Labor Supply Elasticity and Social Security Reform

(Incomplete and very preliminary)

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1 Introduction

It is widely accepted that our current unfunded social security system is in need of reform. Part of the reason is that it may not be an efficient institution to allocate various risks. Previous research argues that partial or complete privatization might enhance the welfare of the individuals. Hubbard and Judd (1987) evaluate the annuity insurance role of social security and İmrohoroğlu, İmrohoroğlu, and Joines (1995) add the insurance for idiosyncratic income risk to unfunded social security, both with an overall negative finding for the unfunded system. İmrohoroğlu, İmrohoroğlu, and Joines (2003) allow unfunded social security to act as a commitment device for individuals with time inconsistent preferences but privatization is still beneficial. With the projected aging of the population, reforming the system is even more urgent. For example, De Nardi, İmrohoroğlu, and Sargent (1999) argue that the projected increase in the dependency ratio and health costs may require an increase of nearly thirty percentage points in the payroll tax if the benefits were to be sustained at current levels.
Then, what are the road blocks to reform? A major issue is the transitional costs to currently alive individuals in the economy.\textsuperscript{1}

Therefore, the extent to which formal or informal insurance arrangements exist and the strength of response in labor supply to reductions in distortions appear to be critical to supporting reform. In this paper, we will extensively analyze the role of labor supply in providing for support for reform and in delivering efficiency gains from reform. In particular, we consider three period utility functions (within the class of preferences consistent with balanced growth) that provide the individuals with differing willingness to substitute labor for leisure over the life cycle and different estimates of the Frisch elasticity of labor supply, ranging from the early estimates of about 0.3 to the more recent estimate by Inai and Keane (2004) of more than 3. For each case, we will analyze i) the macroeconomic effects of social security reform, ii) the effects on the distribution of hours over the life cycle, iii) the fraction of individuals who support the reform, and, iv) overall efficiency gains from reform. In all cases, we compute the transition path to the reformed state, taking into account the projected aging of the population and the expected increases in public expenditures on health services.

Our preliminary experiments indicate that the macro effects of reform toward a funded system are much less sensitive to the micro elasticity of labor supply. However, we find a significant change in the age-hours profile over the life cycle. In particular, after privatization, individuals work much less when young, but spend a more significant fraction of time on market activities during older ages. We conjecture that the different responses across different age groups are mainly driven by the two factors: (1) changes in the intratem-

\textsuperscript{1}Political economy arguments might also play a role to sustain an unfunded system, as argued by Cooley and Soares (1999) and Boldrin and Rustichini (2000), Huang, İmrohorğlu, and Sargent (1997) study social security reform in a stochastic environment, taking into account the transition to the reformed steady-state of the economy. De Nardi et al. (1999) show how different transition policies with similar long run effects generate very different transition effects on currently alive individuals. Kotlikoff, Smetters, and Walliser (1999) also compute the transition path and its impact on exogenously given income groups. Conesa and Krueger (1999) emphasize the transitional effects of reduced consumption and increased labor supply and find that the majority of the individuals would be against the reform. Fuster, İmrohorğlu, and İmrohorğlu (2007) emphasize the role of family insurance and labor supply response in overcoming the transitional costs. Finally, Nishiyama and Smetters (2007) argue that partial (50\%) privatization of social security is likely to bring an overall efficiency loss in an economy populated with individuals facing uninsurable income and longevity risks.
poral margin over the life cycle, and, (2) the intertemporal effect caused by a decline in the interest rate. These factors contribute to even larger differences in labor responses across age groups when we assume a greater degree of micro elasticity. These will induce the tradeoff between income and substitution effects to work differently across age groups. A reform towards a funded system will raise private saving for retirement and reduce the interest rate, which flattens the life-cycle consumption profile. As a result, on one hand, a higher consumption while young will reduce the incentive for work, and this offsets the increase due to the higher wage and the lower payroll taxes. The old, on the other hand, will raise their saving for retirement while reducing their consumption, and the income effect affects the labor supply in the same direction as the substitution effect. Understanding the net effect of these mutually interacting responses calls for a quantitative analysis using a general equilibrium model of overlapping generations calibrated to match the key features of the U.S. economy. Given the different responses of different generations, it is critical for a model to capture the current demographic structure with baby boomers in their 50s in order to assess the aggregate effects. We provide some tables and graphs in the “Preliminary findings” section to highlight our initial computations.

2 Model

2.1 Demographics

At each date \( t = 0, 1, \ldots \) the economy is populated by overlapping generations of individuals of age \( j = 1, 2, \ldots, J \). They face random duration of life that does not exceed the maximum of \( J \) years and work until the mandatory retirement age of \( jR \). At time \( t \), agents of age \( j \) face a conditional probability of \( s_{j,t} \) to survive until the next period. \( s_{j,t} = 0 \) for any \( t \). The unconditional probability that an individual born at \( t + 1 \) survives until age \( j \) is denoted by \( S_{j,t+j} = \prod_{k=1}^{j-1} s_{k,t+k} \). The new cohort at time \( t \) is larger than that at \( t - 1 \) by a fraction \( n_t \).

2.2 Technology

There is a representative firm that runs a constant returns to scale technology of the form \( Y_t = F(A_t, K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha} \), where \( K_t \) and \( L_t \) are
aggregate capital and labor inputs at time $t$ and $\alpha$ is the capital share. $A_t$ is an exogenous labor-augmenting technological progress that grows at a constant rate $g$. Capital depreciates at a constant rate $\delta \in (0, 1)$. The firm rents capital and hires labor from the individuals in competitive markets, where factor prices $r_t$ and $w_t$ are equated to the marginal productivities.

2.3 Social security

Until the social security reform is implemented, the government operates a pay-as-you-go pension system similar to the current U.S. system. Young individuals pay a proportional tax $\tau^p_t$ on their labor income up to the maximum amount of $\bar{y}^p$. Each retired agent receives the benefit $p$, a concave function of an individual’s average lifetime earnings that captures the progressivity of the system in the U.S.

Our model begins at time 0, when the government announces a reform of the social security system. Different reform proposals will be considered, including maintaining the unfunded system while financing the added burden due to aging by an increase in the payroll tax, a reduction in retirement benefits, an increase in the retirement age, and a transition of a fully-funded system (privatization).

2.4 Households

Agents derive utility from their life-time consumption and leisure and from leaving bequests. The individuals are heterogeneous in five dimensions summarized by a state vector $x = \{j, a, \eta, z, e\}$, i.e. age $j$, assets accumulated in the previous period $a$, idiosyncratic labor productivity $\eta$, a fixed ability type $z$ that determines the agent’s lifetime labor earning ability and the vector of conditional survival probabilities, and the cumulated labor earnings $e$ that determines the retirement benefit.

Agents possess “warm glow” altruism and derive utility $\phi(a')$ by leaving a bequest of the amount $a'$. Bequests are collected and distributed as a lump-sum transfer to the entire population.

We compute the households’ problem recursively. We denote by $V_t$ a value function of an individual in period $t$.

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*See, for example, De Nardi et al. (1999) and Attanasio et al. (2007), that study the effect of various social security reforms in different models.*
\[ V_t^s(j, a, \eta, z, e) = \max_{c, \ell, a'} \{ u(c, \ell) + \beta s_{j,z,t} E[V_{t+1}^s(j+1, a', \eta', z, e')] \} + (1 - s_{j,z,t}) \phi(a') \]

subject to

\[ c + a' = (1 + r_t)(a + b_t) + w_t \varepsilon_j(z) \eta \ell + p_t(x) - \Upsilon_{j,t}, \]
\[ a' \geq 0, \]
\[ e' = [(j - 1)e + w_t \varepsilon_j(z) \eta \ell] / j, \text{ for } j < j_R, \]
\[ e' = e, \text{ for } j \leq j_R, \]

where \( c \) and \( \ell \) are consumption and fraction of hours worked, respectively, \( \beta \) is the subjective discount factor, and \( \Upsilon_{j,t} \) denotes the taxes paid by an age-\( j \) individual at time \( t \).

### 2.5 Fiscal policy

Besides the social security tax, the government can raise revenues from taxing on labor income, capital income and consumption at proportional rates denoted by \( \tau_\ell^t, \tau_a^t \) and \( \tau_c^t \) and issue one-period riskless debt \( D_t \). The government borrowing and the tax revenues finance the payment of pensions for the retired, an exogenously given level of public expenditures \( G_t \) and repayment of the debt plus interest. The labor income tax \( \tau_\ell^t \) is set so that the following government budget constraint is satisfied every period.

\[
G_t + (1 + r_t)D_{t-1} + \sum_x p_t(x)\mu_t(x) = \sum_x \left[ \tau_\ell^t w_t \varepsilon_j(z) \eta \ell_t(x) + \tau_a^t \min\{w_t \varepsilon_j(z) \eta \ell_t(x), \bar{y}^p \} + \tau_c^t r_t a_t(x) + b_t \right] + \tau_c^t c_t(x) \mu_t(x) + D_t
\]

(1)

where \( \mu_t(x) \) denotes the measure of individuals in state \( x \) in period \( t \).

### 2.6 Equilibrium

A competitive equilibrium, for a given sequence of demographic variables \( \{\{s_{j,z,t}\}_{j=1}^J n_t\}_{t=0}^\infty \) and a set of multiple sequences of fiscal variables \( \{G_t, D_t, \tau_\ell^t, \tau_a^t\}_{t=0}^\infty \), is households’ decision rules \( \{\{c_{j,t}, \ell_{j,t}, a_{j,t}\}_{j=1}^J \}_{t=0}^\infty \), factor
prices \( \{r_t, w_t\}_{t=0}^\infty \), labor income tax \( \{\tau^f_t\}_{t=0}^\infty \), a lump-sum transfer of accidental bequests \( \{b_t\}_{t=0}^\infty \) and the measure of individuals \( \{\mu_t(x)\}_{t=0}^\infty \) for each state \( x \) that satisfy the following conditions. Note that we have multiple sequences of the equilibrium objects for each possible timing of the reform implementation.

1. Households’ allocation rules solve their recursive optimization problems defined in section 2.4.
2. Factor prices are determined competitively, i.e. \( r_t = F_K(A_t, K_t, L_t) - \delta \) and \( w_t = F_L(A_t, K_t, L_t) \).
3. The labor and capital markets clear.
   \[
   L_t = \sum_x \varepsilon_j(z)\eta^\ell_t(x)\mu_t(x), \quad (2)
   \]
   \[
   K_t = \sum_x r_t a_t(x)\mu_t(x) - D_{t-1}. \quad (3)
   \]
4. The labor tax satisfies the government budget constraint defined in equation (1).
5. The goods market clears.
   \[
   \sum_x c_t(x)\mu_t(x) + K_{t+1} + G_t = Y_t + (1 - \delta)K_t. \quad (4)
   \]
6. The lump-sum bequest transfer is equal to the amount of assets left by the deceased.
   \[
   b_t \sum_x \mu_t(x) = \sum_x a_t(x)(1 - s_{j-1, z, t-1})\mu_{t-1}(x). \quad (5)
   \]

3 Calibration

3.1 Time-line and initial conditions

The time period is annual. The model begins at time 0, where it represents some economic and demographic conditions of the U.S. economy around 2008. Most importantly, we need to define the initial distribution of individuals over the five possible states \( x = (j, a, \eta, z, e) \). For the initial wealth distribution,
we use the Survey of Consumer Finance (SCF) data to calibrate the assets
a held by households across age j and education levels z.\(^3\) The ability type
z is captured by individuals’ educational attainment and we let the initial
distribution match the Census data. For the distribution over the idiosyn-
cratic labor productivity η and average lifetime labor earnings e, we use the
distribution conditional on the other three state variables (j, a, z) that we
obtain in what we call the “initial benchmark economy,” that is, a stationary
(steady state) equilibrium implied by the economic and demographic condi-
tions around 2008. We also use this economy in order to calibrate some basic
parameters of the model.\(^4\)

3.2 Demographics

We assume that individuals enter the economy at age 20 and live up to the
maximum of 100 years old. We calibrate age-specific surviving probabili-
ties that also depend on ability type using the study by Bhattacharya and
Lakdawalla (2006). We then combine the results with the projection of the
surviving probabilities from the SSA (Bell and Miller, 2005) to obtain the
future path of age and type specific surviving rates. We assume that both ed-
ucational types will equally experience the improvement in mortality, which
is consistent with the SSA’s projections on average, while the “ability pre-
mium” measured as the ratio of the surviving rate of the high type to that
of the low type at each age will be maintained.

For the growth rate \(n_t\) of the new entrants to the economy, we set it to

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\(^3\)We define assets as the net worth of households, including the value of households’
financial and non-financial assets of all kinds net of their liabilities and follow the definition
of Bucks, Kennickell, and Moore (2006) . Financial assets include transaction accounts,
CDs, bonds, stocks, investment funds, retirement accounts (IRA/Keogh accounts and cer-
tain employer-sponsored accounts including 401(k), 403(b) and thrift saving accounts from
current or past jobs), cash value life insurance and other managed assets. Nonfinancial
assets include vehicles, residential property and business equity. Liabilities include housing
depts, other lines of credit, installment loans and credit card balances.

We use the definition of “households” in SCF and the associated household “head.”
SCF defines a “primary economic unit” (PEU) which includes the economically dominant
single or couple and all other dependents in the household, and designates a head as the
male in a mixed-sex couple.

\(^4\)Note: We use the initial benchmark economy to calibrate preference parameters (\(\alpha, \beta, B\))
and per-capita government expenditures. We assume that per-capita government
spending is constant over time, while the aggregate expenditure \(G_t\) is time-variant reflect-
ing the changes in population.
1.2% in the initial period, which is the average population growth rate of 1950-2000. According to the SSA, the old dependency ratio defined as the ratio of the population aged 65 and over to that between 20 and 64 will reach 43% by 2080. The growth rate of 0.3% will match the ratio under the conditional survival probabilities at the final steady state. We assume that \( n_t \) will gradually converge to this rate over the next 50 years.

3.3 Preferences, technology and labor productivity

Preference: We assume the instantaneous utility function from consumption and labor takes one of the following three forms

\[
\begin{align*}
    u(c, \ell) &= \frac{[\ell^\nu(1-\ell)^{1-\nu}]^{1-\sigma}}{1-\sigma}, \\
    u(c, \ell) &= \log(c) + A_1(1-\ell)^{1+\gamma}/(1+\gamma), \\
    u(c, \ell) &= \log(c) - A_2\ell^{1+\gamma}/(1+\gamma),
\end{align*}
\]

where \( A_1 \) and \( A_2 \) represent the intensity of leisure and work, respectively, in the two separable period utility functions above. When preferences are given by (8), the Frisch elasticity is constant over the life cycle and is given by \( \gamma \). In (7) and (6), it varies over the life cycle and is a function of consumption and leisure at specific ages.

The utility from leaving bequests \( a' \) takes the form \( \phi(a') = B_1a'^{\nu(1-\sigma)}/(1-\sigma) \) for the case of (6) and \( \phi(a') = B_2\log(a') \) for (8) and (7), where the parameter \( B_1 \) and \( B_2 \) reflect the intensity of bequest motives. We calibrate this parameter so that the fraction of the wealth held by the elderly matches the data in the initial benchmark economy.

The subjective discount factor \( \beta \) is calibrated to match the capital-output ratio is 3 in the initial benchmark economy.

Technology: We assume that the TFP grows at an exogenous rate of \( g = 1.65\% \), close to the average in 1950-2000 and the income share of capital \( \alpha \) is set at 0.33. The depreciation rate \( \delta \) is 0.055 = \( \frac{X/Y}{K/Y} - g - n - gn \) implied by the equilibrium law of motion for the capital in the steady state, where we target an investment-output ratio \( X/Y \) of 25% and capital-output ratio of 3.
**Labor productivity:** We match the profiles of labor efficiency units of college and non-college graduate males, which constitute the two states for the ability $z$, high “$H$” and low “$L$”. We use the data on age and education specific earnings from Census Bureau in 2006 to calibrate $\varepsilon_j(z)$.\(^5\) For the idiosyncratic component, we assume it follows an AR(1) process $u_t(z) = \rho(z)u_{t-1}(z) + \epsilon_t(z)$, where $\eta_t = \exp^{\eta_t(z)}$. We use the estimates of Guvenen (2005), $\rho_H = 0.805$ and $\sigma^2_H = 0.025$ for college graduates and $\rho_L = 0.829$ and $\sigma^2_L = 0.022$ for non-college graduates. We approximate the process of the idiosyncratic component by a Markov chain with two states. The transition matrices are

\[
\begin{bmatrix}
0.9126 & 0.0874 \\
0.0874 & 0.9126
\end{bmatrix}
\text{ for college and}
\begin{bmatrix}
0.9309 & 0.0691 \\
0.0691 & 0.9309
\end{bmatrix}
\text{ for non-college.}
\]

The grid for $u$ is $[-0.2665, 0.2665]$ for college and $[-0.2652, 0.2652]$ for non-college. We set the proportion of type $H$ among the new entrants at 0.30, the average among males at the working age in 2006.

### 3.4 Social security and fiscal policy

Until the reform is implemented, the government continues to run a pay-as-you-go social security system. We set the social security tax rate $\tau_p$ at 10.6\% with the maximum taxable amount of $97,500 as it is in the U.S. in 2007. The benefit is a concave piecewise linear function of the average lifetime earnings (“AIME”). The marginal replacement rate is 90\% for the average earnings up to 20\% of the economy’s average earnings, above which the replacement rate falls to 32\%. For income between 123\% and 202\% of the economy’s average, the replacement rate is 15\%. Additional income above 202\% of the economy’s average does not provide additional pension benefit.\(^6\)

We adjust the benefit schedule proportionally by a factor $\varphi$ so that we achieve an average replacement rate of 45\%.

In the initial benchmark economy, we set the government spending $G$ at 20\% of output, which is the average ratio of government consumption expenditures and investment to GDP in the post-war period. We keep the per-capita expenditures constant across time. The ratio of federal debt held

\(^5\)http://www.census.gov/prod/www/abs/income.html

\(^6\)Based on the formula of Primary Insurance Amount with bend points of $627$ and $3,779$, the maximum monthly benefit of $1,939$ and national average wage index of $36,952$, all in 2005.
by the public to GDP is fixed at 40%, which is the value at the end of 2006. We assume a consumption tax rate of 5% based on Mendoza et al. (1994) and a capital income tax rate of 30%. The labor income tax is set so that the government budget constraint is satisfied.

4 Preliminary findings and next steps

In order to first understand the effect of a reform on the life-cycle profile of labor responses, we first compare two steady states, one implied by the current pay-as-you-go social security system and the other implied by a fully-funded system, while keeping the demographic structures identical.

Table 1 summarizes the aggregate effects of social security reform when we employ the preference of the form \( u(c, \ell) = \log(c) - \frac{A_2 \ell^{1+\gamma}}{(1+\gamma)} \), where \( \gamma \) is the Frisch (or constant marginal utility of wealth) labor-supply elasticity. While the value of the elasticity parameter \( \gamma \) affects the magnitude of the response in the aggregate capital and prices, the effects on the aggregate labor supply and the average work hours are surprisingly small. Hidden, however, behind the relatively small effects are large effects on the distribution of work hours over the life cycle, as shown in Figures 1, 2 and 3.

Given different labor supply responses of different age groups, the aggregate effects of social security reform are likely to be sensitive to the dramatic changes in the demographic structure projected over the next few decades. The next step in our computations will be the transition from the current system toward a partially or completely privatized system while incorporating the projected demographic transition and expected increases in public expenditures on health services, and provide a catalogue of macro responses, welfare effects, and efficiency gains, for different values of labor supply elasticities.
Table 1: Aggregate responses of a social security reform

<table>
<thead>
<tr>
<th>Frisch elasticity $\nu$</th>
<th>0.33</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate output</td>
<td>+7.76%</td>
<td>+6.96%</td>
<td>+5.93%</td>
</tr>
<tr>
<td>Aggregate capital</td>
<td>+25.33%</td>
<td>+22.46%</td>
<td>+18.25%</td>
</tr>
<tr>
<td>Aggregate labor</td>
<td>+0.03%</td>
<td>+0.08%</td>
<td>+0.34%</td>
</tr>
<tr>
<td>Average work hours</td>
<td>−0.54%</td>
<td>−0.90%</td>
<td>−0.78%</td>
</tr>
<tr>
<td>Interest rate (in %-pts)</td>
<td>−1.68%</td>
<td>−1.52%</td>
<td>−1.25%</td>
</tr>
<tr>
<td>Wage rate</td>
<td>+7.73%</td>
<td>+6.87%</td>
<td>+5.57%</td>
</tr>
</tbody>
</table>

Figure 1: Labor supply response over the lifecycle with Frisch elasticity $\gamma = 0.33$
Figure 2: Labor supply response over the lifecycle with Frisch elasticity $\gamma = 1$

Figure 3: Labor supply response over the lifecycle with Frisch elasticity $\gamma = 3$
Figure 4: Consumption over the lifecycle with Frisch elasticity $\gamma = 1$

References


