.74) | |

Descriptives on Residuals from AR(12) for $V$

| | Mean | StdDev | Median | Min | Max | Skew | Kurt | $AR_1$ | $AR_{1:12}$ |
|---|---|---|---|---|---|---|---|---|---|
| | -0.0000 | 0.0088 | -0.0012 | -0.0287 | 0.0952 | 3.7144 | 36.8501 | 0.0010 | 0.0162 |

### AR(12) for $V_{vw}$

| | CNST | $V_{vwt-1}$ | $V_{vwt-2}$ | $V_{vwt-3}$ | $V_{vwt-4}$ | $V_{vwt-5}$ | $V_{vwt-6}$ | $V_{vwt-7}$ | $V_{vwt-8}$ | $V_{vwt-9}$ | $V_{vwt-10}$ | $V_{vwt-11}$ | $V_{vwt-12}$ | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.001 | 0.136 | 0.111 | 0.068 | -0.018 | 0.022 | 0.053 | 0.007 | -0.007 | 0.037 | 0.024 | -0.005 | -0.012 | 2.86% |
| | (3.71) | (2.29) | (2.61) | (3.96) | (-1.41) | (0.78) | (1.53) | (0.60) | (-0.50) | (0.80) | (0.83) | (-0.35) | (-0.68) | |

Descriptives on Residuals from AR(12) for $V_{vw}$

| | Mean | StdDev | Median | Min | Max | Skew | Kurt | $AR_1$ | $AR_{1:12}$ |
|---|---|---|---|---|---|---|---|---|---|
| | 0.0000 | 0.0035 | -0.0006 | -0.0053 | 0.0651 | 14.6045 | 263.0957 | 0.0003 | -0.0261 |

Cross-Correlation

| | $e_{V_{vw}}$ | $e_V$ |
|---|---|---|
| $e_V$ | 0.647 | |
| $r_{vw}$ | -0.310 | -0.045 |

**Panel B: Regression Estimates and Bootstrapped Statistics**

|  | CNST | $V_{vw}$ | $V$ | $\overline{R}^2$ |
|---|---|---|---|---|
| 1. | 0.007<br>(3.21) | -0.597<br>(-1.07)<br>[0.418] | | 0.02% |
| 2. | -0.004<br>(-0.96) | | 0.336<br>(2.57)<br>[0.017] | 1.19% |
| 3. | -0.005<br>(-1.35) | -1.449<br>(-2.70)<br>[0.051] | 0.478<br>(3.86)<br>[0.000] | 2.17% |

## Table 4: Low-Frequency Measures of Volatility

This table presents descriptive statistics on the low-frequency measures of volatility. The low-frequency measure of average stock volatility is computed from equation (5). The high-frequency average stock volatility is computed from equation (2). 'Skew' is the skewness, 'Kurt' is the kurtosis, '$AR_1$' is the first-order autocorrelation, and '$AR_{1:12}$' is the sum of the first 12 autocorrelation coefficients. 'ADF' is the Augmented Dickey-Fuller statistic for presence of unit root calculated with an intercept and 12 lags. The critical values for rejection of unit root are -3.4475 and -2.8684 at 1 percent and 5 percent level. Panel B also reports the cross-correlation between the two measures.

**Panel A: Descriptives (Sample 1926:01-1999:12)**

|  | Mean | Median | StdDev | Min | Max | Skew | Kurt | $AR_1$ | $AR_{1:12}$ | ADF |
|---|---|---|---|---|---|---|---|---|---|---|
| $V^{lf}$ | 0.0246 | 0.0164 | 0.0471 | 0.0022 | 0.7720 | 10.6461 | 144.8416 | 0.4604 | 2.5886 | -4.1489 |
| $SD^{lf}$ | 0.1384 | 0.1280 | 0.0741 | 0.0474 | 0.8786 | 3.9099 | 31.3835 | 0.5955 | 5.1557 | -3.0091 |

**Panel B: Comparison with Time-Series Measures (Sample 1962:07-1999:12)**

|  | Mean | Median | StdDev | Min | Max | Skew | Kurt | $AR_1$ | $AR_{1:12}$ | ADF |
|---|---|---|---|---|---|---|---|---|---|---|
| $V^{lf}$ | 0.0257 | 0.0210 | 0.0206 | 0.0055 | 0.2461 | 4.7815 | 40.5841 | 0.3793 | 3.0857 | -1.8878 |
| $V$ | 0.0286 | 0.0245 | 0.0157 | 0.0099 | 0.1226 | 1.9232 | 8.7414 | 0.7999 | 7.4019 | -1.1031 |
| $SD^{lf}$ | 0.1528 | 0.1450 | 0.0490 | 0.0743 | 0.4961 | 1.9498 | 11.0519 | 0.5341 | 4.6606 | -1.8620 |
| $SD$ | 0.1640 | 0.1564 | 0.0419 | 0.0995 | 0.3501 | 1.0031 | 4.4407 | 0.8426 | 8.3058 | -1.2680 |

$$\text{Corr}(V^{lf},V) = 0.7475, \text{Corr}(SD^{lf},SD) = 0.8166$$

# Table 5: Forecasts of Value-Weighted Portfolio Return Based on Low-Frequency Measure of Average Volatility

This table presents the results of a 1-month ahead predictive regression of the excess value-weighted portfolio return on lagged explanatory variables for a longer sample and for a different measure of average volatility. $V^{lf}$ is the low-frequency measure of average stock variance, and $V_{vw}$ is the value-weighted volatility. $V^{lf}$ is calculated using monthly data according to equation (5). $V_{vw}$ is calculated using CRSP daily data for the sample period 1963:08-1999:12 and using Schwert's daily data for the earlier sample period. The sample period is 1962:07 to 1999:12 (437 monthly observations) in Panel A and 1927:02 to 1999:12 (875 monthly observations) in Panel B. The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

**Panel A: Regressions (Sample 1963:08-1999:12)**

|  | CNST | $V_{vw}$ | $V^{lf}$ | $\overline{R}^2$ |
|---|---|---|---|---|
| 1. | 0.007 | -0.597 |  | 0.02% |
|  | (3.21) | (-1.07) |  |  |
|  |  | [0.426] |  |  |
| 2. | 0.001 |  | 0.195 | 0.59% |
|  | (0.19) |  | (2.18) |  |
|  |  |  | [0.044] |  |
| 3. | 0.001 | -0.926 | 0.237 | 0.91% |
|  | (0.40) | (-1.75) | (2.38) |  |
|  |  | [0.203] | [0.029] |  |

**Panel B: Regressions (Sample 1927:02-1999:12)**

|  | CNST | $V_{vw}$ | $V^{lf}$ | $\overline{R}^2$ |
|---|---|---|---|---|
| 1. | 0.005 | 0.606 |  | 0.37% |
|  | (2.03) | (0.69) |  |  |
|  |  | [0.532] |  |  |
| 2. | 0.003 |  | 0.155 | 1.64% |
|  | (1.52) |  | (2.83) |  |
|  |  |  | [0.010] |  |
| 3. | 0.003 | 0.127 | 0.147 | 1.54% |
|  | (1.17) | (0.15) | (3.23) |  |
|  |  | [0.888] | [0.003] |  |

## Table 6: Forecasts of Value-Weighted Portfolio Return Controlling for Business Cycle

This table presents the results of a 1-month ahead predictive regression of excess portfolio return on lagged explanatory variables. $V$ is the average stock variance, $V_{vw}$ is the value-weighted volatility, and $r_{vw}$ is the value-weighted portfolio return in excess of the 3 month T-Bill rate. $V$ and $V_{vw}$ are calculated using daily data. DP is the logged dividend price ratio calculated as the difference between the log of last 12 month dividends and the log of the current price index of the CRSP value-weighted index. RTB is the relative 3 month Treasury bill rate calculated as the difference between T-Bill and its 12 month moving average. Term Spread is the difference between the yield on long term government bonds and T-Bill. Default Spread is the difference between the yield on BAA- and AAA-rated corporate bonds. The sample period is 1963:08 to 1999:12 (437 monthly observations). The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

|    | CNST   | $V_{vw}$ | $V$     | $r_{vw}$ | DP      | RTB     | Term Spread | Default Spread | $\overline{R}^2$ |
|----|--------|----------|---------|----------|---------|---------|-------------|----------------|------------------|
| 1. | 0.012  |          |         | -0.008   | -0.000  | -7.142  | -0.322      | 1.586          | 4.00%            |
|    | (0.35) |          |         | (-0.13)  | (-0.03) | (-3.05) | (-2.35)     | (2.26)         |                  |
| 2. | 0.012  | -0.958   |         | -0.035   | -0.001  | -7.545  | -0.327      | 1.714          | 4.34%            |
|    | (0.33) | (-1.96)  |         | (-0.70)  | (-0.09) | (-2.94) | (-2.36)     | (2.40)         |                  |
|    |        | [0.167]  |         |          |         |         |             |                |                  |
| 3. | 0.047  |          | 0.363   | -0.004   | 0.010   | -5.617  | -0.440      | 1.648          | 4.92%            |
|    | (1.28) |          | (2.20)  | (-0.07)  | (1.14)  | (-2.26) | (-3.03)     | (2.37)         |                  |
|    |        |          | [0.041] |          |         |         |             |                |                  |
| 4. | 0.074  | -2.230   | 0.665   | -0.065   | 0.018   | -5.285  | -0.553      | 1.997          | 6.97%            |
|    | (1.81) | (-4.93)  | (4.06)  | (-1.35)  | (1.74)  | (-2.06) | (-3.56)     | (2.79)         |                  |
|    |        | [0.002]  | [0.000] |          |         |         |             |                |                  |

## Table 7: Forecasts of Value-Weighted Portfolio Return Based on Low-Frequency Measure of Average Volatility Controlling for Business Cycle

This table presents the results of a 1-month ahead predictive regression of excess portfolio return on lagged explanatory variables for a longer sample and for a different measure of average volatility. $V^{lf}$ is the average stock variance, $V_{vw}$ is the value-weighted volatility, and $r_{vw}$ is the value-weighted portfolio return in excess of the 3 month T-Bill rate. $V^{lf}$ is calculated using cross-sectional monthly data. $V_{vw}$ is calculated using CRSP daily data for the sample period 1962:07-1999:12 and using Schwert daily data for the earlier sample period. DP is the logged dividend price ratio calculated as the difference between the log of last 12 month dividends and the log of the current price index of the CRSP value-weighted index. RTB is the relative 3 month Treasury bill rate calculated as the difference between T-Bill and its 12 month moving average. Term Spread is the difference between the yield on long term government bonds and T-Bill. Default Spread is the difference between the yield on BAA- and AAA-rated corporate bonds. The sample period is 1963:08 to 1999:12 (437 monthly observations) in Panel A and 1927:02 to 1999:12 (875 monthly observations) in Panel B. The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

**Panel A: Regressions (Sample 1963:08-1999:12)**

|  | CNST | $V_{vw}$ | $V^{lf}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.012 | | | -0.008 | -0.000 | -7.142 | -0.322 | 1.586 | 4.00% |
|  | (0.35) | | | (-0.13) | (-0.03) | (-3.05) | (-2.35) | (2.26) | |
| 2. | 0.012 | -0.958 | | -0.035 | -0.001 | -7.545 | -0.327 | 1.714 | 4.34% |
|  | (0.33) | (-1.96) | | (-0.70) | (-0.09) | (-2.94) | (-2.36) | (2.40) | |
|  | | [0.167] | | | | | | | |
| 3. | 0.021 | | 0.120 | -0.015 | 0.002 | -6.689 | -0.349 | 1.535 | 4.03% |
|  | (0.60) | | (1.25) | (-0.25) | (0.28) | (-2.78) | (-2.55) | (2.21) | |
|  | | | [0.243] | | | | | | |
| 4. | 0.025 | -1.284 | 0.194 | -0.056 | 0.003 | -6.952 | -0.373 | 1.675 | 4.73% |
|  | (0.70) | (-2.71) | (1.75) | (-1.13) | (0.38) | (-2.71) | (-2.68) | (2.38) | |
|  | | [0.061] | [0.103] | | | | | | |

**Panel B: Regressions (Sample 1927:02-1999:12)**

|  | CNST | $V_{vw}$ | $V^{lf}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.016<br>(0.93) | | | 0.100<br>(1.54) | 0.003<br>(0.64) | -5.349<br>(-1.90) | -0.056<br>(-0.84) | 0.300<br>(0.49) | 1.62% |
| 2. | 0.018<br>(1.15) | 0.625<br>(0.67)<br>[0.531] | | 0.111<br>(1.62) | 0.004<br>(0.81) | -5.395<br>(-2.00) | -0.032<br>(-0.47) | 0.006<br>(0.01) | 1.86% |
| 3. | 0.025<br>(1.43) | | 0.123<br>(1.71)<br>[0.119] | 0.064<br>(0.97) | 0.005<br>(1.13) | -5.462<br>(-1.99) | -0.056<br>(-0.84) | -0.057<br>(-0.09) | 2.29% |
| 4. | 0.025<br>(1.42) | 0.291<br>(0.30)<br>[0.784] | 0.107<br>(1.46)<br>[0.183] | 0.074<br>(0.97) | 0.005<br>(1.13) | -5.468<br>(-2.02) | -0.045<br>(-0.65) | -0.147<br>(-0.23) | 2.24% |

## Table 8: Forecasts of Equal-Weighted Portfolio Return

This table presents the results of a 1-month ahead predictive regression of excess equal-weighted portfolio return on lagged explanatory variables. $V$ is the average stock variance, $V_{ew}$ is the equal-weighted volatility, and $r_{ew}$ is the equal-weighted portfolio return in excess of the 3 month T-Bill rate. $V$ and $V_{ew}$ are calculated using daily data. DP is the logged dividend price ratio calculated as the difference between the log of last 12 month dividends and the log of the current price index of the CRSP value-weighted index. RTB is the relative 3 month Treasury bill rate calculated as the difference between T-Bill and its 12 month moving average. Term Spread is the difference between the yield on long term government bonds and T-Bill. Default Spread is the difference between the yield on BAA- and AAA-rated corporate bonds. The sample period is 1963:08 to 1999:12 (437 monthly observations). The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

|  | CNST | $V_{ew}$ | $V$ | $r_{ew}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.008 | -0.110 | | | | | | | -0.23% |
| | (2.57) | (-0.13) | | | | | | | |
| | | [0.917] | | | | | | | |
| 2. | -0.005 | | 0.434 | | | | | | 1.22% |
| | (-0.82) | | (2.55) | | | | | | |
| | | | [0.013] | | | | | | |
| 3. | -0.006 | -1.070 | 0.520 | | | | | | 1.32% |
| | (-0.93) | (-1.14) | (2.83) | | | | | | |
| | | [0.328] | [0.009] | | | | | | |
| 4. | 0.088 | | | 0.173 | 0.017 | -8.246 | -0.616 | 2.201 | 10.21% |
| | (1.97) | | | (3.25) | (1.58) | (-3.01) | (-3.28) | (2.25) | |
| 5. | 0.086 | 0.433 | | 0.181 | 0.017 | -8.119 | -0.604 | 2.151 | 10.06% |
| | (1.90) | (0.52) | | (3.11) | (1.54) | (-2.97) | (-3.15) | (2.16) | |
| | | [0.671] | | | | | | | |
| 6. | 0.149 | | 0.651 | 0.170 | 0.036 | -5.555 | -0.834 | 2.341 | 12.26% |
| | (3.05) | | (4.04) | (2.92) | (2.92) | (-2.06) | (-4.39) | (2.46) | |
| | | | [0.000] | | | | | | |
| 7. | 0.177 | -1.555 | 0.854 | 0.138 | 0.043 | -5.169 | -0.944 | 2.564 | 12.56% |
| | (3.17) | (-1.47) | (3.57) | (2.24) | (3.03) | (-1.86) | (-4.24) | (2.55) | |
| | | [0.226] | [0.001] | | | | | | |

## Table 9: Forecasts of Portfolios Based on Characteristics

This table presents the results of the following regression

$$r_{t+1} = a + b\,V_{vwt} + c\,V_t^{lf} + \epsilon_{t+1}$$

where $r$ is the return (in excess of the 3 month T-Bill rate) on various portfolios based on characteristics. $V_{vw}$ is the value-weighted volatility and is calculated using CRSP daily data for the sample period 1962:07-1999:12 and using Schwert daily data for the earlier sample period. $V^{lf}$ is the average stock variance calculated using cross-sectional data. The portfolios selected 25 size and book-to-market portfolios in Panel A, and 17 industry portfolios in Panel B. Sample period is 1927:01 to 1999:12 (875 monthly observations). The $t$-statistics are corrected for Newey-West adjustment.

### Panel A: 25 Size and Book-to-Market Portfolios

| B/M | Small | 2 | — Size —<br>3 | 4 | Large | Small | 2 | — Size —<br>3 | 4 | Large |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | | $b$ | | | | | $t(b)$ | | |
| Low | 0.361 | -0.937 | -0.006 | 0.268 | -0.011 | 0.195 | -1.349 | -0.005 | 0.311 | -0.014 |
| 2 | -1.249 | -0.063 | 0.277 | 0.532 | 0.037 | -0.665 | -0.058 | 0.262 | 0.434 | 0.047 |
| 3 | -0.126 | 0.184 | 0.540 | 0.229 | 0.151 | -0.088 | 0.163 | 0.497 | 0.224 | 0.151 |
| 4 | -0.216 | 0.263 | 0.090 | 0.696 | 0.718 | -0.183 | 0.234 | 0.085 | 0.604 | 0.577 |
| High | -0.240 | 0.103 | 0.535 | 1.142 | 0.424 | -0.211 | 0.083 | 0.422 | 0.733 | 0.235 |
| | | | $c$ | | | | | $t(c)$ | | |
| Low | 0.469 | 0.365 | 0.329 | 0.154 | 0.136 | 1.320 | 3.230 | 3.137 | 3.214 | 3.316 |
| 2 | 0.657 | 0.327 | 0.197 | 0.183 | 0.112 | 4.015 | 2.740 | 2.805 | 3.026 | 2.702 |
| 3 | 0.477 | 0.356 | 0.226 | 0.213 | 0.127 | 4.542 | 2.829 | 3.365 | 3.450 | 2.433 |
| 4 | 0.534 | 0.347 | 0.252 | 0.203 | 0.172 | 2.981 | 2.935 | 2.627 | 2.819 | 2.646 |
| High | 0.466 | 0.304 | 0.332 | 0.266 | 0.199 | 2.557 | 2.810 | 3.450 | 2.763 | 1.691 |
| | | | $\overline{R}^2$ | | | | | | | |
| Low | 3.20% | 3.59% | 3.92% | 1.50% | 1.09% | | | | | |
| 2 | 6.68% | 3.48% | 2.04% | 2.48% | 0.79% | | | | | |
| 3 | 5.20% | 5.02% | 3.11% | 2.59% | 1.02% | | | | | |
| 4 | 7.45% | 4.71% | 2.84% | 2.64% | 2.18% | | | | | |
| High | 4.45% | 2.53% | 3.74% | 3.11% | 0.33% | | | | | |

**Panel B: 17 Industry Portfolios**

| Industry | $b$ | $t(b)$ | $c$ | $t(c)$ | $\overline{R}^2$ |
|---|---|---|---|---|---|
| Food | 0.224 | 0.31 | 0.100 | 2.69 | 0.99% |
| Mining & Minerals | -0.047 | -0.09 | 0.126 | 1.55 | 0.68% |
| Oil & Petroleum | 0.656 | 0.70 | 0.028 | 0.42 | 0.40% |
| Textiles, Apparel, & Footwear | -0.033 | -0.03 | 0.249 | 3.50 | 2.52% |
| Consumer Durables | -0.718 | -0.66 | 0.199 | 3.47 | 1.69% |
| Chemicals | 0.635 | 0.59 | 0.146 | 2.67 | 1.92% |
| Drugs, Soap, Perfumes, & Tobacco | 0.355 | 0.46 | 0.060 | 1.73 | 0.50% |
| Construction | 0.295 | 0.29 | 0.257 | 4.07 | 3.30% |
| Steel | 0.970 | 0.65 | 0.128 | 1.53 | 1.32% |
| Fabricated Products | 0.579 | 0.59 | 0.081 | 1.35 | 0.84% |
| Machinery & Business Equipment | 0.310 | 0.30 | 0.117 | 2.16 | 0.70% |
| Automobiles | 0.699 | 0.45 | 0.233 | 3.18 | 2.78% |
| Transportation | 0.135 | 0.15 | 0.216 | 3.40 | 1.88% |
| Utilities | 0.068 | 0.12 | 0.143 | 2.86 | 1.20% |
| Retail Stores | -0.226 | -0.28 | 0.196 | 4.36 | 1.83% |
| Financial | 0.146 | 0.15 | 0.198 | 3.17 | 1.77% |
| Other | -0.478 | -0.84 | 0.196 | 4.87 | 2.50% |

## Table 10: Trading Strategy Based on Return Forecasts

This table presents descriptive statistics on a trading strategy based on out-of-sample forecasts of value-weighted market return. At time $t$, the trading strategy invests all in the stock index if the forecasted excess return of stocks over the riskfree rate is greater than zero, otherwise invests all in Treasury bills. The parameters are estimated in a regression of excess return based on all the available data upto time $t$. The sample period is 1962:07 to 1999:12, however, the forecasting exercise starts only in 1967:07, to allow for 60 months in the estimation of first of the time series of parameters. The parameters are then reestimated every month. The mean and standard deviation are annualized. N is the number of months the trading strategy is invested in the market portfolio. In Panel B, we report the annualized fees that a quadratic utility investor with relative risk aversion $\gamma$ would be willing to pay a money manager that uses the strategy and attain the same level of utility as buying and holding the market portfolio.

### Panel A: Descriptives

| Forecasting Criterion | Mean | StdDev | N |
|---|---|---|---|
| Market Buy-Hold | 0.0666 | 0.1572 | 390 |
| $V_{vw}$ | 0.0635 | 0.1476 | 374 |
| $V$ | 0.0718 | 0.1457 | 371 |
| $V_{vw}$ and $V$ | 0.0651 | 0.1476 | 372 |
| $V_{vw}$, $V$, and Macro | 0.0562 | 0.1031 | 157 |

### Panel B: Annual Fees

| Forecasting Criterion | $\gamma$ = 1 | $\gamma$ = 5 | $\gamma$ = 10 |
|---|---|---|---|
| $V_{vw}$ | 1.63% | 1.71% | 1.73% |
| $V$ | 2.37% | 2.46% | 2.48% |
| $V_{vw}$ and $V$ | 1.77% | 1.85% | 1.87% |
| $V_{vw}$, $V$, and Macro | 1.25% | 1.49% | 1.54% |

## Table 11: Forecasts of Value-Weighted Portfolio Return Based on Market Model Residuals

This table presents the results of a 1-month ahead predictive regression of excess value-weighted portfolio return on lagged explanatory variables. $V_{vw}$ is the value-weighted volatility and is calculated using CRSP daily data for the sample period 1962:07-1999:12 and using Schwert's daily data for the earlier sample period. $V^{MM}$ is the cross-sectional average of squared residuals. The residuals for each stock are calculated by a monthly time-series regression of the excess stock returns on the CRSP value-weighted portfolio. $r_{vw}$ is the value-weighted portfolio return in excess of the 3 month T-Bill rate. DP is the logged dividend price ratio calculated as the difference between the log of last 12 month dividends and the log of the current price index of the CRSP value-weighted index. RTB is the relative 3 month Treasury bill rate calculated as the difference between T-Bill and its 12 month moving average. Term Spread is the difference between the yield on long term government bonds and T-Bill. Default Spread is the difference between the yield on BAA- and AAA-rated corporate bonds. The sample period is 1963:08 to 1999:12 (437 monthly observations) in Panel A and 1927:02 to 1999:12 (875 monthly observations) in Panel B. The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

**Panel A: Regressions (Sample 1963:08-1999:12)**

| | CNST | $V_{vw}$ | $V^{MM}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.007 | -0.597 | | | | | | | 0.02% |
| | (3.21) | (-1.07) | | | | | | | |
| | | [0.420] | | | | | | | |
| 2. | -0.000 | | 0.251 | | | | | | 0.74% |
| | (-0.00) | | (2.06) | | | | | | |
| | | | [0.053] | | | | | | |
| 3. | 0.001 | -0.655 | 0.257 | | | | | | 0.81% |
| | (0.31) | (-1.28) | (2.05) | | | | | | |
| | | [0.340] | [0.057] | | | | | | |
| 4. | 0.012 | | | -0.008 | -0.000 | -7.142 | -0.322 | 1.586 | 4.00% |
| | (0.35) | | | (-0.13) | (-0.03) | (-3.05) | (-2.35) | (2.26) | |
| 5. | 0.012 | -0.958 | | -0.035 | -0.001 | -7.545 | -0.327 | 1.714 | 4.34% |
| | (0.33) | (-1.96) | | (-0.70) | (-0.09) | (-2.94) | (-2.36) | (2.40) | |
| | | [0.164] | | | | | | | |
| 6. | 0.026 | | 0.191 | -0.023 | 0.004 | -6.502 | -0.371 | 1.562 | 4.21% |
| | (0.73) | | (1.46) | (-0.41) | (0.44) | (-2.72) | (-2.64) | (2.23) | |
| | | | [0.181] | | | | | | |
| 7. | 0.028 | -1.057 | 0.216 | -0.055 | 0.004 | -6.862 | -0.384 | 1.700 | 4.66% |
| | (0.75) | (-2.30) | (1.48) | (-1.12) | (0.43) | (-2.66) | (-2.67) | (2.38) | |
| | | [0.108] | [0.175] | | | | | | |

**Panel B: Regressions (Sample 1927:02-1999:12)**

| | CNST | $V_{vw}$ | $V^{MM}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.005 (1.91) | 0.614 (0.69) [0.509] | | | | | | | 0.39% |
| 2. | 0.002 (0.67) | | 0.268 (2.34) [0.031] | | | | | | 1.47% |
| 3. | 0.001 (0.44) | 0.280 (0.34) [0.746] | 0.245 (2.53) [0.016] | | | | | | 1.45% |
| 4. | 0.014 (0.84) | | | 0.103 (1.58) | 0.003 (0.57) | -5.270 (-1.87) | -0.053 (-0.80) | 0.314 (0.52) | 1.66% |
| 5. | 0.016 (1.04) | 0.635 (0.68) [0.537] | | 0.114 (1.67) | 0.003 (0.74) | -5.318 (-1.97) | -0.029 (-0.41) | 0.015 (0.03) | 1.92% |
| 6. | 0.028 (1.62) | | 0.219 (2.32) [0.030] | 0.068 (1.14) | 0.006 (1.34) | -5.154 (-1.85) | -0.070 (-1.05) | -0.041 (-0.07) | 2.26% |
| 7. | 0.027 (1.58) | 0.401 (0.43) [0.691] | 0.188 (2.24) [0.037] | 0.080 (1.21) | 0.006 (1.31) | -5.201 (-1.93) | -0.052 (-0.75) | -0.179 (-0.30) | 2.28% |

## Table 12: Forecasts of Value-Weighted Portfolio Return Based on Fama-French Factor Residuals

This table presents the results of a 1-month ahead predictive regression of excess value-weighted portfolio return on lagged explanatory variables. $V_{vw}$ is the value-weighted volatility and is calculated using CRSP daily data for the sample period 1962:07-1999:12 and using Schwert's daily data for the earlier sample period. $V^{FF}$ is the cross-sectional average of squared residuals. The residuals for each stock are calculated by a monthly time-series regression of the excess stock returns on the three Fama-French factors. $r_{vw}$ is the value-weighted portfolio return in excess of the 3 month T-Bill rate. DP is the logged dividend price ratio calculated as the difference between the log of last 12 month dividends and the log of the current price index of the CRSP value-weighted index. RTB is the relative 3 month Treasury bill rate calculated as the difference between T-Bill and its 12 month moving average. Term Spread is the difference between the yield on long term government bonds and T-Bill. Default Spread is the difference between the yield on BAA- and AAA-rated corporate bonds. The sample period is 1963:08 to 1999:12 (437 monthly observations) in Panel A and 1927:02 to 1999:12 (875 monthly observations) in Panel B. The first row in each regression is the coefficient, the second row is the Newey-West adjusted $t$-statistic, and the third row is the bootstrapped $p$-value. The bootstrap experiment uses 10,000 replications and is carried out under the null of no predictability of returns from variances.

**Panel A: Regressions (Sample 1963:08-1999:12)**

| | CNST | $V_{vw}$ | $V^{FF}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.007 | -0.597 | | | | | | | 0.02% |
| | (3.21) | (-1.07) | | | | | | | |
| | | [0.414] | | | | | | | |
| 2. | -0.001 | | 0.318 | | | | | | 0.83% |
| | (-0.28) | | (2.41) | | | | | | |
| | | | [0.026] | | | | | | |
| 3. | 0.000 | -0.633 | 0.323 | | | | | | 0.88% |
| | (0.02) | (-1.22) | (2.40) | | | | | | |
| | | [0.363] | [0.023] | | | | | | |
| 4. | 0.012 | | | -0.008 | -0.000 | -7.142 | -0.322 | 1.586 | 4.00% |
| | (0.35) | | | (-0.13) | (-0.03) | (-3.05) | (-2.35) | (2.26) | |
| 5. | 0.012 | -0.958 | | -0.035 | -0.001 | -7.545 | -0.327 | 1.714 | 4.34% |
| | (0.33) | (-1.96) | | (-0.70) | (-0.09) | (-2.94) | (-2.36) | (2.40) | |
| | | [0.159] | | | | | | | |
| 6. | 0.033 | | 0.287 | -0.028 | 0.006 | -6.320 | -0.401 | 1.579 | 4.39% |
| | (0.89) | | (1.83) | (-0.49) | (0.63) | (-2.64) | (-2.75) | (2.26) | |
| | | | [0.099] | | | | | | |
| 7. | 0.035 | -1.053 | 0.313 | -0.059 | 0.006 | -6.688 | -0.414 | 1.719 | 4.84% |
| | (0.91) | (-2.30) | (1.85) | (-1.24) | (0.62) | (-2.59) | (-2.78) | (2.42) | |
| | | [0.115] | [0.092] | | | | | | |

43

**Panel B: Regressions (Sample 1927:02-1999:12)**

|  | CNST | $V_{vw}$ | $V^{FF}$ | $r_{vw}$ | DP | RTB | Term Spread | Default Spread | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 0.005 (1.91) | 0.614 (0.69) [0.520] | | | | | | | 0.39% |
| 2. | 0.001 (0.20) | | 0.362 (1.84) [0.075] | | | | | | 1.29% |
| 3. | 0.000 (0.11) | 0.285 (0.36) [0.734] | 0.326 (2.06) [0.045] | | | | | | 1.27% |
| 4. | 0.014 (0.84) | | | 0.103 (1.58) | 0.003 (0.57) | -5.270 (-1.87) | -0.053 (-0.80) | 0.314 (0.52) | 1.66% |
| 5. | 0.016 (1.04) | 0.635 (0.68) [0.522] | | 0.114 (1.67) | 0.003 (0.74) | -5.318 (-1.97) | -0.029 (-0.41) | 0.015 (0.03) | 1.92% |
| 6. | 0.031 (1.79) | | 0.322 (1.84) [0.097] | 0.068 (1.16) | 0.008 (1.49) | -5.051 (-1.79) | -0.083 (-1.20) | -0.099 (-0.17) | 2.20% |
| 7. | 0.030 (1.69) | 0.391 (0.43) [0.698] | 0.271 (2.00) [0.065] | 0.081 (1.27) | 0.007 (1.41) | -5.115 (-1.88) | -0.063 (-0.91) | -0.217 (-0.36) | 2.21% |

## Figure 1: Average Stock Volatility

This figure plots the average stock standard deviation of stocks for the period 1962:07 to 1999:12. Average stock volatility is calculated using daily data from equation (2). The bottom panel uses a 12 month simple moving average of the top panel to smooth out high frequency components. NBER recessions are represented by shaded bars.
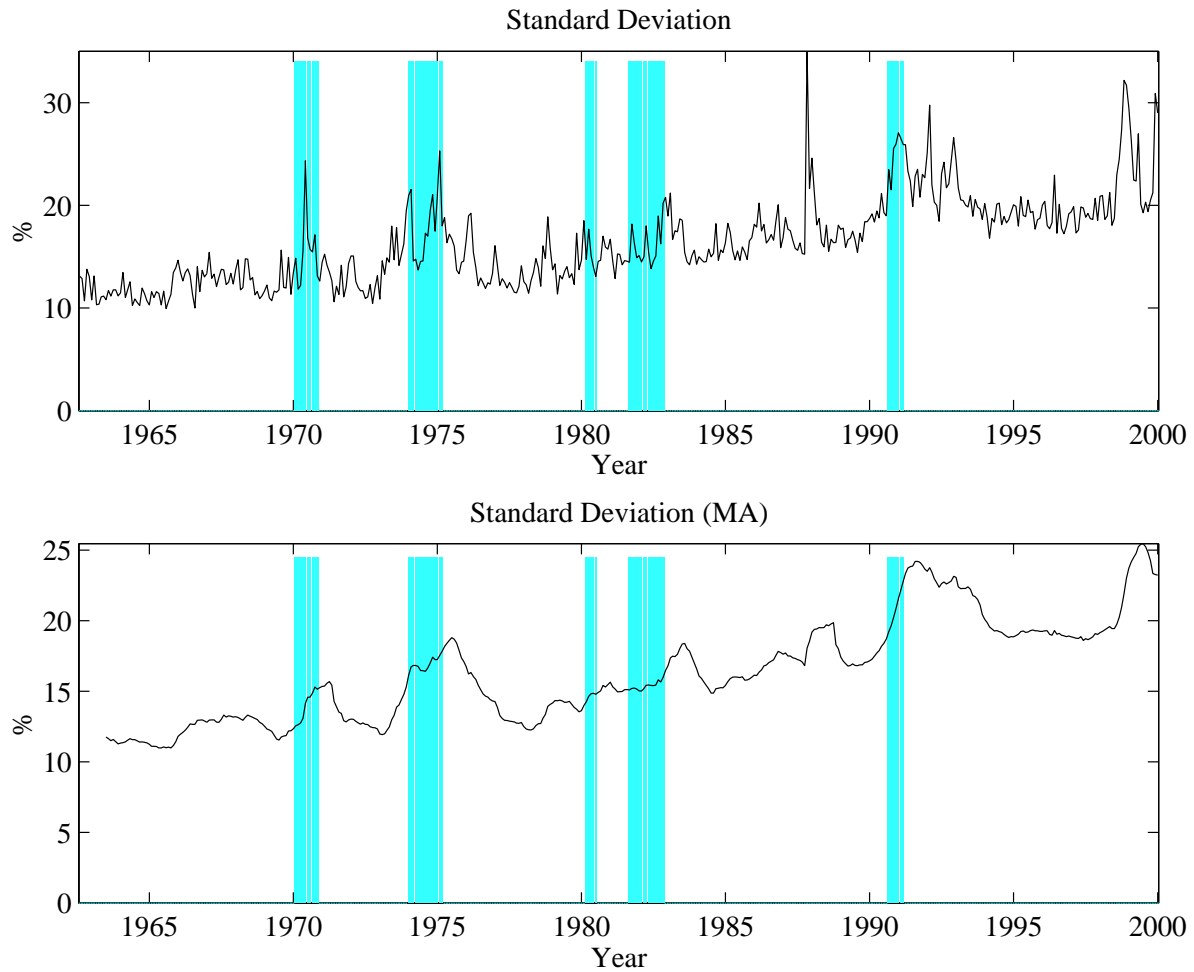
Standard Deviation

Standard Deviation (MA)

45

# Figure 2: Equal- and Value-Weighted Portfolio Volatility

This figure plots the standard deviation for equal- and value-weighted portfolio for the period 1962:07 to 1999:12. Portfolio volatility is calculated using equation (1). The right two panels use a 12 month simple moving average of the left two panels to smooth out high frequency components. NBER recessions are represented by shaded bars.
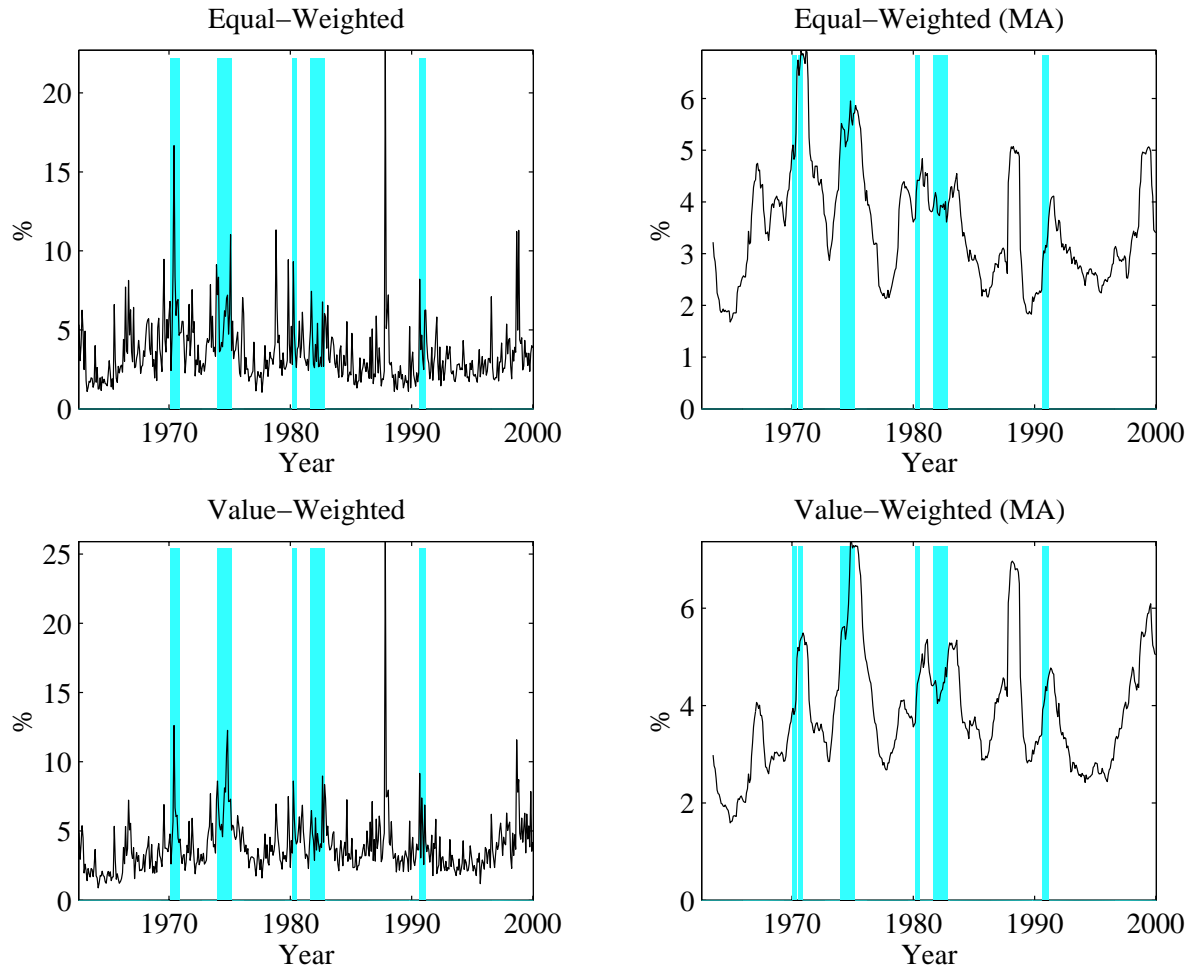
## Figure 3: Average Low Frequency Stock Volatility

This figure plots the average cross-sectional stock standard deviation of stocks calculated using low frequency monthly data from equation (5). The top two panels compare low frequency estimates (solid line) against the high frequency measure (dotted line) for the sample period 1962:07 to 1999:12. The bottom two panels plot the raw time series for low-frequency standard deviation of stocks for the sample period 1926:01 to 1999:12. The right two panels use a 12 month simple moving average of the left two panels to smooth out high frequency components.