ECONOMIC GROWTH CENTER

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Box 1987, Yale Station
New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 313

AN ECONOMIC APPROACH TO BIRTH-ORDER EFFECTS

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May 1979

Notes: The research for this paper was supported in part by the Ford and Rockefeller Foundations, Population and Development Policy Research Program, and in part by the National Institute of Child Health and Human Development, Population and Labor Training Program.

1. Introduction

A simplifying assumption of the "new home economics" fertility demand model is that parents produce the same quality level for each child, i.e. there are neither favorites nor Cinderellas.[1] Yet numerous studies indicate systematic differences in the apparent "quality" of children within families, according to their birth order. Regarding first-born children, there is a near-consensus: they have systematically higher IQ's than their younger siblings; they attend school longer and earn more than middle-born children. Some, but not all, studies report an advantage of last-born children; they stay in school longer and score higher on achievement tests than middle-borns.[2]

Do such findings contradict the model in which utility-maximizing parents jointly "plan" child quantity and quality, seeking to minimize variance in quality?[3] An explanation based on some genetic advantage

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[1] R. J. Willis, "A New Approach to the Economic Theory of Fertility Behavior," Journal of Political Economy, 81:2, Part 2 (March/April, 1973), S-14-S64; Gary S. Becker and H. Gregg Lewis, "Interaction Between Quantity and Quality in Children," JPE, 84 (August, 1976), S143-S162.

[2] Studies of birth-order effects are reviewed in the next section.

[3] The critical feature of the model of Willis and of Becker and Lewis is that of interaction between N and Q (numbers of children, quality of children) in the production of child services. The first-order conditions from their

of being first-born, e.g. to younger parents, could explain some of the findings, but it is virtually impossible to test, and a whole new choice problem arises regarding whether utility-maximizing parents would invest in children according to a rule of complementing genetic advantage or substituting for it.[4] In any event, having younger parents at birth could not explain the advantage of the last-born over middle children.

A more common explanation is that parents fail to see or plan for constraints on spending-per-child which will occur throughout the household's life cycle; first-borns and last-borns benefit from higher average levels of spending because they spend a higher proportion of childhood years in smaller families. This explanation is clearly inconsistent with a model in which parents jointly plan number of children and per-child investment; assuming capital markets permit saving and borrowing, parents could equalize spending on children across periods. Moreover, insofar as parental earnings increase throughout the childrearing years of the household life-cycle, any resource constraint represented by increasing family size could be offset by increased earnings, and in fact the last-born child should

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model include: $U_N = \lambda Q \Pi_C + \lambda p_N$ and $U_Q = \lambda N \Pi_C + \lambda p_Q$. They make clear that the shadow prices of N and Q are each affected by the quantity of the other chosen. However, the derivation of these first-order conditions, and the interactive term in each shadow price, depends on the assumption that parents provide equal inputs to "quality" for each child. Without this assumption, the inclusion of N in the shadow price of Q would make no sense. N in this context simply multiplies the shadow price Π_C by the number of children; the shadow price represents the average cost of increasing the quality of one child by one unit.

[4]Gary S. Becker and Nigel Tomes, "Child Endowments and the Quantity and Quality of Children," JPE, 84 (August, 1976), S143-S162.

be better off than the first-born. Differences across families in the extent of birth-order effects by income could reflect differential access to capital markets; but as long as birth-order effects persist even in families with presumably unlimited access to such markets, we have not explained away the phenomenon of systematic birth-order differences, at least not in a framework of joint planning by parents.

A third line of reasoning explains child quality differences within families in terms of the extent to which a child must share not only the financial resources, but also the time of parents, with siblings. This idea is inherent in the "confluence" theory developed by psychologists, according to which the ratio of all other family members' ages to the child's age influences positively child development.[5]

Is the strong assumption that all parents face imperfect capital markets, and thus financial constraints, necessary for the quality-quantity parent-planning model to hold up in the face of birth-order differences? An objective of the model developed below is to show that even given perfect capital markets, birth-order differences are likely in families in which parents are maximizing utility and seeking to minimize differences in quality amongst their children. The key to such differences is the time constraint parents face--time, unlike money, cannot be saved across periods. In fact, as will be shown, the existence of birth-order effects lends weight to the argument that time inputs are important in the childrearing process.

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[5] R. B. Zajonc, "Family Configuration and Intelligence," Science, 192 (16 April, 1976), 227-236; and R. B. Zajonc and G. B. Markus, "Birth Order and Intellectual Development," Psychological Review, 82 (April, 1975). The confluence theory is explained further, below.

Time can be traded for money in the market; thus mother's participation in the paid labor market is relevant. A notable prediction of the model is that birth-order effects are less likely among children of working mothers; this is because mothers spending time in the labor market outside home throughout the childrearing years can always make the necessary marginal shifts to keep the shadow value of their time of equal value in all periods. They can shift out of the labor market as the family grows (assuming total flexibility of working hours) and back in as older children leave the household.

If the mother does not work, the model indicates birth-order effects are likely as long as goods and mother's time are not easily substituted for each other in rearing children. It suggests that birth-order effects will only occur if childrearing is a sufficiently time-intensive process so that the time constraint parents face is binding. And it implies, given the existence of birth-order effects, limitations on the jointness of production of child quality; at the very least, it is clear that raising two children of given "quality" takes more time than raising one.

The next section is a review of the literature dealing with birth-order effects. A third section presents the model. In a final section, an empirical test of the model is described, and empirical estimates are presented.

2. Prior Literature

Differences in achievement according to order of birth have long been noted. In 1874 Sir Francis Galton, in his English Men of Science, suggested "academic primogeniture" as the reason for the large proportion of first-borns among the eminent men he studied.[6] In 1912, E. L. Clark suggested in his American Men of Letters: Their Nature and Nurture that first-borns' advantage might be due to a depletion of family resources by the arrival of later children; this so-called "economic" explanation was also proposed in a 1968 article in the American Journal of Sociology.[7] Along with the genetic "uterine fatigue" notion,[8] these were, however, generally mentioned as *ad hoc* explanations, rather than as testable hypotheses; indeed they were proposed explanations for a casually-observed but not carefully-measured phenomenon.

Until recently, formal studies of the birth-order question have been plagued with two difficulties. One has been sample selection. Typical groups for study have been college students and eminent scientists.[9] Such studies were based on samples selective in terms of the dependent variable, e.g. education attained. The procedure has been to compare the proportion of first-borns in the sample to

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[6] The Galton study is cited by William D. Altus, "Birth Order and Its Sequelae," Science, 151 (7 January, 1966), 44-49.

[7] Clark's book (Columbia University Press) is also cited by Altus. The 1968 article is that of Bert N. Adams and Miles T. Meidam, "Economics, Family Structure and College Attendance," AJS, 74 (November, 1968), 230-239.

[8] Referred to by Alan E. Bayer, "Birth Order and College Attendance," Journal of Marriage and the Family, 28 (November, 1966), 480-484.

[9] E.g. Stanley Schachter, ["Birth Order, Eminence and Higher Education," American Sociological Review, 28 (October, 1963), 757-767] who studied a small group of University of Minnesota students. Altus refers to analysis by Nichols (unpublished) of the scores of top finalists in the United States National Merit Scholarship Qualification Tests, and to his own data on students at the University of California, Berkeley and Santa Barbara campuses.

that in the population. But this procedure poses several problems. As one observer put it:

> Changes and fluctuations in the marriage rate, age at marriage, completed family size, age of mother at first and last births, spacing of children, age structure of the population and size of the population may all affect the proportion in a given ordinal position in a sample at any point in time.[10]

If all persons in a sample are of the same age, cohort changes in education will affect representation of persons of certain birth-order positions. For example, first-borns of any given age are likely to have younger parents than later-borns of the same age. If younger parents are on average better-educated than older parents, first-borns may be overrepresented in college classes, not because they are first-borns but because they have better-educated parents on average.[11] Similarly, if there is a current annual increase in the proportion of first-borns going to college, first-borns will be overrepresented among college students, <u>even if</u> within families there will be no differences across children, i.e. later children will also attend. And if <u>within</u> families, later-born children actually have a real advantage, but there is an annual increase in the proportion of all high-school graduates going to college, no differences among college students by birth-order will emerge.[12]

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[10] Bayer, p. 483.

[11] Adams and Meidam suggest younger parents are more likely to be white-collar rather than blue-collar workers (p. 238).

[12] Albert I. Hermalin, "Birth Order and College Attendance: A Comment," <u>Journal of Marriage and the Family</u>, 29 (August, 1967), 417-421.

With these types of samples, moreover, the control for family size is critical. As explained below, being first-born is highly correlated with having few siblings; thus an apparent advantage of first-borns may be merely due to the advantage children from small families may enjoy. Thus a second and related difficulty of early studies has been confinement of tests to two and three-way cross-tabulations. Attempts to control for family size (and for socio-economic class, on the grounds that it might be highly and negatively correlated with family size) were often restricted to two or three family size or socio-economic class groups.[13]

For these reasons, until the mid-1960s, the focus of studies was on whether differences by birth-order were a real phenomenon, and attention went primarily to the hypothesized advantage of first-borns. Differences were reported in some studies but not in others, and the question remained unresolved.[14]

More recently, however, the availability of much larger samples and the use of the computer to facilitate analysis of them, have permitted more careful tests. In particular, the large samples have improved the results even using simple cross tabulations, by allowing

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[13]Bayer; Ben Barger and Everette Hall, "The Interrelationships of Family Size and Socioeconomic Status for Parents of College Students," Journal of Marriage and the Family, 28 (May, 1966), 180-187.

[14]Studies reporting no significant relationship between ordinal position and attainment include Barger and Hall; Nichols (cited by Altus); and Altus' study of Santa Barbara college students. Schachter; Adams and Meidam; and Bayer among others report an advantage for first-borns. Bayer mentions a 1933 Handbook of Child Psychology article by Harold Jones (ed. Carl Murchison) in which Jones lists 100 studies of birth-order differences, and no consensus regarding their existence.

examination of birth-order differences within all possible sibship size groups. And in most of these large samples, the dependent variable has been a score on a test of some kind rather than educational attainment;[15] such scores may provide a more finely-tuned (if still imperfect) measure of child "quality."

Bayer examined birth-order differences among 45,000 United States high school students who took several achievement tests.[16] His was one of the early reports that last-borns, as well as first-borns, had an advantage over middle-borns. However, he included children from two-child families, so that all middle children were from larger families, and the family size control was imperfect.

Belmont and Marolla[17] examined 400,000 19-year-old Dutch persons who took a battery of tests; within all family size groups they found monotonically decreasing scores by order of birth.

Zajonc reviewed evidence from large samples of Dutch (the same data as in Belmont and Marolla's work), U.S., Scottish and French children. He attributed the lesser decline in scores with order of birth among the French and Scottish to greater spacing.[18] In those

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[15] Zajonc reports results of studies based on the Raven test (in the Netherlands); the National Merit Scholarship Qualification Test (in the U.S.); an IQ test (Gille) (in France); and the Stanford-Binet test (in Scotland) (p. 228).

[16] These data were collected in 1960 as part of the Project Talent study.

[17] Lillian Belmont and Francis A. Marolla, "Birth Order, Family Size and Intelligence," Science, 182 (14 December, 1973), 1096-1101.

[18] Zajonc, pp. 229-230.

groups, in fact, he reports a U-shaped relationship, i.e. middle-born children do least well.

These samples exposed contradictory results regarding any advantage for later-borns. But in a careful analysis of about 200,000 Israeli eighth-grade students, Davis, Cohan and Bashi[19] proposed a resolution of that question. They divided their sample between children of European and Oriental parents. For the former group they reported the standard result of decreasing scores with increasing order of birth. For the latter group, they found increasing scores for later-borns from families of four or more children. They rejected Zajonc's proposal that greater spacing explains a lesser advantage for early-borns, since birth intervals were probably smaller, not greater, in the larger Oriental families. They proposed instead that later-borns did better in the Oriental families because they had the benefit of help from older siblings; and that the relative value of this help increases the lower the education of parents.[20] Oriental parents were assumed to have lower average educational attainment than European parents. Their idea is of particular relevance for studies of birth-order effects using developing country data, such as **this one for Colombia.** In many developing countries, educational opportunities have been increasing rapidly (as is the case for Colombia),

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[19] Daniel J. Davis, Saul Cohan and Joseph Bashi, "Birth Order and Intellectual Development: The Confluence Model in the Light of Cross-Cultural Evidence," Science, 196 (24 June, 1977), 1470-1472.

[20] This idea might explain Altus' report that in Nichols' data on U.S. high school students who took the National Merit Scholarship Qualification test, statistically significant higher scores for first-borns showed up only within the group of top finalists. Across all students taking the test, no such difference was found. See footnote 14 above.

so that older children's education often exceeds that of their parents.

These analyses have established differences in achievement by order of birth as widespread, if of varying patterns. I know of no effort, however, to develop and test a theory of the determinants of such differences. As noted above, a popular ad hoc explanation is the "economic" one--that parents run out of resources with successive children; why parents would not borrow across periods to equalize spending on different children has not been considered. Social psychologists have proposed that first-borns do better because of greater "dependence" and orientation to "adult norms,"[21] but these ideas are difficult to test empirically.

A more parsimonious explanation is Zajonc's "confluence" theory.[22] Intellectual environment in the home is defined as the average of the absolute intellectual levels of all family members; the intellectual level of family members is simply measured by their age. Children's intellectual development is a function of the home "intellectual environment," and thus of the average age of family members. The average age of the family falls as more children are born, so early-born children are better off. This confluence theory can be reconciled with the reversal of effects when parents have low education, as in the Israeli data, if intellectual environment is defined in terms of average years of education of family members, instead of average age. The confluence theory does not refer explicitly to time inputs to children of different birth orders, though it can

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[21] C. Norman Alexander, Jr., "Ordinal Position and Social Mobility," Sociometry, 31 (September, 1968), 285-293.

[22] Zajonc, p. 227.

clearly be interpreted in terms of time inputs of parents (and siblings). In any event, no explicit test of the theory has been proposed.

In a study of family size and birth-order, Lindert comes closer to a formal test of a theory.[23] He hypothesizes that both financial resources and time determine differences among children.[24] He uses only predicted differences in time inputs in his empirical analysis, with the predicted differences being derived from time use data. Unfortunately, the analysis itself does not provide a test of differences specific to birth-order because of the particular procedure he follows. He measures the difference between first and middle-borns in families with six or more children; he then uses dummy variables to compare this difference to differences for children in groups which combine both other birth-order positions and other sibship sizes. He thus mixes birth-order and family size effects.[25]

In a paper concerned with differences across rather than within families in children's achievement, Hill and Stafford argue that most family background variables that are used to explain such differences, including family size, are actually no more than a reflection of differential parental time inputs to children.[26] Could time inputs

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[23]Lindert, Fertility and Scarcity in America, Ch. 6 and Appendix C. See also his earlier version of that chapter, "Family Inputs and Inequality among Children," University of Wisconsin Institute for Research on Poverty Discussion Paper (October, 1974).

[24]Lindert, Fertility and Scarcity in America, pp. 201-204.

[25]*Ibid.*, Table 6-3, p. 196.

[26]C. Russell Hill and Frank P. Stafford, "Family Background and Lifetime Earnings," paper presented at the Econometric Society meetings, San Francisco, 1974. See also Leibowitz, AER.

explain both differences within families by children's birth-order, and by implication, differences across families by family size? This question is the basis for the model **and the empirical tests which follow.**

3. A Birth-Order Model

The model is specified for mothers working outside the home and those not. For simplicity, it is assumed that the father devotes no time to the care of children (not a terribly strong assumption, based on time use data from household surveys).[27]

Substitutes for mother's care time (including servants, baby-sitters, relatives who help care for children) can be purchased from non-parents, but to raise a child, some input of mother's time is required in every period. Time inputs of older siblings may be important,[28] but are not explicitly modelled.

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[27] Robert E. Evenson and Elizabeth K. Quizon, "Time Allocation and Home Production in Philippine Rural Households," (paper presented at International Center for Research on Women workshop, Elkridge, Maryland, April 1978) report that fathers in a sample of rural Filipino households devote an average of 20 minutes per day to child care (Table 1, p. 4). Further, in their sample, fathers' child care time does not increase with increases in the number of children (Table 4, p. 12). Father's time in nonphysical care of children in a 1967 U.S. sample was .3 hours per day (Kathryn E. Walker, "The Potential for Measurement of Nonmarket Household Production with Time-Use Data," paper prepared for International Sociological Association IX World Congress of Sociology, Uppsala, Sweden, April 1978, p. 17). Gilbert G. Ghez and Gary S. Becker [The Allocation of Time and Goods Over the Life Cycle (New York: Columbia University Press, 1975)] find with U.S. data a slight increase in working hours of men with increases in family size (Table 31, pp. 98-99), presumably because of greater family needs and/or because the wife drops out of the labor force to increase her childrearing time. Such specialization implies men's child care time does not increase as family size increases.

[28] Davis, Cohan, and Bashi.

The father's wage and hours of work are assumed invariant with respect to number of children and the mother's work hours and wage.[29]

An important simplifying assumption of the working-mother version is that she can adjust her hours of work in the market at will across periods. The implication of relaxing this assumption is elaborated on below. Her wage is also assumed constant across periods, though an increasing wage can be built into the model, and the implication for birth-order effects is straightforward. Prices are constant across periods in both versions of the model. Genetic endowment of children is assumed not to vary in any way related to birth order. The model does not allow for joint production in the use of mother's time to raise children, nor does the childrearing productive efficiency of mothers increase with parity or time.

Working mother version

The model takes parents' number of children as given. Recall the model of the preceding chapter, in which parents maximized a utility function

$$U = U(N, Q, Z)$$

where N was number of children, Q average quality, and Z an index of other commodities. This model, in contrast, is conditional on

[29]Orley Ashenfelter and James Heckman, ["The Estimation of Income and Substitution Effects in a Model of Family Labor Supply," Econometrica, 42 (January, 1974), 73-86] find a zero elasticity of men's hours of work with respect to wife's wage (p. 74). Ghez and Becker, pp. 98-101, report that in the U.S. men with more children work more hours, but the increase in their hours is small.

N.[30] Parents maximize utility according to the function

$$U = U(\sum_i q_i, \; Vq_i, \; S)$$

where q_i refers to the quality of the ith child and i (i=1, . . . n) is the order of birth ($\partial U/\partial \sum_i q_i > 0$); Vq_i is the variance in quality among children ($\partial U/\partial Vq_i < 0$); and S represents the parents' standard of living ($\partial U/\partial S > 0$).

The mother produces quality in children according to the production function

$$q_{ijk} = \gamma_k f(t_{ij}, x_{ij}) \; ; \; \Sigma q_i = \sum_{j=1}^{m} q_{ij}$$

where t_{ij} is mother's time inputs to the ith child in the jth period (j=1, . . . m) and x_{ij} are purchased inputs to the ith child in the jth period, including goods and the time of persons other than the mother in child care which mothers purchase. γ_k (k=1, . . . r) is an efficiency parameter which declines with the age of the child such that the marginal product of time and goods is greater the younger the child. This is consistent with the findings of many studies of children's physiological and psychological development indicating the importance of the early years.[31] The efficiency parameter is not required to

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[30] A complete fertility model would explain N jointly with average Q and minimized variance in Q among children. See below, p. 86, for reference to an additional dimension, spacing of children.

[31] Alan Berg, The Nutrition Factor (Washington, D.C.: The Brookings Institution, 1973), discusses malnutrition in infants, the resultant loss of "learning time . . . during the most critical periods of learning" (p. 10) and the question of the reversibility of its damage (pp. 9-10 and references, p. 249).

generate some birth-order effects; in a family of three or more children, the model predicts that middle children are worse off than first and last-born without this "γ-factor." Without the γ-factor we would predict, however, no difference between first and last-born.

To simplify the exposition, in the case shown here a new child is born into the household in each time period, and the duration of time periods corresponds exactly to the duration of developmental phases, or γ-factors. Thus for this case spacing of births is fixed in relation to developmental phases. The actual relation between spacing and birth-order effects can be shown to be a function of the γ-factor, as is explained below.[32]

The production function for S is

$$S = f(x_{s1}, t_{s1}) + \dots f(x_{sm}, t_{sm}) \; ; \; \frac{\partial S}{\partial f} > 0 \; ; \; \frac{\partial^2 S}{\partial f^2} < 0$$

where x_s and t_s represent goods and time in each of the j periods.

Utility is maximized subject to these production functions and the following constraints, numbering m+1:

$$(1) \text{ to } (m): \quad t_j = \sum_i t_{ij} + t_{pj} + t_{sj}$$

and

$$m+1: \quad V = -\sum_j wt_{pj} + p_x x + p_s S$$

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[32]Also, for the case shown, there are enough time periods (j=1, . . . m), and there are not so many developmental periods, such that all children pass through all the developmental phases (k=1, . . . r). Thus the number of time periods equals or exceeds the number of children plus r-1 "γ-factors" ($m \geq n+r-1$); by the last period, the last child has completed the last developmental phase and older children have left the household in sequence. The model is thus outlined for "completed childrearing." In real life, of course, the number of time periods is limited only by the life expectancy of parents and the restriction that a period cannot be shorter in duration than 9 or 10 months. With 5 γ-factors and 10 children born 18 months apart,

where $\sum_i t_{ij}$ is total time devoted to the children present in the jth period; t_{pj} is time spent working in the "paid" labor market; t_{sj} is time spent producing S; V is husband's earned income plus household unearned income; w is the wage; and p_x and p_s are the prices of goods used in production of child quality and of S. Thus in each period, the mother is constrained by total available time in that period. The income constraint, on the other hand, is not period-specific; parents face no capital market imperfections, so that goods can be traded freely across periods.

Given the utility function, production functions and constraints, we wish to show

$$\sum_j q_{i=1} > \sum_j q_n > \sum_j q_{i=2\,.\,.\,.\,n-1}$$ [33]

i.e. that the first-born child receives more time and goods than the last-born and middle-born children, and the last-born more than the middle-born. The model is worked through here for the 3-child case, with 2 efficiency parameters and 4 periods. Appendix A outlines the model for the n-child, r-parameter, m-period case.

With n=3, m=4 and r=2 there are 26 first-order conditions, with the Lagrangean multipliers λ_1, λ_2, λ_3 and λ_4 representing the time constraint in each of the 4 periods, and λ_5 representing the income constraint:

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we thus require at least 14 18-month time periods (21 years) for parents to complete the childrearing process.

[33] It is also true that middle-born children (i=2, . . . n-1) can differ in total quality, depending on the number of γ-factors, their magnitude, and spacing. The relationship between spacing and γ-factors is discussed below.

(1) $\gamma_1 \frac{\partial f}{\partial t_{111}} \quad Uq_{111} = \lambda_1$; $\left[Uq_{ijk} = \frac{\partial U}{\partial \Sigma q_i} \frac{\partial \Sigma q_i}{\partial q_{ijk}} + \frac{\partial U}{\partial V q_i} \frac{\partial V q_i}{\partial q_{ijk}} \right]$

(2) $\gamma_2 \frac{\partial f}{\partial t_{122}} \quad Uq_{122} = \lambda_2$

(3) $\gamma_1 \frac{\partial f}{\partial t_{221}} \quad Uq_{221} = \lambda_2$

(4) $\gamma_2 \frac{\partial f}{\partial t_{232}} \quad Uq_{232} = \lambda_3$

(5) $\gamma_1 \frac{\partial f}{\partial t_{331}} \quad Uq_{331} = \lambda_3$

(6) $\gamma_2 \frac{\partial f}{\partial t_{342}} \quad Uq_{342} = \lambda_4$

(7) $\gamma_1 \frac{\partial f}{\partial x_{111}} \quad Uq_{111} = \lambda_5 p_x$

(8) $\gamma_2 \frac{\partial f}{\partial x_{122}} \quad Uq_{122} = \lambda_5 p_x$

(9) $\gamma_1 \frac{\partial f}{\partial x_{221}} \quad Uq_{221} = \lambda_5 p_x$

(10) $\gamma_2 \frac{\partial f}{\partial x_{232}} \quad Uq_{232} = \lambda_5 p_x$

(11) $\gamma_1 \frac{\partial f}{\partial x_{331}} \quad Uq_{331} = \lambda_5 p_x$

(12) $\gamma_2 \frac{\partial f}{\partial x_{342}} \quad Uq_{342} = \lambda_5 p_x$

(13) $\lambda_1 = \lambda_5 w$

(14) $\lambda_2 = \lambda_5 w$

(15) $\lambda_3 = \lambda_5 w$

(16) $\lambda_4 = \lambda_5 w$

(17) $Ut_{s1} = \lambda_1$; $\left[Ut_{sj} = \frac{\partial U}{\partial S} \frac{\partial S}{\partial t_{sj}} \right]$

(18) $Ut_{s2} = \lambda_2$

(19) $Ut_{s3} = \lambda_3$

(20) $Ut_{s4} = \lambda_4$

(21) $\frac{\partial U}{\partial S} \frac{\partial S}{\partial \sum_j X_{sj}} = \lambda_5 p_s$

(22) $t_{j=1} = t_{111} + t_{p1} + t_{s1}$

(23) $t_{j=2} = t_{122} + t_{221} + t_{p2} + t_{s2}$

(24) $t_{j=3} = t_{232} + t_{331} + t_{p3} + t_{s3}$

(25) $t_{j=4} = t_{342} + t_{p4} + t_{s4}$

(26) $V = -\sum_j wt_{pj} + p_x x + p_s S$.

The solution is straightforward. From (13), (14), (15) and (16)

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 w$$

i.e. the mother's marginal value of time is equated in every period to her wage rate. It follows that conditions (1) to (6) are equal to each other. Thus

$$\gamma_1 \frac{\partial f}{\partial t_{111}} = \gamma_2 \frac{\partial f}{\partial t_{122}} = \gamma_1 \frac{\partial f}{\partial t_{221}} = \gamma_2 \frac{\partial f}{\partial t_{232}} = \gamma_1 \frac{\partial f}{\partial t_{331}} = \gamma_2 \frac{\partial f}{\partial t_{342}} .$$

Also, conditions (7) to (12) are equal to each other, so that

$$\gamma_1 \frac{\partial f}{\partial x_{111}} = \gamma_2 \frac{\partial f}{\partial x_{122}} = \gamma_1 \frac{\partial f}{\partial x_{221}} = \gamma_2 \frac{\partial f}{\partial x_{232}} = \gamma_1 \frac{\partial f}{\partial x_{331}} = \gamma_2 \frac{\partial f}{\partial x_{342}} .$$

It follows that

$$\gamma_1 \frac{\partial f}{\partial t_{111}} + \gamma_2 \frac{\partial f}{\partial t_{122}} + \gamma_1 \frac{\partial f}{\partial x_{111}} + \gamma_2 \frac{\partial f}{\partial x_{122}} =$$

$$\gamma_1 \frac{\partial f}{\partial t_{221}} + \gamma_2 \frac{\partial f}{\partial t_{232}} + \gamma_1 \frac{\partial f}{\partial x_{221}} + \gamma_2 \frac{\partial f}{\partial x_{232}} =$$

$$\gamma_1 \frac{\partial f}{\partial t_{331}} + \gamma_2 \frac{\partial f}{\partial t_{342}} + \gamma_1 \frac{\partial f}{\partial x_{331}} + \gamma_2 \frac{\partial f}{\partial x_{342}}$$ [34]

and there are no birth order effects. Thus with the mother able to adjust her working hours so that the marginal value of her time in every period is the same as the marginal value of her wage, there are no birth-order effects. The time constraints do not affect child quality because the mother can "trade" market time for child care time as child care demands change across periods.

Non-working mother version

Non-working mothers have the same utility function and child quality production functions. They face m+1 constraints:

$$\text{(1) to (m):} \quad t_j = \sum_i t_{ij} + t_{sj}$$

and

$$V = p_x X + p_s S$$

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[34]This equality implies that

$t_{111} = t_{221} = t_{331} > t_{122} = t_{232} = t_{342}$, and

$x_{111} = x_{221} = x_{331} > x_{122} = x_{232} = x_{342}$.

Other combinations are possible, however. If $t_{111} > t_{221}$, then $x_{111} < x_{221}$ in an exactly compensating amount in production, and similarly with time and goods inputs to children in other periods.

For the 3-child, 4-period, 2-efficiency parameter case, first-order conditions (1)-(12) are the same as in the working mother model. Subsequent first-order conditions are:

(13) $Ut_{s1} = \lambda_1$; $\left[Ut_{sj} = \frac{\partial U}{\partial S} \frac{\partial S}{\partial t_{sj}} \right]$

(14) $Ut_{s2} = \lambda_2$

(15) $Ut_{s3} = \lambda_3$

(16) $Ut_{s4} = \lambda_4$

(17) $\frac{\partial U}{\partial S} \frac{\partial S}{\partial \sum_j X_{sj}} = \lambda_5 p_s$

(18) $t_{j=1} = t_{111} + t_{s1}$

(19) $t_{j=2} = t_{122} + t_{221} + t_{s2}$

(20) $t_{j=3} = t_{232} + t_{331} + t_{s3}$

(21) $t_{j=4} = t_{342} + t_{s4}$

(22) $V = p_x x + p_s S$

a. The case of perfect substitution

Only under unrealistic conditions will this model not predict the emergence of birth-order effects. One such unrealistic case is that of perfect substitution between time and goods inputs in the child quality production function.[35] With perfect substitution, first-order condition (1) is equal to (7), as are (2) and (8), (3) and (9), (4) and (10), (5) and (11) and (6) and (12). Since conditions (6) to (12) equal each other, it follows that conditions (1) to (6) equal each other. Thus $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$, i.e. the marginal value of time is the same in each period, even for the non-working mother; since she can substitute goods for time freely in apparently time-short periods 2 and 3, when both children are present, the time constraint built into the model becomes irrelevant.

b. The time-intensity of child quality production

Similarly, birth-order effects would not be predicted if inputs of (mother's) time were insignificant compared to x inputs in the production function. First-order conditions (1) through (6) can be written:

$$\gamma_k \frac{\partial f}{\partial t_{ijk}} \quad U q_{ijk} = \lambda_j \ ; \quad \lambda_j \geq 0 \ .$$

If child quality production is highly goods-intensive, the marginal product of time-inputs is rapidly driven to zero and λ_j equals zero, i.e. the time constraint is not binding. (Strictly speaking, this can only occur where the demand for mothers' labor in the market is zero; otherwise the mother would work outside the home. However, it

35 It is also possible that goods and time are complementary inputs. This is plausible if we include another factor in the production function, e.g. the child's innate ability. Complementarity between goods and time would enhance the advantage of first and last-borns.

is possible to imagine situations in which entry costs (e.g. additional education or training) are high enough or hours in the paid labor market inflexible enough, so that women stay home even when the marginal product of their time in producing S or q is zero. Thus women with teen-age children appear to have time on their hands; indeed it is precisely when children reach older ages that it is likely they are no longer time-intensive.[36])

c. The constant returns to scale assumption

A more plausible assumption is diseconomies of scale in production, whereby a reduction of inputs (to any one child) will reduce quality by proportionately less. Diseconomies in this sense are similar to an assumption of diminishing returns to additional inputs of time and goods to any one child. Imagine that the child's "endowment" (e.g. ability) were explicitly included in the production function; then diminishing returns to additional inputs would seem plausible. These diminishing returns could offset (and even reverse) the predicted advantage of the first and last child.

Increasing returns to scale in inputs, on the other hand, would increase the advantage of the first and last child.

Assuming there are diminishing returns, even imperfect substitution and time-intensity together do not guarantee birth-order effects. On the other hand, if there are increasing returns, we could expect

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[36]Gronau ["Leisure, Home Production and Work--The Theory of the Allocation of Time Revisited," National Bureau of Economic Research Working Paper no. 137 (1976), pp. 30-31] notes that the goods-intensive nature of children becomes more explicit as they grow older. See also his "The Effect of Children on the Housewife's Value of Time," Economics of the Family, ed. Theodore W. Schultz (Chicago: University of Chicago Press, 1974), especially pp. 472-486.

birth-order effects even given perfect substitution and time-intensity.

d. Joint production

Joint production of either S and q_i, or $q_{i=1}$ and $q_{i=2,...n}$ could also offset birth-order effects. If the mother can simultaneously "produce" quality in two children, or simultaneously produce child quality and other commodities, the difference between the first and second periods evaporates. Some element of joint production is not implausible, e.g. if when mothers read to two children, each derives the full benefit of her time, or if mothers combine child care with food preparation.

Here there is no confounding symmetry; joint production could eliminate the predicted advantage of first and last-borns, but would not enhance the situation of middle-borns.

On the production side birth-order effects are thus predicted as long as we accept the production assumptions of the standard (Willis; Becker and Lewis) model of fertility, i.e. the elasticity of substitution between t and x is less than infinite; production of child quality is time-intensive; there are constant returns to scale; and there is not joint production.

e. The utility function

However, even if on the production side, none of the conditions tending to mitigate birth-order effects obtained, sufficient distaste for variance among children in quality could lead parents to trade off higher average quality and/or a higher S to reduce variance and eliminate differences by birth-order. If there were no period-specific time constraints, or if production conditions made the time constraints not binding, parents could automatically minimize variance by simply maximizing average quality--given diminishing returns to parental inputs to any one child, and given that genetic endowment of children does not differ systematically by birth order. However, once the time constraint becomes binding, not only production conditions but also the nature of the utility function will affect the extent of differences among children. Parents, depending on their preferences, may choose differential quality investments, because of the time constraint.

f. One case predicting birth-order effects

An empirical finding of the existence of birth-order effects has this advantage: for families which we can show face no capital constraint, the persistence of birth-order effects implies that certain production conditions do not obtain. For example, if we bar the possibility of increasing returns (to inputs to any one child), then emergence of the predicted birth-order effects eliminates the joint possibility of perfect substitution, goods-intensity, joint production, and diminishing returns. A finding of differences by birth order does not allow us to distinguish among the different production conditions in terms of their relative importance; this would require much more detailed data on actual time and goods inputs to children over a considerable period (consider the difficulties of estimating production functions even for shoes or tractors). But the elimination of certain production possibilities is a finding in itself, particularly insofar as it points up the centrality of time use in childrearing.

To simplify exposition of how the model under certain conditions predicts birth-order differences among children of non-working mothers, a specific case is presented here. We assume constant returns to scale and no joint production. Child quality production is assumed to be sufficiently time-intensive so that the time constraint in each period is binding. Finally, substitution of goods for time is constrained in a certain way. First, note that from first-order conditions (1) and (13); (2), (3) and (14); (4), (5) and (15); and (6) and (16), the following equalities hold:

(a) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{s1}} = \gamma_1 \frac{\partial f}{\partial t_{111}} \cdot U t_{111} = \lambda_1$

(b) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{s2}} = \gamma_2 \frac{\partial f}{\partial t_{122}} \quad U t_{122} = \gamma_1 \frac{\partial f}{\partial t_{221}} \quad U t_{221} = \lambda_2$

(c) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{s3}} = \gamma_2 \frac{\partial f}{\partial t_{232}} \quad U t_{232} = \gamma_1 \frac{\partial f}{\partial t_{331}} \quad U t_{331} = \lambda_3$

(d) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{s4}} = \gamma_2 \frac{\partial f}{\partial t_{342}} \quad U t_{342} = \lambda_4$

For birth-order effects to emerge, we wish to show that $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$, i.e. that the marginal value of time is equal in the second and third time periods, when 2 children are present; and greater in those middle time periods than in the first time period; and greater in the first period, when the only child present is younger (and the marginal product of time greater) than in the fourth period, when the only child present is older. With $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$, it follows from (a) through (d) that

(e) $t_{s2} = t_{s3} < t_{s1} < t_{s4}$, so that

(f) $t_{122} + t_{221} = t_{232} + t_{331} > t_{111} > t_{342}$, and

(g) $t_{221} = t_{331} > t_{122} = t_{232}$.

To demonstrate that

$$t_{111} + t_{122} > t_{331} + t_{342} > t_{221} + t_{232}$$

requires, in addition to (f) and (g), that

$$\frac{t_{111}}{t_{342}} > \frac{t_{331}}{t_{122}} \quad \text{and} \quad t_{342} > t_{232}, \text{ i.e.}$$

that comparing the first and last children $\frac{t_{111}}{t_{342}} > \frac{t_{331}}{t_{122}}$, the advantage of the first in the periods when each is alone (already shown, see (f)) exceeds the advantage of the last, (g), in periods when they share time; and furthermore that at the same (older) age, the last child receives more time than the middle child.[37] The first is true if $\lambda_4 < \lambda_1$, and is shown in Appendix B. The second follows if $\lambda_4 < \lambda_3$, so that, from (c) and (d)

$$\gamma_2 \frac{\partial f}{\partial t_{342}} < \gamma_2 \frac{\partial f}{\partial t_{232}} .$$

However, it is only possible to show $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$ by specifying a limit on the degree of substitutability between goods and time inputs in the production of S and q_{ijk}. Note from first-order

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[37] The assumption is that ordering of inputs implies ordering of outputs.

conditions (7) through (12) that:

$$\text{(h)}\quad \gamma_1 \frac{\partial f}{\partial x_{111}} = \gamma_2 \frac{\partial f}{\partial x_{122}} = \gamma_1 \frac{\partial f}{\partial x_{221}} = \gamma_2 \frac{\partial f}{\partial x_{232}} = \gamma_1 \frac{\partial f}{\partial x_{331}} = \gamma_2 \frac{\partial f}{\partial x_{342}}$$

With $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$, inputs of time to the first and last child will be greater in the first and fourth period than are inputs of time to the middle child in the second and third period. For (h) to hold therefore requires that

$$x_{111} < x_{331} = x_{221}$$

$$x_{342} < x_{232} = x_{122}$$

and since $\gamma_1 > \gamma_2$

$$x_{342} < x_{232} = x_{122} < x_{111} < x_{331} = x_{221}$$

The restriction on substitutability is that the reductions of goods-inputs to the first and last child in the first and fourth periods do not raise the marginal product of time inputs to those children in those periods to the point where those time inputs would be reduced to the level the middle child receives in the second and third periods. In terms of conditions (a) through (d), this assures that if

$$\text{(i)}\quad \gamma_k \frac{\partial f}{\partial t_{ijk}} \quad Ut_{ijk} \lesseqgtr \gamma_k \frac{\partial f}{\partial t_{ijk}} \qquad \text{(where i or j changes)},$$

then $t_{ijk} \gtreqless t_{ijk}$ (where i or j changes). Similarly, in production of S, reductions of goods-inputs in the first and fourth periods cannot raise the marginal product of time-inputs to the point where the following condition does not hold:

$$(j) \quad \frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{sj}} \gtrless \frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{sj}} \qquad \text{(where j changes),}$$

then $t_{sj} \lessgtr t_{sj}$ (where j changes). In Appendix B, it is shown that given conditions (i) and (j), equalities (a) through (d) from the first-order conditions can only hold if $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$.

It can also be shown that for the first and last child, the goods to time ratio is greater in the middle periods than in the first and last periods; and that the goods to time ratio is greater for the first child in the first period than for the last child in the last period, so that some of the last child's relative time loss is made up in the last period. All these results follow because, though the overall goods inputs are constrained to be equal across children, parents optimally choose different ratios of time to goods in different periods, as the marginal value of mother's time changes across periods.

It is also clear from the model that differences between the production efficiency parameters, meant to reflect developmental differences throughout a child's years of growing up, will be reflected in birth-order effects. The greater is γ_1 relative to $\gamma_2 \ldots \gamma_r$, the greater is the advantage of the first-born. The greater γ_r relative to $\gamma_1 \ldots \gamma_{r-1}$, the lesser the disadvantage of the last-born.

The effect of spacing on birth-order effects depends on the number and relative magnitudes of developmental stages through which children pass. At two extremes, there would be no birth-order effects at all: twins (virtually zero spacing) and the situation when the number of years between the birth of two children exceeds the number of developmental periods, so that the second child is in the same position regarding receipts of mother's time as was the first. But in intermediate situations, there is no simple rule. On the one hand, greater spacing (e.g. between the first and second child) increases the time the first child has alone; on the other hand the lower the γ_k of the first-born at the time of the next birth the less the disadvantage of the second child, since the greater the difference between γ_k and γ_1, the greater the inputs to the new child in the critical first period.

There is one clear effect of spacing: for a given number of children, the greater the average spacing among children, the greater the children's average quality. But spacing, as mentioned above, is limited because the childbearing and childrearing years are limited. Spacing is also more limited the greater the number of desired children. Thus a complete model would explain fertility demand in three dimensions: the demand for a certain number of children, the demand for average quality, and the demand for minimum variance among children in quality; and would take into account that parents may make certain accommodations (such as spacing) as they trade off between numbers, quality, and birth-order differences.

The contrast between the working and non-working mother versions of the model indicates what would happen if the flexible-hours assumption of the working-mother version were relaxed, such that mothers could not adjust hours between periods. With the constraint that $t_{p1} = t_{p2} = t_{p3} = t_{p4}$, the results are identical to the (substitution-constrained) non-working version of the model. Mothers could offset birth-order effects by withdrawing from market work altogether in middle periods, but with t_{p1} constrained to equal t_{p4}, the first child retains an advantage over the last, because of the efficiency parameter. A more typical pattern might be for mothers to leave the labor market on the birth of the first child, and to return when all children are older, so that, for example, $t_{p1} = t_{p2} = 0$ and $t_{p3} = t_{p4} > 0$. In this case, the advantage of the first over the last child is accentuated, and early-born middle children will have an advantage over later-born children.

Similarly, the result of an increase in mother's wage rate over time (in the working-mother version) is clear. The marginal value of her time will increase with each period, and later-born children will receive successively less of her time, as $\lambda_m > \lambda_{m-1} \cdots > \lambda_1$. The greater the rate of increase in the wage and the greater the rate of decrease in the γ's, the worse off are later-born relative to middle-born children; their advantage over the middle group can even be reversed.

The fact that a steep age-earnings profile accentuates birth-order differences (in this case, the advantage of early-borns) suggests an explanation on the supply side for the tendency of women to work in occupations with flat age-earnings profiles, even given some loss in the present discounted value of lifetime earnings. Given diminishing returns in utility to income, the decrease in marginal utility due to an income loss associated with such occupations will be relatively smaller the greater the difference between the mother's potential lifetime income and her exogenous income (including in her exogenous income the income of her husband). In theory, then, we would expect the advantage of the last-born to be smaller the greater the ratio of potential lifetime income of the wife to that of her husband.

Women have traditionally opted for jobs with flexible hours,[38] presumably due to childrearing demands which fluctuate over time. Such fluctuations are due not only to changes in the number of children but to changes in the mix of developmental cycles which children go through. Thus it may be minimization of birth-order differences and maximization of the sum of child quality which leads women to seek flexible-hour occupations, and occupations in which temporary withdrawal from the labor market has low opportunity costs in terms of lost experience. If employers view all women as following such a maximization rule, employer-funded training of women will be limited and flexible-hour occupations will have flat earnings profiles. Thus supply and demand effects interact to lead women to such occupations.

4. Empirical Estimates of Birth-Order Effects

Hypotheses tested here, based on the predictions of the model, are:

1. That first-born children are better-off by some measure than later-born children;

2. That last-born children are better off than middle-born children, but somewhat worse off than first-born children, because the extra parental time they receive comes later in their development.

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[38] Jobs which are compatible with childrearing may serve the same purpose.

3. That birth-order effects are attenuated among children of working mothers.

4. That birth-order effects in families in which the mother does not work are not entirely due to parents' inability to equalize spending (for goods) across children, i.e. to imperfections in capital markets, but are at least in part due to the time constraint modelled.

Other testable predictions, e.g. that birth-order effects vary for working mothers as a function of the availability of flexible-hour jobs, and as a function of the age-wage rate profile, are not tested because the data set used does not include the necessary information.

There is support for the idea proposed in prior work on birth-order effects[39] (but not explicitly incorporated into the model) that last-borns' relative advantage is greater in poorer families. If in poorer families, the education of older siblings exceeds that of the mother, then time inputs of older siblings could provide a relatively better substitute for mother's time than would siblings' time inputs in the average family. Educational opportunities expanded rapidly in Colombia in the two decades before the survey; children are especially

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[39] Davis, Cohan and Bashi; and Altus' report of Nichols' data.

likely to be better-educated than their parents in families in which parents migrated to the urban areas sampled from rural areas where schooling was less available.[40]

Finally, differences in birth-order effects between children of working and non-working mothers as a function of mother's education and father's income are discussed. There is some support for the notion that mother's education is a better proxy for her price of time if she works, and that father's income is a better proxy for her price of time if she does not.[41]

The family size problem

Most studies of differences in achievement levels among persons according to the order of their birth are concerned also with the effect of family size and its importance relative to birth order. Unfortunately there are difficulties with combining analysis of

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[40]See Appendix B of Nancy M. Birdsall, "Siblings and Schooling in Urban Colombia," Ph.D. dissertation, Yale University, 1979.

[41]T. Paul Schultz ["Fertility Differences Between Working and Nonworking Wives," paper presented at the annual meetings of the Population Association of America, Atlanta, Georgia, April 1978] suggests husband's income as a proxy for the price of time of non-working wives. The assumption is that market wage offers, a function of education, are independent of hours worked, whereas the shadow value of time in nonmarket activities increases as less time is allocated to them. This does not imply that education has no effect on home productivity, only that for nonworking women, their price

birth-order and family size effects. Parents' demand for number of children is one of a set of demands, another of which is for Q, quality per child. Consider a system of linear equations which represents the N and Q decisions, relating achievement of an individual i to that individual's family size and order of birth:

$$(1)\quad \text{Achievement}_i = \alpha_0 + \alpha_1\text{family size} + \alpha_2\text{birth-order}_i + \sum_i \alpha_i X_i + \sum_j \alpha_j Z_j + \varepsilon_1$$

$$(2)\quad \text{Family size} = \beta_0 + \beta_1\left(\sum_{i=1}^{N} \text{achievement}_i / N\right) + \sum_i \beta_i X_i + \sum_j \beta_j W_j + \varepsilon_2$$

where X_i is a vector of socio-economic variables influencing family size and child achievement; the Z_j are variables which influence child achievement but not family size; the family size of an individual i is N, so that β_1 is the coefficient on average achievement of children in a family; the W_j are variables which influence family size but not child achievement; and ε_1 and ε_2 are error terms.

The two equations together reflect the possibility that parents' decisions regarding family size are affected by their goals for each child's eventual achievement level (or, more crudely in the literature, "quality"). As a result an ordinary least squares estimate of

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of time is captured better by husband's income than their own education. Table 7 (col. 2) implies education does increase home productivity; see discussion below.

equation (1) with actual family size variable entered will result in biased parameter estimates, for two possible reasons: ε_1 and ε_2 are likely to be correlated, though in what direction is not clear. We cannot observe differences across parents in fecundity or taste; a preference for large numbers of children could be positively or negatively related to a preference for child achievement-oriented patterns of spending. Negative correlation of ε_1 and ε_2 would cause family size to be negatively correlated with the error term in the achievement equation and its coefficient would be biased downward; positive correlation would have the opposite effect.

Moreover, the interaction model (Becker and Lewis; Willis) indicates that the shadow price of investment (or achievement) per child is lower for parents with fewer children; if ε_1 and ε_2 are negatively correlated, this interaction effect will increase further the negative correlation between the family size variable in equation (1) and ε_1.

However, simply treating family size as an endogenous variable using appropriate techniques does not resolve the problem as far as analysis of birth-order effects is concerned. If family size is entered into equation (1) as an endogenous variable, the coefficients on birth-order dummies indicating whether the individual was first-born, middle-born or last-born will be biased; the unexplained error in the family size equation (2) is likely to be impounded in the birth-order coefficients, since being a middle-born child is highly correlated with being from a large family. Insofar as large family size has a negative effect on educational achievement,[42] birth-order dummies for first and last-born children will be

[42]Birdsall, Chapter II.

biased upward in a child achievement regression, unless actual family size is controlled for. As a result, it is virtually impossible to obtain consistent estimates in one regression of both family size and birth-order effects.

For analysis of the effects of birth order, two methods of controlling for family size are employed below. The most direct is to examine intra-family differences, e.g. the difference between the educational attainment (age-standardized) of the first or last-born and the average attainment of his or her own siblings. (This procedure has the additional advantage of controlling for other family characteristics, such as parents' education, income and taste for average quality of children, which influence the average level of attainment for all the children.) A simple test of hypotheses 1 and 2 is then whether the intra-family difference in achievement between first-born children is positive and significantly different from that of other children; the analogous test for last-borns is whether the difference is positive and significantly different from that of middle-born children. The result of this test is shown below.

There are two disadvantages of using intra-family differences to test birth-order effects in this sample:

1. The sample size for the former method is small, since the test can only be performed for those families which have a first-born or last-born as well as other children in the age group 6-18. Children from other families whose birth order and education are known but for whom the education of older or younger siblings is not known are thus eliminated. Of 1450 families with children between the ages 6 and 18, 867 families had a first-born and other children; only 336 families

had a first-born and last-born and other children.

2. For these families, first-borns are likely to be near the top of the 6-18 age range, and last-borns near the bottom. This makes results heavily dependent on the manner in which children's educational attainment is standardized for age. The extent and nature of differences in educational attainment varies by age; among the youngest children, enrollment rates are high, and variation in attainment is largely a function of differences in age of beginning school and differences in grade repetition. Among older children, differences in the age of permanently leaving school are probably more important. Thus a direct comparison of older and younger children may not be reasonable.

For these reasons, a second approach is also employed below. It is to use as the units of observation all children for whom birth order and education are known. With a comparison of children across all families, first-borns who are young and last-borns who are old can be included in the analysis. Since both first-borns and last-borns are relatively equally distributed across all ages (with a slightly higher proportion of first-borns among 17 and 18 year olds, and a slightly lower proportion of last-borns among 6 and 7 year olds), results for all children are less dependent on the age standardization. (The age distribution of children by birth order and a more complete discussion of the age standardization problem are available from the author.)

Children's educational index is then regressed on dummies representing birth order; to control for family size, a variable representing each child's actual family size, (ARAT), is included. As explained above, the coefficient on the family size variable (α_1 in equation (1) above) cannot be interpreted as an indication of the effect, in a behavioral sense, of family size on children's achievement. It will capture all associations between an individual child's achievement and family size, including effects of parents' taste for numbers of children and average quality in children, and effects of differences in fecundity. It thus allows a test of the existence of birth-order effects independent of these factors. Parents' average achievement goal for all children obviously does not affect the birth order of any individual child, except insofar as the achievement goal affects the total number of children and children with high orders of birth must come from large families. As long as first and last effects are not associated with family size (and they are not in these data), the coefficients on the birth-order dummies will signal whether birth-order effects exist.

(A third approach to the family size problem is to stratify the sample by family size and examine birth-order effects within family size groups. This requires elimination of children with mothers under age 40, since stratifying by children-ever-born can only be done for families in which mothers have completed childbearing.[43] Using this approach (not shown), dummies for first and last-born children are usually positive but seldom statistically significant, and are in some cases negative.)

The endogeneity of mother's work status

The testing of hypothesis 3 also presents difficulties in estimation. The model predicts differences in the extent of birth-order effects depending on whether the mother works outside the home. However, supply of labor by the mother is jointly determined along

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[43] Stratifying by ARAT is not possible because it is in effect a continuous variable.

with number of children and their quality, and thus, like family size, should not be treated as exogenous. A way to get around this problem would be to predict labor force supply of mothers during the child-rearing years--but this is possible only with identifying variables reflecting demand for mothers' labor, and such variables are not present in this data set. Furthermore, the data include information only on the current labor force status of mothers, not their labor force status over the entire period of childrearing. Thus some mothers counted as working may have spent most of their childrearing years at home; other mothers counted as nonworking may have spent most of their childrearing years working away from home.[44]

In the estimates below, the endogeneity of mothers' working status is ignored; the results are of sufficient interest to warrant more rigorous tests of the model with a better data set.

Description of sample and variables

The data analyzed are from a survey of 2949 households in urban Colombia, in which information was collected on number and ages of children, their educational attainment, and on income and other characteristics of parents.

The variable used as a measure of "quality" across children is educational attainment. **The variable is standardized for the age of children**

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[44] James J. Heckman and Robert J. Willis find using U.S. panel data that there tend to be two groups of women: workers, whose participation probabilities are near unity; and nonworkers, whose participation probabilities are near zero ["A Beta-Logistic Model for the Analysis of Sequential Labor Force Participation by Married Women," JPE, 85 (February, 1977), 27-58].

to permit direct comparison of children of different ages.[45] This dependent variable is defined for children aged 6 to 18. The variable is by no means a perfect measure of the "quality" discussed in the model. It is an even cruder measure of quality than a score on an achievement test of some kind. (Test scores have been the measure used in the more recent large-sample investigations of birth-order differences, discussed **above.**) Educational attainment is in fact a function of the "quality" we seek to measure; the assumption is it reflects parents' inputs of time and goods in the same way actual "quality" of a child would. Yet it may not; it may have a greater goods-component than would be ideal, since schooling is purchased by parents.[46]

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[45] A child's educational attainment is compared to that of other children of the same age and sex group in the sample:

$$EDI_i = \frac{\text{educational attainment}_i}{\text{mean educational attainment}_{js}} : \quad \begin{array}{l} i = \text{child} \\ j = \text{age} \\ s = \text{sex} \end{array}$$

[46] $Q_i = f(Q_i^*, x_{Q_i})$, where Q_i is education of the ith child, Q_i^* is the child's true "quality" and x_{Q_i} is money spent on education for the ith child. But $Q_i^* = f(t_i, x_i)$ where t_i and x_i include all time and goods inputs to the ith child, so that x_i includes money spent on education. If money spent on education affects Q_i more than Q_i^*, then Q_i may be said to have a greater goods-component than would be ideal.

Of the 2949 families, 2405 had children. The birth order of 7223 children in 2288 families could be determined with relative accuracy, given age of children present in the household, number of children born to the mother and still alive, her present age and her age at marriage. Children of women married more than once were excluded, as were children in families where more than two children were no longer in the household. If one or two children of those reported alive were not in the household, other children in that household were included in the sample only if it was clear from mother's duration of marriage and the ages of those children present, that the missing children were the oldest.

Of these 7223 children, 4380 from 1450 families were between the ages of 6 and 18; the sample is confined to children in this latter group by the nature of the dependent variable. It is further reduced by the elimination of children from one-child families and of cases where there are missing values on other variables.

Some children may be from families which are not yet complete. This would not affect results for first-born children. Children at least 6 years of age who had no younger siblings at the time of the survey will probably not subsequently have younger siblings in Colombia where births are seldom so widely spaced; even if they subsequently did, for 6 or more years they would have been "last."[47]

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[47] Is it problematic that first-born and last-born children at older ages are more likely to be living at home and thus in the sample if they are still in school, thus biasing upward achievement levels of first and last-borns? No. Older middle-borns are <u>also</u> more likely to be at home and thus in the sample if they are still in school, and the dependent variable compares a child's educational attainment to that of other children of the same age and sex <u>also still at home</u>. Only if first and last-born children are systematically more likely

Table 1 lists variables used in the cross-children analysis, with their means and standard deviations. (Variables used in the intra-family difference analysis are defined in the tables showing results.) Note that only about 11 percent of mothers worked outside the home at the time of the survey.

The family size variable used is ARAT, for "age ratio." It is a measure of fertility which is standardized for the biological relationship between mother's age and fecundity, using a natural fertility schedule, thereby permitting inclusion of children in the analysis whose mothers may not have completed childbearing.[48]

Results

Table 2 shows the results of a simple test of hypotheses 1 and 2, using the intra-family differences in educational attainment (age-standardized) between first-borns and other siblings (col. 1); first-borns and other siblings excluding last-borns (col. 2); last-borns and other siblings excluding first-borns (col. 3).

Here and in the following tables, results for first-borns only are for families with at least two children; results which also include last-borns are for families with at least three children. The former results compare first-borns to all other children (including last-borns); the latter results compare first-borns and last-borns to middle children.

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to stay at home at older ages for reasons other than schooling is there a problem. Even then it is likely that the bias would reduce the hypothesized effect--since then it would be precisely those older middle-born children still in school who would be more likely to be still living at home. In any event, a cross-tabulation of first and last-born children by age showed they are relatively evenly distributed across all ages, with a slight increase in the proportion of first-borns in the 17 and 18 year groups only.

[48]See Brian Boulier and Mark R. Rosenzweig, "Age, Biological Factors and Socioeconomic Determinants of Fertility: A New Measure of Cumulative Fertility for Use in the Empirical Analysis of Family Size," Demography, 15 (November, 1978), 487-498.

TABLE 1

Variable Definitions and Descriptive Statistics for Samples of Children from Two and Three-Child Families

		Sample of children from families with at least 2 children N = 4296	Sample of children from families with at least 3 children N = 4082
EDI	Educational index: Ratio of child's years of schooling to mean of other children of same age and sex	1.01 (.896)	1.00 (.871)
EDI2	Educational index: Difference of child's years of schooling and mean years of other children of same age and sex	.051 (1.81)	.0329 (1.82)
FRTD	First-born child dummy	.241 (.428)	.222 (.416)
LSTD	Last-born child dummy	---	.091 (.288)
WWD	Dummy indicating a working wife	.112 (.315)	.110 (.313)
FRTWWD	First-born child/working wife dummy interaction term	.0286 (.167)	.0250 (.158)
LSTWD	Last-born child/working wife dummy interaction term	---	.00931 (.0960)
AGE	Age of child	10.92 (3.54)	10.9 (3.52)
SEXD	Sex of child dummy, equals 1 for females	.496 (.50)	.498 (.50)
ARAT	Fertility measure standardized for the age-fecundity relationship using a natural fertility schedule[a]	.665 (.283)	.685 (.273)
SCW	Wife's number of years of schooling completed	6.30 (3.97)	6.29 (3.93)

TABLE 1 (continued)

		Sample of children from families with at least 2 children N = 4296	Sample of children from families with at least 3 children N = 4082
FRTSCWD	First-born child/schooling of wife dummy interaction term	1.61 (3.52)	1.47 (3.37)
LSTSCWD	Last-born child/schooling of wife dummy interaction term	---	.673 (2.48)
YH	Husband's income (1968 pesos, quarterly)	1013. (2255)	1000. (2282)
FRTYHD	First-born child/husband's income interaction term	240. (1168)	213. (1149)
LSTYHD	Last-born child/husband's income interaction term	---	129 (1129)

[a] See Boulier and Rosenzweig (1978).

TABLE 2

Mean Intra-Family Differences in Educational Attainment
(standard error in parentheses)

Types of age standardization	column 1 First-born minus average of other siblings	column 2 First-born minus average of other siblings, excluding last-born	column 3 Last-born minus average of other siblings, excluding first-born
PANEL I (based on EDI: ratio of child's actual attainment to average of child's age-sex group	-.020 (.033) N = 867	.010 (.032) N = 800	.115 (.060) N = 336
PANEL II (Tier I, excluding 6 and 7 year olds)	.038 (.015) N = 671	.039 (.016) N = 635	.019 (.032) N = 218
PANEL III (based on EDI2: difference between child's actual attainment and average of child's age-sex group	.183 (.059) N = 867	.153 (.061) N = 800	-.175 (.091) N = 336

Three panels are shown. In the first, the intra-family difference is based on an educational index which is a ratio of a child's actual attainment to the average for the child's age-sex group (EDI in Table 1); in the second, the same variable is used but 6 and 7 year olds are excluded; in the third, the educational attainment index is a difference (EDI2 in Table 1) and includes 6 and 7 year olds.

The results are mixed, illustrating the problem mentioned above of comparing children within families, i.e. the sensitivity of results to the method of age standardization. There is a tendency for young children--in this sample, likely to be last-borns--who are at normal grade level to have higher scores using the ratio index than older siblings--in this sample, likely to be first-borns--who are also at normal grade level.[49] Because this is especially the case with 6 and 7 year olds, their exclusion "helps" the older first-borns. (Compare panels 1 and 2.)

In the third panel, the intra-family difference is based on the index which is itself a difference between a child's actual educational attainment and the average for the child's age-sex group. Compared to the ratio index, this standardization gives lower relative "scores" to younger children **and favors older children.**
As a result, in intra-family comparisons in which last-borns are younger than first-borns and middle-borns, the last-borns do not

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[49]E.g. a 6-year-old girl who has completed one year of school receives a "score" using the ratio index of 1/.25, where .25 is the average years of education attained of 6 year old girls. **A 10 year old girl who has completed four years of school receives a "score" of 4/2.60. The score of the older child is lower, though both are at grade level.**

appear to have any advantage (the "difference" in column 3 is negative). First-borns (columns 1 and 2) appear to have a significant advantage.

Table 3 indicates the results of regressions using as the units of observation all children for whom birth order and education are known. The individual child educational index is regressed on dummies for being first-born (columns 1 and 2), and last-born (columns 3 and 4), with actual family size entered as a control variable. (Results shown are for EDI, the age-standardization based on a ratio. Results using EDI2, an age-standardization based on a difference, are less pronounced. **They are available from the author.**)

Coefficients on the birth-order dummies (middle-born children being the excluded group) are all positive; in columns 3 and 4 the first-born coefficients are significant at the 10 percent level; the last-born coefficients are significant at the 5 percent level. However, the last-born child dummy coefficient is consistently greater than that of the first-born, contradicting the prediction of the model. Several reasons for this are possible:

1. As mentioned above, the dependent variable has a greater goods-component than would be ideal; it could overstate the total advantage of the last relative to the first, if, for example, first-borns have higher IQ's than last-borns, but do not stay in school longer.[50]

2. Last-borns are somewhat more likely to be at the young end of the age range than at the old end; the opposite is true for

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[50] Lindert, pp. 201-204, argues that especially for the last-born the difference in parental goods-inputs is greater than in time-inputs. See also fn. 46 above.

TABLE 3

Child Education Regressions
(t-statistics in parentheses)

Dependent variable: child's educational attainment relative to other children in his or her age-sex group (EDI)

	Families with at least 2 children, N=4296		Families with at least 3 children, N=4082		
	(1)	(2)	(3)	(4)	
Constant	1.44 (39.5)	1.44 (39.0)	1.40 (34.8)	1.41 (34.4)	
FRTD	.031 (0.97)	.037 (1.11)	.0557 (1.69)	.0620 (1.78)	
LSTD	---	---	.132 (2.72)	.149 (2.91)	
ARAT	-.648 (-13.5)	-.652 (-13.6)	-.624 (-12.3)	-.625 (-12.3)	
WWD	---	-.0541 (-1.10)	---	-.0412 (-0.80)	
FRTWWD	---	-.0542 (-0.55)	---	-.0548 (-0.53)	F=.67 2,4076
LSTWWD	---	---	---	-.170 (-1.10)	
	R^2=.0430	R^2=.0436	R^2=.0457	R^2=.0466	

first-borns. The age standardization used in these tables favors slightly younger children at grade level over older children at grade level.

3. For families which face imperfect capital markets, a number of factors may favor the last-born. The last-born may benefit from additional and unexpected financial resources of parents who are on average older and have thus higher earnings when they are in school. Last-borns in poor families may also benefit from financial transfers of older siblings, now working, and from time inputs of older siblings. Also, families whose future stream of income is uncertain may be more willing to spend heavily on the last child than on earlier children. All these factors imply that in the families (nonworking mothers) with the highest income, that do not face imperfect capital markets, the advantage of the last-born should be reduced. (Birth-order effects for high-income families are shown below.)

In columns 2 and 4, the effect of working mothers is tested. In both cases, the working mother/birth-order dummy interaction terms are negative as expected, suggesting a reduction of the positive birth-order effect among children of working mothers, but they are not significant statistically. Based on the magnitudes of coefficients on FRTD and FRTWWD in columns 2 and 4, and on LSTD and LSTWWD in column 4, the advantage of being first-born or last-born is more or less eliminated when mothers work.

In Table 4, results are shown for the samples split depending on mothers' labor force participation. Birth-order coefficients are positive and in column 3 significant for nonworking mothers; they are not significantly different from zero for working mothers (columns 2

TABLE 4

Child Education Regressions--Split Sample
(t-statistics in parentheses)

Dependent variable: child's educational attainment relative to other children in his or her age-sex group (EDI).

	Families with at least 2 children		Families with at least 3 children			
	(1) Non-Working Mother N=3815	(2) Working Mother N=481	(3) Non-Working Mother N=3632		(4) Working Mother N=450	
Constant	1.43 (37.3)	1.50 (12.9)	1.39 (32.7)		1.53 (12.0)	
FRTD	.039 (1.16)	-.0350 (-0.37)	.0644 (1.85)	F=5.33 2,3628	-.0133 (-0.14)	F=.071 2,446
LSTD	---	---	.155 (3.02)		-.0553 (-0.37)	
ARAT	-0.635 (-12.7)	-0.817 (-5.05)	-.602 (-11.3)		-.870 (-5.03)	
	R^2=.0427	R^2=.0513	R^2=.0459		R^2=.0548	

and 4).

Are such effects due solely to imperfect capital markets (or to parents' failure to plan intertemporally?) rather than to the time constraint which drives the model? If birth-order effects persist even for families we assume have good access to markets for borrowing and saving--the families with the highest incomes--then clearly these effects cannot be due solely to imperfect capital markets.

In Table 5 are shown the results of the Table 4 regressions for nonworking mother families, but with the sample restricted to children from the 20 percent of families with the highest income (in the original sample of 2949 families). Columns 1 and 3 of Table 5 correspond to columns 1 and 3 of Table 4. Birth-order effects are greater in the rich families; in Table 5 the coefficient on the first-born dummy in column 1 is about three times greater than that in Table 4 and is significant (at the 10 percent level). The coefficients on the first-born birth-order dummy in column 3 is also three times greater in the rich-family sample, and the last-born coefficient is twice as great in the rich-family sample. Thus both first-borns and last-borns have a relatively greater advantage in rich families, and the relative advantage for last-borns is not as great as for first-borns. Imperfection in capital markets does not alone explain birth-order effects.

In columns 2 and 4, the age and sex of the child are controlled for. Since the dependent variable is standardized for age (and sex), interpretation of an age coefficient entered linearly is not straightforward. Its negative sign here suggests that younger children in rich families are at a disadvantage compared to older children. Age

TABLE 5

Are Birth-Order Effects Due to Imperfect Capital Markets?
Child Education Regressions for High-Income Families (top 20 percent)

(Non-working mothers)
(t-statistics in parentheses)

	Families with at least 2 children, N= 798		Families with at least 3 children, N= 743	
	(1)	(2)	(3)	(4)
Constant	1.89 (18.8)	2.67 (17.0)	1.69 (15.0)	2.41 (13.8)
FRTD	0.146 (1.68)	.178 (2.09)	0.216 (2.40)	.228 (2.58)
LSTD	---	---	0.292 (2.67)	.180 (1.64)
ARAT	-.786 (-5.04)	-.860 (-4.44)	-.570 (-3.48)	-.704 (-4.33)
AGE	---	-.0660 (-6.67)	---	-.0537 (-5.36)
SEXD	---	-.00178 (-0.02)	---	-.0490 (-0.76)
	R^2=.0379	R^2=.0891	R^2=.0419	R^2=.0784

has no effect when entered in the same way into a regression including children of all nonworking mothers (analogous to the Table 4 regression, not shown). Thus the disadvantage of younger children in rich families does not hold across the population. This is consistent with the idea that young children in poorer families benefit more from time and even financial inputs of older siblings.

Table 6 provides a similar test of the extent to which it is imperfection in capital markets (rather than the time constraint) which causes birth-order effects. Children of nonworking mothers of all families, regardless of income, are included, and interaction terms of income and the birth-order dummies are included to test the effect of income on the extent of birth-order differences. The significantly negative coefficient on LSTYHD (interaction of last-born dummy and income) in column 2 is consistent with the Table 5 results for the highest-income families. The advantage of being last-born is not as great in high-income families, though its net effect is still positive.

Finally, does the effect of mother's education on differences among children by birth-order differ depending upon whether the woman is working or not? If a working woman's education is positively correlated with an increase over time in her wage (i.e. not only with her wage <u>level</u> but with the steepness of her age-earnings profile), then we would expect more education to be associated with a greater attenuation of the last-born's advantage among working women than nonworking women. (This could also be the case if many women currently working had only recently entered the labor market.) Also, insofar as for working women, their own education is a close measure of the

TABLE 6

Child Education Regressions: Effect of Income on Birth-Order Differences

(Non-working mothers)
(t-statistics in parentheses)

	Families with at least 2 children, N = 3815 (1)	Families with at least 3 children, N = 3632 (2)
Constant	1.36 (36.4)	1.32 (30.1)
FRTD	.0358 (0.98)	.075 (2.00)
LSTD	---	.193 (3.49)
ARAT	-.598 (11.9)	-.567 (-10.7)
YH (head's income)[a] (x 10^{-4})	.417 (6.13)	.463 (5.89)
FRTYHD (first-born x head's income) (x 10^{-4})	.140 (0.54)	-.0759 (-0.52)
LSTYHD (last-born x head's income) (x 10^{-4})	---	-.346 (-2.33)
	R^2=.0559	R^2=.0576

[a]Mean income of head in this sample is 1055 pesos (quarterly income). Thus net effect of being first-born (at mean of income) is about .08 (column 1), .07 (column 2); net effect of being last-born is .16 (column 3).

shadow price of their time, whereas for nonworking women, husband's income is a closer measure,[51] we would predict that education reduces the last-born's advantage more (or increases it less) among working women.

Table 7 shows the results of a regression on child education, with schooling of wife (SCW) and interactions of wife's schooling and the birth-order dummies (FRTSCWD, LSTSCWD) added to the variables shown in above tables; the regression is shown for all women in column 1, for nonworking women in column 2, and for working women in column 3. The total difference is summarized in Table 8, which shows that the total effect of being first-born or last-born is much greater among children of nonworking mothers, as seen in earlier regressions above. But the coefficients in Table 7 indicate that much of the difference in the extent of birth-order effects between children of working and nonworking mothers is governed by the differential effects of education between the two groups. For working mothers, the direct positive effect of mother's education on child achievement is greater; but for first-borns of working mothers the greater mother's education, the more is that advantage offset. The direct effect of being last-born is not positive for children of either set of mothers; but among children of nonworking mothers, the greater mother's education, the greater becomes the last-born's advantage. Both results suggest that the price of time of working mothers is increasing over time, and the more so the greater their education. The nonworking mother education effect for the last-born

[51] See fn.41 above.

TABLE 7

Effect of Mother's Education on Birth-Order Differences; Working and Nonworking Mothers with at Least 3 Children

Dependent variable: child's educational index
(t-statistics in parentheses)

	All mothers, N = 4082 (1)	Nonworking mothers, N = 3632 (2)	Working mothers, N = 450 (3)
Constant	.804 (15.6)	.819 (15.0)	.734 (4.71)
FRTD	.0971 (1.66)	.0808 (1.29)	.210 (1.29)
LSTD	.0177 (0.20)	.0252 (0.26)	-.049 (-0.18)
ARAT	-.367 (-7.34)	-.359 (-6.83)	-.475 (-2.82)
SCW	.0681 (16.3)	.0649 (14.6)	.0905 (7.65)
FRTSCWD	-.00799 (-1.04)	-.00404 (-0.49)	-.0376 (-1.73)
LSTSCWD	.0119 (1.12)	.0134 (1.20)	.00121 (0.03)
	R^2=.1321	R^2=.1269	R^2=.1857

TABLE 8

Net birth-order differences by mother's education*
(at mean education; families with at least 3 children)

	All mothers mean education=6.29	Nonworking mothers mean education=6.32	Working mothers mean education=6.02
Net effect of being first-born	.0468[a]	.0553[c]	-.0164[e]
Net effect of being last-born	.0925[b]	.1099[d]	-.0417[f]

[a] .0971 - .00799 (6.29) = .0468

[b] .0177 + .0119 (6.29) = .0925

[c] .0808 - .00404 (6.32) = .0553

[d] .0252 + .0134 (6.32) = .1099

[e] .210 - .0376 (6.02) =-.0164

[f] -.049 + .00121 (6.02) =-.0417

*Source is Table 7.

could also be interpreted as a learning-by-doing phenomenon: women improve at childrearing with experience, and improve more the greater their education; this helps the last-born children of nonworking mothers, but not those of working mothers.

Insofar as women's labor force status is endogenous, and is especially likely to be related to fertility, these education effects must be interpreted with caution. They are shown primarily as suggestive of what we might expect if the data permitted a better test. (The mean of the variable ARAT for working women with at least 3 children is .656 [s.d.: .238], very close to that for nonworking women .689 [s.d.: .277].)

For nonworking women, income effects (where income is that of the husband) seem a better measure of mother's opportunity cost of time than education. Table 9 shows the results of the same regression for nonworking (column 2) and working (column 3) women. The effects of Table 7 for education are reversed for income. The interaction of income and last-born is negative for nonworking mothers (see also Table 6 and discussion there). Thus nonworking women may also experience some increase in the price of their time which reduces the last-born's advantage--but for them this effect is picked up by the variable representing the husband's income.

Conclusions

There is evidence that first and last-born children in this sample have an advantage over middle-borns; among children whose mothers do not work, first-borns score about 6 percent higher than middle-borns, and last-borns about 15 percent higher than middle-borns

TABLE 9

Effect of Husband's Income on Birth-Order Differences;
Working and Nonworking Mothers with
at Least 3 Children

Dependent variable: child's educational index
(t-statistics in parentheses)

	All mothers, N = 4082 (1)	Nonworking mothers, N = 3632 (2)	Working mothers, N = 450 (3)
Constant	1.33 (31.9)	1.32 (30.1)	1.08 (7.60)
FRTD	.069 (1.95)	.0755 (2.00)	.124 (0.98)
LSTD	.168 (3.24)	.193 (3.49)	-.0483 (-0.27)
ARAT	-.586 (-11.58)	-.567 (-10.68)	-.572 (-3.34)
YH (x 10^{-3})	.0513 (6.58)	.0463 (5.89)	.370 (6.01)
FRTYH (x 10^{-3})	-.0103 (-0.71)	-.00759 (-0.52)	-.163 (-1.29)
LSTYH (x 10^{-3})	-.0354 (-2.39)	-.0346 (-2.33)	.0341 (0.23)
	R^2=.0586	R^2=.0576	R^2=.1455

on an age-standardized index of educational attainment. These figures represent respectively about 6 and 14 percent of one standard deviation of the educational index. Results are suggestive, though not definitive; the predicted birth-order effects do not show up within family size classes and are much less strong when a different age-standardization procedure is used.

As predicted, differences by birth order disappear among children of working mothers. But the empirical results of testing the working mother hypothesis cannot be deemed definitive, since labor force participation of mothers should not be treated as exogenous as it is here, and since only mothers' current labor force status is known, whereas the relevant variable to test the model would be labor force status of mothers throughout the childrearing period. The hypothesis that birth-order effects will not obtain among children of working mothers needs to be tested with better data from other settings.

The advantage of the last-born in these data is notable. Results suggest the last-born's advantage is greatest in poorer families; time and financial inputs of older siblings may be important in poor families, and whatever imperfection in capital markets exists would work more to the advantage of the last than the first, especially if parents' earnings increase with age. The advantage of the last-born distinguishes these data from that of most studies of persons in the U.S. and Europe. In Colombia, educational opportunities have expanded greatly from one generation to the next. If older children's education exceeds that of parents, the value of older siblings' help with younger ones may be important.

There is some indication that for working mothers, their own education captures best their price of time, whereas for nonworking mothers, husband's income is a better proxy.

Progress toward explaining birth-order effects is made. The time constraint faced by mothers seems central. The "economic" explanation offered in the sociological and psychological literature to explain the first-born's advantage is that family money resources are depleted successively with additional children; findings above suggest the true "economics" has to do with the price of time, since birth-order effects are prominent and actually greater in high-income families. The birth order model makes explicit the time constraint parents face in raising children. The empirical results imply that time inputs to children, which in certain cases depend partly on their order of birth, do matter.

APPENDIX A

BIRTH-ORDER MODEL, WORKING MOTHER VERSION, WITH n CHILDREN, m PERIODS, AND r EFFICIENCY PARAMETERS

There are 2nr + 3m + 2 first-order conditions, with $\lambda_1 \ldots \lambda_m$ the Lagrangean multipliers corresponding to the time constraint in each period, and λ_{m+1} the Lagrangean corresponding to the income constraint.

(1) $$\gamma_1 \frac{\partial f}{\partial t_{111}} \quad Ut_{111} \quad = \lambda_1 \; ; \left[Uq_{ijk} = \frac{\partial U}{\partial \Sigma q_i} \frac{\partial \Sigma q_i}{\partial q_{ijk}} + \frac{\partial U}{\partial V q_i} \frac{\partial V q_i}{\partial q_{ijk}} \right]$$

(2) $$\gamma_2 \frac{\partial f}{\partial t_{122}} \quad Ut_{122} \quad = \lambda_2$$

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(nr-1) $$\gamma_r \frac{\partial f}{\partial t_{n-1,\ m-1,\ r}} \quad Ut_{n-1,\ m-1,\ r} \quad = \lambda_{m-1}$$

(nr) $$\gamma_r \frac{\partial f}{\partial t_{nmr}} \quad Ut_{nmr} \quad = \lambda_m$$

(nr+1) $$\gamma_1 \frac{\partial f}{\partial x_{111}} \quad Ux_{111} \quad = \lambda_{m+1} p_x$$

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.

(2nr) $$\gamma_r \frac{\partial f}{\partial x_{nmr}} \quad Ux_{nmr} \quad = \lambda_{m+1} p_x$$

(2nr+1) $\lambda_1 = \lambda_{m+1}w$

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.
.

(2nr+m) $\lambda_m = \lambda_{m+1}w$

(2nr + m+1) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{s1}} = \lambda_1$

.
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.

(2nr+2m) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial t_{sm}} = \lambda_m$

(2nr + 2m+1) $\frac{\partial U}{\partial S}\frac{\partial S}{\partial \sum_j X_{sj}} = \lambda_{m+1}p_s$

(2nr + 2m+2) $t_{j=1} = \sum_i t_{i1} + t_{p1} + t_{s1}$

.
.
.

(2nr + 3m+1) $t_{j=m} = \sum_i t_{im} + t_{pm} + t_{sm}$

(2nr + 3m+2) $V = -\sum_j wt_{pj} + p_x X + p_s S$

From first-order conditions 2nr+1 to 2nr+m, it is clear that

$\lambda_{j=1 \ldots m} = \lambda_{m+1}w$, so that

$\lambda_1 = \lambda_2 = \ldots \lambda_m$

and there are no birth-order effects.

APPENDIX B

PROOF OF BIRTH-ORDER EFFECTS IN THE NONWORKING MOTHER VERSION

To show that, for the 3-children, 4-periods, 2-efficiency parameters case, $\lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$ (given the restrictions (i) and (j) in the text, pp. 28-29):

Note that:

(A) $t_{s1} + t_{111} = t_{s2} + t_{122} + t_{211} = t_{s3} + t_{232} + t_{331} = t_{s4} + t_{342}$

1. To show $\lambda_1 < \lambda_2$:

$\lambda_1 \gtreqless \lambda_2$

If $\lambda_1 > \lambda_2$, $t_{s1} < t_{s2}$ from first-order conditions (13) and (14), and $t_{111} > t_{122} + t_{221}$, from (A).

But with $\lambda_1 > \lambda_2$, $\gamma_1 \frac{\partial f}{\partial t_{111}} \quad Ut_{111} \quad > \gamma_1 \frac{\partial f}{\partial t_{221}} \quad Ut_{221}$

[first-order conditions (1) and (3)] ==> $t_{111} < t_{221}$

$\therefore \lambda_1 \leq \lambda_2$

If $\lambda_1 = \lambda_2$, $t_{s1} = t_{s2}$ [(13) and (14)] and $t_{111} = t_{122} + t_{221}$ (A) ==> $t_{111} > t_{221}$, since $t_{122} > 0$, from (b), p. 83.

But with $\lambda_1 = \lambda_2$, $\gamma_1 \frac{\partial f}{\partial t_{111}} \quad Ut_{111} \quad = \gamma_1 \frac{\partial f}{\partial t_{221}} \quad Ut_{221}$ [(1) and (3)]

$\Longrightarrow t_{111} = t_{221}$

$\therefore \lambda_1 < \lambda_2$

2. To show $\lambda_3 = \lambda_2$:

$\lambda_3 \gtreqless \lambda_2$

If $\lambda_3 > \lambda_2$, $t_{s3} < t_{s2}$ [(14) and (15)], and

$t_{232} + t_{331} > t_{122} + t_{221}$ (A).

But with $\lambda_3 > \lambda_2$, $\gamma_2 \frac{\partial f}{\partial t_{232}} \quad Ut_{232} \quad > \gamma_2 \frac{\partial f}{\partial t_{122}} \quad Ut_{122}$

[(2) and (4)] $\Longrightarrow t_{232} < t_{122}$,

and $\gamma_1 \frac{\partial f}{\partial t_{331}} \quad Ut_{331} \quad > \gamma_1 \frac{\partial f}{\partial t_{221}} \quad Ut_{221}$

[(3) and (5)] $\Longrightarrow t_{331} < t_{221}$,

$\Longrightarrow t_{122} + t_{221} > t_{232} + t_{331}$

$\therefore \lambda_3 \leq \lambda_2$

If $\lambda_3 < \lambda_2$, $t_{s3} > t_{s2}$ [(14) and (15)], and

$t_{232} + t_{331} < t_{122} + t_{221}$.

But with $\lambda_3 < \lambda_2$, $\gamma_2 \frac{\partial f}{\partial t_{232}} \quad Ut_{232} \quad < \gamma_2 \frac{\partial f}{\partial t_{122}} \quad Ut_{122}$

[(2) and (4)] $\Longrightarrow t_{232} > t_{122}$,

and $\gamma_1 \frac{\partial f}{\partial t_{331}} \quad Ut_{331} \quad < \gamma_1 \frac{\partial f}{\partial t_{221}} \quad Ut_{221}$

[(3) and (5)] $\Longrightarrow t_{331} > t_{221}$

$$\Longrightarrow \quad t_{232} + t_{331} > t_{122} + t_{221}$$

$\therefore \lambda_3 = \lambda_2$

3. To show $\lambda_4 < \lambda_3$:

$\lambda_4 \gtreqless \lambda_3$

If $\lambda_4 > \lambda_3$, $t_{s4} < t_{s3}$ [(15) and (16)] and

$t_{342} > t_{331} + t_{232}$ (A).

But with $\lambda_4 > \lambda_3$, $\gamma_2 \frac{\partial f}{\partial t_{342}} \quad Ut_{342} \quad > \gamma_2 \frac{\partial f}{\partial t_{232}} \quad Ut_{232}$

[(4) and (6)] ==> $t_{342} < t_{232}$

$\therefore \lambda_4 \leq \lambda_3$

If $\lambda_4 = \lambda_3$, $t_{s4} = t_{s3}$ [(15) and (16)] and

$t_{342} = t_{232} + t_{331}$ (A) ==> $t_{342} > t_{232}$, since

$t_{331} > 0$, from (c), p. 83 above.

But with $\lambda_4 = \lambda_3$, $\gamma_2 \frac{\partial f}{\partial t_{342}} \quad Ut_{342} \quad = \gamma_2 \frac{\partial f}{\partial t_{232}} \quad Ut_{232}$

[(4) and (6)] ==> $t_{342} = t_{122}$

$\therefore \lambda_4 < \lambda_3$

4. To show $\lambda_4 < \lambda_1$:

$\lambda_4 \gtreqless \lambda_1$

If $\lambda_4 > \lambda_1$, $t_{s4} < t_{s1}$ [(13) and (17)] and

$t_{342} > t_{111}$ (A).

But if $\lambda_4 > \lambda_1$, $\gamma_2 \frac{\partial f}{\partial t_{342}} \quad Ut_{342} \quad > \gamma_1 \frac{\partial f}{\partial t_{331}} \quad Ut_{331}$

[(5) and (6)].

Since $\gamma_2 < \gamma_1$ by assumption, $\lambda_4 > \lambda_1 \implies t_{342} < t_{111}$

$\therefore \lambda_4 \leq \lambda_1$

If $\lambda_4 = \lambda_1$, $t_{s4} = t_{s1}$ [(13) and (17)] and

$t_{342} = t_{111}$ (A).

But if $\lambda_4 = \lambda_1$, $\gamma_2 \frac{\partial f}{\partial t_{342}} \quad Ut_{342} = \gamma_1 \frac{\partial f}{\partial t_{111}} \quad Ut_{111}$

[(1) and (6)].

Since $\gamma_2 < \gamma_1$ by assumption, $\lambda_4 = \lambda_1 \implies t_{342} < t_{111}$

$\therefore \lambda_4 < \lambda_1$

$\therefore \lambda_4 < \lambda_1 < \lambda_2 = \lambda_3$ Q.E.D.

For the case of 2 efficiency parameters, but m periods:

$$\lambda_m < \lambda_1 < \lambda_2 = \lambda_3 = \ldots = \lambda_{m-1} .$$

For the case of 3 efficiency parameters, and m periods:

$$\lambda_m < \lambda_1 < \lambda_{m-1} < \lambda_2 < \lambda_3 = \ldots = \lambda_{m-2} .$$

To show $\dfrac{t_{111}}{t_{342}} > \dfrac{t_{331}}{t_{122}}$:

From (1) and (5)

$$\frac{\lambda_1}{\lambda_3} = \frac{\gamma_1 \frac{\partial f}{\partial t_{111}} \quad Ut_{111}}{\gamma_1 \frac{\partial f}{\partial t_{331}} \quad Ut_{221}}$$

From (6) and (2)

$$\frac{\lambda_4}{\lambda_2} = \frac{\gamma_2 \dfrac{\partial f}{\partial t_{342}} \quad U t_{342}}{\gamma_2 \dfrac{\partial f}{\partial t_{122}} \quad U t_{122}}$$

Dividing, and since $\lambda_2 = \lambda_3$

$$\frac{\lambda_4}{\lambda_1} = \frac{\dfrac{\partial f/\partial t_{342}}{\partial f/\partial t_{122}}}{\dfrac{\partial f/\partial t_{111}}{\partial f/\partial t_{331}}} = \frac{\dfrac{\partial f/\partial t_{342}}{\partial f/\partial t_{111}}}{\dfrac{\partial f/\partial t_{122}}{\partial f/\partial t_{331}}}$$

Since $\lambda_4 < \lambda_1$, $\dfrac{\partial f/\partial t_{342}}{\partial f/\partial t_{111}} < \dfrac{\partial f/\partial t_{122}}{\partial f/\partial t_{331}}$

so that $\dfrac{t_{111}}{t_{342}} > \dfrac{t_{331}}{t_{122}}$ Q.E.D.