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A WELFARE ECONOMIC APPROACH TO GROWTH AND DISTRIBUTION IN THE DUAL ECONOMY

Gary S. Fields

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A Welfare Economic Approach to Growth and Distribution In the Dual Economy

I. Introduction

This paper presents a welfare economic analysis of the distributional consequences of growth in the dual economy, a problem which has attracted much attention from development economists of late. We will explore the similarities and differences between the absolute income and poverty and relative inequality approaches for three stylized dualistic development models. It will be shown that these approaches are not always in agreement and, more disturbingly, that the most notable discrepancy is found in the most relevant stylized model--growth via the transfer of population from a backward to an enlarging advanced sector. The fact of these discrepancies raises the important question of how to measure changing income distribution in a manner consistent with the judgments we wish to make about the alleviation of absolute poverty and changes in relative income inequality. Recent controversies over who received the benefits of growth in two less developed countries--Brazil and India--are examined in these terms.

II. Three Stylized Models of Dualistic Economic Development

At the forefront of studies of modern economic growth are the dualistic development models of Lewis [24], Fei and Ranis [12], and Jorgenson [18]. While these models differ one from another in a number of important respects, they have in common the division of the economy into a relatively advanced sector and a relatively backward sector, which we shall call "modern" and "traditional" respectively. As with all dualistic models, the working assumption is that the members of each sector are relatively similar to others in that sector and relatively different from those in the other sector. We shall regard the modern sector as synonymous with high wages and the traditional sector as synonymous with low wages. "Wage" and "income" will be used interchangably.[1]

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[1] This is not to downplay the importance of capital and other sources of income and wealth in determining economic position. Rather, since most people in less developed countries receive most or all of their income from the work they do, and since variation in labor income is the most important source of overall income inequality, a high wage sector-low wage sector dichotomy would appear more relevant than any other dualistic classification.

In the two sectors, workers receive wage rates W^m and W^t respectively.[2] $W^m > P^* > W^t$ where P^* is an agreed-upon absolute poverty line which is constant over time (except for allowing for price changes). The shares of the labor force in the two sectors are f^m and f^t respectively; the total economically active population $f^m + f^t$ is normalized at 1. The models that follow differ with respect to the time paths of W^m, W^t, f^m, and f^t.

The overall growth of the dualistic economy is decomposable into the sum of growth in the two sectors. In turn, each sector's growth (or lack thereof) may be partitioned into two components: one attributable to the enlargement (or contraction) of the sector to include a greater (or lesser) percentage of the economically active population, the other attributable to the enrichment of persons engaged in that sector. If a dualistic economy is growing successfully, one or more of the following must be happening: i) the fraction of workers in the modern sector is increasing; ii) those in the modern sector receive higher average incomes than before; or iii) the incomes of those who remain in the traditional sector may rise. While every successfully-developing country experiences some or all of these phenomena to varying degrees, some pursue more broadly-based or more egalitarian courses than do others.

To capture the essential differences among the alternative growth paths that might be followed, we construct models of three stylized development typologies. In the Modern Sector Enlargement Growth model, an economy develops by enlarging the size of its modern sector, the wages in the two sectors remaining the same. Modern Sector Enrichment Growth occurs when the growth accrues only to a fixed number of persons in modern sector, the number in the traditional sector and their wages remaining unchanged. Finally, we have Traditional Sector Enrichment Growth when all of the proceeds of

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[2] The assumption of identical wages for all workers within a given sector is simply for algebraic and diagrammatic convenience and is not necessary for any of the results above. Intrasectoral wage diversity is allowed for in the model in the Appendix.

growth are divided evenly among those in the traditional sector. For simplicity, these are analyzed separately. The interested reader is invited to explore various combinations. One interesting possibility is modern sector enlargement accompanied by traditional sector enrichment, which might arise when the enlargement of the modern sector labor force leads to competition amongst traditional sector employers for the remaining workers.

In relation to existing literature, the modern sector enlargement growth model most closely reflects the essential nature of economic development as conceived by a number of writers. Fei and Ranis [12], for example, have written: "... the heart of the development problem may be said to lie in the gradual shifting of the center of gravity of the economy from the agricultural to the industrial sector... gauged in terms of the reallocation of the population between the two sectors in order to promote a gradual expansion of industrial employment and output," and this is echoed by Kuznets [22]. Empirical studies of many countries have quantified the absorption of an increasing share of the population into the modern sector; see, for instance, Turnham [36]. Thus, modern sector enlargement comprises a large and perhaps even predominant component of the growth of currently-developing countries.

There are also signs of traditional sector enrichment. The poor in the traditional sectors have not in general been shown to be worse off in absolute terms, and in many countries, their absolute economic position is demonstrably improved.[3] Still the pace of improvement is disappointingly slow, even in the rapidly-growing countries.[4] This may be because substantial elements of modern sector enrichment have taken place also. Nearly everywhere, the wages received by upper-level workers (the skilled,

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[3]In some countries, economic growth has been accompanied by declining relative income inequality, and hence alleviation of absolute poverty; see the studies by Fei, Ranis and Kuo [13] for Taiwan and Ayub [3] for Pakistan. In other countries, relative income inequality did not improve, but the overall income growth was large enough to raise the position of the poor as well; this may be inferred from data contained in the studies of Argentina, Mexico, and Puerto Rico by Weisskoff [37], of Brazil by Fishlow [16], and of Colombia by Berry and Urrutia [6]. Bardhan's [4] country study of India is the one case I have seen where absolute poverty has been shown to increase in severity over time; undoubtedly other "fourth world countries" share a similar plight.

[4]For instance, Fishlow [16] demonstrates that given the existing pattern of income distribution in Brazil, the economy would have to grow at a rate of 5 percent per year for 20 years before the poor would attain incomes of $100 per capita.

government employees, etc.) have risen in real terms.
These wage increases are larger in absolute terms than those received by lower-level workers (the unskilled, self-employed, etc.)[5]

How are we to evaluate these various development typologies? We turn now to an analysis of some of the approaches which have been suggested.

III. Absolute and Relative Approaches for Evaluating Growth and Distribution

Economists are used to regarding social welfare as a positive function of the income levels of the n individuals or families in society before and after development takes place. In empirical studies, the general social welfare function

$$(1) \qquad W = W(Y_1, Y_2, \ldots, Y_n) \; , \; W_1, W_2, \ldots, W_n > 0$$

is too general to be useful, and the Pareto criterion

$$(2) \qquad W^A(Y_1^A, Y_2^A, \ldots Y_n^A) > W^B(Y_1^B, Y_2^B, \ldots, Y_n^B)$$

$$\text{if } Y_i^A \geq Y_i^B \text{ for all } i \text{ and } Y_i^A > Y_i^B \text{ for some } i$$

is too stringent.

For analytical ease, the information contained in the income vector $(Y_1\ Y_2 \ldots Y_n)$ is usually collapsed into one or more aggregative measures. The three classes of measures in most common use are total income (Y) or its per capita equivalent, indices of relative inequality (I), and measures of absolute poverty (P).

The customary approach to studies of distribution and development is to posit (explicitly or implicitly) a social welfare function containing an index of relative inequality as one of its arguments:

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[5] These conclusions are drawn from Berg [5]. He also presents evidence that while skilled-unskilled wage differences widened, skilled-unskilled wage ratios have generally narrowed.

$$(3) \qquad W = f(Y,I), \ f_1 > 0, \ f_2 < 0,$$

where Y is total income and I is an indicator in inequality in its distribution. In what follows, this type of welfare judgment will be termed the "relative inequality approach." Theoretical support for this approach may be found in the welfare economics literature in the writings of Sheshinski [32] and Sen [31]. In the study of distribution and development, exemplary of the relative inequality approach is the Nobel Prize winning work of Professor Kuznets [20] [21], begun two decades ago. Income distribution is said to have 'improved' or 'worsened' according to Lorenz domination (i.e., whether one Lorenz curve lies wholly above or below a previous one (L)) or according to one or more measures of relative inequality, such as the income share of the poorest 40% (S) or the Gini coefficient (G). Thus,relative inequality studies typically make one or more of the following judgments:

$$(4) \qquad \text{(a) } W = f(Y, L), \ f_1 > 0, \ f_2 > 0,$$
$$\text{(b) } W = f(Y, S), \ f_1 > 0 \ \ f_2 > 0,$$
$$\text{(c) } W = f(Y, G), \ f_1 > 0, \ f_2 < 0.$$

A great many studies have made use of this framework. Some of the most influencial recent contributions, which include extensive surveys and bibliographies of prior research studies, are those of Cline [10], Chenery et. al. [9], and Adelman and Morris [1].

As an alternative to the relative inequality approach, some writers have examined the income distribution itself, assigning a lower social welfare weight to income gains of the relatively well-off as compared with the poor. With no loss of generality, we may order the n income recipient units from lowest to highest. The general class of studies which treats social welfare in the form:

$$(5) \qquad W = g\ (Y_1, Y_2, \ldots, Y_n), \ \ g_i > g_j \ \forall \ \ i < j$$

shall be termed the "absolute income approach." In the development literature, the studies of Little and Mirrlees [25], Atkinson [2], and Stern [33] are notable examples. As an extreme version of (5), Rawls [27] has proposed the maximin principle, i.e., maximizing the income of the worst-off person in the economy:

$$(5') \qquad W = g(Y_1), \ g' > 0.$$

Finally, for some purposes, we may wish to define a poverty line P* and concentrate our attention on the group in poverty to the exclusion of the rest of the income distribution. This practice, termed the "absolute poverty approach," is common in studies of growth in the United States; see, for example, Bowman [7] or Perlman [26]. Denoting the extent of poverty by P, absolute poverty studies hold that

$$(6) \qquad W = h\ (P), \ h' < 0.$$

Usual measures of poverty are the number of individuals or families whose incomes are below that line or the gap between the poverty line and the average among the poor. In a paper just published, Sen [30] combines these and argues elegantly for the use of an index $\pi = H[I + (1 - \bar{I})\ G_p]$, where H is the head-count of the poor, $\bar{I}$ is the average income shortfall of the poor, and G_p is the Gini coefficient of income inequality among the poor. Thus, alternative forms of the absolute poverty approach are given by:

$$(7) \qquad \text{(a) } W = h\ (H),\ h' < 0,$$

$$\text{(b) } W = h\ (\bar{I}),\ h' < 0,$$

$$\text{(c) } W = h\ (\pi) = h\ [H[I + (1 - \bar{I})\ G_p]],\ h' < 0.$$

It is not necessary that the relative and absolute approaches be regarded as mutually exclusive. In the following section, we formulate a more general welfare function combining these various approaches.

IV. A General Welfare Approach for Assessing Dualistic Development

The various welfare approaches of Section III were originated largely in a static context. However, since the distribution of benefits in the course of economic development refers to a phenomenon that takes place over time, it is appropriately measured by a dynamic index. It is important, therefore, to establish a suitably dynamic measure. We now posit a general welfare function and a number of properties of that welfare function which are desirable for this purpose.

Consider a welfare function of the form:

$$(8) \qquad W = W(Y, I, P).$$

In the dualistic development models of Section II, total income (Y) is given by:

$$(9) \qquad Y = W^m f^m + W^t f^t.$$

Whichever measure of relative inequality (I) one chooses is functionally related to the distribution of the labor force between the two sectors and the intersectoral wage structure:

$$(10) \qquad I = I\ (W^m,\ f^m,\ W^t,\ f^t).$$

The poverty index (P) depends on the wage in the traditional sector and/or the share of the population in that sector:

$$(11) \qquad P = P\ (W^t,\ f^t).$$

Substituting (9) - (11) into (8), we have:

$$(12) \qquad W = W\ (W^m f^m + W^t f^t,\ I(W^m,\ f^m,\ W^t,\ f^t),\ P\ (W^t,\ f^t)),$$

which we term the "general welfare approach."

We must now specify the relationship between W and its various arguments. In line with the considerations discussed in Section III, it is desirable to posit:

(A) $\underline{\dfrac{\partial W}{\partial Y} > 0}$,

(B) $\frac{\partial W}{\partial I} < 0$,

(C) $\frac{\partial W}{\partial P} < 0$.

Condition (A) relies for its validity on the assumption that the basic goal of an economic system is to maximize the output of goods and services received by each of its members. We should be clear that acceptance of the judgment $\frac{\partial W}{\partial Y} > 0$ does not require us to accept the stronger quasi-Pareto condition $\frac{\partial W}{\partial Y_i} > 0 \; \forall_i$, which in our dualitic development models becomes $\frac{\partial W}{\partial Y_k} > 0$, $k = m, t$. (This is quasi because it is formulated in terms of incomes rather than utilities). The judgment $\frac{\partial W}{\partial Y_m} > 0$ is one which many observers would not want to make, since it implies that even if the richest were the sole beneficiaries of economic growth, society would be deemed better off. No such judgment is imposed in what follows.

Condition (B) requires us first to define what we mean by a more equal relative distribution of income. A generally-accepted (although incomplete) criterion is that one distribtution A is more equal than another B if A Lorenz-dominates B, i.e., if A's Lorenz curve lies above B's at at least one point and never lies below it.

If A Lorenz-dominates B for the same level of income, it means distribution A can be obtained from distribution B by transfering positive amounts of income from the relatively rich to the relatively poor.[6] The judgment that such transfers improve social welfare dates back at least to Dalton [11] in 1920. One possible justification for this principle is diminishing marginal utility of income, coupled with independent and homothetic individual utility functions and an additively separable social welfare function.[7] But these assumptions are not neccesary for the affirmation of the axiomatic judgment $\frac{\partial W}{\partial I} < 0$.

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[6] See Rothschild and Stiglitz [29] and Fields and Fei [14].

[7] See Atkinson [2].

The difficulty with Lorenz-domination as a defining criterion for judgments concerning relative inequality is its incompleteness. When Lorenz curves cross, there is nothing to say. We therefore require a more complete relative inequality measure in order to rank various income distributions when Lorenz curves intersect. For this purpose, many indices of relative income inequality which provide complete orderings have been constructed.

The properties of various inequality indices have been examined by a number of writers (e.g., Champernowne [8], Kondor [19], Szal and Robinson [35], and Fields and Fei [14]). It is agreed that a "good" inequality index should have the following properties: scale irrelevance (if one distribution is a scalar multiple of another, then they have the same relative inequality), symmetry (if one distribution is a permutation of another, then relative inequality in the two cases is the same), and the Daltonian condition (if one distribution is obtained from another by one or more income transfers from a relatively rich person to a relatively poor one, then the first distribution is more equal than the second).

Three other properties of relative inequality measures are desirable for analyzing the growth of a dualistic economy. These are:

(D) $\frac{\partial I}{\partial W^t} < 0$

(E) $\frac{\partial I}{\partial W^m} > 0$

These accord with our intuitive notions about relative inequality (in terms of $W^m - W^t$ or W^m/W^t) and will probably not strike the reader as unusual.

(F) $\frac{\partial I}{\partial f^t} = -\frac{\partial I}{\partial f^m} \geq 0.$

This condition holds that when an increasing fraction of the economically active population is drawn into an enlarged modern sector, then ceteris paribus, relative inequality should be no greater than before. Since the wage differential between modern and traditional sector workers is being held constant, this is hardly an unreasonable property. Many would wish to go one step further and replace (F) by:

(F') $\frac{\partial I}{\partial f^t} = -\frac{\partial I}{\partial f^m} = 0$, which I myself prefer. The choice between (F) and (F') has no bearing on any of the results that follow; what is important is the exclusion of $\frac{\partial I}{\partial f^t} = -\frac{\partial I}{\partial f^m} < 0$. Note that conditions (F) and (F') describe how <u>the inequality index itself</u> varies with the level of development. This does <u>not</u> mean that our <u>feelings</u> about inequality are invariant to income level. For a perceptive analysis of changing tolerance for inequality in the course of economic development, see Hirschman and Rothschild [17].

Finally, we turn to condition (C), which holds that social welfare (W) is increased the less absolute poverty (P) there is. Whatever poverty measure(s) we employ should satisfy the properties:

(G) $\frac{\partial P}{\partial f^t} < 0.$

(H) $\frac{\partial P}{\partial W^t} < 0.$

These conditions state that absolute poverty P is reduced if there are fewer people in the low-income traditional sector and/or if the wage received by those in the traditional sector is increased, i.e., they become less poor. These concepts are equivalent to the 'poverty population' and 'poverty gap' notions used in studies of the United States and the 'headcount' and 'income shortfall' components of the poverty measure proposed by Sen [30]. The appeal of these properties is intuitive and requires no further elaboration.

Function (12) and conditions (A) - (H) constitute the "general welfare approach." Condition (B) may be modified to

$$(B') \quad \frac{\partial W}{\partial I} = 0$$

for observers only interested in absolute poverty, while (C) might be replaced by

$$(C') \quad \frac{\partial W}{\partial P} = 0$$

for those concerned only about relative inequality. The various approaches for analyzing growth and distribution in the dual economy are summarized in Table 1.

As they stand, the welfare functions (4), (5), (7), and (12) are purely static. They are, however, easily made dynamic by differentiating (or differencing) them with respect to time or to their underlying arguments. The growth of the dualistic economy involves changes in W^m, W^t, and/or f^m and f^t. These factors enter directly into (12), indirectly into the others.

The question that then arises is whether the various approaches always give the same qualitative answer when evaluating the distributional consequences of various types of dualistic economic development or whether the judgments differ and, if so, when. We address this question in Section V.

V. Distribution and Development: A Welfare Economic Analysis

This section analyzes the growth and distributional patterns which arise in each of the three stylized models of dualistic development according to the various welfare economic approaches previously discussed. The principal results are summarized in Table 2.

A. Traditional Sector Enrichment Growth

In the traditional sector enrichment growth model, incomes in the traditional sector are assumed to rise, incomes in the modern sector remain the same, and the

TABLE 1. VARIOUS WELFARE ECONOMIC APPROACHES FOR ANALYZING DUALISTIC ECONOMIC DEVELOPMENT

Relative Inequality Approach

General Form: (3) $W = f(Y, I)$, $f_1 > 0$, $f_2 < 0$ ... Inequality index

Specific Applications:

(4) (a) $W = f(Y,L)$, $f_1 > 0$, $f_2 > 0$ ... Lorenz criterion

(b) $W = f(Y,S)$, $f_1 > 0$, $f_2 > 0$ ... Income share of poorest

(c) $W = f(Y,G)$, $f_1 > 0$, $f_2 < 0$ ... Gini coefficient

Absolute Income Approach

General Form : (5) $W = g(Y_1, Y_2, \ldots, Y_n)$, $g_i > g_j$ $\forall i < j$ ... Absolute income

Specific Application: (5') $W = g(Y_1)$, $g' > 0$ ... Rawlsian maximin criterion

Absolute Poverty Approach

General Form : (6) $W = h(P)$, $h' < 0$ ... Poverty index

Specific Applications: (7) (a) $W = h(H)$, $h' < 0$ ... Headcount of poor

(b) $W = h(I)$, $h' < 0$ ... Income shortfall

(c) $W = h(\pi)$, $h' < 0$, ... Sen index

$\pi = H[\bar{I} + (1-\bar{I})G_p]$

General Social Welfare Approach

General Form: (8) $W = W(Y,I,P)$, $\frac{\partial W}{\partial Y} > 0$, $\frac{\partial W}{\partial I} < 0$, $\frac{\partial W}{\partial P} < 0$ ... General welfare

Specific Applications: (12)

$$W = W(W^m f^m + W^t f^t,\ \ I(W^m,W^t,f^m,f^t),\ P(W^t,f^t)),$$

$$\frac{\partial W}{\partial Y} > 0,\ \frac{\partial W}{\partial I} < 0,\ \frac{\partial W}{\partial P} < 0,$$

... General welfare, dualistic

$$\frac{\partial Y}{\partial W^m}, \frac{\partial Y}{\partial W^t}, \frac{\partial Y}{\partial f^m}, \frac{\partial Y}{\partial f^t} > 0,$$

$$\frac{\partial I}{\partial W^m} > 0,\quad \frac{\partial I}{\partial W^t} < 0,\quad \frac{\partial I}{\partial f^m} = -\frac{\partial I}{\partial f^t} \leq 0,$$

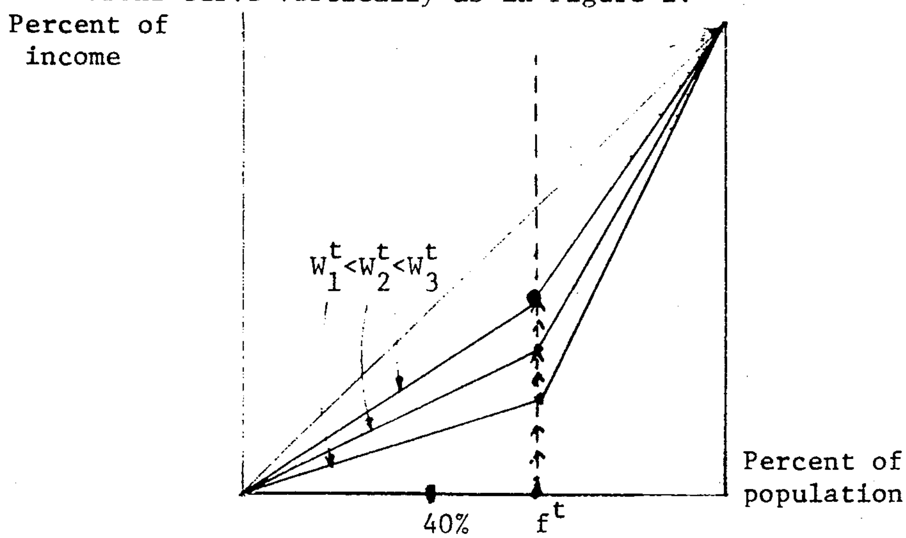
$$\frac{\partial P}{\partial W^t} < 0,\quad \frac{\partial P}{\partial f^t} < 0.$$

allocation of the labor force between the two sectors also remains the same. The following proposition is easily established:

<u>Proposition 1</u>. <u>Traditional sector enrichment growth results in higher income, a more equal relative distribution of income, and less poverty</u>.

The increase in income and the alleviation of poverty (since each of the poor becomes less poor) are evident. Regarding the relative income distribution, we need only observe that traditional sector enrichment growth has the effect of shifting the kink point on the Lorenz curve vertically as in Figure 1:

Figure 1



which establishes Lorenz domination. By inspection, it is apparent that the income share of the poorest 40% (S) increases and the Gini coefficient (G) (the ratio of the area above the Lorenz curve to the entire triangle) decreases. Hence, relative income inequality declines, as was to be shown. By all of the social welfare criteria presented above, this type of growth therefore results in an unambiguous welfare improvement.

B. <u>Modern Sector Enrichment Growth</u>

In modern sector enrichment growth, incomes in the modern sector rise, while incomes in the traditional sector and the allocation of the labor force between the modern sector and the traditional sector remain the same. In this case, we have the following theorem:

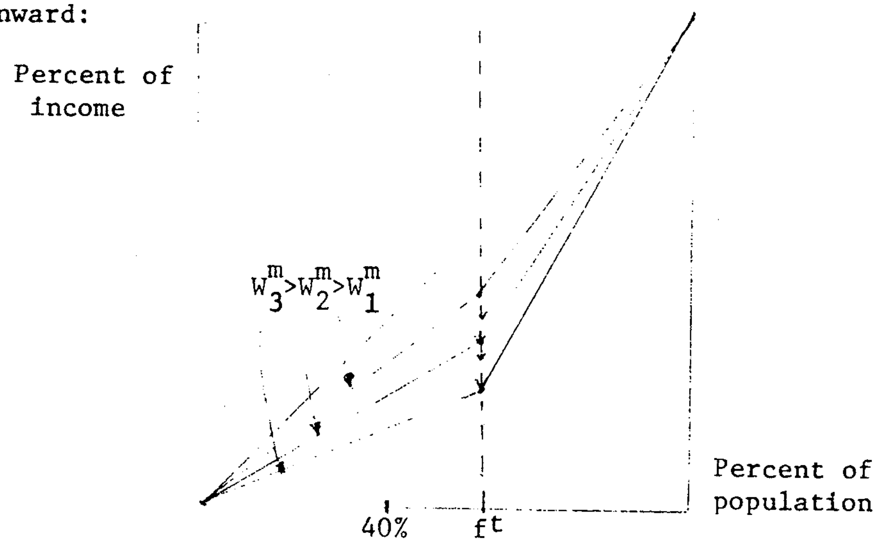
<u>Proposition 2</u>. <u>Modern sector enrichment growth results in higher income, a less</u>

equal relative distribution of income, and no change in poverty.

Adherents of the more general form of the absolute income approach would regard this type of growth as an unambiguous improvement, although they would have preferred a pattern where less of the benefit accrued to the well-to-do. However, Rawlsians and persons who adopt the absolute poverty criterion would be indifferent to this type of growth, since no poverty is being alleviated.

With respect to relative inequality, the gap between the modern sector wage and the traditional sector wage increases. The kink point on the Lorenz curve shifts vertically downward:

Figure 2



In Figure 2, we see clearly the Lorenz-inferiority of the new situation compared with the old, the rising Gini coefficient, and the falling share of the poorest 40%. Those concerned with relative inequality would give positive weight to the growth in income but negative weight to the rising relative inequality. Thus, the judgments rendered by the various welfare economic approaches are in disagreement. The observed discrepancy is not entirely undesirable. It is quite plausible that some observers may wish to regard the rising gap between the rich and poor unfavorably, not because the poor have lower incomes, but rather because the growing income differential might make the poor *feel* worse off. Some might even wish to allow envy of the rich by the poor to more than offset the gain in utility of the income recipients themselves.

This is a defensible position--that income growth concentrated exclusively in the hands of the rich might be interpreted as a socially inferior situation as compared with the rich having less and the poor the same amount--but certainly an extreme one based on the primacy of relative income considerations. In the case of modern sector enrichment growth, therefore, the differing judgments according to the welfare functions (4), (5), (7), and (12) reflect a true difference of opinion.

This is not so in the case of modern sector enlargement growth, to which we now turn.

C. Modern Sector Enlargement Growth

As described by a number of leading writers in the field, countries develop principally by absorbing an increasing share of their labor forces into an ever-enlarging modern sector. As a stylized version of this, in the modern sector enlargement growth model, incomes in both the modern and the traditional sectors remain the same but the modern sector gets bigger. In this case, we may derive the following results:

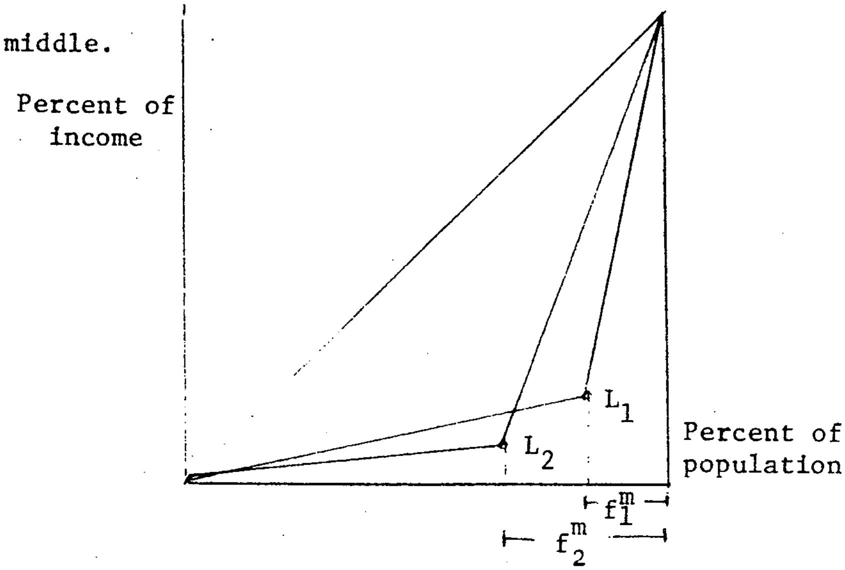
Proposition 3. In modern sector enlargment growth: (a) Absolute incomes rise and absolute poverty is reduced. (b) The Rawlsian criterion shows no change. (c) Lorenz curves always cross, so relative inequality effects are ambiguous. (d) Relative inequality indices first increase and subsequently decline.

Proofs: (a) The proofs of the absolute income and absolute poverty effects are immediate. Clearly, absolute incomes are higher, and since there are fewer poor, poverty is alleviated.

(b) In modern sector enlargement growth, there are fewer poor, but those who remain poor continue to be just as poor as before. Until poverty is totally eliminated, the Rawlsian criterion is completely insensitive to modern sector enlargement growth.

(c) The crossing of Lorenz curves is demonstrated in Figure 3. The explanation is: (i) Those among the poor who are left behind due to the incapacity of the modern sector to absorb everyone have the same incomes, but these incomes are now a smaller fraction of a larger total, so the new Lorenz curve lies below the old Lorenz curve at the lower end of the income distribution; (ii) Each person in the modern sector receives the same absolute income as before, but the share going to the richest f_1^m % is now smaller, and hence the new Lorenz curve lies above the old one at the upper end of the income distribution; (iii) Therefore, the two curves necessarily cross somewhere in the middle.

Figure 3



Of course, when Lorenz curves cross, welfare judgments based on relative inequality considerations are ambiguous.

(d) We shall now demonstrate the inevitability of an initial increase in relative inequality in the early stages of development followed by a subsequent decline for the income share of the poorest 40% (S) and the Gini coefficient (G). This is called the inverted-U hypothesis.

Considering S first, it is evident that in the early stages of modern sector enlargement growth, the poorest 40% receive the same absolute amount from a larger whole, and therefore their share falls. However, in the later stages (i.e., for f^t < 40%), they receive all of the income growth and hence their share rises. This result may be generalized as follows: If our measure of inequality is the share of income accruing to the poorest X%, that share falls continuously until the modern sector has grown to include (1-X)% of the population.

Turning now to the Gini coefficient, the proof is given in footnote 8.[8]

While both measures exhibit the inverted-U pattern in modern sector enlargement growth, the turning points do not coincide. There are three phases:

(I) Initially, both G and S show rising relative inequality; (II) Then, G turns down while S continues fo fall; (III) Finally, S rises while G continues to fall. To indicate the importance of this discrepancy for just these two measures, it is thought that in real terms the modern sector-traditional sector wage gap is something

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[8]The formula for the Gini coefficient in our dualistic model is:

$$G = 1 - \frac{[W^t + W^m - W^t)(f^m)^2]}{[W^t + (W^m - W^t)f^m]} . \tag{13}$$

(13) is a quadratic function. By inspection, $G = 0$ when $f^m = 0$ and $f^m = 1$ and $G > 0$ if $0 < f^m < 1$. Thus, the Gini coefficient follows an inverted-U path. To determine the location of the maximum, find

$$\frac{\partial G}{\partial f^m} = \{\frac{[W^m - W^t]}{[W^t + (W^m - W^t)f^m)]^2}\} \{\frac{-2f^m W^t + W^t}{-(f^m)^2(W^m - W^t)}\}$$

and equate the result to zero. Since the first term in brackets is strictly positive, we need only work with the second term. Setting it equal to zero and applying the quadratic formula to solve for f^m, we find

$$f^m_C = \frac{-W^t \pm \sqrt{W^m W^t}}{W^m - W^t}.$$

It is evident that one of the roots, $(f^m)_C = \dfrac{-W^t - \sqrt{W^m W^t}}{W^m - W^t}$, is negative, so must be rejected. Considering now the other root, $(f^m)_C = \dfrac{\sqrt{W^m W^t} - W^t}{W^m - W^t}$,

the fact that $W^m > W^t$ implies both numerator and denominator are positive and therefore $(f^m)_C > 0$. Likewise, $W^m > W^t$ implies $\sqrt{W^m W^t} < W^m$, and therefore $(f^m)_C < 1$. Thus, G achieves an economically-meaningful critical value at

$$f^m = \frac{\sqrt{W^m W^t - W^t}}{W^m - W^t} .$$

like 3:1. This implies that Phase II ranges from 37% to 60% of the population in the traditional sector. This range is substantial and may well include many LDCs.

In 1955, Kuznets [20] demonstrated this pattern in the historical experiences of a number of then-developed economies. Kuznets' explanation was that the inverted-U pattern was caused by the transfer of workers from the rural sector, where incomes were relatively equally distributed at low levels, to the urban sector, where there was greater income dispersion, owing to the presence of a skilled professional class at the top and poor recent migrants at the bottom. In terms of the development typologies analyzed above, Kuznets' model is basically one of <u>modern sector enlargement growth with within-sector inequality</u>.

In the Appendix, I extend the dualistic development models of this paper to allow for within-sector inequality. There, I prove that the inverted-U pattern <u>always</u> arises in modern sector enlargement growth, even if the <u>traditional sector has a more unequal distribution of income within it</u>. This result has been observed by previous researchers.[9] Where I differ from the others is over the welfare interpretation of these patterns, which we now examine.

<u>Proposition 4.</u> <u>The various welfare approaches give different evaluations of the desirability of modern sector enlargement growth.</u> <u>(a)</u> <u>The absolute income and absolute poverty approaches rate this type of growth as an unambiguous welfare improvement</u>.

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[9] In his original study [20] Kuznets produced a number of numerical examples consistent with the inverted-U pattern in modern sector enlargement growth, using as his measure of relative inequality the difference in percentage shares between the first and fifth quintiles. He did not, however, establish its inevitability (under the same maintained assumptions as those employed here). After the first draft of this paper was completed, I learned that the result in Proposition 3.d had been proven earlier by Swamy [34] using the coefficient of variation. The result has since been reconfirmed, apparently independently, by Robinson [28] using the log variance.

<u>(b) Rawlsians would be indifferent to this type of growth. (c) The relative inequality approach regards this type of growth ambiguously in the early stages but once the turning point is reached, it is a good thing. (d) The general welfare approach (12) considers modern sector enlargement growth as an unambiguous improvement regardless of the stage of development</u>.

The proofs of (a)-(c) are immediate given the respective welfare functions and the patterns established in Propostion 3. Point (d) follows from (12) and conditions (A), (C), (F), and (G). The lack of correspondence between (c) and (d) merits further attention.

Kuznets, Swamy, Robinson, and many others have interpreted the inverted-U pattern as signifying that in a true economic sense "the distribution of income must get worse before it gets better." It would seem at first that a falling share going to the poor (S) or a rising Gini coefficient (G) <u>should</u> receive negative weight in a social welfare judgment, possibly negative enough to outweigh the rising level of income. But why? There are at least two possible answers.

Implicitly, we may have in mind that a falling S or rising G implies that the poor are getting <u>absolutely</u> poorer while the rich are getting absolutely richer, and many of us would regard this as a bad thing indeed. The problem with this notion is that it confuses cause and effect, that is to say, absolute emiseration of the poor would definitely imply falling S and rising G, but as we have just seen, G rises and S falls in the early stages of modern sector enlargement growth without the poor becoming worse off in absolute terms.

Ruling out the necessity of absolute emiseration of the poor as a reason for reacting adversely to a falling S or rising G in modern sector enlargement growth, we may instead have in mind <u>relative</u> income comparisons -- that a growing income differential between rich and poor reduces poor people's utilities. Yet, in the early stages of modern sector enlargement growth, despite the rising Gini coefficient and the falling share of the poorest 40%, the income differential between rich and poor is not changing. Hence:

Proposition 5. For modern sector enlargement growth, the conventional relative inequality measures do not "correctly" measure relative inequality, if the "correct" definition of relative inequality in dualistic development is the intersectoral wage difference or ratio(or a monotonic transformation thereof). In the early stages of modern sector enlargement growth, we may misled into thinking that relative inequality is "worsening" when in fact the wage structure is not changing. This same point holds in reverse for relative inequality "improvements" in the later stages of modern sector enlargement growth. This is because condition (E) is violated.

Proposition 5 implies that rising relative inequality as measured by conventional indices may be a perfectly natural, and even highly desirable, outcome for this type of development. Put differently, the falling share of the lowest 40% and rising Gini coefficient which arise in this case are statistical artifacts without social welfare content. For this type of growth, the specification of social welfare functions like (4) conflicts with our ideas of social well-being as given by (12). This conflict is particularly acute for persons who wish to give heavy weight to relative income considerations. If relative-inequality-averse persons compare Gini coefficients or income shares of the poorest 40% at two points in time when modern sector enlargement growth is taking place, they will be led to social welfare judgments which they themselves would not wish to make. Unfortunately, functions like (4) based on G or S are being used with increasing frequency in current empirical studies of economic development. The use of functions like (12), based on the enlargement and enrichment components of various sectors' growth experiences, would avoid such difficulties.

VI. Conclusions and Implications

This paper has examined the welfare implications of different types of dualistic economic development. Three stylized models of growth in the dual economy were constructed (Section II). Several alternative approaches for assessing the welfare

TABLE 2. SUMMARY OF DISTRIBUTION AND WELFARE EFFECTS IN THREE MODELS OF DUALISTIC DEVELOPMENT

	Traditional Sector Enrichment Growth	Modern Sector Enrichment Growth	Modern Sector Enlargement Growth		
			Phase I	Phase II	Phase III
			$0 \le f^m < \frac{\sqrt{W^m W^t}-W^t}{W^m-W^t}$	$\frac{\sqrt{W^m W^t}-W^t}{W^m-W^t} \le f^m \le 60\%$	$f^m > 60\%$
Growth and Distributional Effects					
f^m	Unchanged	Unchanged	Rises	Rises	Rises
f^t	Unchanged	Unchanged	Falls	Falls	Falls
W^m	Unchanged	Rises	Unchanged	Unchanged	Unchangec
W^t	Rises	Unchanged	Unchanged	Unchanged	Unchangec
Y	Rises	Rises	Rises	Rises	Rises
π	Falls	Unchanged	Falls	Falls	Falls
L	Lorenz-superior	Lorenz-inferior	Lorenz-crossing	Lorenz-crossing	Lorenz-crossing
G	Falls	Rises	Rises	Falls	Falls
S	Rises	Falls	Falls	Falls	Rises
Y_{min}	Rises	Unchanged	Unchanged	Unchanged	Unchangec
Welfare Effects According To:					
Absolute Income Approach	Unambiguous Improvement	Unambiguous Improvement	Unambiguous Improvement	Unambiguous Improvement	Unambiguc Improveme
Rawlsian Maximin Approach	Unambiguous Improvement	Unchanged	Unchanged	Unchanged	Unchangec
Absolute Poverty Approach	Unambiguous Improvement	Unchanged	Unambiguous Improvement	Unambiguous Improvement	Unambiguc Improveme
Relative Inequality Approach:					
L	Unambiguous Improvement	Lorenz-inferior	Lorenz-crossing	Lorenz-crossing	Lorenz-crossing
G	Unambiguous Improvement	Ambiguous	Ambiguous	Unambiguous Improvement	Unambiguc Improveme
S	Unambiguous Improvement	Ambiguous	Ambiguous	Ambiguous	Unambiguo Improveme
fn. (12) and condition (C')	Unambiguous Improvement	Ambiguous	Unambiguous Improvement	Unambiguous Improvement	Unambiguo Improveme
General Welfare Approach (12)	Unambiguous Improvement	Ambiguous	Unambiguous Improvement	Unambiguous Improvement	Unambiguo Improveme

implications of growth were set forth (Sections III and IV). For each stylized development typology, the changes in relative inequality and absolute incomes and poverty and the welfare effects of these changes were derived according to the various welfare criteria (Section V).

A number of conclusions and implications may be drawn:

(1) The time paths of relative inequality and absolute poverty depend on the type of economic development as well as its level. Absolute poverty is diminished in traditional sector enrichment growth and modern sector enlargement growth, but is not alleviated in modern sector enrichment growth. Relative inequality declines in traditional sector enrichment growth and rises in modern sector enrichment growth. The usual relative inequality measures show an inverted-U pattern in modern sector enlargement growth. In short, contrary to Kuznets' [20] suggestion, income distribution need not get worse before it gets better, provided a suitable development strategy is followed.

(2) The absolute income and poverty and relative inequality approaches often do not give the same welfare judgments about the desirability of different patterns of growth. Only for traditional sector enrichment growth and for the later stages of modern sector enlargement growth do these approaches concur in indicating an unambiguous welfare improvement. In the case of modern sector enrichment growth, there is a real substantive disagreement about whether or not growth of that sort is a good thing. However, in the early stages of modern sector enlargement growth, there arises a discrepancy between the various approaches, but it has no apparent welfare economic basis. This is because:

(3) Conventional relative inequality measures show an inverted-U pattern in modern sector enlargement growth despite a constant intersectoral wage structure. This implies that the "worsening" inequality (as ordinarily measured) should not be interpreted as a bad thing, nor should the subsequent "improvement" be regarded as

an economically-meaningful reduction in relative inequality either. Thus, social welfare functions, whether explicitly-stated or implicitly-assumed, of the form $W = W(Y, I)$, $f_1 > 0$, $f_2 < 0$, where I is any of the Lorenz curve-based relative inequality measures in common use, are _invalid_ for this type of growth. In cases of modern sector enlargement growth, it is far better to look only at the rate at which the growth is taking place.

As a corollary of the above:

(4) _Before we can legitimately interpret a rising relative inequality coefficient in a country as an economically-meaningful worsening of the income distribution rather than a statistical artifact, we must know which of the three types of economic development patterns that country has been following._ We have shown that a falling share of income received by the poorest 40% and rising Gini coefficient can be the result of:

(a) _Traditional Sector Impoverishment_, which is clearly _bad_ in social welfare terms; or

(b) _Modern Sector Enrichment_, which is _good_ in absolute income terms, _indifferent_ in absolute poverty terms, and _ambiguous_ in relative income terms; or

(c) _Modern Sector Enlargement_ in the early phases, which is _good_ according to widely accepted axiomatic judgements. Simple calculations of relative inequality patterns cannot distinguish among these causes. This implies:

(5) _Regardless of whether one favors an absolute or relative approach or some combination of them, social welfare judgments about the desirability of a given course of economic development should be made on the basis of the enlargement and enrichment components of that growth._ Equation (12) makes clear that the way we feel about a country's growth pattern depends on changes in its wage structure and occupational structure over the development period. For example, a ten percent rate of growth of income in the modern sector might result from either (i) a 20% rise in the size of sector, coupled with a 10% fall in average incomes, or (ii) a 20% rise in average

incomes, accompanied by a 10% decline in number of persons in the sector. Most observers would have very different qualitative evaluations of the two situations. Hence, examination of the rates of growth of incomes in various sectors of an economy does not provide sufficient information for a welfare judgment.[10]

(6) For persons who wish to give greatest emphasis to the alleviation of absolute poverty, the poverty index proposed by Sen [30] has a number of desirable properties. It avoids the problems associated with the interpretation of relative inequality measures. It is sensitive to the number of poor (the enlargement effect), the severity of their poverty (the enrichment effect), and the degree of income inequality among them. It is easily calculable from microeconomic data or sufficiently disaggregated tabulations. And its axiomatic justification is clearly delineated so that users and non-users alike will know what welfare judgments underlie the measure.

VII. Empirical Significance

The preceding analysis has shown that under certain circumstances the absolute poverty and relative inequality approaches may give very different results concerning the distributional effects of growth in the dual economy. In light of these differences, the choice between the two types of measures should be based on the type of welfare judgments we wish to make. The empirical significance of the choice may be illustrated with reference to two actual cases of particular interest, India and Brazil.

The Brazilian economy achieved a growth in per capita income of 32% over the decade of the 1960s, a substantial accomplishment by the standards of less developed countries. Fishlow [16], Langoni [23], and others have examined the distributional question of who received the benefits of this growth, found greater relative income inequality, and concluded that the poor benefited very little, if at all. Yet when the distributional question is reexamined from an absolute poverty perspective by looking at

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[10] Consider statements of the form "Income of the richest X% grew by A% but income of the poorest Y% grew by only B% (less than A); therefore, income growth was disproportionately concentrated in the upper income groups." This interpretation is correct if average income among those who were originally the richest X% of the people rose much faster than among those who were originally the poorest Y%. However, the interpretation is incorrect if what mainly happened was that the high income sector expanded to include more people. From data on income growth of the richest X% and poorest Y%, we cannot tell which.

the number of very poor and the levels of income they receive, it is found that the average real incomes among families defined as poor by Brazilian standards increased by as much as 60% while the comparable figure for non-poor families was around 25% (Fields [15]). At the same time, the percentage of families below the poverty line fell somewhat. It would thus appear that by assigning heavy weight to changes in the usual indices of relative income inequality and interpreting these increases as offsets to the well-being brought about by growth, previous investigators may have inadvertently overlooked important tendencies toward the alleviation of poverty.

In the India case, the problem is just the opposite. Bardhan [4] reports that relative inequality in India has actually declined in recent years, which some might see as an improvement in income distribution. Yet, due to the lack of growth of the Indian economy, the precentage of people living in absolute poverty increased in both the urban and rural sectors of the economy.

These examples indicate that the choice of an evaluative criterion <u>does</u> make a very real qualitative difference. It comes down to a choice between welfare judgments which emphasize the alleviation of absolute poverty or those focusing on the narrowing of relative income inequality. Personally, I am most concerned about the alleviation of economic misery among the very poorest, and therefore prefer the absolute poverty approach. Others with different value judgments who may be more concerned than I with relative income comparisons or with the middle or upper end of the income distribution may wish to give relatively greater weight to one of the other approaches. The inconsistency between the professed concerns of many researchers for the alleviation of poverty and their usage of relative inequality measures in empirical research is striking and hopefully will be diminished in the not too distant future.

Appendix

In this appendix, we prove the inevitability of the inverted-U pattern in modern sector enlargement growth with within-sector inequality, taking as our measure of inequality the Gini coefficient.[1] The strategy of the proof is to derive an expression for the change in the Gini coefficient with an increase in the size of the modern sector when there is within-sector inequality, and then to demonstrate that a maximum value always exists for a positive fraction of the population, irregardless of the relative sizes of the within-sector inequality coefficients.

Let us suppose that modern sector enlargement growth takes place under the following conditions:

(i) The income distribution within the modern sector is fixed, that is, the frequency distribution of wages in that sector (F^m) remains the same over time, which implies that the mean wage earned by those in the modern sector ($\bar{W}^m$) and the Gini coefficient of those working in the modern sector (G^{m*}) also are constant.

(ii) Similarly, the income distribution within the traditional sector, and therefore F^t, $\bar{W}^t$, and G^{t*} also remain constant.

(iii) The lowest income in the modern sector is greater than the highest income in the traditional sector.[2]

(iv) Population is constant and normalized at 1; the population shares of the modern and traditional sectors are given by f^m and f^t, respectively.

(v) Growth takes place by enlarging the modern sector, i.e., by

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[1] The choice of the Gini coefficient is arbitrary; any other inequality measure might also have been chosen. The Gini coefficient is considered here, because it is the most widely used.

[2] This assumption is not crucial to the analysis, but it greatly eases the algebra.

increasing f^m.

The methodology here draws on a procedure developed by Fei and Ranis [13] for decomposing total inequality into its various component parts. Suppose that we were to array the population in increasing order of income. Then Fei and Ranis show:

$$\text{(A.1)} \qquad G = \sum_i \phi^i G^{i'},$$

$$\text{(A.2)} \qquad G^i \cong G^i R^{i'},$$

and therefore,

$$\text{(A.3)} \qquad G \cong \sum_i \phi^i G^i R^{i'},$$

where G = Gini coefficient of total income,

G^i = Gini coefficient of income from the i'th source, including those who have no income from that source,

ϕ^i = Share of the i'th factor or sector in total income,

$R^{i'}$ = Rank correlation between the total incomes of individuals or groups and their incomes from the i'th source,

$G^{i'}$ = "Pseudo-Gini coefficient" of the i'th income source, obtained by computing a Gini coefficient with the individuals or groups ordered according to total income rather than income from that source.

Fei and Ranis have applied this procedure to the decomposition of total inequality into its various factor components.

The same methodology, appropriately modified, may be applied to the growth of various sectors. Under the conditions of modern sector enlargement growth just assumed, in particular condition (iii), it follows that $G^{i'} = G^i$ and $R^{i'} = 1$ for all i, and therefore (A.1)-(A.3) reduce to

$$G = \sum_i \phi^i G^i,$$

using the true Gini coefficients instead of the pseudo-Ginis.

Suppose now that as in the dualistic development models, we have only two sectors, a modern sector and traditional sector, with respective income distributions F^m and F^t, and comprising f^m and f^t percent of the labor force respectively. The factor share of each sector is the average wage multiplied by the fraction of the labor force in that sector, all divided by total income, which gives us in place of (A.4):

$$\text{(A.5)} \quad G = \frac{\bar{W}^m f^m G^m}{Y} + \frac{\bar{W}^t f^t G^t}{Y}.$$

Recall that the sector Gini coefficients G^m and G^t include persons with no income from that source. Letting G^{m*} and G^{t*} represent the Gini coefficients including only people with income from that sector, and assuming the two sectors to be mutually exclusive, it may be shown that

$$\text{(A.6)} \quad G^{m*} = \frac{G^m - f^t}{1-f^t} \quad \text{and} \quad G^{t*} = \frac{G^t - f^m}{1-f^m}.$$ [3]

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[3] The Gini coefficient of a variable X is equal to 1-2B, where B is the area under the Lorenz curve of X. It is easily established geometrically that $B = \frac{1}{2}\frac{1}{n}(X_1 + X_2 + \ldots + X_n)/Y + \frac{X_{j+1}(\frac{1}{n})}{Y} + \frac{(X_{j+1}+X_{j+2})(\frac{1}{n})}{Y} + \ldots + \frac{(X_{j+1}+\ldots+X_n)(\frac{1}{n})}{Y}$ where n is the total number of persons or families, j is the number who have no income from that source, and Y is total income. The above expression may be rearranged to yield

$$B = \frac{1}{2n} + \frac{1}{nY}[(n-j-1)X_{j+1} + (n-j-2)X_{j+2} + \ldots + X_n].$$

If we now consider only the n-j persons who have positive incomes from that source, and let G^* be the Gini coefficient among those same n-j persons, then $G^* = 1 - 2B^*$, where

$$B^* = \frac{1}{2(n-j)} + \frac{1}{(n-j)Y}[(n-j-1)X_{j+1} + (n-j-2)X_{j+2} + \ldots + X_n].$$

Denote the term in brackets by Z. Then

We now wish to solve for G in terms of the parameters of the model and the proportion of workers in the modern sector labor force. Solving (A.6) for G^m and G^t and substituting the results along with $Y = \bar{W}^m f^m + \bar{W}^t f^t$ and $f^t = 1 - f^m$ into (A.5), we obtain

$$\text{(A.7)} \quad G = \frac{\bar{W}^m f^m (G^{m*} f^m + 1 - f^m) + (\bar{W}^t - \bar{W}^t f^m)(G^{t*} - G^{t*} f^m + f^m)}{\bar{W}^m f^m + \bar{W}^t - \bar{W}^t f^m}$$

$$= \frac{\begin{bmatrix} {f^m}^2 (\bar{W}^m G^{m*} - \bar{W}^m + \bar{W}^t G^{t*} - \bar{W}^t) \\ + f^m (\bar{W}^m - 2\bar{W}^t G^{t*} + \bar{W}^t) \\ + \bar{W}^t G^{t*} \end{bmatrix}}{(\bar{W}^m - \bar{W}^t) f^m + \bar{W}^t}$$

$$= [{f^m}^2 A + f^m B + \bar{W}^t G^{t*}] / [(\bar{W}^m - \bar{W}^t) f^m + \bar{W}^t],$$

where $A = \bar{W}^m G^{m*} - \bar{W}^m + \bar{W}^t G^{t*} - \bar{W}^t$ and $B = \bar{W}^m - 2\bar{W}^t G^{t*} + \bar{W}^t$.

The Kuznets turning point exists if G has an interior maximum, i.e., if the first derivative attains a zero value at a critical value of f_C^m, $0 < f_C^m < 1$. Differentiating (A.7) with respect to f^m, we obtain

$$\text{(A.8)} \quad \frac{\partial G}{\partial f^m} = \frac{Y[2f^m A + B] - [{f^m}^2 A + f^m B + \bar{W}^t G^{t*}][\bar{W}^m - \bar{W}^t]}{Y^2}.$$

Equating (A.8) to zero and rearranging yields

$$\text{(A.9)} \quad {f_C^m}^2 [A(\bar{W}^m - \bar{W}^t)] + f_C^m [2A\bar{W}^t] + [B\bar{W}^t - (\bar{W}^m - \bar{W}^t)\bar{W}^t G^{t*}] = 0$$

$$G = 1 - \frac{1}{n} - \frac{2Z}{nY}$$

and

$$G^* = 1 - \frac{1}{n-j} - \frac{2Z}{(n-j)Y}.$$

Solving these two equations for Z, equating the resulting expressions to one another, and solving the result for G*, we obtain

$$G^* = \frac{Gn-j}{n-j}. \qquad \text{Q.E.D.}$$

Applying the quadratic formula and combining terms, we find

$$\text{(A.10)}\qquad f_C^m = \frac{-2A\bar{W}^t \pm \sqrt{\begin{array}{l}4A^2\bar{W}^{t^2} - 4AB\bar{W}^m\bar{W}^t + 4A\bar{W}^{m^2}\bar{W}^t G^{t*} \\ -8A\bar{W}^m\bar{W}^{t^2}G^{t*} + 4AB\bar{W}^{t^2} + 4A\bar{W}^{t^3}G^{t*}\end{array}}}{2A(\bar{W}^m - \bar{W}^t)}$$

Since $A < 0$, the denominator of (A.10) is negative and the first term in the numerator is positive. If f_C^m is to lie between 0 and 1, the numerator must be negative, and therefore the only potentially meaningful root is

$$\text{(A.11)}\qquad f_C^m = \frac{-\bar{W}^t}{\bar{W}^m - \bar{W}^t} - \frac{\sqrt{\begin{array}{l}A^2\bar{W}^{t^2} - AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^t G^{t*} \\ -2A\bar{W}^m\bar{W}^{t^2}G^{t*} + AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*}\end{array}}}{A(\bar{W}^m - \bar{W}^t)}$$

If the critical value (A.11) is to be economically relevant, it must be positive and less than one. Denoting the term under the square root sign by C, f_C^m will be positive if $C > (A\bar{W}^t)^2$, which is easily demonstrated:

$$\begin{aligned}\text{(A.12)}\qquad C-(A\bar{W}^t) &= -AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^t G^{t*} - 2A\bar{W}^m\bar{W}^{t^2}G^{t*} \\ &\quad +AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*} \\ &= -AB\bar{W}^t(\bar{W}^m - \bar{W}^t) + A\bar{W}^t G^{t*}(\bar{W}^{m^2} - 2\bar{W}^m\bar{W}^t + \bar{W}^{t^2}) \\ &= A\bar{W}^t(\bar{W}^m - \bar{W}^t)[-B + G^{t*}(\bar{W}^m - \bar{W}^t)] \\ &= A\bar{W}^t(\bar{W}^m - \bar{W}^t)(\bar{W}^m + \bar{W}^t)(1 - G^{t*}) > 0.\end{aligned}$$

To show f_C^m is less than one, we require

$$\text{(A.13)}\qquad -\bar{W}^t - \frac{\sqrt{C}}{A} < \bar{W}^m - \bar{W}^t$$

$$\Longleftrightarrow -\frac{\sqrt{C}}{A} < \bar{W}^m$$

$$\Longleftrightarrow \quad \sqrt{C} < -A\bar{W}^m$$

$$\Longleftrightarrow \quad C < A^2\bar{W}^{m^2},$$

which may be shown as follows:

$$A^2\bar{W}^{m^2} > A^2\bar{W}^{t^2} - AB\bar{W}^m\bar{W}^t + A\bar{W}^{m^2}\bar{W}^tG^{t*} - 2A\bar{W}^m\bar{W}^{t^2}G^{t*} + AB\bar{W}^{t^2} + A\bar{W}^{t^3}G^{t*}$$

$$\Longleftrightarrow \frac{A^2(\bar{W}^{m^2}-\bar{W}^{t^2})}{-A} > \begin{array}{l} B\bar{W}^m\bar{W}^t - \bar{W}^{m^2}\bar{W}^tG^{t*} + 2\bar{W}^m\bar{W}^{t^2}G^{t*} \\ - B\bar{W}^{t^2} - \bar{W}^{t^3}G^{t*} \end{array}$$

$$\Longleftrightarrow - A(\bar{W}^{m^2}-\bar{W}^{t^2}) > B\bar{W}^t(\bar{W}^m-\bar{W}^t) - \bar{W}^tG^{t*}(\bar{W}^m-\bar{W}^t)^2$$

$$\Longleftrightarrow - A(\bar{W}^{m^2}-\bar{W}^{t^2}) > \bar{W}^t(\bar{W}^{m^2}-\bar{W}^{t^2})(1-G^{t*})$$

$$\Longleftrightarrow - A > \bar{W}^t(1-G^{t*})$$

$$\Longleftrightarrow - [\bar{W}^m(G^{m*}-1) + \bar{W}^t(G^{t*}-1)] > \bar{W}^t(1-G^{t*})$$

$$\Longleftrightarrow \bar{W}^m(1-G^{m*}) > 0,$$

as was to be proved.

We have therefore shown that when there is within-sector inequality in modern sector enlargement growth, there is always an inverted U-pattern of measured inequality, regardless of whether incomes are distributed more equally, less equally, or the same within the modern sector as in the traditional sector. It should be noted that Proposition (3.d) is the special case $G^{m*} = G^{t*} = 0$.

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