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THE "STANDARD MARKET" OF TRADITIONAL CHINA

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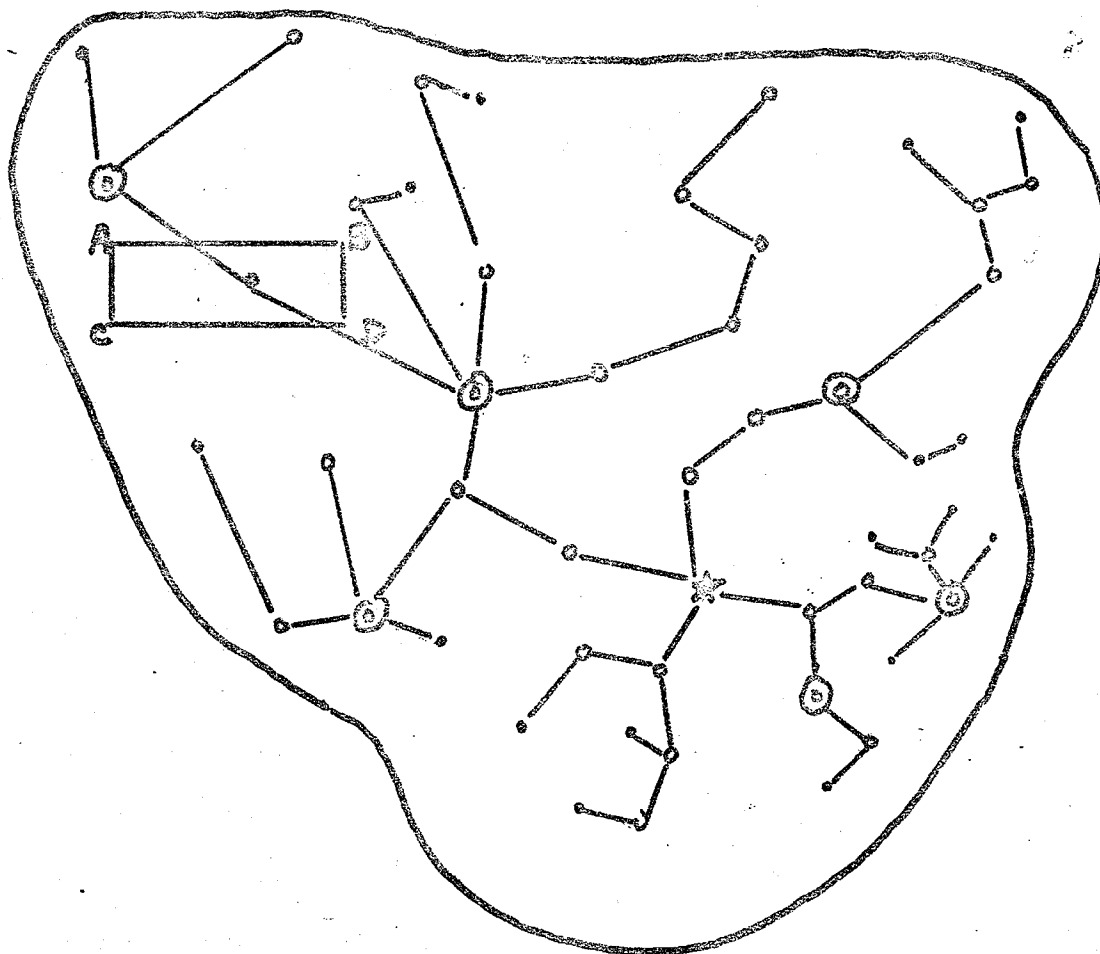
I. Introduction

Not long ago, Professor Skinner discussed the important idea of a "standard market" as a self sufficient unit in the economic and cultural sense in traditional rural China.¹ It is the purpose of this paper to analyze a set of issues, suggested by his paper, in respect to the forces that determine certain quantitative aspects of the standard market: the size, shape, population density, regional variations, process of formation, etc. The theory presented in this paper is essentially an economic theory.

Although the theory can be formulated rigorously, we shall, in the text, concentrate on the major economic ideas--relegating all the technical mathematical details in the appendix. We shall first present Professor Skinner's conclusions in a larger spacial perspective of traditional China. (Section II) The quantitative aspects of the standard market are issues of the economics of space. One major factor that determines the shape of the standard market is the transportation cost, the minimization of which constitutes a rationale of the marketing system. (Section III) The standard market will be next examined as an institution of agrarian dualisms (i.e. coexistence of agricultural and non-agricultural production and exchange) that exhausts agriculture. (Section IV) This viewpoint leads to the identification of the efficiency of large scale production in non-agricultural production as a basic advantage for larger standard markets. This is integrated with the transportation cost argument

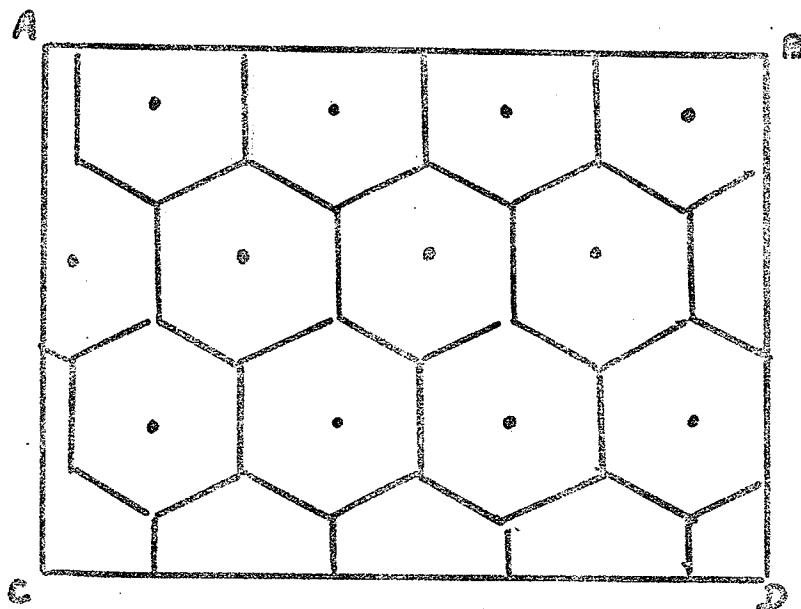
¹ Skinner, G.W., "Marketing and Social Structure in Rural China", Journal of Asian Studies, Volume. XXIV, No. 1, pp. 3-44.

Diagram 1 Spatial Structure of Traditional China

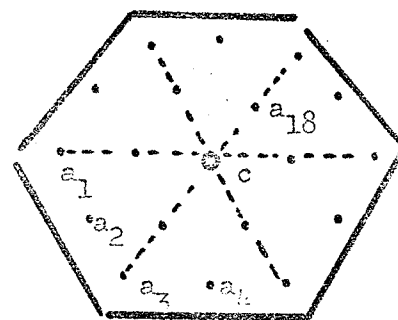


- standard market centers
- Hsien Cheng
- ⊙ Sheng Huai
- ★ Capital

1a) Aggregate Urban Structure
hierarchy of urban centers and their connectivity.

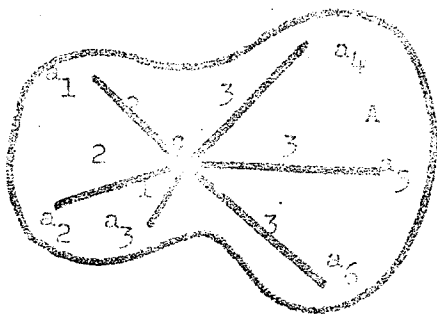


1b) Rural Standard Markets covering the land space

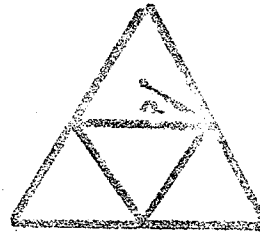


- "c" marketing center
- "a₁, a₂," nucleated villages

1c) Standard Market Structure
- a microscopic view



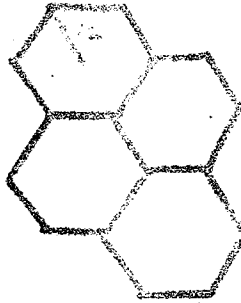
$TC = 2+2+1+3+3+3 = 14$ ton miles
 $TC = 2^2+2^2+1^2+3^2+3^2+3^2 = 36$ ton-mile square



triangle



square

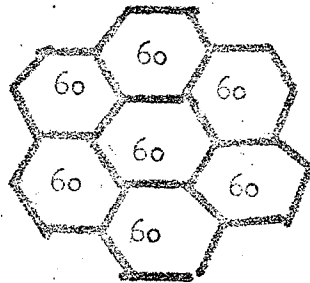


sexogons

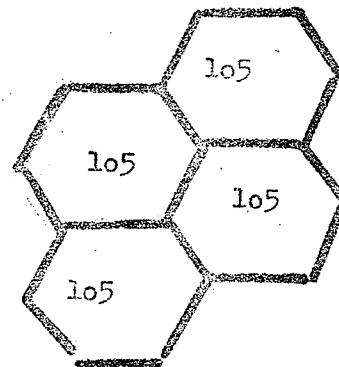
(a) spacial transportation cost coefficient

(b) regular polygon coverage of same total area (and same number of polygons in each case)

Diagram 2

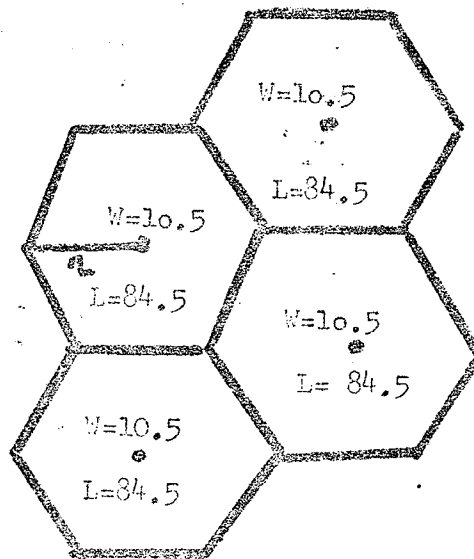
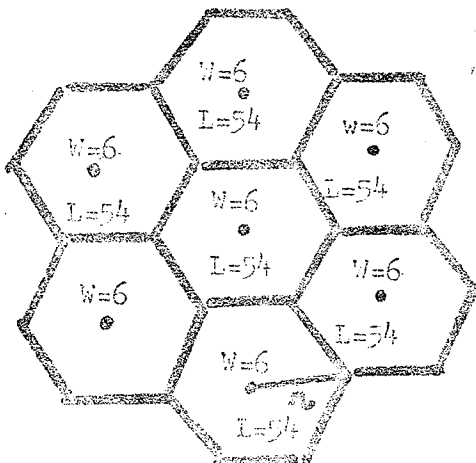


case (i)

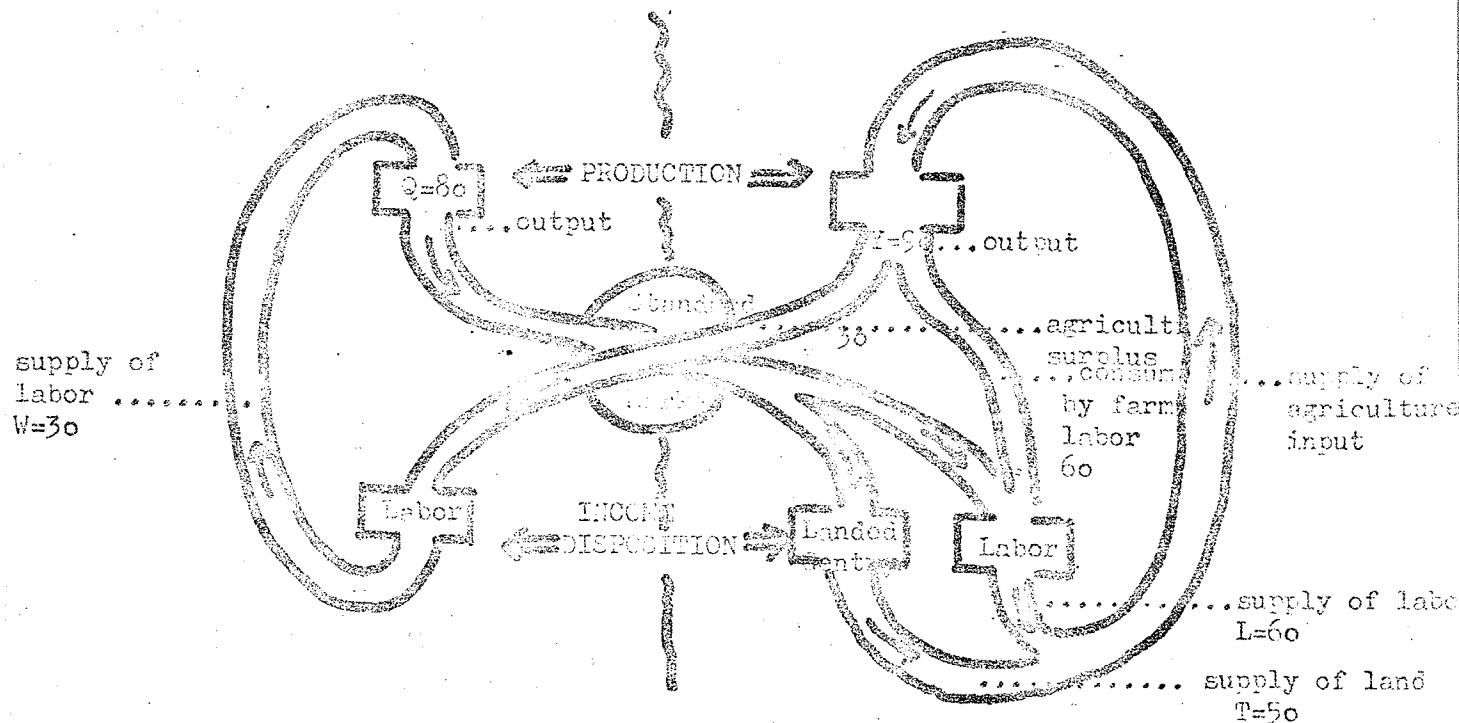


case (2)

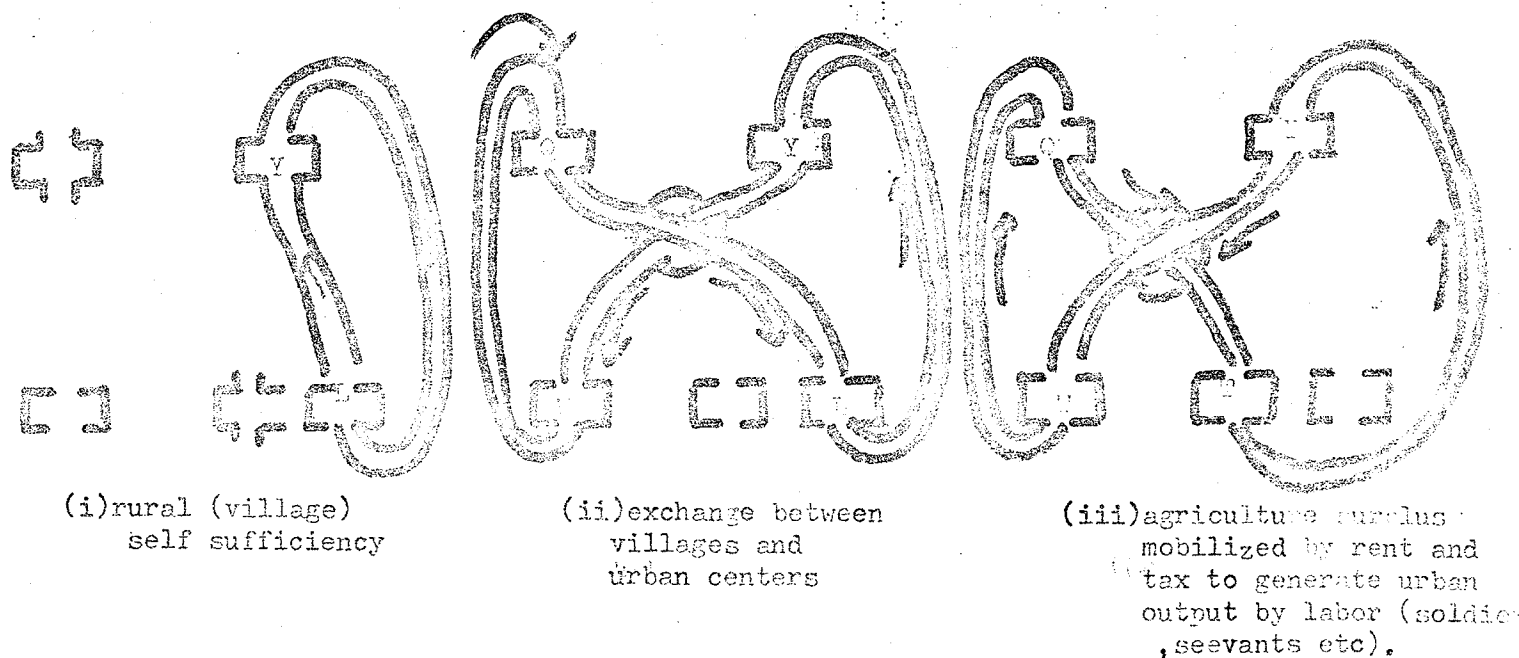
(c) same area covered by more (smaller) sexagons (case i) or by less (larger) sexagons (case ii)



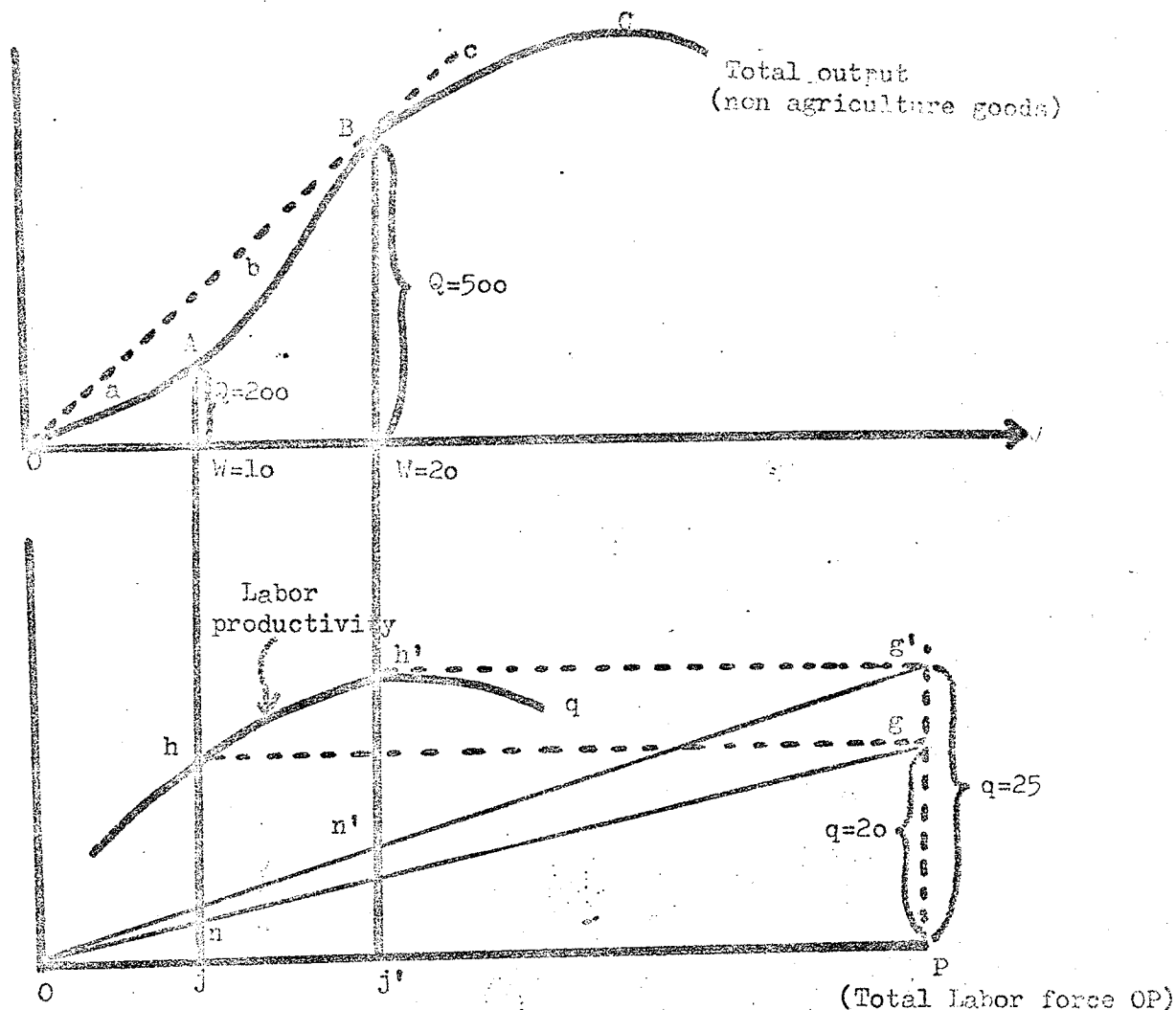
(d) same total area and same population density and same $\theta = W/(W+L)$. (W: labor at urban center L: labor in villages).



(3a) Economic Tableau of Agrarian Dualism



(3b) Circulations of Agrarian Dualism

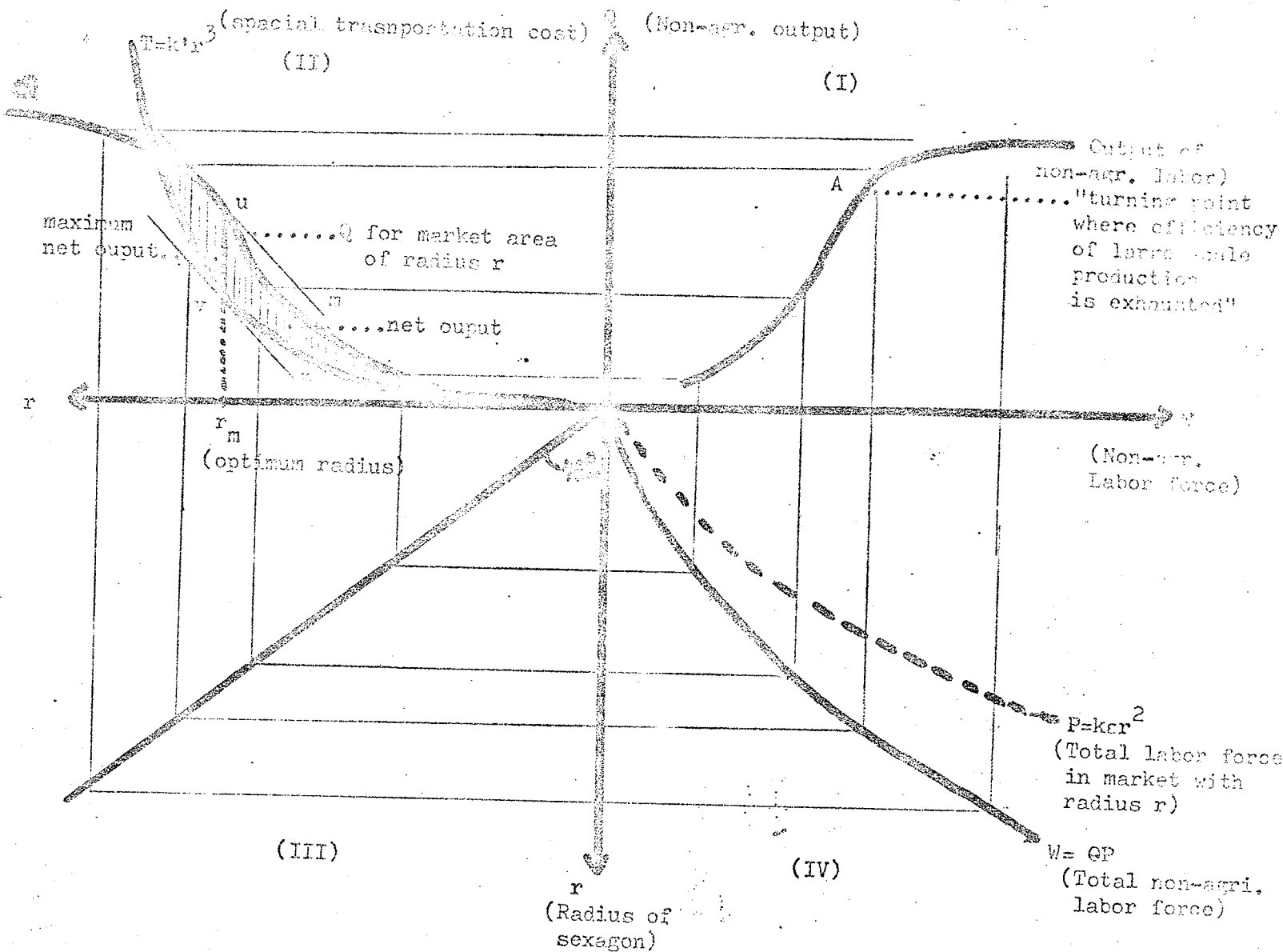


Let OP be total labor force. Increasing agriculture labor productivity leads to an increase of nonagriculture labor force from " j " to " j' " with the following effect on per capita consumption of non-agriculture goods:

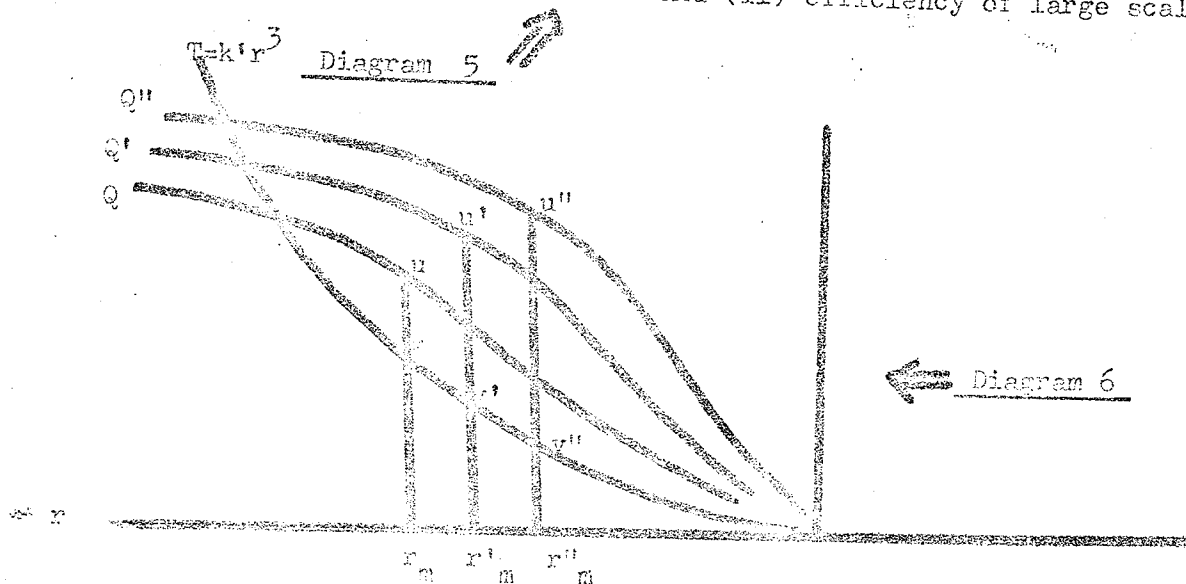
- (i) at j : $\theta = Oj/OP = jn/Pg$ hence $n = \theta q = jn$
- (ii) at j' : $n = j'n'$

Thus $j'n' > jn$ for two reasons: (i) θ becomes higher and (ii) q becomes higher at j' .

Diagram 4



Determination of optimum radius of standard market due to a balancing of (i)spacial transportation cost and (ii) efficiency of large scale production



In more densely populated regions, the Q -curve shifts upward because of higher values for c (population density) and Q (fraction of non-agriculture population). This leads to a decrease of the optimum values for " r "

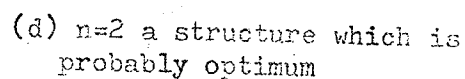
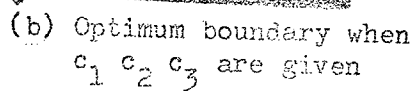
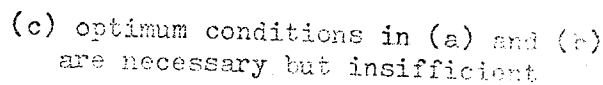
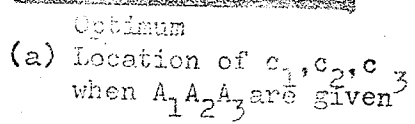
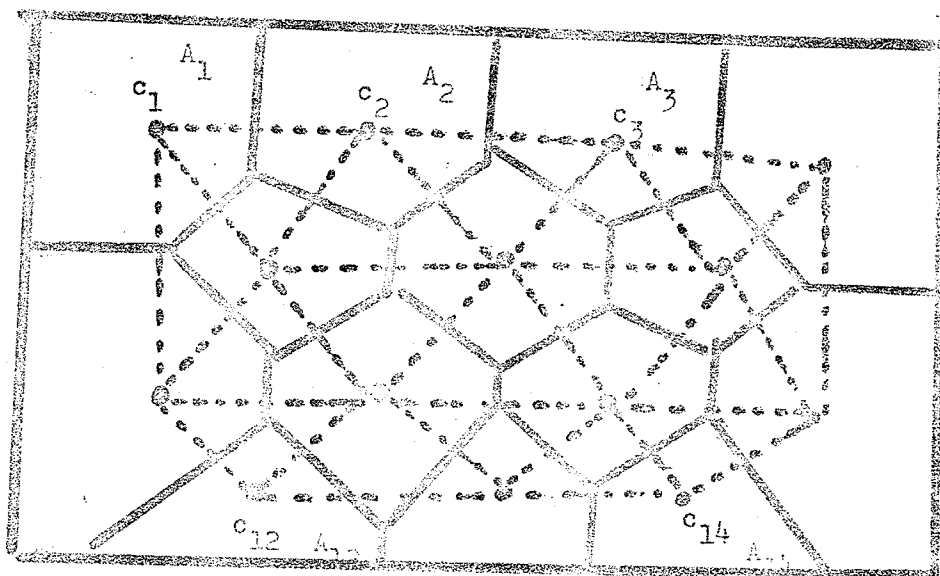


Diagram 7 necessary conditions for Optimum Structure



With increasing n (now $n=14$), an optimum structure leads to hexagonal coverage asymptotically -- a theoretical conjecture.

Diagram 8 Long Run process of formation of standard market s

to form a positive theory of market area. (Section V) The regional variation of the size of the standard market, as traced to differentiated population density, and agricultural production conditions, can then easily be explained as a theoretical consequence. (Section VI) Finally, the dynamic process of the formation of the standard market will be explored. (Section VII)

II. Spacial Perspective of Traditional China

The spacial perspective of traditional China may be presented at three levels of aggregation as shown in Diagrams 1a, 1b and 1c. At the most aggregate level (Diagram 1a), is a transportation network (river and roads) linking urban centers forming a hierarchy which is a coincidence of economic (i.e. from larger manufacture and wholesale trading centers to small scale local manufacture and retail centers) and political (capital, provisional capital, Shiang Hsiang, Cheng, etc.) forces. This aggregate view, portrays the urban aspect of traditional China emphasizing a pattern of connectivity as well as the distance¹ which is one major attribute of space. In Diagram 1b, the rectangle ABCD of Diagram 1a is enlarged. Here we have a more local view of the spacial structure of the standard markets--i.e. the cells of the beehives. Professor Skinner pointed out the hexagonic nature of the cells, the standardization of their size in a homogeneous plane and a tendency for this size to shrink from thinly populated to more densely populated regions. While the standard markets are linked up with the national network (Diagram 1a), there is a second dimension of space which

¹Diagram 1a, in mathematical language is a valued linear graph emphasizing "patterns of connectivity" and "distance". A linear graph consists of vertices (representing urban centers) and edges connecting the vertices. The "value" corresponds to the length of the edge (representing the distance).

is being brought out in Diagram 1b, namely, the area of the space. The area included in a standard market is primarily the area of cultible land--a most important primary factor of production in an agrarian economy. Thus the standard market structure is basically a rural market structure in which agricultural production is a central phenomenon. Diagram 1c is a microscopic view of the standard markets (of diagram 1b) which contains the marketing centers (c) and the nucleated villages ($a_1, a_2, \dots, a_{18}$). An essential fact of life of the standard market, discussed by Professor Skinner, is that while the peasant must live close to the land (i.e. at a_i) they have numerous reasons for making frequent trips to the marketing center (c). While there are cultural purposes¹ for such local trips, the very notion of a standard market implies that the basic purpose is an economic one.

There are really three attributes of space which must be sharply distinguished in the formulation of a positive theory of the standard markets, namely, distance, area and non-homogeneity. In the linear graph of Diagram 1a which portrays the urban part of traditional China, it is the distance of space and transportation cost considerations that are analytically crucial. In Diagram 1b, which portrays the rural part of traditional China, it is the area of space and agricultural production considerations that are crucial. The non-homogeneity of land space will be interpreted mainly as differentiated fertility of land corresponding approximately to the various agricultural regions (e.g. the Buckinized regions) of traditional China. We shall first assume a homogeneity plane and defer agriculture-region related issues in a later section (Section VI).

¹For cultural output (religion, political, recreational, social, etc.) see Skinner.

From Diagram 1c, we see that the marketing center (c) of the standard market is the "nerve end" of the aggregate urban structure (Diagram 1a). Thus they are the focal points of contact between the urban sector and the rural sector of the agrarian dualism of traditional China. The economic significance of the standard markets will be explored from "this" point of view in Section IV. Intuitively, it is obvious that a formal theory of standard market must take into account both agricultural and non-agricultural production--the key phenomenon of any agrarian dualism.

III. Minimization of Transportation Cost

A part of our theory of the size and shape of the standard market area is based on the notion of the minimization of spacial transportation cost in the "densely populated" agrarian society. The notion of "dense population" refers to the fact that the homogeneous plane must be completely covered by the standard market--with no wasteland in between. As a counter example, we may think of the "three field systems" around the 10th century in Western Europe. There were, of course, local self-sufficient units comparable to the standard market. However, there was also wasteland between the units so that the land space was not entirely covered. For traditional China, it is reasonable to assume that the assumption of "dense population" is fulfilled in the Southern Sung dynasty at the latest.¹ It is intuitively obvious that without the dense population assumption, it is

¹The period from Tang to Sung represented a transition phase in traditional China whereby land abundance gave way to labor abundance. This transition was manifested in many dimensions (e.g. ideas on land versus labor as wealth, ideas on population pressure, nature of economic rent, as area related versus location related; agricultural taxation system based on land versus labor; natures of government rural overhead investment; characteristics of population migration etc.). Issues of this type have not been fully explored from the economic theoretical point of view.

impossible to establish the sexagonal shape of the standard market--witnessing the fact that under the three field systems, the spacial unit is three-leaf clover shaped.

In the standard market an important economic cost is the local transportation cost associated with the frequent trips from the nucleated villages to the marketing center (c of Diagram 1c). In the typical (hexagonal) standard market observed by Professor Skinner, the least advantageously located villagers will have to travel 5.4 miles in a round trip to the market center. He will have to perform a "work" (as the physicists would say) of $710 = 150 \times 5.4$ pound-miles--assuming he carries no extra load besides his own weight (150 pounds). When this is multiplied by the number of trips he makes a year (as determined by the frequency of marketing schedules) and the number of population (which is 1350 in a typical standard market) we can have a sense of magnitude of this cost--even some allowances are made for the shorter distances traveled by other villagers (see below). The cost might just have taken on the same order of magnitudes as the other "works" (cultivating the land the manufacturing activities) performed in the standard market area. In traditional rural China where local transportation was supplied by human and/or animal power, there was the phenomenon of "increasing cost"--"the distance of 90 miles is half the distance of 100 miles", as an old Chinese proverb would say. In this case, the simplest assumption is that the cost is proportional to the square of the distance traveled. When the distances traveled are 1, 2, 3, 4.....miles the total transportational cost would be 1, 4, 9, 16.....ton-mile squares, instead of 1, 2, 3, 4.....ton miles. It is intuitively obvious that, other things

being equal, the more pronounced the phenomenon of "increasing cost", the smaller the size the marketing area--in order to economize in the transportation cost.

Let us assume that a market area A and an urban center c (located in A) are postulated arbitrarily as in Diagram 2a.¹ Suppose one unit of commodity (i.e. commodity of one ton) is to be delivered from " c " to every one of the nucleated villages ($a_1, a_2, \dots, a_6$) directly.² With the distance (i.e. miles) indicated in the edges (i.e. 2, 2, 1, 3, 3, 3), the T.C. coefficient (i.e. transport cost coefficient) is 14-ton miles without increasing cost and will be 36-ton miles square with increasing cost. It is evident that the T.C. coefficient depends only on the shape and size of A as well as the precise location of the urban center. For this reason, we may denote the T.C. coefficient by $T(A, c)$. Finally, when the nucleated villages are spreading evenly in A , the value $T(A, c)$ is what the physicists referred to as the first (without increasing cost) and second (with increasing cost) moment of A with respect to c . (See Appendix.)

A fundamental hypothesis which we shall make in this paper is the principle of rationality in the minimizing of location transportation cost (i.e. minimization) of $T(A, c)$. It is hoped that based on this principle, we can establish both a theory of market boundary (i.e. theory of size and shape of A) and a theory of location of location of the marketing centers (c) once A is given. (See Section VII.) In respect to the latter, once A is given, the "principle" (i.e. the principle of minimization of local transportation cost), implies that the optimum urban center " c "

¹The point is now " A " is of arbitrary shape and " c " is located arbitrarily in " A ".

²"Directly" means a special delivery is made between " c " and every a_i separately.

should be located in such a way as to minimize $T(A,c)$. If, furthermore, A is assumed to be a regular polygon, then the optimum location of c is the geographic center of gravity.¹ For the case of regular triangles, squares and hexagons, the optimum location of c is indicated in Diagram 2b.

The fact that the principle of minimization of transportation cost is valid can be supported by the findings of Professor Skinner, namely, the urban centers are indeed located near the geographic center of gravity of the sexagonal marketing area.² For the three regular polygons with radius r when c is optimally located, the values of $T(A,c)$ are:

$$1) a) T(Ac) = \frac{1}{4} [\sqrt{3} + \frac{1}{2} \ln(2 + \sqrt{3})] Y^3 \quad (A \text{ is triangle})$$

$$b) T(Ac) = \frac{8}{3} \frac{1}{\sqrt{2}^3} [\frac{1}{\sqrt{2}} + \frac{1}{2} \ln(1 + \sqrt{2})] Y^3 \quad (A \text{ is square})$$

$$c) T(Ac) = \frac{3\sqrt{3}}{2} [\frac{1}{3} + \frac{1}{2} \ln \sqrt{3}] Y^3 \quad (A \text{ is hexagon})$$

Let us next examine the validity of the Principle from the point of view of the theory of market boundary (i.e. shape and size of A). When the landscape is densely populated, it must be covered by market areas with no wasteland in between. In respect to shape of A, we have to defer a really difficult issue to a later section i.e. the proof that A tends to be regular under the "Principle" (See Section VII). For the time being, let us assume that the A's are in fact regular polygons, of the same size and sides. It is easy to prove that there are exactly three ways to cover a homogeneous plane, as shown in Diagram 2b, namely, triangle, square and hexagon. Next imagine that the total area covered in the three

¹ If A is of arbitrary shape (i.e. not necessarily regular), then the optimum location of "c" may not be the geographic center of gravity when there is no increasing cost. However, when there is increasing cost, the optimum location of c is always the geographic center of gravity. (In other words, the location of c that minimizes the second moment is the geographic center of gravity.)

² In these computations we assume no increasing cost (i.e. we computed the first moment).

cases are the same and that in each case, the same number (e.g. four, as shown) of polygons are used. Thus the areas of the triangle, square and sexagons are the same. From the formula listed in (1) it can be readily proven that the $T(A,c)$ is the smallest for the sexagon coverage. In other words, what we have proven is that, under the principle of minimization of transportation cost, the optimum polygon coverage is the sexagon coverage. The fact that the standard market area indeed tends to be sexagons, as observed by Professor Skinner, thus lends support to the principle of minimization of local transportation cost.

Next, in respect to the size of the market area, Diagram 2c shows that equal total area can be covered by a large number of small sexagons or a small number (7) of large ones. Intuitively, it is obvious that the total transportation cost for the whole region is smaller in the former case. From the formula listed in (1c) we see that $F(A,c)$ is proportional to the third power of the radius i.e. as the radius increases by 1, 2, 3, 4... the $F(A,c)$ increase by 1, 8, 27, 64.....testifying to the disadvantage of standard market with large area. In fact, it can be proven rigorously that, under the principle of minimization of total transportation cost, the optimum size of the (hexagonal) market area tends to be zero. For if the minimization of transportation cost is the only consideration every standard market should shrink to a point reducing the transportation time and cost to zero.

In summary, the principle of minimization of transportation cost can explain (i) the optimum location of market centers at the geographic center of gravity (ii) the optimum shape of marketing area tends to be hexagonal (iii) the optimum size of the standard market tends to be zero. In respect to the last point (iii), contradicted by observed reality, the analysis so far

suggested that we have to search for those economic forces which prevent the standard market from becoming too small. The realistic size is then determined by a balancing of "these" forces (giving advantage to large market areas) and the principle of minimization of transportation cost (giving advantage of small market areas). (Section 5)

IV. Agrarian Dualism and Standard Market

In the land space of traditional China, different regions have different population density. If equal population density contour lines are shown on a map, such lines would run from the south to the southwest--indicating diminishing population density as one moves from the southeast (Lower Yantze Valley) to the "great northwest". Professor Skinner has shown that as population density increases the area of the standard market shrinks. It takes two steps to explain this phenomenon. In the first step, we may raise the question as to the forces that determined the size of the standard market in a homogeneous plane when the population density is given and uniform. The explanation of the regional variation of size, as population density varies, then follows as an easy consequence in a second step.

Referring to Diagram 2c once again, and imagine that, for the two cases portrayed (with the same total area), the population density is the same (i.e. 420 thousand total population in both cases). Thus the large market area contains more population (105 thousand) than the small market area (60 thousand). In spite of the transportation cost advantage, what prevented the market area from becoming too small is the disadvantage of small population living in the same market area. All the non-economic cultural forces point to the advantage of a larger population--which need

not detain us. From the economic standpoint, the basic advantage of a large market area is that it contains a larger population enabling the community to enjoy the efficiency of large scale production.

The role of the standard market in traditional China may be explained with the aid of the Physiocratic Economic Tableau--depicting the regularity of major economic events in the European agrarian economy before the industrial revolution--Diagram 3a. The Physiocrats envisioned an agrarian dualism typified by the coexistence of agricultural production (output $Y = 90$) and non-agricultural production (output $Q = 80$) using, as inputs, land ($T=50$) and labor ($L=60$) in the agricultural sector and labor force ($W = 30$) in the non-agricultural sector. The economic tableau can be decomposed into three types of "circulation" (diagram 3b):

- a) Agricultural Self Sufficiency: part of agricultural input (L) is used to produce agricultural goods (Y) which flow back to feed the agricultural labor force (L).
- b) Dualistic Exchange: part of agricultural input (L) is used to produce agricultural goods (Y) which flow through the market to feed the industrial labor force (W) which, in turn produces non-agricultural goods that flow back to the agricultural labor (L), through the market. This amounts to an exchange of agricultural goods for non-agricultural goods by the two types of labor in the market.
- c) Utilization of Agricultural Surplus: the input of land (T) leads to agricultural output (Q) which is supplied to the non-agricultural labor (W) which produced non-agricultural goods that flow back to the landlord class.

It does not stretch the imagination too far to see that the same pattern of life prevailed in traditional China--and that the role of the standard market can be explained in terms of this dualism. To a large extent, there is self sufficiency in the nucleated villages (Diagram 1c) which

which render frequent trips to the market center unnecessary from the economic standpoint. When such trips are made, it was for the purpose of exchange of agricultural goods for products produced not only by the laborers living in the marketing center (c), but, indirectly through trade, by laborers living in the urban centers of higher hierarchy.¹ Thus the standard market is an institution of exchange and production specialization in a dualistic agrarian economy.

The standard market is also a part of the institution arrangement for the channelization of agricultural surplus to support the urban labor force which produced those goods and services (soldiers, domestic and civil servants, producers of luxury consumer goods, etc.) catering to the demand of the land owning gentry class and/or the government they control. The magnitude of the agricultural surplus in this sense corresponds to the agricultural tax and rental payments which in traditional China may account for as high as 60% of the outputs produced in the nucleated villages. The standard marketing centers are the last link in this institution; the economic functions of which are performed by the landlords, tax collector, civil servants etc. residing in the standard marketing centers.

The Physiocrats not only help us to understand the economic-institutional significance of the standard market but also taught us that a prosperous agricultural sector is a prerequisite for flourishing non-

¹The aggregate view of the urban sector in Diagram 1a merely describes the spacial structure (i.e. location patterns) of non-agricultural production. The pyramid (or hierarchy) of urban centers has its own rules of formation--an aspect of industrial location theory which will not be explored in this paper. Our emphasis is on the "dualistic exchange" between the agricultural and non-agricultural sector and for this reason, the readers may, from now on, assume that there are no urban centers of hierarchy other than the standard marketing centers as a first approximation.

agricultural activities.¹ A modern version of the principle of "exhultation of agriculture" may be stated in terms of agricultural labor productivity ($p = Y/L$) non agricultural labor productivity ($q = Q/W$) the fraction of non-agricultural labor out of the total labor force ($\theta = W/(L + W)$) and the per capita consumption demand of agricultural goods ($c = Y/(L + W)$) and non-agricultural goods ($n = Q/(L + W)$). Since the total supply of food is L_p and the total demand is $c(L + W)$, the equality of supply and demand immediately leads, to:

$$(2) \quad c/p = 1 - \theta$$

For example, if per capita consumption of food (c) is eight percent of agricultural labor productivity (p , such as in China) then 80% of the total population must be farmers to grow enough food for the 20%. Similarly $\theta = n/q$ which, when substituted in (2) leads to:

$$(3) \quad n = q(1 - c/p)$$

which states that the per capita availability of non-agricultural goods (n) can be increased not only due to productivity gain in non-agricultural labor force but also due to productivity gain in the agricultural labor force (p). The later effect comes about because more labor can be released from agricultural to non-agricultural production--after the per capita food demand (c) is met.

To make the argument clearer, in Diagram 4, the non-agricultural labor force W is measured on the horizontal axis and the total output of non-

¹ Thus they refer, somewhat misleadingly, to land as the source of all wealth and regard the non-agricultural labor force as non-productive. We must add that the French Physiocrats, alledgedly, had imported the idea of "exaltation of agriculture" from the Chinese during the early Ching dynasty.

agricultural goods (a) is represented by the total output Q_{AB} . When $W = 10$, total output is 200 leading to a labor productivity of $q = 20$ (plotted by the labor productivity in the lower deck). When W increases to 20, total output increases more than proportionally to $Q = 500$ leading to labor productivity of $q = 25$. Between point A and point B there is efficiency of large scale production--i.e. the total output curve is concave. The efficiency of large scale production is exhausted after point B when the total output curve becomes a straight line (or even becomes convex after point C).

Thus as agricultural productivity (p) expands, the non-agricultural labor force (W) expands accordingly--e.g. through a, A, b, B, c, C.... More output will be available on a per capital basis (n^L). Not only will there be more non-agricultural labor force (higher θ) but their productivity will increase because of the efficiency of large scale production--at least to the point where such efficiency is exhausted. The "exhaustion of agriculture" must basically be argued in "these" economic welfare terms--the standard market is merely an institution to implement such an agrarian system.

V. Theory of the Size of Marketing Area

The various threads of thoughts above may now be integrated to form a thesis which explains the economic forces that determine the size of the standard market. Referring back to the two cases of Diagram 2c which show two hexagon coverages of the same total areas. Let us assume that the population density (ϵ) is fixed so that the total population is 420 in each case. Let us also assume that the agricultural productivity (p) is fixed so that a fraction of non-agricultural labor force (θ) is determined by the argument in

¹The impact on " n " is shown diagrammatically in the lower deck of diagram 4. The curve $hh'q$ is the labor productivity curve (q).

the last section. Suppose $\theta = .1$ so that in each standard market the non-agricultural labor force (W) accounts for 10% of the population. All this is summarized in Diagram 2d which is a reproduction of 2c-- the only added information being labor allocation between L and W .

Now in case there is no efficiency of large scale production in non-agricultural production the smaller labor force ($W = 6$) in case (i) and the larger labor force ($W=105$) in case (ii) will have the same labor productivity (q). The economic welfare of the two cases (in terms of per capital supply of agricultural and non-agricultural goods) will be the same. (See Equation 3.) The Principle of the minimization of transportation immediately implies that under "constant returns to scale in non-agricultural production", the smaller market area case (i) is to be preferred. However, when there is efficiency of large scale production, the larger labor force will have a higher labor productivity (q) implying higher economic welfare. The larger market area will be chosen in any rational society in the absence of transportation cost considerations. Realistically, the optimum market area is that one which maximizes consumer welfare due to a balancing of the advantage of low transportation cost for small area and the efficiency of large scale production in the large area.

The outline of this deterministic theory is sketched in Diagram 5. The two axes in the third quadrant (linked by a 45-degree line) are used to measure r , the radius of the hexagon (i.e. the distance traveled by the least advantageous villagers). The dotted curve in the fourth quadrant measures the total population in the standard market area ($P = kr^2\epsilon$) i.e. population density (ϵ) multiplied by the area of hexagon (Kr^2).¹ The solid curve in the same quadrant is the size of the non-agricultural labor force

¹For any polygon, the area is proportional to the square of the radius.

($W = \theta P$) i.e. a fraction (θ) of the total population. The total output curve of Diagram 4 is reproduced in the first quadrant and is translated to the second quadrant (i.e. the total output curve) with the aid of the rectangles. This curve shows, for each radius (r) the total output of non-agricultural goods produced by W in the market area (with radius r).

In the same quadrant, the total transportation cost, associated with each r , is shown by the transportation cost curve (Equation 1c). With proper adjustment of the unit of measurement¹, the vertical gap between the two curves (shaded) then represents the net gain (i.e. industrial output less total transportation cost). In any rational society, the optimum market areas (or the optimum value of r) is determined at that level (in our case r_m) where the net gain is a maximum (the distance uv).

The statistical works of Professor Skinner gave us the equilibrium (n optimum) observed in a typical standard market in traditional China. Such a typical standard market is located in a region with population density of $\epsilon = 109$ persons per square kilometer. The sexagon market region has a radius of 4.34 kilometers giving an area of 64 km^2 and thus containing total population of $P = 7,037 (=109 \times 64)$ persons. The average family size in traditional China is 5.21 persons and thus there are 1350 families. Assuming $\theta = .2$, there are thus 270 urban families living in the urban centers and 1080 families living in the nucleated villages.² The

¹While the unit of measurement of Q is output, the unit of measurement of transportation cost is labor. However, they may be converted into the same unit of measurement (e.g. labor) by considering the relative price of labor and output e.g. according to the labor theory of value.

²The 64 km^2 of market area amounts to 15808 acres. With a degree of land utilization of .34 (Buck), the cultivated acreage is 4742 ($= 15808 \times .34$) implying 4.3 acres ($4742/1080$) acres per family, etc.

theory which we have constructed thus provides an explanatory framework for these observed magnitudes.

With the theoretical framework, we can now see why the market area cannot be too large. Basically, there are two technological factors which hold back the market size. On the one hand there is a rapid increase of the transportation cost proportional to the third power of the radius " r ". On the other hand the expansion of output (Q) will eventually be much slower, i.e. proportional at most to the second power of r --as the efficiency of large scale production is exhausted. The optimum market radius (r_m) is established at the level whereby the marginal transportation cost is equated with the marginal output as the radius varies, (indicated by the parallel lines m and n).

The reasonableness of our theory may be examined deductively and inductively. In respect to the former aspect, the theory envisions the standard market as a rational economic institution for a dualistic agrarian economy in which technological factors (production and transportation) delimits its operation. The theorem envisions an expansion of the market area associated with every improvement in the transportation facilities (i.e. lowering of the transportation cost curve) and/or gains in the efficiency of large scale production in agricultural production. These theoretical predictions will now be examined in the light of inductive (i.e. statistical) evidence.

VI. Regional Variation of Market Area

Professor Skinner introduced the statistical evidence that, except for a "tail-end exception"¹, the size of the standard market shrinks from the

¹The "tail-end exception" refers to the fact that at the very densely populated large urban centers, the standard market area expands.

thinly populated to the densely populated regions in China. To analyze this phenomenon, we have yet to take into consideration of a third property of land space¹ namely regional non-homogeneity of resources endowment. The fact that traditional China consists of many "Buckinized" agricultural regions is well known. We shall associate with these regions the abstract notion of differentiated fertility of land--as a first approximation.

It is reasonable to assume that the fertile regions are those regions with both a higher population density (ϵ) and a higher agricultural labor productivity (p).² This latter condition, implies a higher fraction of non-agricultural labor force θ (i.e. higher degree of commercial and manufacturing and trading activities for the densely populated "fertile" regions (see Equation 2, last section). Thus, with higher regional agricultural productivity (p) we can associate with higher population density ϵ and higher fraction of non-agricultural labor force (θ).

¹The two other properties are "area and distance" which we have already incorporated in our theory of standard market for a homogeneous plane.

²The formal economic theory which establishes this conclusion is due to Ricardo. His theory of rent formally recognized the differentiated fertility land as a basic factor that determines population allocation and emergence of rent of different magnitudes between regions. It is a queer methodological accident that Ricardo would produce such a theory for post-industrial revolution England giving so much emphasis to differentiation of regional land fertility in the agricultural sector. Such a theory, however, is very useful for the analysis of a large agrarian economy, such as traditional China in which regional differentiation of land fertility is such a prominent feature. Among other things, the theory is the foundation for the analysis of regional population migration under the impact of population growth. In this paper, we have only made use of the "conclusion" cited here--which can be proven rigorously by the Ricardian theory.

Returning to the fourth quadrant of Diagram 5, we see that as (θ, ϵ) increases the two curves shift upwards--meaning, for each " r ", the total population as well as non-agricultural labor force increases in the regions with higher agricultural productivity. This will, in turn raise the total output curve in the second quadrant. The situation is reproduced in Diagram 6 where with a fixed transportation cost curve, the upward shift of the total output curves (Q' , Q'' and Q''' ...) corresponds to higher agricultural productivity in the more densely populated regions. The equilibrium optimum values of the radius is seen to shrink accordingly ($r'_m > r''_m > r'''_m$). This established the phenomenon of shrinkage of market area as observed by Professor Skinner.

Intuitively, the basic cause of the shrinkage of the market area in the more densely populated regions is due to the exhaustion of the efficiency of large scale production as industrial labor force expands. In other words, in an agrarian economy, the traditional technology of non-agricultural production is such that there is a limit to the advantage to be gained from increasing the size of the labor force. When the population density is higher, the same market area will contain more total as well as more non-agricultural labor force than before. Since there is no further advantage that can be gained from the efficiency of large scale production, the market area shrinks in order to economize the transportation cost.¹

¹ The "tail-end exception" phenomenon observed by Professor Skinner can also be explained by the same principle. The large urban centers are principally those placed where the efficiency of large scale production is more conspicuous. Furthermore, these centers are located at river and road junctions where the local transportation cost is lower. Both factors contribute to an expansion of the standard market area.

VII. The Process of the Formation and the Shape of the Standard Market

It takes a different kind of theory to explain the process of formation of the standard market in traditional China. Since the development of such a theory is, as yet, incomplete, we shall only briefly provide an outline.

To begin with, the process of formation is a long process. We can imagine after the wasteland in the agrarian proper part of China was filled up during the Southern Sung dynasty, the process took place for at least 500-600 years. Furthermore, in this time space, the population density increases (because of growing population) so that, according to our analysis in the last section, the number of the standard markets increases (and becomes smaller) in a given homogeneous land space. In this process, the shape of the standard market becomes gradually regularized, i.e. the shape becomes asymptotically sexagon in shape. Finally, the process is an "experimental one" in that deviation from rationality of the spacial structure market will be corrected by a "trial and error" process--so that an irrational (or inefficient) spacial structure will tend to be rebuilt in the direction of rationality, whenever the existing structure was destroyed by war, famine or other calamities causing large scale population dislocation.

Suppose at any point in time in the long process, the population density (^E) warranted the coverage of the land space by n-standard markets $A_1, A_2, \dots, A_n$ with n-marketing centers $c_1, c_2, \dots, c_n$. Since the transportation coefficient are defined for each market, the total transportation cost is

$$(6) \quad T = T(A_1, c_1) + T(A_2, c_2) + \dots + T(A_n, c_n)$$

By a rational coverage (or efficient coverage) we shall mean that the shape and size of the A's as well as the location of the c's must be such that T is minimized.

We can immediately identify two necessary conditions of the efficiency of the coverage:

- (i) When $(A_1, A_2, \dots, A_n)$ are given, the urban centers $c_1, c_2, \dots, c_n$ must be located at the geographic centers of gravity (see Diagram 7a).
- (ii) When $(c_1, c_2, \dots, c_n)$ are given, the market areas $A_1, A_2, \dots, A_n$ must be determined by the perpendicular bisectors to the lines connecting the adjacent urban centers (i.e. the c 's) (see Diagram 7b).

Condition (i) states the principle of the location of the marketing centers when the marketing areas are given while condition (ii) states the principle of the determination of the market boundaries when the location of the city centers are given.¹ The deviations from these principles² will tend to be corrected when the market areas are reconstructed after they are devastated, for example, by war. Thus, through trial and error process, both conditions then are to be fulfilled in the long run.

¹ Both conditions ensure that the transportation cost is minimized within each marketing area individually--a necessary condition for the minimization of total transportation cost. The two conditions, however, are not sufficient: as can be seen from the example in (c) in Diagram 3. Here the two urban centers are too close and the coverage is not as efficient as the one portrayed in (d).

² A marketing center may be wrongly located because it was there historically. A nucleated village may choose to affiliate with the wrong marketing centers for historical loyalty.

Now suppose because of population expansion, the higher population density (ϵ) now warrants a coverage of the same land space with more numerous standard markets ($m > n$). The situation is now depicted in Diagram 8, which magnified Diagram 7 and in which both conditions are fulfilled. Here we begin to sense that the marketing regions ($A_1, A_2, \dots, A_m$) all tend to be sexagonic in shape. This phenomenon suggests the following theorem:

(7) Under the principle of minimization of total transportation cost (6), the coverage of a given land shape (R) by a marketing structure ($A_1, A_2, \dots, A_n$; $c_1, c_2, \dots, c_n$) will be equal-area asymptotically regular-sexagonic coverages as n increases to infinity.

The theorem, purely mathematical in nature, is at yet an unproven conjecture. If proven, the content of the theorem conveys the idea that the study of the process of formation of the standard market is a long run theory emphasizing the rationality of the spacial structure in the context of continuous population pressure. It also firmly established the sexagonic nature of the standard market--a very difficult topic which has eluded us so far in our analysis.¹

¹In Section II of our paper such a coverage was assumed rather than proven.

APPENDIX

Let A be a simply connected region in the two dimension space and let c be a point in A. Let dA be a differential area and let x be the distance between (any point in) dA and c. Suppose the tonnage of shipment is s per unit area, then the shipment from dA to c is xsdA ton-miles (without increasing transportation cost) and is $x^2 s dA$ (with increasing cost). The total special transportation costs are then

$$\text{A1a)} \quad T = s \int_A x dA = s M^{(1)}(A, C) \quad (\text{without increasing cost})$$

$$\text{A1b)} \quad T = s \int_A x^2 dA = s M^{(2)}(A, C) \quad (\text{with increasing cost})$$

where $M^{(1)}$ and $M^{(2)}$ are, respectively, the first and second moment of A with respect to c. If, in particular, A is a regular sexagon with radius r and c is located at the "center," we easily have: (without increasing cost)

$$\text{A2)} \quad T = (r/3) \cos^3(\pi/n) [\tan(\pi/n) \sec(\pi/n) + \ln(\tan(\pi/n) + \sec(\pi/n))] r^3$$

which shows that the special transportation cost is proportional to the third power of the radius. (labc in Section III are special cases of A2 with $n = 3, 4, 6$)

$$\text{A3)} \quad R = k r^2$$

which is proportional to the second power of the radius.

In the theory of determination of the market area of section V, the following equations are postulated:

- A4a) $Q = f(W)$ (production function for nonagriculture output)
- b) $R = k r^2$... (A3)
- c) $P = \epsilon R$ (ϵ is population density)
- d) $W = \theta P$ (θ is fraction of nonagricultural labor force)
- e) $T = k' r^3$ (A2)
- f) $u = (Q-T)/P$ (per capita net consumption of nonagricultural goods)/1

/1 In diagram 5, we have defined u as Q-T to simplify diagrammatic exposition of the main idea.

The problem of the determination of the market area amounts to the determination of that r which maximizes u , i.e.

$$A5) \text{ To maximize: } u = (f(\theta \epsilon k r^2) - k' r^3) / k r^2 \epsilon = f(\theta \epsilon k r^2) / k r^2 \epsilon - (k' / k) (r / \epsilon)$$

which can be solved by differentiating u with respect to r to obtain the first order condition:

$$A6) \quad du/dr = \phi(r; \theta, \epsilon) = 0$$

The first order condition implies that the optimum values of r (i.e. r_m) is a function of θ, ϵ which are being held fixed in the differentiation of A6. Thus conceptually

$$A7) \quad r_m = F(\theta, \epsilon)$$

In order for r_m to be a true maximum, the sufficient condition is

$$A8) \quad d^2u/dr^2 = d\phi/dr < 0$$

The problem of the regional variation of the market area is a comparative static analysis based on the above static equilibrium problem. The basic argument is that as we move from a less fertile to a more fertile region, both θ and ϵ increase. Thus we may postulate on the other hand, the area of the hexagon is:

$$A9) \quad \theta = g(\epsilon) \text{ with } g' > 0$$

We can then differentiate r_m (A7) with respect to ϵ to obtain:

$$A10) \quad dr_m/d\epsilon = \frac{a_F}{a\epsilon} + \frac{a_F}{a\theta} g'$$

The heart of the theory of regional variation of the market area is to prove that A10) is negative -- i.e. r_m decreases as ϵ increases. Notice that in the text, we have not given such a rigorous proof. Instead, we only demonstrate the likelihood and the feasibility of the " r_m -shrinkage" phenomenon (diagram 5b)--as related to exhaustion of the efficiency of large scale production. A rigorous proof would have involved the second order condition (A8) which, in turn, necessitates a careful specification of properties of the production function $Q=f(W)$ (A4) in respect to the efficiency of large scale production.

We may add the remark here that the theory of the determination of the size of the standard market (covered by section V and VI) is a typical economic problem in the Samuelson sense. On the other hand, the theory of the shape of the standard market is an entirely different (and more difficult) analytical problem of space. As formulated in section VII, the theory amounts to the proof of a purely mathematical theorem of two dimension space which, at the present time, is merely a theoretical conjecture. The validity of the elementary discussion on the shape of the standard market given in section III in the text /1 depends on the validity of this unproved conjecture.

/1 In section III we accepted, without proof, that when the number of standard markets is large, they all tend to be regular polygons with the same shape and size. It is then easy to show that the sexagon coverage is the optimum.