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UNCERTAIN LIFETIMES AND THE WELFARE ENHANCING PROPERTIES OF ANNUITY MARKETS AND SOCIAL SECURITY

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Abstract

This paper explores the implications of Social Security programs and annuity markets through which agents, who are characterized by different distributions of length of lifetime, share death related risk. In the absence of annuity markets, a Social Security program can accommodate some risk sharing needs within each generation. However, under these circumstances, it also affects income distributions over time, the structure of observable interest rates and saving levels, by changing the distribution of bequests in the economy. On the other hand, when annuity markets do operate, a non-discriminatory Social Security program will affect only the intragenerational allocation of resources. In the absence of private information regarding individual survival probablities, such a program will lead to a non-optimal intragenerational allocation of resources. However, the presence of adverse selection considerations gives rise to a Pareto improving role for a mandatory non-discriminatory Social Security program.

1. Introduction

The existance of uncertainty with respect to the length of lifetime will lead individuals, in the absence of bequest motive in preferences, to save for their old age via annuities. This form of savings, which shares the risks related to old age, makes possible higher rates of return on foregone consumption by making claims to future payments conditional on individual specific events such as continued life. In contrast, regular modes of savings, where returns are not conditioned on such individual contingencies, pay lower rates of return and generate involuntary transfers of purchasing power across generations in the form of bequests. These considerations, along with the fact that the existing Social Security program has an important annuity like element in the form of reduced survivors' benefits, strongly suggest that both the normative and positive implications of such programs depend very much on the alterntive modes of savings that are available to agents.

Here we show that when a Social Security program is the only life-contingent form of savings available to agents in the economy, it can accomodate some risk-sharing needs within each generation. However, under these circumstances, it also affects income distributions over time by changing the distribtution of bequests in the economy.

On the other hand, when annuity markets do operate but in an environment where there exists diversity with respect to survival probabilities, a non-discriminatory Social Security program will affect only the intragenerational allocation of resources. Moreover, the presence of adverse selection problems created by private information regarding these survival probabilities, gives rise to a Pareto improving role for a manadatory non-discriminatory Social Security program.

The role of annuities as a device for sharing uncertainty about the length of one's life and observations on the absence of complete markets for such contracts constitute an important part of Diamond's (1977) suggestions for evaluating Social Security type programs. The same view underlies the recent work of Kotlikoff and Spivak (1981) who consider the family institution as a substitute, albeit imperfect, for complete annuity markets. Likewise, Sheshinski and Weiss (1981) examine alternative forms of financing a publicly provided, actuarially fair annuity program. A different kind of risk that may be inefficiently allocated in a decentralized equilibrium is considered by Merton (1981), who examines the features of a Pareto improving social security program in an intertemporal model with nontradeable, randomly productive human captial.

A limitation of the above analyses is the absence of an explicit specification of what features in the environment prevents decentralized equilibria from attaining optimal allocations. Thus Kotlikoff and Spivak confine their analysis to the relative efficiency of equilibria when risk sharing opportunities are restricted on a "family" basis, while Merton's Pareto improving policy depends on some unspecified reasons that restricts the tradability of human capital in the first place. On the other hand, Sheshinski and Weiss acknowledge at the outset that their analysis pertains to the Social Security program only to the extent that privately issued annuities are ruled out. Here, we show that the welfare implications of Social Security-type programs depend crucially on whether one explicitly models the features of the environment which inhibit the efficient operation of annuity markets as opposed to imposing exogenous market exclusion restrictions on certain forms of risk sharing. Just as importantly, the impact of such programs on the composition of aggregate savings, the structure of observable rates of return and the distribution of income are shown to depend on precisely the same sorts of considerations. While we concentrate in this work on the uncertainty which is related to length of life, we envision extensions and refinements of our stylized analysis that reflects a variety of individual uncertainties.

Our analysis is conducted with a version of Samuelson's (1958) overlapping generations model which provides a conveneient framework for studying the intergenerational allocation of resources in an intertemporal economy populated by finitely lived agents. In the specific model under consideration, agents live at most two periods. In order to introduce a natural role for annuities, we assume that while life during the first period is certain, death can occur at the beginning of the second period with a positive probability. In addition, we assume that survival probabilities differ across cohorts. These features allow us to examine the role of an annuity-like Social Security program under a variety of assumptions concerning the risk sharing contracts available in the economy.

When individuals survival probabilities are commonly known, Pareto optimal allocations are characterized by equal consumption levels across both periods of life of agents that reach old age. Consequently, ex-ante marginal rates of substitution of consumption across periods are not equal at those optimal allocations for agents with different survivial rates. Rather, with no population growth, marginal rates of substitution associated with optimal allocations must equal group specific actuarially fair rates. A competitive equilibrium, under this public information assumption, will support a discriminatory annuity structure with precisely this return structure, in which each group of cohorts with common survival proability shares the death related risks among its own members. In such an equilibrium, there are no intergenerational transfers of goods in the form of bequests. However, this will not be the case under alternative market structures in which annuities are excluded. Under such a market exclusion restriction, agents save by purchasing unconditional claims to future payments of a paper asset. Untimely death occurances result in a stationary equilibrium with a non-degenerate real bequest distribution which introduces hetrogeneity in agents' initial wealth levels.[1] It is precisely this endogenous randomness that annity markets or social security type programs eliminate. The explicit derivation of the "bequest regime" equilibrium enables us to emphasize the general role of annuities as optimal risk sharing mechanisms as well as to examine the welfare implications of their exclusions.

The optimal sharing of old age risks obtained by private markets is destroyed by considering individual survival proabilties as private information. As was shown by Wilson (1977) and Rothschild and Stiglitz (1976) in their work on insurance equilibira with private information, decentralized equilibria may yield inefficient allocations. In our context the same problem arises because agents with high survival probabilities -- a group which constitutes high risk for annuity issuers -- impose an externatlity which harms other agents without necessarily gaining anything themselves. One way to improve the equilibrium allocation which is consistent with this information structure involves a government-imposed nondiscriminatory annuity program which takes the following form: agents are forced to contribute a prespecified amount to the program when young, and are paid off at a rate of return which equals the economy-wide actuarially fair rate of return if they reach old age. Residual demands for annuities will be supplied in a competitive <u>separ-</u>

ating equilibrium, so that, ex post, agents from different groups will have purchased some annuities with group specific actuarially fair rates of return. The resulting allocation is Pareto optimal and for a large class of economies, Pareto dominates the non-intervention equilibrium. Hence, unlike the analysis of Sheshinski and Weiss, Social Security and private annuities are not perfect substitutes from the point of view of individual agents in environments where there is a clear welfare enhancing role for social insurance. Consequently, aggregate savings will depend, in a systematic way, upon the magnitude of required contributions to the Social Security program. It is shown that the imposition of a Pareto improving social security system can, in some cases, increase the level of aggregate savings. Finally, in contrast to most of the existing literature, our analysis not only distinguishes private from publicly supplied annuities but also derives optimal combinations of these modes of saving.

2. Complete Annuity Markets

In this section, we describe the basic version of the model to be used for examining the implications of uncertain lifetimes on optimal resource allocations and market structures. The main implication of this section regarding the nature of optimal allocations is that, unlike other forms of diversity in individual characteristics such as preferences or endowments, diversity in publicly known survival probabilities affects in a fundamental way the nature of the price system which can support such allocations as decentralized equilibria. Specifically, allocations associated with stationary competitive equilibria will be optimal only if cohorts with different survival probabilities each face actuarially fair intertemporal terms of trade.

The Model

The economy to be studied is a variant of Samuelson's (1958) pure-exchange overlapping generations model. At each period t, $t \geq 1$, the population consists of old members of generation t-1 who all die at the end of that period, and young members of generation t. Each generation t is partitioned into two distinct groups, A and B, whose relative size is fixed for all t, so that for each agent of type A there are γ agents of type B, $\gamma > 0$. Members of each group live at most two periods, the first of which they survive with certainty. Death can occur at the beginning of the second period with probability $(1-\pi_i)$, $0 < \pi_i < 1$, i = A,B. With a continuum of agents, each of whom correctly perceives his death to occur with probabily $1-\pi_i$, where i indicates the agent's group, a proportion $(1-\pi_i)$ of group i, i = A,B passes away after living only one period, and there is no aggregative uncertainty implied by agents' random lifetimes. In this and the following section, the survival probability of any given agent is assumed to be public information.

There is a single nonstorable and nonproducible comsumption good in this economy. Each young agent is endowed at birth with w units of the good. Generations are of equal size so that each member of generation t is viewed as giving birth to one identical agent (member of generation (t+1)) before the uncertainty about his continued life is resolved. Preferences over lifetime consumption (c_1^i, c_2^i) of a representative member of group i are given by the expected utility function

$$(2.1) \quad U^i[c_1^i, c_2^i] = u(c_1^i) + \pi_i u(c_2^i), \ i = A, B$$

with $u' > 0$, $u'' < 0$, $u'(c) \to \infty$ as $c \to 0$ and $u'(c) \to 0$ as $c \to \infty$. Notice that our specification of $U[c_1^i, c_2^i]$ embodies the assumption that preferences are separable over time. In this we follow Yaari (1965) and Barro and Friedman (1977) who utilize this specification of preferences to deal with the problem of parameterizing utility over lifetime consumption bundles when the length of lifetime itself is uncertain. Similar assumptions are made throughout the literature.

Before we discuss specific market structures and the effects of various government interventions, we characterize the set of feasible and optimal stationary allocations of this economy.

Definition:

A stationary allocation $\{c_1^i, c_2^i | i = A,B\}$ is feasible if it satisfies

$$(2.2) \quad c_1^A + \gamma c_1^B + \pi_A c_2^A + \gamma \pi_B c_2^B = w(1+\gamma)$$

Notice that this definition reflects our assumption about the absence of aggregate uncertainty regarding the number of deaths in each group.

Definition:

A feasible stationary allocation $\{\bar{c}_1^i, \bar{c}_2^i | i = A,B\}$ is optimal if there does not exist another feasible stationary allocation $\{c_1^i, c_2^i | i = A,B\}$ such that

$$U^i[c_1^i, c_2^i] \geq U^i[\bar{c}_1^i, \bar{c}_2^i], \qquad i = A,B$$

with strict inequality for some i.

It can be shown that an interior allocation, ($c_k^i > 0$, $k = 1,2$, $i = A,B$), is optimal if for some $\delta_i > 0$, $i = A,B$, it solves the problem

(2.3) Maximize $\delta_A U^A(c_1^A, c_2^A) + \delta_B U^B(c_1^B, c_2^B)$
subject to (2.2).

A necessary and sufficient condition for an interior allocation to be optimal is that it satisfies (2.2) and has the property that

$$(2.4) \quad \frac{u'(c_1^A)}{u'(c_2^A)} = \frac{u'(c_1^B)}{u'(c_2^B)} = 1$$

Notice that (2.4) implies that with strictly concave preferences, optimal allocations have the property that first and second period consumption levels be equal for agents who live for two periods. Given heterogeneity with respect to survival probabilities, (2.4) also implies that optimal allocations have the property that ex ante marginal rates of substitution are not equalized across members of different groups, i.e.,

$$(2.5) \quad \frac{1}{\pi_A} \frac{u'(c_1^A)}{u'(c_2^A)} \neq \frac{1}{\pi_B} \frac{u'(c_1^B)}{u'(c_2^B)} .$$

It follows that in any competitive equilibrium individually planned consumption levels will be the same across both periods of heterogeneous agents' lives only if agents of type i face an intertemporal tradeoff between c_1^i and c_2^i which equals $1/\pi_i$. Consequently, any market structure in which members of the different groups face the same rate of return on savings will not result in an optimal equilibrium.

The above results should be contrasted with those that would arise under different sorts of diversity across agents such as heterogeneity with respect to the preferances and/or endowments. In this light, imagine that π_A and π_B represented different time discount parameters in (2.1) rather than survival probabilities. Under these circumstances, π_A and π_B would not enter the feasibility condition (2.2) which would then be $c_1^A + \gamma c_1^B + c_2^A + \gamma c_2^B = w(1+\gamma)$. The optimality condition would then equate each of the terms in (2.5) to unity so that a unitary intertemporal tradeoff between c_1 and c_2 for both groups would support an optimal allocation. Similarly, heterogeneity with respect to endowments, does not affect the way in which consumption levels enter the feasibility constraint. A more general discussion of the incorporation of agent specific attributes in economy-wide resource constraints and the resulting implications for the existence and optimality of competitive equilibria is provided by Prescott and Townsend, (1982).

The fact that the consumption good is nonstorable and the good endowments occur in the first period of life generate some desired intergenerational transfers which can be facilitated by a paper asset. We follow, therefore, the literature on macroeconomic applications of the overlapping generations models and assume that the surviving old agents of generation zero are endowed with a paper asset (money). The aggregate stock of the paper asset remains fixed through all time periods t, $t \geq 1$. The inclusion of some asset in the model that facilitates intertemporal transactions allows us to separate efficiency problems that are generated by uncertain lifetimes from those related to the intergenerational allocation of resources.[2] In what follows, we describe a competitive stationary equilibrium in which agents have access to competitive annuity markets, while firms that are established by each generation supply these annuities and specialize in storing the paper asset.

We model an annuity bond at period t as a claim to a certain quantity of the consumption good at period t+1 which is payable only if the original purchaser of the annuity is alive. Normalizing the purchasing price of a period t annuity to one unit of the good at t, the annuity's rate of return represents the intertemporal terms of trade faced by its buyer. Given the postulated hetrogeneity of the population with respect to survival probabilities, we can potentially think of two kinds of annuity equilibria; a pooling equilibrium, in which the same annuity is purchased by members of both groups, and a separating equilibirum in which agents with different survival probabilities purchase annuities with different rates of return. Following Rothschild and Stiglitz (1976), we define an equilibrium in this market as a set of contracts such that when agents maximize expected utility: (i) no contract in the equilibrium set makes negative expected profits, and (ii) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. Clearly, the absence of aggregate uncertainty in this economy implies a similar absence of uncertainty regarding the profits of the annuity-supplying firms. Therefore, in either a pooling or separating equilibrium real profits must be equal to zero.

Let $R_i(t)$ be the real payoff, at t+1, to an annuity purchased at time t by a member of group i, $i \in \{A,B\}$, contingent on him being alive at t+1, and let $D_i(t)$ denote the utility maximizing purcahse of such annuities by that agent. The real purchase price of such annuities is normalized to unity.

In the pooling equilibria, $R_A(t) = R_B(t) = R(t)$ and profits at period t+1 are given by:

$$(2.6) \qquad D_A(t+1) + \gamma D_B(t+1) - R(t)[\pi_A D_A(t) + \gamma\pi_B D_B(t)] = 0 .$$

In a stationary equilibrium $D_A(t) = D_A$, $D_B(t) = D_B$ and $R(t) = R$ for all t. Substituting these identities into (2.6), we obtain

$$D_A + \gamma D_B - R[\pi_A D_A + \gamma\pi_B D_B] = 0,$$

or

$$(2.7) \qquad R = \frac{D_A+\gamma D_B}{\pi_A D_A+\gamma\pi_B D_B}.$$

It is clear that this <u>economy wide</u> rate of return lies between $1/\pi_A$ and $1/\pi_B$. Notice, however, that the pooling contract can never be an equilibrium contract because at this rate, given by (2.7), positive profits can be made by restricting the sales of such annuities to only one of the groups. Hence, the equilibrium will necessarily be a separating one with each contract netting zero profits, so that the separating payoffs are given by $R_A = 1/\pi_A$ and $R_B = 1/\pi_B$. But these are precisely the intertemporal rates of return which induce an equilibrium in which (2.4), the necessary and sufficient condition for optimality is satisfied. To see this, notice that the problem of the representative young agent of group i of generation t is:

Maximize

$$u(c_1^i) + \pi_i u(c_2^i)$$

subject to

$$c_1^i = w - D_i(t)$$

$$c_2^i = R_i(t)D_i(t).$$

Given our assumptions on $u(\cdot)$, necessary and sufficient conditions for the solution of this problem are given by

$$(2.8)\qquad \frac{u'(c_1^i)}{\pi_i u'(c_2^i)} = R_i(t),\ i = A,B.$$

These conditions imply (2.4) when $R_i(t) = 1/\pi_i$ for $i = A$ and $i = B$. Notice that given the strict concavity of $u(\cdot)$, (2.8) implies that

$$(2.9)\qquad c_1^i = c_2^i = \frac{w}{1+\pi_i},\ i = A,B$$

for all agents who live for two periods. Finally, since allocation (2.9) satisfies the resource constraint (2.2), the essential features of the competitve annuity equilibrium have been completely described.

It is worthwhile noting that the above equilibrium is essentially one in which the old of group i share the estate of the deceased members of their own group. In effect, competitive annuity markets discriminate between groups in an actuarially fair way, and thereby induce risk sharing <u>within</u> each specific group rather than across the entire generation. The result of this market structure is an optimal transfer of goods both between and <u>within</u> generations. Conversely, any annuity policy which does <u>not</u> discriminate between members of different groups would lead to an inefficient form of risk sharing and a nonoptimal equilibrium. These results are not inconsistent with the existing literature. For example Barro and Friedman and Sheshinski and Weiss assume that agents face an economy-wide actuarially fair rate of return on annuities. To the extent that agents are homogeneous with respect to survival probabilities, the implicit market structures that were considered in these works will lead to optimal allocations. However, when there exists heterogeneity among agents with respect to this attribute, an economy-wide actuarially fair rate of return does not correspond to the decentralized equilibrium-nor would it result in an optimal allocation if it were imposed. Finally, we note that in such environments optimally designed Social Security systems will have no effect on aggregate savings as long as required contributions do not exceed the amount that individuals would save in the absence of such government interventions. Essentially, this follows from the fact that there is no welfare enhancing role for a Social Security program here. Hence, to achieve a Pareto optimal allocation, a Social Security program requires individuals to purchase publicly provided annuities which are perfect substitutes for private modes of saving. Because of this, increases in publicly mandated savings are simply offset, in a one to one way, by decreases in private savings. Aggregate savings will depend on the magnitude of a Social Security program only to the extent that the latter leads to a non-optimal equilibrium.

To clarify the risk sharing role of annuities, we now describe the competitive equilibrium of this economy when no annuities of any kind are available. This will enable us to determine whether an annuity equilibium dominates in some Pareto sense the equilibrium associated with this more restricted market structure, as well as to identify potential groups that will be adversely affected by a decentralized development of more complete annuity markets.

3. Equilibria Without Annuities

In this section, we examine the nature of a more restricted range of markets, which are, in some sense, a natural alternative to the complete market structure of section 2. Consider our basic economy with a market restriction that precludes any asset whose payoff depends on whether its owner is alive or dead. Under such circumstances, even in the absence of any bequest motive, the untimely death of some agents creates an intergenerational transfer in the form of bequests. We posit a "natural" assignment rule for such bequests, in which the offspring of a parent that dies prematurely inherits the value of his parents' savings. We characterize this bequest equilibrium and then compare it to the annuity equilibrium of sections 2 in order to examine both the desirability and implementability of complete annuity markets in environments where there are no natural barriers to the operation of such markets.

The Bequest Equilibrium

Because the main issues of this section can be examined in the context of homogeneous survival probabilities, we assume that all agents are characterized by a common probability of surviving through their second period, i.e., $\pi_A = \pi_B = \pi$. In addition to the common good endowment w, each person receives at birth a bequest consisting of his parent's savings payoff in the event that the parent dies prematurely. Savings in any period t, $t \geq 1$, takes the form of unconditional claims to the consumption good. The claims have a real rate of return R(t), and a selling price that is normalized to unity.

A typical young member of generation t, born with a bequest consisting of claims to b units of the consumption good and a good endowment w, faces the following problem:

$$\max\ u(c_1) + \pi u(c_2)$$
$$\text{s.t.}\quad \text{(i)}\ c_1 + s \leq w + b$$
$$\text{(ii)}\ c_2 \leq sR(t)$$
$$\text{(iii)}\ s \geq 0$$

where s denotes his real savings. The solution to this problem for any $b > -w$ and any $R(t) \geq 0$ satisfies

(3.1) $$-u'[w+b-s] + R(t)\pi u'[sR(t)] \leq 0$$

with equality if $s > 0$.

We can characterize this solution by a saving function $s(b,R)$, defined for $b > -w$ and $R > 0$. The saving function $s(\cdot,\cdot)$ satisfies

(i) $s(b,R) > 0$,

(ii) $0 < \frac{\partial s}{\partial b}(b,R) < 1$,

(iii) $\frac{\partial[Rs(b,R)]}{\partial R} > 0$,

where (i), (ii), (iii) follow from our assumptions about $u(\cdot)$, made in section 2.

Each generation establishes a competitive savings industry which acquires the paper asset from the previous generation's firms, which are then liquidated. Let $p(t)$ be the period t asset price in terms of the period t good. A typical firm of generation t that acquires k units of the paper asset incurs a real cost of $kp(t)$. At period t+1, these k asset units will yield $kp(t+1)$ units of the consumption good units which can be sold as claims at period t for a revenue of $kp(t+1)/R(t)$. A zero profit or no-arbitrage condition for period t implies

$$(3.2) \quad p(t)\,R(t) - p(t+1) = 0, \; t \geq 1.$$

The rate of return on the (unconditional) claims and the asset price are determined by equating the real savings of generation t with the real value of the total paper asset, the supply of which is fixed. Since young members of generation t differ potentially in their real bequests, a measure of aggregate real savings involves a specification of how bequests are distributed each period. Denote by $\psi_t(b)$ the proportion of young agents born at period t with a real bequest of b or less. Then, with M denoting the fixed asset supply per young agent, the equality of real savings and the real value of the asset supply implies

$$(3.3) \quad \int s(b,R(t))d\psi_t(b) = Mp(t), \; t \geq 1.$$

Finally, the law of motion for the bequest distributions can be obtained from a simple argument that exploits the monotonicity of the saving function $s(\cdot,R)$ with respect to the bequest level. Let $h(z,R)$ denote the real bequest level that induces an agent to save z units of the good when the real rate of return is R, so that

$$s[h(z,R),R] = z, \; z \geq 0, \; R \geq 0.$$ [3]

Notice that agents that inherit b or less at t+1 can be divided into two disjoint groups. One group consists of those whose parents live two periods and therefore leave no bequest. The proportion of agents who belong to this group is, by assumption, π. The other group consists of those whose parents died prematurely, leaving a bequest (including interest) of b or less. That, in turn, requires that those parents saved $b/R(t)$ or less, which means that those parents inherited $h[b/R(t),R(t)]$ or less. The proportion of parents that received such bequests <u>and</u> died prematurely is given by $(1-\pi)\psi_t[h(b/R(t), R(t))]$. These considerations imply that

$$(3.4) \qquad \psi_{t+1}(b) = \pi + (1-\pi)\psi_t[h(b/R(t),R(t))], \; t \geq 1.$$

Formally then, we define a fixed asset supply equilibrium as a saving function $s[b,R]$, asset prices $\{p(t)|t \geq 1\}$, rates of return on savings $\{R(t)|t \geq 1\}$ and real bequest distributions $\{\psi_t(\cdot)|t \geq 1\}$, satisfying (3.1), (3.2), (3.3) and (3.4). A <u>stationary</u> equilibrium is defined as an equilibrium with time invariant bequest distributions, i.e., $\psi_t = \psi^*$ for all $t \geq 1$.

In appendix A, we prove the existence and uniqueness of a stationary equilibrium which has the following fairly intuitive characteristics:

In a stationary equilibrium, the asset price is the same in all periods so that the real return on claims is unity at all times. The stationary bequest distribution $\psi^*(\cdot)$ has a bounded countable infinite support, denoted by $\{b_k, k=0,1,2,...\}$, defined recursively by:

$$(3.5) \qquad \begin{aligned} b_o &= 0, \\ b_k &= s(b_{k-1},1),\ k \geq 1, \end{aligned}$$

where $b^* = \sup\{b_k,\ k = 0,1,2,\ldots\}$ satisfies $s(b^*,1) = b^*$.

Finally, $\psi^*(\ )$ is given by

$$(3.6) \qquad \psi^*(b_k) = \pi + \pi(1-\pi) + \ldots + \pi(1-\pi)^k,\ k = 0,1,2,\ldots.$$

An intuitive justification for the above equilibrium is gained by observing that, because savings are positive, no agent inherits a negative bequest and a proportion π of agents inherit zero each period. Hence b_o = 0 and $\psi^*(b_o) = \pi$. Agents that inherit zero save $b_1 = s(b_o,1)$, and $(1-\pi)$ of them bequeath b_1 to their offsprings so that $\psi^*(b_1) - \psi^*(b_0) = \pi(1-\pi)$, etc.. The discrete nature of ψ^* follows from the discrete nature of the shock that impinges upon an agent's life or death in the second period. An alternative specification of uncertain lifetime, such as the one used by Sheshinski and Weiss (1981), might result in a bequest distribution with a continuous support. In any event, given the specification utilized here our intuitive justification for the above steady state bequest equilibrium is formalized in appendix A and is summarized by figure 1.

[INSERT FIGURE 1]

The lifetime consumption allocation of an agent in the above equilibrium clearly depends upon his bequest and is given by

$$(3.7) \qquad \begin{aligned} c_1(b) &= w + b - s(b,1) \\ c_2(b) &= s(b,1) \end{aligned}$$

where $b \in \{b_k,\ k = 0, 1,2,\ldots\}$.

Our explicit characterization of the bequest equilibrium allows us to compare each agent's expected lifetime utility under the two hypothetical market structures: the unrestricted annuity regime of section 2 and the restricted savings opportunities of this section. However, the temporal nature of our model and the diversity in agents' initial wealth in the bequest equilibrium raise a number of interesting conceptual problems in deriving Pareto rankings of the two equilibria. We examine these issues and their implications in the remainder of this section.[4]

Welfare Comparison

Recall that in the stationary bequest equilibrium, the lifetime consumption of an agent whose initial real wealth consists of an endowment, w, and a bequest, b, is given by

$$
\begin{aligned}
&c_1(b) = w + b - s(b,1), \\
&c_2(b) = s(b,1), \text{ if the agent lives through his second period.}
\end{aligned}
\tag{3.8}
$$

In contrast, the stationary annuity equilibrium of section 2 is characterized by the absence of any bequests, and the associated lifetime consumption allocation is given by

$$
\begin{aligned}
&c_1 = \frac{w}{1+\pi} \\
&c_2 = \frac{w}{1+\pi}, \text{ if the agent lives through his second period.}
\end{aligned}
\tag{3.9}
$$

An important aspect of a welfare comparison of these two equilibria clearly involves a comparison of each agent's expected utility under the two regimes. However, in temporal models with agents of different birth dates, there seems to be some latitude in specifying the information on which the expected utilities of unborn agents are conditioned. Not surprisingly, the welfare ranking of allocations (3.8) and (3.9) may indeed depend on information assumptions regarding agents' initial wealth. This sensitivity of the welfare ranking to the information assumptions embedded in the optimality criterion has been demonstrated in a different context by Muench (1977) and Peled (1982). Because of this we compare allocations (3.8) and (3.9) under two optimality criteria and then comment on the appropriateness of each for our model.

Consider first conditioning the expected utility associated with allocations (3.8) and (3.9) on the same information that agents have when making their first period decisions. That is, we evaluate the expected utility of these allocations conditioned on the bequest level. For the bequest equilibrium, these expected utilities are given by

$$(3.10) \quad V(b) = u(w+b-s(b,1)) + \pi\, u(s(b,1)),\ b \in \{b_k,\ k=0,1,2...\}.$$

On the other hand, bequests are uniformly zero in the annuity regime, so that the expected utility of all agents is given by

$$(3.11) \quad EU_a = (1+\pi)\, u(\frac{w}{1+\pi}).$$

We say that allocation (3.9) is <u>conditionally preferred</u> to allocation (3.8) if

$$V(b) \leq EU_a \text{ for } \underline{\text{all}}\ b \in \{b_k,\ k=0,1,2,...\}.$$

It is straight forward, however, to generate examples that demonstrate the absence of any general conclusions which can be made on the basis of such a criterion. Notice that, in essence, this comparison involves the tradeoff between a real bequest $b \geq 0$ along with a real rate of return of unity on the one hand and a zero bequest but a rate of return on savings that equals $1/\pi$ on the other. It is clear that while agents with very low bequests will invariably prefer allocation (3.9), agents with sufficiently high bequests will, in general, prefer (3.8). However, given a sufficiently low survival probability, agents with any bequest level may conditionally prefer allocation (3.9).[5]

Alternatively, allocations (3.8) and (3.9) can be compared in the unconditional expected utilities sense, that is calculated on the basis of the bequest distribution ψ^*. The unconditioned expected utility associated with (3.8) is given by

$$(3.12) \quad EU_b = E_bV(b) = \int \{u[w+b-s(b,1))] + \pi u[s(b,1)]\} \, d\psi^*(b)$$

The relevant unconditioned expected utility associated with (3.9) is still EU_a in (3.11). We say that allocation (3.9) is <u>unconditionally preferred</u> to (3.8) if

$$(3.13) \quad EU_b \leq EU_a.$$

We now show that under certain additional mild restrictions on preferences, allocation (3.9) is, in fact, unconditionally preferred to (3.8). Our discussion takes the form of a sufficient condition under which (3.13) holds with strict inequality. This sufficient condition, though satisfied by a large class of economically relevant specifications of the economy, is not easily interpretable in terms of restrictions on preferences, endowments or survival probabilities, as it involves these features in a rather complex way.

Utilizing the concave, additively separable nature of agents' preferences, we have that for any $b \geq 0$,

$$V(b) = (1+\pi)[\frac{1}{1+\pi}u(w+b-s(b,1)) + \frac{\pi}{1+\pi}u(s(b,1))]$$

$$< (1+\pi)u[\frac{1}{1+\pi}(w+b) + \frac{\pi-1}{1+\pi}s(b,1)].$$

Hence,

$$(3.14) \quad EU_b < (1+\pi)E\{u[\frac{1}{1+\pi}(w+b) + \frac{\pi-1}{1+\pi}s(b,1)]\},$$

where the expectation E{ } is taken with respect to the real bequest distribution ψ^*. Next we would like to further bound the RHS of (3.14) from above by using the Jensen inequality. But even though u( ) is concave, the presence of s(b,1) in (3.14), (with a negative coefficient), may vitiate the concavity in b of the function the expected value of which is carried out in (3.14). Nevertheless, if the concavity of s(•,1) is not "too large," we may proceed by using the Jensen inequality on (3.14) to get:[6]

$$EU_b < (1+\pi)u[\frac{1}{1+\pi}w + \frac{1}{1+\pi}E\{b-(1-\pi)s(b,1)\}].$$

Finally, recall that in the bequest equilibrium, the average bequest per young agent equals the average estate per young agent, so that the expected value of $b-(1-\pi)s(b,1)$ is zero. Consequently, $EU_b < EU_a$, as was asserted.

The fact that (with some restriction on preferences) the annuity equilibrium allocation is unconditionally preferred to the bequest equilibrium allocation can be questioned on the following basis: the unconditional expected utility concept used to prove the above result is an appropriate representation of preferences of agents who are around prior to the realization of their own bequests which are perceived to be distributed according to $\psi^*(\ )$. In contrast, the information and timing specifications of the model endow agents at birth with the knowledge of their bequest. As Peled (1982) has argued elsewhere, this observation suggests a generic nonoptimality in the unconditional sense of any competitive equilibrium that obtains when agents are barred from taking any action before observing the realization of the conditioning event. This is generally the case because some trades which are beneficial in the unconditional sense, such as sharing the risks associated with these realizations, are excluded. Moreover, notice that the annuity equilibrium allocation (3.9) not only dominates the bequest equilibrium allocation (3.8) in the unconditional sense, but it is also optimal in this sense. How is it possible, then, to obtain an unconditionally optimal allocation as a decentralized equilibrium without utilizing the sorts of pre-endowment trades which are physically impossible in our model? The answer is given by the fact that the conditioning event in our model -- the random endowment at birth -- is endogenously determined for each generation by the decision of the previous one. By opening markets for perfect annuities we eliminate bequests altogether and thereby completely degenerate the randomness of the conditioning event. Put differently, the annuity markets remove the distinction between unconditional and conditional expected utilities with respect to bequests. In contrast, the randomness in agents' initial conditions studied by Muench (1977) and Peled (1982) was not endogenous in this sense and consequently results in a generic nonoptimality in the unconditional sense of decentralized equilibria. Likewise, if in our model agents were subject to idiosyncratic exogenous shocks at birth, in addition to the size of their bequest, unconditional optimality would not be displayed by competitive equilibria.

To make these points somewhat more concrete, we now consider the welfare implications of an actual shift from one market structure to another. Notice that the previous analysis was confined to welfare comparisons of allocations associated with the steady state of two alternative equilibria. As such, the analysis abstracted from the implications of a regime shift on the welfare of those generations who are alive during the transition from one steady state to another. It is evident that such an analysis will depend very much on the precise way in which the regime shift is implemented. Here we consider the simplest way of examining such transition effects.

Imagine that at some time t_o-1, $t_o \geq 2$, the economy is in the steady state equilibrium of the bequest regime. At the end of t_o-1 each young agent holds savings in the form of a paper asset which was accumulated with the understanding that the real of return would be unity. In the bequest equilibrium, unborn agents of generation t_o+j, $j \geq 0$, would have inherited real bequests distributed according to $\psi^*(\cdot)$. Consequently their consumption levels would have been described by (3.8).

The policy experiment consists of an announcement by the government at the end of period t_o-1 (after consumption has taken place) that restrictions on annuity markets are removed for all periods t_o+j, $j \geq 0$.

In appendix B we show that the resulting equilibrium has the property that bequests will be eliminated as of time t_o and the associated consumption levels given by

$$(3.15) \quad \begin{aligned} &\bar{c}_{2,t_o-1} = \bar{R}s(b,1), \text{ for some } \bar{R} > 1 \\ &\bar{c}_{1,t} = \bar{c}_{2,t} = \tfrac{w}{1+\pi}, \text{ for all } t \geq t_o. \end{aligned}$$

Hence we may evaluate the effect of this regime shift on the welfare of the unborn generations $t \geq t_o$ by appealing to the results of the previous section. In particular, while we may say that allocation (3.15) is, under certain mild restrictions on preferences, unconditionally preferred to the allocation implied by the continuation of the bequest regime, no general ranking is possible on the basis of the conditional criterion. However, there are no such ambiguities in evaluating the welfare effects regarding the members of generation t_o-1. They are clearly better off under (3.15) because $\bar{R}$ exceeds the rate of return they would receive in the absence of the regime shift, namely unity. Thus the above implementation scheme allows for the transition from the bequest regime to the annuity regime in a decentralized way and in which the transition generation gains from the shift.

Notice, however, that the above scheme precludes any non-trivial announcement effects, in the sense that the policy is implemented in a way that does not allow agents to respond to perceived changes in rates of return. While the analysis of these more general policy shifts is clearly beyond the scope of this paper, we should emphasize that such an exercise is considerably more complicated and may lead to situations in which some members of transitional generations are adversely affected. Consequently, some degree of government intervention may be necessary to prevent adverse effects of an announced regime change on the current old. But as section 2 points out, once private annuity markets have been set up in environments where firms do not face any difficulties in discriminating between members of different groups, there are no welfare gains associated with any maintained government intervention in the annuity markets. In section 4 we examine one such obstacle to discriminatory behavior on the part of firms -- private information with respect to survival probabilities -- and derive its sharply different implications regarding the desirability of government intervention.

4. Annuity Markets and Social Security in the Presence of Private Information

We now turn our attention to the performance of the economy in the presence of private information regarding survival probabilities. Recent developments in the literature regarding the economics of information have pointed out the large differences between the properties of classical Walrasian equilibria and information equilibria, i.e., equilibria in which buyers and sellers have private information regarding the qualitative nature of the good which is being bought and sold. As is well documented in the literature, these differences relate to both the existence and optimality of competitive equilibria. Of particular interest for the problem at hand is the widely known result obtained by Rothschild and Stiglitz [1976] and Wilson [1977] among others, that even when competitive equilibria exist under such circumstances, the associated equilibrium allocations need not be Pareto Optimal.

On the other hand, one of the prime justifications given in the literature for a government-run social security program is the alleged need to correct various sorts of market failures which give rise to inefficient forms of risk sharing. Diamond [1977], for example, asserts that there are a number of market failures in the present U.S. economy which a social security system could help alleviate. While Diamond does not provide a model of the reasons for these alleged market failures, he does discuss in depth the problems of insuring the risks associated with a varying length of working life. Prominent among these are that attempts to insure such risks face severe moral hazard and adverse selection problems. Both problems arise due to the existence of private information in those markets. While this section does not attempt to model the specific phenomena alluded to by Diamond, we try to capture the essence of his arguments in favor of government intervention by considering the nature of competitive annuity markets in the presence of private information regarding survival probabilities. This is done by showing that the framework developed by Rothschild and Stiglitz and Wilson for dealing with the nature of competitive insurance markets in the presence of private information can be easily extended to deal with annuity markets. Once this is done, the resulting competitive equilibrium is shown to be, in general, nonoptimal, so that there is, in principle, a Pareto improving role for the government in such economies. Moreover, it turns out that the set of optimally designed mandatory social security regimes which lead to an equilibrium which Pareto dominates the non-intervention equilibrium allows for the co-existence of private annuity markets and the government-run program. Furthermore, while the government annuities are actuarially fair in an economy-wide sense, the resulting equilibrium in the residual private annuity market is a separating one so that rates of return on privately issued annuities are actuarially fair in a group specific sense. Hence private and public modes of savings will not be perfect subistitutes from the point of view of the individual agents in the system. While one group of agents would always like to invest more in the public program, another group of agents, who view private market rates of return parameterically, would like to opt out of the system. However, were they allowed to do so, the resulting equilibrium would be one in which they would be uniformly worse off.

The Competitive Annuity Market

Excepting our specification of the information sets of agents, the economy to be discussed is the same as that analyzed in section 2. With respect to these information sets, we assume that agents in our economy know their own survival probabilities as well as how many π_A and π_B-type individuals there are in the economy at any given moment. Similarly, the government knows the values of π_A, π_B, and γ, where for each type A agent, there are γ-type B agents. However, no agent, including the government, knows whether any other particular individual belongs to group A or B. For convenience, we assume that $\pi_B > \pi_A$. Analogous to Rothschild and Stiglitz and Wilson, we define an annuity policy α as a two-dimensional vector $[s^\alpha, R^\alpha]$ so that if a young agent purchases the policy α his consumption vector (c_1, c_2) becomes $(w-s^\alpha, R^\alpha s^\alpha)$ if he lives two periods and $(w-s^\alpha, 0)$ if he lives only one perod. Notice the above specification implies that sellers specifiy both "prices and quantities" (R and s) in annuity contracts. Rothschild and Stiglitz argue at length that price and quantity competition coupled with free entry into insurance markets is the appropriate notion of competition in these sorts of markets. In particular, they point out that price competition is clearly a special case of price and quantity competition because nothing in the definition of the latter prevents firms from offering for sale a set of annuities which can be bought in different quantities, but which have the same rate of return if the purchaser survives. Hence, firms which adopt pure price strategies cannot hope to successfully compete with firms who adopt mixed price and quantity strategies. The interested reader is referred to Rothschild and Stiglitz for a more detailed discussion of this point.

Since the consumption vector of each young agent can be represented by the annuity policy that he purchases, the expected utility of agent i associated with various consumption plans can be represented by an indirect utility function V^i, defined over the set of insurance policies, where $V^i(\cdot)$ is given by

$$(4.1) \qquad V^i(R,s) = u(w-s) + \pi_i u(Rs), \; i = A,B$$

[INSERT FIGURE 2]

Hence for any given R and s, the slope of the indifference curve of a young member of group i, is given by

$$(4.2) \qquad \frac{dc_1}{dc_2} = -\pi_i \frac{u'(Rs)}{u'(w-s)}, \; i = A,B$$

Given our assumptions on $u(\cdot)$, for any given contract (s^α, R^α), (represented in figure 2 by α) the slope of a type B indifference curve, I_B, will be in absolute value, greater than a type A indifference curve, I_A. Put alternatively, for any given rate of return R', a type B young person would like to purchase a larger number of annuities than a type A young person (α^A and α^B in figure 2).

Because of the serious and as yet unresolved controversies in the literature regarding the appropriate definition of equilibrium for markets such as these, we consider Wilson's two alternative definitions of equilibrium, both of which are motivated by the desire to describe an equilibrium set of policies for a situation in which firms can costlessly enter the market. Because of the non-random and time independent nature of endowments and paper asset supply, and the fact that the structure of the population does not change over time, we confine ourselves to characterizations of the stationary equilibrium of the economy.

A Rothschild/Stiglitz (E1) equlibrium, which was used in section 2, is a set of contracts such that when agents choose contracts to maximize their expected utility, (i) no contract in the equilibrium set makes expected negative profits, and (ii) there is no contract outside the equilibrium set that, if offered, will make a nonnegative profit. As Rothschild and Stiglitz point out, the E1 equilibrium is of the Nash-Cournot type in that each firm assumes that the contracts its competitors offer are independent of its own actions.

A Wilson (E2) equilibrium is the same as the E1 equilibrium except that firms' expectations are modified by assuming that each firm will correctly anticipate which of those policies that are offered by other firms will become unprofitable as a consequence of any changes in its own policies. The firm then offers a new policy only if it makes nonnegative profits after the other firms have made the expected adjustment in their policy offers.

We first consider the E1 equilibrium and establish the following results:

(i) There cannot be an E1 pooling equilibrium

(ii) If an E1 equilibrium exists, it is a separating equilibrium where type A agents buy the contract $(s_1, \frac{1}{\pi_A})$ and type B buy the contract $(\frac{\pi_B}{1+\pi_B}w, \frac{1}{\pi_B})$. The quantity of annuities, s_1, purchased by a type A agent maximizes his utility given an intertemporal rate of return of $1/\pi_A$, and satisfies the self-selection constraint for type B agents:

$$u(\frac{1}{1+\pi_B}w) + \pi_B\, u(\frac{1}{1+\pi_B}w) \geq u(w-s_1) + \pi_B u(\frac{1}{\pi_A}s_1).$$

(iii) For sufficiently small values of $\gamma > 0$, i.e., a relatively small number of type B agents, there does not exist an E1 annuity market equilibrium.

Because of the similarity of our model to that of Rothschild and Stiglitz and Wilson, we demonstrate (i), (ii) and (iii) primarily via geometric arguments.

A simple graphical argument demonstrates that there cannot be an E1 pooling equilibrum, i.e., an equilibrium in which members of both groups buy the same policy (s,R). Denote by $s^i(R)$ the unconstrained, utility maximizing purchase of annuities that pay R by type i agent, i = A,B. Zero profits in an equilibrium in which both groups face the same return on annuities, $\bar{R}$, requires that

$$(4.3) \quad s^A(\bar{R}) + \gamma s^B(\bar{R}) = \bar{R}[\pi_A s^A(\bar{R}) + \gamma\pi_B s^B(\bar{R})]]$$

Given that $s = s^A = s^B$ in a stationary pooling equilibrium, it is immediately evident from (4.3) that

$$\bar{R} = \frac{1+\gamma}{\pi_A+\gamma\pi_B}, \tag{4.4}$$

so that $1/\pi_B < \bar{R} < 1/\pi_A$. The point α in figure 3 depicts some arbitrary E1 pooling equilibrium.

[INSERT FIGURE 3]

Given our results from Figure 2 and the relative slopes of the type A and B indifference curves, it follows that there always exists some contract A, near α, which if offered, is preferred by group A, but not by group B. Hence, if offered, it will be exclusively bought by members of group A which from the point of view of firms is the low risk group; therefore the firm will earn nonnegative profits. But the existence of such a contract contradicts the second part of the definition of an E1 equilibrium. Because a similar argument can be made for any point α on the $(w,\bar{R}w)$ line, it follows that no E1 pooling equilibrium exists for the industry in question. Therefore, if an E1 equilibrium exists, it must be a separating equilibrium.

[INSERT FIGURE 4]

To establish (ii) we begin by noting that, as in the pooling equilibrium, we require that each contract offered earns zero profits. This in turn implies that in Figure 4 the low risk contract must lie on the $(w, w/\pi_A)$ line while the high risk contract must lie on the $(w, w/\pi_B)$ line. From section 2, the contract on the $(w, w/\pi_B)$ line that is most preferred by members of group B equates planned consumption in both periods of their life. This allocation corresponds to the contract C in Figure 4. On the other hand, members of group A would, of all contracts on the $(w, w/\pi_A)$ line prefer D, which like C, equates planned consumption in both periods of the agent's life.

However, contract D dominates contract C from the point of view of members of group B. Hence, if both C and D are offered, all agents will purchase D. Given private information, all individuals who demand D must be sold D. But since D is actuarially fair for members of group A only, profits will necessarily be negative if members of group B purchase it. Hence, the contract (C,D) cannot be a separating equilbirum. It follows that a separating E1 equilibrium contract for group A must not be more attractive to the members of group B than contract C. Letting $c_1^A(E)$ and $c_2^A(E)$ denote first and second period consumption under some contract E which lies along $(w, w/\pi_A)$ and recalling that $c_1^B = c_2^B = w/(1+\pi_B)$, we require that

$$(4.5) \qquad (1+\pi_B)\ u\left(\frac{w}{1+\pi_B}\right) \geq u(c_1^A(E)) + \pi_B u(c_2^A(E))$$

for the contract C and E to be a separating equilibrium. Hence if an E1 equilibrium exists, the quantity of annuities that pay $1/\pi_A$ purchased by a type A individual, s_1, solve the following problem:

$$(4.6) \qquad \underset{s}{\text{Max}}\ u(w-s) + \pi_A\ u(s/\pi_A)$$

subject to

$$(4.7) \quad (1+\pi_B)u(\frac{w}{1+\pi_B}) \geq u(w-s) + \pi_B u(s/\pi_A).$$

However, because the constraint is clearly binding, we may replace the weak inequality in (4.7) with strict equality. Hence, the set of potential E1 equilibria can be found by examining the solutions of

$$(4.8) \quad (1+\pi_B)u(\frac{w}{1+\pi_B}) = u(w-s) + \pi_B u(s/\pi_A).$$

It is straightforward to verify that there always exist two solutions to (4.8), (corresponding to the points E and E_1 in figure 4), where the indifference curve of a type B agent, I_B, intersects the $(w,w/\pi_A)$ line. Since the indifference curve of the representative type A agent through point E_1 always lies below the one going through point E, the E1 equilibrium is given by (C,E), if it exists. However, for the reasons pointed out by Rothschild and Stiglitz, to be discussed below, (C,E) may not be an equilibrum which, therefore, implies that an E1 equilibrium does not necessarily exist.

[INSERT FIGURE 5]

To establish (iii) we begin by considering the contract F in figure 5 which lies above both I_A and I_B. If F is offered, members of both group B and A will purchase it in preference to contracts C and E, respectively. If it makes nonnegative profit when both groups buy it, F clearly upsets the potential E1 separating equilibrium (C,E). This is the case if the aggregate actuarially fair line is given by $(w,\bar{R}w)$, while this is not the case if that line is given by $(w,\hat{R}w)$. Hence in the former case, there does not exist an E1 equilibrium while in the latter case there does. Notice then that, from equation (4.4), the class of economies for which an E1 equilibrium exists is monotonically increasing in γ, since $\bar{R}$ decreases monotonically in γ.

Before considering the optimality of the E1 equilibrium when it exists, we turn our attention to the class of E2 equilibria. Wilson demonstrates, in a more general context, that if an E1 equilibrium exists, it is also an E2 equilibrium. Given the above results, we need only illustrate that an E2 equilibrium exists when the E1 equilibrium does not. Such a case is displayed in Figure 6. Suppose that F is an E2 equilibrium. Can it be broken by the contract A as was the case with the E1 equilibrium? To see that it cannot, notice that now, before offering A, firms realize that if it is offered only type B people will purchase contract F which will therefore be unprofitable and withdrawn. As such, if A is offered, it is expected that members of both groups will buy it. But under those conditions, the contract A will yield negative profits. As a result, no firm will offer the contract A. In a similar way,

[INSERT FIGURE 6]

it can be shown that there does not exist any contract which will break the proposed equilibrium. Hence the E2 equilibrium is given by the contract F.

In general, then, one can derive the E2 equilibrium as follows: when the parameters of the problem are such that an E1 equilibrium exists, the E1 and E2 equilibria are the same and are given by the solution to (4.6);

when the E1 equilibrium does not exist, the E2 equilibrium is a pooling equilibrium which may be found by solving

$$(4.9) \quad \text{Max } \{u(w-s) + \pi_A u(s/\bar{R})\},$$

and then offering the contract $(s,\bar{R})$ where $\bar{R}$ is given by (4.4).

In summary, this subsection outlines two notions of competitive equilibria that can be shown to exist in our economy. Those two equilibria concepts, suggested by Wilson and Rothschild-Stiglitz, involve strategic considerations on the part of the competing firms. Prescott and Townsend (1979) provide compelling arguments for the nonexistence of a nonstrategic competitive equilibrium in environments of which ours is a special case. We should note, however, that our discussion of competitive equilibria is not ment to be exhaustive. Rather, we require some notion of a competitive equilibrium that can be shown to exist in our adverse selection economy in order to evaluate the desirability of government intervention.

The Welfare Improving Role of Mandatory Social Security

The above examples illustrate the fact that, with private information regarding survival probabilities, the presence of high risk individuals, i.e., type B agents who have a high probability of living and therefore of collecting on their annuity contracts, exerts a negative externality on the type A individuals, i.e., those agents who have a low probability of surviving and therefore of collecting on their annuity contracts. In the case where an E1 equilibrium exists contracts-(C,E) in (Figure 4)-this externality is purely destructive in that while group A is worse off than they would be in the absence of private information (I_A versus I_A'), group B is not better off. On the other hand, in the case where no E1 equilibrium exists and the economy is at an E2 equilibrium (Figure 6), while group A is still worse off, at least the members of group B are better off than they would be in the absence of private information.

Given the existence of these negative externalities, it should not come as a surprise that there exist Pareto improving policies which the government can undertake. Needless to say, in considering such policies, we restrict ourselves, a priori, to interventions which do not require that the government be able to distinguish between agents of different types. Notice that the definitions of the E1 and E2 equilibria impose that each contract which is purchased in equilibrium earn nonnegative profit. But, as Wilson points out, this restriction arises because of the expectations that firms have regarding the effect of an unprofitable policy on its aggregate profits. It does not arise as a consequence of the self-selection problem. The above two observations taken together imply that the search for allocations which are superior to those obtained by the private equilibrium can be restricted to the class of contracts which individually may yield negative profit but which together achieve nonnegative profits. Corresponding to the two possible decentralized equilibria, we consider two cases for a potential role for government intervention in the annuity markets:

Case I E2 Pooling Equilibrium

As discussed above, the E2 pooling equilibrium is characterized by the solution to (4.9), so that the point F in Figure 7 characterizes the equilibrium allocation with members of each group having the planned consumption allocation $\{c_1,c_2\} = \{w-x,\bar{R}x\}$. In general, there exist a continuum of mandatory social security systems that Pareto dominate the E2 pooling equilibrium when combined with private markets which satisfy residual demands for annuities. Here we discuss only two such policies in order to convince the reader of their existence and general characteristics. Imagine that the government introduces a manadatory social security system which requires that all agents contribute $x', 0 < x' < x$, during the first period of their life and receive $\bar{R}x'$ during the second period of their life if they survive, where $\bar{R}$ is the aggregate actuarially fair rate of return that is determined by (4.4). The government then permits the private market to satisfy any residual demand for annuities. Members of both groups can then be viewed as having a new endowment vector, $(w-x', x'\bar{R})$. This creates opportunities for a different private annuity market equilibrium which can be studied by drawing the group specific actuarially fair budget lines originating from the transformed endowment. In figure 7, these correspond to the broken budget lines $(T, x'\bar{R} + \frac{w-x'}{1+\pi_A})$ and $(T, x'\bar{R} + \frac{w-x'}{1+\pi_B})$. For the particular x' considered in figure 7, the resulting equilibrium allocations for groups A and B are given by the points E and C corresponding to the indifference curves I_A' and I_B^o. Notice then that while we start with an E2 pooling equilibrium (at F) the result of combining a mandatory social security contribution x' with private annuity markets is a separating equilibrium in the private sector. That such a separating equilibrium exists is evident from the fact that the aggregate actuarially fair budget line $(w,w\bar{R})$ lies uniformly below I_A'. (Recall that to upset a separating equilibrium, we require that there exist a contract on the aggregate actuarially fair line that is preferred by group A to their self-selected contract). Moreover, in comparing the non-intervention equilibrium allocation (F) and that which is obtained under the social security regime (E and C), we see that while the utility level of group B is unaffected, group A is strictly better off under the intervention regime (I_A' vs. I_A^o). However, as is evident from figure 7, for social security contributions below x', both groups will be made worse off at the resulting equilibrium as compared to allocations F. Thus, x' in figure 7 corresponds to the minimal social security contribution that improves upon the completely decentralized pooling equilibrium. The fact that a contribution of x' to the social security system leaves the welfare level of group B unaffected, implies that x' may be determined by finding the intersection of a budget line with a slope of $1/\pi_B$ tangent to I_B^o and the economy-wide actuarially fair budget line. Formally, I_B^o represents a utility level of $u(w-x) + \pi_B u(\bar{R}x)$, while at point C, the lifetime consumption of a type B agent is given by

$$c_1^B = c_2^B = (w-x'+\pi_B x'\bar{R})\frac{1}{1+\pi_B}.$$

Hence, x' is the solution to

$$(4.10)\quad (1+\pi_B)u[(w-x'+\pi_B x'\bar{R})\frac{1}{1+\pi_B}] = u(w-x) + \pi_B u(\bar{R}x).$$

Alternatively, consider a contribution to the social security program that equals x, the voluntary savings at the initial equilibrium point F. Drawing group specific actuarially fair budget lines from that point, (dotted lines in figure 7), it can be shown that a resulting separating equilibrium occurs at points K and J. Type B agents are on the indifference curve labelled I_B', which intersects the type A agents' budget line and determines point J. As is evident from figure 7, both groups are better off with these allocations than with allocation F. Also, since first period consumption at K and J is lower than at F, it is clear that aggregate savings are higher with social security contribution of size x. While the two levels of social security contributions discussed above Pareto dominate the completely decentralized equilibrium, they are mutually noncomparable. Group A is better off with a smaller social security contribution (point B) than with the larger contribution (point J), while the converse holds for group B. In fact, the equilibria associated with all intermediate levels of contributions are all noncomparable in this way while each of them Pareto dominates the pooling equilibrium F. Although we have demonstrated that aggregate savings level can increase as a result of such a program, this effect may depend on the size of the mandatory contributions.

Case II E1 Separating Equilibrium

Unlike Case I, if we begin from a separating equilibrium, the existence of a Pareto improving social security scheme is not guaranteed. Figure 8 depicts a case where it does exist, but as will become evident below, our success involves a fairly special structure of preferences. We begin with a separating equilibium with group A at point E, and group B at point C. A mandatory contribution to a social security program in which annuities have a real rate of return of $\bar{R}$ gives rise to the group specific actuarially fair (broken) budget lines in figure 8. Private annuities purchased by group B then attain allocation G along indifference curve I_B'. If that curve intersects group A's budget line at a point like H, which is preferred to point E by group A, we have a separating equilibrium that dominates the decentralized one. Although there is nothing that guarantees this possibility in general, one cannot presume the Pareto optimality of the separating decentralized equilibrium.

In concludng this section, we note that while there are still ongoing and unresolved controversies in the literature regarding the appropriate concept of equilibrium for private information economies such as those studied here, our results are quite encouraging in that they suggest an important motive for mandatory social security. Furthermore, the model develops a framework which allows for the co-existence of public and private annuities which are not perfect substitutes from the point of the view of economic agents in the system. In this vein, it is interesting to note that in the social security equilibrium the members of group A obtain a higher rate of return on private annuities than on their contributions to the social security system. As a result, any individual member of group A would like to withdraw from the public plan. This, of course, is why the program must be mandatory since if group A were allowed to opt out the resulting equilibrium would be one in which the members of group A and B are worse off. In sum, then, the analysis not only allows for, but is crucially dependent upon the imperfect substitutability of private and public annuities. Moreover, our ability to decompose total annuities into public and private annuities allows us to analyze the welfare effects of increases in the magnitude of contributions to the social security system with group A desiring smaller (up to some point) mandatory contributions and group B wanting larger (up to some point) contributions.

5. Conclusion

This paper has investigated the nature of market structures that are capable of supporting optimal allocations in environments where there exist diversity with respect to survival probabilities. We find that when individual survival probabilities are public information, optimal allocations have the property that ex ante marginal rates of substitution between consumption in different periods are not equalized across members of different groups. Moreover, decentralized annuity markets support such an allocation by offering group specific actuarially fair annuities as opposed to economy-wide actuarially fair annuities. However, when individual survival probabilities are private information, there can be no presumption that competitive equilibria result in optimal allocations. For a large class of economies, mandatory social security programs, which are actuarially fair in an aggregate sense, when coupled with residual private annuity markets, lead to an equilibrium which Pareto dominates that of the non-intervention regime. Moreover, because the model allows for the co-existence of public and private annuities which are non-perfect substitutes for each other, it will not be true that an increase in savings in the form of contributions to Social Security causes an equal displacement of private, voluntary savings. In fact, it may even increase aggregate savings.

Our results taken together indicate the potential danger in discussing the nature of potential welfare enhancing government interventions by imposing a priori market restrictions rather than conducting the analysis within fully specified models which tie market failures to the fundamental features of the underlying environment which cause them. Thus, the nature of optimally designed social security systems in environments where survival probabilities are public information, but annuity markets are excluded by fiat is very different from the mandatory social security system which is appropriate for environments where survival probabilities is private information.

Finally, the paper provides a framework for analyzing the problems of insuring the risks associated with varying lengths of working life or private information regarding the productivity of human capital. At this point, we can only conjecture that the resulting policy implications will be quite different than those derived from a model in which such markets are a priori excluded.

Appendix A

Before characterizing the unique stationary bequest equilibrium, we verify that every allocation associated with a solution to equations (3.1) - (3.4) is feasible.

The resource constraint of this exchange economy is given by

(A.1) $\int c_{1t}(b)d\psi_{t+1}(b) + \pi\int c_{2t}(t)d\psi_t(b) = w, \qquad t \geq 1$

where

(A.2) $c_{1t}(b) = w + b - s(b,R(t))$

and

(A.3) $c_{2t}(b) = R(t)s(b,R(t))$.

Substituting (A.2) and (A.3) into (A.1) implies

(A.4) $\int bd\psi_{t+1}(b) - \int s(b,R(t+1))d\psi_{t+1}(b) + \pi\int R(t)s(b,R(t))d\psi_t(b) = 0.$

Since savings are nonnegative, $\psi_t(z) = 0$, $z < 0$ for all t. Using this fact and the transition law for the bequest distribution (3.4), the first term in (A.4) can be rewritten as

$$\begin{aligned}\int bd\psi_{t+1}(b) &= \int bd\{\pi+(1-\pi)\psi_t[h(b/R(t),R(t))]\} \\ &= (1-\pi)\int bd\psi_t[h(b/R(t),R(t))] \\ &= (1-\pi)\int R(t)s(z,R(t))d\psi_t(z),\end{aligned}$$

where the last equality comes from defining $z = h[b/R(t),R(t)]$, so that $b = R(t)s(z,R(t))$. Consequently, (A.4) is equivalent to

(A.5) $R(t)\int s(b,R(t))d\psi_t(b) - \int s(b,R(t+1))d\psi_{t+1}(b) = 0,$

which holds whenever (3.2) and (3.3) hold.

Next, we prove that if $\psi_t = \psi$ for all $t \geq 1$ then $R(t) = 1 \ \forall\ t \geq 1$. Substitution of (A.5) into (A.4) and the fact that $\psi_t = \psi_{t+1} = \psi$, yields

(A.6) $\int bd\psi(b) = (1-\pi) \int R(t)s(b,R(t))d\psi(b)$

which equates average real bequests with the average estate of deceased agents per young person. As noted above, the concavity and monotonicity of $u(\cdot)$ is sufficient to deduce that $Rs(b,R)$ is an increasing function of R. Since the LHS of (A.6) is independent of time, if (A.6) has any solution R for a given ψ, this solution is unique. Therefore, $R(t) = R$ for all t. It then follows immediately from (A.5) that $R = 1$.

Denote by $\psi^*(\cdot)$ the stationary equilibrium real bequest distribution. Utilizing (3.4), the nonnegativity constraint on s, and properties (i) - (iii) of $s(\cdot,R)$ from section 2, we can now explicitly solve for $\psi^*(\cdot)$. The stationary real bequest distribution should satisfy

$$\text{(A.7)} \quad \begin{aligned} &\psi^*(z) = \pi + (1-\pi)\psi^*[h(z,1)] && z \geq 0 \\ &\psi^*(z) = 0 && z < 0. \end{aligned}$$

Define the following sequence of real bequest levels:

$$\text{(A.8)} \quad \begin{aligned} &b_0 = 0 \\ &b_k = s(b_{k-1},1), && k = 1,2,\ldots. \end{aligned}$$

Since $0 < \partial s(b,1)/\partial b < 1$ and $s(0,1) > 0$ the above sequence satisfies

$$b_k > b_{k-1} \qquad k = 1,2,\ldots$$

and

$$\lim_{k\to\infty} b_k = b^*,$$

where b^* is such that $s(b^*,1) = b^*$. Notice that for $k \geq 1$, b_k is the real savings of a person inheriting b_{k-1}, so that $b_{k-1} = h(b_k,1)$. Since $s(b,1) \geq s(0,1) = b_1$, for $b \geq 0$, savings levels below b_1 can only be generated by negative bequests. That is, $h(z,1) < 0$ for $z \in [0,b_1)$. From (A.7) it then follows that

$$\psi^*(z) = \pi, \qquad 0 \leq z < b_1.$$

For $z \in [b_1,b_2)$, $h(z,1) \in [0,b_1)$, so that $\psi^*[h(z,1)] = \pi$ in this range, and hence

$$\psi^*(z) = \pi + (1-\pi)\pi, \qquad b_1 \leq z < b_2.$$

In general,

$$\text{(A.9)} \quad \psi^*(z) = \sum_{j=0}^{k-1} (1-\pi)^j \pi, \text{ for } z \in [b_{k-1}, b_k), \; k = 1,2,\ldots.$$

The unique (by construction) stationary distribution of real bequests is then given by (A.8) and (A.9). It is readily verified that

$$\psi^*(b^*) = \lim_{k\to\infty} \psi^*(b_k) = \lim_{k\to\infty} \sum_{j=0}^{k-1} (1-\pi)^j \pi = 1$$

which completes our proof.

Appendix B

The market structure change suggested in the text involves, in effect, reneging on claims to consumption inherited by young members of generation t_0. These agents, therefore, each have a wealth of w in the first period and save via annuities that pay a real rate of return of $1/\pi$ to survivors. On the other hand, old members of generation t_0-1 hold the claims they acquired when young. Specifically, a survivor of generation t_0-1 who inherited b when young, holds claims to $s(b,1)$ units of the good at t_0. We show next that zero profits in the savings industry at period t_0 involve a payoff to these claims at a real rate that exceeds unity.

The real savings level of a young member of generation t is given by $\frac{\pi}{1+\pi}w$. There are π old survivors per young agent, so that if claims to consumption are paid off at a rate R, nonnegative profits in the saving industry require

$$\text{(B.1)} \quad \pi \int R s(b,1) d\psi^*(b) \leq \frac{\pi}{1+\pi} w.$$

We show that (B.1) holds with equality for some $\bar{R} > 1$ by verifying that it holds as a strict inequality for $R = 1$.

First notice that $s(b,1)$, the optimally chosen savings of a member of generation t_0-1 with a bequest b satisfy

$$-u'(w+b-s(b,1)) + \pi u'(s(b,1)) = 0.$$

Hence, since $\pi < 1$,

$$u'(w+b-s(b,1)) < u'(s(b,1)),$$

which implies that

$$w + b - s(b,1) > s(b,1)$$

or that

$$s(b,1) < \frac{w+b}{2}, \quad \forall\, b \geq 0. \tag{B.2}$$

To evaluate the LHS of (B.1) with $R = 1$, we utilize our explicit characterization of ψ^* and (B.2) to get:

$$b_0 = 0;$$

$$b_1 = s(b_0,1) = s(0,1) < \frac{w}{2};$$

$$b_2 = s(b_1,1) < \frac{w+b_1}{2} < \frac{w}{2} + \frac{w}{4}.$$

In general, then,

$$s(b_k,1) < \frac{w}{2} \sum_{j=0}^{k} (1/2)^j = w[1-(1/2)^{k+1}], \quad \text{for } k - 0,1,2,\ldots. \tag{B.3}$$

Consequently, the LHS of (B.1) with $R = 1$ is less than

$$\pi \sum_{k=0}^{\infty} w[1-(1/2)^{k+1}]\pi(1-\pi)^k = w\frac{\pi}{1+\pi},$$

which completes the proof.

The equilibrium consumption levels for generations t_0, t_0+1, t_0+2, etc., then follow the stationary annuity equilibrium of section 2, and are given by (3.15).

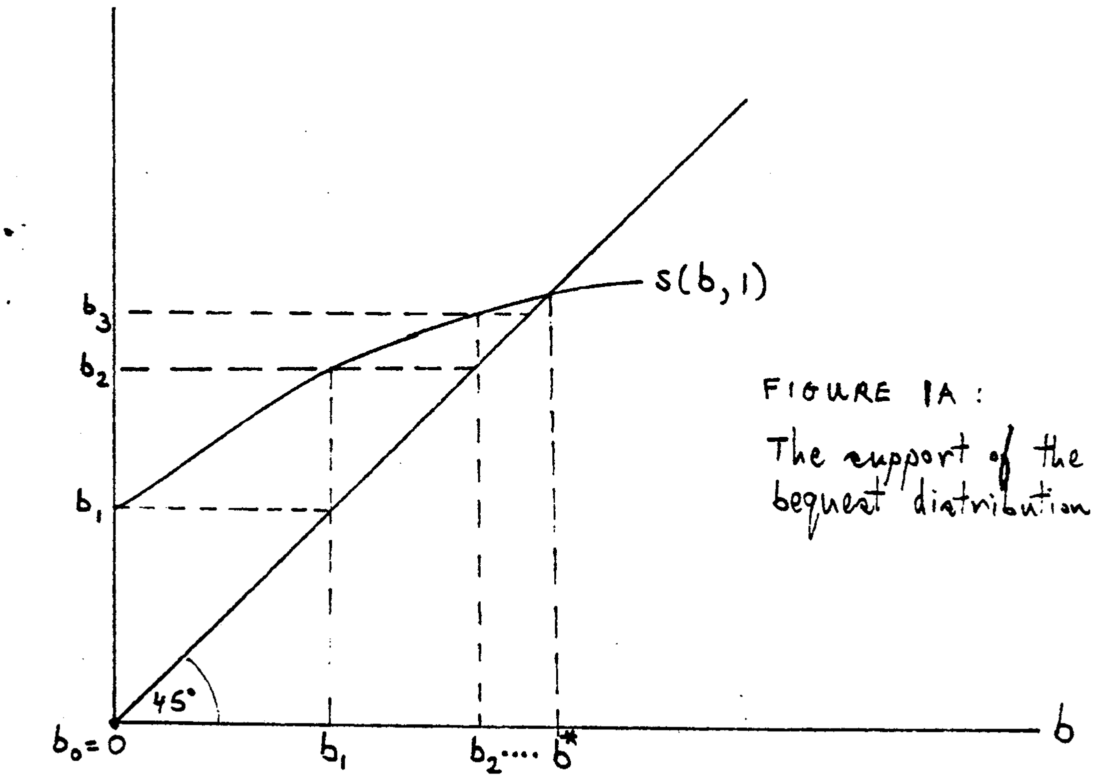


FIGURE 1A: The support of the bequest distribution

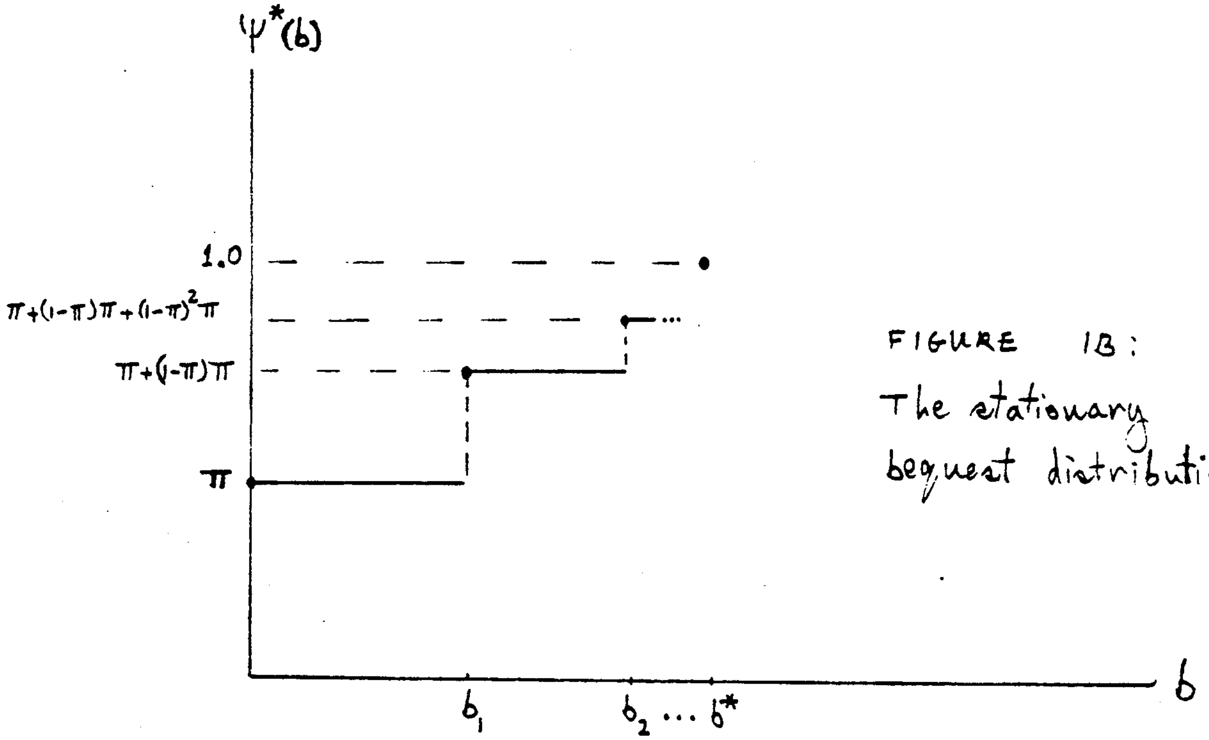
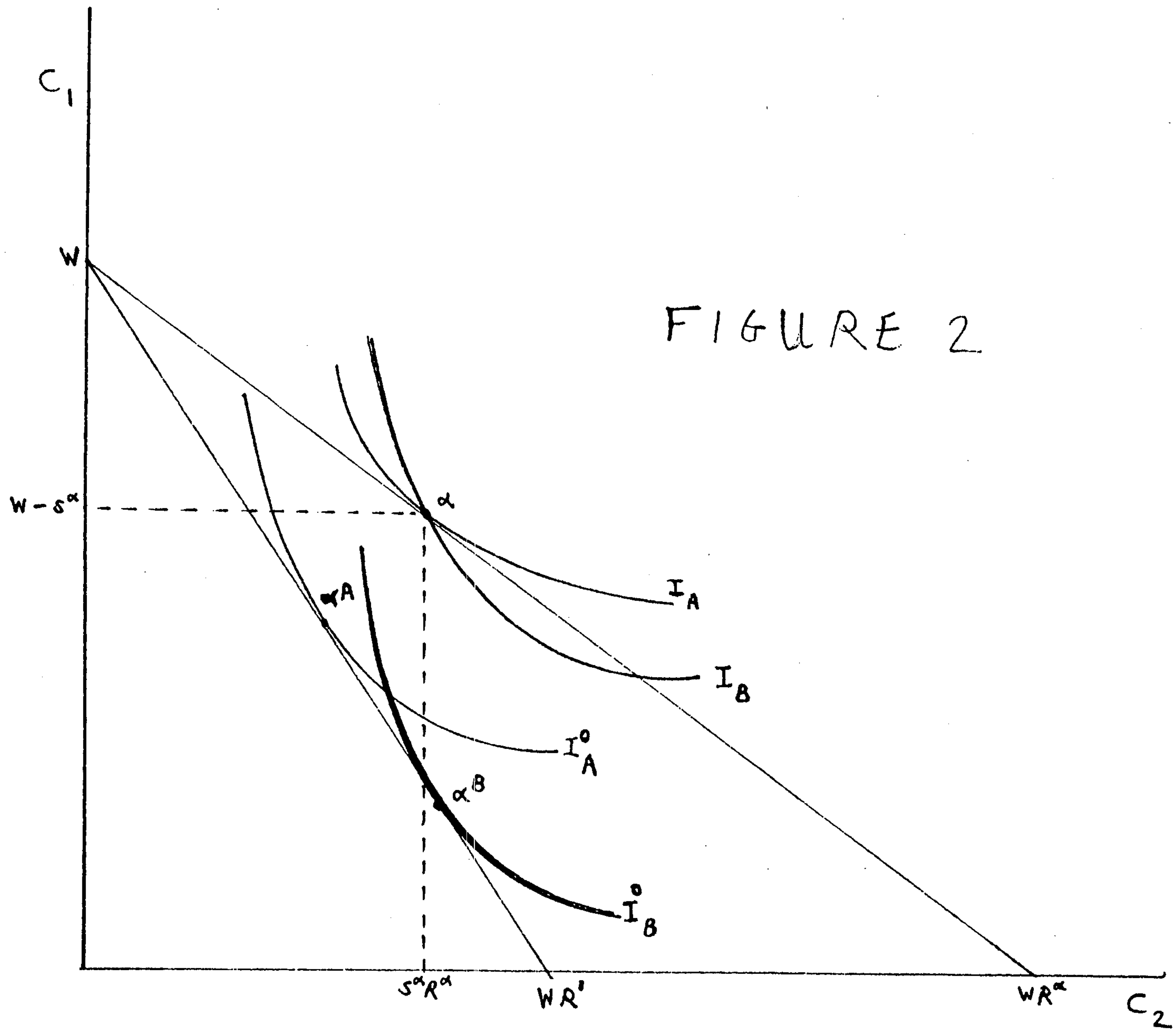


FIGURE 1B: The stationary bequest distribution

FIGURE 2



C_1
W
$W-s^\alpha$
α
α^A
α^B
I_A
I_B
I_A^0
I_B^0
$s^\alpha R^\alpha$
WR'
WR^α
C_2

FIGURE 3

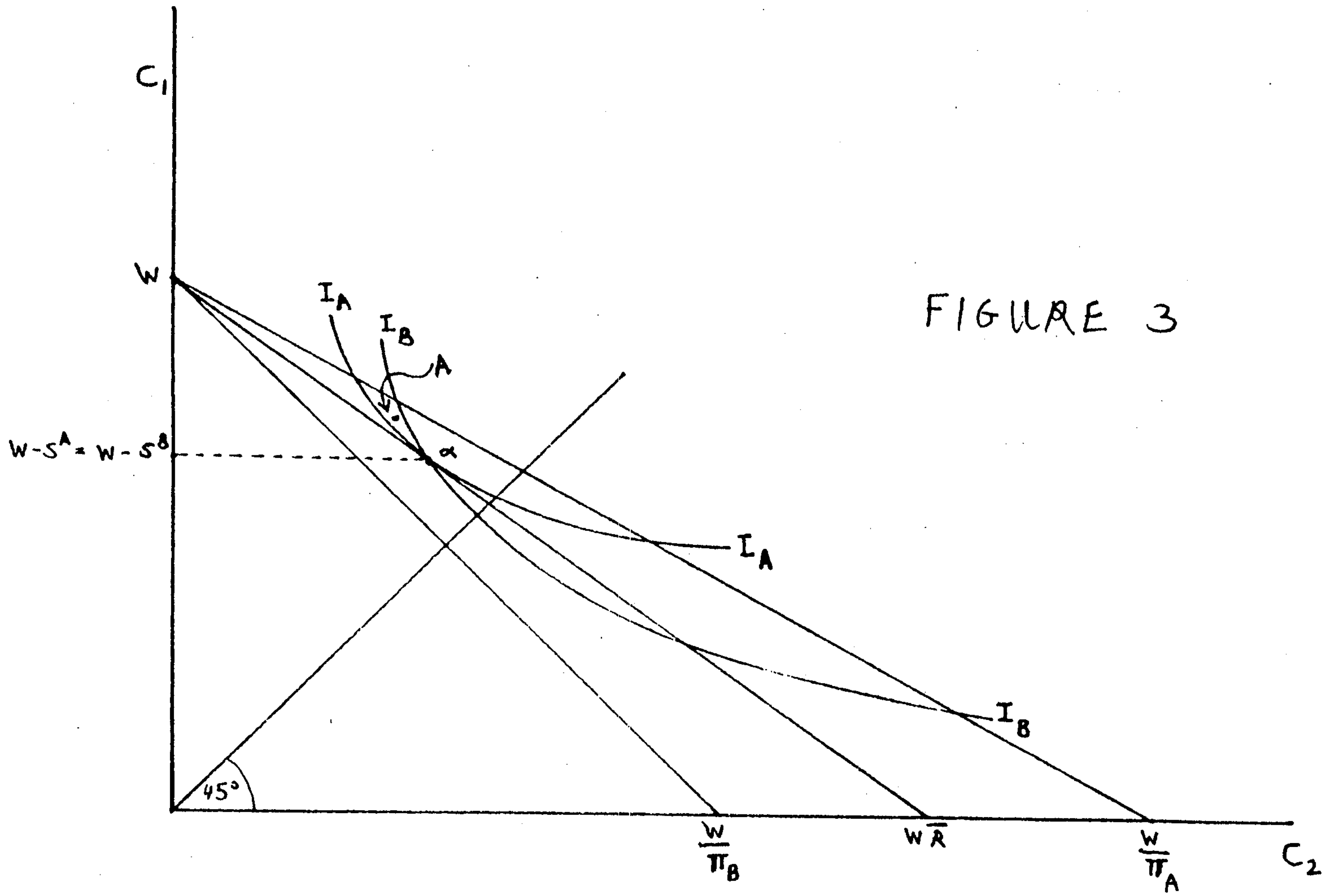


FIGURE 4

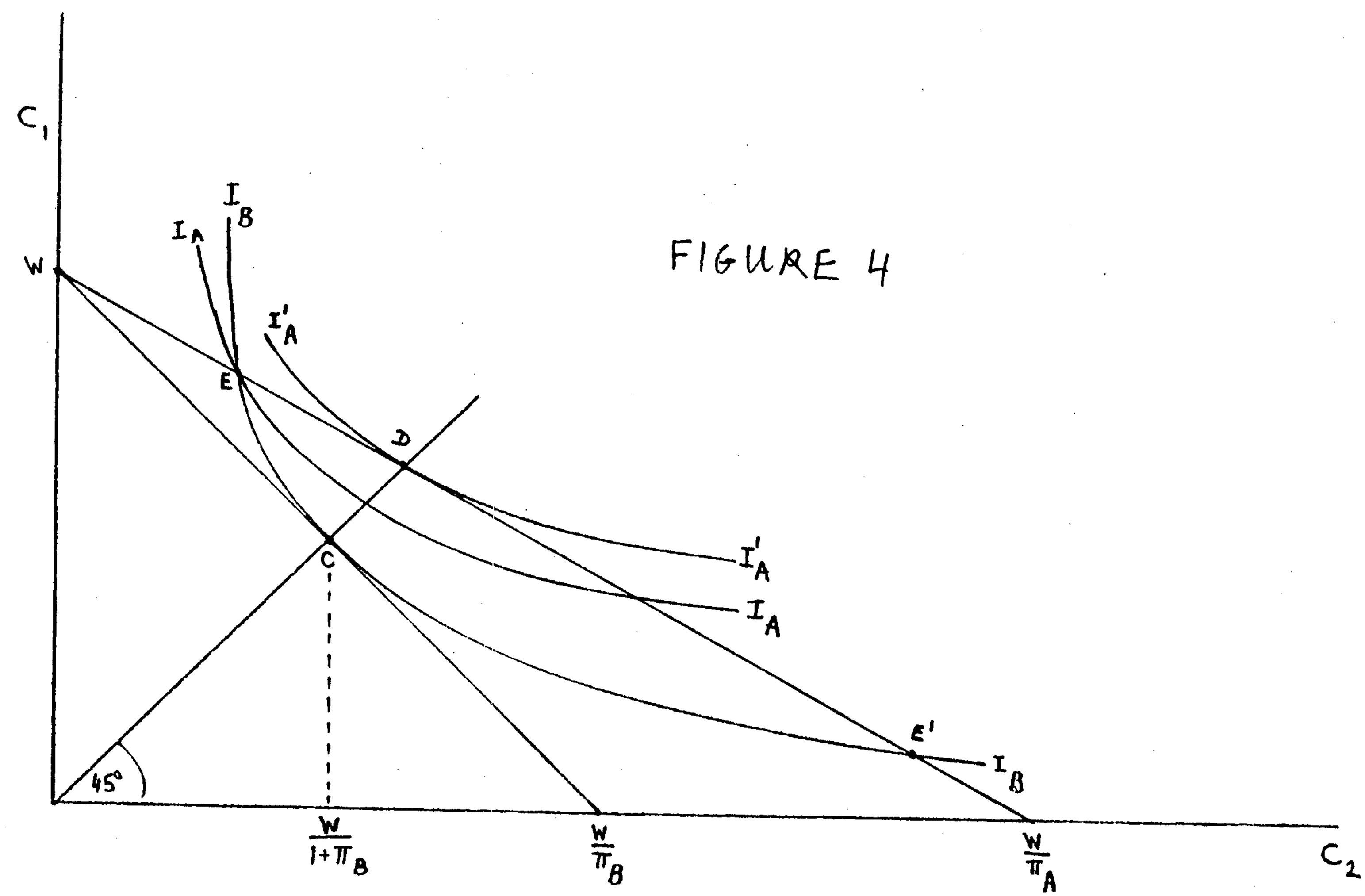


FIGURE 5

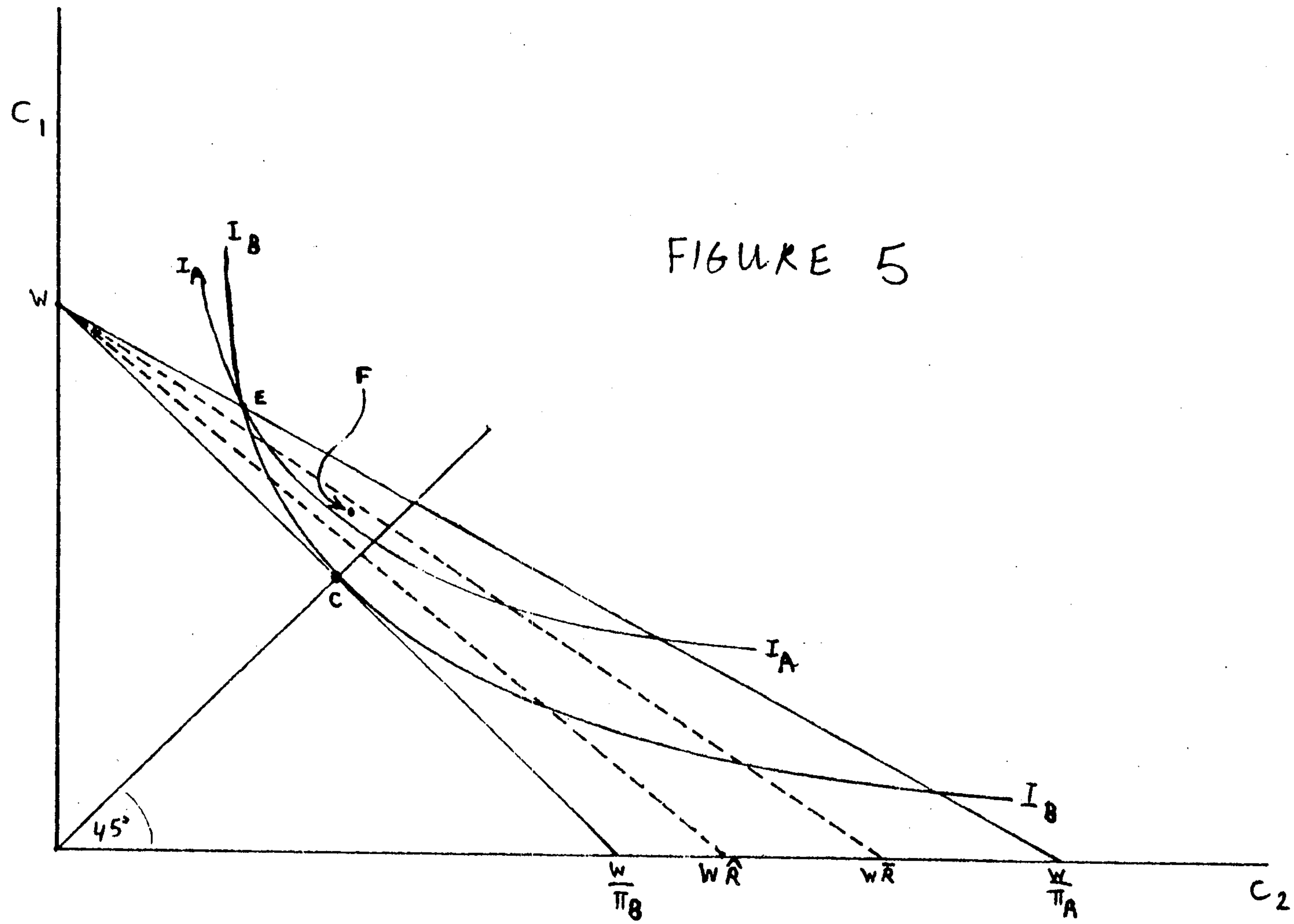
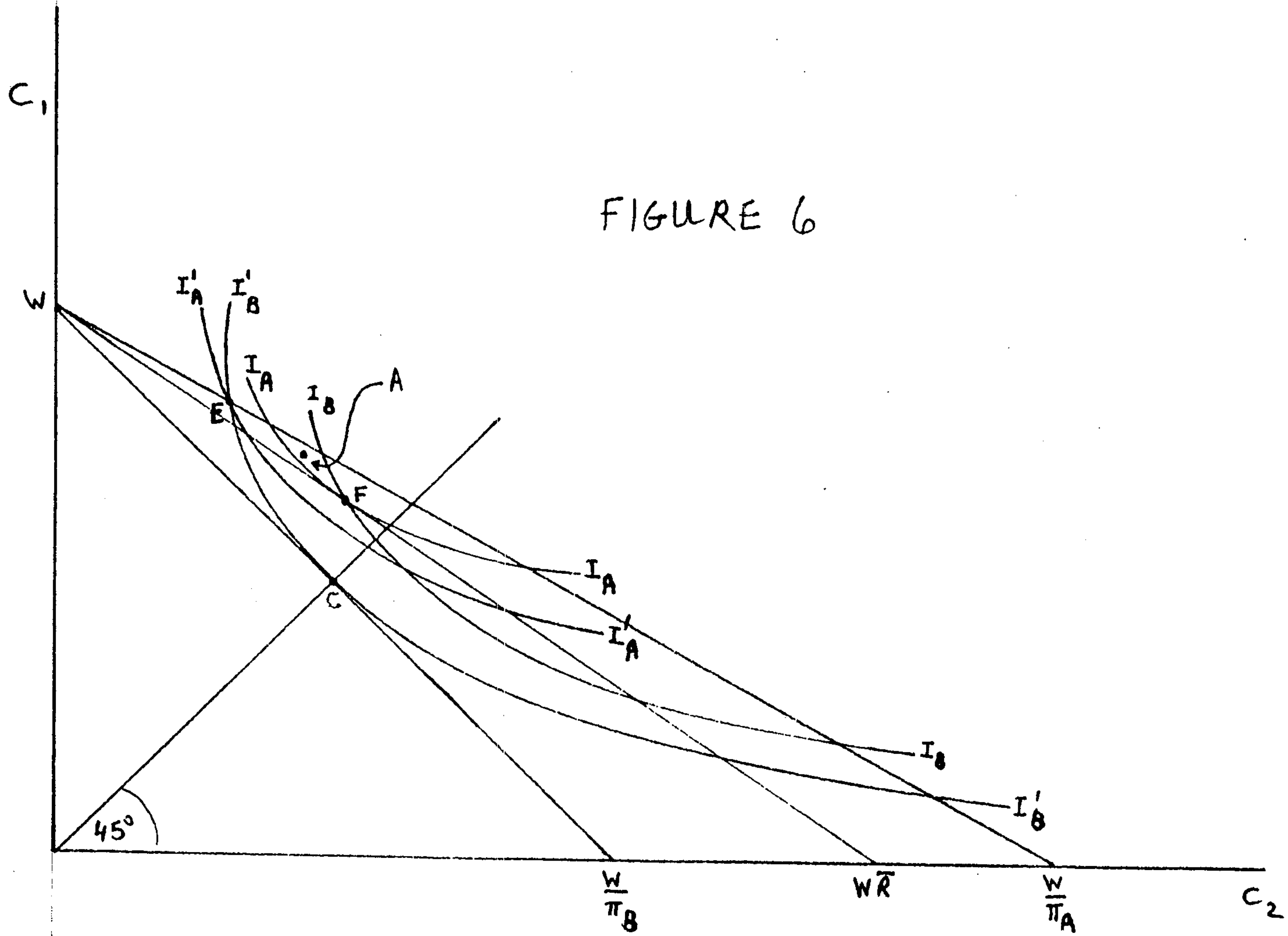


FIGURE 6



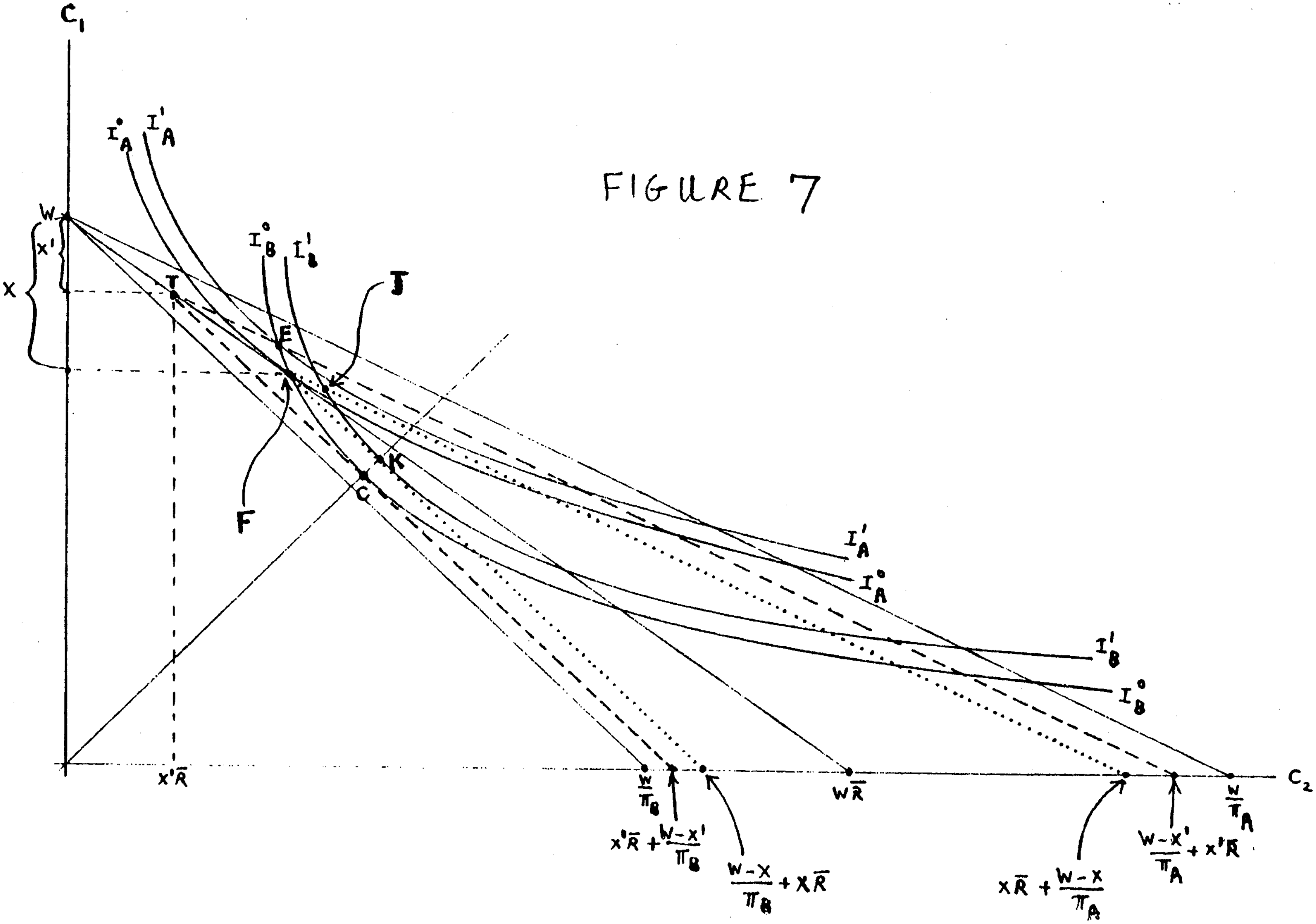


FIGURE 7
C_1
C_2
W
x'
X
T
E
F
J
K
C
I_A^0
I_A^1
I_B^0
I_B^1
$x'\bar{R}$
$\frac{W}{\pi_B}$
$x'\bar{R} + \frac{W-x'}{\pi_B}$
$\frac{W-X}{\pi_B} + X\bar{R}$
$W\bar{R}$
$X\bar{R} + \frac{W-X}{\pi_A}$
$\frac{W-x'}{\pi_A} + x'\bar{R}$
$\frac{W}{\pi_A}$

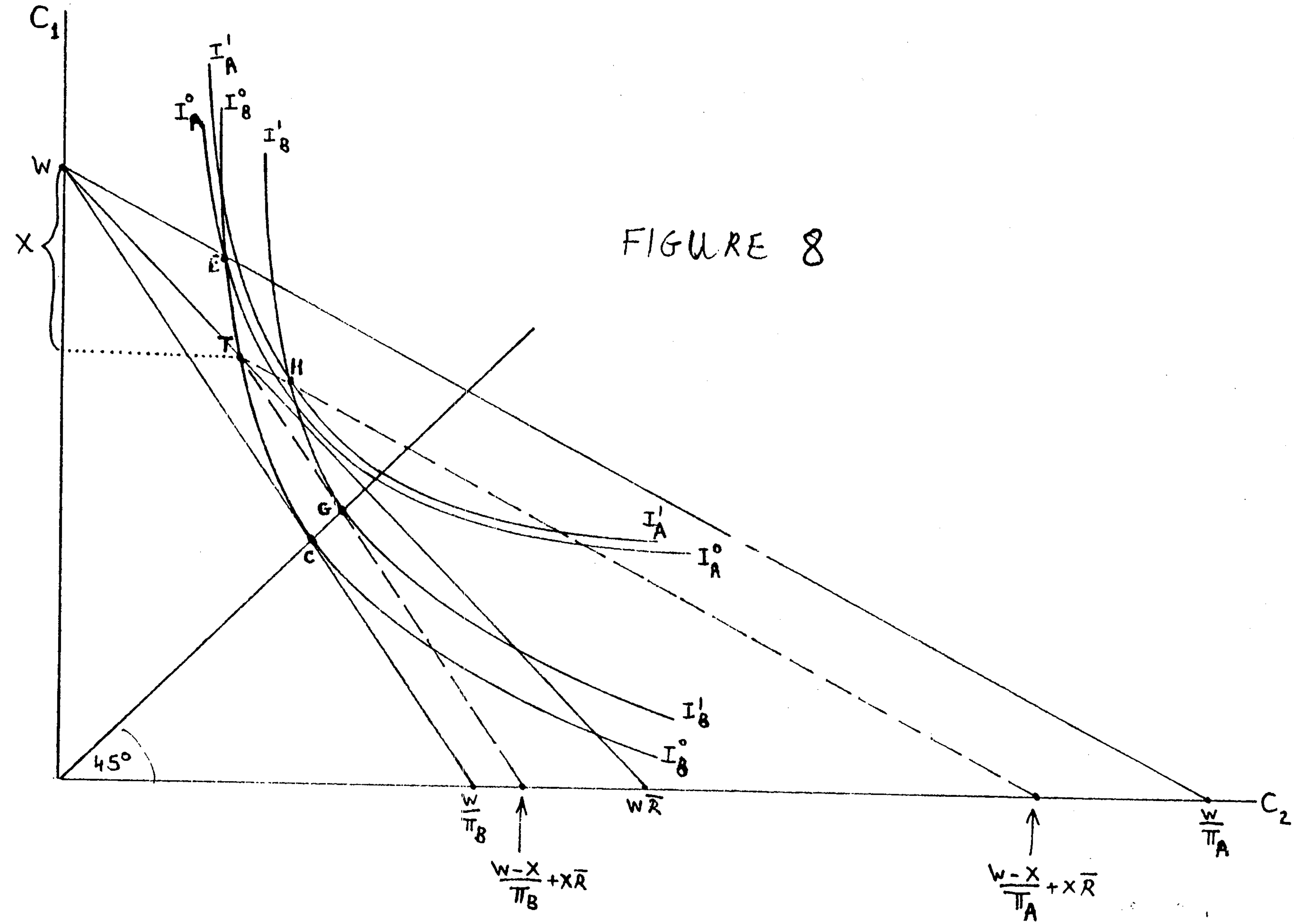


FIGURE 8

FOOTNOTES

1. A similarly derived diversity in agents' initial earning capacity is discussed by Lowry (1981) in a model where markets for sharing risks associated with random human capital productivity are included.

2. An alternative specification of the environment that would achieve the same effect involves endowments in the second period of life, in addition to those at birth. No assets are needed in this case to facilitate intergenerational trades, but the desire to share death related risks is still present.

3. The function $h(\cdot, R)$ exists by property (ii) of $s(\cdot, R)$.

4. With diversity of survival probabilities within generations, ψ^* will be slightly more involved but will display the same qualitative features. Depending on how decendents' survival rates evolve through time, Appendix A can be used with slight modifications to derive the appropriate bequest distribution.

5. For $U(c_1, c_2) = \log c_1 + \pi \log c_2$ it can be shown that $V(b) < EU_a$ for all b in $\{b_k,\ k=0,1,2,\ldots\}$ when $\pi \leq 0.1$.

6. Letting $T(b) = u[\frac{1}{1+\pi}(w+b) + \frac{\pi-1}{\pi+1}s(b,1)]$, we have that $T'(b) > 0$ while

$$T''(b) = -\frac{1-\pi}{1+\pi}u'(\ )\frac{d^2s(b,1)}{db^2} + (\frac{1}{1+\pi})^2 u''(\ )[1-(1-\pi)\frac{ds(b,1)}{db}]^2,$$

which may be positive if $d^2s(b,1)/db^2$ is a large negative number. We could not find general restrictions on $u(\cdot)$ that would rule out this possibility. However, notice that for all utilities of the constant relative risk aversion type, $T''(b) < 0$ since $s(b,1)$ is linear in b.

References

Barro, R. J. and J. W. Friedman, "On Uncertian Lifetimes," Journal of Political Economy 85, (1977), 843-9.

Diamond, P. A., "A Framework for Social Security Analysis," Journal of Public Economics, 8 (1977), 275-298.

Kotlikoff, L. J. and A. Spivak, "The Family as an Incomplete Annuities Market," Journal of Political Economy, Vol. 89, No. 2, (1981) 372-371.

Loury, G.C., "Intergenerational Transfers and the Distribution of Earnings," Econometrica 49, July, 1981, 843-867.

Merton, R. C., "On the Role of Social Security as a Means for Efficient Risk-bearing in an Economy Where Human Capital is not Tradeable," NBER Working Paper No. 743, September 1981.

Muench T., "Optimality, the Interaction of Spot and Future Marekts, and the Nonnutrality of Money in the Lucas Model," Journal of Economic Theory 15, (1977), 325-44.

Peled, D., "Informational Diversity Over Time and the Optimality of Monetary Equilibria," Journal of Economic Theory, Forthcoming (1982).

Prescott, E.C. and R.M. Townsend, "Pareto Optima and Standard Competitive Equilibria with Adverse Selection and Moral Hazzard," Mimeograph, 1979, (revised 1982).

Rothschild, M. and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay in the Economic of Incomplete Information," Quarterly Journal of Economics 90, (1976), 624-649.

Samuelson, P. A., "An Exact Consumption-loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy 66 (1958), 467-482.

Sheshinksi, E. and Y. Weiss, "Uncertainty and Optimal Social Security Systems," Quarterly Journal of Economics, (1981), 189-206.

Wilson, C., "A Model of Insurance Market with Incomplete Information," Journal of Economic Theory 16, (1977), 167-207.

Yaari, M. E., "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer, "Review of Economic Studies 32, (1965), 137-50.