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CREDIT AND SHARECROPPING IN AGRARIAN SOCIETIES

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1. Introduction

A commonly observed feature of agrarian societies in their early stage of development is share-cropping together with a credit arrangement in which landlords provide credit (for consumption, working capital as well as investment) to their share croppers.[1] The extensive literature on sharecropping has not satisfactorily addressed the nature of equilibrium in land, labour and credit markets in such a context.[2] Further, the fact of credit linkage between a landlord and his share-cropper was viewed as a form of exploitation of tenants by landlords.[3]

The purpose of this paper is two fold:

(a) to derive and characterize the equilibrium in a model of a land-scarce, labour-abundant economy under share-cropping, given an infinitely elastic supply of identical share-croppers at a reservation utility. The reservation utility may be determined either by subsistence considerations or by employment opportunities available to a potential share cropper elsewhere in the economy.

(b) to demonstrate that in an imperfect credit market, a landlord may offer credit to his tenant, sometimes even at a subsidized rate of interest, without necessarily insisting that the share-cropper borrow only from him thus precluding an <u>involuntary</u> (from the point of view of the tenant) linkage between credit and land transactions.[4] However, any legally or socially imposed constraints on tenant's share (as for instance, a floor) may provide incentives for a credit-tenancy linkage that may otherwise be absent.

In the following sections we concentrate on a model of linkage between land, labor and credit transactions in the context of sharecropping.[5] In order to explore the implications of policies such as land reform, subsidized credit, taxation and the outlawing of moneylending by landlords, we take it as given that the only form of tenancy is sharecropping.[6] Other crucial assumptions are that a potential tenant is precluded, as part of the tenancy contract, from working outside the farm as a <u>part-time</u> wage laborer and that there are imperfections in the capital market in the form of differing costs of capital to the landlord and to the tenant.

One major conclusion of the paper is valid both in context of credit-cum tenancy contracts and in that of sharecropping contracts alone. It states that, as long as the landlord can vary the size of the plot given to a tenant and there are enough potential tenants, the equilibrium will be characterized by 'utility-equivalent' contracts even if the landlords do not possess any other instrument (e.g., share rent, interest rate). That is, in equilibrium, a tenant's utility obtained through sharecropping will be the same as that which he could have obtained as a full-time wage laborer. Newbery and Stiglitz [1979] assert, without providing a satisfactory proof, the same result in the context of sharecropping alone, while a similar, though not identical, conclusion has been obtained in a different setting by Cheung [1969]. Our proof follows from our result that, _ceteris paribus_, the tenant's optimal effort per hectare is a _decreasing_ function of the size of the plot he cultivates. Our model excludes the possibilities of rationing equilibria in which a tenant obtains a utility level exceeding his reservation utility.[7]

The utility equivalence result has the fundamental implication that policies _other_ than land reform (i.e., reform that confers ownership to the tenant of the piece of land he is cultivating) will leave the welfare of each potential tenant unaltered while affecting the level of output, extent of tenancy and the welfare of landlords.

With the possibility of landlords providing their tenants with credit, it is shown that landlords will resort to that option only if their opportunity cost of capital is lower than the tenants' opportunity cost of capital. If the government offers the tenant subsidized credit at a cost lower than the landlord's opportunity cost of funds, the landlord will move out of the tenant's credit market and allow the tenant to borrow from the government. The increase in surplus due to government subsidization of tenant's credit will _fully_ accrue to the landlord as a consequence of the utility equivalence result. Hence, _government_ subsidization of tenant's credit results only in the subsidization of landlords. Other partial reforms by the government, however, may force the landlord to tie credit and tenancy contracts (even if the government provides the cheaper source of credit) thereby, leaving the tenant's utility unaltered at its pre-reform level while affecting total output and the extent of tenancy. Our model thus provides one theoretical explanation for two almost opposite phenomena that are sometimes observed: low interest consumption loans from landlord to tenant and the opposite, high interest, low volume loans.

We present the model in Section 2, followed by a characterization of the equilibrium in Section 3. Section 4 discusses policies of credit, land and tenancy reforms as well as the impact of taxation and technical progress.

2. The Model

The tenant's choices are limited to the decision to be a sharecropper or not and the level of his work effort, if he decides to be a sharecropper. The landlord has at least one choice variable (plot size) and at most four choice variables: plot size, share rent, interest rate and the amount of tied credit with land contract. The principal constraints are (1) an exogenously available level of utility for tenants at which the supply of tenants is perfectly elastic and (2) tenants and landlords are not free to mix contracts. Given the tenant's choice behavior, the landlord is a Von-Stackelberg maximizer of profits. Formally, we shall first describe the tenant's and landlord's problems and then the equilibrium.

2.1 The Tenant

All workers are identical facing two employment alternatives: first as tenants on landlord's land, or secondly, as wage labourers elsewhere. They cannot mix contracts.[8] Each tenant is offered a plot of land, of size H hectares, in return for which he agrees to pay the landlord a share $(1-\alpha)$ of the harvest. None of the workers possess any savings at the beginning of the production period. Wage workers are paid during the production period and, therefore, have no need to borrow for consumption. The tenant, however, borrows at the beginning of each season his entire consumption needs for the coming season and repays his loan with interest at the end of the season after harvest. He does not store any grain from one season to the next, nor does he have any investment opportunities.[9]

The tenant obtains a proportion ν of his borrowings (either voluntarily or as a part of a "tie-in" package with a tenancy contract) from his landlord at an interest rate r_T per season. He obtains the remaining proportion $(1-\nu)$ of his borrowings from an alternative source (e.g., local moneylender, cooperative, government credit agency) at an interest rate r_A . He treats r_T and r_A as parameters over which he has no influence. We assume that he cannot default partly to simplify the argumentation, and partly because in many areas landlords virtually hold the harvested crop as collateral , thus precluding default. Clearly, if the tenant can borrow the entire present value of his consumption at either r_T or r_A, he will choose to borrow it from the cheaper source.

However, since our discussion focusses on tie-in contracts, we start by assuming that the tenant takes ν as given, so that $\nu > 0$ will represent a tie-in condition over which he has no influence.

Labor provided by the tenant for cultivation (including all operations from land preparation to harvesting) is denoted by eL, where L denotes the number of man-years per season and e denotes the effort per man-year of labor. Thus, eL represents labor in efficiency units. Output Q is a concave function, homogenous of degree one in H and eL.[10] Thus:

$$Q = F(H, eL) \tag{1}$$

Assuming the number of man-year, L, (i.e., labour in natural nunits) to be exogenously fixed, we can set (without loss of generality) $L = 1$. Thus we can rewrite (1) as:

$$Q = \frac{1}{x} F(1, ex) \equiv \frac{f(ex)}{x} \tag{2}$$

where x is man-years of labour per hectare of land, Given that the tenant is endowed with one man-year of labour, x represents the reciprocal of the size of the plot he is allotted. The function f represents the average product per hectare of land. By assumption, f' is positive and f'' is negative where the primes (single and double) denote the first and second derivatives of f, respectively. The tenant's share of the harvest Q is α and his income is therefore αQ.

By our assumption that the tenant borrows his entire consumption needs at the beginning of the season and has no carry-over stock or investment opportunities, it follows that his consumption c in any season equals his income αQ at the end of the season, discounted by $(1 + i)$ where i is the effective interest rate on his borrowing. Of course, i equals $\nu r_T + (1 - \nu)r_A$. Thus:

$$c = \frac{\alpha Q}{1 + \nu r_T + (1 - \nu)r_A} \equiv \beta Q \; . \tag{3}$$

where $$\beta = \frac{\alpha}{1 + \nu r_T + (1 - \nu)r_A} \equiv \text{discounted share of the tenant.} \tag{4}$$

We assume that the tenant's utility function $U(c,e)$ is strictly quasi concave in consumption and leisure, where leisure is defined as $\tilde{e} \equiv -e$. Furthermore, we assume that both consumption and leisure are normal goods.

The tenant's choice or control variable is e. He will not choose to work as a tenant unless $U(c, e)$ is at least as large as $\bar{U}$, the utility he could have assured himself by working as a wage laborer $\bar{U}$ is exogenously given implying that the supply of tenants is infinitely elastic at $\bar{U}$. Thus we can solve his choice problem in two steps. First, let the maximized value of $U(c, e)$ subject to (3) be U^*. If $U^* \geq \bar{U}$, he would work as a tenant, otherwise, as a wage laborer. Thus, the tenant's maximization problem is

$$\max_{e} \; U(c(e),e) \quad \text{s.t. (2) and (3)} \tag{5}$$

It is immediately apparent from (2)-(5) that the parameters α, ν, r_T and r_A enter the tenant's constraint set and utility function only through their effect on his discounted share β. By substituting (2), (3) and (4) in (5), maximizing with respect to e, we get the first order condition:

$$\beta U_1 f'(ex) + U_2 = 0 \tag{6}$$

It can be shown that the second order condition is satisfied from our strict quasi-concavity assumption on U, and the strict concavity of f (see appendix). We note also that (6) can be solved uniquely[11] for e to yield

$$e = e(x, \beta) \tag{7}$$

Define effort per acre as $z \equiv ex$. It follows (see appendix) that:

$$\frac{\partial z}{\partial x} = \frac{1}{x^2 U_1} \cdot \frac{\beta f\left(U_{21}U_1 - U_2U_{11}\right) + z\left(U_{22}U_1 - U_2U_{12}\right)}{\beta f''U_1 + \frac{1}{x^2U_1^2}\left(U_{11}U_2^2 - 2U_{12}U_1U_2 + U_{22}U_1^2\right)} \tag{8}$$

<u>Lemma</u> If U is strictly quasi-concave in $(c,-e)$, f is strictly concave in e, and c and $-e$ are normal goods, then $\frac{\partial z}{\partial x} > 0$.

<u>Proof:</u> Strict quasi-concavity of U and strict concavity of f imply that the denominator of (8) is negative. The normality conditions for c and $-e$ are:

$$U_{12}U_1 - U_2U_{11} < 0 \tag{9}$$

and

$$U_{22}U_1 - U_2U_{12} < 0 \tag{10}$$

They imply that the numerator is negative as well. Hence, $\frac{\partial z}{\partial x} > 0$. Q.E.D.

This lemma states that the tenant's *effort per acre increases with a reduction in his plot* size even if the tenant's effort declines with such a reduction in plot size (increase in x).

Now,

$$\frac{\partial z}{\partial \beta} = - \frac{f'U_1 + \frac{f}{x}\left(\frac{U_1U_{21} - U_2U_{11}}{U_1}\right)}{\beta\, U_1 f'' + \frac{1}{xU_1^2}\left(U_2^2\, U_{11} - 2U_{12}U_1U_2 + U_{22}\, U_1^2\right)} \tag{11}$$

This expression cannot be signed. Hence, effort per acre may either increase or decrease with a <u>ceteris paribus</u> increase in tenant's share. Denoting by U^* the maximized value of $U(c, e)$, it can be shown (noting (6)) that:

$$\frac{\partial U^*}{\partial x} = - \frac{\beta U_1(f(z) - zf')}{x^2} < 0 \tag{12}$$

$$\frac{\partial U^*}{\partial \beta} = U_1 \frac{f}{x} > 0 \tag{13}$$

i.e., *ceteris paribus,* an increase in the plot size and/or the discounted share make the tenant better-off.

2.2 The Landlord

With an infinitely elastic supply of identical tenants, and constant returns to scale in production, maximizing profits is equivalent to maximizing profits per hectare. Hence, our model yields the same results whether different landlords possess different amounts of land or not. Therefore, without loss of generality, we assume that all landlords are identical and possess one hectare of land each, which they divide into plots of size $1/x$ to give each of x tenants. As stated earlier, the landlord may require that each of his tenants get a proportion ν of his borrowings from him at an interest rate r_T. Assuming that an alternative use of funds would have earned the landlord an interest of r_L per season (e.g. deposits in the city's bank), his income g from each tenant is given by:

$$g = \frac{(1 - \alpha)f(ex)}{x} + \nu(r_T - r_L)c$$

$$= \frac{(1 - \alpha)}{x} f(ex) + \nu\,(r_T - r_L)\beta\,\frac{f(ex)}{x} \qquad \text{using (2) and (3)}$$

$$= \frac{f(ex)}{x}\,[(1 - \alpha) + \nu(r_T - r_L)\beta]$$

$$= \frac{f(ex)}{x}\,[1 - \beta\{1 + \nu r_L + (1 - \nu)r_A\}] \qquad \text{using (2), (3) and (4)}$$

Multiplying g by the number, x, of tenants we get the landlord's income G :

$$G = [1 - \beta\{1 + \nu r_L + (1 - \nu)r_A\}f(ex) \quad (14)$$

It is clear from (14) that the interest rate r_T charged by the landlord on his loans to his tenant affects his income only through its effect on β, the discounted share.

The landlord maximizes G with respect to his choice variables given the tenant's effort function $e(x, \beta)$. The choice variables include the plot size $1/x$, and may include the tenant's crop share α, ν (if there are no laws against the landlord providing credit) and r_T, the rate of interest charged.

3. Utility Equivalence and other equilibrium Properties

The equilibrium presented here is a contractual equilibrium, i.e. there is demand and supply for contracts, where a contract consists of a package including plot size, crop share, interest rate and tie-in condition. It is not a competitive equilibrium since the level of tenant's reservation utility is exogenously given, (e.g. by subsistence factors) and, hence, the landlord is facing a profit maximization problem subject to an inequality constraint on tenant's reservation utility. A competitive contractual equilibrium, on the other hand, is characterized by landlord's profit maximization subject to equality constraint on tenant's utility, where this utility level is generated by the competitive market forces.[12]

For the moment, let us focus only on the choice of x (the number of tenants or, equivalently, the plot size per tenant), thus keeping β fixed in particular. Since f is an increasing function of its argument $ex \equiv z$, and since z is an increasing function of x [see (8)], the landlord's income (14) *increases* with x: in other words, a decrease in the tenant's plot size, which therefore leads to the hiring of more tenants, increases the landlord's profits. On the other hand, it follows from (12) that a tenant's utility U^* in sharecropping *decreases* as x increases. Thus, if at any value of x the tenant's utility exceeds his utility $\bar{U}$ in the alternative use of his labor (so that he chooses to be a tenant), the landlord, by increasing x, can increase his income while pushing the tenant towards $\bar{U}$. As long as there are enough potential tenants, that is, as long as there is no upper limit on x, the landlord's choice x will be to push the tenant to a utility level equalling $\bar{U}$.[13] Hence we can state the following basic proposition.

Proposition 1: The equilibrium in the land-labor market will be characterized by utility equivalent contracts.

It should be noted that this proposition does not depend for its validity on the presence or absence of any linkage between tenancy and credit transactions. The landlord's use of plot size as his <u>sole</u> instrument variable is sufficient to result in a utility equivalent contract equilibrium, an outcome obtained by Cheung [1969] under a different structure. Our structure is that initiated by Stiglitz [1974] and utilized by Newbery and Stiglitz [1978]. Assuming a separable utility function, they claimed (Newbery-Stiglitz [1979, p.16]), that competition between landlords will eliminate the less attractive contracts and will drive the inequality $U^* > \bar{U}$ to equality thereby achieving utility equivalence. As demonstrated in Proposition 1, the utility equivalence outcome results from profit maximization and not from competition. Nor is the proposition trivial, arising solely from the fact that there is an infinitely elastic supply of potential tenants at $\bar{U}$, since the possibility of an excess applicants equilibrium at $U^* > \bar{U}$ can occur if only the output share instead of the plot size is the control variable of the landlord. A well-known case of excess applicants equilibrium arose under the efficiency wage hypothesis (e.g., see Leibenstein [1957], Mirrlees [1976] and Stiglitz [1976]), primarily because the landlord is not allowed to use an instrument completely orthogonal to effort to reduce U^* to $\bar{U}$ without affecting effort. In our model, the use of the power to vary the plot size, although non-orthogonal to effort, guarantees the utility equivalent contract result since the tenant's effort per acre increases with a reduction in his plot size.[14] Additional instruments such as cropshare and interest rate are not needed for this purpose.

Of the two assumptions used in deriving our result, namely, that both consumption and leisure are normal goods, and that the tenant is prohibited, as part of his contract, from working as a part-time laborer outside the farm, the latter is perhaps more controversial. Its realism is primarily an empirical issue. It is true that tenants often work as part-time laborers, but the extent of such work is limited. There is also some evidence to suggest that landlords believe that a tenant will put greater effort into cultivation, the smaller his plot size.

From the utility equivalence

$$U\,\{c(x,\beta),\ e(x,\beta)\} = \bar{U} \tag{15}$$

where

$$c(x,\beta) = \beta\,\frac{f\{e(x,\beta)x\}}{x}$$

we can solve for x(the inverse of the plot size) as a function $x(\beta)$ of the discounted share, β. By appropriate differentiation of (15) (see Appendix) we obtain:

$$\frac{dx}{d\beta} = \frac{fx}{\beta(f-f'z)} > 0\ . \tag{16}$$

i.e., in order to maintain the tenant on his iso-utility curve, the landlord must increase the tenant's discounted share if he reduces the plot size. Thus, from now on when analyzing changes in β, unless otherwise specified, we assume that the landlord changes x along the curve $x(\beta)$ so as to maintain the tenant at a welfare level of $\bar{U}$.

Now, denote $\sigma \equiv -\dfrac{f'(f-zf')}{ff''z}$ as the elasticity of substitution between effective labor, e, and land.

It is shown in the Appendix that.

$$\frac{de\{x(\beta),\beta\}}{d\beta} = \frac{\partial e}{\partial x}\,\frac{dx}{d\beta} + \frac{\partial e}{\partial \beta} = \frac{U_1 f'}{\beta f''U_1 + \frac{1}{xU_1^2}\,\{U_{11}U_2^2 - 2U_{12}U_2U_1 + U_{22}U_1^2\}}\ \frac{(1-\sigma)}{\sigma} \tag{17}$$

Hence:

Proposition 2: The tenant's effort e increases, stays the same, or decreases as his discounted share β in output increases, according as the elasticity of substitution σ is greater than, equal to or less than unity.

It is shown further in the Appendix, that effort per hectare, z, satisfies:

$$\frac{dz}{d\beta} > 0 \tag{18}$$

Newbery and Stiglitz [1979] derived (17) assuming a separable utility function in a model that did not feature credit. However, all the results derived so far do not utilize the credit features of the model.

Turning now to the other choice variables of the landlord, (α, ν, r_T), it can be shown by writing his income as

$$G = (1 - \beta\theta)f(ex) \tag{19}$$

where $\theta = 1 + \nu r_L + (1 - \nu)r_A$, that (α, ν, r_T) enter G only through their effect on β and θ, since e and x are functions of β only. Now

$$\frac{\partial G}{\partial \theta} = -\beta f < 0. \tag{20}$$

This means that an income maximizing landlord will choose his optimal θ to be:

$$\theta^* = \text{Minimum feasible } \theta \text{ for any given } \beta \tag{21}$$

and then choose β to maximize $(1-\beta\theta^*)f(ex)$. Since θ depends only on ν (which lies between 0 and 1), if the given value of β does not restrict the choice of ν, then:

$$\left.\begin{aligned} \theta^* = (1 + r_L) \quad &\text{and} \quad \nu^* = 1 \quad \text{if} \quad r_L \leq r_A \\ (1 + r_A) \quad &\text{and} \quad \nu = 0 \quad \text{if} \quad r_L > r_A \end{aligned}\right\} \tag{22}$$

Thus, to minimize θ is to give a weight of 1 to the smaller interest rate, and a weight of 0 to the larger one.

Now, by definition, $\beta = \dfrac{\alpha}{1 + \nu r_T + (1-\nu)r_A}$. The range for β for feasible (α, ν, r_T) (i.e., $0 \leq \alpha \leq 1$, $0 \leq \nu \leq 1$, $r_T \geq 0$)

is therefore $[0, 1]$. And any $\beta \epsilon [0, 1]$ can be reached by a suitable choice of (α, r_T) if $\nu = 1$. This holds true even if there is an institutionally specified floor α_F on α. Thus, in the case $r_L \leq r_A$, the landlord can set $\nu^* = 1$ and $\theta^* = 1 + r_L$ and choose β, (that is α and r_T) to maximize G. In essence, what is happening is that, with $r_L \leq r_A$ the landlord is the cheaper source of credit and by offering credit with tenancy (setting $\nu^* = 1$) the landlord ensures that the tenant uses the cheaper source of credit.

If $\nu = 0$, then values of $\beta > \frac{1}{1+r_A}$ are not attainable through choice of α. Now with $\nu = 0$, any β in $[0, \frac{1}{1 + r_A}]$ can be reached by a suitable choice of α <u>as long as there is no floor on α</u>. And $\beta > \frac{1}{1 + r_A}$ is irrelevant for maximizing G when $r_L > r_A$ since then $\theta \geq 1 + r_A$ so that $\beta\theta > 1$ making $G \leq 0$. Thus we can assert, using (22), that the landlord's optimal choice is $\nu^* = 0$ if $r_L > r_A$. Once again, the landlord ensures that the tenant gets credit from the cheaper source. We can therefore state:

Proposition 3: The landlord, with no restriction on his choice of crop shares, will ensure that the tenant gets credit from the cheaper source. In the event that he is the cheaper source $(r_L \leq r_A)$, he does this by offering a tenancy contract with credit. In the case where $r_A > r_L$, he does this by not offering any credit to the tenant.

Remark: As discussed above, in the case of $r_L \leq r_A$ where offering credit is optimal, it remains optimal even if there is an institutionally imposed floor on the tenant's crop share, the reason being that any given $\beta \equiv \frac{\alpha}{1+r_T}$ (and *a fortiori* the optimal β) can be achieved with an infinite number of pairs (α, r_T), of which, another infinite set will meet the required floor.

Proposition 3 is consistent with empirical observations (Bardhan and Rudra (19)) that landlords frequently offer interest-free loans to their tenants. For example, in the case of $r_L \leq r_A$, with $v^* = 1$, the interest rate r_T charged by the landlord is essentially arbitrary, and it could as well be zero. Hence, if there is no floor on α, the situation observed is not really one of tie-in, since the parties can untie the transactions without altering the outcome. This will not be the case, however, if the environment faced by the parties is subject to certain constraints such as government regulations. This topic will be covered in the next section.

Returning to the case where there is no floor on α_F, we have seen that if $r_L \leq r_A$, $\theta^* = (1 + r_L)$ and with $G = [1 - \beta(1 + r_L)]f(ex)$, the range for β is $[0, 1/(1 + r_L)]$. If $r_L > r_A$, $\theta^* = (1 + r_A)$, with $G = [1 - \beta(1 + r_A)]f(ex)$, the range for β is $[0, 1/(1 + r_A)]$. In either case, G, being a continuous function of β, defined over a compact set, attains its maximum. If this maximum is attained at an interior point, we have

$$\frac{\partial G}{\partial \beta} = -\theta^* f + (1 - \beta^*\theta^*) f' \frac{d}{d\beta}(ex) = 0$$

$$-\theta^* f + (1 - \beta^*\theta^*) f' [e\frac{dx}{d\beta} + x\frac{de}{d\beta}] = 0$$

or

$$\frac{\beta^*\theta^*}{1 - \beta^*\theta^*} = \beta^*\frac{f'}{f}[\frac{fex}{\beta^*(f - exf')}] + \beta^* x\frac{f'}{f}\frac{de}{d\beta} \qquad \text{using (16)}$$

or

$$\frac{\beta^*\theta^*}{1 - \beta^*\theta^*} = \frac{S}{1 - S} + \beta^*\frac{f'}{f}\frac{de}{d\beta} \quad \text{where } S = \frac{exf'}{f} \text{ is the imputed share}$$

of labour in crop output. Using Proposition 2, we can assert that

$$\beta^*\theta^* \gtreqless S \quad \text{according as} \quad \sigma \gtreqless 1 \tag{23}$$

In the case where $r_L \leq r_A$, $\theta^* = (1 + r_L)$ and $\beta^* = \alpha^*/(1 + r_T)$, and in the case where $r_L > r_A$, $\theta^* = (1 + r_A)$ and $\beta^* = \alpha^*/(1 + r_A)$. Since in the first case r_T can be chosen to be r_L , $\beta^*\theta^*$ becomes the crop share α^* in either case. So using (22) we can state:

Proposition 4: If there is no restriction on the landlord's choice of instruments (α , ν , r_T) , and optimal strategy for him involves his offering his tenant a crop share α^* such that $\alpha^* \gtreqless S$ according as $\sigma \gtreqless 1$.

Remark: **In the case of $r_L \leq r_A$ since $\beta^*\theta^* = \dfrac{\alpha^*(1 + r_L)}{(1 + r_T)}$, by choosing (α^* , r_T) with r_T sufficiently less (greater) than r_L , the landlord can offer an α^* which is less (greater) than S , even if σ is greater (less) than unity.**

Newbery and Stiglitz [1979] established Proposition 4 without incorporating credit or its linkage to tenancy. The above remark extends their result to a case where it is optimal for the tenant to borrow from his landlord. It also implies that it is possible to observe crop shares lower than the imputed share of labor even for a production function with an elasticity of substitution larger than 1.

4. Policy Analysis

4.1 Tenancy Reforms

First, consider a reform which imposes a floor, α_F , on the tenant's share α of the harvest. This is a common feature of many agrarian reform laws in India. As discussed earlier in the case where $r_L \leq r_A$, if in an equilibrium $(\alpha^* , 1 , r_T^*)$ prior to the promulgation of the reform law the landlord was offering a crop share below the legal floor α_F , he will raise the crop share after its promulgation to α_F and at the same time raise the interest rate to r_T^{**} so that in the new equilibrium $(\alpha_F, 1, r_T^{**})$, $\frac{\alpha_F}{1 + r_T^{**}} = \frac{\alpha^*}{1 + r_T^*} = \beta^*$. Since output depends only on β^*, it is unaffected by reform. Given utility equivalence, the tenant's welfare is unaffected anyway.

Suppose now that the legal floor is imposed. Consider the following two alternatives: (i) an initial equilibrium in which the landlord is not the cheaper source of credit, i.e., $r_L > r_A$ so that $\nu^* = 0$, $\beta^* = \frac{\alpha^*}{1 + r_A}$ with $\alpha^* < \alpha_F$, or (ii) initially $r_L \leq r_A$ and $\nu^* = 1$, $\beta^* = \frac{\alpha^*}{1 + r_T}$ with $\alpha^* < \alpha_F$. However, as part of a tenancy reform, the interest rate on the tenant's alternative source of credit is brought below r_L. In other words, along with the floor α_F, there is a change in r_A which brings it below r_L. This joint reform of tenancy and credit, could be viewed as two consecutive reforms, *first* a credit reform with no tenancy reform, so that the landlord switches to the equilibrium with one asterisk from one with two asterisks and *then* to a tenancy reform imposing a floor. This way, it suffices to discuss only the tenancy reform.

In such a situation the landlord can partially nullify the tenancy reform by *forcibly* tying the credit and tenancy contracts. In a technical sense, even in this case, the tenancy reform may be made ineffective. For example, consider a sequence of contracts offered to the tenant, the sequence indexed by n: $(\alpha^n = \alpha_F, \gamma^n = \{\alpha_F - \beta^*(1+r_A)\}/n\beta^*, r_T^n = n)$. Clearly, $\nu^n \geq 0$ since in the initial equilibrium $\beta^*(1+r_A) = \alpha^* < \alpha_F$ and for large enough n, ν^n will be less than one. Thus, for large enough n, each member of the sequence is a feasible contract. Now $\beta^n = \frac{\alpha^n}{1 + \nu^n r_T^n + (1-\nu^n) r_A}$. The plot size sequence is $x(\beta^n)$. As

$n \to \infty$, α^n converges to α_F, ν^n converges to zero, $\beta^n \to \beta^*$ and $r_T^n \to \infty$. By choosing n sufficiently large (thereby making r_T^n large, but finite), the landlord can remain as close as he wishes to his income prior to the imposition of the floor even after the reform![15] What this argument suggests is that, after the reform there is no *optimal* policy for the landlord, but there exist policies that will give him an income as close as he wishes to his income *prior* to reform. Since, prior to reform, he was maximising his income without the floor constraint on the tenant's crop share, that income provides an upper bound to his income after reform. Since policies exist, which get as close as one likes to this upper bound, this upper bound is the least upper bound.

The implication of the above discussion is that, if tying is permitted, the landlord can reduce the tenancy and credit reform to insignificance. Suppose now that the government bans tying , *along* with tenancy and credit reforms. Clearly the landlord's income will decline, while the tenant's welfare continues to be at the level he could have achieved while working as a wage labourer. What about the effect on output? Since the landlord no longer has the instrument by which he can maintain the pre-reform discounted share, β^*, of the tenant, the reform will raise β. Since we know from (18) that $d(z)/d\beta > 0$, we can assert that output $f(z)$ will go up.[16] Thus:

Proposition 5: A tenancy reform which imposes a floor on the tenant's share of the crop with or without credit reform (to make credit available to the tenant at a rate lower than the landlord's opportunity cost of capital), will have no effect on output. If it is coupled with a ban on tying of credit and tenancy transactions, it will raise output, reduce the tenant's plot size and increase the number of tenants.

Now consider only a ban on tying of credit and tenancy. This is, of course, meaningless when the landlord is not the cheaper source of credit, since no tying will be observed anyway. Suppose the ban is imposed when the landlord is offering credit, i.e., when $r_L \leq r_A$ and $v^* = 1$.

Clearly, this immediately raises the cost of credit to the tenant to r_A. In the landlord's income maximization problem, fixing v at zero (i.e., preventing linking), fixes θ at $(1 + r_A)$, i.e., raises θ from its optimal value of $1 + r_L$ prior to the ban to $(1+ r_A)$. Since G is a monotonic decreasing function of θ, at any *value* of β, G is lower than before. Clearly, even with the optimal value of β, G is lower. This means that landlord's income definitely goes down. What about output? As long as $f(z)$ as a function of β is concave, optimal β for any specified θ is a decreasing function of θ. Hence, as θ is increased from $(1 + r_L)$ to $(1 + r_A)$, optimal β goes down. This means that firstly, the optimal plot size increases thereby reducing the number of tenants and secondly, output goes down since $f(z)$ is an increasing function of β.

4.2 Land Reform

Suppose starting from an initial equilibrium $[\alpha^*, v^*, r_T^*]$ and $x(\beta^*)$, each tenant is given the ownership of the plot he cultivates and has to forego the opportunity to borrow from one landlord. Clearly, the tenant's welfare improves, for if $r_L > r_A$, $v^* = 0$ and $\beta^* = \dfrac{\alpha^*}{1 + r_A}$. With reform α becomes unity, r_A remains unchanged so that the tenant's (now a land-owning peasant's) discounted share β increases, while the size of the plot remains the same. Hence, without changing, his effort e, (and its dis-utility) he will gain in consumption and, hence, total utility. By optimally adjusting his effort to the changed β, he can raise his utility even further.

Now if $r_L \leq r_A$, initially $\nu^* = 1$. Since, the landlord is indifferent in this case between alternative combinations of (α, r_T) which result in his optimal β^*, we can view the land reform, as if it first changed the interest rate charged by the landlord to r_A with a corresponding change in α to maintain the same β^* and then raised the tenant's crop share to unity. The two moves together imply that the tenant's post-reform discounted share is higher. From this point, the argument is the same as in the previous case.

What about the effect of land reform on output? Land reform increases the discounted share β while keeping the plot size fixed. Thus, output is $f[e(\beta)x]$ where x is fixed. Since the former tenant will choose e to maximize his utility, given any β and x, we know from equation (A.13) in the Appendix that

$$\frac{\partial e}{\partial \beta} = -\frac{f'U_1 + \frac{f}{xU_1}\{U_{21}U_1 - U_{11}\,U_2\}}{\beta x\; f''U_1 + \frac{1}{(xU_1)^2}\{U_2^2U_{11} - 2U_{12}U_1U_2 + U_{22}U_1^2\}} = \tag{24}$$

$$= -\frac{1}{\Delta}\, f'U_1\{1 + \frac{\beta f}{x}\,(\frac{U_{11}}{U_1} - \frac{U_{21}}{U_2})\}$$

where Δ denotes the negative denominator. Now, $\frac{\beta f}{x}$ is the consumption of the tenant, Hence

$$\frac{\partial e}{\partial \beta} \gtreqless 0 \quad \text{according as}$$

$$-c\left(\frac{U_{11}}{U_1} - \frac{U_{21}}{U_2}\right) \lesseqgtr 1 . \tag{25}$$

$\frac{\partial e}{\partial \beta} \gtreqless 0$ implies that output increases, remains unchanged or decreases as β increases. Thus:

Proposition 6: A land reform which confers ownership to the plot of land that a tenant used to cultivate in a sharecropping contract with a landlord will increase, not change, or decrease output, according as $-c\left(\frac{U_{11}}{U_1} - \frac{U_{21}}{U_2}\right) \lesseqgtr 1$.

(25) represents the elasticity of the marginal rate of substitution between consumption and leisure with respect to consumption. The tenant maximizes $U[c, e]$ subject to $c = \beta \frac{f(xe)}{x}$. Now the marginal rate of substitution (MRS) between c and e in U is $-\frac{U_c}{U_e}$ and the marginal rate of transformation (MRT) between c and e through production is $\frac{dc}{de} = \beta f'(xe)$.

$$\text{At } \underline{\text{given}} \ x \text{ and } e, \quad \frac{d \operatorname{Log} (MRT)}{d \operatorname{Log} \beta} = 1$$

and
$$\frac{d \operatorname{Log} (MRS)}{d \operatorname{Log} \beta} = \frac{d \operatorname{Log} (MRS)}{d \operatorname{Log} c} \frac{d \operatorname{Log} c}{d \operatorname{Log} \beta}$$

$$= \frac{d \operatorname{Log} (MRS)}{d \operatorname{Log} c} = - c\left(\frac{U_{11}}{U_1} - \frac{U_{21}}{U_2}\right)$$

Since for optimality MRT = MRS, the impact of a change in β on e is obtained by a comparison of the two elasticities.

Furthermore, consider the case of a separable utility function, i.e., $U(c, e) = u(c) - v(e)$. Then (25) becomes

$$\frac{\partial e}{\partial \beta} \gtreqless 0 \quad \text{according as} \quad - \frac{u''c}{u'} \lesseqgtr 1 \ . \tag{26}$$

The negative of the elasticity of marginal utility $(\frac{u''c}{u'})$ is defined by Arrow [1971] as the measure of relative risk aversion. The intuitive explanation for the value of this elasticity to be of relevance in our case, even though there is no uncertainty, is the following: On the one hand, an increase in β increases tenant's income; hence, the marginal utility of income declines relative to the marginal disutility of effort, and *ceteris paribus,* the new landowner would like to reduce his effort. On the other hand, his share in the marginal productivity of effort increases, with increasing β, thus creating an incentive for more effort. Whether the income effect or the marginal productivity effect is the dominant force depends solely on the elasticity of the marginal utility.

Where land reform distributes the land to more owners than the original cultivators, it may increase total output even if $-c\left(\frac{U_{11}}{U_1} - \frac{U_{21}}{U_2}\right) > 1$ since *ceteris paribus,* output per hectare increases with reductions in plot size.

4.3 Taxation and Technological Progress

Suppose the government imposes a proportional output tax at the rate t on tenants and landlords (i.e., the rural community) in order to raise food to feed the urban workers. Since for any β this tax is equivalent to reducing the discounted share of the tenant from β to $\mu \equiv \beta(1-t)$, the tenant's decision function $e(x, \beta)$ becomes $e(x, \mu)$. It is also easily seen that the landlord's choice set $x(\beta)$ becomes $x(\mu)$. Thus, for any given β (i.e. before tax share of the tenant), the aftertax income of the landlord is:

$$G = (1-t)\ (1-\beta\theta^*)f(z)$$

where $z = e[x(\mu),\mu]\ x(\mu) = z(\mu)$

and $\theta^* = 1 + r_L$ if $r_L \leq r_A$

$= 1 + r_A$ if $r_L > r_A$.

The landlord chooses β to maximize G, implying that:

$$G_\beta = \frac{dG}{d\beta} = (1-t)\ [-\theta^* f + (1-\beta\theta^*)\ f'(z)\ \frac{dz}{d\beta}\ ] = 0. \tag{27}$$

Now by total differentiation of (27) at the optimum we obtain:

$$\frac{d\mu}{dt} < 0 \quad \text{(see Appendix, Part B)} \tag{28}$$

Furthermore,

$$\frac{df(z(\mu))}{dt} = f'\ \frac{dz}{d\mu}\frac{d\mu}{dt} < 0 \tag{29}$$

by (18) and (28).

i.e., output declines due to the imposition of a proportional tax. The implied decline in the aftertax share, μ , necessitates an increase in the tenant's plot size in order to maintain the tenant on his reservation utility $\bar{U}$. The increase in plot size implies both a reduction in the number of tenants , x , and a decline in output. We thus obtain the following proposition:

Proposition 7: The imposition of a proportional output tax on landlords and tenants will cut the aftertax share of the tenant, increase the plot size per tenant, and reduce the number of tenants as well as total output.

Modelling a Hicks neutral technical change is equivalent to modelling a proportional output tax, i.e., a Hicks neutral technological change is a shift in A where the production function is $Af(ex)$. The only difference is the direction of the impact. Hence, considering a Hicks neutral technical change and applying Proposition 7, we obtain:

Proposition 8: A Hicks neutral technical change will increase the aftertax discounted share of the tenant, decrease the plot size per tenant and increase the number of tenants as well as total output.

Now, consider the case of a Cobb-Douglas production function. Given the unit elasticity of substitution, the tenant's effort is independent of $\mu \equiv \beta(1-t)$ (see (17)), i.e., the decline in the aftertax share is totally compensated by the increase in plot size so as to leave the tenant's effort unaltered. Furthermore, it is easily seen using (23) that the optimal β is unaffected by the tax or technical changes. For the Cobb-Douglas case, all factor-augmenting technical changes can be viewed as Hicks neutral changes. Thus considering irrigation as a land augmenting technical change and applying Proposition 8, we obtain:

Proposition 9: If the production function is of the Cobb-Douglas type, introducing irrigation will leave the discounted share contract unaltered, decrease the plot size for tenant and increase the number of tenants as well as total output.[17]

4.4 Increase in the Tenant's Utility Level in an Alternative Occupation

Suppose, for example, that through an increase in the non-agricultural wage rate, the utility that the tenant could obtain (i.e., $\bar{U}$) in an alternative occupation increases. Assuming once again a Cobb-Douglas production function, so that the tenant's effort is independent of β, it is clear that the landlord can meet the higher $\bar{U}$ only by raising the plot size, therefore reducing the number of tenants and output. Equilibrium β is unchanged. Hence:

Proposition 10: If the production function is Cobb-Douglas, any increase in the utility that the tenant can obtain in an alternative occupation will raise the equilibrium plot size, reduce the number of tenants and output, while leaving the discounted crop share unaltered.

5. Conclusions

In conclusion, we summarize our results. Our main result is that in a world in which (i) production takes place under constant returns to scale in land and labor in efficiency units, (ii) a landlord can subdivide his land into as many plots as he chooses, and (iii) a tenant chooses his effort, so as to maximize his utility —equilibrium will be characterized by utility equivalent contracts. In other words, even if a landlord has no power over crop shares or terms of credit, by choosing the plot size appropriately, he will force the tenant to a utility level equal to that which he (the tenant) could have obtained in an alternative occupation as long as there are enough potential tenants. He is able to do this not only because there is a perfectly elastic supply of tenants at this 'reservation' utility level, but also because the *tenant's effort per hectare increases with a reduction in his plot size.*

This result is similar to that found in Cheung's model (1969), where the tenant's effort per unit of raw labor is invariant. Cheung shows that landlords will provide each tenant a plot of land on which the tenant can earn no more than he could have earned in an alternative occupation. Whereas enforcement of the tenant's labor input is necessary in a Cheungian world, it takes a different form in our model: it ensures that the tenant does not split his working time between sharecropping and an alternative occupation.

In this world of utility equivalent contracts, it will be in the interest of the landlord to ensure that the tenant gets his credit from the cheapest source. If the landlord's opportunity cost of capital is lower than that charged by the local moneylender, the landlord will ensure that the tenant gets credit at the cheapest interest cost by offering him a credit contract. This often is not imposed, but chosen, only if it is optimal. The tenant is pushed down to his alternative utility level, not by the credit instrument, but by plot size variations.

Finally, in our model, utility equivalence implies that nothing short of land reform will affect the tenant's welfare, as long as he is a tenant. Indeed, other reforms such as setting a floor on the tenant's share of the crop, making credit available to the tenant at a cost below the opportunity cost of capital to the landlord or banning the tying of credit and tenancy contracts, either have no effect on the equilibrium at all or have an effect on the number of tenants, output and the landlord's income.

APPENDIX

Part A: Properties of the Model

Denote the tenant's utility function as $U(c,e)$ where (c,e) denote consumption and effort, respectively. Define leisure, $\tilde{e} = -e$. Hence, $U(c,e) \equiv V(c, \tilde{e})$, which implies that: $V_1 = U_1, V_2 = -U_2, V_{11} = U_{11}, V_{22} = U_{22}, V_{12} = -U_{12}$. Quasi-concavity of $V(c, \tilde{e})$ means that for the iso-utility $V(c, \tilde{e}) = \bar{V}$, c is a convex function of $\tilde{e}$, i.e.:

$$\frac{\partial c}{\partial \tilde{e}} = -\frac{V_2}{V_1} \tag{A.1}$$

and

$$\frac{\partial^2 c}{\partial \tilde{e}^2} = -\frac{\{V_{21}\frac{\partial c}{\partial \tilde{e}} + V_{22}\}V_1 - \{V_{11}\frac{\partial c}{\partial \tilde{e}} + V_{12}\}V_2}{V_1^2} = \tag{A.2}$$

$$= -\frac{(V_1^2 V_{22} - 2V_1V_2V_{12} + V_2^2 V_{11})}{V_1^3} > 0 .$$

Hence, quasi-concavity of $V(c, \tilde{e}) \equiv U(c, e)$ implies that:

$$U_1^2 U_{22} - 2U_1U_2U_{12} + U_2^2 U_{11} < 0 . \tag{A.3}$$

Now, for c and $\tilde{e}$ to be normal goods the following two conditions must be satisfied:

$$U_1U_{22} - U_2U_{12} < 0 \quad \text{and} \quad U_1U_{12} - U_2U_{11} < 0 . \tag{A.4}$$

We further assume that the tenant's consumption equals his income,

i.e., $c = \beta \frac{f(ex)}{x}$

implying that:

$$\frac{\partial c}{\partial e} = \beta f' > 0 \quad \text{and} \quad \frac{\partial^2 c}{\partial e^2} = \beta x f'' < 0 \tag{A.5}$$

by the strict concavity of the production function $f(ex)$.

Let $\phi(e) = U(c(e), e)$. Hence :

$$\phi'(e) = U_1 \frac{\partial c}{\partial e} + U_2 \tag{A.6}$$

and

$$\phi''(e) = U_{11} \left(\frac{\partial c}{\partial e}\right)^2 + 2U_{12} \frac{\partial c}{\partial e} + U_{22} + U_1 \frac{\partial^2 c}{\partial e^2} .$$

Calculating the second order conditions at the optimum ($\phi'(e) = 0$) we obtain:

$$\phi''(e) = U_1 \beta x f'' + \left(U_2^2 \, U_{11} - 2U_{12} U_2 U_1 + U_{22} U_1^2\right) \frac{1}{U_1^2} < 0. \tag{A.7}$$

By the strict quasi-concavity of U and strict concavity of f, $\phi''(e) < 0$ implies the existence of a maximum to the tenant's problem.

To determine the impact of a reduction in plot size (increase in x) on tenant's effort per acre, we denote $ex \equiv z$.

Thus, (A.6) can be rewritten as:

$$U_1 \left[\frac{\beta f(z)}{x} , \frac{z}{x} \right] \beta f'(z) + U_2 \left[\beta \frac{f(z)}{x} , \frac{z}{x} \right] = 0 . \tag{A.6'}$$

Total differentiation of (A.6') with respect to (z,x) yields:

$$\frac{\partial z}{\partial x} \left\{U_1 \beta f'' + \beta f' \left(U_{11} \frac{\beta f'}{x} + \frac{U_{12}}{x}\right) + \frac{U_{21} \beta f'}{x} + \frac{U_{22}}{x} \right\}$$

$$+ \; \beta f' \left(\frac{-U_{11} \beta f}{x^2} - \frac{U_{12} z}{x^2} \right) + U_{21} \left(\frac{-\beta f}{x^2}\right) + U_{22} \left(\frac{-z}{x^2}\right) = 0$$

Collecting terms and utilizing the first order conditions (i.e. $\beta f' = -\frac{U_2}{U_1}$) we obtain:

$$\frac{\partial z}{\partial x} = \frac{\frac{\beta f}{x^2 U_1} (U_{21} U_1 - U_2 U_{11}) + \frac{z}{x^2 U_1} (U_{22} U_1 - U_2 U_{12})}{\frac{1}{x} \phi''} > 0 \tag{A.8}$$

(A.8) is positive since $\phi'' < 0$ by (A.7), and the numerator is negative by the normality conditions, (A.4).

Now, following the utility equivalence result (Proposition 1) the relation

$$U\left\{\frac{\beta f[e(x,\beta),x]}{x}, e(x,\beta)\right\} = \bar{U} \tag{A.9}$$

determines $x(\beta)$.

Applying the envelope theorem $(\phi'(e) = 0)$, we obtain:

$$x'(\beta) = \frac{fx}{\beta(f-f'z)} . \tag{A.10}$$

The next two terms we shall calculate are $\frac{de}{d\beta}$ and $\frac{dz}{d\beta}$

where

$$\frac{dz}{d\beta} = \frac{\partial z}{\partial x} x'(\beta) + \frac{\partial z}{\partial \beta} = x'(\beta)\; e + x\frac{de}{d\beta} . \tag{A.11}$$

Hence,

$$\frac{de}{d\beta} = \frac{1}{x}\left[\frac{\partial z}{\partial x} x'(\beta) + \frac{\partial z}{\partial \beta} - x'(\beta)e\right] . \tag{A.12}$$

So in order to evaluate (A.11) and (A.12) we have only to calculate $\frac{\partial z}{\partial \beta}$.

($\frac{\partial z}{\partial x}$ is given by (A.8)).

By total differentiation of (A.6') with respect to z and β *holding* x *constant* we obtain:

$$\frac{\partial z}{\partial \beta} = -\frac{f'U_1 + \frac{f}{xU_1}(-U_{11}U_2+U_{21}U_1)}{\frac{1}{x}\phi''(e)} . \tag{A.13}$$

Clearly $\frac{\partial e}{\partial \beta} = \frac{\partial z}{\partial \beta}\cdot\frac{1}{x}$.

Substituting (A.13), (A.8) and (A.10) into (A.11) we obtain:

$$\frac{dz}{d\beta} = \frac{-f'U_1 + \{U_{21}U_1 - U_{11}U_2\}\frac{f}{xU_1}\left(-1+\frac{f}{(f-f'z)}\right) + (U_{22}U_1-U_2U_{12})\frac{z}{xU_1}\cdot\frac{f}{\beta(f-f'z)}}{\frac{1}{x}\phi''(e)} \tag{A.14}$$

(A.14) is positive by the normality conditions (A.4) and since $\frac{f}{f-f'z} > 1$ due to the strict concavity of f and $f(o) = 0$.

In the same manner, calculating A.12 and defining

$\sigma \equiv - \frac{f'(f - zf')}{ff''z}$, we obtain

$$\frac{de}{d\beta} = \frac{U_1 \; f'}{\frac{1}{x} \phi''(e)} \quad \frac{1-\sigma}{\sigma} \; . \tag{A.15}$$

Part B. Comparative Statics: a Proportional Output Tax,

Define the landlord's objective function as $G(\beta,t)$. (We already substituted the condition $x[\beta(1-t)]$ into the objective function). Recall the first order condition in the text (equation (27)) i.e.,

$$G_\beta = (1-t) \; [-\Theta^* f + (1-\beta\Theta^*) \; f'(z) \; z_\beta \;] = 0 \; . \tag{A.16}$$

By total differentiation of (A.16) we obtain

$$\frac{d\beta}{dt} = - \frac{G_{\beta t}}{G_{\beta\beta}} \tag{A.17}$$

and

$$G_{\beta\beta} = (1-t) \; [-2\Theta^* f' z_\beta + (1-\beta\Theta^*) \; f''(z_\beta)^2 + (1-\beta\Theta^*) f' z_{\beta\beta}] \tag{A.18}$$

and

$$G_{\beta t} = - \frac{G_\beta}{1-t} + (1-t) \; [-\Theta^* f' z_t + (1-\beta\Theta^*) f'' z_t z_\beta + (1-\beta\Theta^*) f' z_{\beta t}] \tag{A.19}$$

Now define the aftertax share as $\mu \equiv \beta(1-t)$. Hence:

$$z_\beta = z_\mu (1-t) = \frac{-(1-t)}{\beta} z_t \tag{A.20}$$

$$z_{\beta\beta} = (1-t)^2 \; z_{\mu\mu} \tag{A.21}$$

$$z_{\beta t} = - z_\mu - \beta(1-t) \; z_{\mu\mu} = - \frac{1}{1-t} z_\beta - \frac{\beta}{1-t} \; z_{\beta\beta} \; . \tag{A.22}$$

Substituting (A.20) and (A.22) into (A.19) we obtain:

$$G_{\beta t} = -\frac{G_\beta}{1-t} + (1-t)\ [\ \frac{\theta^* f'\beta}{1-t}\ z_\beta - \frac{(1-\beta\theta^*)f''\beta}{1-t}\ z_\beta^2 \tag{A.23}$$

$$- (1-\beta\theta^*)f'(\ \frac{1}{1-t} z_\beta + \frac{\beta}{1-t} z_{\beta\beta})]$$

By collecting terms and recalling (A.18) we obtain:

$$G_{\beta t} = -\frac{G_\beta}{1-t} - f'z_\beta - \frac{\beta}{1-t}\ G_{\beta\beta} \tag{A.24}$$

At the optimal $\beta = \beta\ (t)$, $G_\beta = 0$. Hence:

$$\frac{d\beta}{dt} = \frac{\beta}{1-t} + \frac{1}{G_{\beta\beta}} f'z_\beta\ . \tag{A.25}$$

The impact of tax policy on μ is:

$$\frac{d\mu}{dt} = -\ \beta + (1-t)\ \frac{d\beta}{dt}\ . \tag{A.26}$$

By substituting (A.25) into (A.26) we obtain:

$$\frac{d\mu}{dt} = (1-t)\ \frac{f'(z)}{G_{\beta\beta}}\ \frac{dz}{d\beta} = \frac{f'(z)}{G_{\beta\beta}}\ \frac{dz}{d\mu} \tag{A.27}$$

$G_{\beta\beta} < 0$ by the second order conditions for maximum, and $\frac{dz}{d\beta} > 0$ (see A.14). Hence, $\frac{d\mu}{dt} < 0$. Furthermore,

$$\frac{df(z(\mu))}{dt} = f'\ \frac{dz}{d\mu}\ \frac{d\mu}{dt} = G_{\beta\beta}(\frac{d\mu}{dt})^2 < 0 \tag{A.28}$$

Footnotes

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[1] See Bardhan [1980], Bardhan and Rudra [1978, 1980a, 1980b and 1981], Bharadwaj [1974], on India and for instance, Ransom and Sutch [1978] on the Post-Bellum Southern United States.

[2] Bell and Zusman [1976], Cheung [1969], Marshall [1959], Newberry [1977], Reid [1973], Newbery and Stiglitz [1979] and Stiglitz [1974].

[3] Bhaduri (1973, 1977). We will be using the terms 'tenant' and share-cropper interchangeably, though strictly speaking, the tenant is one who leases in land at a fixed rent (cash or kind) per season.

[4] Bardhan and Rudra [1978].

[5] We do not discuss in this paper other rationales for interlinking such as uncertainty and asymmetrical distribution of information between landlords and tenants. On these matters, See, Bell and Zusman (1980), Braverman and Stiglitz (1980), Braverman and Guasch (1980) and Mitra (1980).

[6] One economic reason for the emergence of sharecropping contracts is the following: If <u>only</u> incentive problems exist (i.e., the landlord can neither force the worker to contribute a specified level of effort nor can he monitor it), the fixed-rent contract will be best suited to remedy them. It will, in fact, dominate a fixed-wage or a sharecropping contract. The tenant obtains all the fruits of his effort after paying the fixed rent. Fixed rents, however, imply that the tenant must bear all risk resulting from output uncertainty due to exogenous conditions (e.g. weather, illness). If the tenant is risk averse, such a contract will be inefficient, in which case a sharecropping contract will dominate it.

[7] On efficiency wage and rationing equilibria see Leibenstein [1957], Mirrlees [1976] and Stiglitz [1976].

[8] See discussion of this assumption in Section 3 below.

[9]Investment in a distant bank is unattractive for a poor and often, illiterate tenant.

[10]Bell and Braverman (1978) show that, if the production function is of constant returns to scale and there is no uncertainty, landlords will prefer cultivation with wage labor to sharecropping. However, this result does not apply to the present analysis because we do not give the landlord the option of self-cultivation with wage labour and because of other reasons concerning the modelling of tenants' effort and behaviour

[11]Assume $\lim_{c \to 0} U_1 = \infty$ and $\lim_{e \to 0} U_2 = 0$

[12]See Braverman and Stiglitz [1980] for discussion of competitive vs. non-competitive contractual equilibria.

[13]It can also be argued that if at an initial x, U^* is less than $\bar{U}$, the potential tenant will not choose sharecropping. As such, in order to obtain someone to cultivate his land, the landlord will have to increase the plot size, i.e., reduce x. We are ignoring the fact that a tenant is "indivisible" while land is divisible.

[14]The fact that the size of the plot cultivated by the tenant does not change over time does not contradict its use as a policy instrument by the landlord. It only means that in a stagnant situation, once an optimal size has been determined, there is no need to change it.

[15]This is perhaps a rationale for empirical observations of tenants being charged high interest for rather small loans.

[16]Some care is needed in interpreting this result. An increase in β raises the number of efficiency units of labour, i.e., ex supplied by <u>each</u> tenant, and increases the <u>number</u> of tenants through a reduction in plot size. If the elasticity of substitution is less than unity, effort per tenant will decline, so that output per tenant will decline. But the increase in the number of tenants more than offsets this decline.

[17]Since in this model landlords extract all the surplus from their tenants, they have no reason to resist technological innovations. For theoretical discussions of landlords' resistance to technological innovation, see Bhaduri [1973, 1979], Newbery [1975], Srinivasan [1979], Braverman and Stiglitz [1981].

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