Math Camp Syllabus, 2009

Linear Algebra (Lectures 1 - 5, approximately)

Simultaneous linear equations. Gaussian elimination. Row reduced and echelon forms of simultaneous equations.

Vector and matrix representation of simultaneous linear equations. Elementary row operations on matrices. Equivalence of matrices. Row reduced and echelon forms of matrices.

Vector spaces. Linear dependence and independence of vectors. Linear spans. Basis of a vector space. Dimension of a vector space. Subspaces of vector spaces.

Linear transformations. Matrix representation of linear transformations. Composition of linear transformations and matrix multiplication. Invertibility of linear transformations and of matrices. Nullity and rank of linear transformations. Singularity and non-singularity of linear transformations and of matrices. The dual space of a vector space. The annihilator of subset of a vector space. Equality of the row and column ranks of a matrix. N-dimensional Euclidean space and the standard inner product on it. Orthogonality. Orthogonal projections. Orthonormal bases. Gram-Schmidt orthogonalization of a basis. Multilinear forms and determinants. The adjoint of a square matrix. Characteristic values and vectors. Quadratic forms.

Real Analysis (Lectures 5 - 6 approximately)

Open balls and open sets. Complements of sets. Closed sets. Sequences. Convergence of sequences. Continuity of functions.

Completeness of the real numbers. Cauchy sequences. Least upper bound or supremum. Greatest lower bound or infimum. Compactness. Boundedness. Subsequence. Bolzano-Weierstrass theorem. Open covers and subcovers. Heine-Borel theorem. Maximum theorem.

Calculus with One Variable (Lecture 7)

Differentiability. Interior maximum theorem. Rolle's theorem. Mean value theorem. Leibniz's rule. Taylor's theorem. First and second order conditions for local maxima and local minima. Chain rule of differentiation.

Multivariable Calculus (Lectures 7 - 11 approximately)

Differentiability. Directional derivative. Partial derivative. Matrix representation of the derivative. Generalizations of Leibniz's rule. First order condition for local unconstrained extrema. Least squares estimation. Chain rule of differentiation. Mean value theorem. Jacobian matrix. Gradient. Second and higher order derivatives. Taylor's theorem. Second order condition for local maxima and minima. Inverse and implicit functions theorems. Unconstrained envelope theorem.

Constrained optimization. First order condition for extrema. Lagrange multipliers. Constraint qualification. Second order conditions for local maxima and minima. Constrained envelope theorem.

<u>Convex Analysis</u> (Lectures 11 - 12 approximately)

Convexity of sets. Concavity and convexity of functions. Uniqueness of global maxima or minima. Sufficient conditions for convexity or concavity of functions in terms of the second derivative. Kuhn-Tucker theorem. Constraint qualification. Complementary slackness conditions. Lagrangian function. Subgradient. Kuhn-Tucker coefficients. Minkowski separation theorem.

<u>Optimal Control Theory</u> (Lectures 13 - 14 approximately)

Statement of the optimal control problem. Admissible controls. Necessary conditions for an optimum. Hamiltonian function. Dual, conjugate, costate or auxiliary variables. Relation with the Kuhn-Tucker theorem. The maximum principle. A simple growth model example. The turnpike property of optimal solutions in the example.

<u>Dynamic Programming</u> (Lectures 14 - 15 approximately)

The contraction mapping theorem. Fixed points. The dynamic programming principle in discrete time. Value function. A monetary theory example. A growth theory example. Uniform continuity. Euler's equation. The Hahn problem in growth theory. Transversality conditions in growth theory and optimal control theory. The origin of the word "transversality."