Abstract

This paper studies parental allocation of schooling resources among children when family members engage in household production. It shows that the assignment of persons to managerial versus production tasks may result in a complementary effect: the education of the manager not only enhances allocative decisions but also filters through the team work to raise the productivity of other family workers. Under joint production, parents choose to concentrate schooling investment on the manager resulting in a less equal distribution of education among children and to have a larger family size, as compared with labor market participation. The paper shows that the model is consistent with observed family behaviors in developing countries and uses it to analyze gender education gap, birth-order effects, the relationship between household schooling and production efficiency, and the impact of technical change on educational inequality.

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Household Production and Parental Investment in Children

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1 Introduction

Parents play a prominent role in the determination of family size and the allocation of human capital investments in children. These joint decisions of parents have important implications for the welfare of their children, the distribution of individual earnings, the distribution of household income, and the intergenerational transmission of inequality. Among the most important contributions to our understanding of intrafamily allocation of human capital, in particular schooling, are the seminal works of Becker and Tomes (1976), Behrman, Pollark and Taubman (1982), and Becker (1991). Together these works have laid out the analytical foundation for studying parental investment in children.

One of the basic properties underlying the standard framework is that returns to children's schooling is determined by competitive labor markets. With complete and competitive markets, all children face an exgoenously given common schooling-wage schedule, and the returns to individual schooling within a family is independent of decisions on family size and intrafamily allocations of schooling. Combined with parental preference and child endowments, these market outcomes of schooling investment detemine both the level and the distribution of educational expenditures on children (see Behrman, 1997). By now, this framework has been applied extensively to analyze family behaviors in industrialized economies characterized with competitive labor markets.

While very insightful, the standard model of intrafamily allocations faces a difficulty when it is applied to analyzing family behaviors in developing countries. The difficulty lies in the fact that household production, rather than labor market participation, is the main form of organization in many developing countries. Indeed, most people in developing countries earn at least part of their livelihood through work in their own enterprises, including family farming, petty manufacturing and small-scale trading. As a result, some conditions that are central to the determination of optimal family choices under a labor market environment are no longer valid. For instance, under household production, the returns to a child's education is no longer independent of the schooling of other children, and individual schooling returns are also affected by family fertility choices and intrahousehold allocations of schooling. Because of these complications arising from household production, one cannot readily apply the standard results to explain fertility and intrafamily allocation decisions for a large proportion of families in developing countries.

In this paper we develop a model of intrafamily schooling allocations in which family members engage in household production. Hence, the purpose of the paper is to extend the standard framework to analyze family behavior in the developing world. More specifically, our interests lie primarily in two aspects of family choices: the number of children and intrahousehold allocations of educational resources among children. We will show that in comparison with the standard results, family who engage in household production have incentives to choose a less equal distribution of schooling among their children and a larger family size.

The main intuition behind these results can be described briefly. A family that engages in household production needs to make many business decisions. The main contribution of children's schoolings is to enhance the quality of these decisions and therefore increase the efficiency of production. There are two types of decisions: major allocative decisions such as what to produce and whether to adopt a new-seed variety, and task-by-task operational decisions such as water and fertilizer applications. While schooling may improve the quality of both type of decisions, it is optimal for families to allocate more schooling resources to managers, the children who make major allocative decisions, because these decisions generally have long lasting effects on production, and mistakes in these decisions are much more difficult to be overcome by better operational decisions.

Concentration of schooling investment on managers also imply less schooling investment in children who are workers. As a consequence, the marginal cost of these children is low, which gives families incentives to have more children.

Our model of family investments in children under household production have many interesting implications about family behavior. Among others, we will offer explainations from the model for differences in fertility and schooling investment across farm and urban households, the effects of technological change and labor mobility on schooling inequality, gender education gap, birth-order effects on schooling investment, and other puzzles of family behavior in developing countries.

This paper has antecedents in two lines of related research: one on the interactions between child quantity and quality (see Becker and Lewis, 1973; Willis, 1973), and another on intrahousehold human capital investments (see Becker and Tomes, 1976; Behrman et al., 1982). The first set of models take intrahousehold resource allocations as given and emphasize the effects of parental income and child-rearing costs on fertility and child quality. The second set of models take the number of children as given and explore mainly how parental preferences and child endowments are related to the allocation of educational expenditures and bequests among siblings.

We depart from these studies by analyzing the joint decisions of fertility and intrahousehold educational investments in the context of household production. In view of existing theories, we endogenize fertility in a model of intrahousehold resource allocations, or, from a parallel perspective, we consider intrahousehold resource allocations in a framework of quantity and quality interactions. Thus, to some extent, this paper integrates the literature on the determination of the number of children with that of the distribution of resources among children.

The rest of the paper proceeds as follows. Section 2 presents a prototype model of parental choices and analyze the optimal parental choices when children pariticapate in a competitive labor market. Section 3 formulates the household production and managerial technology and derives family optimal choices on family size and allocations of schooling expenditures among children. Section 4 presents several extensions to the basic household production model. Section 5 applies the model to explain a variety of observations in developing countries. Section 6 contains concluding remarks.

2 A Prototype Model of Parental Choices

We start with a prototype model that captures some essential features of the standard theory on intrafamily allocations (e.g. Becker and Tomes, 1976; Behrman, Pollark, and Taubman, 1982; Behrman, 1997). This model will serve as a basis for further analysis in three ways: (1) by defining a common set of parental preferences, constraints, and decisions; (2) by explicating the structure of schooling returns for children under the assumption of competitive labor markets and its implications for intrafamily resource allocations; and (3) by solving quantitatively the solutions for parental choices concerning the number of children, the quality of children, and intrahousehold allocations of educational resources. Incorporating household production into this basic framework and deriving the implications for family behaviors will occupy the remainder of this paper.

Consider a model of intrahousehold allocations with consensus parental preferences in which parents act as if they maximize a single utility function subjective to appropriate constraints. The parents make decisions concerning the number of children (n) and the allocation of their income (I) between own consumption (c) and schooling investment in their children (s_i, which is the schooling level for the ith child).[1] They are altruistic in the sense their utility depends on the level and

[1] Explain the relationship between schooling expenditure and attainment. This paper treats them as the same.

distribution of their children's income. The children are passive: they receive schooling investment from parents and make earnings (y_i) when they become adults, but they do not play a role in the determination of parental expenditures on schooling. In this one-period model, we abstract from timing considerations, such as the birth order of children.

The parents' concensus utility function is

$$U = \ln(c) + \eta \ln(n) + \beta \frac{1}{n}[\ln(y_1) + ... + \ln(y_n)], \tag{1}$$

where the utility weight for the number of children η and the altruism parameter β are both positive. The preferences exhibit equal concern as all children have the same weight in the parents' utility function so that the utility function is symmetric in the space of children's income. Since $\ln(\cdot)$ is a concave function, the preferences also exhibit parental aversion to inequality.

The cost of raising children is divided into a quantity component (Iv) representing living expenses such as food and housing costs and a quality component ($I\mu$) representing the unit cost of schooling, where v and μ are both positive. For a family of n children with schooling attainments s_1, ..., s_n, the parents' budget constraint is

$$c = I(1 - vn - \mu s_1 - ... - \mu s_n), \tag{2}$$

where we assume that the costs of child rearing and schooling are proportional to the parents's income, which captures the emphasis in the fertility literature on the value of parent's time for investing in children. Along with logarithmic utility function, the assumption implies that the parent's decisions on the number of children and schooling investment are independent of income.

Abstracting from income effect helps us to focus the analysis on the role of household production in affecting family choices.

In standard models of intrafamily allocations, the optimal schooling investment in children depends crucially on labor market returns to education. An earnings production function states the relationship between the adult earning of a child and schooling investment in that child. To provide a specific context, consider a market environment with factory-based production in which workers consist of adult children from different families and competitive forces coordinate the assignment of workers to firms. The technology for a representative factory is characterized by a constant-returns-to-scale production function:

$$Y_m = A_m \sum_{j=1}^{N} s_j^{\alpha} L_j, \tag{3}$$

where s_j $(j = 1, ..., N)$ represents a specific level of schooling and, for each j, L_j is the total number of workers with schooling level s_j, and $0 < \alpha < 1$ is a parameter of concave returns to education. In this specification, schooling represents general skills and it augments labor, so that the term $s_j L_j$ can be interpreted as efficiency units of labor. We also assume that the factory output Y_m is affected by a productivity parameter A_m, where the subscript m refers to market production as opposed to home production, which will be analyzed later.

With a competitive labor market, a worker with schooling s would receive a wage rate that equals their marginal product of labor[2]

$$w_m(s) = A_m s^\alpha. \tag{4}$$

Thus all children, both within and across families, would face this earnings function in the labor market, and parents would draw information on schooling returns from this earnings schedule when they make investment decisions in their children. Since the earnings function is given exogenously to individual families, it implies that the schooling returns to a child in a family is independent of the schooling returns to other children in the family. This property is a direct outcome of labor market participation. Under the condition that adult children receive individual wages, it is natural to assume that siblings do not share incomes with each other. Presumably high costs of parental enforcement in income sharing provides partial justifications to this assumption. As a consequence, the income of each child is their market wage rate, *i.e.,* $y_i = A_m s_i^\alpha$ for child i with schooling s_i.

The parental allocation problem outlined in equations (1)-(4), while sharing many characteristics of existing models, has an unique feature. It treats the number of children as a parental choice, a feature that differentiates our formulation from standard models of intrahousehold allocation. Indeed, earlier models focusing on differences among children within a family have taken family size as given.[3] Our approach reflects the belief that the number of children and schooling allocations

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[2]This is the simplest specification of an earnings function that relates one's schooling to income. We choose this simple form to facilitate close comparisons with the structure of schooling returns associated with household production, which will appear in later analysis. Behrman, Pollak and Taubman (1982) also use an earnings function like this, but they allow for child's genetic endowment as another input. Alternatively, we could specify the efficiency unit of labor as $e^{\alpha s_j} L_j$ and $0 < \alpha < 1$, which would lead to an earnings function $\ln(w_m) = \ln A_m + \alpha s$, the familiar Mincer-type formulation.

[3]As Behrman (1997) puts it: "Earlier models considered the determination of the number of children and of average child "quality" together with parental consumption, but not differences among children within a family (see references in Becker (1991); and also see Willis (1973)). The literature concerning, respectively, the determination of the number of children and the distribution of resources among children have not been integrated."

among children are intimately linked. They are joint parental decisions. We will analyze the interactions of these parental choices when we incorporate household production into this basic model.

In comparison with standard models, we have made two simplifying assumptions in the preceding framework. First, to concentrate on schooling investment, we do not model direct transfers from parents to children as a source of children's income. This is a modeling strategy also adopted by other researchers (e.g. Sheshinski and Weiss (1982)). Moreover, empirical evidence supports the view that parental investments in the human resources of children are far more common than transfers of assets (see Becker, 1991). Second, we have assumed implicitly that all children have identical endowments, which include various genetically inherited characteristics that may affect individuals' earning capacity in labor markets. Closely related to this assumption are the specifications in the budget constraint that individual costs of schooling and child rearing are the same for all children. If children have different endowed attributes, their unit costs of education are likely to be different. The main reason behind these assumptions is the fact that systematic analysis on fertility choices conditional on *ex post* realizations of child endowments would require a drastically different framework, which unfortunately will divert the exposition away from the focus of the paper. One important objective of the paper is to show that family behaviors differ significantly under alternative arrangements, labor market participation versus home production, even if children are bestowed with identical endowments. We will leave the generalization of our results to unequal child endowments for future research.

Parental optimal decisions, solutions that maximize utility (1) subjective to the constraints (2) and (4), are solved in two steps. First, following the standard practice of intrahousehold allocation models, we hold the number of children n fixed and derive parental choices for schooling investment.

Then, we solve the joint parental decisions for the number of children and schooling investment in children. In doing so, we will analyze the solutions at each stage, comparing the results of our model with that of the standard model, and more importantly, setting a basis for comparing parental decisions under the alternative production arrangements: labor market participation versus household production.

With equal costs of education, symmetric preferences and a common earnings function for all children, the parents would make the same level of investment in each child. Let s_m denote that level of schooling attainment for children who participate in competitive labor markets, the parents' optimization problem becomes

$$\max_{c_m, s_m} \{\ln(c_m) + \beta \ln(A_m s_m^{\alpha})\}$$

subject to

$$c_m = I(1 - vn - \mu s_m n), \tag{5}$$

which has the following solution for educational investment:

$$s_m(n) = \frac{\alpha\beta(1 - \nu n)}{(1 + \alpha\beta)\mu n}. \tag{6}$$

The result confirms the intuition that the quality of children is positively related to the rate of return to education (α) and the degree of parental altruism (β), but negatively related to the cost of education (μ). With a given budget, the quality of children is also negatively related to the number of children as well as the basic living expenses of raising a child (ν).

In the second step, we solve the parental choice on the number of children conditional on schooling investment in (6), which is equivalent to treating these two variables as joint family decisions. The optimization problem is simply

$$\max_{n_m}\{\ln(c_m) + \eta \ln(n_m) + \beta \ln(A_m s_m^{\alpha}(n_m))\}$$

subject to

$$c_m = I(1 - vn_m - \mu s_m(n_m)n_m),$$

where $s_m(\cdot)$ is given by (6). Note that the parents' investment in the quality (s_m) and quantity (n_m) of children enters the budget constraint in a multiplicative form, which represents the quantity-quality trade-off as in Becker and Becker and Tomes (1976). It can be seen in the first order conditions, this interaction implies that the price of education is proportional to the number of children and the price of the number of children is proportional to the parent's educational investment in each child.

The optimal solution for the number of children is

$$n_m^* = \frac{\eta - \alpha\beta}{v(1+\eta)}. \tag{7}$$

Substituting (7) into (6) gives the level of schooling attainment

$$s_m^* = s_m(n_m^*) = \frac{v\alpha\beta}{\mu(\eta - \alpha\beta)}. \tag{8}$$

With competitive markets, these quantitative solutions for family size and allocations of schooling to children will serve as benchmark values for parental choices. We will show that if instead family members engage in household production, the parental optimal decisions will differ systematically.

3 A Model with Household Production

Family enterprise is a dominant form of economic organization in developing countries. The majority of households in agriculture, and a large-proportion of households in the non-agricultural sector, earn their livelihood through work in their own enterprise.[4] These families form a coherent production unit, make managerial decisions, and use household labor as an important input into the production process of the enterprises.[5]

Based on these institutional arrangements in developing economies, we shall consider a family decision framework in which household production plays a critical role. With household production, children from the same family no longer face the common earnings function in equation (4), which is given exogenously by the labor market. Instead they engage in family-based joint production that determines endogenously the return to schooling of each child. In this new framework, we shall maintain all other properties of the preceding prototype model. After specifying a production and managerial decision function, we shall proceed to solve for parental optimal decisions concerning the number of children and intrahousehold allocations of educational resources.

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[4]Based on the 1970 World Census of Agriculture covering 46 countries, 79 percent of all farms and 61 percent of total farmland are operated by agricultural households. Owner cultivation is also common in advanced industrial economies such as Europe and North America (Otsuka et al. 1992).

[5]Factors contributing to the dominance of family enterprise include moral hazard problems associated with hired labor and various transaction costs associated with information, monitoring, coordination, and enforcement of contracts (see e.g., Osuka et al., 1992 and de Janvry et al., 1991).

3.1 Production and Managerial Technology

Consider that household production involves two separate yet interrelated activities: (i) making managerial decisions, and (ii) execution of production tasks. The former may include major long-run business decisions, such as what to produce and the adoption of new-seed varieties in agriculture; it may also refer to day-to-day operations, such as strategies for a specific product sale, water control, and fertilizer and chemical applications. Examples of routine tasks may include spinning a mill in a food processing operation or watering and weeding in a farming project. We assume that schooling may enhance the quality of allocative decisions (see Welch, 1970; T.W. Schultz, 1975), but play a limited role in affecting the performance of routine tasks.[6] The two activities are closely related because better managerial decisions raise overall output through their interactions with the performance of routine tasks.

A household production function (Y_h) that involves the n children of the family is written as

$$Y_h = A_h q^{\alpha} n, \tag{9}$$

where A_h is the productivity parameter for household production, q is the quality of manageral decisions[7], and $0 < \alpha < 1$ is a parameter that determines the returns to the quality of managerial decisions. Implicit in the equation is the assumption that each child supplies labor inelastically, and their work time is set to be identical and equals to one. In addition, this production function

[6] Although in principle schooling may augment the efficiency of performing routine tasks, a large volume of empirical studies suggest that there are limited returns to education in these activities (see Rosenzweig, 1995, and Huffman, 2001). Building our model on these empirical evidence and without loss of generality, we assume a nil worker productivity effect of education.

[7] For now, we only consider joint production involving all children in the family. The implications of family division and separate production among children will be investigated in later analysis.

shares two similar properties with the factory-based production function in (3): one is the constant-return-to-scale in the number of workers and another is the same value in the parameter of returns to quality (α). These two specifications serve a special purpose. We will show that under certain conditions, the household production function (9) will be exactly the same as the production function (3) with labor market participation.

The quality of managerial decisions depends positively on the schooling of the children

$$q = M(\mathbf{s}) = M(s_1, ..., s_n) \text{ and } M\prime(\mathbf{s}).$$

This managerial decision function is central to our analysis. Our purpose is to explicate the relationship between the education of family workers and the quality of the household managerial decisions, and this structure of schooling returns will serve as a fundamental factor in the determination of fertility and intrahousehold allocation of schooling. Our specification is built on the insights of Welch (1970) and Schultz (1975) on the positive effect of education on making allocative decisions and the insights of Lucas (1978) and Rosen (1982) concerning the assignment of persons into managers and workers within a firm as well as their interactions in the production process.

Consider a family of n children. Among them, child 1 is the designated manager who contributes to major managerial decisions of the household more than each of the remaining $(n-1)$ child. Children other than the manager are considered as regular workers, who also contribute to managerial decisions, but primarily on specific cases as they arise in the execution of routine tasks. In general, problems facing the manager are more complex and their decisions may have a greater impact on output. Since the decisions facing the workers are relatively simple, there is a greater degree of substitutability among them in dealing with the decisions. For both the manager and workers, we use the schooling of each person to approximate the quality of their managerial inputs that individuals contribute to the decision making process.

A managerial decision function with constant-elasticity-of-substitution (CES) properties is written as

$$\begin{aligned} M(s_1, ..., s_n) &= \left[s_1^\rho + g^\rho(s_2, ..., s_n)\right]^{1/\rho} \text{ and} \\ g(s_2, ..., s_n) &= (s_2^\gamma + ... + s_n^\gamma)^{1/\gamma}, \text{ where } \rho \leq \gamma. \end{aligned} \tag{10}$$

This is a double-CES specification in which the parameter ρ reflects the substitutability between the manager (child 1) and other workers (child $2 \rightarrow n$), while γ reflects the substitutability among workers and the contribution from each worker is perfectly symmetric. The elasticity of substitution measures the extent to which one productive input can be replaced by another input but still producing the same level of output. The higher the value of ρ or γ, the higher the elasticity of substitution, and the easier for substitution among the inputs. Thus, the assumption $\gamma \geq \rho$ represents the fact that it is easier to substitute one worker for another worker than to substitute workers for manager.

The manageral decision function in (10) has one technical property that needs appropriate adjustments. As it stands, $M(\mathbf{s})$ is an increasing function of the number of children, and the scale of return to n depends on the substitutability parameters ρ and γ. Economically, however, there is no strong reason to justify why the quality of manageral decisions should be an increasing function of n, especially given the usual concern that efficiency of communication declines with the size of the team. Moreover, it is hard to justify why the scale effect should depend on the substitutability parameters ρ and γ. To adopt a scale-neutral specification, we multiple the managerial decision function in (10) by $\lambda(n) = \left[1 + (n-1)^{\rho/\gamma}\right]^{-1/\rho}$ so that the adjusted function is independent of the number of children.[8]

This managerial decision function provides a fairly general structure of manager-worker interactions through which education contributes to decision making. It contains several familiar specifications of production as special cases:

Case 1: when $\rho = \gamma = 1$, the function becomes the average schooling level of all children in the family, $M(\mathbf{s}) = (s_1 + ... + s_n)/n$.

Case 2: when $\rho < \gamma = 1$, the function becomes the manager-worker type of hierarchical production function similar to the one analyzed by Lucas (1978) and Rosen (1982), $M(\mathbf{s}) = [s_1^\rho + (s_2 + ... + s_n)^\rho]^{1/\rho}\lambda(n)$. With this form, the education of the manager and workers play a differential role in raising the quality of management. While the workers are perfectly substitutable among themselves, the manager and the workers are not. The manager has a more important role. One can interpret the situation as one in which the manager has to solve difficult tasks, but other children all perform routine tasks – one worker is just as good as another. These descriptions correspond to a common production setting in which there are some division of labor, but not excessively.

Case 3: when $\rho = \gamma < 1$, the function becomes the O-Ring type production function similar to the one analyzed by Kremer (1993), $M(\mathbf{s}) = [s_1^\rho + s_2^\rho + ... + s_n^\rho]^{1/\rho}\lambda(n)$. In this case, the education of all children plays a symmetric role, but contributions from each child are not perfectly substitutable. Economically, it is consistent with high degree of specialization, where tasks performed by each child are all important. The strong complementarily among inputs applies to a modern,

[8] In the context of endogenous growth models, Benassy (1998) points out a similar scale effect associated with modeling product variety specifications using CES functions. Benassy, as well as Aghion and Howitt (1998) in a later analysis, all make similar an adjustment similar to ours for controlling the scale effect.

complicated production technology in which each individual worker has to make their own decisions and each decision has an effect on the whole production process.

Case 4: when $\rho = \gamma = +\infty$, the function becomes an inverse Leontief technology, $M(\mathbf{s}) = \max(s_1, ...s_n)$. This is the case in which the most schooled child makes all allocative decisions and his level of education serves as is a good proxy for the quality of management. Family incentives to share information and the convenience of making collective decisions when family members live and work together provide support for the outcome. Indeed, empirical studies have given support for this structure of schooling returns for farm families engaging in traditional agricultural production.[9] Within-household information sharing is also seen in Basu and Foster (1998) who argue that the presence of a literate person in a household may have positive externalities on other illiterate family members. For instance, being able to read pamphlets of agricultural extensions and the label on a new fertilizer packet by a literate family member may raise the productivity of all family workers.

It is clear from the preceding discussions that different values of ρ and γ represent a range of production environments in which the manager interacts with the workers in making managerial decisions. Under the condition $0 \leq \rho \leq \gamma < 1$, the schooling of the manager is a complementary input to schooling of the workers, and this condition conforms with the standard assumptions made for productive inputs. Henceforth, we shall assume these parameter values. It is worth emphasis that the value of γ relative to ρ actually defines the complexity of production technology. Holding ρ constant, high values of γ would imply low production complexity so that workers are easily

[9] Yang (1997) proposed and tested hypotheses concerning the effects of schooling composition on the efficiency of household production. Empirical findings based on small-scale farming in China indicate that returns to education come primarily from the highest level of family schooling through improvements in allocative efficiency. This result corroborates with the findings by Foster and Rosenzweig (1996) that whether anyone in the household has primary education is a powerful predictor for the adoption of high-yield seed varieties in the early stages of the Green Revolution in India. Jolliffe (2002) also finds the maximum level of education important for farm household efficiency.

substitutable. For the case of agriculture, the condition would be associated with a traditional, static environment in which workers engage primarily in simple tasks. Alternatively given ρ, low values of γ would imply high production complexity so that there is high degree of specialization and workers are hardly substitutable. For agriculture, the condition would be associated with a modern, dynamic environment in which workers engage in complicated tasks. In short, changes in the value of parameter γ may reflect technical changes in our model of managerial decision making.

Substituting the decision function with the scale-adjustment factor into equation (9), the household production function becomes:

$$Y_h(s_1, ..., s_n, n) = A_h \left[s_1^\rho + (s_2^\gamma + ... + s_n^\gamma)^{\rho/\gamma} \right]^{\alpha/\rho} \left(1 + (n-1)^{\rho/\gamma} \right)^{-\alpha/\rho} n. \tag{11}$$

This function incorporates the interactions of family workers in the presence of differential duties under household production, thus setting a framework for analyzing family fertility and schooling investment decisions that could internalize the returns to the quantity and quality of children. The production function is also in a flexible form such that if all children have the same level of education s, the function becomes $Y_h(s, ..., s, n) = A_h s^\alpha n$, which is comparable with the one in equation (3) under the condition of labor market participation. The common properties shared by these two functions help us focus the analysis on differences in family behaviors concerning fertility and intrahousehold allocations that are results of household production.

Unlike the assumption made for competitive markets with individual earnings, we shall assume for family production that the joint output is shared equally by the n children, even though they may have different schooling attainments and make differential contributions to production. This income sharing arrangement is consistent with parental aversion to inequality among their children and the fact that parents have some power to apply social and family pressures to achieve optimal transfers among the siblings.[10] Accordingly, the per capita income of the children under household production is

$$y_h = Y_h/n = A_h \left[s_1^{\rho} + (s_2^{\gamma} + ... + s_n^{\gamma})^{\rho/\gamma} \right]^{\alpha/\rho} \left(1 + (n-1)^{\rho/\gamma} \right)^{-\alpha/\rho}. \tag{12}$$

3.2 Allocation of Schooling Resources

Parents under household production have the same preference and budget constraint as parents who have children participating in labor markets, as in equations (1) and (2). The difference is that the returns to schooling for each child is no longer represented by the common wage schedule dictated by the labor market, as in equation (4). Instead, schooling returns to each child reflects the organizational arrangement of household production of assigning adult children to managerial versus production tasks. To examine the implications of household arrangements on intrafamily schooling investments among children, we first take fertility as exogenous. Fixing the number of children helps us highlight the differences in sibling educational investment across the two types of institutional structures.

Under household production, the parents would choose a different schooling level for child 1 from all other children because the schooling of child 1 plays a differential role comparing with the schooling of other children in household production. The parents would invest equally in the schooling of children other than child 1 because the schooling those children enters into the production function symmetrically. Let s_{h1} denote the schooling attainment of child 1 who takes

[10] See Ranis (1988) for additional explanations for output sharing in traditional agricultural households. This income sharing arrangement, which differs from those who participate in labor markets, is viewed as a central feature of the organizational dualism across the traditional and modern sectors of production.

the managerial assignment and s_{h2} denote the common level of schooling in all other children who take the responsibility of carrying out regular production tasks. Then, the parent's optimization problem is

$$\max_{c_h, s_{h1}, s_{h2}} \left\{ \ln\left(c_h\right) + \beta \ln\left(A_h \left[s_{h1}^{\rho} + (n-1)^{\rho/\gamma} s_{h2}^{\rho} \right]^{\alpha/\rho} \left(1 + (n-1)^{\rho/\gamma} \right)^{-\alpha/\rho} \right) \right\}.$$

subject to

$$c_h = I\left(1 - vn - \mu s_{h1} - \mu(n-1)s_{h2}\right), \tag{13}$$

which has the following solution for schooling investment:

$$s_{h1}(n) = \frac{\alpha\beta(1-vn)}{\mu(1+\alpha\beta)\left[1+(n-1)^{\frac{\rho(1-\gamma)}{\gamma(1-\rho)}}\right]}, \tag{14}$$

$$s_{h2}(n) = \frac{\alpha\beta(1-vn)(n-1)^{\frac{\rho-\gamma}{\gamma(1-\rho)}}}{\mu(1+\alpha\beta)\left[1+(n-1)^{\frac{\rho(1-\gamma)}{\gamma(1-\rho)}}\right]}, \tag{15}$$

where $s_{h1} > s_{h2}$ when $n > 2$, because $0 < (n-1)^{(\rho-\gamma)/\gamma(1-\rho)} < 1$. Therefore, under household production with various work assignments, the family chooses to invest more schooling in the child who takes managerial duties than in regular family workers, even if the costs of education are the same for all children. This result departs from the usual implications of intrahousehold allocation models in which variations in schooling investments among siblings are generally attributable to differences in child endowment that affects costs of schooling or parental preference toward individual children. Our model shows that comparing with families who participate in labor markets, families under household production choose a less equal distribution of schooling for their children even if they have otherwise equal economic conditions, including preferences, child endowments, parental incomes, and costs of education and child rearing. This behavioral difference reflects the fact that families under household production both have incentives and are able to fully internalize the productivity gains from efficient assignments of workers to production tasks.

Based on solutions in (14) and (15), we have:

Proposition 1 *Holding the number of children constant, the average level of schooling for a family under household production is the same as that for a family with labor market participation. However, when the number of children is greater than two, the schooling attainment of child* 1 *is greater than that of other children in a family under household production, and this schooling gap increases with both the number of children and the worker substitutability parameter* γ*.*

Proof: All proofs are given in the appendix.

The reason for equal average schooling reflects the fact that the parameter for schooling returns α is set the same for production technologies across the two family types (see equations (3) and (9)). Therefore, holding the number of children constant, the parents would allocate the same amount of total resources to children's schooling.

The comparative static results that schooling inequality increases with the number of children and the worker substitutability parameter γ also conform with intuition. It is easy to verify that the marginal productivity of schooling is a function of n and $MP_{s_{h1}}/MP_{s_{h2}} = (n-1)^{(\gamma-\rho)/\gamma}$, which is increasing in n when $n > 2$ and $\gamma > \rho$. Therefore, when the given amount of resources is distributed to more children, it would be optimal to implement a smaller reduction in s_{h1} than in s_{h2}, which results in a larger schooling gap. In addition, an increase in γ is associated with simpler production tasks and higher substitutability among workers, thus raising the relative value of the manager's schooling. Consequently, the parents would reduce schooling expenditures on workers and raise investment in the manager, again resulting in a larger schooling gap. On the other hand, increases in the complexity of production tasks, for instance due to mechanization in family farming, would have an effect of reducing schooling inequality among children.

3.3 Fertility and Schooling Investments

The preceding analysis has followed the standard modeling strategy taken by the literature on intrahousehold allocations (e.g. Becker and Tomes, 1976; Behrman et al., 1982) of treating fertility as an exogenous variable. This approach helps highlight the role of household production in affecting the distribution of schooling within families. However, casual observations and systematic analysis both suggest that fertility and human capital investments are intimatedly linked; they are joint family decisions (see Becker and Lewis, 1973; Becker, 1991). In an attempt to introduce fertility choice in an intrahousehold allocation model, we build on solutions obtained so far and solve for optimal family size and intrafamily schooling allocations jointly. The results show that the properties of household production determining within-family educational allocations also have direct influence on decisions on family size and average educational attainment.

Given the solutions for $s_{r1}(\cdot)$ and $s_{r2}(\cdot)$ in equations (14) and (15), the optimal fertility decision for the family under household production can be formulated as

$$\max_{c_h, n_h} \left\{ \ln(c_h) + \eta \ln(n_h) + \beta \ln \left(A_h \left[s_{h1}^{\rho}(n_h) + (n_h - 1)^{\rho/\gamma} s_{h2}^{\gamma}(n_h) \right]^{\alpha/\rho} \left(1 + (n_\rho - 1)^{\rho/\gamma} \right)^{-\alpha/\rho} \right) \right\}$$

subject to

$$c_h = I[1 - v n_h - \mu s_{h1}(n_h) - \mu(n_h - 1) s_{h2}(n_h)],$$

which is equivalent to solving the choice variables jointly. In absence of a close-form solution for

n_h, the appendix proves the following proposition:

Proposition 2 *When $0 \leq \rho < 1$, the optimal fertility for the family with household production is always higher than for the family with labor market participation. Moreover, the difference in fertility across these two family types increases with the worker substitutability parameter γ.*

The two aspects of this proposition deserve emphasis. First, the fertility difference across the two family types is a direct result of their distributional differences in educational investments in children. Given the same total schooling expenditures, families with household production make more investment in the manager and less investment in regular workers, when compared with families who participate in the labor market. This decision affects the fundamental trade-off between the quantity and quality of children. With family-based production, the lower educational spending on production workers reduces the price of child quantity (μs_{h2}) through the interaction term $\mu(n_h - 1)s_{h2}$ in the budget constraint. As a result, those families would choose a larger sibling size.

By contrast, if there is no distributional differences in the educational investments in children, families would not choose to have more children. Consider the special parameter specifications presented earlier in *Case 1* and *Case 3*, $\rho = \gamma \leq 1$. Since individual schooling enters the decision function symmetrically, each child would receive the same schooling expenditure (s_h), and per child income under household production would be $y_h = A_h s_h^\alpha$. The parents' optimization problem becomes $\max_{c_h, n_h, s_h}\{\ln(c_h) + \eta \ln(n_h) + \beta \ln(A_h s_h^\alpha)\}$ subject to $c_h = I(1 - vn_h - \mu n_h s_h)$, which is the same problem confronted by families with labor market participation, as illustrated in the prototype model. Consequently, without the distinction between the manager and workers, the optimal schooling and fertility choices converge to the set of solutions in (7) and (8), the choices made by families who participate in the labor market.

Second, the proposition also suggests that sibling education gap tends to be larger when families under household production face simple production tasks, and the gap tends to be small when facing complicated production tasks. The underlying reason for the result is that the parameter γ is closely related to job complexity and degree of specialization for production workers. For instance, in a traditional and static agricultural environment as exemplified by *Case 4* when $\rho = \gamma = +\infty$, the family only needs to educate one child. This is an efficient solution because all production workers are perfectly substitutable and the best schooled child could achieve perfect knowledge sharing with other siblings. In contrast, when production tasks become more complicated with a declining value of γ, say due to mechanization in family farming, job specialization will deepen and workers will become less substitutable. Consequently, families would have incentives to reduce educational inequality.

Combining the findings in Propositions (1) and (2), we are now in a position to summarize other key results. Since average schooling attainment is decreasing in fertility and families with household production have more children, their average schooling expenditures on children are lower than families who participate in labor markets. Since educational inequality is increasing in fertility, the larger number of children for families under household production implies that their sibling education gap is even larger than that predicted in equations (14) and (15) by the model with exogenous fertility. In summary, we have shown that household production has caused a larger family size, lower average schooling investment, and a greater sibling education gap. Because we have controlled for other economic conditions, such as parental preference, individual costs of education and family income, these fundamental behavioral differences are attributable to the special properties of household production, including prominently the deferential roles played by the education of individual children.

4 Extensions

To illustrate the importance of joint household production in affecting parental investments in children, we made some simplifying assumptions in the previous section. In particular, we assume that parents can perfectly enforce equal income sharing among siblings, that children who engage household production will never participate in the labor market, and that parents care about children's schooling only because they increase production efficiency. In this section, we show that our main conclusions remain valid even if we allow for imperfect income sharing, potential participation in labor market, and the fact that parents derive utilities from having educated child directly.

A. Imperfect Income Sharing and labor mobility

We assume that siblings initially work together and share their joint output equally, (which can be interpreted as the case when their parents are still alive), and, in the second stage, they split and produce output by themselves, or alternatively, they earn competitive wages in the labor market. Accordingly, the lifetime incomes of the n siblings are

$$y_i = \pi A M^{\alpha}(s_1, ..., s_n, n) + (1-\pi) A s_i^{\alpha}, \quad i = 1, ..., n, \tag{16}$$

where π is the length of the first stage, and $1-\pi$ is the second stage.

B. Parental Preferences in Schooling

We also assume that the parent cares about the children's schooling attainments as well as income. In particular, we revise the parental utility function as follows:

$$U = \ln(c) + \eta \ln(n) + \beta \frac{1}{n}[\ln(y_1) + ... + \ln(y_n)] + \delta \frac{1}{n}[\ln(s_1) + ... + \ln(s_n)]. \tag{17}$$

So, the parent's optimal schooling investment problem is to choose $s = (s_1, ..., s_n)$ to maximize U above subject to the budget constraint (2) and the income constraints in (16).

Proposition 3 *Assume that $\gamma \geq \rho > 0$. The average level of schooling in a family is a decreasing function of the number of children in the family, independent of the parameters γ and π, but the sibling education gap increases with γ and π, and the number of children. In addition, the optimal fertility increases with γ and π.*

5 Applications

Family behaviors implied by the model are consistent with a series of empirical observations from developing countries. While each of the observations may be due to a variety of factors, taken together, they suggest that an intrafamily allocation model with household production is empirically relevant, providing novel explanations for many observed behaviors.

5.1 Evidence on Family Decisions

The preceding model of intrafamily allocations suggest that families in self-employed activities would have a larger sibling size, lower average education, and greater inequality in sibling education than families who participate in wage activities. In addition, with the development of labor markets and when the technology of self-employed activities becomes more complicated, these families would reduce schooling inequality of their children and adjust other behaviors to that of families with labor market participation. While to our knowledge these systematic differences in behaviors are not yet documented in the existing literature, we attempt to fill this void.

Table 1 presents information on sibling size, average schooling and sibling education inequality for two types of families, self-employment versus wage employment, using data from the 1989 Taiwan Women and Fertility Survey.[11] This data set has two strengths particularly suited for the current study: (1) it provides information on the schooling of children for a large number of sibling units over a wide range of ages, and (2) it contains employment history records of the parents, which enable us to identify the types of production environments in which the children were raised. More specifically, we use the information on the fathers' longest-held jobs to designate the sibling units as either belonging to self-employment or wage-employment families. This is the best data set we are aware of that contains complete information on sibling education with parents' employment history covering a long-period of economic development.[12]

We use three measures to document within-family schooling inequality. The first is a measure on sibling education gap,

$$g = \frac{s_{\max} - \overline{s}_o}{\overline{s}}, \tag{18}$$

where $s_{\max}$ is the highest schooling attainment among children, $\overline{s}_o$ is the average schooling of other children, and $\overline{s}$ is the average schooling of all siblings. This measure captures closely the distinction made between the manager and workers in the theoretical model with household production. The

[11] This survey was designed by researchers at the University of Chicago and was conducted in March 1989 with the collaboration of researchers at the National Opinion Research Center and the National Taiwan University. Yang and Zhu (2001) provides detailed data descriptions closely related to the analysis in this paper, while Parish and Willis (1993) provides general information about the survey and the data.

[12] By contrast, the other well known longitudinal surveys, such as the Malaysian and the Indonesian Family Life Surveys, do not have complete sibling information.

second measure is coefficient of variation ($C.V.$), which gives the standard deviation ($S.D.$) as a proportion of the mean, $C.V. = S.D./\bar{s}$. Therefore, the two measures are both neutral to possible changes in schooling level in measuring inequality. Finally, to provide a reference for comparison, we also report the standard deviation of sibling education, which does not adjust for changes in the level of attainment.

Remarkable differences in behavior across self-employed and wage-employed families emerge in Table 1. To see a broad picture of family choices and their changes over time, we group sibling units into ten-year age cohorts based on the age of the respondents at the time of the survey. The observed patterns in behavior are consistent with our theoretical results. Most importantly, for any given cohorts, families engaging in self-employment have a larger family size,[13] lower average schooling investment in children, and a greater inequality in sibling education than families engaging in wage-employment. The inequality result is robust to all three measures of within-family variations.[14]

Conventional explanations for these patterns include differences in income for the two family types, differential costs of rearing children (e.g. Becker, 1991), rates of return to education unique to each activity, and children's contribution to family production (e.g. Rosenzweig, 1977). While these considerations shed light on differences in fertility and average schooling attainment, limited insights can be drawn to explain differences in the variability of sibling education. The household production model provides a coherent explanation for all three behaviors, since within-family work

[13]Note that the average family size appears to be large. This might be due to one of the two factors: (1) larger families have a higher probability to be selected into this random sample, and (2) the observations are from families who have at least two children. Sibling units with one child are taken away because of our interest in the distribution of sibling education.

[14]One potential explanation for the differences in education inequality across family types and over time is the existence of gender-education gap. However, the data reveal that education inequality among male siblings is similar to patterns of all siblings. For instance, the gap measure for male children of self-employed families are 0.88, 0.69, 0.64, and 0.39 for the four respective cohorts, and the measure for wage-employed families are 0.74, 0.43, 0.39, and 0.31. Therefore, the sibling education variability presented in the table goes beyond gender-specific inequality.

assignments induce self-employed families to concentrate educational resources on certain children, and as part of joint decisions, they also have a larger sibling size and lower average education.

Table 1 also reveals an interesting trend that schooling inequality within self-employed families have declined rapidly over time, especially when using the gap measure. While other factors, such as compulsory schooling laws, may contribute to this result, the household production model provides another perspective. Since technology in household production became more complicated over time and labor markets were also expanded during this period in Taiwan, both raising the value of the technology parameter γ, families would have incentives to reduce sibling education gap. Accordingly, they would also reduce fertility and increase average schooling level. These two behavioral changes are also revealed in the data.

5.2 Gender Education Gap

It has been well documented that at aggregate level, gender education gap decline rapidly and persistently in the process of economic development (see Schultz, 1985). Existing explanations have attributed this long-run trend to parental preference in favor of boys, differential prices of acquiring education, and labor market discrimination against women (e.g. Behrman, Pollak and Taubman, 1986).

Our household production model suggests another explanation. To the extent that women are biologically different from men, for instance because of child bearing, they would spend less working time in family production, which would significantly reduce the value of their education. More importantly, in many agrarian societies in which daughters are married out to men's families, it further reduces the value of women's education to their own families. Since economic development inevitably involves the structural change from household production to labor market participation, gender education gap declines as a result of changes in family composition.

5.3 Birth-Order Effects

In developing countries, families generally invest more education in children of lower birth-orders, especially in their oldest son. Inheritance laws and limited family budget have been suggested as explanations. Our model provides an alternative explanation: families do so because the oldest son often makes the longest time commitment to joint family production.

5.4 Some Puzzles Explained

(1) It has been well documented that rates of return to education are generally larger in developing countries (see Becker, 1993). It is also well known that average schooling attainment in less-developed countries are much lower than in developed economies. Why people in developing countries do not invest more in their education?

Our intrafamily allocation model emphasizing household production provides an explanation. Although the returns to schooling for the household manager may be high, the returns to education for household workers may be low. If estimates for schooling returns are taken across the manager and worker groups, the estimates could be higher that the rates of return in developed countries. But, this does not imply that workers in developing countries have incentives to invest more in schooling.

(2) One of the persistent puzzles in empirical research on estimating schooling returns in agriculture is: whose education matters for farm household efficiency? Empirical results reveal an inconsistent pattern. Depending on the nature and environments of agricultural production, the important determinants could be the average schooling of farm workers, the highest level of school-

ing within the family, or other schooling measures (see Lockheed, Lamison and Lau, 1980; Yang 1997). What factors have caused these variations?

Our model suggests that the structure of schooling returns depends crucially on the complexity of household production technology. In traditional agriculture with relatively simple technology, there is a distinction between the role of a manager and workers. Under this environment, returns to schooling may be observed for selected members of the household, but not necessarily for all farm workers. In contrast, in a modernizing agriculture with sophisticated technology, there is a greater degree of specialization and workers are less substitutable. Under this environment, positive schooling returns are likely to be observed for many if not all farm members. Therefore, a flexible empirical specification that takes into account individual contributions to farm efficiency is necessary to provide accurate estimates for schooling returns.

6 Summary and Concluding Remarks

To be completed.

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Insert Table 1 here.

A Proofs of Propositions

Proposition 1: The average schooling level of the family is $n^{-1}[s_1(n)+(n-1)s_2(n)]$. From (**??**) and (15) we can see that it equals $\beta(1-vn)\,[\mu(1+\beta)n]^{-1}$, which is the same as the average schooling level of a family whose children participate in a competitive labor market. Also, from (**??**) and (15) we see that

$$\frac{s_1(n)}{s_2(n)} = (n-1)^{\frac{\gamma-\rho}{\gamma(1-\rho)}}.$$

When $\gamma > \rho \geq 0$ and $n > 2$, the ratio above is greater than one because $n-1 > 1$ and $(\gamma-\rho)[\gamma(1-\rho)]^{-1} > 0$. Apparently, the ratio also increases with n and γ. Q.E.D.

Proposition 2: The first part of this proposition is a direct application of Proposition 3 for $\pi = 1$ and $\pi = 0$. The second part of this proposition is a special case of Preposition 3 when $\pi = 1$ and $\delta = 0$. Q.E.D.

Proposition 3: Since $\rho < 1$, parents will always choose the same schooling level for children 2 to n. Let s_1 be the schooling level of child 1, and let the common schooling level of the other children be s_1/z. If z is increasing in π and γ, so is the sibling education gap. Let $x = n-1$. Then, from (16) we have

$$\begin{aligned} y_1 &= As_1^{\alpha}\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + 1 - \pi\right], \\ y_2 &= As_1^{\alpha}\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + (1-\pi)/z^{\alpha}\right]. \end{aligned}$$

So, the parental utility function can be written as

$$\begin{aligned} V &= \ln\left[1 - v(1+x) - \mu s_1(1+x/z)\right] + (\alpha\beta+\delta)\ln s_1 + \eta\ln(1+x) - \frac{\delta x}{1+x}\ln z \\ &+ \frac{\beta}{1+x}\ln\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + 1 - \pi\right] + \ln\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + (1-\pi)/z^{\alpha}\right]. \end{aligned}$$

The first order condition for s_1 is

$$\frac{\mu(1+x/z)}{1 - v(1+x) - \mu s_1(1+x/z)} = \frac{\alpha\beta+\delta}{s_1},$$

which implies that

$$s_1 = \frac{(\alpha\beta+\delta)\left[1 - v(1+x)\right]}{(1+\alpha\beta+\delta)\mu(1+x/z)}.$$

The average level of schooling of the n children is

$$\frac{s_1(1+x/z)}{1+x} = \frac{(\alpha\beta+\delta)\left[1 - v(1+x)\right]}{(1+\alpha\beta+\delta)\mu(1+x)},$$

which is a decreasing function of $n = 1+x$, independent of π and γ. Now, substituting the expression for s_1 into the utility function V, we have

$$\begin{aligned} V &= F(z,x,\pi,\gamma) = F_0 + \\ &(1+\alpha\beta+\delta)\ln\left[1 - v(1+x)\right] - (\alpha\beta+\delta)\ln(1+x/z) + \eta\ln(1+x) - \frac{\delta x}{1+x}\ln z \\ &+ \frac{\beta}{1+x}\ln\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + 1 - \pi\right] + \ln\left[\pi\frac{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}}{(1+x^{\frac{\rho}{\gamma}}/z^{\rho})^{\frac{\alpha}{\rho}}} + (1-\pi)/z^{\alpha}\right], \end{aligned}$$

where F_0 is some constant. It is straightforward to check that cross partials of function F with respect to any two arguments are positive over the domain $\Omega = [1, +\infty) \times (1, \infty) \times [0, 1] \times [\rho, 1)$. In other words, F is a strictly super modular function on Ω. From Theorem 5 of Milgrom and Roberts (1990), then, we know that

$$(z^*, x^*) = \arg \max_{z \geq 1, x > 1} F(z, x, \pi, \gamma)$$

is increasing in π and γ. Q.E.D.