

ECONOMIC GROWTH CENTER

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CENTER DISCUSSION PAPER NO. 16

THE WELFARE ECONOMICS OF EXTERNAL BORROWING

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December 14, 1966

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THE WELFARE ECONOMICS OF EXTERNAL BORROWING

I. Introduction

1.1 In a recent work on "Foreign Assistance and Economic Development" /3/, Hollis B. Chenery and Alan M. Strout use a model that does not take explicit account of the cost of external borrowing. Since the bulk of "assistance" as they define it /Ibid., p. 679 n/ actually constitutes loans and foreign private investment at various contractual terms, the omission of the cost of external "assistance" to recipients of such assistance is unrealistic. It has, moreover, resulted in an analysis that is logically unsatisfactory.

The logical confusion is evident in the following statement (Ibid., pp. 686-687):

"....we need some minimal assumptions as to the objectives of the recipient country and the conditions under which aid is provided. We assume that aid is sufficiently limited---or expensive---to make the recipient country unwilling or unable to increase aid merely to increase consumption without also securing some rise in GNP. Second, we assume that the country tries to maximize consumption until the target growth rate is attained. Under the assumption made, there is is no incentive to....increase aid by reducing saving." (Italics added)

The italicized statement plays a crucial role in the Chenery-Strout analysis which underscores the welfare-efficiency of pushing savings to its maximum---given either by the saving limit or by the minimum trade gap whichever happens to be binding---, a policy, in other words, of minimizing consumption every year to the extent feasible (see, e.g., Ibid., Section C, pp. 697-701). The statement, however, does not logically follow from the assumptions made. It is not clear how a country which tries to maximize consumption and which would presumably

be both able and willing to increase aid to increase consumption after securing some rise in GNP, nevertheless should have no incentive to reduce saving. Under the assumptions made, in fact, the question seems to be wide open, to be settled only by an explicit consideration of the cost of external borrowing.

1.2 The omission of the cost of external borrowing as an explicit argument has led, it appears, to another confusion that has critically impaired the practical usefulness of the model and its analysis as a guide to policy. What has been called gross national product (GNP, defined in p. 683) throughout the analysis actually stands for gross domestic product (GDP) as it is understood in national accounting. The use of GDP, instead of GNP proper, as the growth argument has taken away the welfare relevance of the analysis considerably. Insofar as external borrowing involved a contractual obligation to pay interest that is respected by the recipient country, the relevant growth argument for welfare analysis is not GDP but GNP proper, defined as GDP minus interest charges on external debt.¹

Failure to recognize the above distinction between GDP and GNP has resulted in missing a significant aspect of the economics of external borrowing: as long as a net capital inflow continues into a country, all interest charges are, in effect, compounded.² This has a significant bearing on national welfare accounting for the recipient country, arising from the fact that the same cumulative total of net capital inflows over any given period (number of successive years) gives different cumulative totals of net borrowing (total borrowing inclusive of interest payments but exclusive of debt retirement)³ according to different time-distributions of the former, and vice versa. Since interest is payable on the second item, the argument with which the rate of growth of national income over the period in question---given the rate of growth of domestic product---varies is the

second, irrespective of the magnitude of the first. For example, given the rate of growth of GDP, the rate of growth of GNP over any given period will be lower the more biased is the time-distribution of any given total net capital inflows towards the earlier years. Thus, if we regard national, and not domestic, income as the relevant welfare argument---a principle that is actually practiced in most countries and for sound reasons---, it is not possible to logically formulate a determinate policy of external borrowing without explicitly bringing interest charges, and hence total external borrowing, in the model.

1.3 This brings out the difficulty of logically deducing the Chenery-Strout strategy of pushing domestic saving to its maximum from the assumptions postulated in the authors' own paper. The authors also seek to draw support (see Ibid., p. 701) for their strategy from another work /4/ where a linear social welfare function has been used to deduce the "optimum" time-pattern of growth and capital inflows.⁴ With no non-linearities in the system, optimization would necessarily require going to one extreme or the other in every period; the use of a rate of time-discount that is lower than the rate of return on investment, and an index of psychic disutility from capital inflows⁵ that is higher than the index of utility of consumption, ensures that it is advantageous always to postpone consumption in favour of investment, and to refrain from using external resources for consumption as long as the limit to saving is not reached. But linearity in the social welfare function is a strong assumption that is unsupported either by reason or by evidence. A strategy derived from a linear welfare function, while it may be instructive as

an analytical exercise, cannot claim any normative character, and cannot be accepted as the basis of a general welfare theory of capital inflows and growth.

1.4 In this paper we reconstruct the Chenery-Strout model so that the main element of choice lies in regulating the time-distribution of the flow of external resources into a developing country over a stipulated "perspective-plan" period while keeping fixed the cumulative total of external debts over the whole period. The problem differs from the Chenery-Strout specifications in (a) fixing a target plan-terminal national income as it is actually understood in national accounting, and (b) using a non-linear welfare function to evaluate the stream of consumption during the perspective-plan period. The analysis brings out "optimum" time-patterns of net-external borrowing that are strikingly different from that indicated in the Chenery-Strout strategy and shows in particular how welfare-maximizing behaviour in recipient countries should involve external borrowing not only for investment but for consumption as well if it is possible to borrow in sufficient quantities. The results have significant bearing on the question of rationing external "assistance" to underdeveloped countries if the supply of "assistance" is scarce instead, and suggest that one should use the total savings function, and not marginal savings rates alone as suggested by Chenery and Strout, as the "standard of performance", *if 'self-help' is to be enforced.*

II. The Problem Stated

For analytical convenience the problem is considered as one of continuous time.⁶ The model has the following definitional relations:

(1) $V_t = C_t + I_t - F_t$: Gross domestic product equals consumption plus investment minus net capital inflows, defined as imports minus exports; imports do not include interest payments on outstanding external debt;

(2) $Y_t = V_t - iB_t$: Gross national income equals gross domestic product minus interest payments;

(3) $F_t = \dot{B}_t - iB_t$: Net capital inflows equal change in external indebtedness (net total borrowing) minus interest payments.

These three relations yield:

$$(4) \quad C_t = V_t - I_t + \dot{B}_t - iB_t.$$

The production function is given by the familiar Harrod-Domar equation:

(5) $\dot{V}_t = bI_t$: Marginal output-capital ratio is a constant, 'b'.

Investment has to satisfy the following:

(6) $I_t \leq I_0 \cdot e^{\beta t}$: Upper limit to investment (absorptive capacity) grows at a constant rate 'β', starting from initial investment (historically given as an initial condition).

The problem, presented in a general form, is to maximize $\int_0^T U(C_t) dt$, subject to

(7) $Y_T = Y_0 \cdot e^{gT}$: maximize the integral of utility derived from (instantaneous) consumption over a plan-period covering T time-units, subject to the condition that over this period national income should grow at a given average rate, 'g'.

Requirement (7) imposes only a terminal condition on the growth of national income. In order to allow the model maximum freedom to reveal its basic character in relation to maximizing the postulated functional, no constraint is imposed on the time-path of national income within the plan-period, nor any on the amount of consumption that the nation can physically absorb during any given interval of time.

A time-discount is not used for standard ethical reasons.⁷

III. Analytical Solution

It is easy to see that if 'b' is higher than the rate of interest 'i' on external borrowing, it would be efficient to invest up to investment-absorptive capacity always.⁸ In other words, relation (6) would hold as a strict equality in the optimum programme throughout. Assuming $b > i$, we can then proceed with

$$(6.1) \quad I_t = I_0 \cdot e^{\beta t}.$$

Substituting (6.1) into (5) and integrating we obtain the optimum time-path of gross domestic product:

$$(8) \quad V_t = \frac{b}{\beta} \cdot I_0 e^{\beta t} + (V_0 - \frac{b}{\beta} \cdot I_0).$$

Normalizing V_0 as unity, we have

$$(8.1) \quad V_t = \frac{b}{\beta} \cdot I_0 e^{\beta t} + (1 - \frac{b I_0}{\beta})$$

Using (6.1) and (8.1) in (4), we have

$$(4.1) \quad C_t = \lambda_1 e^{\beta t} + B_t - i B_t + \lambda_2; \text{ where } \lambda_1 = I_0 (\frac{b}{\beta} - 1); \lambda_2 = 1 - \frac{b I_0}{\beta}$$

Choice of time-path of consumption, C_t , now reduces to choice of the time-path of external borrowing, B_t .

We now have a problem in the classical calculus of variations where the time-path of B is to be found so as to maximize the functional $\int_0^T U(C_t) dt$, with C_t a function of B_t and $\dot{B}_t$ as given by (4.1). The initial and terminal conditions are given by

(9) $B_0 =$ given (outstanding indebtedness at the beginning of the plan-period); and

(10) $B_T = \frac{1}{i} (V_T - Y_T)$, with V_T and Y_T given by (8) and (7) respectively.

The Euler equation is $U_B - \frac{d}{dt} U_{\dot{B}} = 0$, or $-i \cdot U'(C_t) = U''(C_t) \cdot \dot{C}_t$ (using 4.1), whence

$$(11) \quad \frac{\dot{C}_t}{C_t} = \frac{i}{\epsilon}, \text{ where } \epsilon = - \frac{C_t}{U'(C_t)} \cdot U''(C_t)$$

Assuming $U''(C_t) < 0$, i.e., diminishing marginal utility from instantaneous consumption, ϵ stands for the absolute value of the point-elasticity of marginal utility. Equation (11) then says that optimum rate of growth of consumption at every point of time equals the ratio of the rate of interest on external borrowing over the absolute value of the elasticity of marginal utility from consumption at that point of time.

For simplifying the rest of the analysis, we shall assume that the elasticity of marginal utility from consumption is a constant, so that $U(C_t)$ may be represented as

(12) $U(C_t) = \frac{1}{1-v} C_t^{1-v}$, a form that is already familiar in the optimum-growth literature, with 'v' standing for the constant elasticity of marginal utility from instantaneous consumption.

The optimality condition (11) then becomes

$$(11.1) \quad \frac{\dot{C}_t}{C_t} = \frac{i}{v}$$

Combining (11.1) with (4.1) and arranging terms, we have the differential equation

$$(13) \quad \frac{d^2 B}{dt^2} - \frac{i(1+v)}{v} \frac{dB}{dt} + \frac{i^2}{v} B_t - \frac{(1-v\beta)}{v} \cdot \lambda_1 \cdot e^{\beta t} - \frac{i}{v} \lambda_2 = 0,$$

with initial and terminal conditions given by (9) and (10).

The solution of this differential equation is given by

$$(14) \quad B_t = \frac{1}{\Delta} (e^{\frac{i}{v}T} \alpha_1 - \alpha_2) e^{it} + \frac{1}{\Delta} (\alpha_2 - e^{iT} \alpha_1) e^{\frac{i}{v}T} - \frac{\lambda_1}{\beta-i} e^{\beta t} + \frac{\lambda_2}{i}, \text{ where}$$

$$\Delta = e^{\frac{i}{v}T} - e^{iT};$$

$$\alpha_1 = B_0 + \frac{\lambda_1}{\beta-i} - \frac{\lambda_2}{i};$$

$$\alpha_2 = B_T + \frac{\lambda_1}{\beta-i} e^{\beta T} - \frac{\lambda_2}{i}$$

Equation (14) gives, under the assumptions of the problem, the optimum time-path of external borrowing during the stipulated perspective-plan period. Correspondingly, the optimum time-path of consumption is given by

$$(15) \quad C_t = C_0 \cdot A_2 e^{\frac{i}{v}t}, \text{ where}$$

$$C_0 = i\left(\frac{1}{v}-1\right) \cdot A_2.$$

IV. Quantitative Solutions

In this section quantitative solutions are presented for two different values of 'v', using Pakistan's figures as the initial conditions, and the Chenery-MacEwan specifications in /3/ about the structural parameters for this country. The target growth rate for GNP is arbitrarily fixed at 7 per cent, and the rate of interest on external borrowing likewise at 6 per cent. The qualitative characters of the two solutions and the lessons obtained therefrom are believed to be of fairly general validity.

The various parameters in the two exercises have the following values:

Initial conditions (normalized)¹⁰

$$V_0 = 1;$$

$$I_0 = .15842 \quad B_0 = .05383; \quad Y_0 = 1 - iB_0 = .99677$$

Incremental output-capital ratio: $b = .33$;

Rate of growth of investment-absorptive capacity: $\beta = .13$

Target growth rate for GNP: $g = .07$;

Rate of interest on external borrowing: $i = .06$;

Elasticity of marginal utility from instantaneous consumption: $v = 2, .6$.¹¹

The solutions for the two values of 'v' are given in Tables 1 and 2 respectively.

Main features of the quantitative solutions

Solution 1 : $v = 2$

1) The rate of borrowing (B_t , the rate of change in outstanding indebtedness) is heavy from the beginning, and rises slowly. Net capital inflows (F_t) is also heavy initially and rises for the first few years. After a point it starts to decline, and actually becomes negative in a last phase of the plan.

(Since initially interest payments are low, the bulk of total borrowing consists of net capital inflows. Gradually, interest payments mount up, and a larger and larger portion of total borrowing is used up only to pay interest charges so that less and less net capital inflows are obtained. Towards the end of the plan total borrowing is not enough even to cover all interest payments, so that a reverse flow of net capital is generated.)

2) Consumption (C_t) is heavy from the beginning, and rises slowly (at the rate i/v , i.e., 3 per cent a year). It exceeds gross domestic product (V_t) for about the first 12 years, after which GDP catches up and surpasses consumption. Consumption exceeds gross national income (Y_t) until the last two years of the plan.

Gross domestic saving, defined as gross domestic product minus consumption (S_t^d), is accordingly negative in the first 12 years and positive thereafter. The rate of domestic saving reaches the high figure of around 55 per cent at the end of the plan. Gross national saving (S_t^n , column), defined as gross national product minus consumption, is negative until the last two years of the plans, when it becomes positive and rises sharply to reach a plan-terminal rate of about 18 per cent.

(3) Gross national income (Y_t) initially falls and then rises, slowly at first and accelerating later, reaching a plan-terminal rate of growth of approximately 16 per cent per annum.

Solution 2: $v = .6$

1) The rate of borrowing is low initially and rises rapidly over time. The time-distribution of net capital inflows is skewed in the opposite direction, (i.e., biased in favour of later years) than in solution 1.

2) Consumption is lower initially than in solution 1, but grows at a much faster rate (10 per cent a year). It exceeds both domestic and national product from after the second year, so that both gross domestic saving and gross national saving are negative from this time on.

3) Gross national income rises throughout the period, accelerating less rapidly than in solution 1, and reaches a plan-terminal growth rate of approximately 11 per cent per annum.

The cumulative total of net capital inflows, $\int_0^T F_t dt$, is substantially higher in solution 2 than in solution 1.

Table 1
(V = 2)

t	y_t	V_t	I_t	B_t	$\hat{B}_t$	IB_t	F_t	C_t	$\frac{S_t^d}{V_t}$	$\frac{S_t^D}{V_t}$	$\frac{V_{t+1}-V_t}{V_t}$	$\frac{Y_{t+1}-Y_t}{Y_t}$
0	.997	1.000	.158	.054	.986	.003	.983	1.725	-.725	-.730		
1	.991	1.056	.180	1.082	1.069	.065	1.004	1.880			.056	-.006
2	<u>.987</u>	1.119	.205	2.193	1.155	.132	1.023	1.937				<u>-.003</u>
3	.988	1.192	.234	3.392	1.242	.204	1.038	1.996				<u>+.001</u>
4	.993	1.274	.266	4.677	1.330	.281	1.049	2.057				
5	1.005	1.368	.303	6.051	1.418	.363	1.055	2.120	-.550	-1.109	.074	+.012
6	1.024	1.475	.346	7.512	1.505	.451	<u>1.055</u>	2.184				
7	1.053	1.597	.394	9.061	1.591	.544	1.047	2.250				
8	1.094	1.736	.448	10.693	1.673	.642	1.032	2.320				
9	1.150	1.894	.510	12.406	1.751	.744	1.007	2.391				
10	1.221	2.073	.581	14.193	1.822	.852	.970	2.462	-.188	-1.164	.095	+.062
11	1.315	2.278	.662	16.047	1.884	.963	.921	2.537				
12	1.435	2.512	.754	17.957	1.935	1.077	.858	<u>2.616</u>				
13	1.582	2.777	.859	19.911	1.970	1.195	.776	<u>2.694</u>				
14	1.766	3.080	.978	21.893	1.988	1.314	.674	2.776				
15	1.991	3.424	1.113	23.880	<u>1.983</u>	1.433	.550	2.861	+.164	-1.437	.112	.121
16	2.266	3.817	1.268	25.850	<u>1.950</u>	1.551	.400	2.949				
17	2.598	4.264	1.444	27.770	1.885	1.666	.218	3.038				
18	2.997	4.773	1.645	29.605	1.779	1.776	<u>.003</u>	3.131		-.045		
19	3.473	5.352	1.873	31.312	1.625	1.879	-.253	3.226		+.071		
20	4.042	6.012	2.133	32.837	1.415	1.970	-.555	3.324	+.553	+.178	.123	.164

$\int_0^T \dot{F}_t dt = 14.668$

Table 2
(V = .6)

t	Y _t	V _t	I _t	B _t	B _t	IB _t	F _t	C _t	$\frac{S_t^d}{V_t}$	$\frac{S_t^n}{V_t}$	$\frac{V_{t+1} - V_t}{V_t}$	$\frac{Y_{t+1} - Y_t}{V_t}$
0	.997	1.000	.158	.054	.057	.003	.054	.896	.104	.101		
1	1.047	1.056	.180	.143	.123	.009	.114	.990	.062	.054		
2	1.101	1.119	.205	.303	.198	.018	.180	<u>1.094</u>	.022	.006	.056	.050
3	1.160	1.192	.234	.542	.284	.032	.252	<u>1.210</u>	-.015	-.043		
4	1.222	1.274	.266	.873	.381	.052	.329	1.337				
5	1.290	1.368	.303	1.308	.490	.078	.412	1.477			.074	.056
6	1.363	1.475	.346	1.859	.614	.112	.502	1.631	-.080	-.145		
7	1.445	1.597	.394	2.540	.752	.152	.600	1.803				
8	1.534	1.736	.448	3.369	.908	.202	.706	1.994				
9	1.632	1.894	.510	4.362	1.081	.262	.819	2.203				
10	1.741	2.073	.581	5.538	1.274	.332	.942	2.434	-.174	-.398	.095	.067
11	1.863	2.278	.662	6.918	1.489	.415	1.074	2.690				
12	2.001	2.512	.754	8.525	1.727	.511	1.216	2.974				
13	2.154	2.777	.859	10.380	1.990	.623	1.367	3.285				
14	2.329	3.080	.928	12.512	2.280	.751	1.529	3.681				
15	2.527	3.424	1.113	14.950	2.600	.897	1.703	4.014	-.172	-.588	.112	.085
16	2.754	3.817	1.268	17.722	2.950	1.063	1.887	4.436				
17	3.013	4.264	1.444	20.850	3.333	1.251	2.082	4.902				
18	3.309	4.773	1.645	24.402	3.754	1.464	2.290	5.418				
19	3.649	5.352	1.873	28.380	4.210	1.703	2.507	5.986				
20	4.042	6.012	2.133	32.837	4.708	1.970	2.738	6.617	-.110	-.637	.123	.108

$$\int_0^T F_t dt = 21.888$$

V. Lessons from the Exercises

From the purely economic point of view the two exercises demonstrate how very wide the time-distribution of net capital inflows, and also the cumulative total of these inflows, can be corresponding to the same cumulative total of external borrowing (B_t) over a given time-interval (20 years in our problem).. This brings out the necessity for planners in the recipient country to face squarely the choice that has been designed in this paper in any rational approach to external borrowing that uses a target growth rate in terms of national income over a sufficiently long period.

Insofar as variation of the time-distribution of external borrowing varies the time-path of consumption accordingly--given the time-path of investment--, rational planning must inevitably rest on some sort of ordering of alternative consumption programmes over time. The use of a conventional non-linear valuation of the stream of consumption, with two widely different but entirely plausible values of the elasticity of marginal utility from instantaneous consumption, indicates one common feature of the resulting "optimum" policies that is radically different from what is suggested in the Chenery-Strout paper: In both the situations optimum consumption exceeds both GNP and GDP, and optimum savings is far less than any conceivable 'saving limit', for significant stretches of the plan-period. This indicates unequivocally that external borrowing would be used for increasing consumption during significant time stretches under any reasonable welfare-maximizing behavior on the part of recipient countries where the incremental output-capital ratio is significantly higher than both the rate of interest on external borrowing and the target growth rate for national income. The underlying argument is easy to see intuitively:

With a plan-period sufficiently large, the high productivity of investment would make terminal national income substantially higher than the target if investment were always maintained at the rapidly growing level of absorptive capacity and domestic saving maintained at its 'upper limit'. The margin thus made available can be used, as it were, to increase consumption, i.e., to reduce saving, by further external borrowing during the plan-period. In other words, the "incentive to increase aid by reducing saving" would be very much there: if the country is able to increase aid to increase consumption after securing the stipulated rise in GNP, it will clearly be rational for it to do so, unless it were unwilling to do so for wholly extra-economic reasons.

How exactly the above margin will be "distributed" for consumption over the different time-phases of the plan-period will depend on how fast marginal utility from consumption at every point of time falls with additional consumption. As is well-known, and is also demonstrated in the exercises in this paper, a higher elasticity of marginal utility calls for a higher initial consumption and a lower rate of growth of consumption than a lower elasticity. It is also of interest to note that the "discrimination" against earlier consumption that a lower elasticity indicates also implies a higher cumulative total of net capital inflows over the given plan-period (i.e., a higher total real transfer of resources over the period) out of the same cumulative total of external borrowing.

VI. Conclusion

Three extreme assumptions--(1) the target growth rate of national income is concerned only with the average growth rate over the whole perspective-plan period;

(2) that it is possible to absorb any amount of consumption during any time interval; and (3) a perfectly elastic supply of external finance--have certainly made our model unrealistic. The analysis nevertheless highlights a moral that is of general validity: the availability of external resources enables a country to have any given rate of growth of national income with less austerity on its own part. There is no reason why developing countries should not see this possibility and act accordingly.

Coming back to the Chenery-Strout analysis, the emphasis laid on maximizing domestic savings and thereby minimizing the cumulative total of net capital inflows during a country's transition to "self-sustaining growth" is understandable from the point of view of donours to external "assistance", but is not convincing as an efficient strategy from the point of view of the recipient country's welfare. The harmony of interests that the Chenery-Strout analysis presumes does not exist in reality. Empirical evidence can be cited from the Pakistan case as it has been noted in the Chenery-Strout paper itself (Ibid., p. 694,n), over the period 1954-55 to 1959-69, the period which marks the beginning of a steady flow of external resources into the country, Pakistan shows a negative marginal rate of saving, indicating that a part of foreign aid must have been used to increase consumption.

In the presence of such a conflict of interests between donours and recipients of external "assistance" the wisdom of rationing scarce external capital among the latter on the basis of marginal savings rates as suggested by Chenery and Strout is open to question. If self-help is to be forced on the recipients--from the analysis of welfare-maximizing behaviour in this paper forced, it seems, it has to be--external "assistance" should be related to the total savings function and not

the marginal rate of saving alone. For, abstracting from other differences, a country with a higher total savings function albeit with a lower marginal rate of saving will be showing more self-help than one with a lower total savings function albeit with a higher marginal savings rate in the relevant income range; the former will also need less cumulative total of net capital inflows to reach the stage of "self-sustaining growth",¹² and will reach this stage earlier. This point is missed in the Chenery-Strout analysis because of implicitly defining the saving limit -- this definition is explicit in the Chenery-MacEwan paper--as taking off from the historically given trend savings in the base-year, as can be seen from the quantitative exercises in the paper. The choice of saving is presented as choice of the marginal savings rate only, assuming away a large part of the problem of optimizing the rate of saving¹³ in a developing country both from its own point of view and also from the point of view of the donors of external "assistance". Quite apart from this, as an efficiency criterion the idea of relating "assistance" to marginal savings rates alone and not to the total savings function is liable to be self-defeating; for it would encourage a country motivated by understandable worldly desires to consume lavishly until it confronts the aid-giving "consortium", thereby pushing down the historical trend of saving as much as it can, and then promise a high marginal rate of saving in order to get "whatever amount of aid it requested". And as a principle of allocation of "assistance" it is liable to be inequitable, discriminating as it would against a country showing greater self-help but failing if it does to show a high enough marginal rate of saving.

REFERENCES

1. Andrews, J.R. and Mohammed, A.F., Trade, Finance and Development in Pakistan, Oxford, 1966.
2. Chakravarty, S., "Optimal Savings with Finite Planning Horizon," International Economic Review, Vol. 3, No. 3., September, 1962, pp. 338-355.
3. Chenery, Hollis B. and Strout, Alan M., "Foreign Assistance and Economic Development," American Economic Review, Vol. 56, September, 1966, No. 4, Part 1, pp. 679-733.
4. Chenery, Hollis B., and MacEwan, Arthur, "Optimal Patterns of Growth and Aid: The Case of Pakistan," Pakistan Development Review, Vol. 6, Summer, 1966, No. 2, pp. 209-242.
5. Frisch, R., New Methods of Measuring Marginal Utility, Verlag Von J.C.B. Mohr (Paul Siebeck), 1932.
6. Goodwin, R.M., "The Optimal Growth Path for an Underdeveloped Economy," The Economic Journal, Vol. 71, No. 284, December, 1961, pp. 756-774.
7. Hamada, Koichi, "Economic Growth and Long-term International Capital Movements," Yale Economic Essays, Vol. 6, No. 1, Spring, 1966, pp. 49-98.
8. IBRD, "Economic Growth and External Debt--An Analytical Framework", Proceedings of the United Nations Conference on Trade and Development, Vol. 5, New York, 1964.
9. Ramsay, F.P., "A Mathematical Theory of Saving," Economic Journal, December, 1928, pp. 543-559.
10. Tinbergen, J., "Optimum Savings and Utility Maximization Overtime," Econometrica, Vol. 28, No. 2, April, 1960, pp. 481-489.

FOOTNOTES

1. For expositional simplicity, we assume in this paper that interest payment is the only element of factor payments to foreigners. The general lessons derived from the analysis are not crucially dependent on this assumption.

2. Interest charges may be paid as they accrue, individually, but a positive net capital inflow implies that the country must have borrowed to pay interest charges also, in addition to borrowing to finance an excess of imports (excluding interest payments) over exports.

3. It is necessary to distinguish between at least three different concepts: (1) an excess of imports, exclusive of interest payments, over exports: this is called net capital inflows in this as in the Chenery-Strout paper; (2) the amount by which outstanding indebtedness changes, i.e., net capital inflows as defined above plus interest payments on outstanding debts: this will be called net borrowing, often simply borrowing; (3) net borrowing as defined above plus debt retirement: this does not appear in the present analysis but the term gross borrowing may be recorded to complete the picture.

4. The welfare function used is of the form $W = \sum_{t=1}^{\infty} D_t (C_t - \gamma F_t)$, where C and F stand for consumption and foreign aid, γ the index of disutility of aid, and D a discount factor which follows the familiar compounding principle except that the rate of discount is raised (from .08 to .10) as t passes T, the last year of the "Perspective Plan" period.

5. Curiously, the cost of foreign aid in terms of interest charges is not considered in this work either, while the bulk of foreign 'aid' for Pakistan with which this work is specifically interested, consist of loans and foreign private investment.

6. The model is in some respects similar to one used by Hamada /7/, but differs from the latter in (a) using a finite instead of infinite plan-horizon; (b) imposing a target national income for the plan-terminal year; and (c) following the Chenery-MacEwan-Strout specification about relation between output and investment.
7. See Ramsay /9/ and Tinbergen /10/.
8. The proof is left to the reader.
9. See Chakravarty /2/; also Tinbergen /10/ and Goodwin /6/.
10. The actual values are (at current prices): V_0 (1964-65) = Rupees 46906 million, using c.s.o. wholesale price deflator on an estimate of GDP at constant prices kindly supplied by Gustav Papanek (unpublished); I_0 (1964-65) = Rupees 7431 million, from Pakistan's Third Five-Year Plan; B_0 (outstanding at the beginning of 1964) = Rupees 2525 million, from Andrews and Mohammed /1/.

An error in specification of the initial structure of Pakistan is because of the discrepancy between the time argument of B_0 and that of V_0 and I_0 . This is not of major importance in the exercises designed to provide qualitative insights only.

11. Frisch's 1932 study /5/ of individual elasticities range from well below 1 to well above 6, rising as the level of consumption falls. Although the elasticity of marginal utility from consumption in the social welfare function need not follow directly from observed individual elasticities, the latter may be presumed as a guide to a plausible range for the former.
12. The Chenery-Strout definition of "self-sustaining growth" defining it as "growth at a given rate with capital inflows limited to a specified ratio to GNP which can be sustained without concessional financing," (italics added) is logically overdefined. Of the two conditions in italics, only one will in general be binding. A more satisfactory definition is provided in /8, p. 105/: "Self-sustained growth is defined to mean a rate of income increase of, say, 5 per cent p.a., financed out of domestically generated funds and out of foreign capital which flows into the country because it wants to do so."

13. As a corollary, the problem of maximizing the rate of saving is conceived as a problem of maximizing the marginal rate of saving only. A particularly serious logical difficulty follows from this: Suppose, given the historical ex-post saving in some (arbitrary) base year, t_0 , an upper-limit saving function has been estimated by estimating the upper-limit to the marginal rate of saving only. Suppose now that in some subsequent year, t_1 , the trade-limit rather than the saving-limit thus defined is binding, so that ex-post saving in year t_1 is less than estimated upper-limit saving for this year. A serious question arises whether the upper-limit saving function for years subsequent to t_1 would be given by the function estimated with reference to ex-post saving in the arbitrarily chosen "base year" or with reference to that in year t_1 . It has got to be the latter, if the maximum marginal rate of saving is defined to be independent of historical saving prior to the year in question, as Chenery and Strout appear to have defined it.

This indicates that the whole concept of saving-limit, and the concomittant theory of the two-resource gaps as presented in the Chenery-Strout paper, rest on elusive definitions. It also questions the validity of the saving-limits used in the quantitative exercises in the said paper: the historical "base-year" saving from which the saving-limit-functions for the respective countries start may, for some of them at least, actually have been the result of a historically binding trade-limit, a possibility that Chenery and Strout seem to have missed by dropping the import-content of investment from equation 13 (p. 689) in their paper (in the corresponding equation in the Chenery-MacEwan paper --- equation 10, p. 217 --- the marginal import rate on investment is estimated to be as high as .35, while it is zero in equation 13 of the Chenery-Strout paper.) Even for countries where this has not happened, the question may be raised whether historically such countries have indeed been saving their utmost! On the other hand, a parallel question may be asked for countries where historically the trade-limit has not been binding: have such countries been minimizing imports historically? If not, what is the validity of including ex-post imports of an arbitrary base year as an argument in the minimum import function?