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ON INEQUALITY AND ECONOMIC DEVELOPMENT

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Introduction

The purpose of this paper is to develop an explicitly dynamic and growth-relevant framework for relating changing income distributions in less developed countries to their economic development performances. To introduce the basic point of this paper let us consider two hypothetical situations.

First, let us suppose that two initially identical countries have the following development histories:

	Percentage of Labor Force in:		
Country	High Wage Jobs (W=2)	Low Wage Jobs (W=1)	Rate of Growth of Modern Sector ("Modern Sector Labor Absorption Rate")
Both countries initially	10%	90%	
Country A later	20%	80%	100%
Country B later	30%	70%	200%

In both countries, the poor received the benefits of growth; but in country B, twice as many poor benefited.
Development economists would almost certainly rate country B as superior, and development planners would seek to find out what had brought about that country's favorable experience and adopt those policies in their own countries. It is hard to imagine that anyone would not prefer B to A.

Now let us consider another example. Suppose we had some other information about the development histories of two hypothetical countries:

		Share of Lowest 40%:		Gini Coefficient:	
Country	Rate of Growth	Level	% Change	Level	% Change
Both countries initially		.363		.082	
Country C later	11%	.333	-8%	.133	+62%
Country D later	22%	.307	-15%	.162	+97%

Country D grew twice as fast as country C. However, its income distribution, as measured by the Gini coefficient and income share of the lowest 40%, seems to be "worse" than in country C; that is, it would appear that the rich benefited at the expense of the poor, whose relative income share deteriorated. A development economist might question whether the higher rate of growth in country D was "worth it" in terms of income distribution, and a well-meaning development planner seeking to give very high weight to alleviation of inequality might go so far as to choose country C's policies over country D's. In any case, the issue is apparently open to doubt here, whereas in choosing between countries A and B, it was very clear cut.

In point of fact, country C is the same as country A and country D the same as country B. Real-world economic development histories and policy projections are often presented in both ways. Yet, as this example illustrates, how the information is presented can have a dramatic influence on how we feel about the outcome.

How can it be that the two sets of answers are so different? It is reasonable to assume that development economists and planners have intertemporally consistent judgments about social welfare, i.e., each time we ask for a comparison of two situations, one is always judged as either better than, worse than, or equally good as the other, or we admit that we cannot choose.[1] Assuming consistency of true preferences, then, it must be that

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[1]For instance, in choosing between two alternative distributions of the <u>same amount of income,</u> many people would adopt the following decision rule:

a) If one Lorenz curve lies wholly above another, then the first situation is preferred to the second;
b) If the two Lorenz curves coincide, then the two situations are deemed equally good (or bad);
c) If the two Lorenz curves intersect, we require further information before making a decision.

there is something about the way we process the two data sets in our minds which causes the problem. More precisely, if we were to specify more carefully the decision rule we used in arriving at each of the two judgments for the type of economic development illustrated by this example, we would probably find an inherent conflict between them.

This paper seeks: (1) To explore these decision rules more carefully in a growth-relevant framework; (2) To determine whether there is a "natural" or "inevitable" relationship between inequality in the distribution of income in a country and its economic development; (3) To ascertain when a rising degree of inequality (as measured by a rising Gini coefficient or falling share of income received by the poorest 40% of the population, for example) can be interpreted as a true "worsening of the income distribution", receiving negative weight in the social welfare judgments or persons who prefer economic development patterns which favor the poor, and when it cannot; and (4) To propose an alternative methodology for analyzing the distribution of the benefits of economic development; this methodology decomposes total growth into its component parts, indicating growth in average incomes within well-defined sectors and the growth in size of those sectors, thus telling us who gets the benefits of development.

The specification of the rules for arriving at judgments about the distributional effects of economic development is the subject of Section I. In Section II, we examine how the rules are or are not in conflict with each of three very different stylized types of dualistic economic development: Modern Sector Enlargement Growth, illustrated above, where an economy grows by enlarging the size of its modern sector; Modern Sector Enrichment Growth, where the growth accrues only to a fixed number of persons in the modern sector; and Traditional Sector Enrichment Growth, where all of the proceeds of growth go to those in the traditional sector. Section III analyzes how measured inequality changes as each type of development proceeds, thereby telling us if the income distribution must inevitably "worsen" in the initial stages of economic development. Then, in Section IV, we demonstrate how these three stylized cases can be synthesized and present an alternative methodology for measuring the distribution of the benefits of economic growth. Section V shows how the basic methodology can be extended to treat more than two sectors and to recognize explicitly population growth; the methodology derived here is also compared with an approach recently suggested by Ahluwalia and Chenery (1974). The paper concludes in Section VI by summarizing the main findings of the paper as they relate to empirical research.

I. The Inequality Index and Axiomatic Approaches for Analyzing the Distribution of the Benefits of Economic Growth

A. The Inequality Index Approach

A prime issue of discussion in economic development and in other branches of economics is the question of who receives the benefits of growth, or more precisely, which economic or demographic groups receive how much. There is considerable agreement about the inadequacy of aggregate GNP growth as the sole measure of economic development, for it fails to show whether the poor share in the benefits or are left behind. A number of distributionally-oriented studies have appeared in recent years, taking as their criteria for inequality such indices as a Gini coefficient or the share of income received by the lowest 40%.[1] On the basis of comparisons of these measures at two or more points in time, larger measured inequality than before is

[1] In addition to the many studies conducted in individual countries, a now considerable number of studies have examined changing income distributions in a number of countries. The interested reader is referred to the work of Adelman and Morris (1971) Chenery et. al (1974), Cline (1973), Kuznets (1963), Musgrove (1974), Paukert (1973), and Weisskoff (1970).

reported in many countries.

Implicit in many if not all of the recent studies is the judgment that social welfare (W) is a positive function of the level of national income (Y) and a negative function of the inequality in the distribution of that income (I). For example, taking the share of income of the lowest 40% of the population (S) as an index of equality and the Gini coefficient (G) as an index of inequality, these studies would hold that:

$$\text{(1a)}\quad W = f\,(Y,S),\quad f_1 > 0,\quad f_2 > 0$$

or

$$\text{(b)}\quad W = f\,(Y,G),\quad f_1 > 0,\quad f_2 < 0.$$

The existence of more than one easily calculable inequality index has generated a great deal of discussion of which is the best to use, and the various indices have undergone intensive examination.[1] With an occasional exception, the investigations of the properties of inequality measures have taken a static perspective, that is, they have focused on the measurement on inequality at a point in time. However, the distribution of economic benefits in the course of economic development is inherently dynamic, referring to a phenomenon that takes place over time, and is appropriately measured by a dynamic index.

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[1]One good review of the properties of these various inequality measures may be found in Sen (1973). For an attempt to characterize these various indices in terms of their compatability with widely-shared judgments about economic welfare, see Fields and Fei (1974).

Do the customary indices retain their validity in a dynamic development context? To answer this question, we must have some independent criteria other than the inequality measures themselves for deciding whether a given index does or not satisfy them. If there is a direct correspondence between changes in a particular inequality measure and changes in social welfare according to these independent criteria, then there is no problem in using that measure. However, if there is disagreement between the independent criteria, and a given inequality measure--for example, one registering an unambiguous improvement with the other suggesting ambiguity---then to resolve the conflict we are impelled to reject either the measure or the criteria

I.B. The Axiomatic Approach[1]

In this subsection, I shall propose a set of criteria which have substantial support in the econcmics literature. The criteria proposed here will be regarded as an axiomatic system, in that their validity will be accepted without proof.[2] In Section II, we will see how these axioms relate to the three stylized types of economic growth mentioned above.

Let us begin by defining one income distribution as the same as another if the two have the same Lorenz curve, and one as more equal than another if it Lorenz-dominates the other.[3] I will now suggest three propositions

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[1]Everything in this section should be understood as pertaining to real incomes. Anything which changes relative prices paid by some groups but leaves their money incomes the same does not satisfy the ceteris paribus conditions of the three axioms presented below.

[2]An axiomatic system must have two other characteristics besides plausibility: the axioms must be consistent with one another and they must be independent. Consistency is easily established by numerical example. Independence requires that it be possible to satisfy each two-way combination without necessarily satisfying the third; this is also established by example.

[3]One Lorenz curve dominates another when it lies above the other at at leas one point and never lies below it. Since Lorenz curves are defined according to income shares, the use of the Lorenz curve as the means of defining inequalit implies that measured inequality is independent of the level of income. Note that this does not mean that our feelings about inequality are invariant with income level. For a perceptive analysis of changing tolerance for inequality in the course of economic development, see Hirschman and Rothschild (1973).

about social well-being, which receive considerable support in the literature. These ideas are rather uncontroversial, and I imagine others would share them as well:

(2a) <u>For any given Lorenz distribution, social welfare is greater the higher is the level of national income</u>.

This axiom holds that if everybody is made better off by exactly the same percentage, then society has achieved a higher level of social well-being. It relies for its validity on the assumption that the basic goal of an economic system is to maximize the output of goods and services received by each of its members, and the more each receives, the closer the economic system is toward fulfilling that goal.

(2b) <u>For any given level of national income, social welfare is greater the more equal is the distribution of income, i.e., if one Lorenz curve dominates another</u>.

If the Lorenz curve of one income distribution A dominates that of another distribution B for the same level of income, it means distribution A can be obtained from distribution B by transferring positive amounts of income from the relatively rich to the relatively poor.[1] The judgment that such transfers improve social welfare dates back at least to Dalton (1920). One possible justification for this principle is diminishing marginal utility of income, coupled with independent and homothetic individual utility functions and an additively separable social welfare function.[2] But these assumptions are not necessary for the affirmation of this axiom.

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[1]See Rothschild and Stiglitz (1973) and Fields and Fei (1974).

[2]See Atkinson (1970).

(2c) Social Welfare is greater for any Pareto improvement.

Ordinarily, we think that if somebody is made better off economically and nobody is made worse off, then the sum total of happiness in society is greater than before. We should be clear just what this axiom implies, for the key word in this axiom is the word "any". Even if the richest man in the country were the beneficiary of the Pareto improvement, the axiom would hold that society is better off. In other words, whatever weight we give to relative income notions and envy of one's neighbors or compatriots, acceptance of this axiom implies that the envy is more than counterbalanced in our social welfare judgments by the increased happiness of the income recipient.[1]

The converses of each of these propositions are also assumed to hold. Thus, for instance, a lower level of national income for a given Lorenz distribution would imply reduced social welfare.

It is probably worth noting explicitly that the axiomatic system given by criteria (2a-c) is incomplete in the sense that it does not tell us how to compare all possible combinations of growth rates and distributional patterns For example, if comparing two initially identical countries M and N, M had achieved a higher rate of growth than N but its income distribution was less equal (i.e., Lorenz-inferior) to that of N, we could not use the above criteria to determine which we would prefer unless the less equal income distribution were the result of one or a series of Pareto improvements.
This is because M would be better by (2a),

[1]For example, if a sudden increase in the world price of coffee increases the income of a Latin American coffee-grower, and he buys a transistor radio to celebrate, this axiom holds that there is a net gain in social welfare, even if the grower's day-workers are envious of his good fortune.

worse by (2b), and (2c) does not apply in this comparison, and the three rules do not provide a basis for balancing the various effects.

It is evident that the implicit social welfare function rule (1) based on inequality indices and the explicit but incomplete social welfare rules (2) based on axioms are not identically equal. They may, however, be the same in practice, always yielding the same qualitative judgment for any given growth pattern. Whether or not they do depends on the nature of the economic development and is the subject of Section II.

II. Changing Inequality in Three Economic Development Typologies

In this section, we construct models of three types of economic development and examine what happens to measured inequality when growth takes place. We do not ask what produces this growth; there is a voluminous literature on that. Rather, we ask: given that growth proceeds according to one of these three patterns, what happens to inequality (a) as measured by conventional indices, and (b) according to the three axioms.

Each of the models developed here is a simple dualistic model, divided into two sectors: modern and traditional.[1] The three stylized development typologies considered here are defined as follows:

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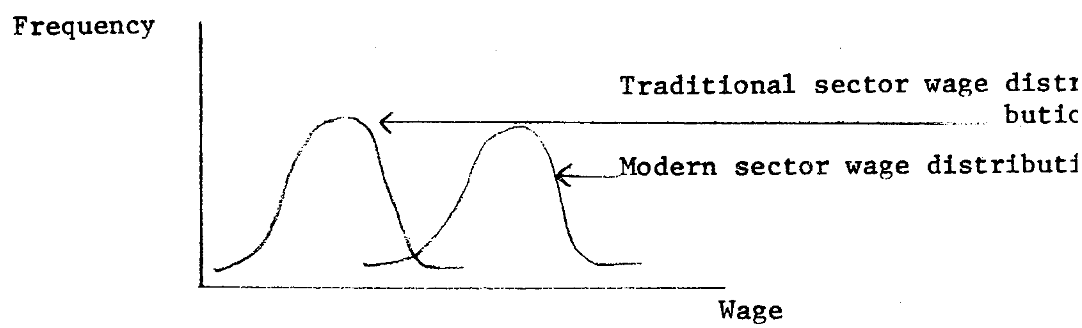
[1] The two sectors might alternatively be thought of as skilled vs. unskilled labor, or urban vs. rural, or capital vs. labor.

As with all dualistic models, the working assumption being made is that the members of each sector are relatively similar to others in that sector and relatively different from those in the other sector. That is, for a two sector model to be useful, there must be a strong, although not necessarily perfect, division between the sectors, as pictured:

TABLE 1. DEFINITION OF DUALISTIC DEVELOPMENT TYPOLOGIES

Development Typology	Distribution of the Labor Force Between the Modern Sector and the Traditional Sector	Modern Sector Income	Traditional Sector Income
Traditional Sector Enrichment	Remains the same	Remains the same	Rises
Modern Sector Enrichment	Remains the same	Rises	Remains the same
Modern Sector Enlargement	More workers in modern sector	Remains the same	Remains the same



If, however, within-sector inequality is very important relative to between-sector inequality, then a dualistic model loses much of its usefulness. Recent work by Fishlow (1972) for Brazil and Fei and Ranis (1974) for Taiwan suggest that variation in labor income is the most important source of income inequality. This suggests that a high wage sector-low wage sector dichotomy would be most relevant, and that a division according to functional shares (labor vs. capital, for example) would be somewhat less useful. Accordingly, in the models in this paper, we shall regard the modern sector as synonymous with high wages and the traditional sector as synonymous with low wages.

There is no solid factual basis for determining which countries or regions of the world most closely fit each typology. However, general impressions suggest that "traditional sector enrichment" may come closest to describing some of the socialist countries (China, Cuba, Tanzania) and some Asian countries (Taiwan, Korea); "Modern sector enrichment" may typify other parts of Asia (the Philippines, Thailand) and the oil-rich countries of the Middle East (Iran); and "modern sector enlargement" might be the case in much of sub-Saharan Africa (Kenya, Nigeria at present).[1] Whether or not the reader agrees with these particular impressionistic illustrations, the actual classification of countries or regions is a matter of degree, and can only be carried out after much more intensive analysis of more comprehensive data than are now available.

We now consider each type in turn and then analyze their similarities and differences.

A. Traditional Sector Enrichment Growth

In the traditional sector enrichment growth model, incomes in the traditional sector are assumed to rise, incomes in the modern sector remain the same, and the allocation of the labor force between the two sectors also remains the same. Intuitively, it would seem that the enrichment of the poorer class would lead to greater equality in the distribution of income, and that the

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[1]In addition to these development typologies, it may be helpful to consider two types of non-development: the so-called "Fourth World" countries (Bolivia, Chad, Bangladesh), which if they are developing at all, are growing at a very slow rate; and the so-called "development disasters" (Uganda at present, Nigeria in the late 1960s) which by all accounts seem to have had negative growth.

faster this type of growth proceeds, the "better" things are, both in terms of level of income and in terms of its distribution. In fact, as is demonstrated in Appendix A, the inequality index approaches (eq. (1a) and (1b)) and the axiomatic approach (2) to social welfare are in full agreement for traditional sector enrichment growth.

B. Modern Sector Enrichment Growth

In modern sector enrichment growth, incomes in the modern sector rise, while incomes in the traditional sector and the allocation of the labor force between the modern sector and the traditional sector remain the same. With this type of growth, there arises a conflict between the inequality index approach and the axiomatic approach. In specific, we have the following theorem: The higher the rate of modern sector enrichment growth:

(a) The higher is social welfare according to the axiomatic approach;

(b) There is an ambiguous effect on social welfare according to the inequality index approaches.

This theorem is proven in Appendix B.

Why should this discrepancy arise? When modern sector enrichment growth takes place, three things happen:

(a) There is a Pareto improvement (in favor of those at the top);

(b) The rising incomes of those at the top imply a falling share of the total for those at the bottom, whose incomes are not growing;

(c) The rising incomes of those at the top and constant incomes of those at the bottom imply a rising Gini coefficient.

The axiomatic approach rates (a), the Pareto improvement, as a gain in social welfare. Since (b) and (c) do not signal less social welfare according to one of the other axioms, the axiomatic approach registers an improvement. On the other hand, according to the inequality index approach, even if the absolute level of income received by the poor remains the same, this approach gives negative weight to a falling share received by the poor and the rising Gini coefficient, and in this lies the source of the ambiguity.

C. Modern Sector Enlargement Growth

In the modern sector enlargement growth model, incomes in both the modern and the traditional sectors remain the same but the modern sector gets bigger. In this case, we may derive the following result:

When the modern sector is small relative to the total population, the higher the rate of modern sector enlargement growth:

(a) The higher is social welfare according to the axiomatic approach;

(b) There is an ambiguous effect on social welfare according to the inequality index approaches.

However, once the modern sector is sufficiently large, the two approaches are in agreement. This theorem is proven in Appendix C.

Essentially, the ambiguity of the inequality index approaches in early modern sector enlargement growth comes about because modern sector enlargement affects only some of the poor, not all. Consequently, those whose situations are not improved by this type of growth, and who therefore remain as poor as before, receive the same amount, but it is a smaller part of a larger whole. Furthermore, the faster the rate of this type of growth, and hence the higher the level of national income, the smaller the fraction of that income received by those left behind. Since the inequality index approaches give negative weight to the falling share of income received by the lowest 40% (the composition of which is changing with modern sector enlargement growth) or to the rising Gini coefficient, and both of these inequality indices register a "worsening" of the income distribution the faster this type of growth proceeds, somebody evaluating this type of economic development according to one of the inequality index approaches <u>might</u>, depending on his particular decision rule f, claim to prefer a situation with <u>less</u> modern sector enrichment growth, because the measured income distribution would be "better."

It seems to me that the more modern sector enlargement growth a country experiences, the better its economy has performed, and I doubt very many readers would disagree. In terms of the examples of the introduction, this means that whatever decision rule we use should rate the first situation (A and C) as inferior to the second (B and D). In this light, there is cause for concern over the lack of correspondence between the social welfare and inequality approaches, on two grounds.

First of all, there is the academic point that a "worsening" of the measured income distribution during modern sector enlargement growth should <u>not</u> be interpreted as a bad thing. Rather, the falling share of the lowest 40% (S) and rising Gini coefficient (G) which arise in this case are <u>statistical artifacts without social welfare content.</u> Thus, social welfare functions, whether explicitly-stated or implicitly-assumed, of the form

(1a) $W^S = f(Y, S), \; f_1 > 0, \; f_2 > 0$

and

(1b) $W^G = f(Y, G), \; f_1 > 0, \; f_2 < 0$

which most of us use, are invalid for this type of growth, and it would be far better to set $f_2 = 0$ and look only at income levels. We shall return to these points in Section III.C.

Second, from a policy perspective, it is quite disturbing to consider even the possibility that a real-world development economist or planner might write scholarly papers or recommend particular economic measures in support of a policy of less of this type of growth. Yet, unfortunately, we do find instances of writers who advocate that a certain type of economic development or industrialization not be undertaken, because although the project would add to national income and create high-paying jobs, it would result in a less equal measured income distribution.

D. Summary of the Three Development Typologies

The changes in measured income inequality and the various social welfare effects for each of the three economic development typologies are given in Table 2.

There is definite agreement between the inequality index and the axiomatic approaches to social welfare only for traditional sector enrichment growth. As this type of development takes place, the higher is the level of income (Y), the higher is the share of income received by the poorest 40% of the labor force (S), and the lower is the Gini coefficient (G), and hence the higher is the level of social welfare according to both approaches.

For <u>modern sector enrichment growth</u>, there arises a discrepancy. During this type of development, Y is higher, but there is also greater measured inequality (lower S, higher G), and therefore an ambiguous effect on social welfare according to the inequality index approach. However, the three axioms postulated above indicate an unambiguous welfare improvement for this type of growth.

Finally, for <u>modern sector enlargement growth</u>, there is also a discrepancy. While this type of development is taking place and Y increases steadily, inequality follows the familiar "inverted-U" pattern, first rising and then falling. During the first stage when measured inequality is rising, the social welfare effect of economic growth is ambiguous according to the inequality index approach. As with the other cases, the axiomatic approach shows an unambiguous welfare gain.

We turn now to an analysis of these effects.

TABLE 2

SUMMARY OF WELFARE EFFECTS FOR THREE TYPES OF ECONOMIC DEVELOPMENT

WELFARE EFFECT	TYPE OF ECONOMIC DEVELOPMENT		
	Traditional Sector Enrichment Growth	Modern Sector Enrichment Growth	Modern Sector Enlargement Growth
Income Level (Y)	Rises	Rises	Rises
Share of Income Received by Poorest 40% (S)	Rises	Falls	Falls while traditional sector is more than 40% of labor force, rises thereafter
Gini Coefficient (G)	Falls	Rises	Rises while modern sector is small, then falls
Effect on Social Welfare According to Inequality Index Approach (1)	Unambiguous improvement	Ambiguous	Ambiguous
Effect on Social Welfare According to Axiomatic Approach (2)	Unambiguous improvement	Unambiguous improvement	Unambiguous improvement

III. On the Inevitability of a "Worsening" Income Distribution in the Course of Economic Development

A. Is It Inevitable?

Kuznets (1955) observed that in a number of currently-developed countries, it appears that measured income inequality seems to rise at first with economic growth and then falls at higher levels of development, producing an "inverted-U" pattern. Ever since, economists have wondered whether such a relationship exists in the currently-developing countries as well. Initial evidence comparing measured inequality across countries was consistent with the "inverted-U" pattern, although only a small percentage of the variance in inequality can be explained by income level alone.[1] Further study of individual less developed countries over time has in many cases confirmed this pattern.[2]

These findings have led many development economists to ask whether an "inverted-U" pattern is inevitable in the course of economic development, or as the question is usually phrased: "Must income distribution get 'worse' before it gets 'better'?" The time paths of measured inequality for each of the three pure models of dualistic economic development are shown in Figure 1. It is apparent that the path of measured inequality depends on the <u>type</u> of economic development as well as its <u>level</u>. More specifically, on the inevitability issue, we have:

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[1] Kuznets (1955) and Oshima (1962) originally proposed the "inverted-U" pattern and presented evidence from several countries. Since then, several investigators have compiled additional cross-country data. Such data typically support the "inverted-U" pattern. Paukert (1973), for instance, found that "there is an increase in inequality as countries progress from the below $100 level to the $101-200 level and beyond...the peak of inequality is reached on attainment of the level of development and the structural pattern characterized by the countries...which in the neighbourhood of 1965 had a GDP per capita in the $201-500 range." Before regarding such a pattern as inevitable, though, even in the cross-section, we should note that the fit is far from perfect: using Paukert's data, I regressed the Gini coefficient on GDP-per-capita and GDP-per-capita squared (i.e., a parabolic regression) and found that income level can explain only 11% of the inter-country variance in inequality as measured by the Gini coefficient.

[2] The latest compilation of data for within-country comparisons may be found in Ahluwalia (1974).

(i) Traditional Sector Enrichment Growth. If a country chooses to develop along the lines of traditional sector enrichment growth, then ceteris paribus its income distribution will get progressively "better". over time.

(ii) Modern Sector Enrichment Growth. If a country chooses to develop along the lines of modern sector enrichment growth, then ceteris paribus its income distribution will get progressively "worse" over time.

(iii) Modern Sector Enlargement Growth. If a country chooses to develop along the lines of modern sector enlargement growth, then ceteris paribus its income distribution will follow an "inverted-U" pattern.

(iv) Switch from Modern Sector Enrichment to Traditional Sector Enrichment Growth. If a country first chooses to develop along the lines of modern sector enrichment growth, and then switches to a strategy of traditional sector enrichment, then its income distribution will also follow an "inverted-U" pattern, ceteris paribus.

The question of an inevitable initial stage of "worsening" then comes down to the inevitability of development strategies (iii) and (iv) as opposed to (i) and (ii). At the present time, there is no hard evidence on which patterns are being followed in which countries. Further research now under way on Taiwan, where measured inequality has fallen in recent years,[1] and on Colombia, where it has risen,[2] may shed some light on this issue.

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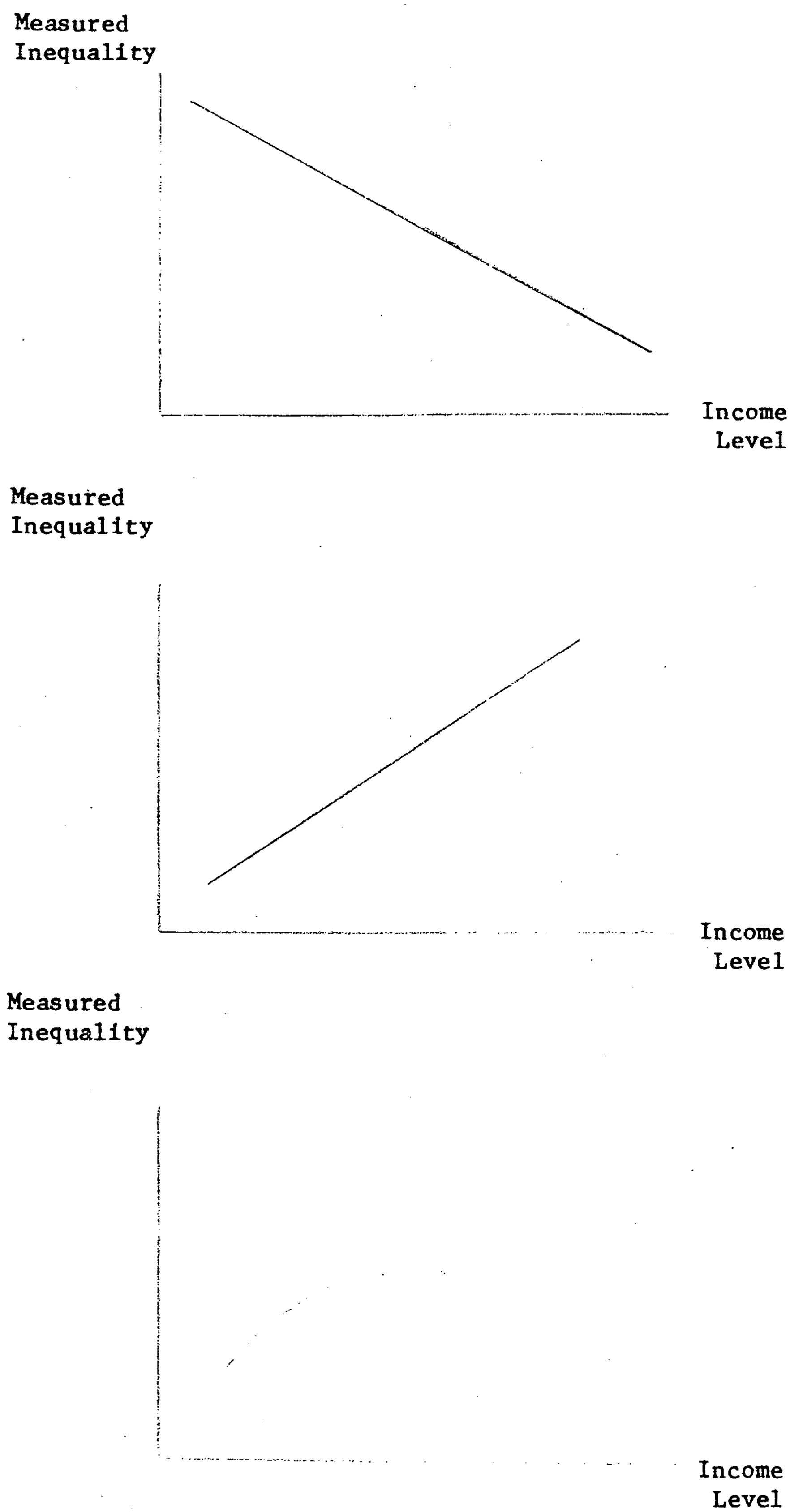
[1] See Fei, Ranis, and Kuo (1975).

[2] See Berry (1974).

FIGURE 1 CHANGING INEQUALITY DURING THREE TYPES OF ECONOMIC DEVELOPMENT.



(a) Traditional sector enrichment growth

(b) Modern sector enrichment growth

(c) Modern sector enlargement growth

B. If It Happens, Why?

Kuznets' original hypothesis was that the "inverted-U" pattern is caused by the transfer of workers from the rural sector, where incomes are relatively equally distributed at low levels, to the urban sector, where there is greater income dispersion, owing to the presence of a skilled professional class at the top and poor recent migrants at the bottom. In terms of the development typologies analyzed above, Kuznets' model is basically one of modern sector enlargement growth with within-sector inequality. Allowing for within-sector inequality in the context of the modern sector enlargement growth model, and taking the Gini coefficient as our measure of inequality, it may be demonstrated that the "inverted-U" pattern always arises in modern sector enlargement growth, regardless of the relationship between the Gini coefficients within the modern sector and the traditional sector; this is proven in Appendix D.[1]

These results add another dimension to Kuznets' explanation for the "inverted-U" pattern, namely, that the transfer of workers from the low income sector to the high income sector produces an "inverted-U" pattern which is a statistical consequence of modern sector enlargement growth. This leads us to ask whether this statistical pattern in modern sector enlargement growth truly signifies in an economic sense that "the distribution of income must get worse before it gets better." We take this up in Section C.

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[1]Probably the most important assumption producing this result is that the within-sector income distributions remain the same. See Appendix D for a description of the assumed conditions.

C. <u>If It Happens, Is It Bad</u>?

If in the course of economic development, measured inequality in the distribution of income rises, is it necessarily bad in social welfare terms for those who prefer a more equal distribution of any given amount of income to a less equal one? To answer this question, we must examine the various factors which affect our judgments based on inequality indices and the axiomatic approach respectively. These basic welfare effects have been summarized in Table 2 and Section II.D.

The reader may find the ambiguities in the cases of modern sector enrichment and enlargement at least somewhat discomforting. At first, it might seem that a falling share going to the poor (S) or a rising Gini coefficient (G) should receive negative weight in a social welfare judgment, possibly negative enough to outweigh the rising level of income. But why? There are at least two possible answers.

Implicitly, we may have in mind that a falling S or rising G implies that the poor are getting absolutely poorer while the rich are getting absolutely richer, and many of us would regard this as a bad thing indeed. The problem with this notion is that it confuses cause and effect, that is to say, absolute emiseration of the poor would definitely imply falling S and rising G, but as we have just seen, S can rise and G can fall without the poor becoming worse off in absolute terms.

Ruling out the necessity of absolute emiseration of the poor as a reason for reacting adversely to a falling S or a rising G, we may instead have in mind something of a relative income notion, that a rising gap between rich and poor is in and of itself a bad thing, not because the poor have lower incomes but rather because a wider percentage gap between rich and poor might make the poor feel worse off. As we observed when we first stated the third axiom, if we accept its universality, the axiom implies that any Pareto improvement leads to higher social welfare, so that any negative weight we give to envy of the rich by the poor is more than offset in social welfare terms by the gain in utility of the income recipients themselves.

In the case of modern sector enrichment growth, this relative income argument has some validity. It is plausible that, contrary to the third axiom, income growth concentrated exclusively in the hands of the rich might be interpreted as a socially inferior situation as compared with the rich having less and the poor the same amount. However, in the case of modern sector enlargement growth, there is not even this defense to fall back on.

As we have seen, a rising G and falling S may be a perfectly natural, and even highly desirable, outcome for this type of development. In this case, the specification of social welfare functions of forms like (1a) and (1b) apparently conflicts with our ideas of social well-being.

From this analysis of the various types of economic development and their relationships to measured income inequality and social welfare, we may observe that a falling share of income received by the poorest 40% (S) and rising Gini coefficient (G) can be the result of:

(a) Traditional Sector Impoverishment, which is clearly bad in social welfare terms; or

(b) Modern Sector Enrichment, which is good according to the axioms presented above, but can be plausibly challenged on relative income grounds; or

(c) Modern Sector Enlargement, which is good according to widely acceptable axiomatic judgments.

The practical implication of this finding is clear: before we can legitimately interpret a rising inequality coefficient in a country as an economically-meaningful "worsening" of the income distribution rather than a statistical artifact, we must know which of the three types of economic development patterns that country has been following. The converse of this statement does not hold. That is, for a country which has been growing and has also experienced falling measured inequality, we need not resort to development typology to evaluate it -- declining measured inequality with growing income is unambiguously a good thing in terms of our axioms.

Of course, no real-world country is a pure case of any of these three types, and it is necessary, therefore, to devise a methodology for determining which of the three typologies it most closely fits, and hence how to analyze the distributional consequences of growth. This is the task of Section IV.

IV. A Synthesis of the Three Pure Cases and an Analytical Methodology

In this section, we synthesize the three pure models of Sections II and III and suggest a methodology for analyzing the distribution of the benefits of growth. Letting year 1 be the base year and year 2 the terminal year, denoting the labor force frequencies in the modern and traditional sectors by f^m and f^t respectively, and their respective wages by W^m and W^t, we have:

	Year 1	Year 2
Number of persons in traditional sector	f^t_1	f^t_2
Number of persons in modern sector	f^m_1	f^m_2
Wage in traditional sector	W^t_1	W^t_2
Wage in modern sector	W^m_1	W^m_2

In each year, national income (Y) is:

$$(3)\quad Y = W^m f^m + W^t f^t.$$

Taking the first difference of (3), the change in income between the two years is the sum of four terms which have the following economic meaning:

$$(4)\quad \Delta Y = \underbrace{(f^m_2 - f^m_1)\ (W^m_1 - W^t_1)}_{\substack{\text{Modern sector}\\ \text{enlargement effect}\\ (\alpha)}} + \underbrace{(W^m_2 - W^m_1) f^m_1}_{\substack{\text{Modern sector}\\ \text{enrichment effect}\\ (\beta)}}$$

$$+ \underbrace{(W^m_2 - W^m_1)\ (f^m_2 - f^m_1)}_{\substack{\text{Interaction between}\\ \text{modern sector enlargement}\\ \text{and enrichment}\\ \text{effects}\\ (\gamma)}} + \underbrace{(W^t_2 - W^t_1)\ f^t_2}_{\substack{\text{Traditional sector}\\ \text{enrichment effect}\\ (\delta)}}.$$

It is easily verified that for the three pure cases of Section II,

$\alpha = \Delta Y,\ \beta = \gamma = \delta = 0$ for modern sector enlargement growth,

$\beta = \Delta Y,\ \alpha = \gamma = \delta = 0$ for modern sector enrichment growth, and

$\delta = \Delta Y,\ \alpha = \beta = \gamma = 0$ for traditional sector enrichment growth.

In the mixed case, the percentage of growth attributable to each of the pure cases depends on changes in the economy's wage structure and occupational structure over the development period.

A comparative static analysis of equation (4) reveals:

(a) The modern sector enlargement effect (α) is greater:

(i) The greater the increase in modern sector employment; and

(ii) The greater the difference between modern sector and traditional sector wage rates.

(b) The modern sector enrichment effect (β) is greater:

(i) The greater the rate of increase of modern sector wages; and

(ii) The more important the modern sector in total employment.

(c) The traditional sector enrichment effect (δ) is greater:

(i) The greater the rate of increase of traditional sector wages; and

(ii) The more important the traditional sector in total employment.

Note that negative enlargement and enrichment effects are both possible. Negative enlargement would occur when a sector shrinks in size, while negative enrichment would result when real incomes in that sector fall.

Total income growth can be positive while either of these effects are negative. For example, a ten percent growth rate in a sector might result from either (i) a 20% rise in the size of the sector, coupled with a 10% fall in average wages, or (ii) a 20% rise in average wages, accompanied by a 10% decline in number of persons in that sector. This example should make clear that our qualitative judgments about the desirability of any particular sector growth rate depend crucially on the enlargement and enrichment components of that growth; examination of the sector growth rate is not enough.[1]

To illustrate how these ideas might be applied, suppose we had obtained the following data from a less developed country at two different points in time:

Year 1	Year 2
$f_1^t = 80$	$f_2^t = 40$
$f_1^m = 20$	$f_2^m = 60$
$W_1^t = \$1$	$W_2^t = \$1\ 1/2$
$W_1^m = \$2$	$W_2^m = \$3$
$Y_1 = \$120$	$Y_2 = \$240$

(Here the wages W^m and W^t should be thought of as the average wages paid in the modern and traditional sectors respectively.) Before calculating either the conventional inequality indices or the various enlargement and enrichment effects given by equation (4), let us examine the raw data themselves. We note three "facts":

[1]Consider statements of the form "Income of the richest X% grew by A% but income of the poorest Y% grew by only B% (less than A); therefore, income growth was disproportionately concentrated in the upper income groups." This interpretation is correct if average income among those who were originally the richest X% of the people rose much faster than among those who were originally the poorest Y%. However, the interpretation is incorrect if what mainly happened was that the high income sector expanded to include more people. From data on income growth of the richest X% and poorest Y%, we cannot tell which.

(a) The size of the modern sector tripled.

(b) Wages paid in the modern sector and in the traditional sector both increased by 50%.

(c) National income doubled.

From these facts, we might expect to find the following. First the tripling in size of the modern sector suggests that modern sector enlargement will be found to have been an important component of this hypothetica country's economic growth. Also, the fact that wages increased substantially in both sectors suggests that both the modern sector and the traditional sector enrichment effects will prove to be important; however, since the percentage increase in wages was less than the percentage growth of the modern sector, these effects will probably not be as large as the modern sector enlargement effect. Furthermore, since the two wage rates increased by the same percentage, their respective contributions to growth should be about the same. Finally, since the ratio of modern sector to traditional sector wages stayed the same (two to one), measured inequality should remain at more or less the same level.

Turning now to the calculations, it turns out that each of these expectations holds except one---that relating to measured inequality. Figure 2 demonstrates that the two Lorenz curves cross. Despite this, the Gini coefficient registers a percentage increase of 50% and the share of the lowest 40% falls by 25%. Measured inequality is markedly greater by both measures.

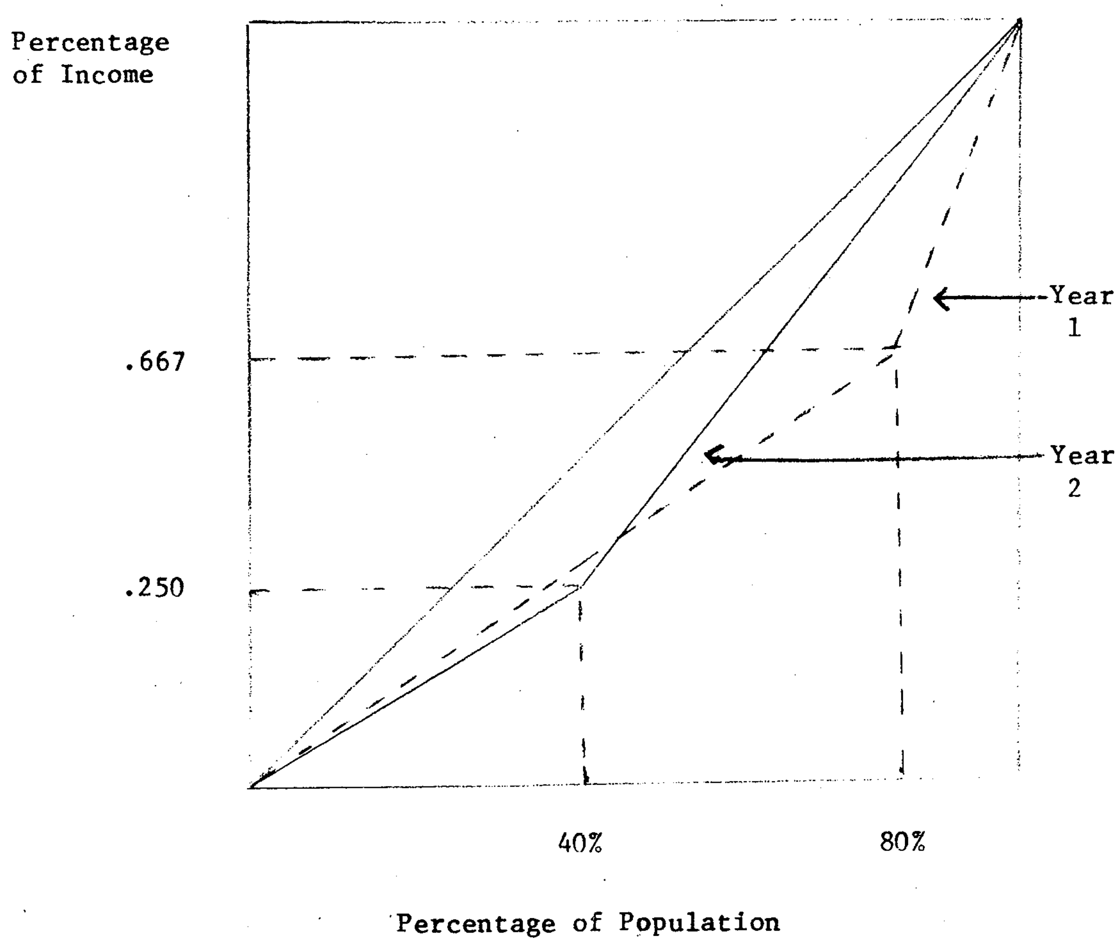
Should we interpret the higher measured inequality as an economically -meaningful worsening or a statistical accident? This depends on whether or not modern sector enlargement is an important part of the growth pattern. Decomposing the income growth, we find:

α =	Modern sector enlargement effect	= \$40
β =	Modern sector enrichment effect	= 20
γ =	Interaction between modern sector enlargement and enrichment effects	= 40
δ =	Traditional sector enrichment effect	= 20
ΔY	Total income growth	= \$120

Note that the modern sector enlargement effect (α) is equal to the sum of the other pure effects (β and δ) combined. This suggests that the modern sector enlargement growth model better characterizes the country's development pattern than either of the others, and that we should therefore interpret changes in the conventional inequality measures in this hypothetical case with a great deal of care.

We turn now to some extensions of the model.

FIGURE 2. LORENZ CURVES AND INEQUALITY INDICES FOR A HYPOTHETICAL ECONOMY.



Inequality Measure	Year 1	Year 2	% Change
Gini coefficient	.10	.15	+50%
Share of poorest 40%	.333	.250	-25%

V. Extensions of the Methodology

In this section, we show how the general methodology may be extended to allow for division of the economy into more than two sectors and to recognize population growth explicitly. We also compare the methodology derived here with that suggested in a recent paper by Ahluwalia and Chenery (1974).

A. Extension to n Sectors

In practical applications, the strict division of an economy into a modern sector and a traditional sector may be unsatisfactory, and a finer breakdown may be more desirable, for instance, into a modern urban sector, a traditional urban sector, and a traditional agricultural sector. In general with n sectors, national income (Y) is

$$(5)\quad Y = \sum_{i=1}^{n} W^i f^i .$$

The change in national income between the initial year (year 1) and terminal year (year 2) is

$$(6)\quad \Delta Y = Y_2 - Y_1 = \sum_{i=1}^{n} W_2^i f_2^i - \sum_{i=1}^{n} W_1^i f_1^i$$

which, when re-written as

$$(7)\quad \Delta Y = Y \sum_{i=1}^{n} (W_2^i f_2^i - W_1^i f_1^i),$$

enables us to measure the contribution of the i'th sector to total growth. To distinguish each sector's enlargement and enrichment effects and the interaction between them, (7) may be manipulated to yield

$$(8)\quad \Delta Y = \sum_{i=1}^{n} [\underbrace{W_1^i (f_2^i - f_1^i)}_{\substack{\text{Sector } i\\ \text{enlargement}\\ \text{effect}}} + \underbrace{(W_2^i - W_1^i) f_1^i}_{\substack{\text{Sector } i\\ \text{enrichment}\\ \text{effect}}} + \underbrace{(W_2^i - W_1^i)(f_2^i - f_1^i)}_{\substack{\text{Interaction of}\\ \text{sector } i \text{ enlargement}\\ \text{and enrichment effects}}}]$$

The results of the comparative static analysis of the two sector case (equation (4)) carry over to the n-sector case in an analogous manner.

Besides extensions to more than two sectors, the methodology developed here may be carried over as well to more than two income sources, or to a hybrid classification of sectors and sources. For example, it might be useful to measure income growth in the following six groups:

(i) Labor income among modern sector workers in urban areas

(ii) Labor income among traditional sector workers in urban areas

(iii) Labor income among traditional sector workers in agriculture

(iv) Capital income in urban areas

(v) Capital income in rural areas.

(vi) Other income

With such an extended methodology, we are limited only by restrictions of data and our own ingenuity.

B. Explicit Recognition of Population Growth

It is a straightforward matter to give explicit recognition to population growth. Total income growth (ΔY) may be thought to have two components: (i) A population growth effect (P), defined as the expansion of the economy to absorb a growing population at the initial occupational and wage structure, and (ii) A net growth effect (N), which results from higher wages and a higher proportion of the population employed in high-paying activities. Let f^i be the number of persons in sector i and p the rate of growth of population between years 1 and 2. Then net growth (income growth net of population) is given by:

$$(9)\ N = \Delta Y - P$$

$$= \sum_{i=1}^{n} (W_2^i f_2^i - W_1^i f_1^i) - \sum_{i=1}^{n} W_1^i f_1^i p$$

and can be decomposed into its various net effects as

$$(10)\ N = \sum_{i=1}^{n} [W_2^i f_2^i - W_1^i f_1^i (1+p)]$$

$$= \sum_{i=1}^{n} [\underbrace{[W_1^i [f_2^i - f_1^i (1+p)]}_{\substack{\text{Sector i net} \\ \text{enlargement effect}}} + \underbrace{(W_2^i - W_1^i)\ f_1^i\ (1+p)}_{\substack{\text{Sector i net} \\ \text{enrichment effect}}}$$

$$+ \underbrace{(W_2^i - W_1^i)\ [f_2^i - f_1^i\ (1+p\ )]]}_{\substack{\text{Interaction of sector i} \\ \text{net enlargement and enrichment} \\ \text{effects}}}.$$

It is interesting to note that a sector can have a negative net enlargement effect if its labor absorption rate is less than the rate of growth of population over the same period.

C. The Ahluwalia-Chenery Growth Index in Three Development Typologies

In a recent paper, Ahluwalia and Chenery (1974) have constructed an explicitly dynamic and distributionally-oriented technique for evaluating economic growth. The essence of their approach is to give greater weight to income growth if it is received by the relatively poor than by the relatively rich. Specifically, they divide the economy into quintile groups, ordered from lowest income to highest, and assign non-increasing weights to the income growth of each successive quintile, i.e.,

(11) Ahluwalia-Chenery Index = $g_1w_1 + g_2w_2 + g_3w_3 + g_4w_4 + g_5w_5$,

where $g_i = (Y_2^i - Y_1^i)/Y_1^i$ for the i'th quintile

and $w_1 \geq w_2 \geq w_3 \geq w_4 \geq w_5 \geq$, $\sum_i w_i = 1$.

The Ahluwalia-Chenery Index works very well for the two enrichment development typologies, but less well for modern sector enlargement. To investigate these relationships, let us consider a simple ten person economy with an initial income of twelve, an income growth of one dollar, and three alternative distributions among individuals (see Table 3). The Ahluwalia-Chenery measures, the Gini coefficients, and the rankings according to the inequality index and axiomatic approaches for the three development types are given below.

Let us begin by examining the two enrichment types of growth. The ideal data base for analyzing the distribution of the benefits of economic growth would be data on the same people at two points in time (so-called longitudinal or panel data). If we had such data, as in Panel (A) of the table, we could calculate the income growth of those persons who were originally in each income quintile, and weight these by the appropriate welfare weight w_i to obtain the Ahluwalia-Chenery Index. In the hypothetical traditional sector enrichment pattern of Table 3, we see that persons in each of the four lowest quintiles experienced a 12.5% income growth, and the index would equal $.125\ (w_1+w_2+w_3+w_4)$.

If, for example, our subjectively chosen weights were w_1=.40, w_2=.30, w_3=.20, w_4=.06, w_5=.04, the Ahluwalia-Chenery index would have the numerical value of 0.12.[1] The corresponding value for modern sector enrichment growth is 0.02. Thus, the Ahluwalia-Chenery Index considers the traditional sector enrichment pattern superior to modern sector enrichment. This accords with both the axiomat: approach and the inequality index approach (see Panel (C)).

In pзactice, however, census data or sample surveys in less developed countries do not generally chart the same people over time. At best, we have two comparable cross sections. Panel (B) is based on the same information as Panel (A), except that it is in the familiar form of incomes received by decile groups. For the two enrichment growth types, Panels (A) and (B) are identical, as are the Ahluwalia-Chenery indexes computed from them. In this sense, the Ahluwalia-Chenery index serves very well for these types of development, for both longitudinal and cross-sectional data.

Turning now to modern sector enlargement growth, the results are disappointingly mixed. For the longitudinal data (Panel (A)), modern sector enlargement growth shows up at least as well as the same amount of traditional sector enrichment growth, which conflicts with both the inequality index and axiomatic approaches.[2]

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[1]The choice of strictly monotonically declining weights is deliberate. Ahluwalia and Chenery's "equal weights" scheme would have produced the same index for each development type.

[2]Since it is arbitrary <u>which</u> poor person's income was increased by being drawn into the enlarged modern sector, we should probably use the average weight which we give to income growth among the four lowest quintiles, which is $(w_1+ w_2+ w_3+ w_4)/4$. If we were to use the welfare weight for the poorest quintile, since all are equally poor, the Ahluwalia-Chenery index would actually be greater for modern sector enlargement growth than for the same amount of traditional sector enrichment growth.

If, instead, our data had come to us as comparable cross sections, we would have gotten a very different result (see Panel (B)). The income growth appears to occur in the fourth quintile only, with no growth in the first three. This results in a substantially lower value for the Ahluwalia-Chenery Index than from the "true" longitudinal data, and results also in the placement of modern sector enlargement in an intermediate position.

The intermediate ranking of modern sector enlargement growth depends on the specific numerical values. In the extreme case mentioned by Ahluwalia and Chenery, in which $w_1=1$ (concern only with the poorest quintile), modern sector enlargement growth would receive a weight of zero irregardless of whether no poor persons, one poor person, or six poor persons were absorbed into an enlarged modern sector. This is hardly a desirable property for a growth index to have.

In general, however, the Ahluwalia-Chenery index does assign an intermediate rank to modern sector enlargement growth, which accords with the other approaches. A number of other problems remain. First, the lower value found from longitudinal data tends to bias our judgments away from modern sector enlargement patterns and in favor of modern sector enrichment patterns. We can imagine for instance that if traditional sector enrichment had been infeasible due to political or resource constraints and if a modern sector enrichment policy had yielded a slightly higher income, we might have mistakenly been led to choose that strategy over one of modern sector enlargement. Second, it seems unreasonable that the income growth should be recorded in the fourth quintile, with zero income growth among the lowest 60%. And third, we are left with the nagging suspicion that in modern sector enlargement growth the quintile income growth rates and the resulting Ahluwalia-Chenery Index do not seem to be measuring what they are really intended to measure -- the extent of income growth among persons (or families) in various positions in the income distribution.

In summary, for the three development typologies, we may conclude that the Ahluwalia-Chenery index is useful for both types of enrichment growth (modern sector and traditional sector) and for both types of data (longitudinal and comparative cross section). However, the validity for modern sector enlargement growth remains to be proven.

TABLE 3. HYPOTHETICAL DATA ILLUSTRATING THE AHLUWALIA-CHENERY INDEX.

Panel (A) - Longitudinal Data

Original Income Pattern		New Income Pattern Under:		
Person	Income Originally	Trad. Sector Enrichment	Modern Sector Enrichment	Modern Sector Enlargement
A	1	1.125	1	2
B	1	1.125	1	1
C	1	1.125	1	1
D	1	1.125	1	1
E	1	1.125	1	1
F	1	1.125	1	1
G	1	1.125	1	1
H	1	1.125	1	1
I	2	2	2.5	2
J	2	2	2.5	2
Total	12	13	13	13

Ahluwalia-Chenery Index Computed from This Data			
General	$.125(w_1+w_2+w_3+w_4)$	$.5w_5$	$\frac{.5(w_1+w_2+w_3+w_4)}{4}$
Specific (w_1=.40, w_2=.30, w_3=.20, w_4=.06, w_5=.04)	.12	.02	.12

TABLE 3 CONTINUED

Panel (B) - Data from Comparable Cross Sections

Original Income Pattern		New Income Pattern Under:		
Decile	Income Originally	Trad. Sector Enrichment	Modern Sector Enrichment	Modern Sector Enlargement
1	1	1.125	1	1
2	1	1.125	1	1
3	1	1.125	1	1
4	1	1.125	1	1
5	1	1.125	1	1
6	1	1.125	1	1
7	1	1.125	1	1
8	1	1.125	1	2
9	2	2	2.5	2
10	2	2	2.5	2
Total	12	13	13	13

Ahluwalia-Chenery Index Computed from This Data			
General	$.125(w_1+w_2+w_3+w_4)$	$.5w_5$	$.5w_4$
Specific ($w_1=.40$, $w_2=.30$, $w_3=.20$, $w_4=.06$, $w_5=.04$)	.12	.02	.03
Rank according to the Ahluwalia-Chenery Index	1	3	2

Panel (C) - Inequality Indices

Rate of growth	8.3%	8.3%	8.3%
Gini coefficient[1]	.024	.184	.162
Rank according to inequality index approach	1	3	2
Rank according to axiomatic approach	1	3	2

[1]The Gini coefficient of the original distribution is .134.

VI. Conclusion

This paper has sought to develop an explicitly dynamic and growth-relevant framework for analyzing the distribution of the economic benefits of growth. Three stylized models of dualistic economic development were considered: (a) Traditional sector enrichment, in which a country develops by raising the incomes of workers in its traditional sector; (b) Modern sector enrichment, where growth accrues to those already in the modern sector; and (c) Modern sector enlargement, wherein development proceeds by absorbing an ever-increasing number of traditional sector workers into an enlarged modern sector.

The distributional consequences of each of these types of growth were analyzed according to two alternative approaches: an inequality index approach and an axiomatic approach. The inequality index approach holds that social welfare is a positive function of the level of national income and a negative function of inequality in the distribution of that income. The specific inequality measures considered in this paper were the Gini coefficient and the share of income accruing to the poorest 40%.

The axiomatic approach, in contrast, sets forth specific qualitative propositions about social well-being. In this paper, we postulated three such value judgments: (a) For any given income distribution pattern, social welfare is greater the higher the level of income; (b) For any given income level, social welfare is greater the more equally national income is distributed (equality being defined in terms of Lorenz curves); and (c) Any Pareto improvement improves social welfare.

The results of the comparison of the inequality index and axiomatic approaches in the three development typologies were decidedly mixed.

In the case of traditional sector enrichment growth, there was no problem. For this type of development, the faster growth proceeds, the better things are, according to both approaches.

In modern sector enrichment growth, there was a discrepancy. This type of economic development is unambiguously a good thing according to the axiomatic approach. The inequality index approach, however, is ambiguous --- it registers a welfare gain from the rising income level and a welfare loss from the rising measured inequality, and without further specifying the social welfare function, there is no basis for weighing the two. It seemed, though, that relative income considerations could perhaps justify the resulting ambiguity.

For modern sector enlargement growth, there was the same ambiguity, but in this case the discrepancy could not be justified. It appeared that the rising Gini coefficient and falling share of the poorest 40% in the early stages of this type of growth were nothing more than statistical artifacts without social welfare content. In other words, the rising measured inequality in this type of growth reflects a natural and highly desirable pattern when modern sector enlargement is taking place.

The practical implication of this finding is that we should not automatically interpret a rising Gini coefficient or other inequality measure as an economically-meaningful worsening of a country's income distribution until we know what type of economic development pattern that country has been following.[1] Rising measured inequality could result from either: Traditional sector impoverishment, which is clearly undesirable; Modern sector enrichment, which the axiomatic approach holds is desirable but can reasonably be challenged on relative income grounds; or the early stages of Modern sector enlargement, which is clearly desirable, and is probably the way in which many countries are developing.

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[1] This point pertains to the Ahluwalia-Chenery growth index as well. See belo

To ascertain which of these three economic development typologies a real-world country most closely fits, the three pure cases were synthesized, and formulas were given for determining the percentage contribution of each sector's growth to the overall total. This methodology provides a practical means of arriving at a qualitative assessment of the distribution of the benefits of growth for any given country.

In addition, the methodology developed here also affords a means of testing various hypotheses about the relationship between income inequality and growth across countries for all economic development typologies. Among the questions that might be addressed in a cross section of less developed countries are the following:

(a) Is there a systematic relationship between the percentage importance of each type of growth in various countries (as measured by α, β, and δ) and the level of growth? In particular, is a higher percentage of modern sector enrichment growth correlated with a higher growth rate, or is the reverse the case?

(b) Is there a systematic relationship between the percentage importance of each type of growth and the level of income, perhaps analogous to the "inverted-U" pattern?

(c) Is there a systematic relationship between (i) the percentage importance and (ii) the level of each type of growth, and the types of policies followed, e.g., exchange rate policy, tax reform, land redistribution, or educational policy?

Past investigations have met with a notable lack of success in answering questions of this sort, particularly in relating the distribution of the benefits of growth to the level of growth.[1] Perhaps tests using the methodology devised above will meet with a better fate.

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[1]See, for instance, the study by Ahluwalia (1974), who concluded: "We do not really know what relationships exist between growth and income distribution."

Finally, the methodology of this paper was compared with that recently suggested by Ahluwalia and Chenery. The two approaches differ in a number of respects, most importantly in the fact that the Ahluwalia-Chenery index has both the advantage and the disadvantage of relying on obviously arbitrary weights assigned to income growth of different quintiles. This is advantageous insofar as it makes welfare judgments explicit, and it is certainly a great improvement over a simple GNP approach for evaluating economic growth. However, as with all explicitly arbitrary measures (for instance, that suggested by Atkinson (1970) , we do not yet have a firm theoretical basis for arriving at the specific weights to be used.

A potentially fruitful direction for future study might be to try to merge the growth decomposition methodology devised in this paper with the Ahluwalia-Chenery procedure for numerically comparing various development strategies. This could conceivably result in a subjectively-defined welfare index based upon the growth in size and income of the various sectors or income sources of the economy. This awaits additional research.

APPENDIX A

This and the following three appendices prove several theorems from Sections II and III in the text concerning the relationship between various types of dualistic economic development and measured inequality.

We suppose there are two sectors: a high-paying modern sector (m) and low-paying traditional sector (t). The wages paid in each sector are denoted by W^m and W^t, respectively. Their respective sizes are denoted by f^m and f^t, where $f^m + f^t = P$, total population.

The relevant economic magnitudes are national income (Y), share received by the lowest 40% (S), and the Gini coefficient (G). In terms of the above notation, we have:

$$\text{(A.1)} \quad Y = W^m f^m + W^t f^t,$$

$$\text{(A.2) a)} \quad S = \frac{40\% P W^t}{Y} \quad \text{if} \quad f^t \geq 40\%,$$

$$\text{b)} \quad S = \frac{f^t W^t + (40\%P - f^t)W^m}{Y} \quad \text{if} \quad f^t < 40\%,$$

$$\text{(A.3)} \quad G = \{\frac{1}{2} - [\frac{1}{2}\frac{f^t}{P}\frac{W^t f^t}{Y} + \frac{1}{2}\frac{f^m}{P}\frac{(W^m f^m)}{Y} + \frac{f^m}{P}\frac{W^t f^t}{Y}]\}/1/2$$

$$= 1 - \frac{[W^t f^t (f^t + 2f^m) + W^m (f^m)^2]}{[P(W^m f^m + W^t f^t)]}$$

$$= 1 - \frac{[W^t P^2 + (W^m - W^t)(f^m)^2]}{[W^t P^2 + P(W^m - W^t) f^m]} .$$

For simplicity, we will henceforth assume that the traditional sector always comprises at least 40%, so that (A.2.a) always holds.

In this appendix, we demonstrate the validity of four propositions concerning traditional sector enrichment growth:

Proposition A.1: A higher rate of traditional sector enrichment growth leads to a higher level of national income (Y) and a higher share of national income accruing to the poorest 40% (S), and therefore an unambiguously higher level of social welfare (W) according to (1a) $W^S = f(Y,S)$, $f_1 > 0$, $f_2 > 0$.

Proof: Substituting (A.1) into (A.2.a), we have:

$$(A.4) \quad S = \frac{40\%PW^t}{W^m f^m + W^t f^t} .$$

Differentiating (A.4) with respect to W^t gives

$$\frac{\partial S}{\partial W^t} = \frac{\partial}{\partial W^t}\left[\frac{40\%PW^t}{W^m f^m + W^t f^t}\right] = \frac{40\%PW^m f^m}{Y^2} > 0.$$

Also, $\frac{\partial Y}{\partial W^t} = f^t > 0$. Since

$$\frac{\partial W^S}{\partial W^t} = \frac{\partial W^S}{\partial Y}\frac{\partial Y}{\partial W^t} + \frac{\partial W^S}{\partial S}\frac{\partial S}{\partial W^t}$$

and we have just seen that all four right hand side terms are positive, it follows that $\frac{\partial W^S}{\partial W^t} > 0$.

Proposition A.2: A higher rate of traditional sector enrichment growth leads to a higher level of national income (Y) and a lower Gini coefficient (G), and therefore an unambiguously higher level of social welfare (W) according to (1b) $W^G = f(Y,G)$, $f_1 > 0$, $f_2 < 0$.

Proof: Partial differentiation of (A.3) with respect to W^t yields

$$\frac{\partial G}{\partial W^t} = \frac{[-2P(W^m-W^t)(f^m)^3 - W^tP^2f^m(P-f^m) - P^2(W^m-W^t)f^m(P-f^m)]}{[W^tP^2 + P(W^m-W^t)f^m]^2}$$

which is clearly negative. As before, $\frac{\partial Y}{\partial W^t} > 0$. From (1b), we have $\frac{\partial W^G}{\partial W^t} = \frac{\partial W^G}{\partial Y}\frac{\partial Y}{\partial W^t} + \frac{\partial W^G}{\partial G}\frac{\partial G}{\partial W^t}$. We see that the first two terms of the right hand side are positive, the second two negative, and therefore $\frac{\partial W^G}{\partial W^t} > 0$.

Proposition A.3: A higher rate of traditional sector enrichment growth leads to higher social welfare according to the axiomatic system (2a-c).

Proof: A higher rate of traditional sector enrichment growth is a Pareto-superior situation, which by (2c), signals rising social welfare. Since social welfare is not reduced by one of the other axioms, social welfare is increased by a higher rate of traditional sector enrichment growth.

Proposition A.4: The inequality index approaches to social welfare (eq. (1a) and (1b)) and the axiomatic approach (2) are in complete agreement for traditional sector enrichment growth.

Proof: This follows directly from Propositions (A.1)-(A.3).

APPENDIX B

In this appendix, we prove the two parts of the theorem of Section II.B.

Proposition B.1: The higher the rate of modern sector enrichment growth, the higher is social welfare according to the axiomatic approach.

Proof:

Same argument as in proof of Proposition A.3.

Proposition B.2: The higher the rate of modern sector enrichment growth, there is no unambiguous change in social welfare according to the inequality index approach based on the share of income received by the poorest 40%:

(1a) $W^S = f(Y,S),\ f_1 \geq 0,\ f_2 \geq 0.$

Proof: Differentiating (A.4) with respect to W^m, we have $\frac{\partial S}{\partial W^m} = \frac{-40\%PW^t f^m}{Y^2} < 0$. Also, $\frac{\partial Y}{\partial W^m} = f^m > 0$. Hence, $\frac{\partial W^S}{\partial W^m}$ is the sum of one positive and one negative term, and therefore unless we further specify the social welfare function, the social welfare effect of modern sector enrichment growth is ambiguous according to the income share approach.

Proposition B.3: The higher the rate of modern sector enrichment growth, there is no unambiguous change in social welfare according to the inequality index approach based on the Gini coefficient:

(1b) $W^G = f(Y,G),\ f_1 \geq 0,\ f_2 \leq 0.$

Proof: Partially differentiating (A.3) with respect to W^m, we obtain

$$\frac{\partial G}{\partial W^m} = \frac{[2P(W^m-W^t)(f^m)^3 + W^t P^2 f^m(P-f^m)]}{[W^t P^2 + P(W^m-W^t)f^m]^2},$$

which is the ratio of two positive numbers. Since $\frac{\partial W^G}{\partial W^m} = \frac{\partial W^G}{\partial Y}\frac{\partial Y}{\partial W^m} + \frac{\partial W^G}{\partial G}\frac{\partial G}{\partial W^m}$ and $\frac{\partial W^G}{\partial Y}$, $\frac{\partial Y}{\partial W^m}$, and $\frac{\partial G}{\partial W^m}$ are positive but $\frac{\partial W^G}{\partial G} < 0$, we have the same ambiguity as with the income share approach for this type of growth.

APPENDIX C

In this appendix, we prove the theorem of Section II.C. We begin with:

Proposition C.1: The higher the rate of modern sector enlargement growth, the higher is social welfare according to the axiomatic approach.

Proof:

Same argument as in proof of Proposition A.3.

Proposition C.2: (a) In the early stages of modern sector enlargement growth, a higher rate of this type of growth leads to a higher level of national income (Y), a lower share of national income accruing to the poorest 40% (S), and therefore an ambiguous effect on social welfare according to (1a) $W^S = f(Y,S)$, $f_1 \geq 0$, $f_2 \geq 0$. (b) Only when the traditional sector is less than 40% of the population is a higher modern sector enlargement growth rate unambiguously better according to (1a).[1]

Proof: (a) From the definition of S for $f^t \geq 40\%$ (eq. (A.2.a)), it is evident that in the early stages of modern sector enlargement growth, the poorest 40% receive the same absolute amount from a larger whole, and therefore

[1]This result may be generalized as follows: If our measure of inequality is the share of income accruing to the poorest X%, that share falls continuously until the modern sector has grown to include (1-X)% of the population.

their share falls. (b) In the later stages of modern sector enlargement growth (i.e., for $f^t < 40\%$), equation (A.2.b) applies. Differentiating (A.2.b) with respect to f^t, we derive

$$\frac{\partial S}{\partial f^t} = -\left[\frac{(W^m-W^t)}{Y^2}\right][W^m(f^m-40\%P) + W^m f^t] < 0,$$

since the bracketed terms are both positive. By the chain rule, $\frac{\partial S}{\partial f^m} = \frac{\partial S}{\partial f^t}\frac{\partial f^t}{\partial f^m} = -\frac{\partial S}{\partial f^t}$, we see that $\frac{\partial S}{\partial f^m} > 0$ for $f^t < 40\%$, as was to be proved.

Proposition C.3: (a) In the early stages of modern sector enlargement growth, a higher rate of this type of growth leads to a higher level of national income (Y), a higher Gini coefficient (G), and therefore an ambiguous effect on social welfare according to (1b) $W^G = f(Y,G)$, $f_1 > 0$, $f_2 < 0$. (b) Only when the modern sector is greater than $\left[\frac{\sqrt{W^m W^t}-W^t}{W^m-W^t}\right]$ percent of the population does a higher modern sector enlargement growth rate lead to an unambiguous improvement according to (1b).

Proof: These two propositions may be proven simultaneously by demonstrating that G reaches a maximum for some value of $\frac{f^m}{P}$ strictly between zero and one. This requires that $\frac{\partial G}{\partial f^m} = 0$ for $0 < \frac{f^m}{P} < 1$ and $\frac{\partial^2 G}{\partial (f^m)^2} < 0$. Differentiating (A.3) with respect to f^m and rearranging, we obtain

$$\text{(C.1)}\quad \frac{\partial G}{\partial f^m} = \left\{\frac{[W^m - W^t]}{[W^t P^2 + P(W^m-W^t)f^m]^2}\right\}\left\{\begin{matrix}-2f^m W^t P^2 + P^3 W^t \\ -(f^m)^2 P(W^m-W^t)\end{matrix}\right\}.$$

A maximum, if it exists, occurs at $\frac{\partial G}{\partial f^m} = 0$. Since the first term in brackets is strictly positive, we need only work with the second term. Setting it equal to zero and applying the quadratic formula to solve for f^m, we find

$$f^m_C = \frac{-W^t P \pm P\sqrt{W^m W^t}}{W^m-W^t}.$$

It is evident that one of the roots, $(\frac{f^m}{P})_C = \frac{-W^t - \sqrt{W^m W^t}}{W^m - W^t}$, is negative, so must be rejected. Considering now the other root, $(\frac{f^m}{P})_C = \frac{\sqrt{W^m W^t} - W^t}{W^m - W^t}$, the fact that $W^m > W^t$ implies both numerator and denominator are positive and therefore $(\frac{f^m}{P})_C > 0$. Likewise, $W^m > W^t$ implies $\sqrt{W^m W^t} < W^m$, and therefore $(\frac{f^m}{P})_C < 1$. Thus, G achieves a critical value for some $(\frac{f^m}{P})_C, 0 < (\frac{f^m}{P})_C < 1$. This critical value is a maximum provided the second order condition, $\frac{\partial^2 G}{\partial (f^m)^2} < 0$, is satisfied. Differentiating (C.1) again and rearranging, we find

$$\frac{\partial^2 G}{\partial (f^m)^2} = \frac{-2PW^t(W^m - W^t)[(W^m - W^t) + W^t P]}{[W^t P + (W^m - W^t)f^m]} < 0,$$

so $(\frac{f^m}{P})_C = \frac{\sqrt{W^m W^t} - W^t}{W^m - W^t}$ is indeed a maximum as claimed.

Proposition C.4: In modern sector enlargement growth, there are three phases, with the following discrepancies:

	Phase I $0 \leq \frac{f^m}{P} < \frac{\sqrt{W^m W^t} - W^t}{W^m - W^t}$	Phase II $\frac{\sqrt{W^m W^t} - W^t}{W^m - W^t} \leq \frac{f^m}{P} \leq 60\%$	Phase III $\frac{f^m}{P} > 60\%$
Income Level (Y)	Rises	Rises	Rises
Gini Coefficient (G)	Rises	Falls	Falls
Income Share of Poorest 40% (S)	Falls	Falls	Rises
Effect on Social Welfare According to Gini Coefficient Approach	Ambiguous	Unambiguous Improvement	Unambiguous Improvement
Effect on Social Welfare According to the Income Share Approach	Ambiguous	Ambiguous	Unambiguous Improvement
Effect on Social Welfare According to the Axiomatic Approach	Unambiguous Improvement	Unambiguous Improvement	Unambiguous Improvement

Proof: Follows directly from Propositions (C.1)-(C.3).

APPENDIX D

In this appendix, we prove the validity of the theorems of Section III, taking as our measure of inequality the Gini coefficient.[1] The strategy of the proof is to derive an expression for the change in the Gini coefficient with an increase in the size of the modern sector when there is within-sector inequality, and then to demonstrate that a maximum value always exists for a positive fraction of the population, irregardless of the relative sizes of the within-sector inequality coefficients.

Let us suppose that modern sector enlargement growth takes place under the following conditions:

(i) The income distribution within the modern sector is fixed, that is, the frequency distribution of wages in that sector (F^m) remains the same over time, which implies that the mean wage earned by those in the modern sector ($\bar{W}^m$) and the Gini coefficient of those working in the modern sector (G^{m*}) also are constant.

(ii) Similarly, the income distribution within the traditional sector, and therefore F^t, $\bar{W}^t$, and G^{t*} also remain constant.

(iii) The lowest income in the modern sector is greater than the highest income in the traditional sector.[2]

(iv) Population is constant and normalized at 1; the population shares of the modern and traditional sectors are given by f^m and f^t, respectively.

(v) Growth takes place by enlarging the modern sector, i.e., by

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[1]The choice of the Gini coefficient is arbitrary; any other inequality measure might also have been chosen. The Gini coefficient is considered here, because it is the most widely used.

[2]This assumption is not crucial to the analysis, but it greatly eases the algebra.

increasing f^m.

The methodology here draws on a procedure developed by Fei and Ranis (1974) for decomposing total inequality into its various component parts. Suppose that we were to array the population in increasing order of income. Then Fei and Ranis show:

$$\text{(D.1)} \quad G = \sum_i \phi^i G^{i'},$$

$$\text{(D.2)} \quad G^i \cong G^i R^{i'},$$

and therefore,

$$\text{(D.3)} \quad G \cong \sum_i \phi^i G^i R^{i'},$$

where G = Gini coefficient of total income,

G^i = Gini coefficient of income from the i'th source, including those who have no income from that source,

ϕ^i = Share of the i'th factor or sector in total income,

$R^{i'}$ = Rank correlation between the total incomes of individuals or groups and their incomes from the i'th source,

$G^{i'}$ = "Pseudo-Gini coefficient" of the i'th income source, obtained by computing a Gini coefficient with the individuals or groups ordered according to total income rather than income from that source.

Fei and Ranis have applied this procedure to the decomposition of total inequality into its various factor components.

The same methodology, appropriately modified, may be applied to the growth of various sectors. Under the conditions of modern sector enlargement growth just assumed, in particular condition (iii), it follows that $G^{i'} = G^i$ and $R^{i'} = 1$ for all i, and therefore (D.1)-(D.3) reduce to

$$(D.4)\quad G = \sum_i \phi^i G^i,$$

using the true Gini coefficients instead of the pseudo-Ginis.

Suppose now we have only two sectors, a modern sector and traditional sector, with respective income distributions F^m and F^t, and comprising f^m and f^t percent of the labor force respectively. The factor share of each sector is the average wage multiplied by the fraction of the labor force in that sector, all divided by total income, which gives us in place of (D.4):

$$(D.5)\quad G = \frac{\bar{W}^m f^m G^m}{Y} + \frac{\bar{W}^t f^t G^t}{Y}.$$

Recall that the sector Gini coefficients G^m and G^t include persons with no income from that source. Letting G^{m*} and G^{t*} represent the Gini coefficients including only people with income from that sector, and assuming the two sectors to be mutually exclusive, it may be shown that

$$(D.6)\quad G^{m*} = \frac{G^m - f^t}{1-f^t} \quad \text{and} \quad G^{t*} = \frac{G^t - f^m}{1-f^m}.$$ [3]

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[3] The Gini coefficient of a variable X is equal to 1-2B, where B is the area under the Lorenz curve of X. It is easily established geometrically that

$$B = \frac{1}{2}\frac{1}{n}(X_1 + X_2 + \dots + X_n)/Y + \frac{X_{j+1}(\frac{1}{n})}{Y} + \frac{(X_{j+1}+X_{j+2})(\frac{1}{n})}{Y} + \dots + \frac{(X_{j+1}+\dots+X_n)(\frac{1}{n})}{Y}$$

where n is the total number of persons or families, j is the number who have no income from that source, and Y is total income. The above expression may be rearranged to yield

$$B = \frac{1}{2n} + \frac{1}{nY}[(n-j-1)X_{j+1} + (n-j-2)X_{j+2} + \dots + X_n].$$

If we now consider only the n-j persons who have positive incomes from that source, and let G^* be the Gini coefficient among those same n-j persons, then $G^* = 1 - 2B^*$, where

$$B^* = \frac{1}{2(n-j)} + \frac{1}{(n-j)Y}[(n-j-1)X_{j+1} + (n-j-2)X_{j+2} + \dots + X_n].$$

Denote the term in brackets by Z. Then

We now wish to solve for G in terms of the parameters of the model and the proportion of workers in the modern sector labor force. Solving (D.6) for G^m and G^t and substituting the results along with $Y = \bar{W}^m f^m + \bar{W}^t f^t$ and $f^t = 1 - f^m$ into (D.5), we obtain

$$(D.7)\quad G = \frac{\bar{W}^m f^m(G^{m*}f^m+1-f^m) + (\bar{W}^t-\bar{W}^t f^m)(G^{t*}-G^{t*}f^m+f^m)}{\bar{W}^m f^m + \bar{W}^t - \bar{W}^t f^m}$$

$$= \frac{\begin{bmatrix} f^{m^2}(\bar{W}^m G^{m*} - \bar{W}^m + \bar{W}^t G^{t*} - \bar{W}^t) \\ +f^m(\bar{W}^m - 2\bar{W}^t G^{t*} + \bar{W}^t) \\ +\bar{W}^t G^{t*} \end{bmatrix}}{(\bar{W}^m - \bar{W}^t)f^m + \bar{W}^t}$$

$$= [f^{m^2}A + f^m B + \bar{W}^t G^{t*}]/[(\bar{W}^m-\bar{W}^t)f^m + \bar{W}^t],$$

where $A = \bar{W}^m G^{m*} - \bar{W}^m + \bar{W}^t G^{t*} - \bar{W}^t$ and $B = \bar{W}^m - 2\bar{W}^t G^{t*} + \bar{W}^t$.

The Kuznets turning point exists if G has an interior maximum, i.e., if the first derivative attains a zero value at a critical value of f^m_C, $0 < f^m_C < 1$. Differentiating (D.7) with respect to f^m, we obtain

$$(D.8)\quad \frac{\partial G}{\partial f^m} = \frac{Y[2f^m A + B] - [f^{m^2}A + f^m B + \bar{W}^t G^{t*}][\bar{W}^m-\bar{W}^t]}{Y^2}.$$

Equating (D.8) to zero and rearranging yields

$$(D.9)\quad f^{m^2}_C[A(\bar{W}^m-\bar{W}^t)] + f^m_C[2A\bar{W}^t] + [B\bar{W}^t - (\bar{W}^m-\bar{W}^t)\bar{W}^t G^{t*}] = 0$$

$$G = 1 - \frac{1}{n} - \frac{2Z}{nY}$$

and

$$G^* = 1 - \frac{1}{n-j} - \frac{2Z}{(n-j)Y}.$$

Solving these two equations for Z, equating the resulting expressions to one another, and solving the result for G*, we obtain

$$G^* = \frac{Gn-j}{n-j}. \qquad \text{Q.E.D.}$$

Applying the quadratic formula and combining terms, we find

$$\text{(D.10)}\quad f_C^m = \frac{-2A\bar{W}^t \pm \sqrt{\begin{array}{l}4A^2\bar{W}^{t^2} - 4AB\bar{W}^m\bar{W}^t + 4A\bar{W}^{m^2}\bar{W}^tG^{t*} \\ -8A\bar{W}^m\bar{W}^{t^2}G^{t*} + 4AB\bar{W}^{t^2} + 4A\bar{W}^{t^3}G^{t*}\end{array}}}{2A(\bar{W}^m - \bar{W}^t)}$$

Since $A < 0$, the denominator of (D.10) is negative and the first term in the numerator is positive. If f_C^m is to lie between 0 and 1, the numerator must be negative, and therefore the only potentially meaningful root is

$$\text{(D.11)}\quad f_C^m = \frac{-\bar{W}^t}{\bar{W}^m-\bar{W}^t} - \frac{\sqrt{\begin{array}{l}A^2\bar{W}^{t^2} - AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^tG^{t*} \\ -2A\bar{W}^m\bar{W}^{t^2}G^{t*} + AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*}\end{array}}}{A(\bar{W}^m - \bar{W}^t)}$$

If the critical value (D.11) is to be economically relevant, it must be positive and less than one. Denoting the term under the square root sign by C, f_C^m will be positive if $C > (A\bar{W}^t)^2$, which is easily demonstrated:

$$\begin{aligned}\text{(D.12)}\quad C-(A\bar{W}^t) &= -AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^tG^{t*} - 2A\bar{W}^m\bar{W}^{t^2}G^{t*} \\ &\quad +AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*} \\ &= -AB\bar{W}^t(\bar{W}^m-\bar{W}^t) + A\bar{W}^tG^{t*}(\bar{W}^{m^2}-2\bar{W}^m\bar{W}^t+\bar{W}^{t^2}) \\ &= A\bar{W}^t(\bar{W}^m-\bar{W}^t)[-B+G^{t*}(\bar{W}^m-\bar{W}^t)] \\ &= A\bar{W}^t(\bar{W}^m-\bar{W}^t)(\bar{W}^m+\bar{W}^t)(1-G^{t*}) > 0.\end{aligned}$$

To show f_C^m is less than one, we require

$$\text{(D.13)}\quad -\bar{W}^t - \frac{\sqrt{C}}{A} < \bar{W}^m - \bar{W}^t$$

$$\Longleftrightarrow - \frac{\sqrt{C}}{A} < \bar{W}^m$$

$$\Longleftrightarrow \quad \sqrt{C} < -A\bar{W}^m$$

$$\Longleftrightarrow \quad C < A^2\bar{W}^{m^2},$$

which may be shown as follows:

$$A^2\bar{W}^{m^2} > A^2\bar{W}^{t^2} - AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^tG^{t*} - 2A\bar{W}^m\bar{W}^{t^2}G^{t*} + AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*}$$

$$\Longleftrightarrow \frac{A^2(\bar{W}^{m^2}-\bar{W}^{t^2})}{-A} > \begin{array}{l} B\bar{W}^m\bar{W}^t - \bar{W}^{m^2}\bar{W}^tG^{t*} + 2\bar{W}^m\bar{W}^{t^2}G^{t*} \\ - B\bar{W}^{t^2} - \bar{W}^{t^3}G^{t*} \end{array}$$

$$\Longleftrightarrow - A(\bar{W}^{m^2}-\bar{W}^{t^2}) > B\bar{W}^t(\bar{W}^m+\bar{W}^t) - \bar{W}^tG^{t*}(\bar{W}^m-\bar{W}^t)^2$$

$$\Longleftrightarrow - A(\bar{W}^{m^2}-\bar{W}^{t^2}) > \bar{W}^t(\bar{W}^{m^2}-\bar{W}^{t^2})(1-G^{t*})$$

$$\Longleftrightarrow - A > \bar{W}^t(1-G^{t*})$$

$$\Longleftrightarrow - [\bar{W}^m(G^{m*}-1) + \bar{W}^t(G^{t*}-1)] > \bar{W}^t(1-G^{t*})$$

$$\Longleftrightarrow \bar{W}^m(1-G^{m*}) > 0,$$

as was to be proved.

We have therefore shown that when there is within-sector inequality in modern sector enlargement growth, there is always an inverted U-pattern of measured inequality, regardless of whether incomes are distributed more equally, less equally, or the same within the modern sector as in the traditional sector. It should be noted that Proposition (C.3) is the special case $G^{m*} = G^{t*} = 0$.

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