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A NOTE ON THE CONSTANCY OF THE REAL WAGE

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Development models often assume an unlimited supply of labour from agriculture to industry at a constant real wage. In fact, there are several wage definitions available depending on whether real wages are specified in terms of agricultural goods, manufactured goods, or in "constant utility combinations" of food and manufactured goods. The purpose of this note is to explore some implications of three of these definitions for the growth of the manufacturing sector and the welfare of its workers.[1]

The usual assumption about wages, attractive for its simplicity, is that they are constant in terms of agricultural goods. Money wages are assumed equal to $P_f \bar{f}$ where $\bar{f}$ is a fixed amount of food and P_f is the price of food. Historically, the idea that wages are fixed in terms of food (corn), probably stems from a notion of a subsistence minimum. More recent usage, however, derives from the assumption that the wage rate in manufacturing is determined more or less by the productivity in agriculture. In the Fei-Ranis version, for example, the institutional wage is the historical average agricultural productivity of the population and the wage remains constant as workers are withdrawn. This definition is also relevant for models where there is a plentiful supply of land and the opportunity cost of urban employment is the constant average product on this unlimited land.

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[1] Harry G. Johnson, "Optimal Trade Intervention in the Presence of Domestic Distortions," in Baldwin et. al., Trade, Growth, and the Balance of Payments, analyses the very different implications of assuming these three different definitions of rigid wages in tariff theory.

A second possible definition is that the wage rate is fixed in terms of manufactured goods or, the money wage in the manufacturing sector is $P_m \bar{m}$ where $\bar{m}$ is a "fixed bundle of industrial goods" and P_m is the price of manufactured goods. A constant real wage in these terms is therefore $\bar{w} = \frac{w}{P_m} = \bar{m}$. As far as we have been able to ascertain, there is no empirical grounds for assuming that the wage rate is rigid in terms of manufactured goods (except perhaps in the short run) and we know of no model of underdeveloped countries that has used this assumption for long term analysis.

The final definition is in terms of a wage based on constant utility. The money wage that approximately keeps utility constant is defined by $w = P_f \bar{f} + P_m \bar{m}$, where $\bar{f}$ and $\bar{m}$ are the appropriate weights for a worker's price index. This type of wage definition would be most useful for migration theories where it may be hypothesized that the worker's decision to move depends upon his expected utility in the city.[2] It is then possible to imagine a theory of labour mobility postulating an unlimited supply at a constant real wage in terms of the utility foregone by leaving the rural area.

As long as the intersectoral terms of trade are constant, various definitions of the real wage give the same result, but if they vary (as, of course, is true in practice), the course of the real wage depends on whether it is defined in terms of one good or the other or in some combination of the two. Our problem is analyzed from the point of view of a manufacturing sector facing given supply functions of food and labour from an

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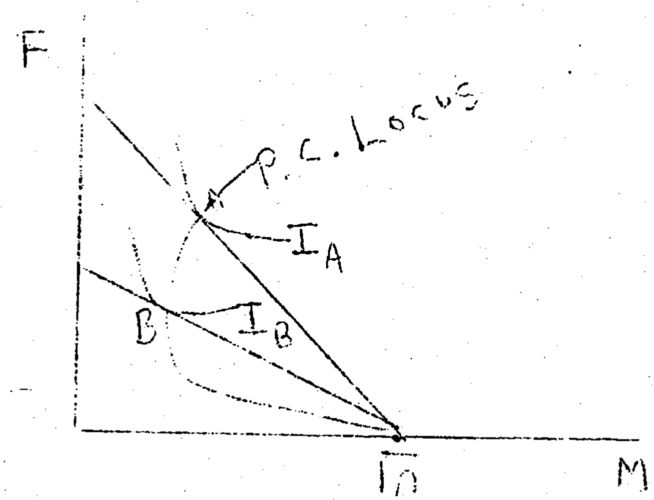
[1]A migration theory should also consider the migrant's probability of finding work in the city as well as some variable reflecting his ability to tide himself over while seeking a job.

agricultural sector.[3] The manufacturing sector is assumed to produce one good, M, using capital and labour with a constant returns to scale production function. All profits are reinvested in the manufacturing sector and all wages are spent on consumption goods.

Wage Defined in Terms of M Goods Only

It is simplest to begin with the case in which real wages are "fixed" in terms of industrial goods. As the manufacturing sector expands because of positive capital accumulation, the demand for labour increases proportionally, and there is a corresponding shift in the demand curve for food. As $\frac{P_f}{P_m}$ increases, the welfare of the individual worker declines along the path indicated in the diagram below. With the real wage fixed at $\bar{m}$, and P_f in-

FIGURE 1



creasing, the worker is placed on successively lower indifference curves (I_B).

[3] In general we shall assume that the supply curve for food (as a function of the intersectoral terms of trade) is positively sloped while the supply of labour is perfectly elastic at some given real wage. We assume further, that these two supply curves are independent, i.e., the migration of labour has no effect on the supply curve for agriculture, and vice-versa. This is not the most realistic assumption but it is convenient for the problem at hand and does not affect in any essential way the major points of this note. The reader can imagine that the manufacturing sector draws labour from one region of the country and food from another and that the two regions are distinct and have little trade between them.

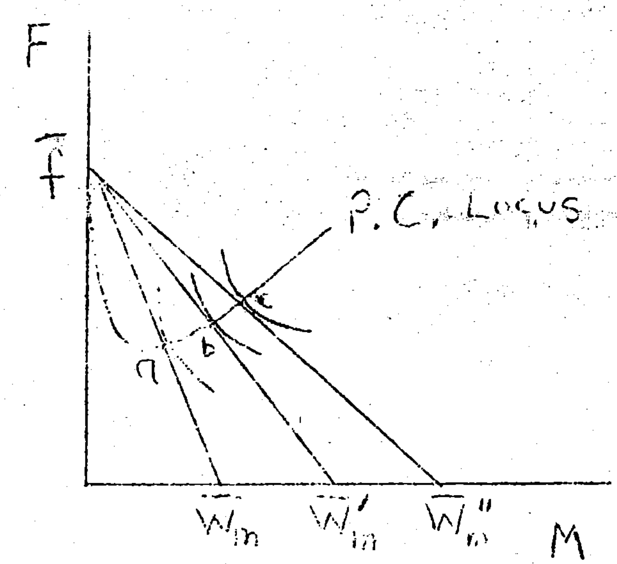
Although individual welfare is reduced, more workers are employed in the manufacturing sector so that the effect on total welfare of the urban workers is not unambiguous. Since real wages are defined in terms of M goods only, the assumed food price increase does not affect the cost of labour to the manufacturing sector, and therefore does not affect industrial profits and growth. Since the real wage is always constant, and since we are assuming constant returns to scale, the reinvestment of profits will allow output to grow with a corresponding increase in labour demanded from the agricultural sector. Therefore, if wages are defined only in terms of M goods, an increase of P_f/P_m leaves the growth rate of manufacturing unaltered but reduces welfare per worker.

Wage Defined in Terms of F Goods Only

In the second case we examine, the worker is always paid sufficient money income to buy a fixed amount of food. It is "as if" the worker was paid in a fixed bundle of corn (or arrived in the city "carrying their bundle of consumer goods with them") and then proceeds to trade whatever food he does not want for the manufactured goods he can obtain.

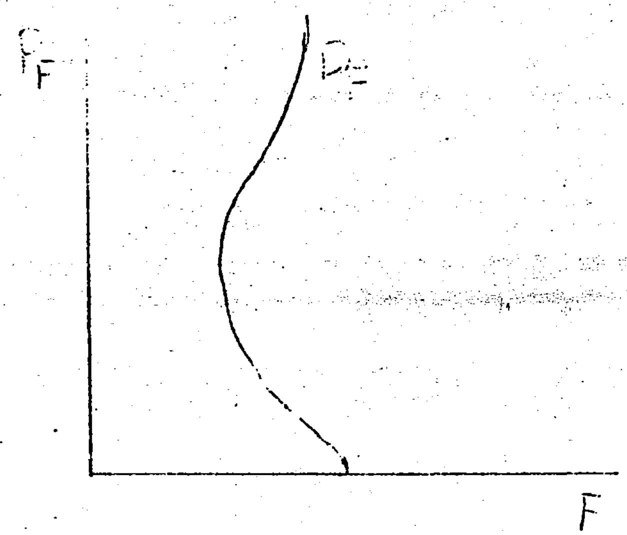
The worker's indifference map is pictured in Figure 2A; $\bar{f}$ on the vertical axis is the constant quantity of food that determines the wage rate. As the terms of trade (P_f/P_m) rise, the budget line rotates through this fixed point intersecting the horizontal axis at successively higher wage rates in terms of manufactures ($\bar{w} = \frac{P_f}{P_m}\bar{m}$), and traces out the Price Consumption locus (P. C.). The difference between the wage defined in terms of manufactured and food goods is now clear. In the former, the budget line rotated through a constant point on the horizontal axis and higher food prices were associated with lower wages in terms of food and with decreased consumption possibilities. In the present definition, as the price of food increases, the real wage in terms of manufactured goods rises and the consumption possibilities of the worker increases.

FIGURE 2A



The demand curve for food per worker (pictured in 2B) that is derived from the price consumption locus has the following characteristics. When the price of food is zero, it lies on the horizontal axis at the point where the amount of food demanded is $\bar{f}$ (i.e., when the price of manufactured goods is infinite, the worker spends all his income on food and therefore consumes his initial bundle). As the price of food increases, the demand for food drops as the worker substitutes manufactured goods for food, and the demand curve in this range is normal. Eventually, a price is reached (corresponding to the turning point in the price consumption expansion path) when the amount of food demanded increases as the price rises. At this point the income effect outweighs the substitution effect.

FIGURE 2B



The process is most clearly understood in terms of a Slutsky equation. The worker maximizes $U(f, m)$ subject to the constraint that $y = P_f \bar{f}$. The worker's constraint is therefore $P_f(\bar{f} - f) = P_m m$. It can easily be shown that the Slutsky equation is as follows:

$$\frac{df}{dP_f} = \frac{\partial f}{\partial P_f}\Bigg|_{\text{compensated}} + (\bar{f} - f)\,\frac{\partial f}{\partial y}\Bigg|_{P\ \text{constant}}$$

The first term is negative, but the second is positive (unless food is an inferior good) and acquires higher and higher weight as food consumption drops. Therefore, even if we start with a normal demand curve under which the amount of food demanded decreases as price rises, the second term becomes increasingly important and may eventually outweigh the first. The worker, endowed with a fixed amount of food or guaranteed sufficient income to buy a fixed amount, becomes richer as the price of food increases and eventually he may take some of his added income out in higher food consumption.

A demand curve such as the one pictured in Figure 2B could, of course, lead to a situation where the market equilibrium does not exist or is unstable. The fact that observed markets do not behave this way suggests that real wages do not stay constant in terms of agricultural goods or that the backward bending range is not reached.

Notice that the above analysis refers only to the demand curve for the individual worker. The aggregate demand curve for the manufacturing sector must be multiplied by the number of workers, which is also changing, because a rise in the price of food increases the real wage in terms of manufactures. A general formula taking this feature into account can be derived as follows:

Let F equal the total amount of food demanded so that[4]

$$F = f\ L$$

then $\frac{df}{dP} = \frac{df}{dP}\ L + f\ \frac{dL}{dw}\ \frac{dw}{dP}.$

Assume $f = f\ (w, P)$

then $\frac{df}{dP} = \frac{\partial f}{\partial P}\frac{dw}{dP} + \frac{\partial f}{\partial P}$

Therefore $\frac{df}{dP} = \frac{\partial f}{\partial P}\ L + \frac{dw}{dP}\ [\ \frac{\partial f}{\partial w}\ L + \frac{dL}{dw}\ f].$

This can be rewritten in terms of elasticities as:

$$\varepsilon_{FP} = \varepsilon_{fP} + \varepsilon_{wP}\ [\ \varepsilon_{fw} + \varepsilon_{Lw}].$$

ε_{fP} which is the effect of price on consumption when the money wage rate is held constant is negative, e.g., the substitution and income effects work in the same direction.

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[4] P is the terms of trade where P_m is assumed to be constant and set equal to unity by definition.

In the present case in which wages are defined in terms of food goods, ε_{wP} is positive, e.g., $\frac{dw}{dP} = \bar{f} > 0$ and $\varepsilon_{wP} = 1$. We have two effects, therefore, to weigh against ε_{fP}: the pure income effect, ε_{fw}, due to the change in money income, which is positive, and the elasticity of labour demand with respect to a wage change, ε_{Lw}, which is negative. It is then an empirical question as to the final sign of ε_{FP}.

It is important to note, however, that the possible phenomenon of an abnormal demand curve is not restricted to the case where the real wage is set solely in terms of food goods. Suppose the more realistic case in which the wage rate is set in terms of a bundle of fixed amounts of food and manufactured goods, $\bar{f}$ and $\bar{m}$ respectively. Money wages will then be set equal to $P_f \bar{f} + P_m \bar{m}$; a likely occurrence if they are adjusted continuously in terms of some price index where the weights $\bar{f}$ and $\bar{m}$ are consumption in some initial period.[5] The same Slutsky equation prevails as above and a backward bending demand curve is a possibility as long as the food weight $\bar{f}$ exceeds the actual amount of food consumed. This is simply another way of stating the well known proposition that wages derived using weights from the base period overcompensate for price changes. The resulting overcompensation yields a positive income effect which may outweigh the negative substitution effect.

We may note also that the problem of instability is aggravated when it is assumed that the constant wage in agricultural goods adjusts to average

[5] If the government did change commodity weights continuously as prices changed, then worker's utility in terms of a bundle of food and manufactured goods would approximately remain constant (see Figure 3). However, it is usually the case that commodity weights remain fixed over long time intervals. Also, there are probably decision and statistical lags allowing income effects to be realized.

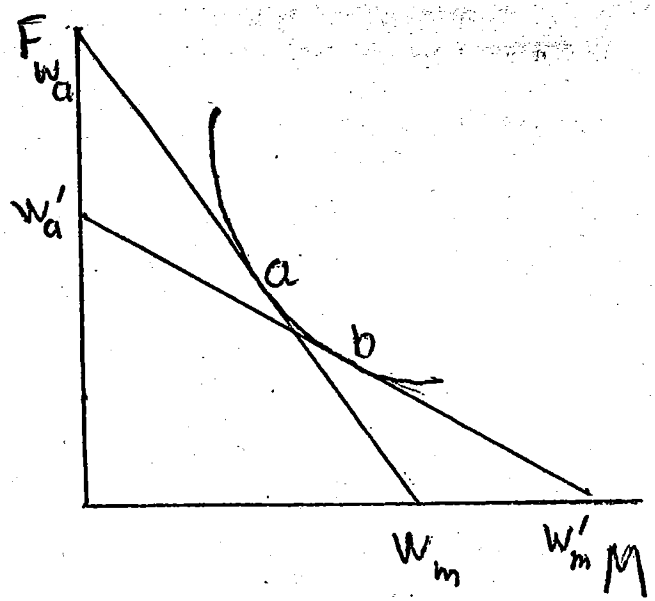
productivity in the rural area. As the price of food rises, the rural area becomes better off on average; if the wage rate is tied to average productivity $\bar{f}$ will increase and the tendencies discussed above are intensified. Finally, as the increase in the price of food causes a rise in the real wage to the manufacturing sector, profits are decreased and growth in this sector too is constrained. Instead of balanced growth, we get $M^* > L^*$ (where a star indicates the percentage change of the variable) and both decreasing over time.

Utility Wage

The problem associated with a definition of real wages in terms of food only suggests a wage based on utility. Here money wages would be equal to $P_f \bar{f} + P_m \bar{m}$ and as P_f changes the weights $\bar{f}$ and $\bar{m}$ are altered to keep the worker on the same indifference curve as shown in Figure 3. Alternatively, we might assume that worker's productivity is determined by his utility and that a certain level of welfare is needed to maintain efficiency in the industrial sector. As long as this level of utility is above that prevailing in rural areas, an unlimited supply of labour will be forthcoming. Thi wage rate will not fall because this would impair productivity. Instead, jobs are rationed by the unemployment pool, for example.

In Figure 3, $W_a W_m$ is the initial wage line. As the price of food changes, a new line must be found tangent to the old level of utility. This is $W_a' W_m'$ which implies a wage rate equivalent to W_a' in terms of food or W_m' in terms of manufactured goods. If wages are adjusted in this manner, the change in food consumption per worker consequent upon a change in price is the pure substitution effect in the Slutsky equation and is always negative. Since real income is kept constant along the demand curve for food, the possibility of a positively sloped demand curve is removed.[6] Here, the different "substitution" points serve directly to trace out the demand curve whose final shape depends on the elasticity of substitution between food and manufactured goods.

FIGURE 3



Finally, as the price of food increases, a wage constant in terms of utility combinations of goods implies that manufacturing growth is constrained but not as much as in the previous case. The real wage measured in manufactured goods does not rise as much because there is not overcompensation. Balanced growth is not possible in the absence of technological change in agriculture.

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[6]M. Friedman, "The Marshallian Demand Curve," reprinted in Essays in Positive Economics.