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EFFECTS OF CHANGES IN THE LEVEL AND DISTRIBUTION OF CAPITAL ON INCOME INEQUALITY

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Summary measures of inequality in the size distribution of income tend to increase during the early stages of development and subsequently decline as a country matures. One explanation of this observed trend, adopted by Kuznets and others [6] and [7], is based on factor-price distortions. These are characterized by the fact that per capita income in general, and wage income in particular, tends to be higher in the urban than it does in the rural sector, and that this disparity widens as a country develops. It is well known that, in the simple two-sector case where such an income discrepancy exists, the variance of per capita income is maximized when the number of recipients residing in the two sectors is equal. In the initial stages of development, the share of the population residing in the rural sector is substantially greater than 50 percent. It often becomes less than 50 percent as a country matures. Consequently, simply as a result of normal migration, as well as growing per capita income disparity, an almost inevitable parabolic relationship between summary measures of inequality and per capita GDP will exist.

This explanation, which makes per capita GDP the main determinant of changes in size distribution, ignores the tendency of open unemployment and underemployment rates to rise during the early and fall during the later stages of growth process. In cases where the number of laborers in each primary economic unit (e.g., the household) is small, the frequency distribution of units by employment rate is likely to be highly uneven almost binomial. Moreover, we would expect the variance of this distribution to increase as the mean employment rate falls. Now assume that laborers whose occupational status has an associated income below the mean also have a higher probability of being unemployed. Then, as Schultz indicates [8], given these conditions, increases in the aggregate employment rate will cause the overall size distribution to become more equal not only by reducing the discrepancy in mean incomes between owners of labor and capital but by making the size distribution of wage income more even.

In this paper, we present a model designed to examine the impact of changes in the aggregate capital-labor ratio on the size distribution of income through its relationship to the employment rate. In part, our analysis does reflect the implication of the Harrod-Domar model that, in a one-good world with fixed proportions, increases in capital stock will increase employment when capital is scarce. In addition, however, we examine the effect of output composition on the employment rate in a model with factor-price rigidity. Such analysis necessarily involves a general equilibrium system, and must allow for the possibility that changes in income dispersion feedback on the employment rate, however small this effect may be.

Income dispersion affects the employment rate through its influence on the labor-intensity of the output mix in our system. A major determinant of income dispersion, in addition to the employment rate, is the variance of household capital holdings. This variable, as well as the variance of household wage income, may affect the employment rate through an association with the bill of goods demanded. Consequently, to make our analysis complete, we do examine the impact of asset redistribution on the employment rate, although the direct effect of such a policy on income distribution may well predominate in most cases.

In section I, aggregate savings and consumer demand functions are derived.

In section II an equilibrium production point in a two-sector model based on these functions is explained. In section III, the impact of exogenous changes in the level and dispersion of capital per household on aggregate output, employment and income inequality is investigated. The final section involves a summary of the results and a discussion of the possible implications of relaxing certain critical assumptions.

I. Commodity Demand Functions

The model is constructed on the assumption that there are two-traded goods. Good 1, the import-competing commodity, is both a capital and consumption good. Good 2, the exportable, is strictly a consumption good. In this section, we shall derive private consumption demand function for goods 1 and 2 with both level and disparity of household wage income as separate arguments. Before deriving these functions, however, we first demonstrate how savings is determined; the composition of consumption demand is generally assumed to be an explicit function of total consumption expenditure, i.e., income less savings. Income, in most cases, enters individual commodity demand functions only implicitly through the expenditure argument.

Household and Aggregate Savings

Assuming constant household size and a total population of N households, let us define the following quantities for the j-th household

(1.1) $c_j \triangleq$ consumption

(1.2) $s_j \triangleq$ savings

(1.3) $k_j \triangleq$ capital stock (excluding human capital)

(1.4) $w_j \triangleq$ wage income

(1.5) $\quad y_D^j \triangleq$ disposable capital income

Then we have savings, s_j, equal to the rate of change of capital, i.e.

(1.6) $\quad s_j =$ gross changes in capital stock k_j (assuming zero net foreign capital inflow)

But,

(1.7) $$s_j = \text{total income} - \text{consumption}$$
$$= y_D^j + \hat{w}_j - c_j$$

We assume that the function determining the consumption of the j-th household is of the form

(1.8) $$c_j = \alpha k_j + \hat{w}_j$$

where

$$0 \leq \alpha \leq 1$$

i.e. all wage income--which is untaxed--and a constant fraction α of capital income are allocated completely to consumption.

Equation (1.8) is consistent with a number of theories of consumption behavior. If it is assumed that the ratio of real cash balances to capital assets remains constant, then the relationship is similar to one proposed by Tobin [9] which makes consumption proportional to real wealth. If, on the other hand, individual households save in order to maintain a fixed ratio of capital assets to normal income, then, given no adjustment lag, α may be interpreted as the product of the reciprocal of this ratio and the marginal propensity to consume out of normal income.

The expression for savings s_j now becomes from (1.7) and (1.8)

$$(1.9) \qquad s_j = y_D^j - \alpha k_j$$

Denote aggregate domestic savings per household by s, disposable capital income by y_D, and capital stock per household by k. Then, from (1.9) we obtain the following aggregate savings function

$$(1.10) \qquad s = y_D - \alpha k$$

Private Consumption Demand

Given these formulations, consumption demand functions for goods 1 and 2 in the model which include the employment rate explicitly may be derived in the following manner. Clearly aggregate consumption per household of commodity i , given a non-linear household demand function, will depend not only on aggregate consumption per household but on the entire size distribution of household expenditure. Under certain conditions, we need only consider two moments of this distribution and the function determining aggregate private consumption per household of commodity i may be written as

$$(1.11) \qquad c_i = c^i (c, \sigma_c^2, P)$$

where σ_c is the standard deviation of distribution of household consumption, and P is the ratio of the price of good 2 to the price of good 1. An exact derivation of this function exists in the case of a quadratic demand function.[1] Suppose that the household demand function has the form

$$(1.12) \qquad c_i^j = a_{i0} + a_{i1}\, c^j + a_{i2}\, (c^j)^2 + a_{i3}\, (c^j \cdot P) + a_{i4}P + a_{i5}P^2$$

where the j superscripts designate the value of consumption of the j-th household. Taking expected values of this expression yields

$$(1.13) \qquad c_i = a_{i0} + a_{i1}\, c + a_{i2}\, (c^2+\sigma^2 c) + a_{i3}\, (c_i P) + a_{i4}P + a_{i5}P^2$$

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[1]The aggregate commodity demand function may also be derived from a household demand function of the form

$$c_i^j = A_i (c^j)^{\eta_{ic}} (P)^{\eta_{ip}}$$

where the exponents represent partial elasticities, which are assumed constant. In this case, the form of the distribution of expenditure per laborer must be restricted to be log normal. It can be shown that, under these assumptions, the function determining the aggregate value of c_i may be written as

$$c_i = e^{\mu/2} A_i\, (c)^{\eta_{ic}} (P)^{\eta_{ip}}$$

where

$$\mu = \eta_{ic}\, (\eta_{ic} - 1)\, \left(\frac{\sigma_c}{c}\right)^2$$

Well-behaved household demand functions relating consumption of good i to expenditure are monotone of the range being considered and pass through the origin. It can be shown that the expenditure elasticities are greater than (less than) unity according as the functions are convex (concave) to the origin. Denoting the expenditure elasticity of demand of the jth household for ith commodity by η_i^j, we have

$$(1.14)\quad \partial c_i/\partial\sigma_c^2 \gtreqless 0$$

$$\text{as } \eta_i^j \gtreqless 1$$

i.e., according as the household commodity demand functions are convex or concave.

The expression for total private consumption per household has the form in the aggregate as it does in the individual household case, which is shown in equation (1.8). Moreover, given that kj and ŵj are independently distributed from (1.8), it is clear that

$$(1.15)\quad \sigma_c^2 = \alpha^2\sigma^2 + \sigma_w^2$$

where σ^2 is the variance of household capital holdings and $\sigma^2 w$ is the variance of household wage income. By substituting (1.15)and the expression for aggregate private consumption per household into the commodity demand function, we obtain

$$(1.16)\quad c_i = c_i'\,(\hat{w},k,\sigma^2,\sigma^2 w,\ p)$$

We assume that the distribution of employment per household binomial and independent of household capital holdings. Under these conditions, σ_w^2, the variance of wage income per household is seen to be

$$(1.16) \qquad \sigma_w^2 = w_1^2 \, e(1 - e)$$

where w_1 is the wage rate in terms of good 1 and e is aggregate employment rate per household. It is seen that

$$(1.17) \qquad \frac{d\sigma_w^2}{de} \leq 0 \quad \text{as} \quad e \geq 0.5$$

Aggregate wage income per household, $\hat{w}$, is given by the relation

$$(1.18) \qquad \hat{w} = ew_1$$

We can therefore write (1.16) in the form

$$(1.19) \qquad c_i = c_i''(k, P, \sigma, e)$$

where

$$\frac{\partial c_i''}{\partial k} = \frac{\partial c_i}{\partial c} \alpha, \quad \frac{\partial c_i''}{\partial P} = \frac{\partial c_i}{\partial P}$$

$$\frac{\partial c_i''}{\partial \sigma} = \frac{\partial c_i}{\partial \sigma_c^2} \alpha^2 \, 2\sigma, \quad \frac{\partial c_i''}{\partial e} = \frac{\partial c_i}{\partial c} w_1 + \frac{\partial c_i}{\partial \sigma^2 c} \frac{\partial \sigma_w^{2}}{\partial e}$$

Given that neither good is inferior,

$$(1.20) \qquad \partial c_i / \partial c > 0$$

Thus, it is clear that

$$(1.21) \qquad \partial c_i'' / \partial k > 0$$

If the demand function is quadratic and convex in case of good 1 and concave in the case of good 2, then it can be shown that

$$\frac{\partial c_i''}{\partial e} > 0$$

with non-inferiority. The sign conditions on $\frac{\partial c_i''}{\partial \sigma}$ are given by (1.14) since both α and σ are assumed positive.

II. Supply and Demand Equilibrium

Our model is based on the assumption that, in addition to two commodities, there are two factors of production, capital and labor. It is assumed that real wage expressed in terms of good 2, w_2, is a set equal to an exogenously determined minimum, $\bar{w}_2$. This implies open unemployment of labor. Since the production function in the two sectors are homogenous of degree one and the system is assumed to be in competitive equilibrium, the rental rate on capital, expressed in terms of either good, is given by this specified factor price. Government purchases are equated to taxes and restricted to good 1. From (1.10), this implies that investment and government consumption is equal to total capital income less αk, which, with the rental rate on capital given, depends only on the capital-labor ratio. Thus, capital dispersion affects the output mix only through its association with the composition of consumer demand. Because good 2 is more labor intensive than good 1 in the sense that its fixed ratios of capital stock to employment and output are lower, these changes in the bill of goods demanded will influence aggregate real output and employment.

We shall now demonstrate formally, by referring to the consumer demand functions derived in the previous section, as well as classical supply relationships, the precise mechanism by which these changes are accomplished. This analysis will involve the effects of changes in both the level and dispersion of capital. Initially, we shall consider only a closed economy and then later on demonstrate how the results are affected by the introduction of final good exports and imports.

Under the assumptions made, the transformation surface T_2T_1 in Figure 1 relating output in sector 2 to output in sector 1 is linear; with the real wage (expressed in terms of either one or a combination of commodities) fixed, relative factor prices and proportions will be fixed. Output in each sector divided by the total number is (x_i) of households measured along the axes of the transformation surface T_2T_1. The commodity price line PP , whose slope is uniquely determined by the real wage, cuts the transformation surface from below, since good 2 is assumed to the lower capital-employment ratio. This has been proven by Brecher [2]. Owing to the assumed relative labor intensity of good 2,movements along the transformation surface from T_1 to T_2 correspond to increases in both the employment rate and constant-price gross domestic product (GDP) per laborer.

Each x_1-x_2 combination on the transformation surface determines a unique aggregate employment rate. Given a specified value for the standard deviation of capital holding and the employment rate, the private consumption

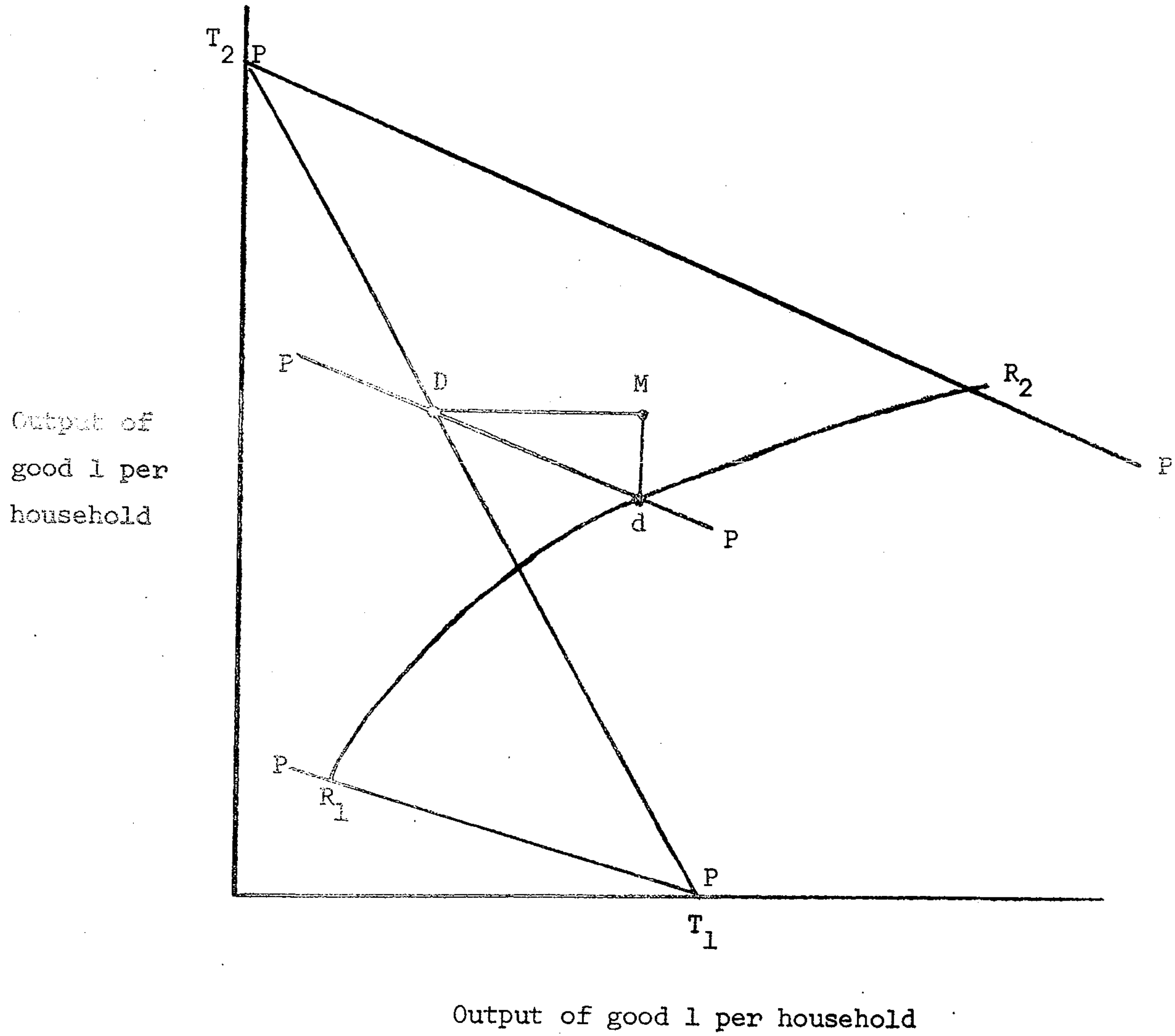
T2
P
P
D
M
R2
P
Output of
good 1 per
household
d
P
P
R1
P
T1
Output of good 1 per household

FIGURE 1

demand for both commodities may be derived from the functions (1.19). In these latter relationships, changes in the employment rate affect commodity both through their influence on wage income per labor and on the variance of wage income and hence σ_c^2. Since the sum of public consumption and investment per household is given by the relation

$$(1.22) \qquad g + i = (r_1 - \alpha)k$$

The total final demand for good 1 is simply

$$(1.23) \qquad c_1 + (r_1 - \alpha)k$$

while c_2 represents total final demand for good 2.
Hence curve R_1R_2 giving total final demand for both goods corresponding to each point on the transformation surface may be derived from (1.19) and (1.23). If commodity demand functions are quadratic, this curve will always be positively sloped in the region of non-feriority if the expenditure elasticity for good 1 is greater than unity. Each point on the curve represents a fixed standard deviation of household capital (σ), but a different mean and variance of wage income due to the change in the employment rate along the transformation surface. In the case where there is no foreign trade, the intersection of the demand curve with the transformation surface constitutes the equilibrium production point.

If trade is restricted to final goods, relaxing closed-economy assumption will result in only a minor modification of the system. The home offer curve is constructed by means of offer triangles, such as DMd in Figure 1, formed by the demand curve R_1R_2, the transformation surface T_1T_2, and different commodity price lines (each with the same slope). The intersection of the commodity price line with the transformation surface representing a given employment rate and the demand curve forms the offer triangle. From these triangles the home offer curve in the region of incomplete specialization is derived. This curve, depicted as OA in Figure 2, is of the straight line Ricardian variety, whereas the foreign offer curve OF has the conventional shape. The intersection at point S gives the equilibrium level of imports and exports. The offer triangle DMd determining the equilibrium product point, D, has the same dimension as the triangle OSJ shown in the offer curve diagram (Figure 2).

III. Comparative Statics

Precise mathematical conditions determining the direction of response of the employment rate and real GDP per household to changes in k and σ may now be derived. Before doing so, however, we shall first examine how the transformation surface and the demand curve are affected by changes in k and σ. In the case where there always is open unemployment of labor, an increase in k will cause the transformation surface to shift outward in a parallel manner, as, for example, illustrated by the shift from T_2T_1 to $T_2'T_1'$ in Figure 3. An increase in capital per household implies that, when the country is completely specialized in either commodity, output per household and the employment rate will be higher than they were before. The employment rate, which increases along the

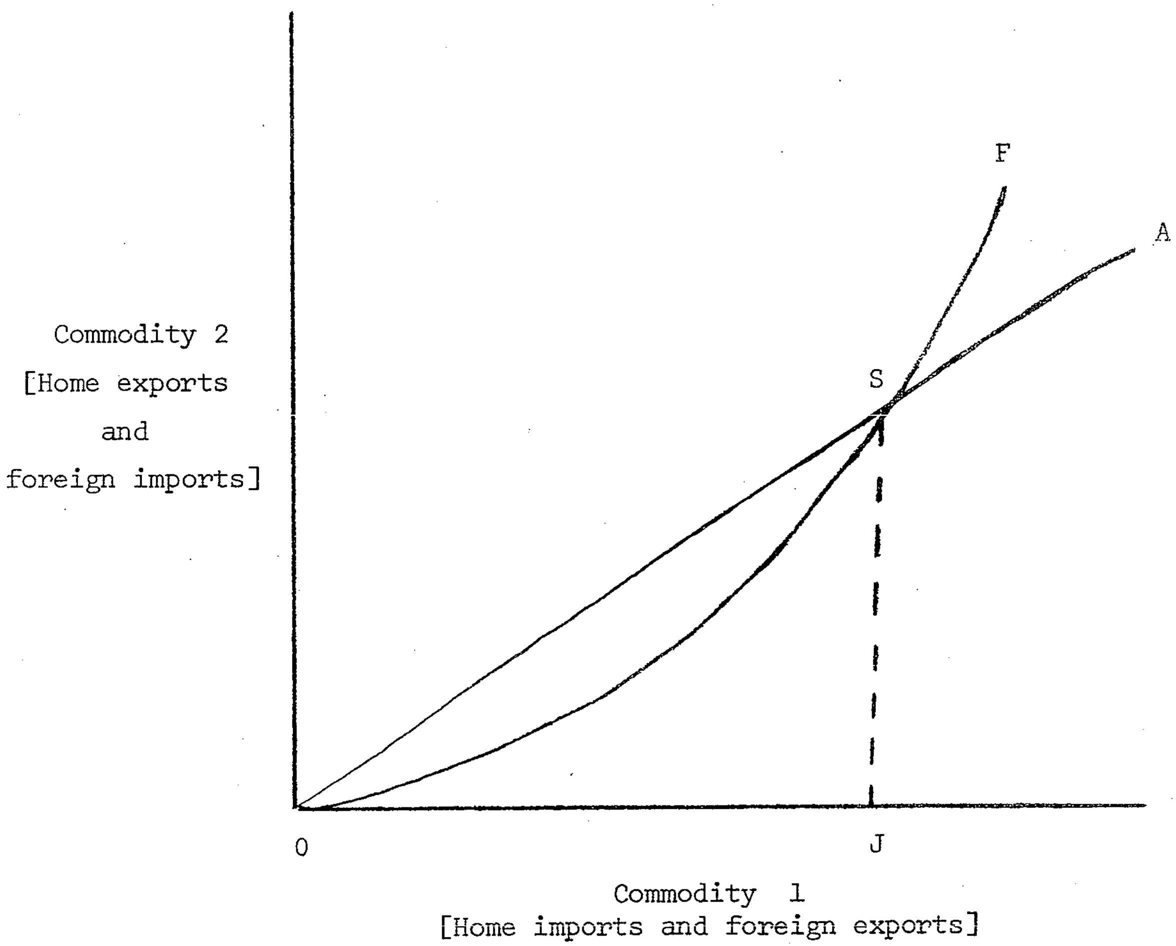
Commodity 2
[Home exports
and
foreign imports]
F
A
S
0
J
Commodity 1
[Home imports and foreign exports]

FIGURE 2

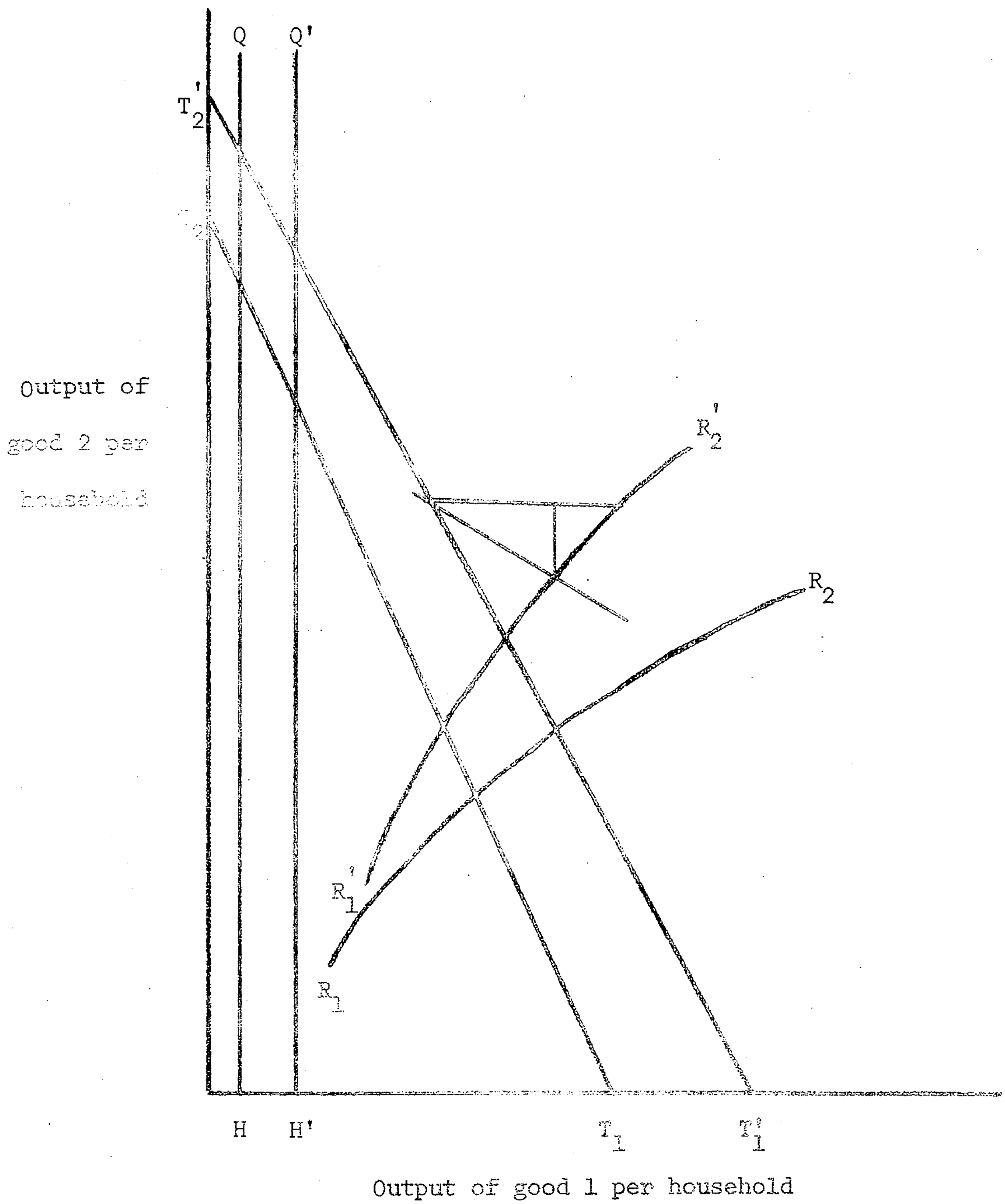
Q
Q'
T_2'
T_2
Output of good 2 per household
R_2'
R_2
R_1'
R_1
H
H'
T_1
T_1'
Output of good 1 per household

FIGURE 3

transformation surface with increases in output share of good 2, is higher at point T_1' than at point T_1. Consequently, given (1.17), and e greater than .5 , the variance of wage income per household will be lower at this point. This implies that, with σ held fixed and no change in investment and public consumption demand for good 1, the demand curve will shift to the left (right) as k increases provided that the expenditure elasticity for good 1, η_1, is greater (less) than for good 2. However, if r_1 is greater than α, the vertical line HQ representing the sum of investment and public consumption demand per household will shift to the right. If we assume that σ is adjusted in such a way as to keep the variance of consumption per household constant, then the demand curve will cross the horizontal axis at point H as the employment rate goes to zero. Consequently, the rightward movement in HQ associated with an increase in k will have an effect opposite to that attributable to a more even distribution of wage income. While the new demand curve $R_1'R_2'$ will have a steeper slope than the old one demand curve it may also intersect the curve R_1R_2 from below. But k and e are both higher at point T_1' than they are at point T_1. For this reason, the demand curve $R_1'R_2'$ derived from points along $T_1'T_2'$, will always be above R_1R_2 provided that neither good is inferior.

Note that, with $\bar{w}_2$ constant, factor proportions, in particular employment-output ratios, will remain constant. Denote the employment-output ratio in sector i by $\tilde{e}_i$, output per household of good 2 by x_2, GDP per household expressed in terms of good 1 by q_1, and the ratio of the price of good 2 to the price of good 1 by P. Then the change in aggregate employment per household (e) is given by the relation

$$(3.1) \qquad de = (\tilde{e}_2 - \tilde{e}_1)\, p dx_2 + \tilde{e}_1\, dq_1$$

The slope of the home offer curve determining the dimensions of the offer triangle at the equilibrium production point is uniquely determined by $\bar{w}_2$, and hence the dimension of this triangle will be unaltered by an increase in k. Further, given non-feriority, the demand curve $R_1'R_2'$ lies above R_1R_2. For these reasons, q_1 and x_2 will both be higher at the new equilibrium production point. With $(\tilde{e}_2 - \tilde{e}_1)$ positive, this implies that both employment and GDP per household will increase as a result of a rise in k.

The impact of change in σ on employment results only through a shift in the curve determining final demands, not the transformation surface, and hence is considerably easier to evaluate qualitatively than a change in k . It is quite clear that when expenditure elasticity of demand for good 1 is greater (less) than that for good 2, a decrease in σ, k fixed, will cause the final demand curve R_1R_2 to shift to the left (right). Since the dimensions of the equilibrium offer triangle are unaltered, a shift of these curves to the left (right) implies an equilibrium production point with higher (lower) level of real GDP and employment.

Let us now examine the exact expressions for the partial derivatives of e with respect to k and σ derived in the appendix. Denote $\partial c_1/\partial c$, the marginal propensity to consume commodity 1 out of total expenditure, by M_1c. Denote the first commodity's share in the general expansion of output due solely to an increase in the employment rate by B_1e and that due solely to and increase in capital per household by B_1k. Recall that $\frac{\partial c_1''}{\partial e}$ and $\frac{\partial c_1''}{\partial \sigma}$ are the partial derivatives of consumer demand function for good 1 with respect to e and σ respectively. Then we may write

$$(3.2) \qquad \frac{\partial e}{\partial k} = \frac{-[-a(1-M_{1c}) + (1-B_{1k})\, r_1]}{\Delta}$$

$$(3.3) \qquad \frac{\partial e}{\partial \sigma} = \frac{\frac{\partial c_1''}{\partial \sigma}}{\Delta}$$

where

$$\Delta = \frac{\partial c_1''}{\partial e} - w_1 B_{1e}$$

The denominator, Δ, will be positive if the expenditure elasticity of demand for good 1 is greater than unity at least in the case of the quadratic demand function (1.12). Under these conditions, it can be shown that

$$(3.4) \qquad \frac{\partial c_1''}{\partial e} > 0$$

Further as Kemp has shown,

$$(3.5) \qquad B_1e < 0$$

provided that sector 2 is relatively labor intensive.

Since B_1k is greater than unity the numerator of (3.2) will only be negative in case where good is inferior, implying that M_1c is greater than unity. Given that estimated expenditure elasticities are generally positive, we would in most cases expect $\partial e/\partial k$ to be positive. The sign of the derivative $\partial e/\partial\sigma$ will be negative if $\frac{\partial c_1''}{\partial\sigma}$ is positive which is assured in the case where $\eta_1 > 1$. However, the sign of this derivative is less clear in the case where $\eta_1 < 1$, since $\frac{\partial c_1''}{\partial e}$ may be negative which makes the sign of the numerator Δ ambiguous.

Real GDP expressed in terms of good 1, q_1, is given by the expression

$$(3.6) \qquad q_1 = r_1k + w_1e$$

where r_1 and w_1 are respectively the returns on capital and the wage rate expressed in terms of good 1. Since r_1 and w_1 are uniquely determined by the specified minimum wage, q_1 depends on k and on σ through its effect on e. Define the function determining the employment rate as

$$(3.7) \qquad e = g(k,\sigma)$$

The partial derivative of this function are given by the expressions (3.2) and (
Then we may write

$$(3.8) \qquad q_1 = f(k,\sigma) = r_1k + w_1g\ (k,\ \sigma)$$

where

$$f_k = r_1 + w_1 g_k$$

and

$$f_\sigma = w_1 g_\sigma .$$

Thus

$$(3.9) \qquad f_k \gtreqless r_1 \quad \text{as} \quad g_k \gtreqless 0$$

and

$$(3.10) \qquad f_\sigma \gtreqless 0 \quad \text{as} \quad g_\sigma \gtreqless 0$$

From (3.2) and (3.9) it is clear that in the case where good 2 is inferior and g_k is negative, it is possible for an increase in capital per household to cause a rise in real GDP per household and a decline in employment per household. Assume that income dispersion is measured by the coefficient of variation (the ratio of the standard deviation of household income) to aggregate income (equal to GDP in this model). It is obvious that, in the case where g_k is positive, a rise in k will cause this coefficient to decline, the numerator will as a consequence of a decline in the variance of wage income (with the variance of capital income held constant), and denominator will increase. If, on the other hand, g_k is negative, the coefficient of variation may well rise. In this case a rise in k will cause the standard deviation of total household income to rise due to a fall in the employment. From (1.16) it is clear that this effect can be substantial if the initial value of e is close to unity.

Similarly, if g_σ is negative, it is clear that increases in e will reinforce the direct depressive effect of decreases in σ on the coefficient of variation of household income. The benefits of asset redistribution in terms of income equality may be the opposite of what one normally expects if g_σ is positive.

In the appendix, sufficient conditions are derived for $f(k,\sigma)$ to be concave. In the case of a quadratic demand function, $f(k,\sigma)$ will be concave if the household expenditure elasticity for good 1 is greater than unity and neither good is inferior. It will also be concave in the case where good 2 is inferior and g_k is negative provided that the condition described in the appendix is met. The importance of the expenditure elasticity of good 1 being greater than unity is intuitively obvious, since this implies that the output share of the capital intensive good will increase as k increases. Under these conditions, the marginal impact of a change in σ on the employment rate will decline as k rises. Further, as σ decreases, the marginal impact of change in σ on the variance of expenditure declines, and from (1.19) it is clear that $\frac{\partial c_i''}{\partial \sigma}$ decreases. Thus, since $M_1 c$ and Δ remains constant in the case of a quadratic demand function, the marginal impact of change in σ on employment rate will decrease as σ falls.

IV. Conclusions and Extensions

In this paper, we have developed a model which shows the impact of changes in the level and dispersion of capital on the size distribution of income. Following Brecher, open unemployment has been assumed to exist as a result of a binding minimum real wage but the standard neo-classical assumptions, other than full employment, have been retained. Under these conditions, we have shown that capital accumulation will cause the size distribution of income to become more even, as well as increase output and employment, provided that the labor-intensive good is not inferior. Greater equality in asset holdings will generally cause the income distribution to become more even except possibly in the case where the expenditure elasticity of demand for the labor-intensive good is greater than unity. In the latter case, the variance of the size distribution of wage income will rise due to a fall in the employment rate.

These results are robust in the sense that they may be derived under a variety of assumptions. A model similar to the one presented in this paper may be constructed under the assumption that the rental on capital, rather than the real wage, is exogenously specified. In a country which imports most of its equipment, such an assumption may be quite realistic. In this case, the rental rate on capital may well be determined by the world rate of interest and price of imported equipment. Since the real wage is uniquely determined by the rental rate on capital, such a modification will have no effect on our results provided that real wage and the rental rate on capital are not both exogenously specified.

Whether or not the introduction of a factor-price distortion into the model will have a significant effect on the results depends on how the distortion is created. If the wage rate in the capital-intensive sector is higher by a fixed proportion than wage rate in the labor-intensive sector, an increase in the employment share of the capital-intensive sector may cause the distribution of wage income to become less even, despite the fact that aggregate employment has risen. On the other hand, there may be no open unemployment in the system, only underemployment in a third sector such as traditional agriculture. Suppose that the only variable factor in this sector is labor and that its product is consumed only domestically (thus its internal price is free to change). Suppose, further, that the expenditure elasticity of demand for this good is positive and that wage income per laborer equals the average product of labor in the third sector. Then under certain conditions, this wage rate (expressed in terms of good 1) will be an increasing function of the proportion of laborers employed in the other two sectors.[1] In this case, an increase in labor absorbed by the two other sectors where the real wage

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[1]An example may be easily constructed. Denote the wage rate received by an underemployed laborer in sector 3 by w', the ratio of the price of good 3 to the price of good 1 by p', the ratio of sector 3 output to total labor force by x_3, and the proportion of laborers employed in sector 1 and 2 by $\hat{e}$. Assume the demand function for good 3 takes the form

$$\frac{1}{p'} w'(1 - \hat{e}) = x_3 = m_3 \left(\frac{c + w'}{p'} \right)$$

where m_3 is a positive constant. Then we have

$$w' = \left[\frac{1}{(1 - \hat{e})} \right] \left[\frac{m_3 \alpha k}{(1 - m_3)} + \frac{m_3 w_1 \hat{e}}{(1 - m_3)} \right]$$

(expressed in terms of good 1) is constant and above that in the third sector will make the overall size distribution more even if these sectors initially employ more than 50 percent of the labor force. Here our main qualitative results provided they are confined to relationships among modern-sector employment, capital stock and overall size distribution of income remain intact.[1] If most of the labor force is employed in the third sector initially, there will be conflicting effects: a reduction in the wage gap leading to greater inequality. In any case, however, the assumption of a fixed wage gap obscures an important equity advantage of policies designed to reduce underemployment.

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[1]The share of the labor force employed in the traditional sector may be less than 50 percent even though the share of the agricultural labor force is substantially greater than 50 percent. Many developing countries are characterized by a modern plantation sector specializing mainly in export crops. For this reason, the analysis of Kuznets [6], [7], which emphasizes the rural-urban income discrepancy, may not be relevant.

APPENDIX

Notation:

c = average private consumption per household

p = the ratio of the price of good 2 to the price of good 1

w_i = the real wage rate (expressed in terms of good i)

r_i = the rental rate on capital (expressed in terms of good i)

$\hat{w}$ = average wage income per household (expressed in terms of good 1)

e = average employment per household

k = average capital per household

σ = the standard deviation of capital per household

M_{1c} = the marginal propensity to consume good 1 out of total expenditure, i.e., the partial derivative of (1.11) with respect to c

M_{1cc} = the second partial derivative of the commodity demand function (1.11) with respect to private consumption

$M_1''\sigma$ = the partial derivative of the commodity demand function (1.19) with respect to σ

$M_1''\sigma\sigma$ = the second partial derivative of the commodity demand function (1.19) with respect to σ

$M_1''e$ = the partial derivative of the commodity 1 demand function (1.19) with respect to employment per household.

$M_1c\,\sigma$ = the cross partial derivative of the commodity 1 demand function (1.11) with respect to σ

$M_1''e\,\sigma$ = the cross partial of (1.19) with respect to e and σ

N = the specified number of households in the home country

N^* = the specified number of households in the rest of the world

B_{1e} = the first commodity's share in the general expansion of output due solely to an increase in employment per household

B_{1k} = the first commodity's share in the general expansion of output due solely to an increase in capital per household

Recall that commodity 1 is assumed to be the import-competing good. The output of this commodity per household, x_1, depends on the commodity price ratio, the employment rate, and capital per household. IMports of commodity 1 per household, z_1, are given by the expression

$$\text{(A.1)} \qquad z_1 = c_1'' (k, \sigma, e, p) + (r_1 - \alpha)k - x_1 (p, e, k) = z_1(p, k, \sigma, e)$$

where $x_1(p, e, k)$ is the function determining x_1. The balance of payments conditionmay be written as

$$\text{(A.2)} \qquad Nz_1 (p, k, \sigma, e) - N^*p\ z_2(p) = 0$$

where the function $z_2^*(p)$ determines the rest of the world's net imports commodity 2 per household.[1] It can be shown that the endogenous variable in this equation will be unaffected by growth in the number of households provided that the ratio of N^* to N remains fixed.

By differentiating this equation totally, we obtain expressions for the

[1]This formulation is a simple extension of Kemp's [5 , Chapter 4], and his general approach to comparative statics applies here. We have retained his assumption and notation as much as possible.

partial derivatives of the employment rate with respect to k and

$$\frac{\partial e}{\partial k} = \frac{-[-\alpha(1-M_{1c}) + (1-B_{1k})\, r_1]}{\Delta}$$

$$\frac{\partial e}{\partial \sigma} = \frac{M''_{1\sigma}}{\Delta}$$

where $\Delta = M''_1 e - w_1 B_1 e$

In the case of a quadratic demand function it is clear that the cross partials $M_{c\sigma}$ and $M''_1 e_\sigma$ are zero. Under this assumption, it can be shown that

$$\frac{\partial^2 e}{\partial k^2} = \frac{-[\alpha^2 M_{1cc}\Delta - w_1\, \alpha\, M_{1cc}\, u]}{\Delta^2}$$

$$\frac{\partial^2 e}{\partial \sigma^2} = \frac{-[M''_{1\sigma\sigma}\, \Delta]}{\Delta^2}, \quad \frac{\partial^2 e}{\partial k \partial \sigma} = 0$$

where $u = -\alpha(1 - M_{1c}) + (1 - B_1 k)\, r_1 << 0$. Consequently if the household expenditure elasticity for good 1 (η_i^j) is greater than unity, implying that M_{1cc} and $M''_{1\sigma\sigma}$ are both positive in the quadratic demand case, then $\partial^2 e/\partial k^2$ and $\partial^2 e/\partial\sigma^2$ will both be negative. This, along with $\partial^2 e/\partial k\partial\sigma$ being zero, implies that the Hessian for the function determining the employment rate will be negative definite.. Therefore, since GDP per household is linearly related to k and e, the function $f(k,\sigma)$ determining this output variable will be concave.

If u is positive (implying that $g_k < 0$), then the function will still be concave provided that $\alpha\Delta > w_1 u$.

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