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TRADE, GROWTH, AND THE HECKSCHER OHLIN THEOREM

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Trade, Growth, and the Heckscher Ohlin Theorem

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Although recent work has cast theoretical doubts upon the

applicability of the factor price equalization theorem to a dynamic

economy,[1] the other pillar of the Heckscher-Ohlin, comparative cost

analysis--that a country will export the good using its relatively

abundant factor[2]--has not been subject to similar scrutiny. Certainly

one reason for this neglect is the major theoretical qualification that

demand conditions may nullify the theorem.[3] As Ohlin himself stated,

> "Differences in relative commodity prices depend upon
> the state of supply of industrial agents and upon demand con-
> ditions." (p. 15) "In a loose sense we may say, as we have
> said above, that differences in equipment of factors of pro-
> duction are the cause of trade. But we must be careful to
> remember the qualification which lies in the possible influence
> of differences in demand conditions." (p. 17)

In other words, abundant must be interpreted not in the physical sense, but

in the economic sense, which makes the theorem tautological.

The purpose of this paper is to investigate the validity of the

Heckscher Ohlin theorem within a dynamic model. The dynamics are provided

through growth in the labor force and capital accumulation out of each

country's income, with the rate of investment out of income assumed to be

constant. The Marxian saving variant and the rational saving model with

a constant rate of time preference will also be considered briefly. As

previously shown,[4] either of these two assumptions will usually force the two economies to attain different long run interest rates, prevent factor price equalization, and lead one or both countries to specialize completely. By contrast the Keynesian saving behavior assumed in the main portion of this paper permits, though it does not guarantee, factor price equalization in the long run. More importantly for the purposes of this paper, the Keynesian saving assumption permits a long run equilibrium with incomplete specialization in both countries. Thus with Keynesian saving behavior it is possible to investigate the long run patterns of incomplete as well as complete specialization.

The paper demonstrates that unless the demand-saving as well as the production functions are exactly the same in the two countries, there is some possibility that the capital abundant country will export labor intensive goods intitially, due to the different intensities of demand. However the dynamics of growth lead to a unique long run equilibrium in the two countries, with the capital intensive country always exporting the capital intensive good (unless there are factor intensity reversals). This is due to the fact that differences in the rate of demand for investment goods out of income will always be dominated by the corresponding accumulation of capital, and a country with a high saving rate will eventually accumulate enough capital to export the capital intensive good. Therefore the Heckscher Ohlin theorem holds in the long run. As Ohlin (1933) argued, initial levels of producible factors are irrelevant. Rather, as this paper shows, it is the relative rates of accumulation which eventually determine physical abundance and in the long run this coincides with economic abundance.

I. The Model

A. Supply

As is usual in the Heckscher Ohlin models of trade[5] it is assumed that identical constant returns to scale production functions prevail in the two countries and that the production functions satisfy the usual assumptions regarding concavity and differentiability. Further it is assumed that production in one of the industries is always capital intensive to prevent factor intensity reversals.[6] Assuming full employment, per capita supply for country j can be written as

1. $Y_c^j = m_c^j + f_c[k_c](k_I - k^j)/(k_I - k_C)$

2. $Y_I^j = m_I^j + f_I[k_I](k^j - k_c)/(k_I - k_C)$

where Y_i^j = per capita supply of good i in country j

$f_i = f_i[k_i[W]$ per laborer production function of industry i,

$f_i' \geq 0, f_i'' < 0, k_I[W] \gtrless k_c[W]$

W = wage rentals ratio

m_i^j = per capita imports of good i by country j, exports are negative imports

k_i = capital per laborer in industry i, with superscript omitted in the region of incomplete specialization

j = A, B superscript omitted for A, i - I (investment goods), c (consumption goods) and the brackets indicate a function.

B. Factor Allocation

Again following traditional Heckscher Ohlin lines the economies are assumed to be competitive, with factors earning their marginal products. Thus:

3. $r \geq pf_c'$ $\qquad\qquad w \geq p(f_c - k_c f_c')$

with equality if $k_c > 0$

4. $r \geq f_I'$ $\qquad\qquad w \geq f_I - k_I f_I'$

with equality if $k_I > 0$,

where $\quad$ r = rate of interest or rental on capital

$\qquad$ w = the wage rate

$\qquad$ $p = p_c/p_I$ = the relative price of consumption goods

By Euler's rule we obtain:

5. $k_i + W = f_i/f_i'$ if good i is produced.

From Equations 3 and 4 we obtain:

6. $p = f_I'/f_c'$ when there is incomplete specialization. Otherwise relative prices are determined solely by the market clearing conditions and have no relation to the ratio of marginal products.

C. Saving-Investment Demand and Market Equilibrium

A Keynesian model of saving is assumed with all markets cleared. Therefore:

7. $sy = sf_I'(k + W) = m_I + f_I'(k_I + W)(k - k_c)/(k_I - k_c)$

where s = rate of saving, y = per capita income = $f'(k + \text{!?})$ by Euler's rule.
Some alternative assumptions regarding saving-investment demand are briefly
explored in Section V.

II. Reciprocal Demand

In addition to demand balancing supply in individual countries world
demand must equal world supply. This can be ensured by using a relation
expressing the equality of reciprocal demands with no capital flows:

8. $\quad Vm_I + V^B m_I^B = 0$

where

$$V = \frac{L}{L + L^B} \qquad\qquad V^B = \frac{L^B}{L + L^B}$$

and noting that by the assumption of reciprocal demand or barter with no
capital flows:

9. $\quad m_I + pm_c = 0 = m_I^B + pm_c^B.$

The price ratio, p, determined by the equality of the two countries' reciprocal
demands in turn determines whether each country will be incompletely specialized,
or completely specialized in investment or consumption goods production. For
example, if the relative price of consumption goods is high and for the over-
all factor proportions of the country are relatively unsuited to investment
goods production, then the investment goods industry will not be able to cover
the opportunity cost of the factors of production i.e. what they could earn
in the consumption goods industry As a result production of investment goods

will disappear. On the other side, at the same overall factor proportions, there is a low relative price of consumption goods at which that industry cannot cover opportunity costs. We define these two prices as p_{max} and p_{min}.[6a] Thus:

$p \geq p_{max}$ complete specialization in consumption goods

$p_{min} \geq p$ complete specialization in capital goods

$p_{max} > p > p_{min}$ incomplete specialization

Now it is easy to show that the price ratio determined by the equality of the two countries' reciprocal demand is unique in the short run. Define the reciprocal demand for per capita imports of manufactures by country A as

$$\Theta = \Theta[k, p] = sf_I'(k + W) - f_I'(k_I + W)(k - k_c)/(k_I - k_c) = -(1 - s)f_I'(k + W)$$
$$+ pf_c(k_c + W)(k_I - k)/(k_I - k_c).$$

(to pay for its imports country A exports $pm_c = -m_I$ by the reciprocal demand Equation 9). Differentiating Θ partially with respect to p in the three regions of specialization we obtain:

10. $\frac{\partial \Theta}{\partial p} = 0$ in the region of complete specialization in capital goods.

In the region of complete specialization in consumption goods we use the reciprocal demand expression for Θ to obtain

11. $\Theta = +s\, pf_c'(k + W)$

and

$$\frac{p}{\Theta}\, \frac{\partial \Theta}{\partial p} = 1 > 0$$

In the region of incomplete specialization we use the reciprocal demand expression for Θ to obtain

12. $\dfrac{\partial \Theta}{\partial W} = \dfrac{f'_I}{(k_I - k_c)^2} \{ + k'_I(k_c + W)(k - k_c)$

$+ k'_c(k_I + W)(k_I - k)$

$+ s(k_I - k_c)\ (k_I - k)/(k_I + W)\} \gtrless 0$ as

$k_I - k_c \gtrless 0$

By differentiating Equation 6 logarithmically and using Equation 5 we obtain:

$\dfrac{p}{W}\ \dfrac{dW}{dp} = \dfrac{1}{W}\ \dfrac{(k_I + W)(k_c + W)}{(k_I - k_c)} \gtrless 0$ as $k_I - k_c \gtrless 0,$

therefore $\dfrac{\partial \Theta}{\partial p} > 0.$

The reciprocal demand function $V\Theta$ is graphed in Figure 1 as a positively sloped line in the p, m_I plane. Since the imports of country A are the exports of country B, $V^B\Theta^B$ is graphed in Figure 1 as a negatively sloped line and the intersection of the two reciprocal demands is unique. As shown in Figure 1 B the uniquely determined relative price, p, determines the wage interest ratio uniquely and, correspondingly, the capital labor ratios in the industries as shown in Figure 1C for the case $k_c < k_I$.

III. Comparative Statics Analysis and the Patterns of Specialization

If we were to consider one of the economies in isolation we would generally expect accumulation to change the relative prices of factors and goods. The corresponding change in an open economy is a shift in the reciprocal demand curve, $\Theta^j[p, k^j]$ at every relative price. The direction of the shift
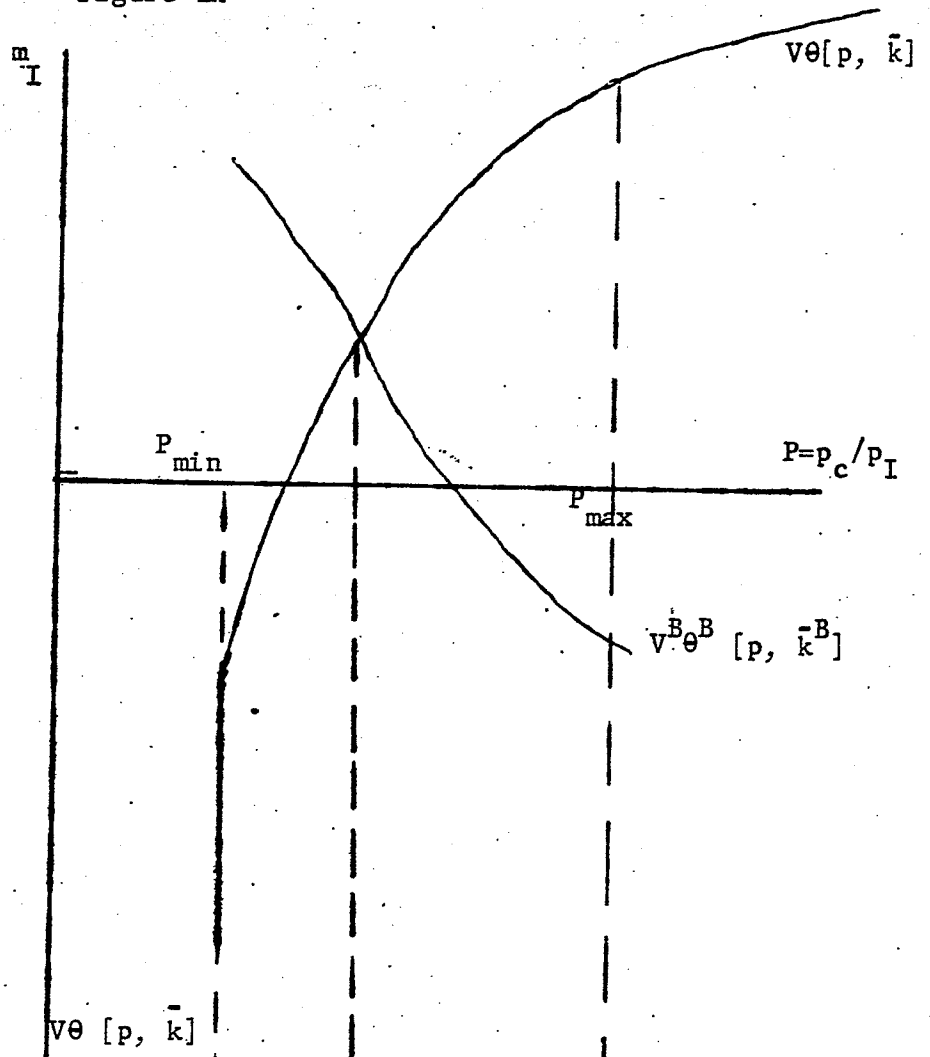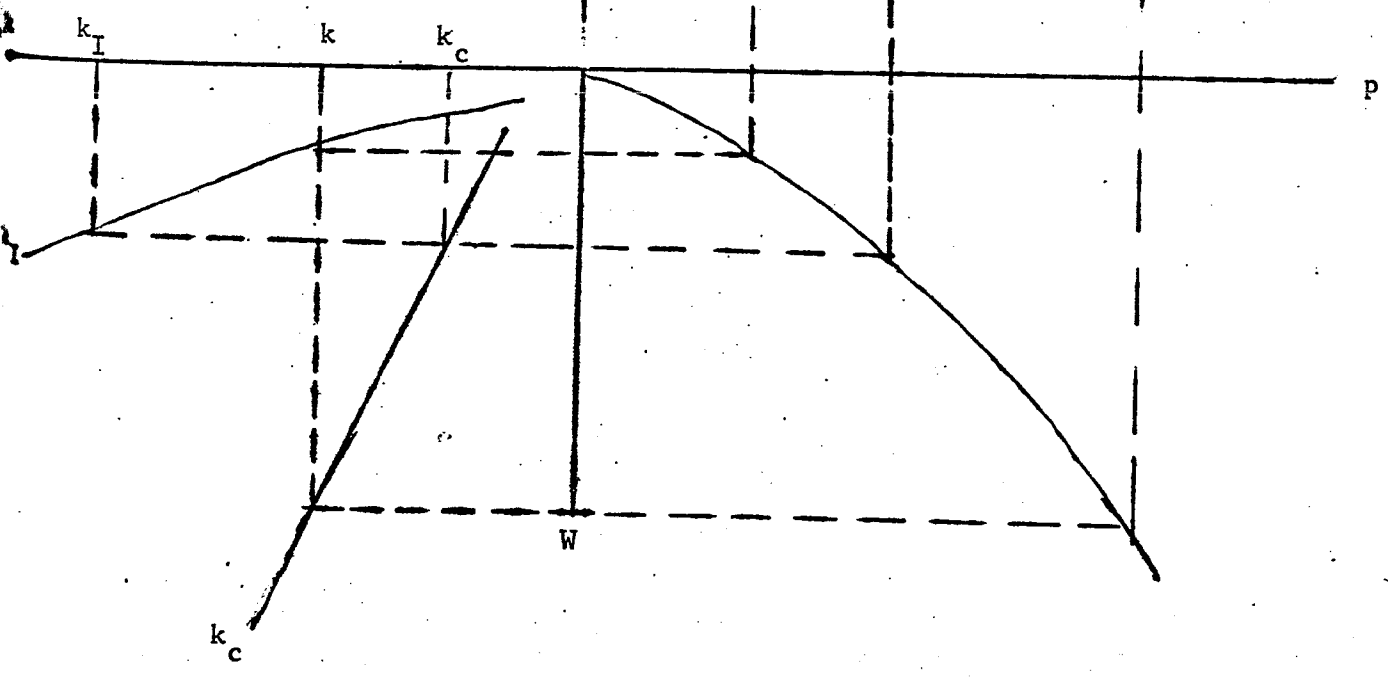
Figure 1A

Figure 1C

can be easily determined by the well known Rybczynski theorem.[7] Since there is a unique relation between relative prices, relative factor costs, and factor proportions, an increase in capital per head, p constant, may only be absorbed at constant factor proportions. In turn this implies a proportionate release of factors from the labor intensive industry and a combination of them with the new capital per head to yield the (constant) factor proportions of the capital intensive industry. Therefore per capita production of the labor intensive industry falls _absolutely_, while per capita production of the capital intensive industry rises. Since demands for both products rise, due to the rise in income, there is an increase in the excess demand for labor intensive goods and a corresponding reciprocal decrease in the excess demand for capital intensive goods.

Mathematically these results are obtained by differentiating Θ partially with respect to k. We obtain:

13. Case 1 $p \leq p_{min}$ (complete specialization in capital goods)

$$\frac{\partial \Theta}{\partial k} = - p (1 - s) f_I' < 0$$

14. Case 2 $p_{min} < p < p_{max}$ (incomplete specialization)

$$\frac{\partial \Theta}{\partial k} = f_I' (-(1 - s) - (k_c + W)/(k_I - k_c)) \gtrless 0 \quad k_I \gtrless k_c$$

15. Case 3 $p_{max} \leq p$ (complete specialization in consumption goods)

$$\frac{\partial \Theta}{\partial k} = s p f_c' > 0$$

The comparative statics result of Equation 14 leads immediately to the determination of the regions of specialization. Define the no trade locus

as the set of pairs of capital labor ratios which imply autarchy. For

example, imagine that demand and production functions were exactly the same

in two countries and that capital labor ratios were also exactly the same.

Clearly there would be no difference in the relative prices in the two closed

economies and thus no incentive to trade. Any increase in the capital labor

ratio in country A would shift the offer curve in the manner described by

Equations 13-15. In particular it would raise the relative price of the labor

intensive good at which no trade would occur and, as pointed out by Heckscher

and Ohlin, would provide an incentive for country A to export the capital

intensive good at the old relative price. To prevent trade and remain on the no

trade locus a corresponding increase in B's capital labor ratio would be

necessary. Thus the no trade locus would be a simple 45° line in the (k, k^B)

plane. Above the locus country B would be more capital intensive than A

and trade the capital intensive good for the labor intensive good, while below

the locus country A would be more capital intensive and export the capital

intensive good, giving the Heckscher Ohlin result.

As described above, the Heckscher Ohlin theorem does not hold generally

for all parts of capital labor ratios because of differences in demand in

the countries. Therefore the 45° line is not in general the no trade locus:

however it is easy to see that the no trade locus lies wholly above or below

the 45° line if, when capital-labor ratios are the same, one country can

always be identified as demanding capital intensive goods more intensely.

This assumption simply means that if country A has an intense demand for

capital intensive goods—for example $s > s^B$, $k_I > k_c$ in the Keynesian

case—then, with $k = k^B$ A would export labor intensive goods to get capital

intensive goods. Starting from the situation $k = k^B$, Equation 14 shows that
continued accumulation raises country A's productive capacity in capital
goods more rapidly than its consumption of them. Eventually country A
would become incompletely and finally completely specialized in the
production of capital intensive goods.[8] Thus a positively sloped no trade
locus exists below the 45° line and above the region of complete specialization.[9]
Below the no trade locus country A would export the capital intensive good,
above it the labor intensive good.

We graph the no trade locus $\hat{k}^B = F(\hat{k})$ (where $\hat{k}$, $\hat{k}^B$ signify a pair
of capital labor ratios which result in autarchy) for the case $s > s^B$,
$k_I < k_c$ as the positively sloped line OK in Figure 2, lying wholly above the
45° line. The line OK and the 45° line divide the figure into three regions.
In the first region, below the 45° line, country A is more capital intensive
than B. Since it has little demand for consumption goods and great ability to
produce them, it specializes in the capital intensive consumption good (perhaps
completely) and the Heckscher Ohlin theorem holds. Between the line OK and the
45° line country A is more labor intensive than B. However the intensity of its
demand for labor intensive capital goods more than offsets its relatively
slight productive advantage and continues to force their import in exchange
of exports of consumption goods, nullifying the Heckscher Ohlin theorem as
the above quotes from Ohlin point out. Finally, above the line OK, country
A's intensity of demand for labor intensive goods is offset by a significant
advantage in their production, causing A to specialize, perhaps completely,
in the labor intensive capital good, as the theorem would predict.

IV. Dynamics and the Heckscher Ohlin Theorem (Keynesian Saving)

From Equation 7 and 8 our dynamic equation, expressing the growth rate of the capital labor ratio, is

$$16. \quad h^j = \frac{s^j f_I'(k^j + W)}{k^j} - g$$

where g is the growth rate of population. It is assumed that g is equal in the two countries to prevent one country from becoming infinitely large relative to the other.[9a]

If the countries have attained their long run equilibrium capital labor ratio (k^*, k^{B*}), then

$$17. \quad \frac{s^* f_I'^*(k^* + W^*)}{k^*} = g = \frac{s^B f_I'^{B*}(k^{B*} + W^{B*})}{k^{B*}}$$

Assume $s > s^B$ but that the difference in savings rates is not so great as to cause complete specialization at long run equilibrium. Therefore $W^* = W^{B*}, f_I'^* = f_I'^{B*}$ by our assumption that production functions are the same in the two countries, and we may solve for k^{B*}/k^*.

$$18. \quad \frac{k^{B*}}{k^*} = \frac{s^B(k^{B*} + W^*)}{s(k^* + W^*)} < 1 \quad \text{since}$$

$$\frac{s^B}{s} = \frac{1 + \dfrac{W^*}{k^*}}{1 + \dfrac{W^*}{k^{B*}}} \quad \text{implies } 1 > \frac{k^{B*}}{k^*}$$

To locate the point k^*, k^{B*} in relation to the no trade locus we now use the function $\hat{k}^B = K[\hat{k}]$.

In the simple Keynesian case no trade implies

$$19. \quad sf'_I(\hat{k} + W) = f'_I(k_I + W)(\hat{k} - k_c)/(k_I - k_c)$$

No superscripts are necessary for f'_I, k_i, and W because lack of trade implies that the price ratio, p, is the same in both countries and p determines these variables uniquely. In fact, since these variables are the same in the two countries we have

$$20. \quad (\hat{k}^B - k_c)/s^B(\hat{k}^B + W) = (k_I - k_c)/(k_I + W) = (\hat{k} - k_c)/s(\hat{k} + W)$$

or

$$21. \quad \frac{k^B + W}{k + W} = \frac{1 - s\,\dfrac{\dfrac{k_I - k_c}{k_I + W}}{\dfrac{k_I - k_c}{k_I + W}}}{1 - s\,\dfrac{k_I - k_c}{k_I + W}} \quad \gtrless \quad 1 \text{ as } k_I \gtrless k_c$$

If investment goods are labor intensive, $(k_I < k_c)$ then $s > s^B$ implies A has an intense demand for labor intensive goods. In this case, by our argument of Section III, or by Equation 21 above, the no trade locus will lie <u>above</u> the 45° line as shown in Figure 2. Below the 45° line country A's higher capital labor ratio and its corresponding production advantage in capital intensive goods will coincide with a lack of demand for them. Since the unique[10] point of long run equilibrium (k^*, k^{B*}) lies in the region below the 45° line by Equation 18, long run growth will eventually lead to a situation which satisfies the Heckscher Ohlin proposition. Of course initial factor endowments might yield a temporary situation of demand reversal--for example (k_0, k_0^B) could lie in the region above the 45° line and below the

no trade locus as shown in Figure 2. However in the long run the world

economies would approach the pair (k^*, k^{B*}) which, as shown in Figure 2,

lies in the region in which the Heckscher Ohlin theorem holds.

Alternatively, suppose investment goods are capital intensive

$(k_I > k_c)$, a case which has not been given much consideration in dynamic

trade models owing to the difficulty of proving a unique equilibrium with

Keynesian saving.[11] Then Equation 21 and the argument of Section III imply

that the no trade locus lies below the 45° line, as shown in Figure 3.

However we may determine the relation between (k^*, k^{B*}) and $\hat{k}^B = K[\hat{k}]$ by solving

Equation 20 for $\hat{k}^B/\hat{k}$:

$$
22. \quad \frac{\hat{k}^B}{\hat{k}} = \frac{s^B(\hat{k}^B + W)}{s(\hat{k} + W)} - \frac{k_c}{\hat{k}} \quad \frac{s^B(\hat{k}^B + W)}{s(\hat{k} + W)} - 1 \quad > \quad \frac{s^B(\hat{k}^B + W)}{s(\hat{k} + W)}
$$

Equation 22 implies that although the long run equilibrium pair of capital

labor ratios (k^*, k^{B*}) (which is shown to be unique in the Appendix) and the

no trade locus both fall below the 45° line, (k^*, k^{B*}) falls below the no

trade locus. This means that country A is both capital rich and specializes

in the capital intensive good. Thus while initial conditions might permit

a demand reversal—(k_0, k_0^B) might fall in the region between the 45° line

and above the no trade locus, meaning A imports investment goods as shown

in Figure 3—eventually the Heckscher Ohlin theorem would be verified. The

world economy would move toward the pair of capital labor ratios (k^*, k^{B*})

which lies below the no trade locus.

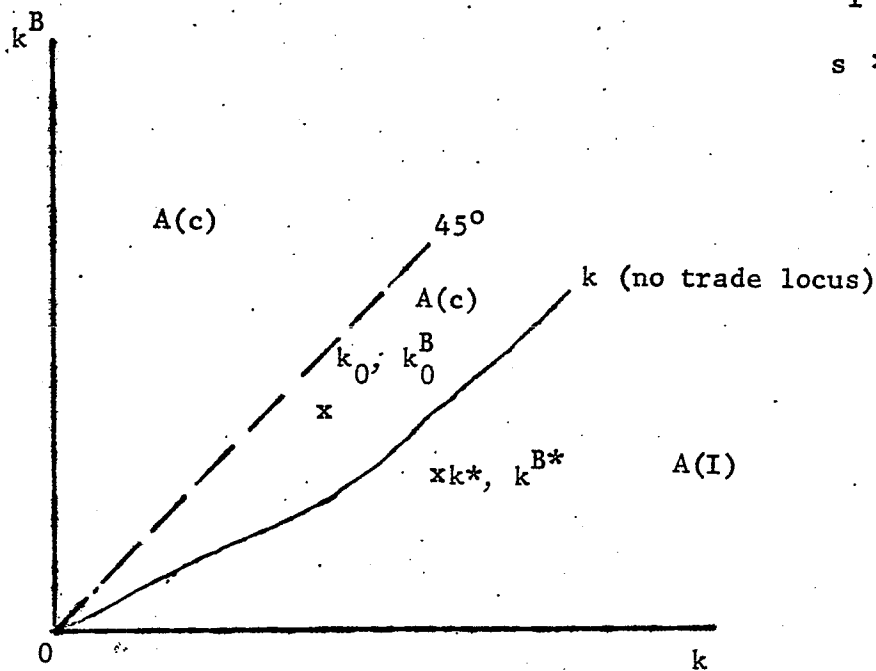If saving rates (or country sizes[12]) are sufficiently different to

Figure 2
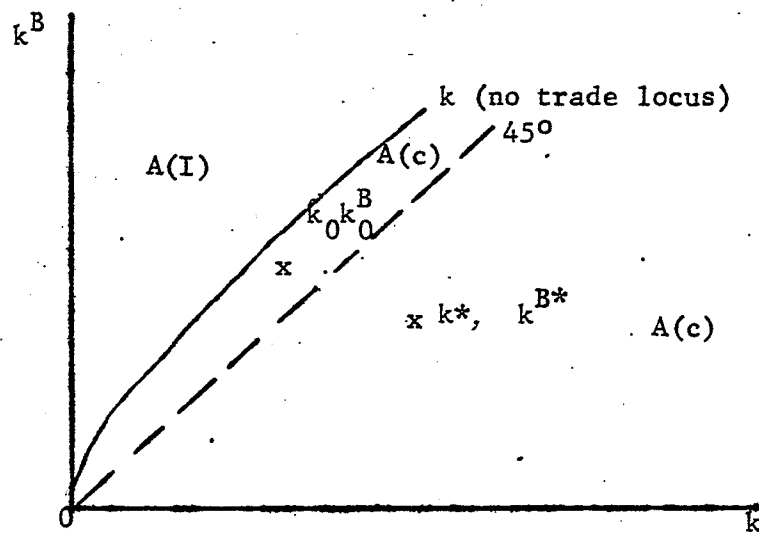
$k_I \lessgtr k_c$

$s \gtrless s^B$

A(i) means A exports;
B imports good i,
i=I, C.

Figure 3

$k_I > k_c$

$s \gtrless s^B$

cause complete specialization in at least one country, the above results still hold. Since the average product of capital in the economy is a decreasing function of the capital labor ratio[13] we know that in the long run $s > s^B$ implies $k^* > k^{B*}$. Now if $k_I > k_c$ ($k_c > k_I$) and there is complete specialization, in one country then in the long run only country A can export capital (consumption) goods while only B can export consumption (capital) goods,[14] verifying the theorem once again.

V. Alternative Saving Behavior

Instead of the simple Keynesian saving assumption a Marxian behavioral equation could have been used:

$$s = sf'_I k/f'_I(k + W).$$

Another possible behavioral assumption is the adjustment to a rational saving policy, using the constant pure rate of time preferences described by Stiglitz (1970). Both assumptions tie each economy to the interest rate in the long run ($f'^j_I = \frac{g}{s^j}$ in the Marxian version, $f'^j_I = \delta^j$ in the rational saving version). Unless this interest rate happens to be the same in the two countries, at least one and perhaps both of the economies must become completely specialized in the long run.[15] In the long run the country with the higher saving rate (lower rate of time preference) will attain the lower long run interest rate. In turn this means that with $s > s^B$ or $\delta < \delta^B$ country A must specialize in capital intensive goods in the long run.[16] Since in long run equilibrium specialization in capital intensive goods means

$k^A = \ell_I^A \, k_I^A + (1 - \ell_I^A)k_c^A > \ell_I^B k_I^B + (1 - \ell_I^B)k_c^B = k^B$, then country A will also have

a higher capital labor ratio in the long run, once again verifying the

Heckscher Ohlin theorem.[17]

Appendix

A Demonstration of the Uniqueness of the Two Country Equilibrium
In the Case Where $k_I > k_c$

To prove the uniqueness of the balanced growth pair of capital labor ratios, $(k^{\alpha*}, k^{\beta*})$, we use the method of Oniki and Uzawa (1965), basically deriving the shape of the two loci of pairs of capital labor ratios which imply a stable value for $k^j (h^j [k^\alpha k^\beta] = 0)$ and then demonstrating that the intersection of $h^\alpha = 0$ and $h^\beta = 0$ is stable and unique. Mathematically we take the total derivative of h:

$$\frac{\partial h^j}{\partial k^\beta} dk^\beta + \frac{\partial h^j}{\partial k^\alpha} dk^\alpha = 0 \qquad h^j = 0$$

and solve for

$$\frac{dk^\beta}{dk^\alpha} = - \frac{\dfrac{\partial h^j}{\partial k^\alpha}}{\dfrac{\partial h^j}{\partial k^\beta}} \qquad h^j = 0$$

To obtain the total derivative we note:

$$\frac{1}{h^\alpha} \frac{dh}{dk^\beta} = \frac{1}{k^\alpha + W} - \frac{1}{k_I + W} \frac{\dfrac{dp}{dk^\alpha}}{\dfrac{dp}{dW}} - \frac{1}{k^\alpha} - \frac{1}{k^\alpha + W} < 0$$

$$\frac{1}{h^{\alpha}} \quad \frac{dh^{\alpha}}{dk^{\beta}} = \frac{1}{k^{\alpha} + W} - \frac{1}{k_I + W} \quad \frac{\frac{dp}{dk}}{\frac{dp}{\partial W}} > 0,$$

assuming stability in the one country case (either $\sigma_c^j \geq 1$, or $\sigma_c^j \geq f_I' k_I / f_I'(k_I + W)$, $\sigma_I^j \geq f_I' k_c / f_I'(k_c + W)$.[18] Consider next the set of pairs of capital labor ratios which imply long run equilibrium in country j, $h^j[k^{\alpha}, k^{\beta}] = 0$. Along this locus in the k^{α}, k^{β} plane

$$\frac{dk^{\beta}}{dk^{\alpha}}\bigg|_{h^j = 0} = - \frac{\frac{dh^j}{dh^{\alpha}}}{\frac{dh^j}{dk^{\beta}}} > 0$$

If the intersection of $h^{\alpha} = 0$, $h^{\beta} = 0$ is to be unique then

$$\frac{dk^{\beta}}{dk|h^{\alpha} = 0} > \frac{dk^{\beta}}{dk|h^{\beta} = 0}$$

Let $\sigma^j = \frac{dk^j}{dw} \quad \frac{W}{k} = \frac{v^{\alpha} \frac{\partial \Theta^{\beta}}{\partial W} + v^{\beta} \frac{\partial \Theta^{\beta}}{\partial^{\beta}}}{-v^j \frac{\partial \Theta^j}{\partial k}} \quad \frac{W}{k}$

Then we obtain the condition for uniqueness

$$\sigma^{\alpha} \frac{(k_I + W)}{k_I - k^{\alpha}} + \sigma^{\beta} \frac{(k_I + W)}{k_I - k^{\beta}} - \sigma^{\alpha}\sigma^{\beta} \frac{(k_I + W)^2}{(k_I - k^{\beta})(k_I - k^{\alpha})} < 0$$

which can be rewritten using the definition of σ^j, as

$$-v^\alpha \left(\frac{\partial\Theta^\alpha}{\partial k^\alpha} \; k^\alpha \; (k_I - k^\alpha) + \frac{\partial\Theta^\alpha}{\partial W} \; W \; (k_I + W) \right) - v^\beta \left(\frac{\partial\Theta^\beta}{\partial k^\beta} k^\beta (k_I - k^\beta) + \frac{\partial\Theta^\beta}{\partial W} W(k_I W) \right) < 0$$

Using Equations 11 and 14 we obtain two terms of the form

$$\frac{v^j f_I'}{(k_I - k_c)^2} \quad \{ (k_c + W)(k_I - k_c) k(k_I - k) + (1 - s)(k_I - k_c)^2 k(k_I - k)$$
$$- \sigma_I k_I (k_c + W)(k - k_c)(k_I + W)$$
$$- \sigma_c k_c (k_I + W)^2 (k_I - k)$$
$$- sW (k_I - k_c)^2 (k_I - k) \} \qquad < 0$$

where the superscript above the capital labor ratio k^j is understood to be the same as for v and omitted.

Adding and subtracting $v^j f_I' (k_c + W)(k_I - k^j)^2 (k_I - k_c)$ and combining terms we obtain

$$- \Theta v^j (k_I - k^j)$$
$$+ \frac{v^j f_I'}{(k_I - k_c)^2} \quad \{ (k_c + W)(k_i - k)(k_I - k_c) k_I$$
$$- \sigma_I k_I (k_c + W)(k - k_c)(k_I + W)$$
$$- \sigma_c k_c (k_I + W)^2 (k_I - k)$$
$$- W(k_I - k_c)^2 (k_I - k) \}$$

The term in brackets can be shown ≤ 0 if $\sigma_c \geq f_I'(k_I)/f_I'(k_I + W)$, in other words if the closed economy conditions for uniqueness hold. Combining the first term for countries α and β and noting k_I are the same in region of incomplete specialization, and that $\Theta^\alpha v^\alpha = \Theta^\beta v^\beta$ by the equality of reciprocal demand, we obtain: $-\Theta v^\alpha(-k^\alpha + k^\beta)$ which is <0, if $\alpha \lessgtr 0$ when $\frac{k^\alpha}{k^\beta} \gtrless 1$, i.e.

if the Heckscher Ohlin theorem is satisfied at the point of intersection. However

Section IV demonstrates this is true in general. Moreover, since all inter-sections must fall into the region in the (k^α, k^β) plane where the Heckscher Ohlin theorem is satisfied, and the intersection of the locus $h^\alpha = 0$ with the no trade locus is above the intersection of the locus $h^\beta = 0$ there can only be one intersection of h^β and h^α, $(k^{\alpha*}, k^{\beta*})$.

The remaining possible intersections, where one or both countries are completely specialized, can be easily shown to be stable, since in these regions the slopes of the loci $h^j = 0$ $(dk^\beta/dk^\alpha \Big|_{h^j = 0}$ are either opposite in sign or take on the values zero or infinity, when the other is positive. Since

$$k^\alpha \Big|_{k^\beta = k^{-\beta}} \gtrless k^{\alpha*} \Big|_{k^\beta = k^{-\beta}} \quad \text{imply} \quad \frac{\dot{k}^\alpha}{k^\alpha} \lessgtr 0 \text{ and similarly for } k^\beta, \ k^\alpha = k^{-\alpha},$$

stability is assured in these regions.

FOOTNOTES

[1] Stiglitz (1970).

[2] Heckscher (1919), Ohlin (1933).

[3] Ohlin (1933), Valavanis-Vail (1954), Jones (1956).

[4] Stiglitz (1970).

[5] Ohlin (1933).

[6] See Johnson (1958) for a description of the importance of this assumption. The well known CES function provides the most common example of a production function which may not satisfy this condition.

[6a] This formulation follows Oniki and Uzawa (1965).

[7] Rybczynski (1955).

[8] This paper does not attempt to define the shape of the regions of complete specialization. For some attempts at this difficult task see Oniki and Uzawa (1965), Bardhan (1966), Hanson (1967) and Stiglitz (1970).

[9] The 45° line must lie between the regions of complete specialization. Otherwise we arrive at the contradiction:

$$k^B = k = k_I > \ell_I^B k_I + (1 - \ell_I^B) \, k_C \text{ with } 0 \le \ell_I^B < \ell = k^B$$

$$k^B = k = k_C < \ell_I^B k_I + (1 - \ell_I^B) \, k_I \text{ with } 0 < \ell_I^B \le \ell = k^B$$

ℓ_I = fraction of labor in investment goods production

[9a] See Bardhan (1965).

[10] See Oniki and Uzawa (1965).

[11]Bardhan (1965), Bardhan (1966) and Stiglitz (1970) deal with Marxian saving, Oniki and Uzawa (1965) with Keynesian saving and the case $k_c > k_I$.

[12]See Hanson (1967) and Stiglitz (1970) p. 471 for a discussion of the role of relative size.

[13]Assuming $k_I < k_c$ or $k_I > k_c$ together with either $\sigma_I \geq f'_I k_c / f'_I (k_c + W)$,

$$\sigma_c \geq f'_I k_I / f'_I (k_I + W) \text{ or } \sigma_c \geq 1.$$

[14]To prove this result we assume the converse and demonstrate a contradiction. If we have long run equilibrium and $k_I > k_c$, then assuming the reverse gives B specialized completely in capital goods, A incompletely or completely specialized in consumption goods or B incompletely specialized in capital goods, A completely specialized in consumption goods, i.e.

$$0 = \ell_c^B, \quad 0 < \ell_c \leq 1$$

$$0 < \ell_c^B < 1, \quad 1 = \ell_c$$

Alternatively, if $k_c > k_I$, then $1 = \ell_c^B$, $0 \leq \ell_c < 1$

$$0 < \ell_c < 1, \quad 0 = \ell_c$$

Therefore in all cases $k^{B*} = (1 - \ell_c^B)k_I + k_c \ell^B > (1 - \ell_c)k_I + \ell_c k_I = k^*$

However this result contradicts the requirement for long run equilibrium that $k^* > k^{B*}$. Therefore we see our assumption was wrong and that country A must be the exporter of capital intensive goods in long-run equilibrium with one of the countries completely specialized.

[15]This result also assumes the existence of a unique equilibrium. See Stiglitz (1970) for the conditions, assumed throughout Section V.

[16]Otherwise we have $k_I > k_c$ A exporting consumption goods, completely specialized or incompletely specialized with B completely specialized $pf_c^{'B} \le f_I^{'B} \le f_I^{'A} \le pf_c$. $k_I < k_c$ A exporting capital goods etc. $pf_c^{'B} \le f_I^{'B} \le f_I^{'A}$. However with $s^A > s^E$ these inequalities mean that long run equilibrium cannot be attained in both countries. Thus A must be specialized in capital intensive goods in the long run.

[17]Two other assumptions on saving-investment behavior also deserve brief mention: Ricardian saving and rational saving with a non-constant rate of time preference. We shall consider only the case of incomplete long run specialization.

Ricardian saving may be interpreted as a saving function of the Marxian form with $s^j = s^j [f_I' \text{ or } k^j]$. However in the former case, k^*, k^{B*} occurring in the region of incomplete specialization is _not_ unique. To see this we note that all capital labor ratios which satisfy $k^B = C + -kV/V^B$ will yield the same world capital to labor ratio, the same wage rentals ratio, and the same interest rate (Hanson 1967). Some of the multiple equilibria may obviously lie in the region of demand reversal.

In the case of $s^j = s [k^j]$ $s' < 0$ the long run equilibrium is unique. Assuming $s [k = k^B] < s^B [k^B = k]$ means country A imports (exports) capital goods along the 45° line. Since long run equilibrium with incomplete specialization requires $s = g/f_I = s^B$ this means $\frac{\partial sk}{\partial ks} = e_{s:k} \gtrless e_{s^B:k^B}$ and long

run equilibrium lies below (above) the 45° line. Now $k_I < k_c$ means that the area below (above) the 45° line is a region in which country A imports (exports) capital goods since it has little (great) ability to produce them and great (little) demand. Thus the Heckscher Ohlin theorem is satisfied since country A imports (exports) capital goods and $k^* \gtrless k^{B*}$. For the case in which $k_I > k_c$ the no trade locus lies above (below) the point where $s[k] = s^B[k^B]$ by the argument of Section IV.

With rational saving $\delta^j = \delta^j[y^j]$, $\delta^j < 0$ we note that in long run incompletely specialized equilibrium $\delta = \delta^B$. Assuming $k^* > k^{B*}$ we find $s = \bar{s} = \bar{s}^B > s^B$ for the set of capital labor ratios which keep y constant. Therefore the arguments of Section IV hold along the locus of pairs of capital labor ratios keeping y constant, again verifying the theorem.

[18]Sato (1969), Hanson (1970).

REFERENCES

Bardhan, P.K. "Equilibrium Growth in the International Economy." O.J.E.
    79 (August 1965): 455-64.

_____ "On Factor Accumulation and the Pattern of International
    Specialization." Rev. Econ. Studies 33 (January 1966): 39-44.

Hanson, J.A. The Terms of Trade and Economic Growth. (Unpublished Ph.D.
    dissertation), Yale University, 1967.

Heckscher, E. "The Effect of Foreign Trade on the Distribution of
    Income." Ekonomisk Tidskrift 21 (1919): 497-512. Reprinted in
    Readings in the Theory of International Trade, edited by F.S.
    Ellis and L.A. Metzler. New York: Unwin, 1949.

Jones, R. "Factor Proportions and the Heckscher Ohlin Model." Rev. Econ.
    Studies, Vol. 24, (October 1956): 1-10.

Johnson, H.G. International Trade and Economic Growth. London: Allen &
    Unwin, 1958.

Ohlin, B. Interregional and International Trade. Cambridge, Mass.: Harvard
    University Press, 1933.

Oniki, H. and Uzawa, H. "Patterns of Trade and Investment in a Dynamic
    Model of International Trade." Rev. Econ. Studies, Vol. 32, (January
    1965).

Sato, R. "Stability Conditions in Two Sector Models of Economic Growth."
    J.E.T. 1, (June 1969): 107-117.

Stiglitz, J. "Factor Price Equalization in a Dynamic Economy." J.P.E. 78,
    (May/June 1970).

Valavanis-Vail, S. "Leontief's Scarce Factor Paradox." J.P.E. 62, (December
    1954): 523-28.