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Measuring Asset Values for Cash Settlement in Derivative Markets: Hedonic Repeated Measures Indices and Perpetual Futures

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ABSTRACT

Two proposals are made that may facilitate the creation of derivative market instruments, such as futures contracts, cash settled based on economic indices. The first proposal concerns index number construction: indices based on infrequent measurements of nonstandardized items may control for quality change by using a hedonic repeated measures method, an index number construction method that follows individual assets or subjects through time and also takes account of measured quality variables. The second proposal is to establish markets for perpetual claims on cash flows matching indices of dividends or rents. Such markets may help us to measure the prices of the assets generating these dividends or rents even when the underlying asset prices are difficult or impossible to observe directly. A perpetual futures contract is proposed that would cash settle every day in terms of both the change in the futures price and the dividend or rent index for that day.

THIS PAPER CONCERNS PROBLEMS in the measurement of asset values or present values, problems that, if solved, may facilitate the creation of new hedging vehicles, new cash-settled futures, options, and other derivative financial products.

The greatest components of world wealth are not hedgeable at all, and measurement problems may be an important reason why they are not. Human capital is by far the greatest component of wealth, properly measured, and yet there are no liquid markets on which human capital risk can be hedged. Real estate represents most of the wealth as conventionally measured, and yet here there are no liquid derivative markets (except on mortgages).[1] Privately held financial assets are also of great importance, especially outside the United States, yet there are no derivative markets specifically designed for hedging the risks of this class of investments. There would logically be derivative markets representing the major components of

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[1] See Case, Shiller, and Weiss (1993).

the producer price index or consumer price index, but none exists. A futures market in the aggregate consumer price index has succeeded only in Brazil, a country where the volatility of the consumer price index has been very high. There are many other economic indicators that represent specific risks faced by economic agents, risks for which there is no hedging market today.

There are two separate (but related) measurement problems that may be an important part of the reason why these cash-settled derivative markets have not been established and successful.

The first problem is that the underlying cash market prices, in response to which a derivative market would cash settle, may be observed only infrequently and, when they are observed, the observations at different times are on assets of different type of quality, so that their average price does not allow us to infer the changes through time in the price of an existing asset. This problem may account for the slowness to develop derivative markets on real estate prices or prices of privately held financial assets. These assets tend to be held for years or even decades between sales, making the nontrading problem of near-astronomical proportions. There are secular changes in the stock of outstanding real estate, such as a gradual increase in the size or quality of properties.

The second problem is that in many cases the best (or only) measurements we have on the cash market are not on asset prices at all, but are better thought of as measurements of the dividend or rent on the asset. This problem may be an important part of the reason for the absence of futures markets in such things as labor costs, commercial real estate, the consumer price index, or components of the producer price index. The measurements we have on these are more on cash flows than on asset values or present values. No prices exist at all for the asset value of labor costs: people are not bought and sold. Now, of course, we could easily construct a conventional futures market to be settled on a wage index, but that would be a market settled on the flow return on human capital rather than its price.

Most financial futures contracts today settle on the price of an enduring asset, rather than on a dividend or rent paid on the asset. One might imagine that the exchanges could easily have created conventional futures markets on the dividend accruing to a stock price index as well as the index itself, but there has apparently been no interest in such contracts. While we cannot rule out that some people will want to trade conventional futures on the dividend accruing to a portfolio of stocks, there is probably reason to suspect that most would prefer to trade on the portfolio price itself. Present values collapse information about the indefinite future into today's price.

For the first problem, a hedonic repeated measures price index number construction method is proposed here. There has been much discussion of the problem of quality change in the literature on the construction of producer and consumer price indices. But this literature does not solve the problem of unobserved quality change for the purpose of constructing indices for the settlement of contracts. Discussions of these indices presume that they are trying to measure the price of the flow of newly produced commodities. The problem of quality change is then fundamental and deep, since the qualities of the commodities change through time in unobserved or unquantifiable ways; the indices are in effect pricing apples one period and oranges the next. The hedonic repeated measures price index proposed here follows the price of existing assets through time taking account of their quality. Many of the components of the producer price index (such as airplanes, railroad equipment, computers) could be priced in this way. This may produce indices that are radically different; in some cases prices might fall dramatically through time even though the conventional producer price index component does not. The constructors of the producer price index may be resistant to changing their methods so radically, but for the purpose of constructing hedging instruments we may need to do just this.

The hedonic repeated measures index number construction method generalizes the repeat sales price indices of Baily, Muth, and Nourse (1963), Case and Shiller (1987, 1989), Webb (1988), and Goetzman (1990). The generalization here is to take account of hedonic variables (variables measuring the quality of each asset sold or time measured) within the context of a repeated measures index number construction method. The repeat sales price indices were improvements over earlier indices (simple averages or medians of prices of items sold) in that the repeat sales index number construction methods followed individual properties through time, so changes in the index occurred only in response to changes in prices of individual properties sold. The models that give rise to these indices assumed that all kinds of properties had the same price path (up to a scalar multiple and an idiosycratic error term) through time. Consider in contrast financial indices such as stock price indices. The constructors of these financial indices recognize that different stocks (items of different quality) may have different price paths through time, and seek to construct an index representing the price path of a standard portfolio of stocks. The method discussed here allows us to do this with assets where sales occur infrequently and where the measure of quality may not be categorical. It is a response to some critics of repeat sales indices (Abraham (1990), Case, Pollakowski, and Wachter (1992), and Clapp and Giaccotto (1992)) who expressed concern that those properties represented by repeat sales may not always be representative of the market.

The second proposal, for the problem that dividends or rents rather than prices may be well measured, is to create markets that will provide measures of asset prices: to create markets for perpetual claims on cash flows, paid from shorts to longs, representing the dividends or rents on assets. Specifically, perpetual futures contracts are proposed here that would cash settle on indices of dividends accruing to an asset in such a way that the futures price should tend to track the value of the asset that generates the dividend. Each contract is perpetual so that it can price the entire stream of dividends accruing to an asset, and thereby provide price discovery for that asset. Also, with perpetual contracts, any given contract does not grow shorter term with time, and so a single contract can be adopted as a standard through time. The futures markets institutions of margin accounts and daily resettlement allow perpetual contracts to be traded even though no market participant, short or long, can assure credit worthiness in the distant future and each participant has only a finite horizon interest in the market. Once a perpetual futures market is established and liquid, the futures price may be used as a measure of asset value in other derivative markets, such as forwards or options.

Past perpetual claims contracts that are analogous to perpetual futures have traded only where the cash market is very liquid and cash prices are easily observable. It is proposed here that some perpetual futures contracts, such as a market for human capital, could even be traded "blind," the market at first never even having observed the cash price. This may seem like a radical proposal, but, of course, every initial public offering of stock is first traded with little knowledge of the ultimate market price, and there is no conceptual difference here in the problems facing traders.

I. Hedonic Repeated Measures Index Number Construction

A. Repeated Measurement Design

Students of experimental design have long advocated repeated measures methods for their greater "design power" (see for example Lee (1975), and Dunn and Clark (1987)).[2] For example, in testing whether a drug has an effect on human subjects, it is often better to use information only on subjects who were in both the experimental and control groups at different times, thereby ruling out the possibility that the results were confounded by unobserved differences between the control and experimental groups. This is especially important when it is impossible to control membership in the two groups completely (as when some subjects drop out of the study). The importance of repeated measures methods will be especially important in the financial applications intended here, as we may have little or no control over the dates when prices, dividends, or rents are observed.[3]

Constructors of financial price indices also use repeated measures methods, without calling them by this name. They follow the individual stocks or bonds through time, linking them out or replacing them with new ones when they disappear or become unsuitable for the index, thereby maintaining a repeated measures design. No one would advocate computing a stock price index that was the average price of one sample of stocks for one period and of a different sample of stocks the next period, even if some objective measures of the firms (book values, industry category, etc.) were the same across periods. That is, however, essentially what is done for the most widely quoted price indices for residential real estate. The median sale price of a single family home published by the National Association of Realtors is based on

[2] For the same reason, researchers have long appreciated the importance of panel data for empirical work (see for example Hsiao (1986)).

[3] The literature on hedonic regressions has shown some ways of using data on sales in the used market to correct for quality changes (see Cagan (1971) and Hall (1971)).

selling prices of whatever houses were sold each time period, and makes no effort to standardize houses from period to period. The constant quality index published by the Department of Commerce is based on a regression-per-period hedonic regression involving a number of characteristics, and the regression method assures that in a sense, in terms of these measurable characteristics, standard houses are priced. But, especially since the constant quality index prices only new houses, there is a distinct possibility for bias because of unobserved quality changes. Consider a period when housing prices in an area have dropped very far, so far as to be below construction costs. There will still be *some* construction, as houses are built for special purposes and in special areas, and the prices of these houses will not be representative of all houses; indeed they will certainly tend to sell for prices at or above the construction costs. These houses may not differ from most houses in terms of objective measurements like the number of rooms.[4]

B. Hedonic Repeated Measures Indices

It is instructive first to review the conventional hedonic methods. The conventional regression-per-period hedonic index is produced by regressing, each time period, log price (or log dividend or log rent) on a constant and a number of variables, called hedonic or quality variables, that characterize the property sold or rented at the time of observations.[5] The dependent variable for time t, call here Y_t, is an N_t-element column vector, where N_t is the number of observations of prices or rents at time t, and where the ith element of Y_t is the ith log price or (or log dividend or log rent). The matrix of independent variables Z_t is an $N_t \times K$ matrix whose ith row consists of (a constant and) a vector of hedonic variables for the time of observation of that sale or rent. The regression model is $Y_t = Z_t\gamma_t + \epsilon_t$, where γ_t is a K-element vector of regression coefficients and ϵ_t is an N_t-element vector of error terms with mean zero and variance matrix Ω_t. The generalized least squares estimate of γ_t is $\hat{\gamma}_t \equiv (Z_t'\Omega_t^{-1}Z_t)^{-1}Z_t'\Omega_t^{-1}Y_t$, which simplifies in the case where Ω_t is proportional to the identity matrix to ordinary least squares $\hat{\gamma}_t \equiv (Z_t'Z_t)^{-1}Z_t'Y_t$. A regression-per-period index I_{rpp} for time t may be taken as a fitted value of the regression for some standard property whose characteristics are given by the $1 \times K$ element vector $\bar{Z}$: $I_{rppt} = \bar{Z}\hat{\gamma}_t$. Or, chain indices could be constructed from the regression coefficients, where the quality of the standard property is updated through time.

For example, in the context of real estate prices, if p_{it} is the price of property i observed sold at time t, if $N_t = 3$, and Z_t consists of a constant

[4] The problem of unmeasured quality changes is an important reason why hedonic methods are not used more widely (see Triplett (1990)).

[5] Using the log price as a dependent variable creates a log price index whose antilog is essentially a geometric average of prices or rents. For financial market applications, it may be preferable to use arithmetic indices; arithmetic analogues of all the indices described here are possible (see Shiller (1991, 1993)).

and the number of square feet for that property at time t, then the matrice are:

$$Y_t = \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} \qquad Z_t = \begin{bmatrix} 1 & s_{1t} \\ 1 & s_{2t} \\ 1 & s_{3t} \end{bmatrix} \tag{1}$$

where s_{it} is the square feet of floor space in property i at time t.

It is convenient, for purposes that will become clear in a moment, to assemble these regression matrices for T time periods into one giant regression to be run where the price indices are computed for all T times for which we will have the index. We construct the N-element vector Y, where $N = \Sigma N_t$; Y is the stacked Y_t vectors. We also construct the $N \times TK$ matrix Z which is block diagonal, with the Z_t matrices along the diagonal. Now, the combined regressions can be written as a single regression model $Y = Z\gamma + \epsilon$ where γ is the stacked γ_t vectors, so that γ has TK elements, and ϵ is an N-element vector of error terms with mean zero and variance matrix Ω. If Ω is block diagonal, then since the Z matrix is block diagonal, the estimated coefficient vector $\hat{\gamma} = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}Y$ is just the stacked per period regression coefficient vectors.

For example suppose that there are only T = three time periods, times 0, 1, and 2, and the matrices are:

$$Y = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \end{bmatrix} \qquad Z = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \tag{2}$$

As time goes on, we would keep expanding the Y and Z matrices by appending the latest Y_t to Y and the latest Z_t to the bottom right corner of a matrix whose rows and columns have been augmented from Z with a row and column of zero matrices: of course, so long as Ω is block diagonal, this entails no revisions in past values of the index computed before.

The fundamental problem with hedonic price indices such as these is, as noted in the introduction, that we do not have all characteristics as hedonic variables; there are likely to be omitted hedonic variables. Thus, we seek a repeated measures method.

The repeated measures approach that we shall pursue here consists merely of adding additional dummy variables, variables that proxy for the omitted hedonic variables, to the giant regression. We will, following the literature on experimental design, call them subject dummies, even though they might better be called property dummies or asset dummies in many of our intended applications. There will be one dummy for each subject: one for each individual property (if we are estimating a real estate price index or, say, the railroad equipment component of the producer price index), individual person (if we are estimating a labor cost index), or individual plot of land (if we are estimating a yield per acre index), in each case a dummy that identifies that subject. Each dummy is zero everywhere except for an observation where the dependent variable element corresponds to that property or individual; there the dummy is 1.00. These subject dummy variables are indicators of the unique quality of each property or individual. To avoid multicollinearity, we will, when adding the subject dummies, drop the first column of Z_0, so that we will have an augmented matrix Z_A which is $N \times TK - 1 + m$ where m is the number of subjects. The regession model is now $Y = Z_A B + \epsilon$ where B is a $TK - 1 + m$ element vector of coefficients, and ϵ is, as before, an N-element vector of error terms with mean 0 and variance matrix Ω; the generalized least squares estimate of B is $\hat{B} \equiv (Z_A' \Omega^{-1} Z_A)^{-1} Z_A' \Omega^{-1} Y$.

For example, we may augment the Z matrix in (2) to produce the augmented matrix Z_A:

$$Z_A = \begin{bmatrix} \hat{Z}_0 & 0 & 0 & D_{10} & D_{20} & \cdots & D_{m0} \\ 0 & Z_1 & 0 & D_{11} & D_{21} & \cdots & D_{m1} \\ 0 & 0 & Z_2 & D_{12} & D_{22} & \cdots & D_{m2} \end{bmatrix} \tag{3}$$

Where D_{kt} is the $N_t \times 1$ subject dummy for subject k at time t. For example, with the matrices (1) above D_{2t} equals $[0, 1, 0]'$ and D_{4t} equals $[0, 0, 0]'$. Since the sum over k of the subject dummies equals the vector 1, there would be multicollinearity in this regression if we included all Z_t matrices completely, so we will suppose that the constant term is dropped from Z_0 (hence the ^ above the Z_0 in (3)). Adding the subject dummies as columns to produce the Z_A matrix above will generally break the block diagonality of the matrix, and unless the subject dummy variables have a certain conformation, will cause the index number production method to produce revisions in lagged values of the index.[6]

Note the analogy of this regression model to the conventional fixed effects analysis of covariance model (or, in the special case here where there are no quantitative hedonic variables, only constant terms in the Z_t, the analysis of variance model). By this interpretation, the subject dummies correspond to the "experimental factors" and the hedonic variables to the "concomitant factors." The fixed effect formulation embodied in the subject dummies, rather than the random effect formulation, is adopted here to allow for the possibility that the mix of properties or items changes through time. A conventional random effect model, for which we would substitute in place of the subject dummies a variance component to the error term related to the

[6] Even if we did not include subject dummies, we would then properly need to take account of subject variance components in Ω, which would cause revisions even in regression-per-period methods. Ways of handling revisions in data from the standpoint of contract settlement are discussed in Shiller (1993).

property, would normally assume that the distribution of the property-specific noise did not change through time.

This $N \times (TK - 1 + m)$ matrix Z_A may be very large, both in terms of numbers of rows and numbers of columns. Particularly problematic is that there is a column for every individual property or item and then some; this means that the $Z'_A \Omega^{-1} Z_A$ matrix may be of very large rank, and thus very hard to invert. When estimating residential real estate price indexes, for example, we may have data on millions of houses, resulting in a $Z'_A \Omega^{-1} Z_A$ matrix whose rank is in the millions. Fortunately, however, since our interest is in the coefficients of the original Z_t variables and not in the coefficients of the dummy variables, we can exploit the structure of the subject dummies in such a way as to reduce the size of the matrix that must be inverted.

To do this, construct a matrix S_1 such that $y = S_1 Y$ is the vector of differences of consecutive observations of the dependent variable for each subject. For example, following the real estate example above, if individual property i appears three times, in times t_1, t_2, and t_3, then there will be two rows in $y = S_1 Y$ corresponding to i, a row with element $p_{it3} - p_{it2}$ and a row with element $p_{it2} - p_{it1}$. Now S_1 is of dimension $n \times N$ where n is the number of pairs of consecutive observations of individual subjects that can be constructed out of these N observations. Now construct an $m \times N$ matrix S_2 such that $\overline{Y}_2 \equiv S_2 Y$ is the vector of all first observations of individual subjects. Note that $N = m + n$ and that the matrix $S = (S'_1, S'_2)'$, is nonsingular. Call $\overline{Y} = SY$, $\overline{Z} = SZ_A$, and $\overline{\Omega} = S \Omega S'$. Let us denote the upper left $n \times (TK - 1)$ corner of $\overline{Z}$ by z, the upper $(TK - 1)$ – element vector of B by β, and the upper left $n \times n$ block of $\overline{\Omega}$ by $\overline{\Omega}_{11}$.

It will be shown now that the generalized least squares estimate $(z' \overline{\Omega}_{11}^{-1} z)^{-1} z' \overline{\Omega}_{11}^{-1} y$ is the same as first component $\hat{\beta}$ of $\hat{B} = Z'_A \Omega^{-1} Z_A)^{-1} Z'_A \Omega^{-1} Y$. The former expression involves matrices of smaller dimension than the latter.

To show this, note that $\overline{Z} \equiv SZ_A$ is block triangular. The upper right $n \times m$ block of $\overline{Z}$ consists of zeros. The lower right $m \times m$ block of $\overline{Z}$ is the identity matrix. The generalized least squares estimator $\hat{B} = (Z'_A \Omega^{-1} Z_A)^{-1} Z'_A \Omega^{-1} Y$ equals $(\overline{Z}' \Sigma \overline{Z})^{-1} \overline{Z}' \Sigma \overline{Y}$ where $\Sigma \equiv \overline{\Omega}^{-1} = S'^{-1} \Omega^{-1} S^{-1}$; partition Σ into 4 square parts, with upper left $n \times n$ part called Σ_{11}, etc. From the last m normal equations for $\hat{B}$ (the last m rows of $\overline{Z}' \Sigma \overline{Z} \hat{B} = \overline{Z}' Y$) one finds that $\hat{B}_2 - \overline{Y}_2 + \overline{Z}_{21} \hat{\beta} = \Sigma_{22}^{-1} \Sigma_{21} (y - z \hat{\beta})$; substituting this into the first $TK - 1$ normal equations one finds that $z'(\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}) z \hat{\beta} = z'(\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}) y$, and so $\hat{\beta} = (z' \overline{\Omega}_{11}^{-1} z)^{-1} z' \overline{\Omega}_{11}^{-1} y$ as was to be shown.[7]

For an example of this hedonic repeated measures regression, suppose that the matrices Z_t contain two columns, a vector of ones and the variable s_{it} (square feet of floor space of ith property at time t) as in (1), and let us

[7] Incidentially, had we begun with a random effect rather than fixed effect formulation in (3), the single-sale properties would not have dropped out of the estimation as they have here, reflecting the implicit assumption of the random effect model that the mix of properties does not change through time.

premultiply the regression independent variables and dependent variable by the matrix S described above. We may then be left with the regression (for a sample of six houses) which is:

$$z = \begin{bmatrix} 1 & 0 & -s_{10} & s_{11} & 0 \\ 1 & 0 & -s_{20} & s_{21} & 0 \\ -1 & 1 & 0 & -s_{31} & s_{32} \\ -1 & 1 & 0 & -s_{41} & s_{42} \\ 0 & 1 & -s_{50} & 0 & s_{52} \\ 0 & 1 & -s_{60} & 0 & s_{62} \end{bmatrix} \qquad y = \begin{bmatrix} p_{11} - p_{10} \\ p_{21} - p_{20} \\ p_{32} - p_{31} \\ p_{42} - p_{41} \\ p_{52} - p_{50} \\ p_{62} - p_{60} \end{bmatrix} \tag{4}$$

The hedonic repeated measures regression model $y = z\beta + \epsilon$ (where ϵ is an n-element vector of error terms with mean zero and variance matrix $\overline{\Omega}_{11}$) has a simple interpretation.[8] In (4), the change in log price between any two periods depends on the time periods of the two observations, and on the size of the house at the two times; it may depend on the size of the house either because different size houses have different price paths or because the size of this house changed between sales. The hedonic repeated measures index will be constructed using the coefficient vector $\hat{\beta} = (z'\overline{\Omega}_{11}^{-1}z)^{-1}z'\overline{\Omega}_{11}^{-1}y$. The hedonic repeated measures index at time t could then be taken, in this example, as the fitted value for a standard size (square feet $= \bar{s}$) house at time t: for example, a fixed weight index at time t would be the coefficient of the constant term for time t (or zero if $t = 0$) plus $\bar{s}$ times the coefficient of the square foot variable for time t. (The index may be adjusted by an additive constant to make it equal an assigned value in the base period.) Other kinds of hedonic repeated measures indices could be constructed from the estimated regession coefficient $\hat{\beta}$: chain indices or indices representing the value of a portfolio reinvested through time to be representative of the market in terms of the hedonic variables.

Note that even though we have not used any information about the characteristics of individual properties per se, our set of subject dummy variables spans the set of any characteristics that are constant for each individual asset through time and that have a constant proportional effect on price or rent. That is, continuing our real estate example, suppose that at a later date, after the regression was run, someone discovers that a very important hedonic variable was left out of the regression, a variable that

[8] The transformation that produced (4) is analogous to the differencing employed in many panel data analyses (see for example Hsiao (1986)), but here the differencing interval may be determined by random dates of sales and may not be constant across properties or subjects. Note that if we dropped the last three columns of z, corresponding to the hedonic variable, then this regression reduces to that of Baily, Muth, and Nourse (1963). The method using (4) for regession differs from the hedonic repeat sales regression method proposed by Case and Quigley (1991) in that (4) does not make coefficients linear in time, and in that (4) makes no use of single-sales data. Problems in bias due to errors and variables may be magnified by the differencing here; see Griliches and Hausman (1983).

measures the quality of a house independently of the number of square feet, and it was found that at some times higher quality houses are sold than at other times. Because of concern that the price index might jump up erroneously in periods when the mix of sales is relatively tilted towards the high quality houses, we consider adding a single column to the Z_A matrix in (3) that represents this quality variable for each property. But, if we tried doing that, we would quickly discover that we would have to take that additional column out of the Z_A matrix. So long as quality is constant through time for each house, there is always a weighted sum of the subject dummy columns of the Z_A matrix (3) that will equal the particular hedonic variable. This means that the repeated measures regression had already in effect taken this new hedonic variable into account. In fact, so long as we can assume that the coefficient of the hedonic variable is constant through time, we have in the matrices (3) accounted for all such hedonic variables, all nonlinearities in these variables, and all interaction effects between them; there is no arbitrariness in our methods. No one can ever claim to have found another characteristic variable or interaction effect that was left out of this regression. However, the additional hedonic variable may still be of use for us in gauging whether the relative price of the higher quality homes varies through time: if we add not a single column to the Z_A matrix in (3) for this variable, but a single column to each of the Z_t matrices exemplified by (1), then generally none of the columns that are created in (3) will be collinear with the subject dummy variables.

If any characteristic is constant through time for all subjects (houses), then the sum of the hedonic variable columns in z corresponding to these characteristics (e.g., the sum of the last three columns in z in (4)) is zero, and so the regression cannot be run due to multicollinearity. This poses no particular problems however; if all characteristics are constant through time for each subject, we need only drop one of these columns. There is no reason to drop all of them. Keeping the others in this example allows us to correct for possible changes through time in the pricing of individual characteristics. For example, in constructing a real estate price index, there may be concern that the price path of big houses is different from that of little houses, and that there are times when there are many big houses sold and times when few big houses are sold. An index number based on the fitted value of this regression could take account of a standard house in terms of square feet. Alternatively, there could be in each single period hedonic regression a "big house" dummy variable in place of the square foot variable, or there could be dummies for various types of houses.[9] (This would make our method essentially that of computing an index for each type of house, so that a fixed weight aggregate index can be computed from these indices, just as aggregate stock price

[9] Clapp and Giacotto (1992) have stressed that assessed value might be used in a repeat sales real estate price index where assessed value in effect replaces one of the sale dates: it may be even more useful to use assessed value in a given year in place of square feet in (4) as a hedonic variable.

indexes are produced from prices of individual stocks using weights representative of quantities outstanding.) All these methods will allow us to construct indices that control for time variation in the price of characteristics.

Applying this method as exemplified in (4) might produce very different index values than do conventional index number construction methods. The price or rent index produced may tend to decline through time due to depreciation of the individual assets. This decline in the index is in no way a problem for an index whose purpose is to serve as the basis of trade for the settlement of contracts. In fact, having depreciation included as part of the index could be considered an advantage, as it could allow hedgers to deal with the intrinsic uncertainty about depreciation. We might also want to include as a hedonic variable the age of the asset, and use an expression like (4) to produce an index of prices of or rents on an asset of standard age, although this would not allow us to correct for any overall downward bias in the index due to depreciation. One would have to drop one of the columns in the z matrix corresponding to the age of the asset, since the sum of these columns would equal the sum (for the columns of z corresponding to constant terms, the first two columns in (4)) of i times the ith column of z. One cannot gauge the absolute effect of asset age on price or rent in a hedonic repeated measures regression, since all assets age the same amount between the same intervals of time; there is no way to distinguish the effects of age from the effects of price or rent change of a standard age.[10] But it is not necessary to drop all of the columns in the z matrix corresponding to age variables: leaving all but one of them in allows us to account for the possibility that assets of different ages have different price paths through time, which could affect the index if the age mix of assets sold changes through time.

C. Applications of Hedonic Repeated Measures

The simple framework exemplified by (4) appears not to be in the index number literature; its use might have had many applications. It is immediately applicable, for example, to real estate price indices, where there are long sales histories of individual properties available in deeds offices. It could be used to produce an index of prices of infrequently traded bonds; the hedonic variable could be time to maturity, so that the hedonic repeat sales method could use data on adjacent maturities to price a bond of a standard maturity. It would no longer be necessary to base indices only on that subset of bonds that are frequently traded in a fixed maturity category.

The framework would also be helpful in constructing labor cost indices. The Bureau of Labor Statistics Employment Cost Indices are computed according to a standard Laspeyres index formula from data on wages in occupational categories (see Sheifer (1975) and Wood (1982)). No attempt is made to adjust for quality characteristics of the workers in the occupational categories, or to

[10] This point was made by Bailey, Muth, and Nourse (1963) and Palmquist (1979); see also Hall (1971).

base the index on changes in individuals' wages. Blanchard and Katz (1992), who sought to develop, from Current Population Survey data on individuals, a wage index that controls for industry and worker characteristics, regressed log hourly wages on linear, quadratic, cubic, and quartic experience terms, dummy variables capturing an individual's education level, race, urban-rural residence, and full- or part-time work status, as well as the occupation and industry in which the individual is employed. Although their method shows some improvements over the Bureau of Labor Statistics procedures, they did not use a repeated measures formulation either. When data are collected with a rotation method, sampling at intervals from the same households as with the Current Population Survey, hedonic repeated measures indices would be possible.

II. Perpetual Futures

A. Definition and Settlement Procedures

To create a market for the present value of a cash flow represented by some dividend or rent index, we need to create a perpetual claim on a cash flow represented by the index. A daily dividend index, ideally based on dividends actually paid each day, should be constructed for this purpose, even if there are many zeros in the dividend series, and dividend payments are measured as occurring in bigger, infrequent, lumps. On dates where the daily dividend index is nonzero, shorts in the perpetual claims market pay a quantity of money proportional to the index to longs.

One kind of perpetual claim will be called here perpetual futures. With perpetual futures, on any day t, the next daily resettlement s_{t+1} received by a long from the short in the perpetual futures contract is defined to be:

$$s_{t+1} = (f_{t+1} - f_t) + (d_{t+1} - r_t f_t) \tag{5}$$

where f_t is the perpetual futures price at day t, d_{t+1} is the index, representing dividends actually paid to owners of the underlying asset on day $t + 1$, and r_t is a return on an alternative asset between time t and time $t + 1$. The alternative asset would be any liquid asset, though in practice it would most likely be the return on risk-free short debt, such as an overnight repo rate, debt that is either nominal or indexed to a consumer price index.

We will generally expect that d_t and r_t are both small relative to the price of the asset on which the dividends are paid. The dividend d_t consists only of dividends actually paid on date d_t, a small amount in general. The alternative asset return r_t is a daily interest rate (something like, say, 0.0005). Thus, the daily resettlement will likely be primarily just the change in futures price since the preceding day. Still, the component of the daily resettlement due to the difference between dividend d_{t+1} and $r_t f_t$ is of fundamental importance, and in fact it is this component that ultimately determines the futures price.

The terms on the right hand side of the expression are grouped so that the first term $(f_{t+1} - f_t)$ corresponds to the daily resettlement of a conventional futures contract and the second term $(d_{t+1} - r_t f_t)$ corresponds to the final cash settlement of a conventional futures contract. In a conventional cash-settled futures contract with a fixed maturity, there is every day except for the last day a daily resettlement $f_{t+1} - f_t$ based only on the futures price; this daily resettlement is replaced by a final cash settlement $c_T - \mathrm{f}_{T-1}$ on the maturity date T, where C_T is the final cash price. In a perpetual futures contract, both types of settlement occur every day. In a perpetual futures contract, the term corresponding to the final cash settlement is not $d_{t+1} - f_t$ but d_{t+1} minus something akin to a "permanent" dividend on an asset with price f_t, inferred by multiplying f_t by r_t.

There is also a different interpretation of expression (5). Suppose we took f_t to be the price of an asset that is a perpetual claim, i.e., a contract,[11] promising the buyer of the contract a perpetual stream of dividends (paid by the writer of the perpetual claim) equal to the index $d_{t+k} k = 1, 2, \ldots$ Long-term indexed government bonds, such as the index-linked gilts in the United Kingdom which are linked to the U.K. Retail Price Index, are, although they are finite term, essentially such contracts between the government and the public. Then s_{t+1} would just be the excess return to a one-period investment in one unit of the perpetual claim, the one-period return to a portfolio that is long one perpetual claim and short an equivalent value of the competing asset. Expression (5) is the return a long would obtain if he or she bought the perpetual claim and borrowed the entire purchase price on margin, and minus the return a short would obtain if he or she sold short the perpetual claim with 100 percent margin, where the rate earned on margin accounts is r_t. In practice, longs could not borrow 100 percent on margin and uncovered shorts would be required to hold more than 100 percent margin.[12] I have emphasized here the kind of perpetual claims called here perpetual futures because of the expectation that the contract might best be handled in a futures market without any short sales of securities.

B. Relation between Cash and Futures Price

Since the perpetual futures contract is essentially a perpetual claim purchased with a margin loan, a long can undo this loan by investing an amount f_t in the alternative asset at the time of purchase of the futures contract, and a short can undo his or her side of the loan by shorting the alternative asset. If this is done every period forever, then, so long as the futures price f_t does

[11] This is not a forward contract, not a contract to pay f_t in the future; the distinction between forwards and futures stressed by Cox, Ingersoll, and Ross (1981) does not arise here.

[12] Regulation T provides that 150 percent margin is required for uncovered short sales of securities; the regulation of course does not specify what would constitute adequate cover for one who is short a perpetual claim on an economic index. For some of the contracts considered here such cover might be hard to define.

not diverge in such a way that the present value at t of f_{t+k} does not tend to zero as k goes to infinity, the long may be thought of as receiving (or the short paying) the same dividend payments as the holder of the asset that pays d_t. This suggests that, if there is an observed price p_t on a liquid asset that pays the dividend stream $d_{t+k} k = 1, \ldots$, then the futures price f_t will tend to equal the price p_t.[13]

Note that the argument that the futures price f_t will tend to equal the cash price p_t does not hinge on what alternative asset, whose return r_t is part of the basis for cash settlement, is chosen by the exchange for the contract specification, so long as r_t is the return on a marketable, liquid asset. However, the choice of the alternative asset is not irrelevant, as it affects the cash settlement. A perpetual futures contract is analogous to a swap of the dividend stream d_{t+k}, $k = 0, \ldots$ with the return of another asset, and while traders can undo the swap by other portfolio transactions, the lowest cost thing to do is to just buy or sell the futures contract. For this reason, the usual alternative asset would probably be a repo rate, or other nearly riskless obligation. If riskless indexed short debt is available, then it might be preferable to use this as an alternative asset, since then hedgers would be hedging the real, rather than nominal, value of the asset.

This argument that f_t will tend to equal p_t does not quite have the same force as a no-arbitrage condition. At time t, anticipating any finite holding period of length k, one always faces the risk that the terminal futures price f_{t+k} will not equal the cash price p_{t+k}. This suggests that f_t might track p_t (supposing that the latter is observed) less well than do other futures price track their cash prices.

C. Liquidity and Long Investing Horizons

The perpetual futures contract is designed so that there can be liquidity in a single long-term contract. There would perhaps be no other contracts traded for this asset, no shorter horizon futures contracts, thereby forcing the volume of trade to concentrate on a contract with a long horizon. In contrast, there is a risk that a conventional futures market involving long horizon maturities will wind up having most of the volume of trade and open interest in the short maturities, even if hedgers would, other things equal, prefer to have long horizon contracts. The long horizon futures contract might fail to obtain liquidity, and thus not serve the needs of hedgers. Perpetual futures contracts may tend to attract liquidity, even if there are shorter horizon contracts, since it represents a fixed reference point on which investors' attention might be directed.[14]

[13] Another kind of perpetual futures contract might involve announcing r_t at something other than a market return; if r_t were set above a market interest rate, this might have the effect of shortening the effective maturity of the perpetual futures contract.

[14] Essentially the same advantage was claimed for undated futures by Gehr (1988) and perpetual currency options by Garman (1987).

The futures exchanges have in effect given away the business of long-dated contracts to the over-the-counter markets. This may be because the futures exchanges do not have any means to insure that in market equilibrium it will be the long-term market that is selected to have the high volume of trade. Their policy of scheduling contract months at frequent intervals means that there is no natural single long-term contact on which to base trading; thus trading gravitates to the shortest term contract. It is natural for long horizon customers to do business in the over-the-counter markets: for them there is no liquidity advantage in the long horizon conventional futures markets to offset the extra services and tailoring of the contract to the customer's needs found in the over-the-counter markets.

The clearinghouse mechanism at futures exchanges would naturally facilitate perpetual contracts. In contrast, if a commercial bank that has taken one side of a perpetual swap gets out of the swap by a reversing trade with a third party, it is still locked into two contracts forever. The long-term credit risks of the counterparties is an essential worry for them in these circumstances, discouraging the writing of perpetual swaps.

D. Applications of Perpetual Futures

Regional or occupational category labor cost perpetual futures contracts ought to fill important hedging needs. Firms may be especially interested in hedging the present value of their labor costs and workers, on the other side of futures contracts, interested in hedging the present value of their wages. A futures price that represents such a present value of wages in the future is especially advantageous in the price discovery that it affords. Firms deciding where to locate their operations, and individuals deciding where to move, can use the price discovery afforded by the present value futures price.

Regional commercial real estate perpetual futures contracts may also fill important hedging needs. Ordinary futures contracts cash settled on commercial real estate prices are problematical; cash price indices used to settle futures contracts are unreliable since it is notoriously difficult to infer price changes in commercial real estate from observed transaction prices for the real estate. Transactions of commercial real estate are relatively infrequent. Often the property is altered between sales, other property is transmitted at time of sale, and financial deals (such as seller financing at nonmarket rates) accompany the transfer. For these reasons, published indices of commercial real estate prices are based on appraisals. Unfortunately, appraisals are not unbiased estimates of the price of the property; they are just guesses made by certain parties, who often have an indirect interest in the property. A hedonic repeated measures price index might be constructed from transactions data that takes account of changed characteristics between sales, but doing this requires that characteristics be measured. It may be more advantageous to create a perpetual futures contract based on some measure of rents of commercial property. Rents are observed regularly, and so many of them are paid that it is possible for the constructor of a rental index to be choosy and select only those rental properties for which the rents observed are relatively clean (or to use a hedonic repeated measures index method to correct for unrepresentativeness).

Note that rents on commercial properties tend to be established as parts of contracts that may extend for several years or more. This means that commercial rents are sluggish and do not respond at all quickly to new information, all the more reason why any futures contract settled on rents should be a perpetual futures contract (in contrast to the commercial property rental index futures that was tried at the London Futures and Options Exchange in 1991). Some indices of commercial property rents report on rents of newly negotiated contracts, to try to make their index more forward looking; in doing this they lose sample size, and run the risk that the newly negotiated contracts are somehow systematically different from all contracts. For a perpetual futures contract, even though rents may be set in multiyear contracts, the rental index could be nothing more than the average rent actually in force on all unchanged properties, or, better, it could be derived from data on all rents from a hedonic repeated measures method like that described in the preceding section.

Farmers may find that it is more in their interest to hedge the price of their farms than the price of the next crop. But agricultural land prices also pose problems for constructors of indices, especially as farm sizes contract with time, so that the component of agricultural land sales that is due to the land itself, and not the house situated on the land, is declining through time. Anyway, farm sales occur less frequently than rental payments, and so there are more data on rents. Moreover, it might be possible to create a land rent index from rental or sharecropping contracts in which only the land is rented, and not the house situated on it.

Other commodity cash prices might also in some circumstances be used as the variable d_t in the settlement of a perpetual futures contract. For farms that produce single crops, the cash price of the crop may be a reliable indicator of the rent on the land. The perpetual futures contract may thus better serve the hedging needs of owners of such farms, who with today's contracts have the option only of hedging the next "dividend" rather than their farming operation.

The consumer price index may be considered more nearly a dividend or rent than a price of an asset, and so consumer price index futures might best be handled as a dividend in the perpetual futures contract. There may be little uncertainty about the next few values of the consumer price index, just as there may be little uncertainty about the next few dividends to be paid on corporate stocks, and so a conventional futures market may show little volatility. A perpetual futures contract might be especially useful to people hedging the present value of long-term nominal contracts, such as long-term bonds. And, someone who has purchased a perpetual futures contract based on any other index might, if the perpetual futures contract uses the nominal risk-free rate to settle, want to go short in the consumer price index perpetual futures contract, so as to convert the other perpetual futures contract from a nominal edge into a real hedge.

Perpetual futures might even, in some circumstances, be settled on asset prices rather than dividends or rents. This might be advantageous in cases where the cash market is very illiquid, as with the market for single family homes. If single-family home prices move sluggishly in response to news, then a short horizon futures contract based on an index of selling prices may not reflect the news when it arrives, since the news may not be incorporated into the cash price before the contract matures. Rolling over conventional futures contacts on residential real estate may thus not hedge against such news; subsequent futures prices will already have taken account of such news, so that it is not hedgeable later.

E. Antecedents of Perpetual Futures

Perpetual futures should not be confused with undated futures, such as the undated gold futures traded at the Chinese Gold and Silver Exchange of Hong Kong (see Gehr (1988)). These undated futures are really essentially one-day futures, that are automatically rolled over every day unless the trader opts to discontinue the contract.[15] The daily settlement of the undated futures is the change in the spot price minus an "interest" that was determined the previous day in the futures market. Clearly, the interest is equivalent to a one-day futures price minus the spot price. These undated futures may be viewed as at the other end of a spectrum from perpetual futures, the shortest rather than the longest futures.

Some commodity swaps have features resembling perpetual claims, but they are not perpetual; these tend to have maturities ranging from one month to five years. Commodity swaps have been contracts to provide, in effect, a fixed flow of a commodity (such as oil or gasoline) in exchange for fixed flow of cash payments; in practice the contracts are usually cash settled each period. If the cash price in the contract is capitalized into a present value by a riskless interest rate of matching duration, then that transformed price may be considered a sort of price for the present value of future commodity prices, though only for the duration of the contract.[16]

The closest we have come to perpetual futures are the index participations (IPs) traded May to August 1989 at the American Stock Exchange (AMEX) and the Philadelphia Stock Exchange (PHLX). These IPs were attempts to make stock price indices marketable as a basket, following recommendations made in response to the 1987 stock market crash by the Securities and

[15] The Toronto Stock Exchange 300 Composite Spot Index contract at the Toronto Futures Exchange is also a one-day futures market, but differs from the Chinese Gold and Silver Exchange undated gold futures contract in that the contract has to be reestablished every day. (Trade has virtually disappeared in this market.)

[16] Equity swaps, whose use has been growing rapidly in recent years, have actually generally used the capital gains component as measured using stock price indices, rather than dividend component of equity returns, and hence these do not closely resemble perpetual futures.

Exchange Commission (SEC).[17] Those traded at the AMEX were called Equity Index Participations and those traded at the PHLX were called Cash Index Participations.[18] The short time span that these were traded was defined by regulators and the courts: the IPs began trading when they won SEC approval and stopped trading when the U.S. Court of Appeals in Chicago rules that they were under the jurisdiction of the Commodity Futures Trading Commission rather than the SEC.

The AMEX and PHLX IPs were fundamentally different from other actual and proposed market basket products (the New York Stock Exchange's Exchange Stock Portfolios, the Toronto Stock Exchange Toronto 35 IP Units, the Dresdner Bank's Deutscher Aktienindex IPs and the SuperUnits and the Standard and Poor's Depositary Receipts traded at the AMEX). A person who is long in one of the latter contracts is a beneficial owner of the shares in the underlying index. With the AMEX and PHLX IPs, dividends were paid from the shorts to the longs rather than by the companies comprising the index; no ownership of shares was involved. To the longs, the contract was like an investment in the shares comprising the index: the long paid the IP price and received amounts proportional to the dividends accruing to the stocks comprising the index until he or she closed out the contract; the long needed to put up no margin. The shorts, who were obligated to give to the longs cash payments proportional to the dividend equivalent on the stock price index, had to put up 150 percent margin and see the margin account credited or debited as the IP price changed.

There was an important difference between the IPs at the AMEX and PHLX and the perpetual futures discussed here. The underlying asset for IPs was very liquid and had a price measured by an index; the exchanges added a provision to the contract, a cash-out option for the longs, that enforced some correspondence between the IP price and the index. The option represented another way longs could close out their position (other than selling their IP): they could opt to receive the underlying value proportional to the index, rather than the IP price. When a long exercised this option, there was a random assignment to a short who had to take the other side and pay an amount proportional to the index. This option was created out of a fear that the IP price might not track the stock price index; the option would have the effect of keeping the IP price from falling below the index value.[19] In practice, the specialist assigned to the IPs at the AMEX kept the IP price very close to the index value when these IPs were traded. There was thus no important price discovery afforded by the AMEX and PHLX IP markets; in contrast,

[17] See *The October 1987 Market Break*, U.S. Securities and Exchange Commission (1988), pp. 3–18.

[18] See Kupiec (1990) for a description of these instruments.

[19] Shorts did not have a cash-out option; the exchanges wanted the IPs to be viewed as investment media by longs, and did not want them to be bothered with cash-outs initiated by shorts.

such price discovery might even be described as the raison d'etre for the perpetual futures markets proposed here.

F. Starting Perpetual Futures

The IPs at the AMEX and the PHLX had an advantage over proposed perpetual futures contracts in that there the cash price was very well known. Getting a perpetual futures contract launched where the cash price is not well observed might require some different methods than with conventional futures markets, since there will be great uncertainty about the initial market price. Procedures analogous to those used in the underwriting of initial public offerings, which help the market deal with the inherent uncertainty at the offering date, may be transferred and modified here: publicity programs by underwriters and elicitations of initial interest by underwriters to help them gauge offering price. Participants in the perpetual market may also require conditions limiting amounts of cash settlement in the event of a market failure, since, given that these are in perpetual contracts, participants may have heightened concerns that the market may become illiquid or be closed down.

III. Summary and Conclusion

Both proposals made here are intended as solutions to measurement problems, and so their application must hinge on the nature of the measurement problems encountered.

In deciding how and whether to apply the first proposal, that of hedonic repeated measures indices, one must assess the amount of repeated measures data, the measurability of quality characteristics, and the nature of the measurement problems afforded by the time variation in prices of quality characteristics. There is no point in using the repeated measures method unless there are a large number of repeated measures; ultimately, if measures are infrequent, this means a long enough historical sample period. There is no point in trying take account of a quality characteristic unless this characteristic is both measurable and has a changing impact on asset price or rent through time.

In deciding whether to apply the second proposal, deciding whether to establish a market in perpetual futures rather than conventional futures, one must assess which is easier to measure: the true asset price or the dividend on the asset. If the former, then conventional futures contracts should suffice; if the latter, perpetual futures. Failure to measure true asset price may occur for any of a number of reasons: unrepresentativeness of the assets sold, infrequency of sales, difficulty in measuring what is sold (e.g., financing or service deals with the sale), or illiquidity and inefficiency of the cash market. On the other hand, failure to measure dividend accurately can also occur, as when only an unrepresentative sample of the asset passes through the rental market, or when tax considerations pertaining to the asset are not reflected in rents. Unrepresentativeness of the sample of assets passing through the sale or rental markets could be dealt with using the hedonic repeated measures index number construction method, but only if the characteristics of the assets are available as hedonic variables.

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