

ECONOMIC GROWTH CENTER

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CENTER DISCUSSION PAPER NO. 290

THE SIZE AND GROWTH OF INDUSTRIAL FIRMS IN MEXICO

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Note: Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

Gustav Ranis made valuable suggestions for the improvement of this paper.

INTRODUCTION

The size-distribution of firms in the industrial sector of an economy like Mexico's is positively skewed with a small number of large firms and many small ones. Whether output, value added, number of employees, or value of assets are used as the size measure, the observed distribution is always highly skewed. Tables 1 and 2 show, for 1970, the size distribution of firms in the industrial sector of Mexico using the number of employees and the value of output as a measure of firm size. In 1970 more than 60% of the industrial firms employed 5 workers or less and 40% of firms produced an output worth only 25,000 pesos (about 1000 dollars at the 1978 rate of exchange) or less. Figures 1 and 2 show the skewed size distribution of firms for 1965 and 1970 when size is measured according to the number of employees.

Explanations about the degree of concentration and about the growth of firms and their size distribution in an industry have almost always assumed that the causal mechanism for these phenomenon is the degree of economies or diseconomies of scale. For an application of this approach to the study of the industrial concentration in Mexico see Gollas (1978). However, the explanations of the causes of the growth of firms and of the observed size distributions based on arguments on the shape of the long run average cost curve exhibit a number of limitations. Although we do not agree with the extreme position that "economic theory has little to say about the distribution of firm sizes" (Simon and Bonini 1958), we are aware that traditional economic theory gives limited guidance about the nature of

TABLE 1

Size Distribution of Firms
In Terms of The Number of Employees
1970

Size Class No. of Employees	Number of Firms	Percentage	Cumulative Percentage
≤ 5	39222	62.86	62.86
6 - 15	10959	17.56	80.42
16 - 25	3329	5.34	85.75
26 - 50	3532	5.66	91.42
51 - 75	1582	2.54	93.96
76 - 100	910	1.46	95.42
101 - 175	1217	1.95	97.37
176 - 250	579	0.93	98.30
251 - 350	366	0.59	98.89
351 - 500	287	0.46	99.35
501 - 750	200	0.32	99.67
751 and +	212	0.33	100.00
TOTAL	62395		100

Source: VIII Censo Industrial, 1965 and IX Censo Industrial, 1970
Direccion General de Estadistica, Secretaria de Industria
y Comercio, Mexico, D.F.

TABLE 2
Size Distribution of Firms
In Terms of Output

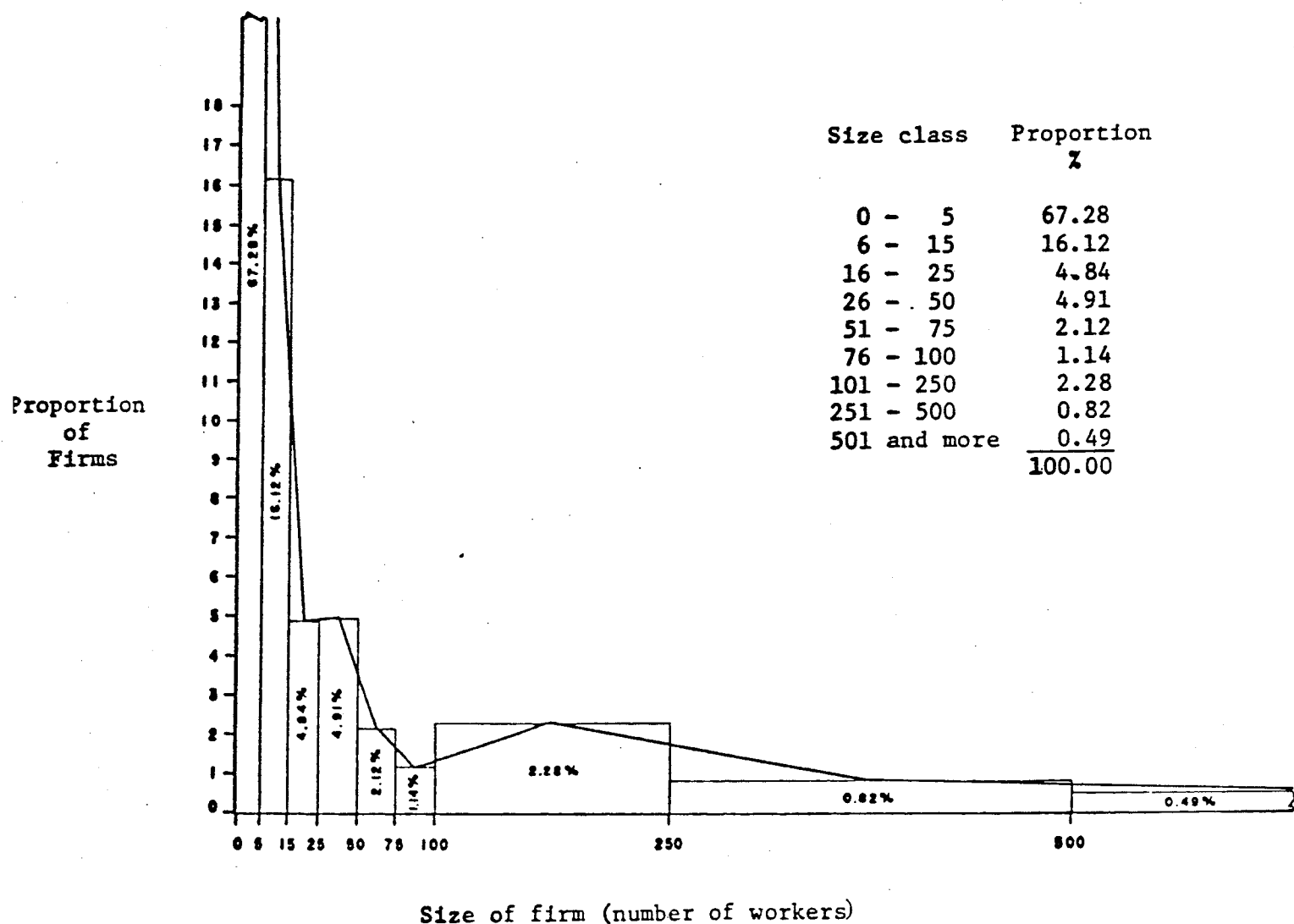
1970

Size Class (Thousands of pesos)	Number of Firms	Percentage	Cumulative Percentage
≤ 25	48575	40.5	40.5
26 - 100	38838	27.4	67.9
101 - 500	22431	18.7	86.6
501 - 1500	6439	5.4	92.0
1501 - 3000	2888	2.4	94.4
3001 - 5000	1640	1.4	95.8
5001 - 10000	1867	1.6	97.4
10001 - 20000	1365	1.1	98.5
20001 - 35000	801	0.6	99.1
35001 - 50000	348	0.3	99.4
50001 - 75000	263	0.2	99.6
75001 - 100000	142	0.1	99.7
100001 - 150000	164	0.1	99.8
150001 and +	202	0.2	100.0
TOTAL	119963		100.0

Source: VIII Censo Industrial, 1965 and IX Censo Industrial 1970
 Direccion General de Estadística, Secretaria de Industria
 y Comercio, Mexico, D.F.

Figure 1

Mexico: Size Distribution of Firms in Terms of Employment
1965

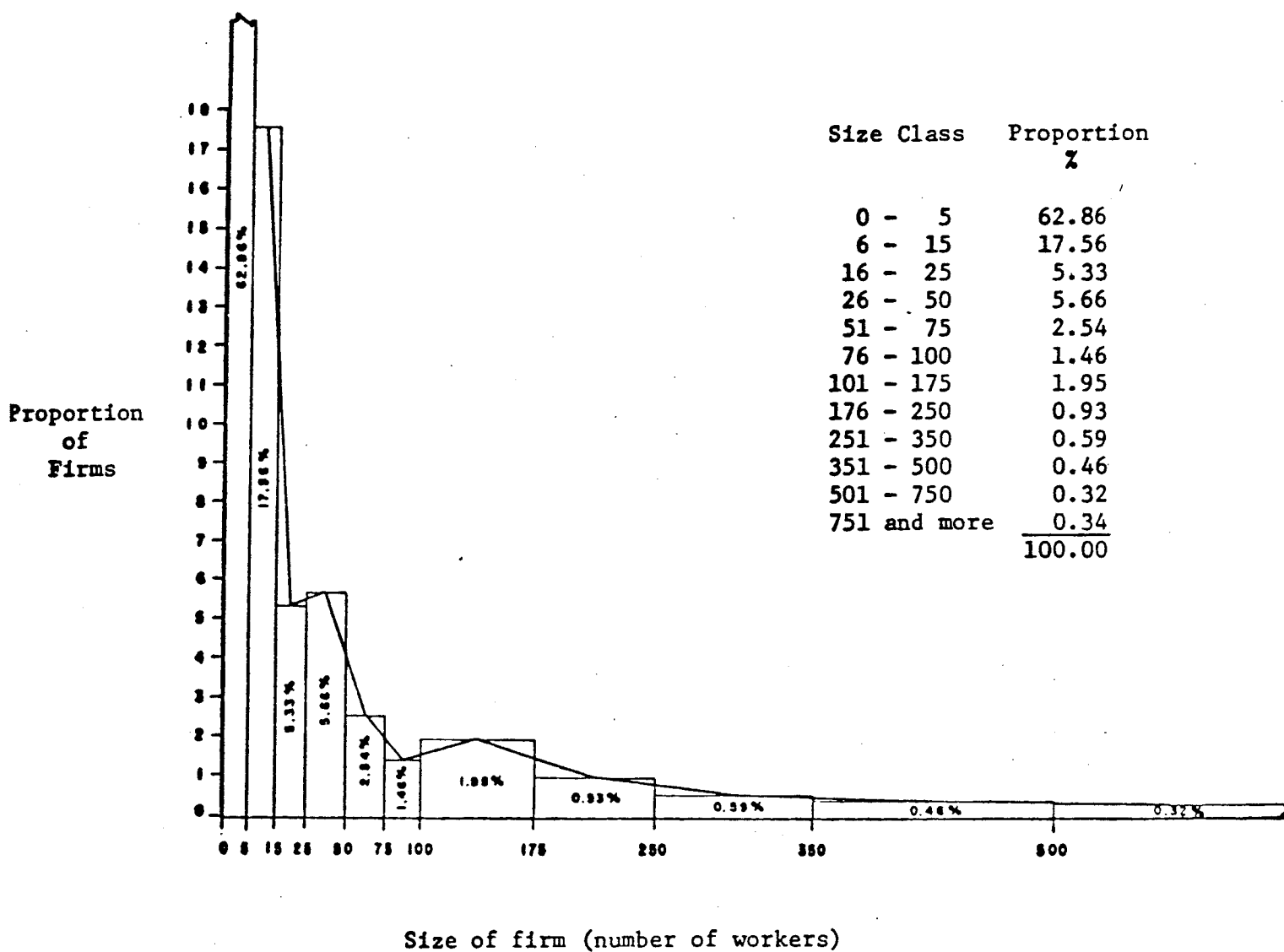


Source: Manuel Collas, "Reflexiones sobre la concentracion economica y el crecimiento de las empresas", El Trimestre Economico, Vol. XLII, Abril-Junio, 1975.

Figure 2

Mexico: Size Distribution of Firms in Terms of Employment

1970



Source: Manuel Gollas, Loc. cit

the relationship between size and growth of firms.

Let us first explore the case of a long run average cost curve of the popular U type which first exhibits increasing, then constant and finally decreasing returns to scale. The long run equilibrium for a firm in perfect competition occurs at the point where price equals minimum long run average cost. And at this point the minimum short run average cost equals the minimum long run average cost. This position of long run equilibrium is characterized by a "no profit" situation. Under these circumstances and if (a) the scale corresponding to minimum cost was the same for different firms in the industry (which is never the case), and if (b) each firm were to participate in the market with the amount of output needed to attain the long run minimum optimum size, then we would expect all firms to be more or less of the same size. In other words, one would anticipate, with certain variations, that the distribution of firms by size would be symmetrically distributed around the long run minimum optimum size. However, the empirically observed size distribution of firms is highly asymmetrical.

If one assumes that the long run average cost curve takes the form of an L which indicates that after a certain minimum size there exist constant returns to scale, then all size firms, after that minimum, would have the same probability of occurring. In this case one would expect a uniform size distribution of firms, but again, one empirically observes that the size distribution of firms is highly skew.

In short, the theory of cost predicts that, in the long run, the distribution of firms by size must be either (1) symmetric (may be normal)

around the long run minimum optimum size if the long run average curve has the form of a U or (2) uniform if the long run average cost exhibits constant returns to scale over a wide range, i.e., has the form of an L. However, the empirically observed size distributions of firms are not symmetric nor uniform, but highly skew. Later in the paper we discuss the implications of a continuously declining long run average cost curve.

We must explore alternative approaches since the static cost theory fails to help us to understand or predict the growth and size distribution of firms in the real world. In this paper we apply a stochastic model of growth to explain the observed growth and size distribution of industrial firms in Mexico between 1965 and 1970. We then test the consistency of our empirical findings with alternative shapes of the long run average cost curve of firms.

STOCHASTIC MODELS OF THE GROWTH OF FIRMS

A number of studies (Hart and Prais 1956, Simon and Bonini 1958, Hymer and Pashigian 1962, Hart 1962, Mansfield 1962, Ijiri and Simon 1964, Steindl 1965, Scherer 1970, and Singh and Whittington 1975, among others) have sought to explain the growth and size distribution of firms in terms of stochastic processes with varying degrees of restrictive assumptions. The basic common argument is that a highly skewed statistical distribution - like the observed size distribution of firms - may be generated by a stochastic process in which the "law of proportionate effect" also known as "the law of proportionate growth" or "Gibrat's Law" is in operation. According to this law the firm size has no effect on its expected percentage growth. One important implication of this law is that the distribution of the percentage of firm size changes

in a given firm size class is the same in all size classes. In other words, the probability of a firm growing at a certain percentage is independent of its size: if we randomly select a firm from a class of say, 500 million pesos in assets, it has the same probability for growing at, for example, 10%, as a firm randomly selected from a class of 5 million pesos in assets. Some economists (Simon and Bonini 1958) believe that this outcome is more likely to occur if there exist constant returns to scale above a critical minimum of optimum firm size (an L shape long run average cost curve). The argument is that with this kind of cost curve firms in each class above the minimum size will have the same probability, on the average, of increasing or decreasing in size in proportion to their actual size. Other models (Mansfield 1962) make use of a random process and of the law of proportionate effect to explain the size distribution of firms but do not assume the strong assumption of an L-shaped long run average cost curve.

THE LAW OF PROPORTIONATE EFFECT AND THE LOGNORMAL DISTRIBUTION

The most commonly used statistical distributions (normal, exponential, Poisson, etc.) can be generated from simple stochastic models like the simplest random walk (Feller 1950). When a stochastic process is applied to the logarithm of a variable a lognormal distribution is generated. The lognormal distribution in its simplest form is defined as the distribution of a random variable whose logarithm conforms to the normal probability density. Let Y be a random variable that is normally distributed with parameters μ_y and σ_y . If $Y = \ln X$ or $X = e^Y$ then X is

said to have a lognormal distribution. The skewness of the lognormal distribution arises when X is affected by random causes which produce small effects proportional to the variate X . The operation of these random causes producing small proportionate effects is known as the law of proportionate effect or law of proportionate growth. This random process was first formulated by Kaptyn in 1903 (Aitchison and Brown 1963). Applied to the variate size of firms, this law may be interpreted to mean that firm size is a random variable which is the outcome of other independent random variables acting upon the size of firms in a multiplicative manner. Aitchison and Brown (1963) define the law of proportionate effect as follows: "A variate subject to a process of change is said to obey the law of proportionate effect if the change in the variate at any step of the process is a random proportion of the previous value of the variate." If we let X be the random variate at time t and $t - 1$ we may write

$$X_t - X_{t-1} = \epsilon_t X_{t-1} \quad (1)$$

where ϵ_t are mutually independent factors and independent of the X .

If the initial value of X is X_0 and if after n steps its final value is X_n we have

$$X_n = X_0(1 + \epsilon_1)(1 + \epsilon_2) \dots (1 + \epsilon_n) \quad (2)$$

The prolonged operation (when n is large) of these effects on X generates a lognormal distribution of the variate X . In other words, a normal distribution is generated when a large number of small, independent, random forces operate on a variate in an additive manner; and a lognormal distribution may be

generated if they act multiplicatively.¹ In the case where X is the firm size we may say that some forces make for an increase and others for a decline but all act randomly in such a way that their combined effect does not show any tendency to favor or disfavor firms of any particular size. In this view the growth of firms is regarded as a purely stochastic phenomenon resulting from the cumulative effect of the random operation of a large number of forces acting independently of each other.

The probability density function of the lognormal distribution generated by the operation of the law of proportionate effect is

$$f(x) = \begin{cases} \frac{1}{x\sigma_y\sqrt{2\pi}} \exp \left\{ -\frac{1}{2\sigma_y^2} (\ln x - \mu_y)^2 \right\} & , \quad \begin{matrix} x > 0 \\ \sigma_y > 0, \quad -\infty < \mu_y < \infty \end{matrix} \\ 0 & \text{elsewhere} \end{cases}$$

The two parameters which define the density function completely, μ_y and σ_y , locate the relative position of the mean and the degree of dispersion. It is obvious that the variate X can not take zero or negative values since the transformation $Y = \ln X$ is not defined for $X = 0$ and negative values do not have logarithms. The lognormal distribution is positively skewed and the greater the value of σ_y the greater is its skewness.

¹The lognormal distribution arises from a theory of elementary errors combined by a multiplicative process, just as the normal distribution originates from a theory of errors combined by addition. The relevant issue is discovering whether in real world situations the nature of the growth process is multiplicative or additive. According to Aitchison and Brown (1963) "The problem here is formally similar to that of the choice of the geometric or the arithmetic mean as the more appropriate measure of location. Man has found addition an easier operation than multiplication, and so it is not surprising that an additive law of errors was the first to be formulated. Had man been more adept at multiplication the 'exponential-lognormal', or normal, might then have been the derivative distribution."

The properties of the lognormal distribution have been examined in great detail by Aitchison and Brown (1963) who also discuss its wide range of applications. The lognormal distribution was first used in the study of business concentration by Gibrat (1931) using French data. In his study Gibrat shows that the lognormal distribution gives a good description of firm sizes and that the variance of the logarithm of the sizes (the variance of the lognormal distribution) is a measure of the degree of concentration. Furthermore, not only is the variance of the lognormal distribution an appropriate measure of dispersion, but the commonly used Gini coefficient of concentration may be expressed as a function of this statistic. One of the advantages of the lognormal distribution is that the classical tests of statistical significance are immediately applicable to the measurement of changes in business concentration. Since the time of Gibrat the lognormal distribution has been used to study the distribution (concentration) of other economic variables like personal income and consumption (Cramer 1969).

THE LOGNORMAL SIZE DISTRIBUTION OF FIRMS IN MEXICO

There are at least two ways to approach the problem of whether the observed size distribution of firms in the industrial sector of Mexico is lognormal: (a) to apply one of the statistical methods devised to test for lognormality of a distribution and/or (b) to verify whether the law of proportionate effect applies to the observed growth distributions of different size firms. In this section we follow (a) and in the next one we test whether the law of proportionate effect is in operation and explore the implication of our findings.

There is a quick and fairly accurate graphical method (Aitchison and Brown 1963, Cramer 1969) to test whether a given distribution conforms to the lognormal distribution. The form of the data required for the application of this method is a group cumulative frequency table like Tables 1 and 2, which group firms according to the number of employees and according to the value of output. If, when we plot the upper value of each class on the logarithmic scale of a lognormal probability paper and the cumulative frequency proportions on the probability scale the resulting curve is nearly a straight line, then the distribution in question is approximately lognormal. Moreover, the parameters μ and σ that completely define the density function may be estimated from the straight line fitted by eye to the resulting points.

Figures 3 and 4 show that when firm size is measured either by the value of output or by the number of employees, the 1970 size distribution of industrial firms in Mexico conforms with a lognormal distribution. Table 3 shows the estimated values of the parameters of the lognormal size distributions of firms according to output and employment. ¹

¹ Once a straight line has been fitted to the data plotted on a lognormal probability paper we can estimate the parameters μ and σ . We first read off the values of firm size that correspond to the 16%, 50%, and 84% percentage points. Since these quantities are equal to

$$Q_{16\%} = e^{\mu - \sigma}$$

$$Q_{50\%} = e^{\mu}$$

$$Q_{84\%} = e^{\mu + \sigma}$$

and

$$\mu = \log Q_{50\%}$$

$$\sigma = \log \left\{ \frac{1}{2} \left(\frac{Q_{50\%}}{Q_{16\%}} + \frac{Q_{84\%}}{Q_{50\%}} \right) \right\}$$

we obtain in this way the estimators of the parameters μ and σ .

FIGURE 3
Size Distribution of Industrial Firms in Mexico in 1970 using the value of output
as a Measure of Firm Size. Logarithmic Probability Graph

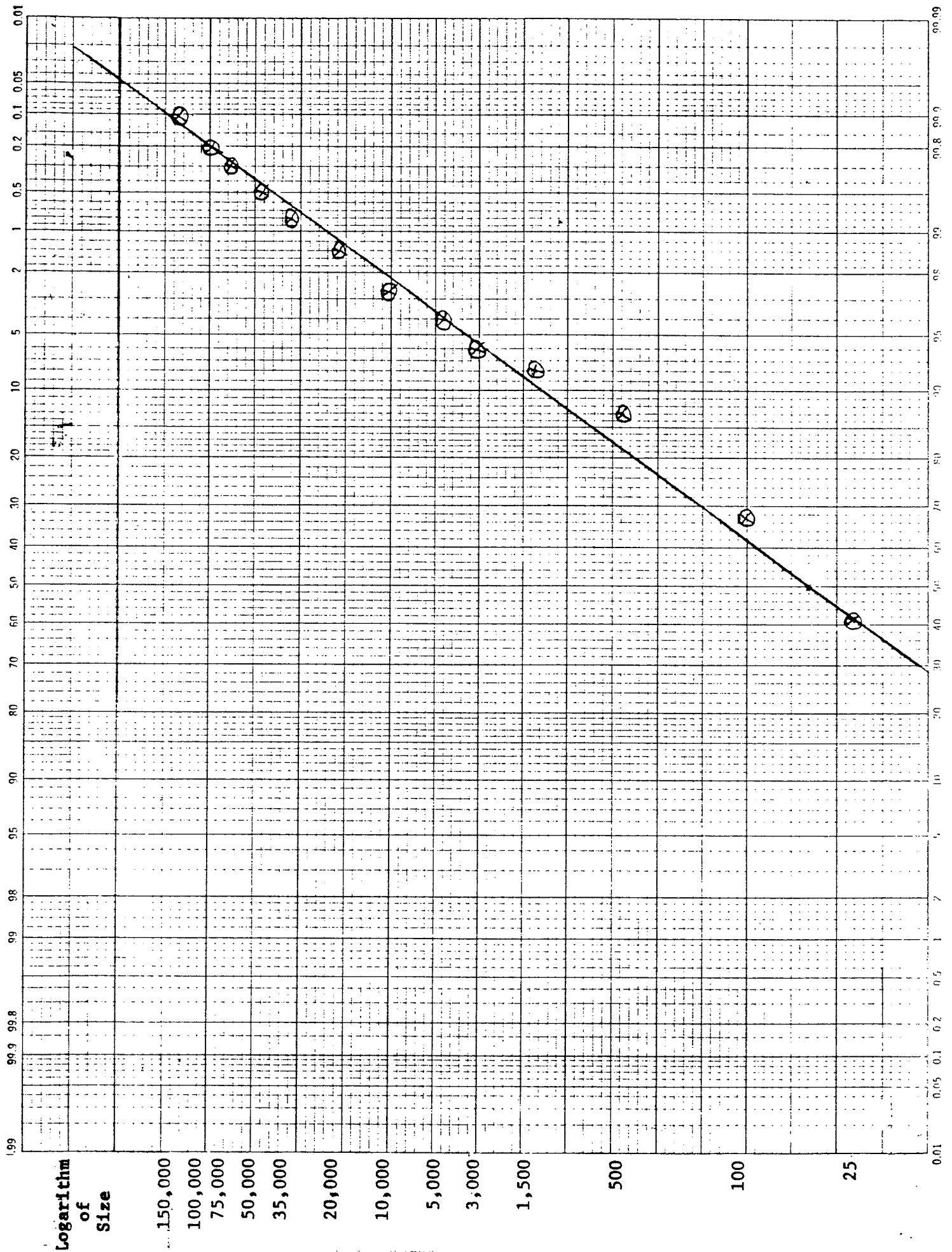


FIGURE 4 Size Distribution of Industrial Firms in Mexico in 1970 Using the Number of Employees as a Measure of Firm Size, Logarithmic Probability Graph

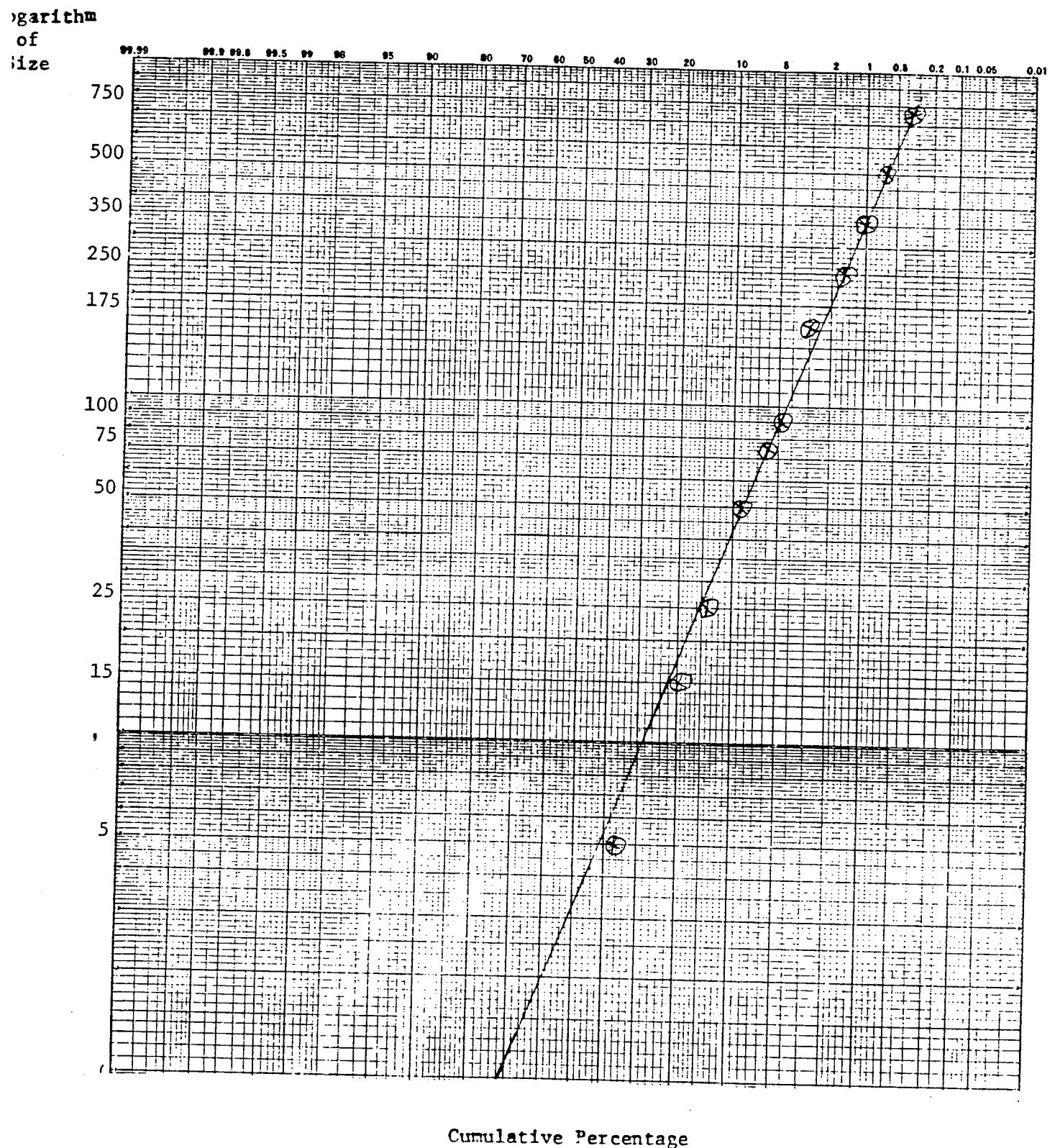


TABLE 3

Estimates of μ and σ of the Lognormal Size Distributions
According to the Number of Employees and
the Value of Output
1965-1970

Parameter	Size Distribution in Terms of			
	Employees		Output	
	1965	1970	1965	1970
Mean (m)	0.20	0.51	1.20	1.65
Standard Deviation (s)	1.03	0.90	1.28	1.15

Between 1965 and 1970 the means of these distributions have increased and their variances have decreased, suggesting that a reduction of industrial concentration took place during this period (see Table 3).

Our graphical analysis show that the highly skewed size distribution of firms in the industrial sector of Mexico fits a lognormal distribution reasonably well. Before we explore the economic implications of the above findings, we will discuss the general economic implications of the law of proportionate effect and investigate whether the observed lognormal size distribution of firms may have been generated by this stochastic process.

ECONOMIC IMPLICATIONS OF THE LAW OF PROPORTIONAL EFFECT FOR THE SIZE DISTRIBUTION OF INDUSTRIAL FIRMS

As stated above, the lognormal distribution results from the action of a large number of small random factors acting independently upon a variable in a multiplicative manner. If the variable is the firm size, the effect of event i on a firm of size x would change it to $\epsilon_i x$, but would change a firm of size z to $\epsilon_i z$. This implies that while a large firm may have a better chance of increasing its size by a given absolute amount, the chance of a given proportionate increase is the same for all firms of all sizes. This is the law of proportionate growth also known as the law of proportionate effect or Gibrat's law.

Furthermore, it can be shown that if the growth of firms is determined by the law of proportionate effect, the distribution of firms by size would tend towards a lognormal distribution (Steindl 1965).

The following are some of the economic implications of the law of proportionate effect when applied to the growth process of firms.

1. Large, medium and small firms have the same average proportionate growth, and there is no optimum size of firm.

Some economists (Marris and Wood 1971) argue that in a modern corporation, characterized by a divorce of ownership from control, managers are more interested in maximizing the rate of growth of firms and less in maximizing the rate of profits. It is also suggested that there is no limit to the absolute size of the firm, but there is a limit to its growth rate per unit of time. Within this framework one is likely to observe a positive relationship between size and growth of firms, since, as the firm increases its size, its control will be in the hands of managers mainly interested in maximizing its growth rate. Small firms, on the other hand, are more likely to be controlled by their owners, who are less interested in growth for growth's sake and more interested in maximizing profits.

The law of proportionate growth conforms to the above description of the growth of firms in that there is not an optimum size of firm, but differs in that there is a positive relation between firm size and firm growth.

2. The dispersion of growth rates around the common average rate of growth (see first implication) is the same for large, medium and small firms.

This implication of the law of proportionate growth is more in agreement with the assumption that the long run average cost curves are horizontal beyond some minimum optimum size (Simon and Bonini 1958) than with the assumption that the unit cost curves are U-shaped. That is, if constant returns to scale beyond some minimum size exist, one may expect that firms in each size class beyond the minimum will have the same chance, on the average, of increasing or decreasing in size in proportion to their present size.

3. The longer the law of proportionate growth is in operation on a variable the larger will be the variance of its lognormal distribution, i.e., the skewness (concentration) of the distribution will be more pronounced (Aitchison and Brown 1963).

When studying the size distribution of income, this implication predicts that its inequality (as measured by the σ parameter) must continually increase through time. Kalecki (1945) has suggested some modifications of the law of proportionate growth in order to be able to explain, using this law, the contrary evidence that in the advanced industrialized countries the inequality of income has not increased, or has increased very little. Kalecki modifies some assumptions of the law of proportionate effect to show that variations in the inequality of incomes are, up to a great extent, determined by well defined economic forces. In other words, there are, according

to Kalecki, detectable changes in σ which are partly systematic and partly induced by random events.

In our study of the distribution of firms by size, the law of proportionate effect predicts that the degree of industrial concentration will increase through time.

4. The distribution of proportionate growth rates is also lognormal.

This implication of the law of proportionate effect tells us that a small rate of growth has the same probability of increasing by a certain proportion than a higher rate of growth.

THE EMPIRICAL TEST OF THE LAW OF PROPORTIONATE EFFECT

If the size of the i firm at time t is S_{it} and at time $t-1$ is $S_{i,t-1}$ the law of proportionate effect states that

$$S_{it} - S_{i,t-1} = \epsilon_{it} S_{i,t-1}$$

$$\frac{S_{it} - S_{i,t-1}}{S_{i,t-1}} = \epsilon_{it}$$

where ϵ_{it} is a random variable distributed independently of $S_{i,t-1}$.

One way of testing the law of proportionate growth is to classify firms by their initial size $S_{i,t-1}$ and then compare the means and the standard deviations of the distributions of the growth rates $\left(\frac{S_{it} - S_{i,t-1}}{S_{i,t-1}}\right)$ of firms in different size classes. If the law of proportionate effect holds, each class should have the same growth rate distribution.

The four digit level data of the 1965 and 1970 Industrial Census was used to classify firms by size in terms of the number of employees and of the value of output. The rate of growth for each class and in each industry was measured by the percentage change of employment or output between 1965 and 1970. We assume this rate of change to be the growth rate of the average firm in that class. We then proceeded to estimate the mean and the standard deviation of the growth rate distributions for each class across the entire industrial sector (208 industries). Tables 4 and 5 show these estimates.

Columns (4) and (5) of Table 4 suggest that there is no visible systematic relationship between size of firm measured by the number of employees (employment size of firms) and rates of growth, nor between employment size of firms and the standard deviation of growth rates. On the other hand, when value of output is used to measure the size of firm (output size of firm) we observe (Table 5) that small firms tend to have smaller rates of growth and larger standard deviations than large firms.

Before reaching a more definite conclusion we must statistically test whether the observed differences in the means and the standard deviation of the growth rate distribution of different classes are significantly different.

An analysis of variance was used to test the hypothesis that the mean growth rates of firms in different classes, 1 through 8, are not significantly different. That is, we tested the hypothesis that $\mu_1 = \mu_2 \dots = \mu_8$.

When the number of employees is used as a measure of firm size the estimated F value does not exceed the critical F theoretical value at 5% or 1% level of significance. Hence we accept the hypothesis that

TABLE 4

Means and Variances of the Growth Rate Distributions
of Firms of Different Sizes, According to the Number of
Employees
1965-1970

Upper Limits of Size Classes (Number of Employees) (1)	Number of Firms (2)	Percentage of Firms (3)	Growth Rate (4)	Standard Deviation (5)	Variance
1 ≤ 5	39222	62.86	0.70	9.39	88.24
2 15	10959	17.56	1.30	9.22	85.00
3 25	3329	5.34	1.80	8.86	78.52
4 50	3532	5.66	1.91	8.99	80.90
5 75	1582	2.54	1.74	9.89	97.88
6 100	910	1.46	0.50	11.31	127.97
7 250	1796	2.88	1.66	10.59	112.19
8 >250	1065	1.70	5.26	8.08	65.35
TOTAL	62395	100.00			

TABLE 5

Means and Variances of the Growth Rate Distributions
of Firms of Different Sizes According to the Value
of Output
1965-1970

Mean value of size classes (thousands of pesos) (1)	Number of Firms (2)	Percentage of Firms (3)	Growth Rate (4)	Standard Deviation (5)	Variance
1 ≤ 125	39222	62.86	7.45	14.23	202.68
2 650	10959	17.56	6.31	12.88	166.07
3 2000	3329	5.34	9.24	10.89	118.74
4 4000	3532	5.66	7.15	10.85	117.77
5 8000	1582	2.54	8.68	13.47	181.61
6 12000	910	1.46	7.54	13.10	171.60
7 24000	1796	2.88	11.15	8.17	66.80
8 100000	1065	1.70	12.88	7.92	62.86
TOTAL	62395	100.00			

there is no significant difference among the mean growth rates of firms of different employment sizes. Employment size of firms and their growth rates do not have a significant degree of association.

On the other hand, when value of output is used as a measure of size of firm the estimated F values exceed the critical F value at 5% level of significance. We therefore reject the hypothesis that the output growth rates are the same in all classes. The output firm size and its growth rate show a significant positive association: the greater the size of firm, the larger its expected growth rate.

Next, we tested whether the variances of the growth rate distributions of different classes were significantly different. That is, our hypothesis was that $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \dots \sigma_8^2$. The statistical test we used is known as the Bartlett Test for homogeneity of variances.¹ The test consists of estimating χ^2 values with the estimated

¹As stated by Hoel (1963) this test amounts to creating with the estimated variances a new variable which has a χ^2 distribution. The variable in question is:

$$\lambda = \frac{-2 \cdot (\ln \mu)}{1 + \frac{1}{3(k-1)} \left(\sum_{i=1}^k \frac{1}{n_i - 1} - \frac{1}{n - k} \right)}$$

where

$$\mu = \frac{\prod_{i=1}^k \left(\frac{n_i S_i^2}{n_i - 1} \right)^{\frac{n_i - 1}{2}}}{\left(\frac{\sum_{i=1}^k n_i S_i^2}{\sum (n_i - 1)} \right)^{\sum \frac{n_i - 1}{2}}}$$

and S_i^2 are the estimated variances; k is the number of independent normally distributed variables (in our case 8 independent growth rate distributions), and n is the number of observations in each distribution.

variances and then comparing them with the critical values of χ^2 , corrected for degrees of freedom.

When the number of employees is used as a measure of the size of firms, the estimated χ^2 does not exceed the critical value of χ^2 at the 5% significance level. This result indicates that the hypothesis of homogeneity of variances of employment size growth rate distributions is a reasonable one. In other words, our results suggest there is no statistically significant association between the employment firm size and the variance of its growth rates.

On the other hand, when value of output is used as a measure of firm size, the estimated χ^2 value exceeds the critical χ^2 value at the 5% level of significance. We therefore reject the hypothesis that there is no significant difference among the variances of the output growth rate distributions of different firm sizes. Our results suggest that when size is measured in terms of value of output, one may anticipate a larger growth rate variance for small firms than for the large ones, (see column 5, Table 5).

The results of our test may be summarized as follows:

1. When number of employees is used as a measure of firm size, there is no relationship between size and proportionate growth rates nor between size and the dispersion (variance) of growth rates.
2. When value of output is used as a measure of firms size, there is a systematic and significantly positive relationship between size and proportionate growth rates, and an inverse

relation between size and the dispersion (variance) of growth rates.

As to the validity of the law of proportionate effect we may conclude that:

- (a) When firm size is measured by the number of employees, our results are consistent with the two most important implications of the law of proportionate effect, i.e. that the expected proportionate growth rate is the same for large and small firms, and that the dispersion of growth rates is also the same. These results are, however, contrary to what one would expect of the employment behavior of small and large industrial firms in Mexico. Since small firms in Mexico tend to use a more labor intensive technology than do large firms (Gollas 1978), it is not unreasonable to expect larger proportionate increases or decreases in their level of employment, as well as a greater variability in those changes. On the other hand, because the influence of labor unions is stronger and the enforcement of labor laws more effective in large firms, one also expects that the dispersion of employment size growth rates would be smaller in large firms. Our results, however, indicate that during 1965-1970 small size firms did not, proportionately, increase their employment more than large firms, nor was their employment rate more variable. These results question the importance that the small firms are claimed to have had in providing industrial employment during 1965-1970.

(b) When size of firm is measured by the value of output, our results do not conform with the predictions of the law of proportionate effect: we found that the average proportionate growth rates tend to be high for large firms, and there is a systematic pattern for the small firms to exhibit larger growth rate variability than the large ones. These results, although inconsistent with the law of proportionate effect, are more in agreement with the following economic arguments. Large industrial firms in Mexico are usually in a better position than small firms to take advantage of government policies that stimulate growth: credit for the acquisition of capital, fiscal preferential treatment, facilities to import capital, and technical assistance programs. This being the case, it is reasonable not to expect an equi-proportionate rate of output growth for large and small firms. Moreover, large firms are usually able to offset unfavorable market conditions since they are capable of a more diversified production. Because of these advantages, one also expects less variability in the output growth of large firms than in small ones.

In summary, our results bring evidence to support the view that the law of proportionate effect is in operation when employment is the variable measuring the rate of growth of firms; but the same law fails to hold when output is the variable that measures the growth of firms.

The other implications of the law of proportionate effect - that the longer this law is in operation the larger would be the degree of industrial concentration, and that the distribution of proportionate growth

rates is also lognormal - are not verified by our results. The estimated variances of the lognormal size distributions of firms by employment and by value of output (see Table 3) show a smaller value of σ for 1970 than for 1965: this result means that the size distribution of firms in Mexico was less concentrated in 1970 than in 1965. On the other hand, the graphic test of lognormality of the distributions of employment and output growth rates did not show a lognormal distribution pattern.

IMPLICATIONS OF OUR RESULTS FOR THE SHAPE OF COST CURVES

In this section we explore the implications of our findings for the shape of a given firm's long run average cost curve in the industrial sector of Mexico between 1965 and 1970.

In particular, we analyze the consequences to cost curves of our findings that output size growth rates increase with the size of firm and that the variability of those growth rates decreases as the firm becomes larger. Our findings indicate (see Table 2 and the statistical test of the preceeding section) that the average growth rate of small firms (μ_s) is significantly lower than the average growth rate of large firms (μ_L). We also found that the variances of the growth rate distributions of small firms (σ_s) are larger than the variances of the growth rate distributions of large firms (σ_L).

We now analyze whether these empirical findings are, or are not, compatible with several alternative shapes of the long run average cost curve.

a. Increasing Unit Costs (decreasing returns to scale).

If we suppose that diseconomies of scale exist, then further expansion of the firm will result, ceteris paribus, in higher unit costs, in a reduction in profits and, probably, in large variations in the firm's growth rate. Our empirical findings, however, indicate the contrary: large firms grow faster than small ones and their growth rate variation is smaller. We therefore reject the possibility that the long run average cost curve in the industrial sector of Mexico between 1965 and 1970 exhibited decreasing returns to scale. The headings of columns 2 and 3 of Table 6 show the relations we empirically found to exist between the growth rates of small and large firms and also the relationship between their variances. In Table 6 we indicate that the empirically observed relations between the growth rates and the variances of different firm sizes are inconsistent with decreasing returns to scale cost curves, (see first row columns 2, 3 and 4 of Table 6).

b. Constant Unitary Costs (constant returns to scale).

Let us now suppose that constant unit costs exist within a wide range of very small and very large industrial firms. Given this equality of unitary costs between small and large firms, there is no obvious reason for large firms to grow more slowly or faster than small ones. Our findings (that large firms grow faster than small ones) are, therefore, not incompatible with this type of cost curve, although it is difficult to find reasons that explain this behavior. We may accept then, as a possibility, that large firms grow faster than small ones despite equal unitary costs, (see column 5, Table 5).

However, our findings also indicate that the variances of the growth rates of large firms are smaller than the variances of growth rates

TABLE 6

Growth Rates of Different Firm Sizes, Their Variances
and the Shape of the Long Run Average Cost Curve a/
1965-1970

Assumption About the Shape of the Long Run Cost Curve	Empirical Results		Shape of Long Run Average Cost Curve
	$\mu_s < \mu_L$	$\sigma_s^2 > \sigma_L^2$	
Increasing costs	Inconsistent	Inconsistent	Not possible
Constant costs	Consistent	Inconsistent	Not possible
Decreasing costs	Consistent	Consistent	Possible

a/ μ_s, σ_s^2 refer to the mean and the variance of the small firms and μ_L, σ_L^2 to the same parameters for large firms.

of the small firms: large firms show less variability in their growth rate than the small ones. The usual explanation of the small dispersion of large firms' growth rates is as follows. The output of large firms is usually more diversified and they operate in several markets. Hence, the adverse growth rate in one market may be offset by a good performance in another. However, due to our assumption of constant unitary costs there is no reason why the very small and the very large firms should not have the same capacity to diversify their production, and therefore have fairly similar growth rate distribution variances.

Our results, however, indicate that small firms have larger variances than the large firms, this finding being inconsistent with the assumption of constant unit cost, (See second row, column 3 of Table 6).

To summarize; although our finding, that large firms have larger growth rates than small ones have, is compatible with the assumption of constant unit costs, we showed that unequal growth rate variances are incompatible with this assumption. Therefore, we exclude the possibility that the long run average cost curve in the industrial sector in Mexico between 1965 and 1970 exhibited constant returns to scale for large and small firms, (See second row, column 4 of Table 6).

c. Decreasing Unitary Costs (increasing returns to scale).

Finally, let us suppose that there are continuously declining unit costs for small and large firms; given this assumption let us see whether our findings, that small firms have lower rates of growth and higher growth rate variances than large firms do, are compatible with this assumption.

By definition, along the assumed continuously declining average cost curve there are small high cost firms and large low cost firms. Among the small high cost firms we may distinguish the following factors affecting their growth. First, it is probable that small high cost firms would have lower profit rates, their survival rate would be lower and/or they would grow more slowly than large firms. But, simultaneously, it could be that small high cost firms would have stronger incentives to grow than large firms since they realize larger economies of scale on a given proportional size increase. The final outcome, as to whether small or large firms will have the largest rate of growth, depends upon which of these tendencies dominates. If the small firms' high cost considerations are stronger than the economies of scale incentives to expand, then small firms would grow at a slower rate than large firms. The opposite occurs if the expansion incentives prevail over the high cost restrictions. The important conclusion to be underlined is that both of these outcomes are compatible with a situation of a declining (increasing returns to scale) long run average cost curve.

Our findings that the small firms in the Mexican industrial sector have a lower growth rate than the large ones is, therefore, consistent with the existence of a continuously declining long run average cost curve, (see third row, column 2 of Table 6). Our finding that large firms grow faster than small ones is more easily understood when one examines the preferential treatment that large firms have had in Mexico's industrial development programs.

We now examine whether our finding that there are significant differences between the variances of the growth rates of small and large firms (the small firms having a larger variance than the large ones) is

compatible with the existence of a continuously declining long run average cost curve. As explained in (b) above, large firms tend to have a more diversified output and to operate in more markets, and this makes it possible for them to offset the negative effects on their growth rate of their miscalculations and of the fluctuations in the prices of output and the factors of production. Small firms, on the other hand, do not generally have the financial and managerial resources to produce a more varied output, are less protected by government industrial policies, and are more directly affected by changing product and factor market conditions. The combined action of these factors determines highly variable growth rate of small firms.

Our findings that large firms tend to have larger and less variable growth rates than small firms is, therefore, consistent with a continuously declining long run average cost curve, (see third row, columns 3 and 4 of Table 6).

To recapitulate the discussion of this section: We argue that our empirical findings concerning different size firms' growth rates, and the variance of those growth rates, are compatible with a continuously declining long run average cost curve. That is, the findings that large firms tend to grow faster than small ones, and that the growth rates of the latter are more variable than those of the former, strongly suggest the existence of a continuously declining (increasing returns to scale) long run average cost curve in the industrial sector of Mexico. Our analysis does not, of course, say anything about the degree of economies of scale (the steepness of the slope of the cost curve) only that there are economies of scale over a wide range of firm sizes. We reject as inconsistent with our findings the alternative situations of a constant or of a decreasing returns to scale long run average cost curve.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The following are some tentative conclusions from our findings.

1. The highly skewed employment and output size distribution of industrial firms in Mexico in 1965 and 1970 fits the lognormal distribution reasonably well. (see Figures 3 and 4)
2. The stochastic process, known as the law of proportionate effect, which may have generated these highly skewed lognormal distributions is rejected when size is measured by value of output and verified when size is measured by the firm's number of employees. We found, contrary to what the law of proportionate effect predicts, that the output size of firm has a significant positive effect upon its expected growth rate and a negative effect upon the dispersion of these growth rates. In other words, the output growth rates of large firms tend to be higher and have less variability than the growth rates of small firms. It is possible then, that a stochastic process, different than the law of proportionate effect, is at work, and/or that some other economic factors need to be taken into account for explaining the highly skew output size distribution of Mexico's industrial firms.

On the other hand, as predicted by the law of proportionate effect, our results suggest that there is no systematic relationship between the employment size of firms, their growth rates, and the variability of these growth rates. That is, we did not find significant differences in the proportional employment growth rates of

large and small firms. This evidence does not support the argument that small firms have proportionately given in the past, or that *ceteris paribus*, will give in the future, more industrial employment than large firms. If, as our findings indicate, the law of proportionate effect is indeed the stochastic process that generates the observed lognormal employment size distribution of industrial firms, then we must accept the implication that the observed differences in employment growth rates of large and small firms are the result of chance and not of any basic difference in their production processes.

3. Contrary to predictions of the law of proportionate effect, the 1965 and 1970 variances of the lognormal employment and output size distributions of firms suggest a slowly decreasing employment and output concentration in the industrial sector as a whole (see Table 3).
4. As indicated above, our findings show that large firms tend to experience high and less variable output growth rates than small firms, and, because of this evidence, we reject the law of proportionate effect that predicts equal growth rates and variances for large and small firms. Furthermore, we argue that the empirical evidence of increasing growth rates and declining variances as firms increase their size, is inconsistent with either constant or increasing unit costs. The fact that growth rates increase and that dispersion declines with an increase in firm size, is compatible with, and strongly suggests, a

continuously falling (increasing returns to scale) long run average cost curve for large and small industrial firms in Mexico. In other words, the assumption that there exists a continuously declining (increasing returns to scale) long run average cost curve, makes it possible to explain the growth behavior of small and large firms, as well as to predict the observed highly skew size distribution of firms in the industrial sector of Mexico. It would not have been possible, on the other hand, to explain the observed growth pattern of small and large firms nor the highly skew size distribution of firms had we assumed a constant or decreasing returns to scale cost curve.

5. Our findings of a positive relationship between size and growth (see 2 above) bring evidence that supports the view (Marris and Wood 1971) that because large firms tend to be managerially dominated it is more likely that they will be interested in maximizing, above all, their rate of growth. In other words, our results do not support the view (Penrose 1959) that because growth requires a number of special conditions (new or surplus resources, management awareness, willingness to accept risks, planning, etc.) it is subject to some upper limit over a period of time. Our findings do not suggest that for large firms the stimulus for growth, created by the advantages of economies of scale, was offset during 1965-1970 by the negative effects of the factors suggested by Penrose. The "Penrose effect", if it existed, was not strong

enough during that period to exert a decisive influence in a direction opposite to that of economies of scale thereby slowing down the growth rate of the largest firms.

Finally, we now indicate some areas we think are important for further research. The lognormal is only one of many theoretical distributions which may be used to explain the growth of firms. Some other theoretical distributions may better describe and explain the observed size distribution and the growth of firms. For example, it has been argued (Simon 1955, Simon and Bonini 1958, Ijiri and Simon 1964) that the lognormal distribution is only a special case of the Yule distribution. The Yule distribution is generated, as is the lognormal distribution, by the stochastic process of the law of proportionate effect. However, the generation of the Yule distribution also assumes a constant birth rate of new firms in the classes of small size firms. The application of the Yule distribution which assumes the law of proportionate effect and some assumption about the birth of firms, may provide useful insight for the understanding of the growth process of the industrial firms in Mexico.

The models which make use of the assumption that a large number of small independent disturbances acting in a random and multiplicative fashion (law of proportionate effect), can explain the observed size distribution of firms, should explicitly incorporate economic assumptions (Kalecki 1945, Steindl 1965) so as to better understand the firm's growth process. In other words, with the help of stochastic models that incorporate some aspect of the law of proportionate effect as well as economic assumptions, we may be able to distinguish the changes in the size of

firms which are partly systematic from those which are induced by random events. Moreover, the use of stochastic models should only be complementary to other tools of economic analysis in order to avoid a situation where only a small proportion of the growth of firms is explained by economic considerations of price changes, managerial ability, technique of production, and government policies. Something must be left for economic theory to explain!

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