ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 410

SHORT RUN FLUCTUATIONS IN FERTILITY AND MORTALITY IN PREINDUSTRIAL SWEDEN

Zvi Eckstein

T. Paul Schultz

Kenneth I. Wolpin

June 1982

Notes: A preliminary draft was presented at a Conference on British Demographic History, Asilomar, California, March 7-9, 1982. We have benefited from comments on the earlier draft from participants at the conference and Jonathan Eaton, Wallace Huffman, Christopher Sims and David Weir. We acknowledge the able research assistance of Thomas Frenkel and Paul McGuire and the financial support of a grant from the Hewlett Foundation to Yale's Economic Demography Program.

Short Run Fluctuations in Fertility and Mortality in Preindustrial Sweden*

Zvi Eckstein
T. Paul Schultz
Kenneth I. Wolpin

Yale University

June 1982

*A preliminary draft was presented at a Conference on British Demographic History, Asilomar, California, March 7-9, 1982. We have benefited from comments on the earlier draft from participants at the conference and Jonathan Eaton, Wallace Huffman, Christopher Sims and David Weir. We acknowledge the able research assistance of Thomas Frenkel and Paul McGuire and the financial support of a grant from the Hewlett Foundation to Yale's Economic Demography Program.

1. Introduction

There are basically three phenomena that economic demographers seek to understand. First, what governs the long swings in fertility in industrially advanced countries, such as the United States, after they have completed their demographic transition. Second, what initiates and explains the pace of the demographic transition, during which the level and short-run variability of birth and death rates decreases. Only the third phenomena will be studied in this paper: how have societies before they entered into the demographic transition achieved a balance between resources and population. Malthus most notably addressed this third topic. He characterized the factors underlying the preindustrial economic demographic equilibrium in terms of wages, death rates and birth rates, and the diminishing marginal productivity of labor in traditional agriculture.

One can attempt to translate the insights of Malthus into expectations as to the sign of correlations in coincident series or into a structural equation econometric model and estimate parameters from historical time series (Thomas, 1941 Lee, 1973). The theoretical basis for imposing a particular structure on such data is, however, in our view limited. Consequently, it would be preferable to summarize historical data and then use this unrestricted summary representation of the data to explore the questions Malthus considered, and even to interpret the data as tentatively testing certain of Malthus' technical and behavioral hypotheses regarding the short run effects of the real wage on birth and death rates.

Vector autoregression is a statistical methodology for summarizing data that has been recently employed to study macroeconomic time series and to make projections. It has special appeal in those areas in which macroeconomic dynamic theory is unable to identify statistically the underlying structural system (Sargent, 1979; Sims, 1980). If this statistical methodology is applied to historical aggregate time series on weather, crops, wages, deaths and births, the resulting economic-demographic equation system is in one way more tractable than modern macroeconomic systems. We have strong a priori knowledge that weather is determined outside the system, or isstrictly exogenous, and this information reduces the number of parameters to be estimated. But we also have little theoretical basis for ordering the other variables and treating any one endogenous variable as predetermined with respect to another. Researchers have, nonetheless, regressed one endogenous variable on several others and interpreted the distributed lagged estimates as a technical or behavioral causal relationship (Lee, 1981). These single equation structural formulations implicitly posit many assumptions and restrictions that do not appear justified at this stage in our research. Thus, we have opted for the less restrictive vector autoregression framework, even though it requires the estimation of many parameters. These more restricted studies are nested within our more general representation.

Sweden is our case study. The annual demographic data for Sweden are good after 1750, and a variety of time series are available to characterize weather conditions, crops, commodity prices and wages.

The paper is ordered as follows. Section 2 discusses the data and Section 3 the statistical model. The empirical results are reported in Section 4 and interpreted in Section 5. A concluding section summarizes our findings. Three appendices provide more detail on data sources, the econometric methodology and the statistical specification tests.

2. Data

The registered figures of births and deaths for all of the counties of modern Sweden, as well as the annual number of Swedish inhabitants, are widely regarded as a reliable basis for calculating Swedish birth and death rates after 1749. The historical statistics series (Sweden, 1955, Table B.2) are supplemented by those reported by the United Nations after 1950 (United Nations, 1979). For several reasons we examine here the crude birth rate (CBR), or the number of births occurring in the calendar year per thousand inhabitants at the end of that year. Our measure of fertility is not adjusted for changes in the age composition of the population, since our primary goal is to characterize short run fluctuations in birth rates rather than slow changes in their levels related to the changing age composition. Before Swedish emigration increases in the 1860's, the short run effects of migration on the age composition are also negligible at the national level, even though they may be more important at the level of county or other subnational unit (Thomas, 1941). Changes in fertility are not decomposed into changes in (1) the proportion of women married in the childbearing ages, (2) marital fertility rates, and (3) extra-marital fertility rates. Thomas and others have noted the short run responsiveness of all three components of the Swedish fertility are strongly correlated with each other and with the harvest cycle, particularly in the 18th and early 19th centuries (Thomas , 1941, p. 87 and Table 25). It is not our current objective to consider how fertility changes were accomplished among these three routes.

Since the level of mortality is substantially higher in the first year of life than in subsequent years, fluctuations in births will tend to affect deaths, in the same direction, in the current and following year. This demographic linkage from births to deaths, by way of the age schedule of mortality, suggests the need to disaggregate deaths of infants from those occurring to persons over the age of one. The causes of mortality among infants and older persons may also be substantially different, since many infants are breastfed and, thereby, derive immunities to certain diseases. Consequently, mortality experienced by infants and older persons may respond differently to conditioning variables. Deaths to members of these two populations may also elicit different patterns of fertility response.

Infant deaths are registered in the year of their occurrence; these infants, under one year of age, may have been born in either the current or previous year. We analyze, therefore, an adjusted infant death rate (IDR) that divides the number of infant deaths in a particular year by a weighted average of the number of births in the current and previous year, where the weights depend simply on the level of the unadjusted infant mortality rate (Shryock, 1971, p. 441).

Other (non-infant) deaths are divided by the current year non-infant population (NIDR). There may still be a slight tendency for the NIDR to increase one to four years after the birth rate increases, since mortality among one to four year olds is greater than at subsequent ages, at least in the early years of our study. But the severity of this problem is discounted by historical demographers (e.g., Lee, 1977), and we do not adjust the series to account for this second-order demographic feedback.

The general crop index (CROP) reported in the Historical Statistics of Sweden (1959, Table E.12) starts in 1786, but is available from 1748 in Sundbärg's (1907, Table C) original monograph on the Swedish population.[1] The real wage (RWAGE) is the nominal wage in agriculture divided by the price of basic foodgrains or a cost of living index. Crop variation presumably affects real wages, but also influences payments to land and other factors of production in agriculture. Over time, moreover, improvements in the transportation system and storage facilities for grains should have weakened the coincident and lagged relationship between the crop index and the price of foodgrains. Therefore, both the traditional crop index and a new measure of real wages are employed in our exploration of Swedish time series.

Although the composition of basic foodstuffs produced and consumed in Sweden changed in this period, rye was the predominate food grain in Sweden until 1860 (Thomas, 1981). Moreover, the prices of alternative major grains--barley, oats and later wheat--are highly correlated annually at .95 to .99 from 1750 to 1913 (Jörberg,1972). Our measure of the real agricultural wage from 1750 to 1870 is, thus, constructed from Jörberg's (1972) series on the daily male agricultural worker's wage divided by the price of a

[1] The Sundbärg index is divided by two to be consistent with the later historical statistics series, in which 3.0 is an average crop year. There does not appear to be a general crop index after 1955, and,therefore, projections are based on an agricultural output index for Sweden from the United Nations Statistical Office.

hectolitre of rye. Since this agricultural wage series is discontinuous after 1913, Jungenfelt's (1966) estimate of annual earnings of workers in agriculture is divided by Phelps-Brown's (1968) cost of living index to define the real agricultural wage (RWAGE) for the entire later period, 1870 to 1955.[2]

Five series are selected to summarize weather. The average annual rainfall is from the average of three Swedish meteorological stations in Lund, Stockholm, and Uppsala (Sweden, 1959, Table C.7).[3] The average annual temperature was also used initially, though it is available only for Stockholm (Sweden, 1959, Table C.2). This is undoubtedly a blunt measure of climate; moderately cold winters were sometimes beneficial for grains, but they increased mortality, while hot summers may have increased mortality, while nonetheless improving the harvest (Le Roy Laduire, 1971). Preliminary exclusion (F) tests led us to replace a single annual or July temperature with the average temperature for each of the four seasons of the year. The winter temperature refers to the average of January, February and March, and so on. The temperature series are published from 1756 and they determine the beginning of our time series analysis.

[2] Where the two real agricultural wage series overlap, 1870-1913, their logarithms are correlated at .94, though the annual earning series is relatively less volatile than the daily wage, i.e., the standard deviation of the logarithms are .18 and .27, respectively.

[3] One annual observation is missing for Lund (1806), 25 are missing from 1761 to 1835 for Uppsala and reports for Stockholm start in 1784. Rather than rely only on Lund or omit the first 29 years of our series, multiple regressions are fit to the existing overlapping data for 1750 to 1955 and used to predict values for the missing observations on rainfall. Using only the Lund series does not change in any noted way the results that are later reported.

The sources and definitions of all the data series are reported in Appendix A. The final data used in our study are plotted in Figures 1a through 1k and are summarized in Table 1 in absolute form and in natural logarithms in Table 2: they illustrate the transition in Sweden from the preindustrial era of high and unstable death and birth rates to the industrial period of low mortality and low fertility,with the pronounced swing of the postwar baby boom following the depression. The fraction of Sweden's population in urban areas is virtually constant at 10 or 11 percent until 1860, while the fraction of the labor force employed outside of agriculture is roughly twice that amount but growing slowly until the late 19th century (Mosher, 1980). Legislation enacted in the middle of the 18th century sought to modernize Swedish agriculture according to the English example, but the redistribution and consolidation of land holdings associated with the abolition of the common field system and enclosures met with resistance and proceeded slowly. Only by the middle of the 19th century had the process run its course. During this time of increasing rural population density, the proportion of the agricultural labor force without land increased substantially. Migration of workers out of agriculture facilitated after 1850 the expansion of rural industrial centers and urban employment. Later in the 1860s workers leaving agriculture began to leave Sweden, emigrating mostly to North America. These large scale emigrations continued for half a century until internal rural-urban flows of population were more or less again in balance.

Figure 1

Basic Data Series

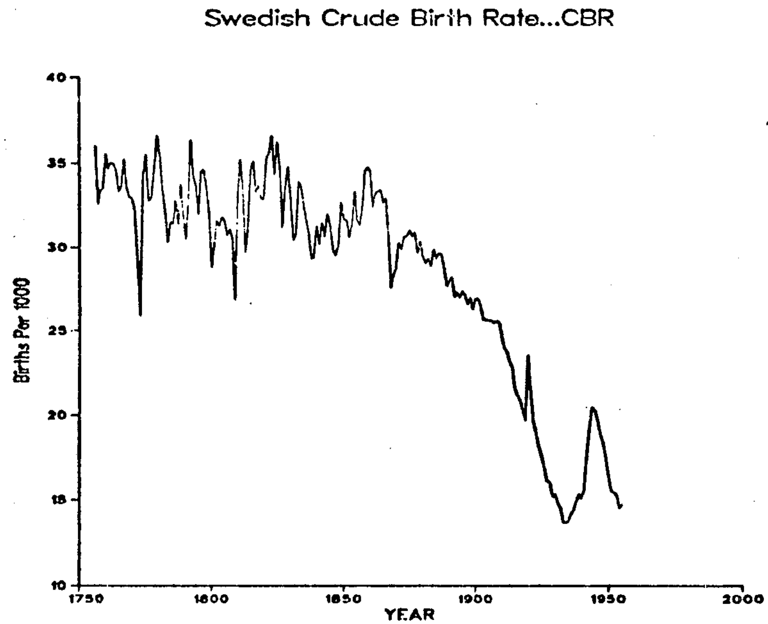

Figure 1a

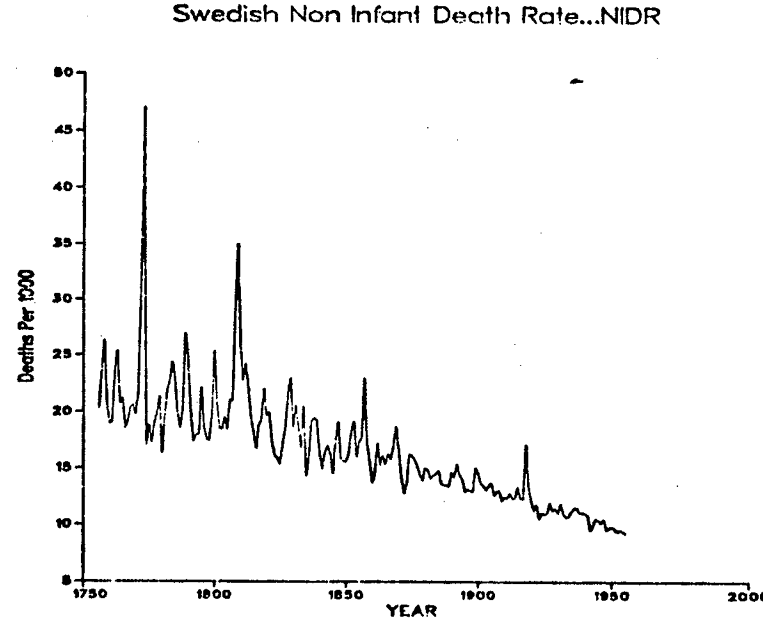

Figure 1b

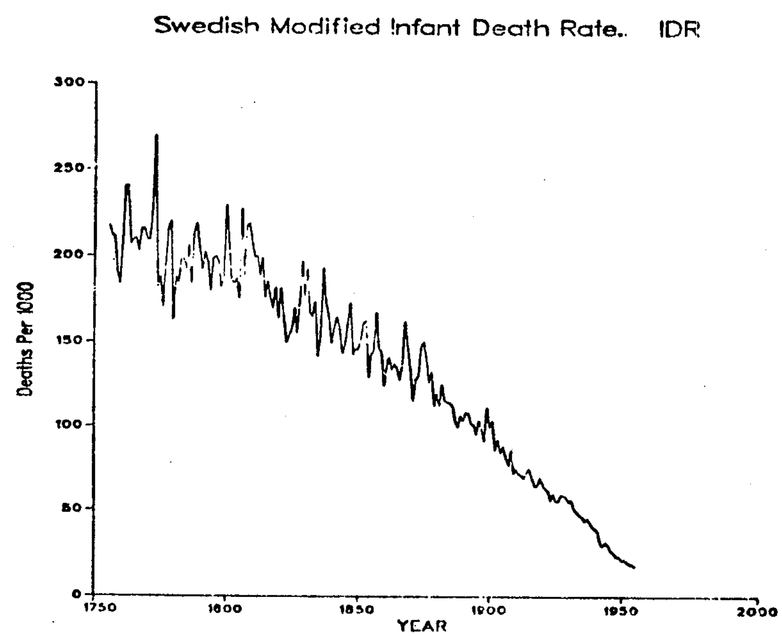

Figure 1c

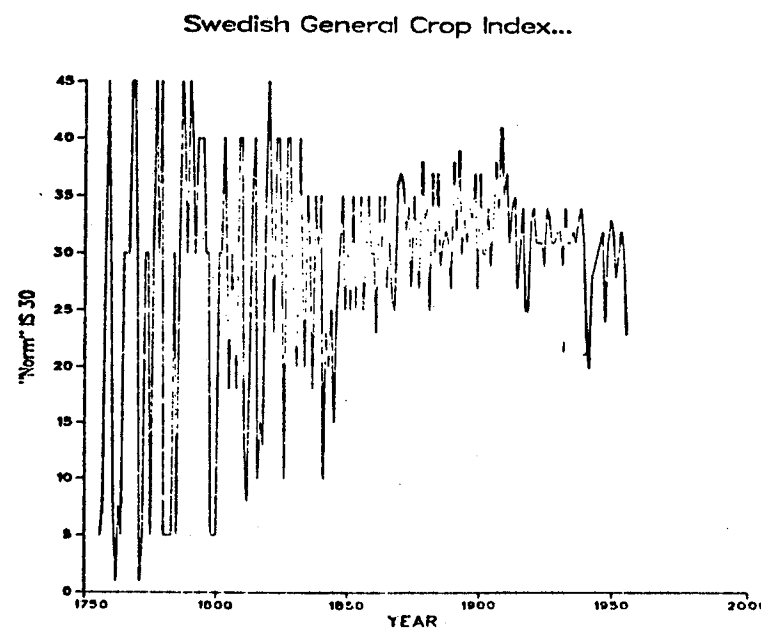

Figure 1d

Figure 1 continued

Basic Data Series

Swedish Average Temperature ...ATEMP

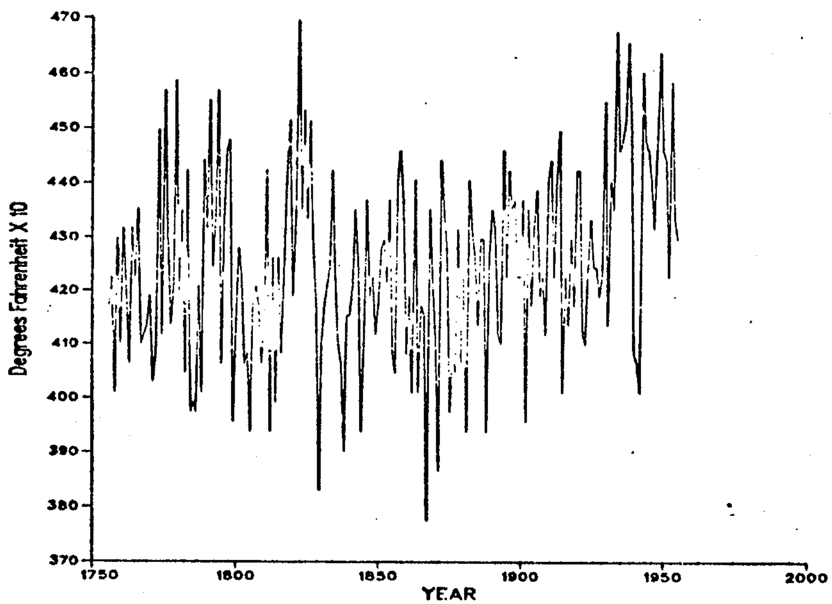

Figure 1e

Swedish Average Precipitation... RAIN

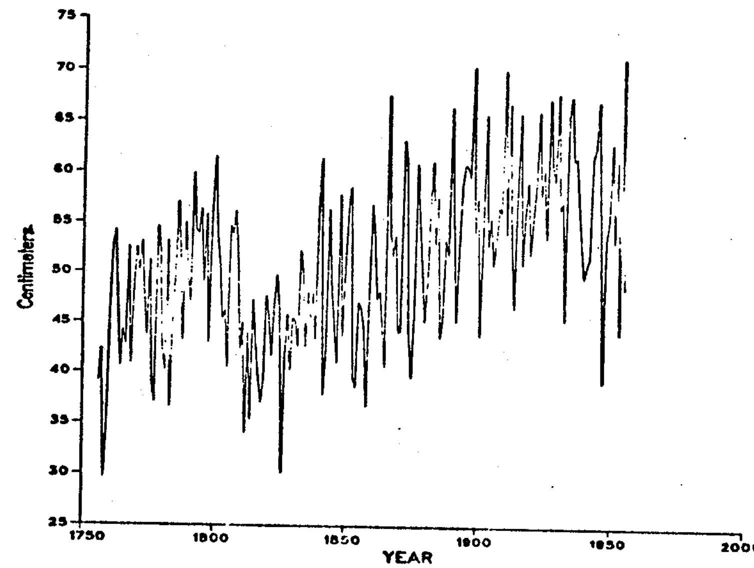

Figure 1f

Swedish Real Wage ...WAGE

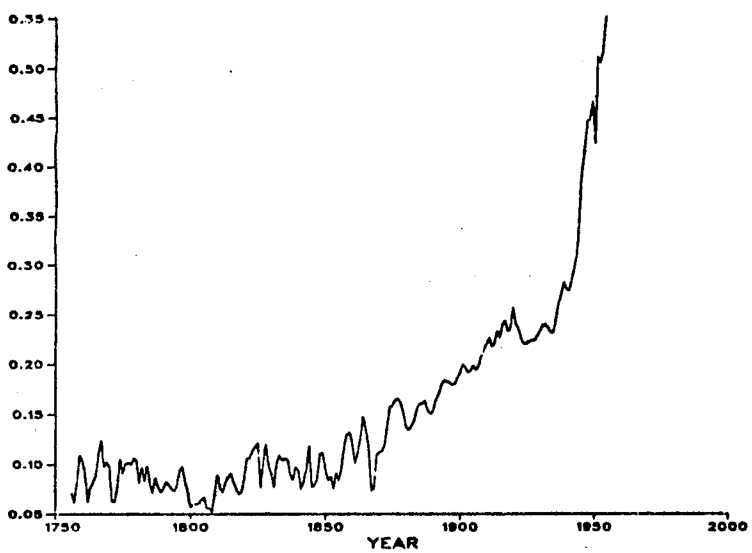

Figure 1 continued

Basic Data Series

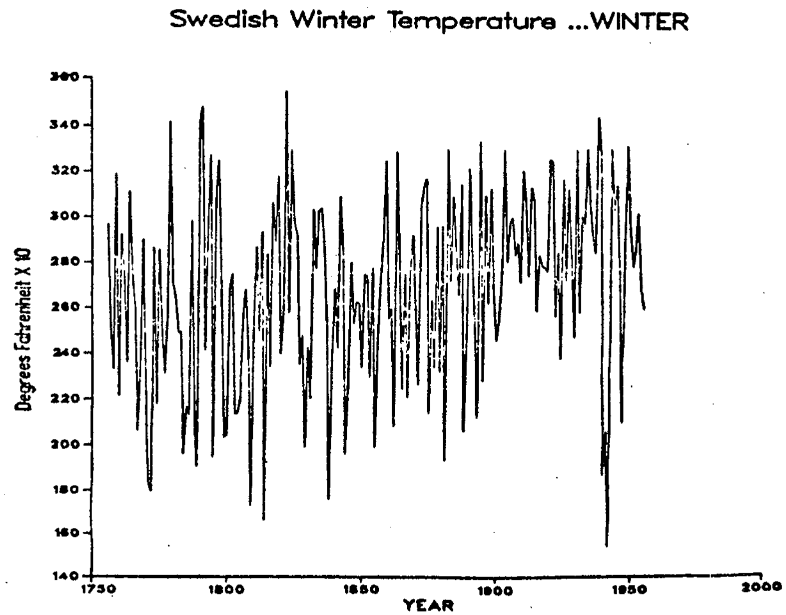

Figure 1h

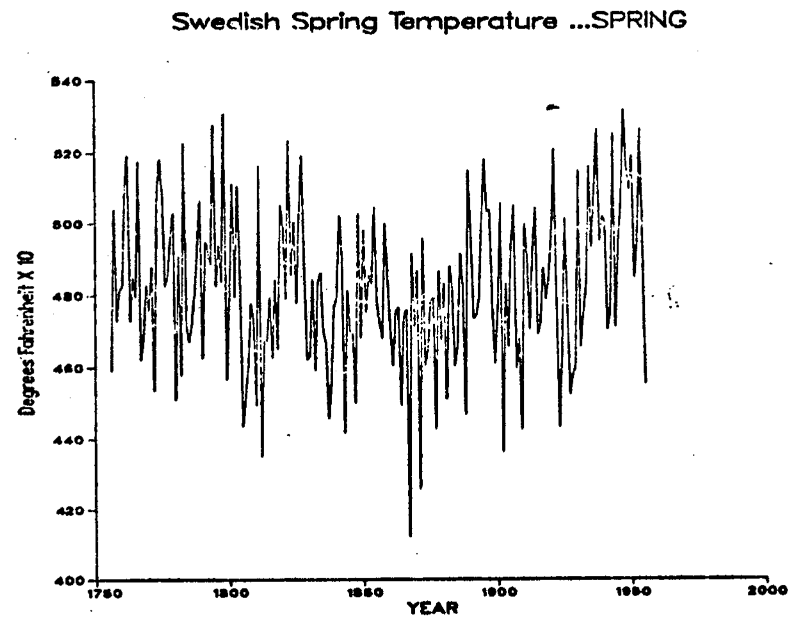

Figure 1i

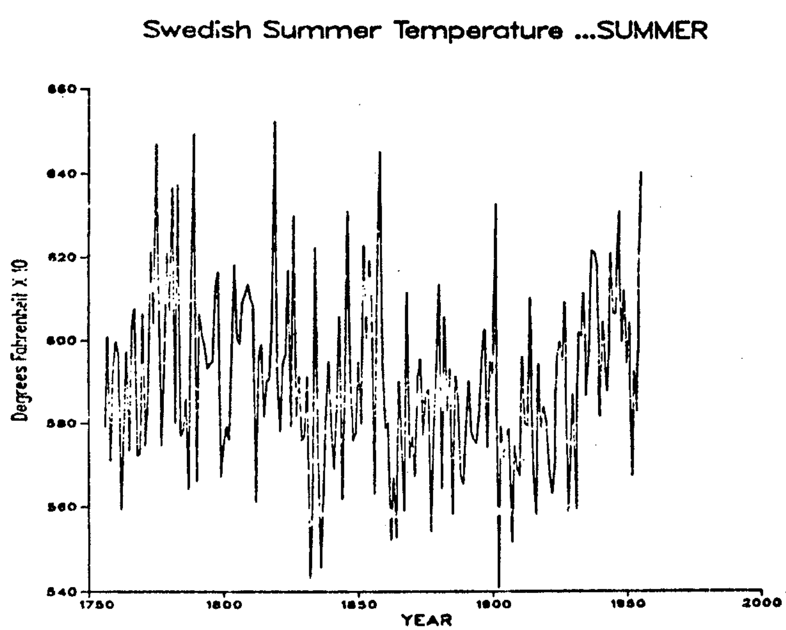

Figure 1j

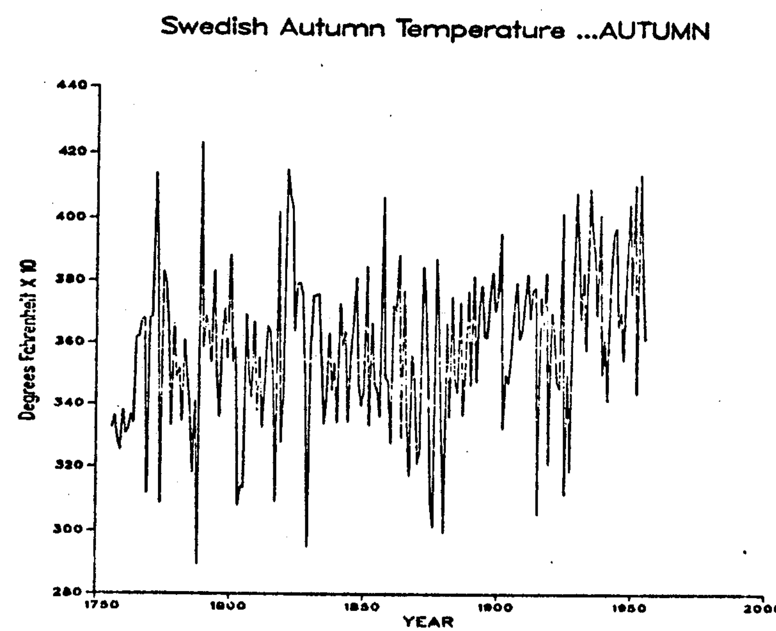

Figure 1k

Table 1

Simple Correlations among Contemporaneous Variables in Absolute Form and Sample Statistics: 1756-1869 and 1870-1955

Variable Symbols	Crude Birth Rate (1)	Infant Death Rate (2)	Non-Infant Death Rate (3)	General Crop Index (4)	Real Agricultural Wage (5)	Spring Temperature (6)	Summer Temperature (7)	Autumn Temperature (8)	Winter Temperature (9)	Precipitation (10)
					Period 1: 1756-1869					
CBR(1)	-	-.016	-.361	-.037	.359	.196	.075	-.075	.286	-.171
IDR(2)	.909	-	.750	-.180	-.536	.133	.165	-.004	-.147	.003
NIDR(3)	.819	.908	-	-.093	-.415	..021	.186	.108	-.205	-.020
CROP(4)	.309	.312	.225	-	.363	.043	-.064	-.045	.117	.039
RWAGE(5)	-.693	-.842	-.781	-.315	-	-.025	-.145	.062	.223	-.088
SPTEMP(6)	-.296	-.345	-.353	.010	.363	-	.401	.112	.244	.022
SMTEMP(7)	-.327	-.331	-.348	-.158	.366	.413	-	.142	.095	-.164
AUTEMP(8)	-.350	-.402	-.417	-.120	.330	.298	.152	-	.091	.051
WNTEMP(9)	-.108	-.097	-.138	.400	.068	.322	.156	.158	-	-.087
RAIN(10)	-.272	-.256	-.285	.199	.091	..059	-.159	.282	.396	-
					Period 1: 1756-1869					
Mean	32.5	181.0	19.7	26.5	.090	48.1	59.2	35.4	25.9	472.8
Standard Deviation	2.10	28.8	4.36	12.2	.019	2.12	2.25	2.51	4.18	70.2
					Period 2: 1870-1955					
Mean	22.7	75.7	12.5	31.6	.234	48.4	58.7	36.4	28.1	559.3
Standard Deviation	5.77	35.6	1.89	3.82	.097	2.29	18.5	2.59	3.86	75.1

Note: correlations below the diagonal (vertical label "Period 2: 1870-1955") refer to Period 2: 1870-1955; those above the diagonal refer to Period 1: 1756-1869.

Table 2

Simple Correlations Among Contemporaneous Variables in Logarithmic Form and Sample Statistics: 1756-1869 and 1980-1955

Variable Symbols	Crude Birth Rate (1)	Infant Death Rate (2)	Non-Infant Death Rate (3)	General Crop Index (4)	Real Agricultural Wage (5)	Spring Temperature (6)	Summer Temperature (7)	Autumn Temperature (8)	Winter Temperature (9)	Precipitation (10)
					Period 1: 1756-1869 (above diagonal); Period 2: 1870-1955 (below diagonal)					
CBR(1)	-	-.163	-.328	-.077	.360	.189	.066	-.092	.289	-.176
IDR(2)	.832	-	.788	-.279	-.540	.134	.181	-.012	-.158	-.020
NIDR(3)	.812	.920	-	-.184	-.465	.015	.190	.067	-.243	-.050
CROP(4)	.300	.333	.230	-	.384	-.030	-.012	-.030	.112	.019
RWAGE(5)	-.755	-.956	-.850	-.322	-	-.092	-.138	.062	.231	-.099
SPTEMP(6)	-.283	.362	-.351	.024	.379	-	.400	.112	.236	-.022
SMTEMP(7)	-.335	-.385	-.357	-.157	.359	.420	-	.135	.085	-.162
AUTEMP(8)	-.337	-.379	-.411	-.105	.363	.291	.141	-	.084	.059
WNTEMP(9)	-.086	-.046	-.100	.427	.089	.304	.140	.140	-	-.085
RAIN(10)	-.259	-.173	-.264	.213	.144	.048	-.167	.295	.379	-
					Period 1: 1756-1869					
Mean	-3.42	-1.72	-8.56	3.08	-2.43	6.18	6.38	5.86	5.55	6.15
Standard Deviation	.0657	.161	.188	.776	.215	.0443	.0380	.071	.166	.152
					Period 2: 1870-1955					
Mean	-3.82	-2.72	-8.99	3.45	-1.52	6.18	6.38	5.90	5.63	6.32
Standard Deviation	.270	.572	.153	.126	.357	.0474	.0331	.0729	.150	.138

Non-infant mortality may have been decreasing slowly in the late 18th and early 19th centuries, but the extreme variability in deaths makes it difficult to extract the secular trend with much confidence. Smallpox was brought under control after 1809, and serious outbreaks of dysentary subsided after 1818. Many other epidemic diseases, however, showed no tendency to diminish until well into the 19th century, e.g., measles, whooping cough, typhus and typhoid (Utterström, 1954). After 1880, the only resurgence in the decreasing level of non-infant deaths occurred during the world influenza pandemic of 1918-1920.

Infant mortality rates were decreasing throughout our period, though the rate of decline may have accelerated over time; this pattern in Sweden is similar to that observed in France (Blayo, 1975), but may be contrasted with stability in infant deaths rates in England throughout the 19th century where urbanization proceeded more rapidly than in Sweden (Wrigley and Schofield, 1981). The volatility of the Swedish series is much reduced after 1880, as epidemics receded. While the birth rate and non-infant death rate decreased about 50 percent in our period of 200 years, the infant death rate decreased 90 percent, from one-in-five to one-in-fifty.

The general crop index shows a tendency to vary less after 1850 than before that date. This may be a result of applying scientific knowledge to agriculture, progress in plant breeding, rotational schemes and increased use of fertilizers , or due to a change in the composition of output that reduced its sensitivity to the weather, or an artifact of how the series was constructed, such as shifting from price to quantity series.[4] The real wage in agriculture declined in the last half of the 18th century, particularly after 1775. Deflating the wage by more comprehensive cost of living indexes reduces the deterioration, but does not change the direction of trends or turning points (Jorbärg, 1972, II p. 186). Real agricultural wages increased during and after the Napoleonic wars, 1806 to 1823, regaining their trend upward only after 1854 and continuing until 1913. Overall, the level of real wages in agriculture approximately doubled from 1800 to 1875, and tripled in the next 75 years to 1950.

Rainfall and annual average temperature are highly variable in both subperiods, as is to be expected of the weather. There are, nonetheless, clues of longer run swings. Temperatures tended downward in the 1800s, up in the 1820s, down through the 1860s, and upward thereafter for nearly a century. Rainfall diminished from the 1790s to the 1830s, and increased thereafter to a higher level in the first half of this century.

---

[4] Official crop yield reports were not available before 1865 (Thomas, 1941), and thus Sundbärg's general crop index must have relied heavily in this earlier period on annual grain price series (Utterstrom, 1954). In this case, it may be particularly interesting in this early period to include the wage series to disentangle changes in the price level of crops from changes in real wages (wage/grain prices). This general crop series has been widely used since Sundbärg (1907) incorporated it into his classic analysis of population developments in Sweden. Utterstrom (1954) doubts whether this series was derived entirely from representative data on harvest yields for he surmises that, at least in the 18th century, only grain price series were available. This he notes may have confounded in the series both variation in real grain prices and also changes in the general price level that had little to do with the abundance of the harvest. If the demand schedule for foodgrains was inelastic with respect to price, reliance on price rather than quantity data might have imparted a bias toward greater variance in the index in earlier years.

3. An Econometric Framework: Vector Autoregression

The methodology we adopt in this paper originated in the work of Sims (1980), and has been applied mainly in the analyses of macroeconomic time series. Sims argued against structural macro-econometric modeling because the identifying restrictions of existing models are "incredible", because the dynamic elements of the models are not well specified, because there is only a weak distinction between endogenous and exogenous variables, and because of the incomplete treatment of expectations. Instead, he proposed estimating unrestricted vector autoregressions (VAR) which can be interpreted as the reduced form relationships that arise from macro-econometric structural models. Sims also developed methods for describing or summarizing the content of the vector autoregression from which hypotheses could be formulated.

Another focus of research on interpreting economic time-series, exemplified in the work of Sargent (1981), argues that in a well formulated equilibrium framework based on optimizing agents who form expectations in a manner consistent with the equilibrium model, restrictions on the parameters across the equations of the VAR will be implied. The underlying structural parameters in this context are those related to preference functions and technological constraints. Structural econometric models are not structural in this sense. Demographic and economic time series should be viewed similarly as having a microeconomic basis. We do not present such a theoretical foundation, although we hope to learn about the important ingredients of such a theory from the descriptive analysis. In this section, we discuss a simplified version of the econometric model actually estimated. The more general and rigorous discussion may be found in Appendix B.

Assume we have time-series observations for a particular country on birth rates, infant mortality rates, and a measure of weather. Further, assume that we can "best" represent the system of these three variables (detrended and as deviations from means) in the following manner:[5]

(1) $B_t = \alpha_1 B_{t-1} + \alpha_2 M_{t-1} + \alpha_3 W_t + \alpha_4 W_{t-1} + \varepsilon_{1t}$

(2) $M_t = \beta_1 B_{t-1} + \beta_2 M_{t-1} + \beta_3 W_t + \beta_4 W_{t-1} + \varepsilon_{2t}$

(3) $W_t = \gamma_1 W_{t-1} + v_t$

where B_t is the birth rate at time period t, M_t the death rate at t, and W_t is weather at t. This system is assumed to arise from a complex structural dynamic model of behavior that is conditioned by biological and technological constraints. In other words, the α's and β's are interpreted as composites of more fundamental biological, technical and behavioral parameters. We will therefore refer to this representation as unrestricted, since the fundamental parameters appearing in the α's and β's are not delineated and the restrictions that could be imposed in the estimation are ignored.

The innovations or random shocks, namely ε_{1t}, ε_{2t}, and v_t, are assumed uncorrelated with the demographic variables or weather. In addition, they are assumed to be serially uncorrelated; all correlations of one error with the lagged values of itself or with the lagged values of the error in other equations ar zero. Neither of the innovations in the demographic variables is permitted to be contemporaneously correlated with the weather shock, although in principle they may be correlated with each other. The force of these

[5]See Appendix B for a more rigorous definition of "best."

assumptions, given that lagged demographic variables do not enter the weather equation, is to ensure that weather is strictly exogenous (see Appendix B). Having estimated this system, we can test statistically for the possible presence of lagged demographic variables in the weather equation. This is a test of causality in the sense of Granger (1969). In addition, we will perform Sims's(1972) exogeneity test which is based on examining future weather effects in the demographic equation; should we find that future weather "affects" current births and deaths, this would imply that the random shocks in the demographic variables are contemporaneously correlated with the random weather shock. Since it seems logical to assume that weather is truly strictly exogenous to the demographic outcomes, if one finds that future weather appears to affect the demographic variables this may be viewed as evidence that explanatory variables are omitted from the system. In other words, exogeneity tests in this context are tests of the completeness of the specification of the model. For example, suppose equations (1)-(3) represent the true model but M_{t-1} is omitted from equation (1). Then estimating $B_t = \alpha_1 B_{t-1} + \alpha_3 W_t + \alpha_4 W_{t-1} + \alpha_5 W_{t+1} + \varepsilon_t$ may give rise to a significant estimate of $\alpha_5 \neq 0$ while exogeneity requi[res] that $\alpha_5 = 0$. This arises since W_{t+1} is correlated with $M_{t+1}(\beta_3)$ and M_{t+1} with M_t (β_2)

This system of equations can be efficiently estimated by ordinary least squares (OLS), equation by equation; these OLS estimates are identical with joint conditional maximumlikelihood estimates[6], even though ε_{1t} and ε_{2t} may be correlated. The lag length adopted, such as "one" in the example, need not be arbitrary, since statistical tests for alternative lag lengths can be readily performed. However, as the number of parameters expands much more quickly than the number of lags, it is necessary to restrict the lag length.[7]

[6] Conditional maximum likelihood in the sense that it is conditioned on the initial observations, since the system includes lags.

[7] There are several tests for the lag length. We used Sims' (1980) "modified" likelihood ratio tests (see Tables C.1 and C.2 in Appendix C).

A useful way to describe the system, once the parameters have been estimated, is to observe the system's response to random shocks (Sims, 1980). We refer to these as impulse responses. Consider one standard deviation shock in weather σ_v, at time t. In period t, the birth rate will change by $\alpha_3\sigma_v$ and the death rate by $\beta_3\sigma_v$. In period t + 1, the birth rate changes by $(\alpha_1\alpha_3 + \alpha_2\beta_3 + \alpha_3\gamma_1 + \alpha_4)\ \sigma_v$ and the death rate by $(\beta_1\alpha_3 + \beta_2\beta_3 + \beta_3\gamma_1 + \beta_4)\ \sigma_v$. In like manner, we can continue to trace out the impact of the t^{th} period weather shock on births and deaths at t + 2, t + 3,... . If the system is stable, the impulse responses will dampen. Similar responses can be obtained for shocks in the demographic variables.

The interpretation of these impulse responses critically depends upon the extent to which the random shocks that generate the responses are distinct. In the interpretations we choose to give for the impulse responses, we assume the contemporaneous cross equation correlation in shocks to be small as if they are distinct, i.e., we assume the variance-covariance matrix of the residuals to be diagonal. Thus, if the shock to the birth rate (ε_{1t}) is significantly correlated with the shock to the death rate (ε_{2t}), the impulse response to the birth rate shock will ignore the response to the coincident death rate shock.

An alternative approach to the problem of contemporaneous error correlation pursued by Sims (1980) is to apply an orthogonalization transformation of the variance-covariance matrix of the errors so as to make it the identity matrix. One set of possible transformations is to triangularize the variance-covariance matrix, which transforms the unrestricted system to a block-recursive system. For example, M_t might appear in the B_t equation but not vice-versa. Since the variance-covariance matrix of the system we actually estimate does not appear to be diagonal as we assumed, we report several orthogonalizations to check for robustness in the pattern of impulse response

To illustrate these ideas more concretely, let us suppose that there is a common component to the random elements in the birth and death rates such that

(4) $\varepsilon_{1t} = \theta_t + \delta_{1t}$

(5) $\varepsilon_{2t} = \beta\theta_t + \delta_{2t}$

where δ_{1t} and δ_{2t} are independently distributed of each other and of θ_t. As an example, θ_t might represent an epidemic that reduces conceptions and increases mortality, i.e. $\beta < 0$. The existence of this common error causes a contemporaneous correlation between the birth rate and the death rate. If one could distinguish θ_t from δ_{1t} and δ_{2t}, then the impulse responses of interest would be those to innovations in the δ's. An innovation in δ_{1t}, for example, would correspond to an unpredicted change in the birth rate alone. However, an innovation in ε_{1t} comes from two sources and impulse responses based upon the false premise that ε_{1t} and ε_{2t} are uncorrelated would neither correctly characterize the response to a shock only in δ_{1t} nor to a shock in ε_{1t}, since ε_{2t} would also change. However, under the assumptions given above, the no alized variances of the three independent errors and β could be determined from knowledge of the variance-covariance matrix of the ε_{1t}, ε_{2t} error vector. Thus, the appropriate one standard deviation shock in δ_{1t} and δ_{2t} could be ascertained and impulse responses generated. The assumption we maintain, however, is that $\sigma_\theta^2 = 0$, i.e., that the composite shocks in ε_{1t} and ε_{2t} are independent.

Consider an alternative assumption about the error structure, namely where the birth rate shock consists only of a common component, while the mortality rate shock has both a common and specific part.

(6) $\varepsilon_{1t} = \theta_t$

(7) $\varepsilon_{2t} = \beta\theta_t + \delta_{2t}$

It is easy to verify that this error structure is equivalent to a recursive model in the birth rate and the death rate. Ignoring other regressors we may write the corresponding system as

(8) $B_t = \theta_t$

(9) $M_t = \beta B_t + \delta_{2t}$

This recursive system therefore is implied by a particular error structure for the system given by (1) - (3). Normalizing the variance-covariance matrix of the errors in (8) and (9) to be the identity matrix yields one particular orthogonalization that permits contemporaneous correlations between endogenous variables. Clearly, an alternative (more restricted) structure is placed on the errors in a recursive system. A shock in the birth rate must now be interpreted as a shock also in the death rate; it would not be surprising to find that a recursive model yielded quite different impulse responses than would a less structured model, particularly when the contemporaneous correlation is of consequence.

To complement the impulse responses, we also calculate the proportion of the forecast error variance in each variable k^{th} period in the future that is produced by a particular shock or innovation. For example, an initial shock at time t of one standard deviation in weather, births and deathseach causes the birth rate to deviate from its mean at each future period. The fraction of the total variance in the birth rate caused by this set of standardized innovations k periods ahead, for relatively large k, is called the variance decomposition of the birth rate. The variance decomposition of each dependent variable measures the degree of interaction among the variables in the system. If the variance in a dependent variable created by innovations in all of the variables of the system is explained mostly by its own innovation, it would not appear interdependent with the other system variables. This lack of interdependence was assumed in the case of weather in the above simplified system.

Since the parameters of the unrestricted system are functions of more fundamental parameters reflecting preferences, biology, and technology, any change in these latter structural parameters will, in general, induce changes in all of the parameters of the unrestricted system. If there is reason to believe that, within the sample period, structural relationships have changed, then it would be important to estimate the system within the appropriate subperiods. Statistical tests for structural change are, therefore, conducted and are described in the next section, although they are not general in the sense of determining what subperiods should be examined for structural change.

4. Estimation and Specification Tests of the Model[8]

The system consists of five endogenous variables: CBR, IDR, NIDR, CROP and RWAGE. The five exogenous variables are the four seasonal temperatures and annual precipitation. All of the variables are expressed in logarithms and we include an annual time trend and its square in each equation of the endogenous variables. Each of the exogenous variables is assumed to be a function of lagged values of itself, that is, they are not detrended. The lag length in the exogenous variable equations is assumed to be the same as the endogenous variable equations.

There are several reasons to think that the parameters of this system of equations may have changed during the period of 1756 to 1955. One noticeable watershed occurred in the late 19th century. First, the secular decline in fertility appeared to start about 1870, although the timing of this development may be affected somewhat by the surge in emigration that begins in the 1860s (Mosher, 1980). Second, not only are the demographic trends more noticeable after 1870, the fluctuations around these trends that we want to account for become smaller, both absolutely and relatively (see Tables 1 and 2). Third, by the last half of the 19th century, Sweden had become closely integrated into world agricultural markets, importing a growing share of its foodgrains and exporting mainly animal products. With improvements in transportation, local crops ceased to determine food prices and to affect as strongly the real wage. Finally, after about 1870 the rate of industrialization increased in Sweden, and the economic-demographic system became more responsive to conditions in the nonagricultural economy. Indeed, the ebb and flow of the business-trade cycle became a

[8] The estimation used the RATS computer package, version 4.01,1980, written by T.A. Doan and R. B. Litterman.

major short run perturbation to the demographic system in the 20th century, if not earlier (Thomas, 1941 ; Galbraith and Thomas, 1941). Consequently, we will statistically evaluate the hypothesis that our econometric representation of the economic-demographic system is structurally different for Sweden before and after the onset of the roughly coincident demographic transition and industrialization. We chose 1870 as the year separating these two periods. The null hypothesis in the test is that there is no structural change between these two periods.

Prior to the structural change test it is necessary to establish the approriate lag length for the endogenous and exogenous variables. Appendix Table C.1 reports the (modified) likelihood ratio tests of the number of lags to include in our model.[9] The evidence suggests that the hypothesis of four annual lags over the entire period is supported. The structural change tests are then shown in Table 3, conditional on four lags. The results indicate that the hypothesis of no structural change is distinctly rejected for the entire system as well as for the subset of endogenous variable equations. Tests for structural change of the individual equations indicate that structural changes are most marked in the birth rate and infant death rate equations. We also tested whether the structural change is due solely to different trends in the two periods by allowing for different trends in the restricted specification (case (b), Table 3). In this manner we test whether the deviations from trend behave differently over the two periods. The results indicate the presence of structural change in the system other than trend. The data, therefore, support the hypothesis that the structure of the economic-demographic system is

[9]The modified likelihood ratio test is defined in Table C.2.

Table 3

Tests of Structural Change Between the Two Subperiods

	$\log\lvert V_u\rvert$ (1)	$\log\lvert V_R\rvert$ (2)	χ^2 (3)	d.f (4)	Marginal Significance Level (5)
Entire System[1]					
a)	-53.02	-50.70	343.0	265	$.003\text{x}10^{-1}$
b)	-53.02	-50.99	299.6	22.4	$.002\text{x}10^{-1}$
Subsystem of Endogenous Variables					
a)	-26.70	-24.61	308.7	240	.001
b)	-26.70	-24.90	265.2	225	.029
Separate Endogenous Variables					
CBR a)	-7.40	-6.73	98.60	48	$.002\text{x}10^{-2}$
b)	-7.40	-6.76	94.73	45	$.002\text{x}10^{-2}$
IDR a)	-6.00	-5.41	87.18	48	$.005\text{x}10^{-1}$
b)	-6.00	-5.56	65.41	45	.025
NIDR a)	-5.18	-4.81	55.31	48	.22
b)	-5.18	-4.85	49.38	45	.31
CROP a)	-2.01	-1.79	32.34	48	.96
b)	-2.01	-1.82	28.76	45	.97
RWAGE a)	-5.25	-4.84	61.88	48	.09
b)	-5.25	-4.92	50.17	45	.28

1. Row a) treats the trend and its square exactly as the other variables in the system. Row b) assumes the trend and its square to differ between the two periods as a maintained hypothesis and therefore tests for structural change of the remaining variables only. V_u and V_R are explained in Table C.1 and $\chi^2 = T((2)-(1))$.

dissimilar in the two periods.

As noted at the outset, the methodology and variables examined here to describe the interplay of economic and demographic processes in a preindustrial and pretransition society are probably less adequate for describing transitional and modern trends for a variety of reasons. In particular, we expect that changes in health technology, the general growth in wealth levels (i.e., human and physical capital), the more rapid growth in women's wages than men's wages, and the incentive effects of the modern tax-transfer system, have all altered the short-run and long-run responses of birth rates and death rates to current economic conditions and fluctuations in weather. Estimates of the model's parameters for the later period, 1870–1955, implied dynamic patterns that are substantially different from those of the early period. Small changes in the model's specifications implied substantial changes in the system **outcomes and often unstable processes were estimated for some (or all) variables. Therefore, we restrict our analysis to the first period.**

To specify the model for the earlier period, we again perform the test of lag length. Due to the smaller sample size (109 observations), and to the number of parameters, the lag length tests as modified by Sims (1980) cannot reject any lag length less than five (see Appendix Table C.2), supporting the choice of a single year lag.[10] Conversely, if we do not adopt Sims' conservative modification of the conventional χ^2 statistical significance test, it rejects all lags less than five. Hence, we have adopted the four lag specification accepted above for the entire sample.

[10]The modified likelihood ratio test (see also Table C.1) reduces the χ^2 statistic by subtracting the number of coefficients in an unrestricted equation from the number of observations in calculating the likelihood ratio statistic.

Given the lag length, we performed the tests of exogeneity due to Sims as described in the previous section. The test of exogeneity of the weather variables should be viewed as a test for omitted variables. Following Sims (1972), four leading values of the exogenous variables were included in the endogenous variable equations. Table 4 shows that there are no important omitted variables in the demographic and the crop index equations. However, the results for the agricultural real wage equation suggest that there are omitted variables correlated with weather that are also part of the RWAGE process.[11] Nonetheless, a test for the entire system does not reject exogeneity of weather at the conventional 5 percent confidence level.[12]

Table 5 presents the estimated parameters of the endogenous variables while the estimates for the exogenous variables are in Table C.3. Many of the tests for excluding each variable (all lags) from specific equations do not support inclusion of this explanatory variable at usual confidence levels. Overall F statistics are, however, significant for the entire system, and for the NIDR and RWAGE equations separately.

The zero-order contemporaneous correlation between the exogenous and the endogenous variables residuals is due to the inclusion of current exogenous variables in the endogenous variables equations. There is a large positive correlation between the two

---

[11] For example, if the level of employment should be included in the system as an endogenous variable, weather might appear endogenous as the example in the previous section demonstrated. To repeat that argument, future weather is correlated with future employment, which is correlated with past employment and the current wages.

[12] Table C.5 reports results of exclusion tests (Granger (1969) causality) which indicate a general support for the "no" omitted variables hypothesis except for the results with respect to winter temperature.

Table 4

Sims' Exogeneity Tests

Leads of	CBR F	CBR Marginal Significance	IDR F	IDR Marginal Significance	NIDR F	NIDR Marginal Significance	CROP F	CROP Marginal Significance	RWAGE F	RWAGE Marginal Significance
SPTEMP	1.14	.35	.76	.56	.21	.93	1.26	.31	2.87	.03
SMTEMP	2.30	.07	1.01	.41	.99	.42	.76	.56	.47	.75
AUTEMP	.65	.63	.29	.88	.80	.53	2.73	.04	2.55	.05
WNTEMP	.78	.54	.35	.84	1.14	.35	.97	.44	1.90	.13
RAIN	.94	.45	.46	.77	.77	.55	1.53	.21	2.99	.03
ALL	1.31	.22	.55	.92	.84	.64	1.55	.11	1.75	.06

χ^2 test for the entire subsystem: $\chi^2(100) = (110-68)(-24.98-(-27.23)) = 94.5$, marginal significance level = .63

Table 5

The Endogenous Variables Equations: 1756-1869

Regressor	lag	CBR	IDR	NIDR	CROP	RWAGE
		Dependent Variables				
Constant	-	-5.91	8.08	7.23	-36.06	-11.90
Trend	-	$-.5\times10^{-4}$	$-.15\times10^{-2}$	$-.3\times10^{-2}$	$.34\times10^{-2}$	$-.5\times10^{-2}$
$Trend^2$	-	$.3\times10^{-5}$	$-.25\times10^{-4}$	$-.13\times10^{-4}$	$.27\times10^{-4}$	$.5\times10^{-4}$
CBR	1	.163***	.040	.521	-3.855	-1.373**
	2	-.055	-.281	.142	4.892	1.248
	3	.188	.382	.275	.353	-.321
	4	.156	.363	.262	-2.703	-.036
IDR	1	.071	.078	-.205	-.939	.359
	2	.084	-.141	-.409	.637	-.335
	3	-.083	-.083	.031	.113	.217
	4	.011	-.172	-.432	-1.31	-.386
NIDR	1	-.019	-.013	.315	1.045	.040
	2	-.022	.041	.152	-.756	.109
	3	.043	-.003	-.137	.339	-.789
	4	.050	.141	.191	1.099	.269
CROP	1	.005	-.149	-.048	.360*	.004
	2	.013	.184	.014	.229	.017
	3	-.006	0	.032	-.136	.021
	4	.005	.019	.009	-.357	-.051
RWAGE	1	.13**	-.074	-.137***	-1.138	.073*
	2	.009	-.106	-.304	.167	-.016
	3	-.104	.155	.098	-.977	-.240
	4	-.008	-.166	-.073	.922	.230
WNTEMP	0	-.089	-.072	-.198***	1.046***	.328***
	1	.017	-.123	-.286	-.054	.065
	2	.002	-.054	-.096	-.317	.064
	3	-.009	-.079	-.062	.550	.107
	4	-.011	.006	.018	1.046	.066
SPTEMP	0	.065**	-.131	-.362	-2.574	-.470
	2	.431	-.215	-.125	-.113	.723
	2	.102	.032	-.418	-.334	.051
	3	.219	.081	-.791	1.531	-.091
	4	-.013	-.259	-.419	1.301	.065
SUTEMP	0	.025	.089	.434	2.642	.247
	1	-.094	-.235	-.046	-.595	-.333
	2	.051	-.280	-.124	-3.175	.014
	3	.004	.366	.557	-.041	-.185
	4	.170	-.217	-.177	2.177	.649
AUTEMP	0	-.15	.058	.289	.280	.225***
	1	-.040	-.144	.105	1.582	.271
	2	.015	-.013	.066	2.028	.308
	3	.035	-.032	.045	1.102	.413
	4	-.073	-.158	-.146	-.556	-.226
RAIN	0	-.020	-.035	.001	.645	.008 **
	1	-.010	.115	.021	-.880	-.360
	2	-.001	-.018	-.036	-.150	-.056
	3	-.011	.031	.099	.235	.082
	4	-.051	.028	.007	.174	-.016
R^2		.82	.87	.74	.60	.81
Significance Level		.8710	.8490	.9699	.7379	.9997

*, **, ***, indicate that the F-test for excluding this variable (all lags) is rejected at the 1%, 5%, and 10% level respectively.

TABLE 6

Decomposition of Variance: Percentage of Forecast Error Variance 25 Years Ahead Produced by Each Innovation (ρ^2_{ij} (25))

	Innovation in: (j)									
Response in: (i)	CBR	IDR	NIDR	CROP	RWAGE	WNTEMP	SPTEMP	SUTEMP	AUTEMP	RAIN
CBR	27	4	4	3	15	13	15	3	8	6
IDR	3	44	2	4	6	16	3	8	8	5
NIDR	4	4	34	6	5	23	6	6	5	4
CROP	6	1	2	48	10	8	8	4	8	5
RWAGE	5	4	3	3	38	10	3	4	17	13

death rates, IDR and NIDR (i.e., .73), the correlation between IDR and CBR is -.31 and the correlation between NIDR and CBR is -.42. Hence, the shocks to the demographic series do not appear to be independent, as we had hoped, in order to confirm the working assumption of our approach. Furthermore, the innovation in RWAGE is positively (.4) correlated with the innovation in the crop index and negatively correlated with the death rates. In interpreting the results we, nevertheless, maintain the assumption of zero contemporaneous correlation among the variables (Σ_v and Σ_ε in Appendix B are diagonal), rather than impose a temporal ordering on the endogenous variables. In addition, we have computed the results for various other orthogonalizations (see Appendix B for explanation) and report the results of those that have some plausibility and which are notably different from those implied under the assumption of a diagonal covariance matrix.

Table 6 reports the decomposition of the variance of each variable due to a one standard deviation shock in each variable. Table C.4 reports the estimated variance-covariance matrix of the innovations upon which these shocks are based. The variance decompositions emphasize the magnitude of the importance of each variable in each endogenous variable equation. Each variable accounts for less than 50 percent of its own variance. The winter temperature is especially important in accounting for the variance of the demographic variables. The real wage and spring temperature account for much of the variance in the birth rate. Interactions between the demographic variables are not significant. That does not imply, however, that the impact of a shock in one demographic variable on another is small, in any absolute sense. Alternative decompositions of the covariance matrix of endogenous variables according to alternative triangularizations of the contemporaneous covariance matrix **reveal only minor differ-** ences. Appendix Table C.6 reports two alternative triangularized decompositions of the variance. The main difference is with respect to the responses of the two death rates, although the sum of the two is not greatly affected. Hence, our interpretation of the covariance between the innovations gives rise to a decomposition of variance which is almost identical to alternative interpretations of the covariance.

5. Description and Interpretations of Impulse Responses

Impulse responses are presented in figures 2a-g, 3a-c, and 4a-d. The first set shows demographic reactions to shocks in weather and economic variables, the second set shows demographic reactions to demographic shocks, and the third set shows economic reactions to weather shocks. We will discuss each set in turn after first establishing several general features of all of the responses.

The first characteristic to note is the overall stability of the system. Responses to shocks in time period 1 tend to dampen quite rapidly, with convergence to zero (i.e,to mean values) occurring within a 10 to 15 year period. The second notable feature is the relatively short cycle of the responses. There is little evidence of persistence; fluctuations around zero are sharp and freqaent.

The reactions of the demographic variables to exogenous and economic (CROP,RW shocks exhibit a consistent pattern. For every individual response depicted in figure 2, the crude birth rate reacts in an opposite fashion as do both of the death rates. Thus, for example, a positive innovation in the general crop index or in the real wage increases fertility for several years and decreases the infant and non-infant death rates over the same period, as hypothesized by Malthus. Also, each seasonal temperature shock that initially reduces the death rates also increases fertility, while an increase in precipitation subsequently increases mortality and reduces fertility. In particular, warm winters have especially beneficial effects on survival. Increases in wealth, broadly defined, tend to increase fertility and reduce both infant and non-infant mortality at

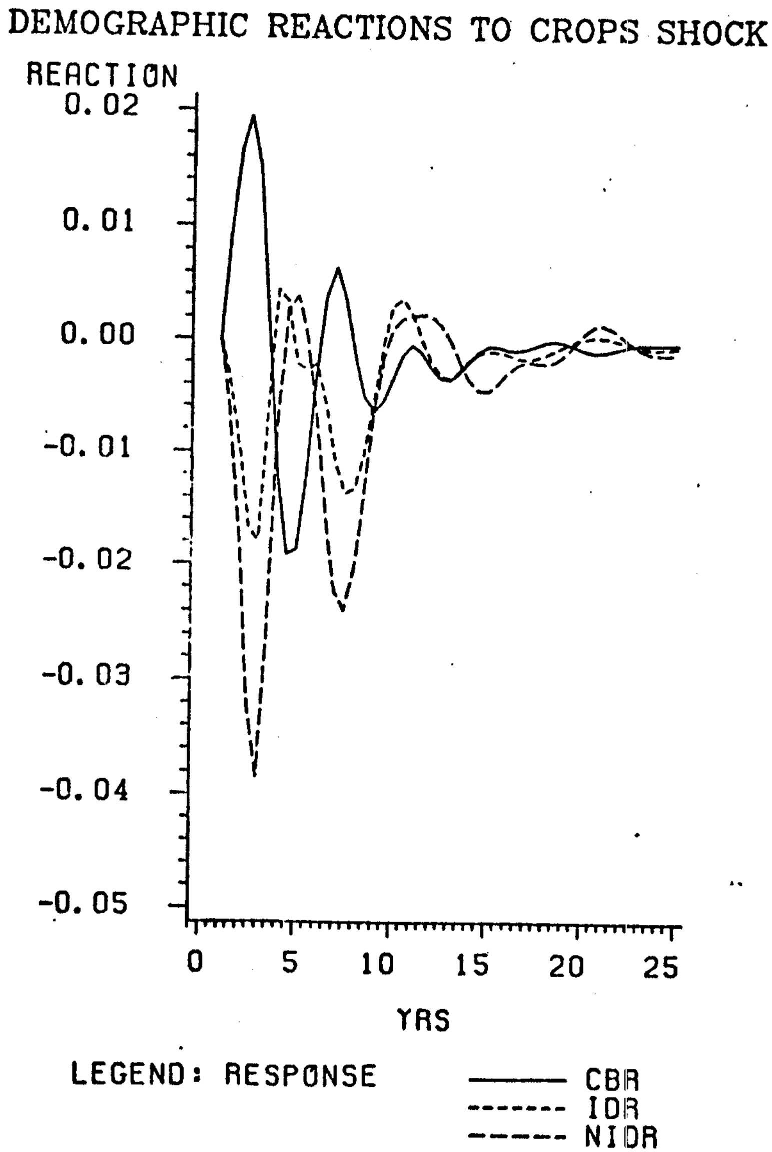

Figure 2a

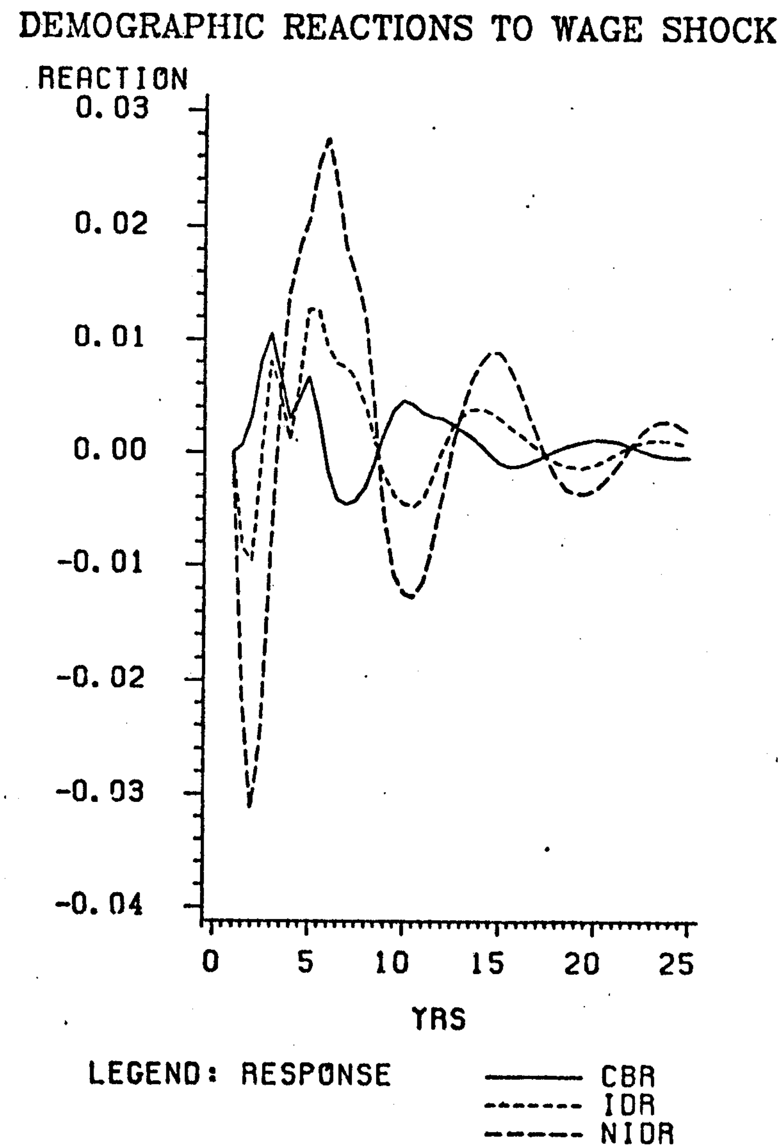

Figure 2b

DEMOGRAPHIC REACTIONS TO WINTER TEMPERATURE SHOCK

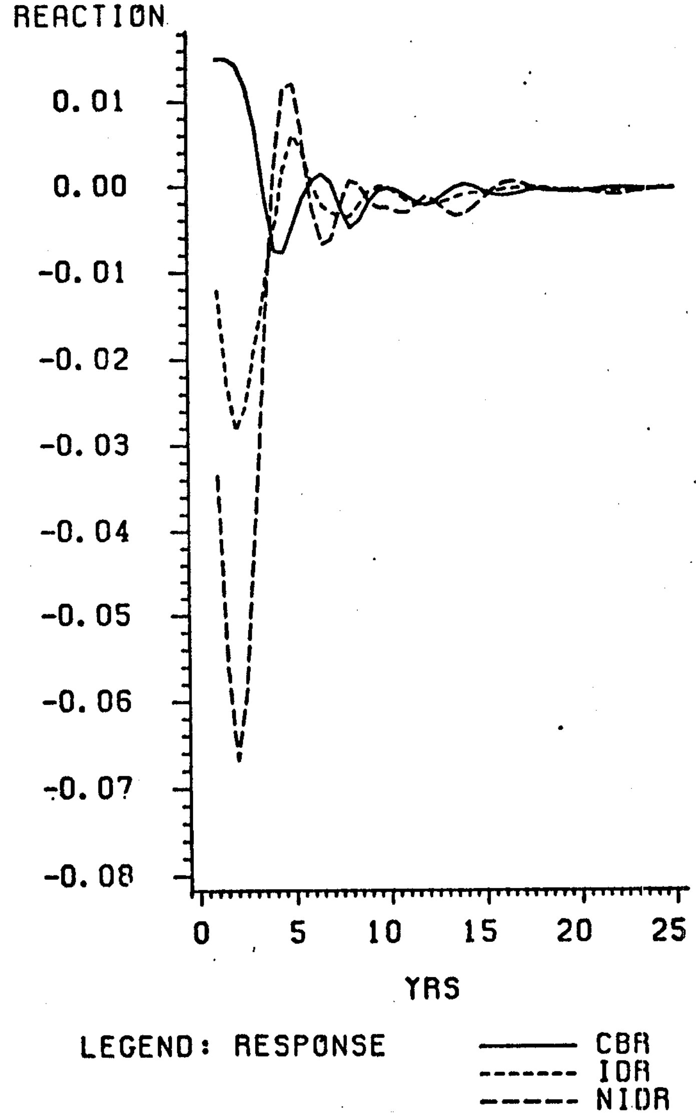

Figure 2c

DEMOGRAPHIC REACTIONS TO SPRING TEMPERATURE SHOCK

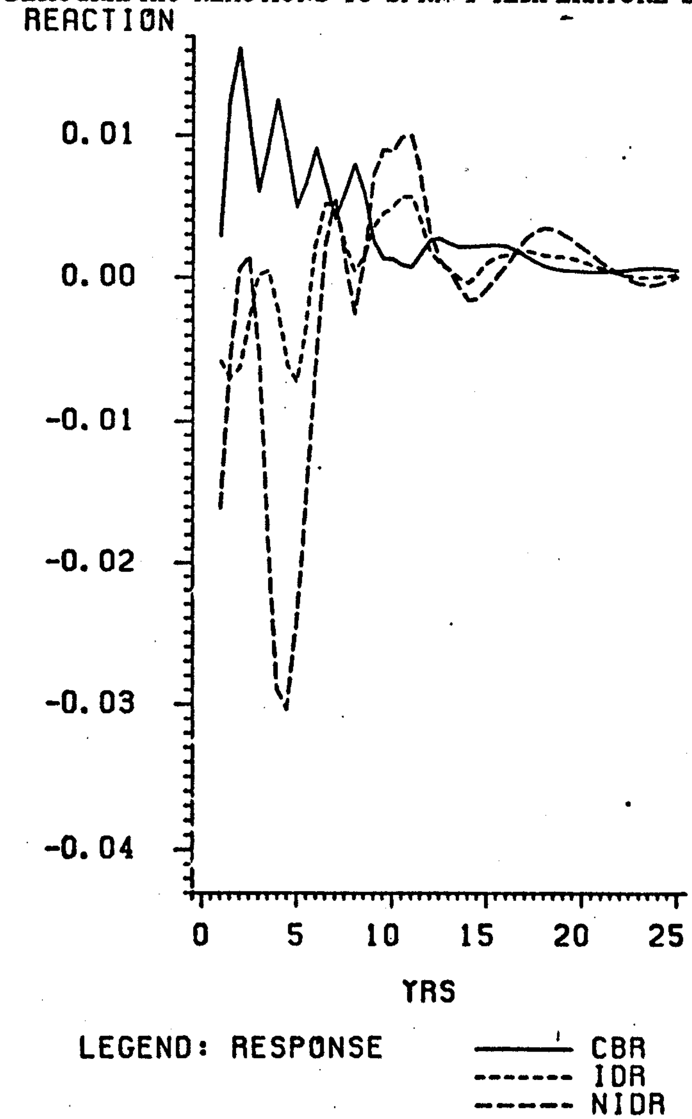

Figure 2d

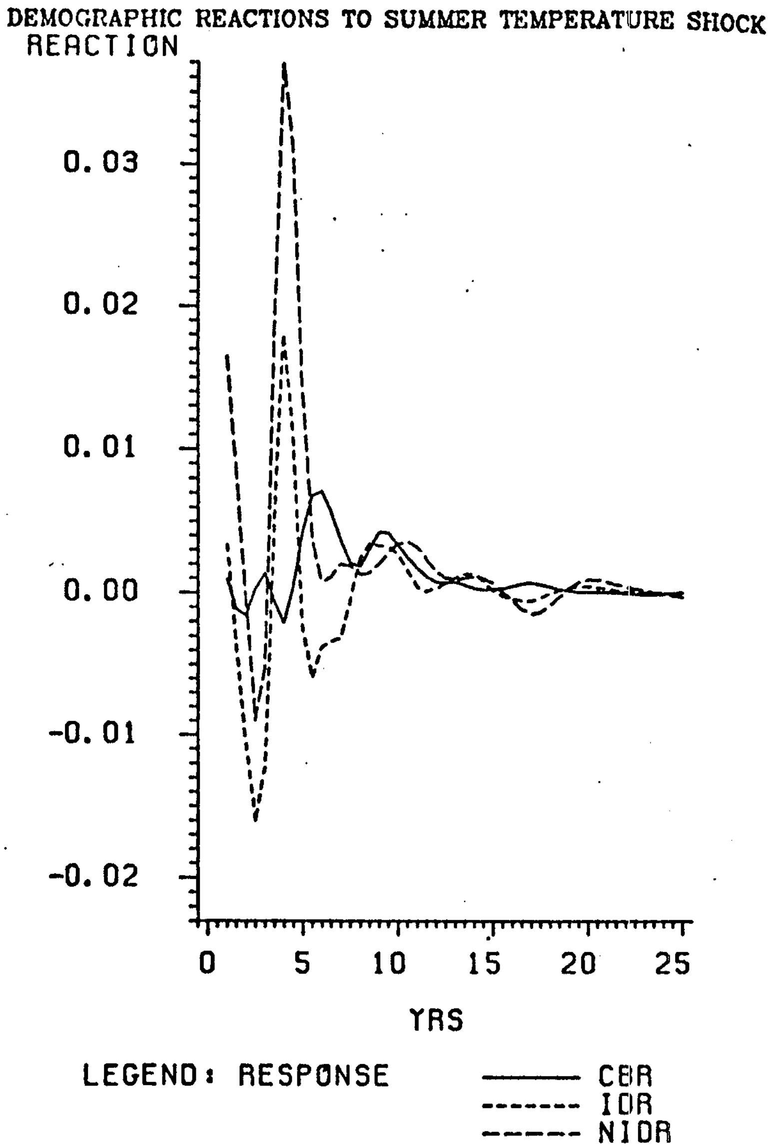

Figure 2e

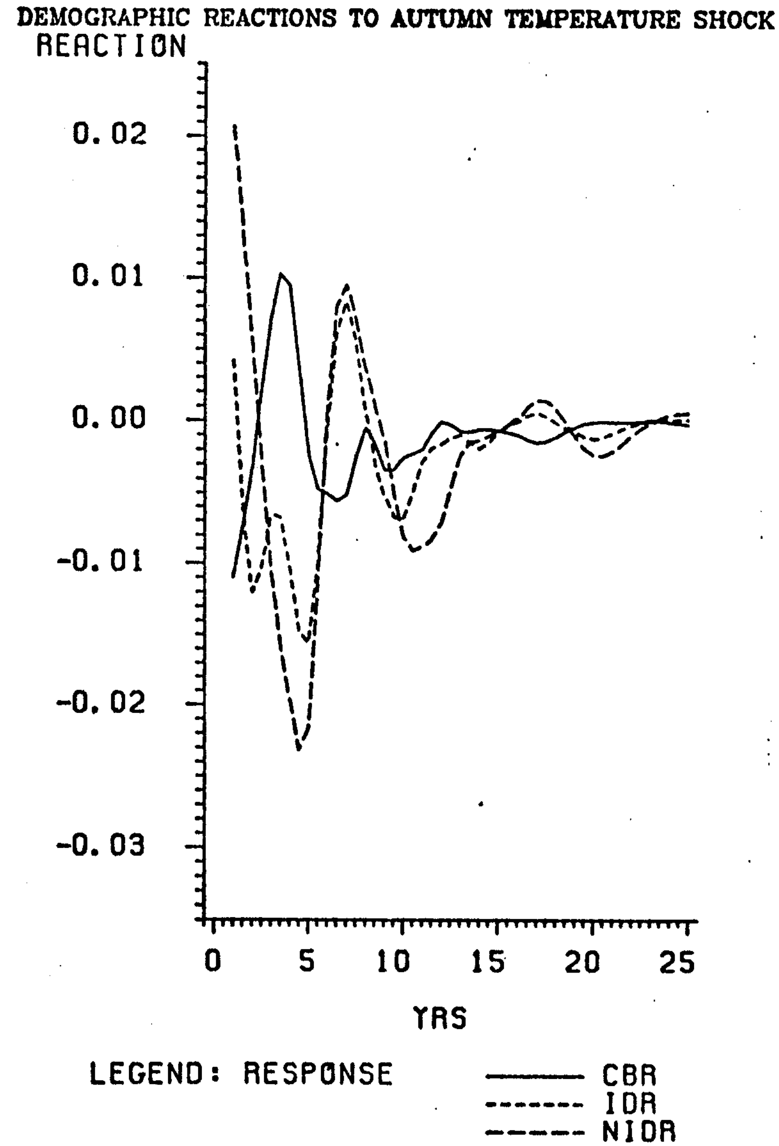

Figure 2f

DEMOGRAPHIC REACTIONS TO RAIN SHOCK

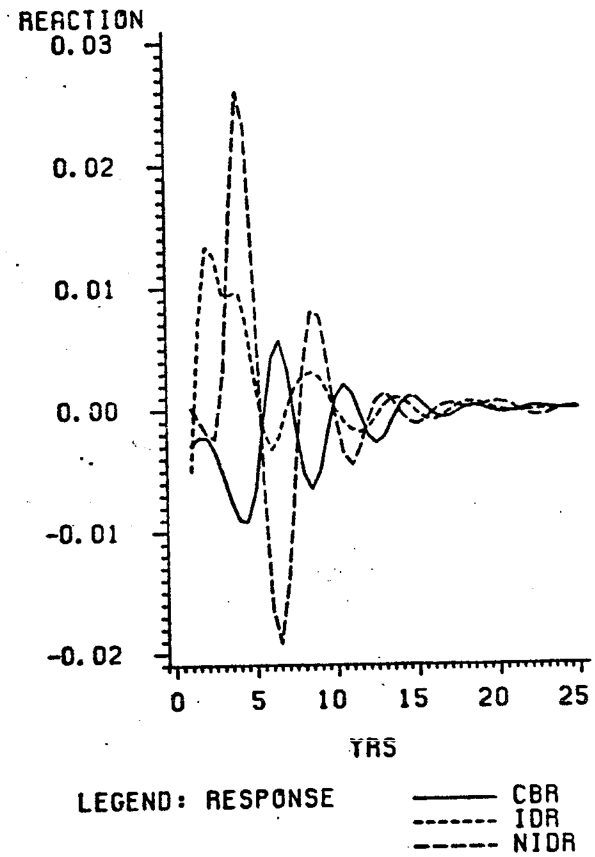

Figure 2g

least for several years. In addition, with a few exceptions, the cumulative responses of mortality are in the same direction as the initial response, i.e., the negative response to increased wealth outweighs the positive components of the ensuing cycles. The positive components are explicable as postponement or selection effects,where increased survival of inherently weaker individuals due to, for example, favorable crop outcomes, merely delays some of the deaths that would otherwise have occurred earlier . Note, also, the larger responses in amplitude of the non-infant death rate than of the infant death rate.

The demographic responses to demographic shocks display different patterns of interaction. The birth rate reaction to its own innovation (figure 3a) reveals a three year cycle that seems to be compatible with the biological reproductive cycle, a finding that is also apparent in the previous figures and in the Bengtsson(1981) study of southern Sweden. The birth rate response to mortality rates appears consistent with a replacement strategy. An increase in the infant death rate is followed by an increase in the birth rate with the peak increase occurring in two years. The cumulative response, however, appears negligible,implying a change in the timing of children rather than in completed fertility. An increase in the non-infant death rate first reduces fertility as would be anticipated if the proportion of child bearing population in marital unions is thereby reduced. But it is then followed by a rise in fertility peaking after approximately five years. This latter response is consistent with the delayed "replacement" that would occur as new households were formed in response to the loss of spouse or parent.

Both the infant and non-infant death rates (figures 3b, 3c) respond

CBR REACTION TO DEMOGRAPHIC SHOCKS

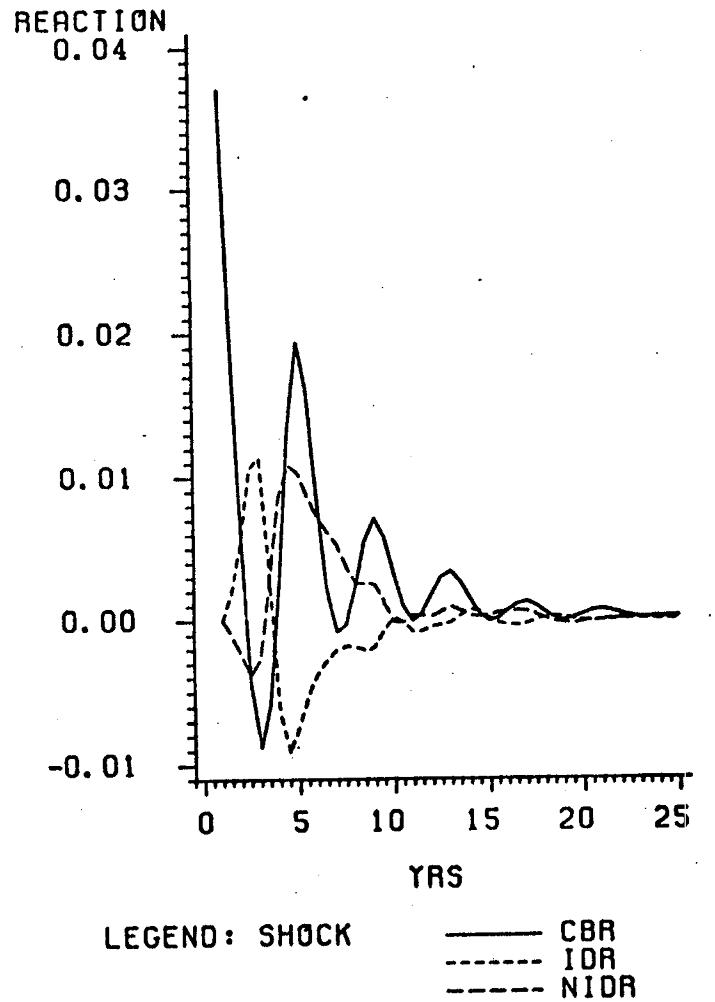

Figure 3a

IDR REACTION TO DEMOGRAPHIC SHOCKS

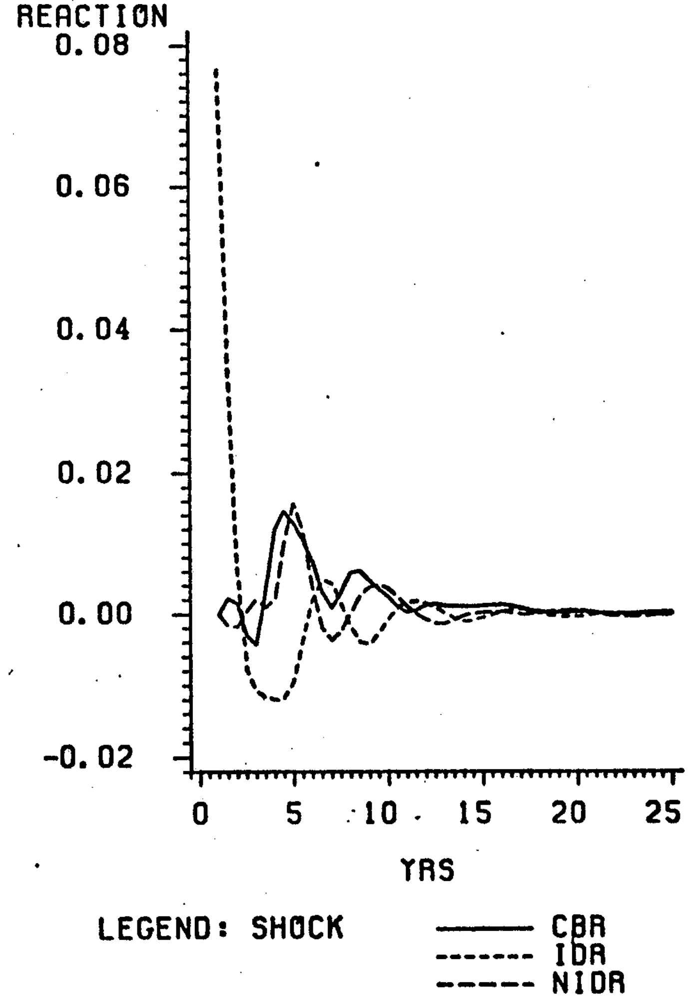

Figure 3b

NIDR REACTION TO DEMOGRAPHIC SHOCKS

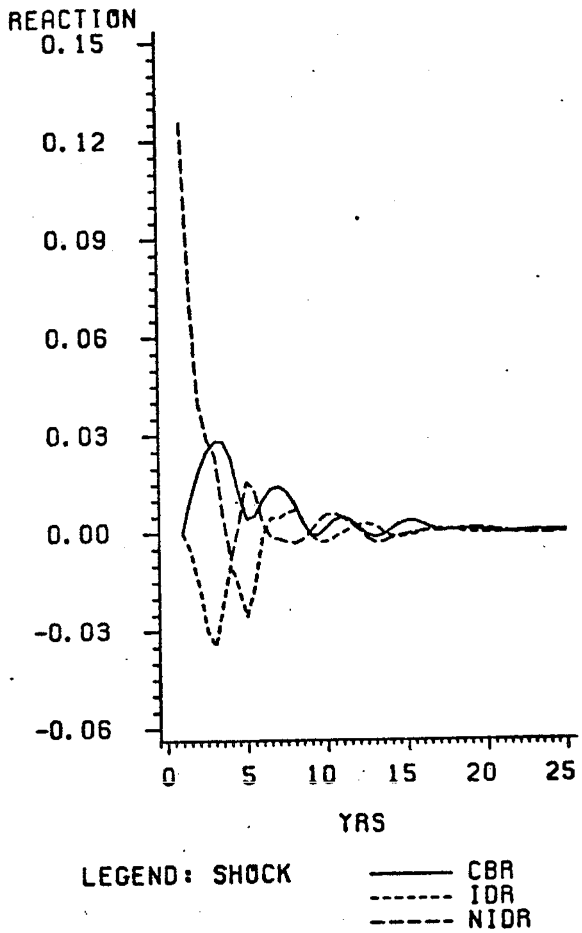

Figure 3c

positively to an increase in fertility, although the former response is delayed for several years. These may reflect crowding effects as children compete for the limited resources within the family. Interestingly, death rates cycle in opposite fashion when responding to **shocks in** the death rate themselves. Although we do not completely understand these interactions, some have plausible interpretations. For example, the observed fall in the non-infant death rate due to an increase in the infant-death rate may represent a selection or "survival of the fittest" process whereby the death of the weakest infants reduces the mortality rate of those that survive. Also, the impacts of the death rate shocks on themselves possibly reveal the short-lived nature of epidemics in this period, with the non-inf death rate responding to its own shock with somewhat more persistance than in the case of the infant death rate.

Figures 4a and 4b illustrate the reaction of crops to the exogenous and economic variables. Except for warm springs, higher temperatures initially increase agricultural output followed by a sharply fluctuating pattern somewhat similar to that obtained from a shock in crops itself (figure 4b). Rainfall also increases crops initially with the same kind of subsequent oscillatory pattern as observed for temperature. An increase in the real wage has a discernible (negative) impact on crops, but only five to seven years later, for reasons that we do not understand.

The real wage responds to seasonal temperatures in a qualitatively similar way as do crops (figures 4c and 4a). The response of wages to rainfall is, however, negligible. The wage response to an increase in crops could not reflect an increase in the marginal product of labor, since the initial response is negative, but would be consistent with an exogenous increase in the supply of agricultural labor.

CROPS REACTION TO SEASONAL TEMPERATURE SHOCKS

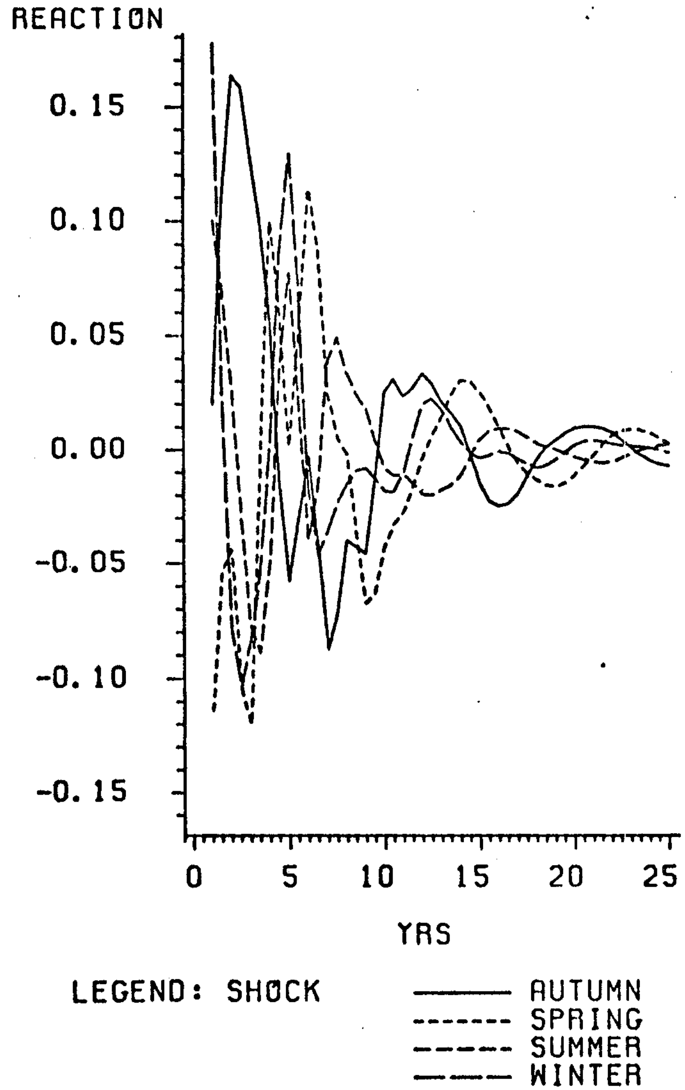

Figure 4a

CROPS REACTION TO WEATHER AND ECONOMIC SHOCKS

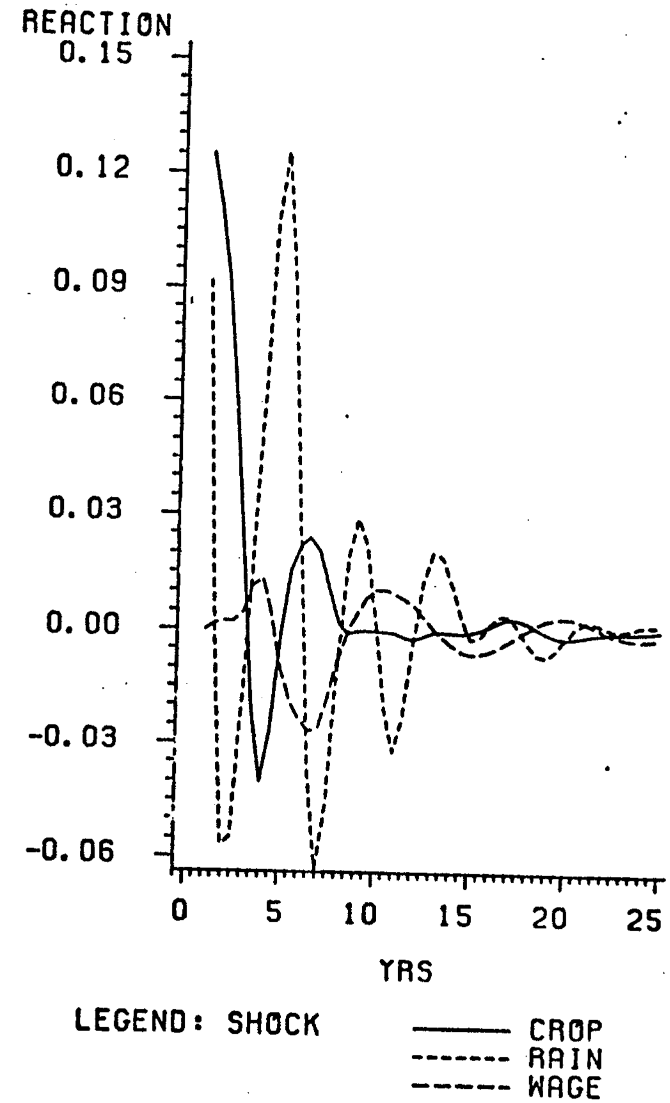

Figure 4b

WAGE REACTION TO SEASONAL TEMPERATURE SHOCKS

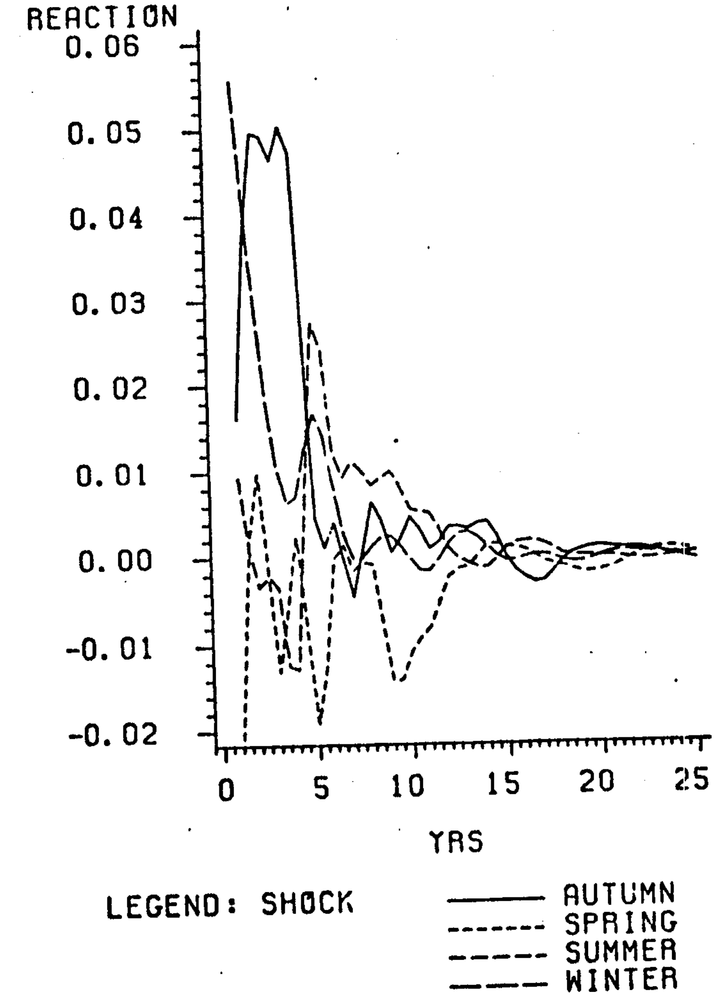

Figure 4c

WAGE REACTION TO WEATHER AND ECONOMIC SHOCKS

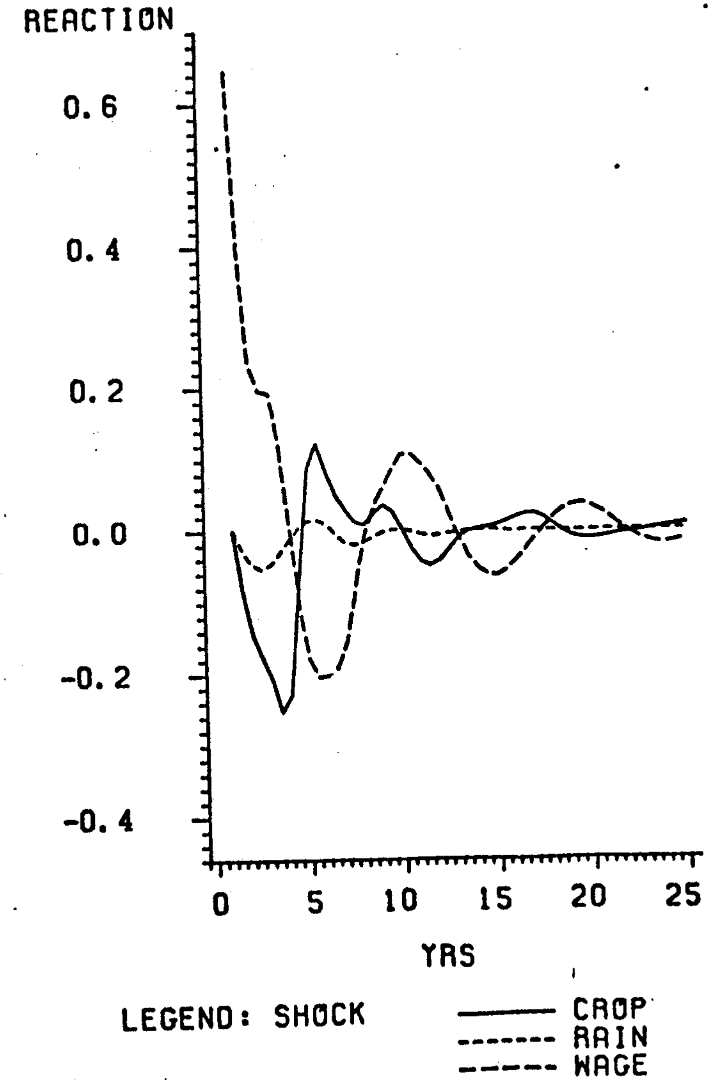

Figure 4d

We compared the above results to impulse responses where the contemporanous correlations between the errors of the endogenous variables were triangularized as suggested by Sims (1980). We used two alternative orderings of variables which were different only with respect to the order of the pairs NIDR-IDR and RWAGE-CROP. The responses of the demographic variables to weather shocks as well as to shocks in RWAGE and CROPS were basically unchanged. The shape and scale of the responses were insensitive to these different triangularizations of the covariance matrix; the initial sign and cycle of the responses of these variables were particularly robust. However, responses of the demographic variables to shocks in demographic variables showed significant changes with respect to alternative interpretations of the covariance matrix. Specifically, the sign of the initial response, the pattern and the scale were different. Hence, we have less confidence in the robustness of our results with respect to alternative specifications and interpretations of the contemporaneous relationships between the birth rate and the death rates. The example in Section 3 demonstrates a particular rationale for the existence of this type of sensitivity.

The above impulse responses are scaled in logarithmic or proportional changes for each endogenous variable outcome. To compare the magnitudes of these impulse responses and to facilitate their aggregation, responses in the three demographic variables are expressed in common units as they contribute to the natural rate of increase of the population, that is the difference between births and deaths. Table 7 reports the cumulative response from a standard deviation innovation of the residuals contemporaneously, after one year, five years and ten years. Since the responses tend to dampen rapidly, the cumulative response after ten years tends to approach an asymptote and thereafter is constant.

Although we previously observed that the response pattern of demographic variables to innovations in real wages and crops were basically similar, their cumulative effects on population growth differ, as seen in Table 7. An unanticipated rise of 12.5 percent in real wages in one year is associated with increase in popul tion growth in the next year by almost one per thousand (.91), and by more than two per thousand (2.24) by the second year. But after the second year the effect of raising birth rates for the first two years is offset by a shortfall in births. The effect of wages reducing deaths, however, continues to accumulate for nearly ten years. Thus, wages affect only the timing of births, whereas the persistent effect of wages on population growth arises from the reduction in mortality, and quantitatively the reduction in noninfant mortality is the bulk of the demographic response (88% of the reduction in deaths after ten years). The importance of the mortality response is consistent with Malthus' supposition and does not accord with Wrigley and Schofield's (1981) conclusion or Lee's (1977) analysis of English data.

On the other hand, with an innovation in the crop index, the response of birth rates cumulates steadily to .44 per thousand in the next year, to .70 after five years, and persists at .77 after ten years. In the case of unanticipated variation in crops, however, infant deaths aee little affected, and noninfant mortality falls for only two years, with a more than offsetting reversal in later years, not unlike Lee's (1981) finding for the effect of wheat prices on mortality in England. Thus, abundant

Table 7

Cumulative Impulse Response in Demographic Variables

on Rate of Population Increase Per Thousand Inhabitants Per Year

Variable Shocked by One Standard Deviation (Percent of Mean)		Cumulative Effect on Population Growth after a Certain Number of Years				
		0	1	2	5	10
Real Wage	Crude Birth Rate	*	.54	1.02	-.07	-.25
(12.5%)	Infant Death Rate	*	.05	.16	.16	.30
See figure 2b	NonInfant Death Rate	*	.33	1.06	1.25	2.14
	Population Growth	*	.91	2.24	1.34	2.19
Crop Index	Crude Birth Rate	*	.10	.44	.70	.77
(65%)	Infant Death Rate	*	.06	.01	-.13	-.13
See figure 2a	NonInfant Death Rate	*	.60	.76	-.43	-.43
	Population Growth	*	.76	1.21	.15	.21
Infant Death Rate	Crude Birth Rate	*	.18	.55	-.02	-.20
(7.65%)	NonInfant Death Rate	*	.24	.96	1.65	1.36
See figures 3a and 3b						
NonInfant Death Rate	Crude Birth Rate	*	-.08	-.16	.68	1.03
(12.6%)	Infant Death Rate	*	-.01	.00	.13	.16
See figures 3a and 3b						
Winter Temperature	Crude Birth Rate	.49	.95	1.17	.81	.57
(16.9%)	Infant Death Rate	.07	.24	.34	.33	.38
See figure 2c	NonInfant Death Rate	.64	1.91	2.63	1.95	2.19
	Population Growth	1.20	3.10	4.14	3.09	3.14

*Assumed to be zero

harvests in Sweden are associated with only a transitory remission in mortality and the persistent source of population growth linked to good crops accrues through an elevated fertility level.

Among the impulse responses of demographic variables to each other, Table 7 reports the responses to shocks coming from the two death rates. An innovation in infant mortality rate is equal to a 7.7 percent increase, which would itself reduce population growth by .45 per thousand. The increase in the birth rate in the following year adds .18 to the rate of population growth and consequently "replaces" 40% of the additional infant deaths. After two years the cumulative replacement reaches 122 percent, but is completely offset in the next three years by below average fertility, leaving the net effect negative after five or ten years. The replacement of infant deaths is apparently only one of timing, not of raising completed fertility. Noninfant deaths decrease, however, after a rise in infant mortality, perhaps because the more stringent selection of infants who survive improve their health endowments and augment their survival through childhood. This effect cumulates for five years and then tapers off.

As we have noted, shocks in noninfant deaths are associated with a decline in births for two years, followed by a substantial "replacement," cumulating after ten years to one per thousand or 43 percent of the initial number of unanticiapted noninfant deaths. Infant deaths, on the other hand, are not greatly affected by shocks in noninfant deaths.

Finally, the least difficult to interpret relationships are those linking weather innovations to demographic outcomes. The largest effects are associated with winter mean temperature for which a standard deviation rise involves an increase of 17 percent or 8 degrees Fahrenheit. As shown in Table 7, this shock leads to a rise in birth rates and a decline in death rates, with the contemporary rate of population growth increasing 1.5 per thousand, cumulating to an effect of 3.1 by the following year that is more or less persistent. The temperature of the other three seasons have smaller and offsetting effects, suggesting that a general warming of the climate in one year is linked to a substantial increase in population over the following five years, that is not counterbalanced by a later shortfall.

6. Conclusions

We have described and interpreted Swedish historical demographic, economic and weather annual data for the entire country using vector autoregression. Our particular emphasis has been on short run interactions in the preindustrial period, as characterized by the impulse responses of the estimated system from 1756 to 1869. We found that unexpected increases in wealth, whether this occurred through changes in real wages, agricultural output, or weather, led to increased fertility and decreased mortality, at least for several years, and thus to an increased rate of population growth cumulatively over a five to ten year period. We observed a short-run replacement phenomenon in that an unanticipated increase in infant deaths increased sharply fertility for one or two years, although only a negligible cumulative effect remained after five or ten years, indicative of a timing response in fertility that did not modify lifetime fertility patterns. An unanticipated increase in non-infant deaths also evoked a fertility response several years later, consistent with a delayed replacement effect, but this response appears to persist for at least a decade.

Although vector autoregression is not designed to account for long term trends and their consequences, our analysis of short term fluctuations suggests the need for further study of how longer trends and swings in weather variables could contribute to persistent changes in population growth, operating principally, perhaps, through variation in mortality rates. Many persons have hypothesized a link from long cycles in weather to swings in mortality; our short run evidence could be seen as consistent with this conjecture.

The other long term relationship we would like to understand better is that between the wage rate and population growth, but both of these variables are endogenous and detrended in our analysis. In the short to medium term, say less than ten years, real wage innovations contribute to population growth, mainly by reducing death rates. But the response of mortality or fertility to a secular change in real wage may not be as we have discovered here.

These results, like all of the results detailed in the paper, are not interpreted as stemming from a single structural relationship, whether it is a biological or technological constraint, or a function of people's preferences. Our findings are presumed to be derived from a complex behavioral and biological system, and should not be interpreted as distinguishing between particular hypotheses that relate to the existence or importance of particular structural components of the system. Such a task must be left for future work.

Appendix A

Table A-1

Definitions and Sources of Data

Symbol	Definition	Source and Notes
CBR	Crude Birth Rate: The number of births registered per thousand inhabitants during calendar year.	1750-1950, Sweden (1955) Table B.2; 1951-1955 United Nations (1979)
IDR	Infant Death Rate: The number of deaths of children under one year of age per thousand live births during calendar year. See text for minor adjustment of births included in denominator to include some births of previous calendar year.	1750-1950, Sweden (1955) Table B.2; 1951-1955, United Nations (1979)
NIDR	NonInfant Death Rate: The number of deaths of persons one year and older per thousand inhabitants one year and older during calendar year.	1750-1950, Sweden (1955) Table B.2; 1951-1955, United Nations (1979)
CROPS	A general index of Swedish crop yields: The relative abundance of crops in the calendar year season, constructed by G. Sundbarg from Royal Commission estimates and subsequent crop yield information. Not strictly available for last few years, when U.N. index of agricultural output in Sweden was substituted.	1750-1800, Sundbarg (1907, Table C); 1800-1955, Sweden (1959) Table E12
RWAGE	The real wage in agriculture: From 1750-1869 series is the daily wage for a male agricultural worker divided by the price of a hectolitre of Rye (representative of foodprices). Alternative cereal prices varies together ($r > .98$). Beginning in 1870 an overall agricultural annual wage is available (Jungsfeldt), which is deflated by a GNP deflator (Phelps-Brown).	1750-1869, Joburg (1972); 1870-1955 Jungsfeldt (1966) and Phelps Brown (1968)
WNTEMP	Mean Winter Temperature: The monthly mean temperatures for January, February and March, averaged for the calendar year in Fahrenheit divided by ten.	1856-1955 Sweden (1959). Table C2

Table A-1 continued

SPTEMP	Mean Spring Temperature: The monthly mean temperatures for April, May and June, averaged for the calendar year in Fahrenheit, divided by ten.	as above for WNTEMP
SUTEMP	Mean Summer Temperature: The monthly mean temperature for July, August and September, averaged for the calendar year in Fahrenheit, divided by ten.	as above for WNTEMP
AUTEMP	Mean Autumn Temperature: The monthly mean temperature for October, November and December, averaged for calendar year in Fahrenheit, divided by ten.	as above for WNTEMP
RAIN	Precipitation in centimeters during calendar year: The average reported at the meteorological stations at Lund, Stockholm and Uppsala with exceptions noted in text.	1750-1955, Sweden (1959). Table C7-C8

Table A-2

Basic Data Series in Natural Logaritims

CALENDAR YEAR	CBR	MIDR	NIDR	GINDX
1756-	-3.32471	-1.52649	-8.49858	1.60944
1757-	-3.42403	-1.55017	-8.35781	2.01490
1758-	-3.39887	-1.55698	-8.24454	3.40120
1759-	-3.39592	-1.65838	-8.49160	3.80666
1760-	-3.33767	-1.69242	-8.57031	3.40120
1761-	-3.36209	-1.57196	-8.56282	2.01490
1762-	-3.35181	-1.42501	-8.35118	.0
1763-	-3.35383	-1.42596	-8.27968	2.01490
1764-	-3.36452	-1.57497	-8.48342	1.60944
1765-	-3.40153	-1.56257	-8.45720	3.40120
1766-	-3.39136	-1.55979	-8.59187	3.40120
1767-	-3.34662	-1.59483	-8.55834	3.40120
1768-	-3.39576	-1.53400	-8.49620	3.80666
1769-	-3.41203	-1.53366	-8.48686	3.80666
1770-	-3.41471	-1.56440	-8.52921	3.40120
1771-	-3.43645	-1.56373	-8.43921	.0
1772-	-3.53983	-1.47646	-8.06147	1.60944
1773-	-3.65440	-1.31156	-7.66289	3.40120
1774-	-3.37388	-1.70150	-8.67454	3.40120
1775-	-3.33959	-1.67169	-8.57703	1.60944
1776-	-3.41860	-1.77338	-8.66469	3.40120
1777-	-3.41409	-1.62920	-8.56618	3.80666
1778-	-3.36160	-1.53063	-8.52207	3.40120
1779-	-3.30893	-1.51488	-8.45317	3.80666
1780-	-3.33925	-1.81367	-8.71845	1.60944
1781-	-3.40073	-1.67041	-8.53204	1.60944
1782-	-3.44236	-1.68824	-8.42634	1.60944
1783-	-3.49632	-1.61392	-8.39313	1.60944
1784-	-3.45731	-1.61744	-8.32043	3.40120
1785-	-3.46103	-1.64777	-8.37999	1.60944
1786-	-3.41769	-1.57997	-8.52262	3.40120
1787-	-3.46203	-1.69323	-8.58998	3.80666
1788-	-3.38459	-1.54269	-8.50299	3.68888
1789-	-3.44087	-1.52007	-8.22239	3.40120
1790-	-3.49027	-1.58752	-8.30057	3.80666
1791-	-3.42599	-1.64728	-8.51910	3.68888
1792-	-3.31424	-1.59835	-8.65782	3.40120
1793-	-3.37486	-1.63064	-8.62647	3.68888
1794-	-3.39245	-1.71470	-8.62158	3.68888
1795-	-3.44270	-1.61645	-8.41624	3.68888
1796-	-3.36593	-1.61102	-8.59433	3.40120
1797-	-3.36384	-1.62598	-8.64932	3.40120
1798-	-3.39543	-1.70323	-8.65648	1.60944
1799-	-3.44401	-1.67452	-8.53482	1.60944
1800-	-3.54808	-1.47318	-8.28109	1.60944
1801-	-3.50683	-1.57133	-8.49102	3.40120
1802-	-3.45459	-1.67876	-8.59820	3.40120
1803-	-3.46574	-1.69282	-8.59491	3.68888
1804-	-3.44827	-1.67575	-8.54191	3.40120
1805-	-3.45439	-1.73954	-8.59893	2.89037
1806-	-3.48324	-1.48256	-8.46481	3.40120
1807-	-3.47094	-1.66599	-8.46961	3.13549
1808-	-3.49109	-1.52540	-8.14120	2.89037
1809-	-3.61720	-1.51943	-7.95670	3.68888

Table A-2 continued

Basic Data Series in Natural Logaritims.

CALENDAR YEAR	CBR	MIDR	NIDR	GINDX
1810-	-3.41332	-1.56939	-8.25246	3.69999
1811-	-3.34693	-1.61221	-8.39088	2.56495
1812-	-3.39559	-1.61031	-8.32624	2.07944
1813-	-3.51620	-1.66648	-8.41738	2.99573
1814-	-3.47045	-1.61390	-8.54043	3.55535
1815-	-3.36451	-1.73575	-8.61062	3.68889
1816-	-3.34976	-1.68461	-8.70001	2.30259
1817-	-3.40390	-1.73978	-8.57794	2.70805
1818-	-3.39142	-1.77245	-8.55713	2.56495
1819-	-3.41467	-1.70485	-8.42276	3.55535
1820-	-3.41658	-1.80897	-8.53605	3.80666
1821-	-3.34487	-1.70737	-8.51982	3.55535
1822-	-3.33435	-1.80962	-8.66400	3.13549
1823-	-3.30933	-1.90052	-8.73743	3.68888
1824-	-3.37219	-1.87537	-8.75129	3.68888
1825-	-3.31874	-1.84842	-8.78137	3.40120
1826-	-3.36313	-1.77353	-8.67256	2.30259
1827-	-3.46832	-1.86179	-8.59356	3.68969
1828-	-3.39635	-1.74477	-8.43707	3.68888
1829-	-3.35963	-1.62333	-8.37966	3.40120
1830-	-3.41842	-1.73243	-8.58566	3.13549
1831-	-3.49262	-1.64665	-8.49051	2.99573
1832-	-3.48202	-1.78978	-8.58093	3.68368
1833-	-3.38440	-1.80157	-8.69295	3.40120
1834-	-3.39323	-1.75201	-8.49574	2.99573
1835-	-3.42838	-1.95551	-8.85521	3.55535
1836-	-3.45272	-1.88461	-8.77060	3.40120
1837-	-3.48162	-1.64479	-8.55931	2.89037
1838-	-3.52994	-1.75535	-8.54455	3.55535
1839-	-3.52640	-1.80377	-8.55635	3.40120
1840-	-3.46502	-1.90357	-8.72791	3.55535
1841-	-3.50091	-1.84585	-8.80757	2.30259
1842-	-3.45814	-1.80411	-8.71753	3.13549
1843-	-3.48561	-1.84220	-8.68028	2.99573
1844-	-3.44308	-1.94292	-8.72969	3.21888
1845-	-3.46556	-1.90971	-8.83701	2.70805
1846-	-3.51241	-1.82613	-8.65672	3.21888
1847-	-3.52339	-1.75504	-8.56475	3.40120
1848-	-3.50068	-1.94837	-8.75291	3.55535
1849-	-3.42239	-1.92460	-8.76581	3.21888
1850-	-3.45142	-1.92835	-8.77010	3.40120
1851-	-3.45515	-1.88135	-8.71989	3.21888
1852-	-3.48705	-1.82520	-8.61116	3.55535
1853-	-3.46526	-1.81693	-8.56058	3.21888
1854-	-3.40256	-2.04559	-8.74251	3.40120
1855-	-3.45446	-1.94616	-8.66118	3.55535
1856-	-3.46312	-1.93273	-8.63960	3.21888
1857-	-3.43068	-1.78738	-8.38136	3.40120
1858-	-3.36518	-1.92180	-8.66685	3.55535
1859-	-3.35972	-1.93758	-8.76832	3.40120
1860-	-3.36679	-2.08637	-8.89619	3.40120
1861-	-3.43187	-2.00273	-8.84825	3.13549
1862-	-3.40584	-1.95878	-8.66691	3.55535
1863-	-3.39976	-2.01374	-8.78625	3.40120
1864-	-3.39873	-1.98560	-8.73405	3.55535

Table A-2 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	CBR	MIDR	NIDR	GINDX
1865-	-3.42225	-2.00615	-8.78202	3.29584
1866-	-3.41353	-2.05775	-8.72642	3.43399
1867-	-3.48330	-1.98588	-8.75661	3.25810
1968-	-3.59187	-1.82086	-8.68746	3.21888
1869-	-3.56503	-1.91805	-8.58370	3.58352
1870-	-3.54918	-2.01959	-8.71201	3.61092
1871-	-3.49703	-2.15956	-8.86523	3.58352
1872-	-3.51072	-2.05459	-8.96851	3.46574
1873-	-3.48580	-2.03796	-8.90443	3.52636
1874-	-3.48377	-1.91546	-8.72734	3.29584
1875-	-3.47321	-1.89754	-8.73695	3.55535
1876-	-3.48424	-1.96484	-8.76121	3.40120
1877-	-3.47769	-2.06914	-8.79567	3.29584
1878-	-3.51759	-2.01839	-8.84485	3.63759
1879-	-3.49443	-2.18800	-8.88146	3.49651
1980-	-3.52653	-2.12371	-8.90363	3.52636
1881-	-3.53889	-2.18537	-8.81642	3.21888
1882-	-3.52919	-2.07990	-8.96673	3.61092
1883-	-3.54518	-2.15960	-8.84978	3.40120
1884-	-3.51079	-2.16315	-8.83814	3.61092
1885-	-3.52942	-2.17143	-8.82109	3.36730
1886-	-3.51817	-2.19181	-8.89903	3.43399
1887-	-3.51987	-2.27157	-8.91432	3.46574
1888-	-3.54957	-2.30616	-8.91212	3.43399
1889-	-3.58770	-2.24102	-8.92162	3.29584
1890-	-3.57841	-2.26940	-8.82980	3.63759
1891-	-3.56785	-2.22274	-8.96419	3.46574
1892-	-3.61193	-2.22600	-8.78129	3.66356
1893-	-3.60064	-2.28657	-8.94386	3.40120
1894-	-3.61319	-2.29395	-8.87694	3.49651
1895-	-3.59861	-2.35122	-8.95731	3.43399
1896-	-3.60954	-2.27059	-8.93820	3.52636
1897-	-3.62878	-2.31950	-8.94714	3.49651
1898-	-3.61321	-2.39297	-8.95560	3.61092
1899-	-3.63953	-2.19723	-8.80117	3.29584
1900-	-3.61586	-2.31038	-8.83782	3.61092
1901-	-3.61451	-2.27131	-8.90379	3.40120
1902-	-3.63354	-2.45294	-8.91712	3.40120
1903-	-3.66344	-2.38332	-8.94733	3.52636
1904-	-3.66312	-2.47075	-8.91648	3.36730
1905-	-3.66620	-2.42614	-8.89835	3.49651
1906-	-3.66523	-2.51066	-8.98275	3.63759
1907-	-3.67155	-2.56602	-8.95521	3.52636
1908-	-3.66605	-2.45631	-8.94907	3.71357
1909-	-3.67011	-2.62763	-9.02141	3.55535
1910-	-3.70667	-2.59542	-8.99188	3.61092
1911-	-3.73350	-2.63479	-9.00167	3.43399
1912-	-3.74191	-2.64722	-8.96644	3.52636
1913-	-3.76832	-2.66753	-9.00488	3.55535
1914-	-3.78128	-2.62008	-8.99444	3.29584
1915-	-3.83829	-2.59294	-8.92621	3.40120
1916-	-3.85688	-2.66549	-9.00381	3.52636
1917-	-3.87117	-2.74221	-9.00728	3.21888
1918-	-3.89770	-2.74430	-8.67827	3.21888
1919-	-3.92708	-2.66933	-8.92471	3.49651

Table A-2 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR		CBR	MIDR	NIDR	GINDX
1920-	1	-3.75077	-2.72556	-9.02574	3.52636
1921-	1	-3.94201	-2.76566	-9.09546	3.43399
1922-	1	-3.93572	-2.79100	-9.04826	3.43399
1923-	1	-3.96924	-2.88455	-9.15870	3.43399
1924-	1	-4.01366	-2.81673	-9.11152	3.36730
1925-	1	-4.04221	-2.89355	-9.12145	3.52636
1926-	1	-4.08679	-2.89154	-9.11594	3.49651
1927-	1	-4.12916	-2.82560	-9.03523	3.43399
1928-	1	-4.13328	-2.83381	-9.09360	3.43399
1929-	1	-4.18823	-2.84879	-9.07363	3.46574
1930-	1	-4.17730	-2.90239	-9.11345	3.46574
1931-	1	-4.21456	-2.87791	-9.04037	3.36730
1932-	1	-4.23340	-2.98401	-9.11805	3.52636
1933-	1	-4.29128	-3.01656	-9.14508	3.43399
1934-	1	-4.29389	-3.05377	-9.14081	3.43399
1935-	1	-4.28716	-3.07967	-9.09951	3.46574
1936-	1	-4.25510	-3.13121	-9.07153	3.43399
1937-	1	-4.24193	-3.09418	-9.06953	3.49651
1938-	1	-4.20720	-3.15079	-9.11033	3.52636
1939-	1	-4.17622	-3.22477	-9.10998	3.43399
1940-	1	-4.19755	-3.24148	-9.11564	3.13549
1941-	1	-4.16263	-3.29021	-9.13173	2.99573
1942-	1	-4.03725	-3.51187	-9.25942	3.33220
1943-	1	-3.95162	-3.52972	-9.23404	3.36730
1944-	1	-3.88921	-3.45699	-9.15810	3.40120
1945-	1	-3.89790	-3.50791	-9.17336	3.43399
1946-	1	-3.93201	-3.63313	-9.19823	3.46574
1947-	1	-3.97274	-3.67836	-9.16408	3.17805
1948-	1	-4.00119	-3.76712	-9.25894	3.43399
1949-	1	-4.05365	-3.76691	-9.23893	3.49651
1950-	1	-4.11110	-3.87193	-9.23060	3.46574
1951-	1	-4.16567	-3.84286	-9.24624	3.33220
1952-	1	-4.17273	-3.91056	-9.27398	3.40120
1953-	1	-4.17898	-3.97716	-9.25854	3.46574
1954-	1	-4.23177	-3.98127	-9.27153	3.40120
1955-	1	-4.21860	-4.04817	-9.28347	3.13549

Table A-3

Basic Data Series in Natural Logarithms

CALENDAR YEAR	RAIN	RWAGE	ATEMP
1756-	5.97064	-2.63906	6.03357
1757-	6.04664	-2.78037	6.04643
1758-	5.69047	-2.51231	5.99396
1759-	5.83120	-2.22162	6.06332
1760-	6.12898	-2.26868	6.01616
1761-	6.26471	-2.40695	6.06750
1762-	6.29479	-2.77778	6.04216
1763-	6.15436	-2.56495	6.00734
1764-	6.00715	-2.50041	6.06750
1765-	6.09170	-2.42591	6.04643
1766-	6.06016	-2.21102	6.07581
1767-	6.26370	-2.09597	6.01616
1768-	6.01241	-2.32728	6.02054
1769-	6.18983	-2.27727	6.02490
1770-	6.26353	-2.33289	6.03787
1771-	6.21794	-2.76777	5.99944
1772-	6.27384	-2.76777	6.01176
1773-	6.17174	-2.56495	6.10836
1774-	6.08016	-2.25533	6.02054
1775-	6.23736	-2.39790	6.12425
1776-	5.96546	-2.29590	6.06332
1777-	5.91449	-2.28238	6.02490
1778-	6.30244	-2.30882	6.04216
1779-	6.25876	-2.24469	6.12818
1780-	6.04658	-2.25785	6.03787
1781-	5.99489	-2.51231	6.07581
1782-	6.27683	-2.33336	6.00290
1783-	5.90272	-2.49010	6.09221
1784-	6.05627	-2.32116	5.98494
1785-	6.21749	-2.53072	5.98946
1786-	6.34553	-2.63906	5.98494
1787-	6.25831	-2.45101	6.04216
1788-	6.06940	-2.56495	5.99396
1789-	6.30946	-2.62949	6.09628
1790-	6.21461	-2.58525	6.06750
1791-	6.15486	-2.49010	6.12030
1792-	6.39359	-2.54207	6.05068
1793-	6.29767	-2.58289	6.07166
1794-	6.29095	-2.61624	6.12425
1795-	6.33356	-2.55334	6.00734
1796-	6.19577	-2.37601	6.07166
1797-	6.32496	-2.32834	6.10032
1798-	6.06533	-2.48491	6.10435
1799-	6.33387	-2.63631	5.98040
1800-	6.42162	-2.86958	6.02054
1801-	6.28164	-2.80768	6.05912
1802-	6.22983	-2.80896	6.04643
1803-	6.11810	-2.79484	6.00734
1804-	6.13556	-2.73244	6.01176
1805-	6.00471	-2.69607	5.97584
1806-	6.30492	-2.88480	6.03787
1807-	6.28972	-2.87356	6.04216

Table A-3 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	RAIN	RWAGE	ATEMP
1808-	6.33091	-2.95491	6.03357
1809-	6.24352	-2.64636	6.00734
1810-	6.04737	-2.42037	6.02490
1811-	6.10951	-2.57671	6.09221
1812-	5.82993	-2.63223	5.97584
1813-	6.08526	-2.50611	6.05491
1814-	5.86930	-2.44107	5.98946
1815-	6.15768	-2.40057	6.05491
1816-	6.06456	-2.51113	6.01176
1817-	5.97720	-2.59135	6.04643
1818-	5.91350	-2.65854	6.08814
1819-	5.97381	-2.61675	6.11235
1820-	6.16612	-2.46783	6.03797
1821-	6.12395	-2.24420	6.07166
1822-	6.03229	-2.23845	6.15146
1823-	6.15060	-2.17881	6.07581
1824-	6.20926	-2.14696	6.11633
1825-	6.13988	-2.11860	6.07166
1826-	5.70932	-2.56953	6.11235
1827-	6.02667	-2.30408	6.05912
1828-	6.12687	-2.12503	6.03357
1829-	5.99894	-2.32440	5.94804
1830-	6.11956	-2.40838	6.01616
1831-	6.11325	-2.56174	6.03357
1832-	6.05653	-2.29976	6.04216
1833-	6.25801	-2.21890	6.05068
1834-	6.21212	-2.27155	6.09221
1835-	6.05492	-2.24543	6.04216
1836-	6.18002	-2.25567	6.01616
1837-	6.10777	-2.41169	6.00734
1838-	6.18002	-2.48259	5.96666
1839-	6.07458	-2.33493	6.02924
1840-	6.33387	-2.36627	6.02924
1841-	6.41999	-2.59300	6.04216
1842-	5.93754	-2.49596	6.07581
1843-	6.04501	-2.36211	6.05068
1844-	6.33505	-2.14409	5.97594
1845-	6.17725	-2.56602	6.01616
1846-	6.10777	-2.54662	6.07993
1847-	6.01616	-2.46097	6.03787
1848-	6.36015	-2.20313	6.04643
1849-	6.08146	-2.19722	6.02054
1850-	6.22654	-2.36393	6.03357
1851-	6.34095	-2.48391	6.05912
1852-	6.37218	-2.44065	6.06332
1853-	5.97720	-2.58710	6.04216
1854-	5.95670	-2.39898	6.07993
1855-	6.15415	-2.48898	6.01176
1956-	6.14132	-2.34260	6.00290
1857-	6.09432	-2.14702	6.08814
1858-	5.90536	-2.04272	6.10032
1859-	6.16612	-2.03175	6.07993
1860-	6.34329	-2.11727	6.01176
1861-	6.28351	-2.29985	6.03787

Table A-3 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	RAIN	RWAGE	ATEMP
1862-	6.16171	-2.20332	5.99396
1863-	6.17794	-2.07834	6.08814
1864-	6.09507	-1.91249	5.99396
1865-	6.00881	-2.00585	6.03357
1866-	6.51669	-2.14908	6.02924
1867-	6.29095	-2.61496	5.93384
1868-	6.24933	-2.57782	6.07581
1869-	6.28724	-2.21954	6.04216
1870-	6.09980	-2.18811	6.01616
1871-	6.09206	-2.18338	5.95739
1872-	6.44889	-2.13212	6.09628
1873-	6.41728	-1.97966	6.07581
1874-	6.08829	-1.85107	6.05912
1875-	5.98141	-1.84194	5.98494
1876-	6.11883	-1.81080	6.01176
1877-	6.41127	-1.80207	6.00290
1878-	6.30992	-1.82577	6.06750
1879-	6.21261	-1.89706	6.00734
1880-	6.11294	-1.99352	6.04643
1881-	6.19780	-2.01193	5.97584
1882-	6.35495	-1.97866	6.08814
1883-	6.41619	-1.94298	6.06750
1884-	6.27225	-1.86629	6.05491
1885-	6.35611	-1.83178	6.02490
1886-	6.07688	-1.83318	6.06332
1887-	6.11736	-1.81435	6.06332
1888-	6.27915	-1.87637	5.97584
1889-	6.25255	-1.89861	6.05491
1890-	6.49929	-1.88009	6.07581
1891-	6.30749	-1.81494	6.06750
1892-	6.11368	-1.78253	6.02054
1893-	6.26276	-1.72973	6.01616
1894-	6.37899	-1.69555	6.10032
1895-	6.41127	-1.70168	6.04643
1896-	6.40908	-1.70415	6.09221
1897-	6.39248	-1.72018	6.07581
1898-	6.55725	-1.71950	6.07993
1899-	6.29588	-1.68472	6.04643
1900-	6.35495	-1.65801	6.04643
1901-	6.08146	-1.60999	6.07993
1902-	6.27351	-1.62599	5.99040
1903-	6.48870	-1.65227	6.07581
1904-	6.26973	-1.64417	6.03357
1905-	6.32197	-1.61749	6.06750
1906-	6.23245	-1.63999	6.08404
1907-	6.27915	-1.62012	6.03787
1908-	6.33977	-1.57328	6.04643
1909-	6.33150	-1.54342	6.02054
1910-	6.55298	-1.51433	6.08814
1911-	6.29219	-1.48761	6.09628
1912-	6.50578	-1.52725	6.04643
1913-	6.26593	-1.51242	6.08814
1914-	6.14275	-1.45809	6.10836

Table A-3 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	RAIN	RWAGE	ATEMP
1915-	6.36015	-1.48955	5.99396
1916-	6.49123	-1.43175	6.04643
1917-	6.23376	-1.41431	6.02490
1918-	6.32675	-1.45892	6.06332
1919-	6.38294	-1.44621	6.03357
1920-	6.25383	-1.35928	6.09221
1921-	6.31897	-1.42860	6.09221
1922-	6.37616	-1.45006	6.02054
1923-	6.49426	-1.49628	6.01616
1924-	6.36245	-1.51574	6.05068
1925-	6.40302	-1.51264	6.07166
1926-	6.28413	-1.50385	6.05068
1927-	6.51175	-1.49606	6.05068
1928-	6.41072	-1.49857	6.03797
1929-	6.38576	-1.47799	6.04643
1930-	6.52062	-1.45683	6.12030
1931-	6.33742	-1.43305	6.02490
1932-	6.34972	-1.43078	6.08814
1933-	6.11810	-1.44493	6.07581
1934-	6.48870	-1.46453	6.14761
1935-	6.51619	-1.46279	6.10032
1936-	6.41945	-1.41213	6.10435
1937-	6.42325	-1.34431	6.11235
1938-	6.35205	-1.30583	6.14376
1939-	6.26593	-1.26493	6.10336
1940-	6.20725	-1.28950	6.01176
1941-	6.23048	-1.29284	6.00734
1942-	6.24869	-1.25514	5.99396
1943-	6.42325	-1.21665	6.13210
1944-	6.44148	-1.17144	6.10435
1945-	6.50976	-1.04573	6.10032
1946-	6.32197	-.935458	6.08814
1947-	5.97211	-.884722	6.06750
1948-	6.28040	-.814397	6.10435
1949-	6.31897	-.809453	6.13988
1950-	6.44572	-.770854	6.10032
1951-	6.27727	-.865677	6.09628
1952-	6.42162	-.677762	6.04643
1953-	6.08829	-.691493	6.12818
1954-	6.57042	-.669470	6.07166
1955-	6.18552	-.600499	6.06332

Table A-4

Basic Data Series in Natural Logarithms

CALENDAR YEAR	WINTER	SPRING	SUMMER	AUTUMN
1756-	5.69238	6.12949	6.36165	5.80694
1757-	5.53022	6.22297	6.39826	5.81771
1758-	5.45104	6.15910	6.34704	5.79423
1759-	5.76456	6.17670	6.38317	5.78506
1760-	5.40087	6.17919	6.39626	5.82305
1761-	5.67812	6.23245	6.39125	5.80151
1762-	5.61895	6.25229	6.32686	5.80513
1763-	5.46383	6.15910	6.36268	5.81949
1764-	5.74172	6.18415	6.39225	5.81054
1765-	5.61677	6.17295	6.35123	5.89164
1766-	5.56299	6.24882	6.40225	5.89164
1767-	5.33078	6.13600	6.40519	5.90645
1768-	5.46637	6.15018	6.34914	5.90808
1769-	5.46988	6.18043	6.35019	5.74172
1770-	5.41965	6.16542	6.40721	5.90808
1771-	5.21058	6.19032	6.35333	5.90971
1772-	5.19073	6.11766	6.37707	6.02490
1773-	5.65739	6.23009	6.43165	5.95739
1774-	5.38450	6.24558	6.40324	5.73205
1775-	5.65529	6.23127	6.47235	5.94804
1776-	5.52545	6.17919	6.41313	5.93859
1777-	5.44328	6.18539	6.35333	5.89990
1778-	5.54674	6.20617	6.38823	5.80874
1779-	5.83364	6.22059	6.43165	5.90318
1780-	5.60138	6.11235	6.40820	5.85450
1781-	5.57671	6.19644	6.45646	5.86306
1782-	5.51826	6.12687	6.36268	5.81234
1783-	5.51626	6.25920	6.45740	5.88832
1784-	5.28015	6.15528	6.35750	5.85278
1785-	5.38174	6.14633	6.35957	5.82305
1786-	5.36223	6.15783	6.37298	5.76268
1787-	5.69843	6.17420	6.33541	5.83188
1788-	5.35659	6.20132	6.41706	5.66781
1789-	5.25227	6.22772	6.47605	6.04784
1790-	5.83540	6.13730	6.33859	5.88165
1791-	5.35278	6.20496	6.40721	5.90971
1792-	5.48646	6.19889	6.39926	5.89660
1793-	5.69238	6.19154	6.39426	5.86817
1794-	5.79057	6.26834	6.38519	5.94804
1795-	5.27402	6.17919	6.38722	5.85965
1796-	5.73786	6.20254	6.38823	5.81771
1797-	5.78321	6.18167	6.41804	5.88999
1798-	5.60138	6.27401	6.42390	5.91620
1799-	5.31616	6.12425	6.34071	5.87155
1800-	5.32203	6.17420	6.35333	5.96204
1801-	5.59248	6.23715	6.36165	5.86817
1802-	5.61677	6.17295	6.35541	5.89165
1803-	5.36504	6.23598	6.38923	5.73010
1804-	5.36784	6.18043	6.42681	5.74939
1805-	5.39544	6.09492	6.39726	5.74748
1806-	5.54908	6.11368	6.39526	5.91134

Table A-4 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	WINTER	SPRING	SUMMER	AUTUMN
1807-	5.59024	6.13210	6.41215	5.85622
1808-	5.48646	6.16919	6.41510	5.83540
1809-	5.15329	6.15910	6.41902	5.90482
1810-	5.54440	6.10836	6.41313	5.92305
1811-	5.65948	6.24549	6.41017	5.87324
1812-	5.52066	6.07581	6.33008	5.80694
1813-	5.68222	6.14761	6.38823	5.85105
1814-	5.11439	6.14761	6.39526	5.90154
1815-	5.64897	6.17295	6.36578	5.89495
1816-	5.45618	6.13730	6.38012	5.85105
1817-	5.72620	6.18291	6.38114	5.73399
1818-	5.66781	6.14247	6.40324	5.99695
1819-	5.76268	6.22535	6.48066	5.79240
1820-	5.48147	6.21341	6.38114	5.83890
1821-	5.55606	6.17170	6.35957	6.02780
1822-	5.97155	6.26035	6.38722	6.00881
1823-	5.55373	6.18539	6.39125	5.99993
1824-	5.79788	6.21581	6.42487	5.99660
1825-	5.69642	6.16919	6.36165	5.93859
1826-	5.67607	6.22178	6.44604	5.93859
1827-	5.46129	6.25229	6.36578	5.92746
1828-	5.51343	6.18539	6.38215	5.84932
1829-	5.29531	6.13600	6.35541	5.68630
1830-	5.49388	6.13859	6.35750	5.87830
1831-	5.39544	6.18291	6.38215	5.92746
1832-	5.71637	6.12949	6.29748	5.92906
1833-	5.62546	6.18167	6.33008	5.92906
1834-	5.71439	6.18662	6.43358	5.87324
1835-	5.71934	6.15273	6.35228	5.81054
1836-	5.66157	6.14633	6.30189	5.83540
1837-	5.54440	6.10032	6.33221	5.89495
1838-	5.17048	6.12162	6.36784	5.84238
1839-	5.46890	6.16668	6.38823	5.86817
1840-	5.59248	6.17295	6.36061	5.81234
1841-	5.49141	6.21940	6.34388	5.92104
1842-	5.73593	6.19644	6.36990	5.88332
1843-	5.64686	6.09096	6.40622	5.89660
1844-	5.28015	6.17670	6.33115	5.81234
1845-	5.42759	6.15273	6.37911	5.87493
1846-	5.63622	6.15146	6.44794	5.90318
1847-	5.53497	6.10969	6.38215	5.94332
1848-	5.57215	6.22059	6.35541	5.84759
1849-	5.56987	6.14890	6.35854	5.82876
1850-	5.45618	6.21100	6.38823	5.84238
1851-	5.61677	6.16416	6.36268	5.95272
1852-	5.61459	6.18539	6.43455	5.80874
1853-	5.43808	6.18043	6.39726	5.90482
1854-	5.62762	6.22416	6.42875	5.84759
1855-	5.29531	6.16919	6.41116	5.84238
1856-	5.51101	6.15783	6.33328	5.81771
1857-	5.61240	6.14890	6.42390	6.00881
1858-	5.67607	6.21461	6.46956	5.85450
1859-	5.78321	6.18291	6.38722	5.85105

Table A-4 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	WINTER	SPRING	SUMMER	AUTUMN
1860-	5.54205	6.15273	6.36061	5.79240
1861-	5.56068	6.13210	6.36372	5.91943
1862-	5.34233	6.16416	6.31391	5.91458
1863-	5.79606	6.16668	6.34071	5.96204
1864-	5.61459	6.10836	6.31500	5.79788
1865-	5.41699	6.16036	6.38012	5.93225
1866-	5.61895	6.16542	6.35541	5.81592
1867-	5.40087	6.02199	6.32579	5.76079
1868-	5.62978	6.19767	6.41608	5.87493
1869-	5.67812	6.15528	6.34909	5.84759
1870-	5.55141	6.18785	6.35750	5.77206
1871-	5.42495	6.05491	6.34071	5.78506
1872-	5.72424	6.20617	6.38114	5.95272
1873-	5.75130	6.13210	6.38923	5.91943
1874-	5.76079	6.14118	6.35750	5.86136
1875-	5.36784	6.17045	6.37502	5.73399
1876-	5.57443	6.17295	6.37707	5.70844
1877-	5.45618	6.09357	6.31716	5.95894
1878-	5.69036	6.18785	6.36061	5.89990
1879-	5.44846	6.14761	6.39621	5.79970
1880-	5.69238	6.18043	6.41902	5.70245
1881-	5.26786	6.11235	6.33541	5.90482
1882-	5.79970	6.19032	6.40622	5.81771
1883-	5.60580	6.18043	6.36784	5.92746
1884-	5.73593	6.13210	6.38519	5.85278
1885-	5.65319	6.14376	6.32472	5.84064
1886-	5.58124	6.19767	6.38215	5.92265
1887-	5.75321	6.17919	6.37400	5.81949
1888-	5.33078	5.10301	6.34071	5.86647
1889-	5.51926	6.24417	6.33753	5.93225
1890-	5.77393	6.21100	6.35957	5.84759
1891-	5.64049	6.16036	6.38012	5.94490
1892-	5.52545	6.16163	6.35957	5.85105
1893-	5.35941	6.17170	6.35541	5.91620
1894-	5.91054	6.21581	6.35437	5.93701
1895-	5.43022	6.24998	6.36784	5.89330
1896-	5.73786	6.22178	6.39225	5.89164
1897-	5.56987	6.22297	6.40125	5.92586
1898-	5.74748	6.16668	6.35228	5.94804
1899-	5.59248	6.13340	6.38823	5.91458
1900-	5.50614	6.16416	6.38215	5.92586
1901-	5.53969	6.22654	6.45079	5.98040
1902-	5.61020	6.07856	6.29305	5.80694
1903-	5.79970	6.18043	6.36165	5.85793
1904-	5.63835	6.14376	6.34809	5.84932
1905-	5.69642	6.20254	6.35645	5.87324
1906-	5.70445	6.22535	6.36061	5.91458
1907-	5.64897	6.13079	6.31282	5.94017
1908-	5.66781	6.14890	6.35333	5.88999
1909-	5.60359	6.09357	6.34388	5.89660
1910-	5.77206	6.21461	6.34071	5.92426

Table A-4 continued

Basic Data Series in Natural Logarithms

CALENDAR YEAR	WINTER	SPRING	SUMMER	AUTUMN
1911-	5.72228	6.19522	6.39024	5.94647
1912-	5.61459	6.15401	6.36269	5.90809
1913-	5.74939	6.19767	6.36165	5.92384
1914-	5.73010	6.22416	6.41411	5.93542
1915-	5.55837	6.15146	6.34177	5.72228
1916-	5.64897	6.15783	6.32472	5.92746
1917-	5.63193	6.19032	6.38722	5.89825
1918-	5.62978	6.17170	6.36061	5.94804
1919-	5.62330	6.18291	6.36990	5.77206
1920-	5.78690	6.20738	6.36372	5.91458
1921-	5.78321	6.25575	6.34177	5.89660
1922-	5.54908	6.17295	6.33328	5.85105
1923-	5.65319	6.09492	6.34493	5.84412
1924-	5.47395	6.13730	6.39024	5.99695
1925-	5.76079	6.21820	6.39626	5.74172
1926-	5.60801	6.16919	6.38823	5.85793
1927-	5.74748	6.11501	6.41215	5.76456
1928-	5.64474	6.12687	6.32579	5.92265
1929-	5.51343	6.13079	6.35019	6.01323
1930-	5.79970	6.24417	6.37502	5.96666
1931-	5.55606	6.14376	6.32686	5.90809
1932-	5.70844	6.16163	6.40026	5.94804
1933-	5.69642	6.18043	6.39826	5.87997
1934-	5.80151	6.24649	6.41608	6.01616
1935-	5.72031	6.20132	6.37400	5.98343
1936-	5.67401	6.23598	6.39626	5.96204
1937-	5.65319	6.26606	6.43262	5.91134
1938-	5.84238	6.20496	6.43165	5.99546
1939-	5.79423	6.21940	6.42681	5.85793
1940-	5.23325	6.21581	6.36578	5.88959
1941-	5.33078	6.15401	6.40424	5.83364
1942-	5.03955	6.16299	6.38621	5.94175
1943-	5.80151	6.26378	6.37605	5.97889
1944-	5.71637	6.15528	6.43165	5.98494
1945-	5.75321	6.20375	6.40721	5.90154
1946-	5.62546	6.22654	6.40721	5.91458
1947-	5.35091	6.27627	6.44794	5.87155
1948-	5.66988	6.24533	6.39626	5.94960
1949-	5.80513	6.23363	6.41706	6.00439
1950-	5.69441	6.25229	6.38722	5.92906
1951-	5.63193	6.18415	6.40424	6.01908
1952-	5.65739	6.20617	6.34071	5.84064
1953-	5.71241	6.26606	6.38418	6.02635
1954-	5.59248	6.20738	6.36784	5.95428
1955-	5.56299	6.12162	6.46209	5.88999

Appendix B: The Econometric Framework

The econometric model is a system of equations involving a number of endogenous variables (variables determined by the model), exogenous variables (variables that affect the system but are not affected by it) and random shocks (variables that are unobserved and uncorrelated with either the exogenous or the endogenous variables). The idea is to use historical aggregate data to estimate the model/ The linear econometric model should be viewed as an unrestricted linear specification of a (or several) structural model(s).

Let Y_t be an n x 1 vector of endogenous variables such as the birth rate, the mortality rate etc., and let X_t be an m x 1 vector of exogenous variables such as precipitation and temperature. If we subtract from Y_t and X_t the deterministic parts, such as the level (constant) and the trend, then we may define the vector $Z_t = [y_t, x_t]'$, where y_t and x_t are the non-deterministic parts of Y_t and X_t, respectively. We can regard the vector of time observations, Z_t, as a time series stochastic process, that is $[Z_t]_{t=0}^{\infty}$ is a set of random vectors indexed by time together with a joint distribution functions for the Z's. In particular, past observations may be correlated with current observations of Z_t. Z_t is a ℓ x 1 ($\ell = n + m$) vector. In general, the econometric model can be written as

$$\text{(B.1)} \quad y_t = A_{11}y_{t-1} + A_{12}y_{t-Z} + \dots + A_{1g}y_{t-g} + A_{20}x_t + A_{21}x_{t-1} + \dots$$

$$+ A_{2g}x_{t-g} + \varepsilon_t$$

$$\text{(B.2)} \quad x_t = A_{41}x_{t-1} + A_{42}x_{t-2} + \dots + A_{4g}x_{t-g} + v_t$$

Here ε_t and v_t are (n x 1) and (m x 1) vectors, respectively, of random disturbances. The matrix A_{1j} is (ℓ x ℓ); A_{2j} is (ℓxm); A_{4j} is (m x m). The disturbance processes of ε_t and v_t are assumed to be serially and contemporaneously uncorrelated with $E(\varepsilon_t) = E(v_t) = 0$, $E(v_t \varepsilon_t') = E(v_t v_s') = E(\varepsilon_t \varepsilon_s') = 0$ for all $t \neq s$, $E(v_t v_t') = \Sigma_v$ and $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$. The definition of the exogenous variables x_t is that they are uncorrelated with the ε's at all lags, that is $E(\varepsilon_t x_g') = 0$ for all t and s. The above specification completely describes the first and the second moments of the $Z_t = [y_t, x_t]'$ process. The equivalence of lags, g, across all variables and equations is assumed for convenience. By the above assumptions x_t is a strictly exogenous vector of variables.[1]

The model (B.1) and (B.2) can be written as a vector autoregression (VAR) for Z_t

(B.3) $\quad A(L)Z_t = U_t \qquad\qquad U_t = (\varepsilon_{t+1} v_t)'$

Where $A(L) = A._0 - A._1 L - A._2 L^2 - \ldots - A._g L^g$ and $A._j$ is an (ℓ x ℓ) matrix, j=0,..g. Given the above assumptions on the error term (U_t) and the equal lag structure across the model, ordinary least squares (OLS) for each equation turns out to be identical with joint conditional maximum likelihood even for unrestricted variance covariance matrices Σ_u and Σ_ε.[2] Furthermore, given the strict exogeneity assumption with respect to the x's, we can set the lag structure and estimate (B.1) independently of (B.2). The lag length of the VAR (or B.1))is initially unspecified, and may be determined using an asymptotic χ^2 test for alternative lag lengths fitted to the model. An increase of lag increases the number of parameters by $(n+m)^2$

[1] Strict exogeneity is defined in Sims (1972) and implies that the vector of all observations on $x_t (x_1, x_2, \ldots x_T)$ is orthogonal to the error in the regression equation for x_t.

[2] This is a special case of Zellner's seemingly unrelated regression method.

(or (n + m)xn). Therefore, we must restrict the number of lags subject to the number of observations and variables, in order to apply statistical tests.

Observe that (B.1) and (B.2) imply that the typical $A._j$ matrix is (B.3) can be written as

$$\text{(B.4)} \quad A._j = \begin{bmatrix} \underset{(n \times n)}{A_{1j}} & \underset{(n \times m)}{A_{2j}} \\ \underset{(m \times n)}{A_{3j}} & \underset{(m \times m)}{A_{4j}} \end{bmatrix} \quad \text{for } j = 0, \ldots, g$$

and we assume that $A_{2j} = 0 \quad \forall_j \geq 0$, $A_{10} = I_m$ and $A_{40} = I_n$. The assumption that $A_{3j} = 0$ is equivalent to assuming that y_t does not cause x_t in the sense define by Granger (1969), which is a necessary condition for x_t to be exogenous (see Sims, 1972). F-tests of this assumption can be applied to the set of equations (B.2). Given that we assume that $E(v_t \varepsilon_t') = 0$, it follows from a theorem in Sims (1972) that x_t is strictly exogenous in (B.1). Sims' exogeneity test can be applied by inserting lead variables of x_t in (B.1) and statistically evaluating whether the coefficients are zero. The above tests evaluate the specification of the econometric model. The x's in our model are weather variables that are undoubtedly exogenous. Hence, the exogeneity tests should be viewed as indicating omitted variables. If an important omitted variable is correlated both with the x's and the y's, the exogeneity test could fail, because the assumption that $E(v_t \varepsilon_t') = 0$ is violated.

Once the A's in (B.3) are estimated, we can express Z_t as a linear combination of current and past innovations (U's), in other words, as a distributed lag on U_t. Then we can write the Wold moving average representation

(MAR) for Z_t as

(B.5) $$Z_t = \sum_{s=0}^{\infty} B_s U_{t-s}$$

where B_s is an $(\ell \times \ell)$ matrix of parameters and we use a partition of the B's that is equivalent to that of the A's. Observe that the B's are written as independent of t, which is the result of the A's in (B.3) being independent of t.

A useful way to describe the economic system during the sample period is by looking at the system's response to random shocks.[3] Except for scaling, this is equivalent to tracing out the system's MAR by matrix polynomial long division. In order to see that, we can rewrite the MAR as

$$Z_t = [A(L)]^{-1} U_t \quad .$$

Therefore,

$$\sum_{s=0}^{\infty} B_s L^s = [A(L)]^{-1} \quad ,$$

i.e., finding the B_s coefficients is equivalent to inverting the matrix polynomial. Suppose we simulate the VAR of Z by setting for a particular equation j, $U_{jt} = 1$ and $U_{it+s} = 0$ for all $i \neq j$ and $s = 0, 1, 2, 3, \ldots$, together with the initial conditions $Z_{t-r} = 0$ for $r = 0, 1, 2, \ldots, g,$. This procedure generates infinite Z_{t+s} vectors for $s = 0, 1, 2, \ldots$, which are equal to the j'th column of the corresponding B_s matrix.[4] Hence, the inversion of the matrix polynomial A(L) is equivalent to the above simulation.

[3] This was suggested and implemented by Sims (1980).

[4] For example, take the VAR $Z_t = aZ_{t-1} + U_t$, where $|a| < 1$. Set $Z_{t-1} = 0$, $U_t = 1$ and $U_{t+r} = 0$ for all $r > 0$. Then $Z_{t+s} = a^s$, where it is easy to see that

$$Z_t = \sum_{s=0}^{\infty} a^s U_{t-s} \; .$$

One can regard the i, j'th component of B_s, $b_{ij}(s)$ as the "average" response, s periods ahead, of the i'th variable, to an initial shock in the j'th variable. However, the components of U may be contemporaneously correlated and the above simulation does not take this possibility into account. In describing and summarizing the data using the above simulation we ignore the effect of a shock in one variable on the current observation of other variables if Σ_v and Σ_ε are not diagonal. In what follows we explain one way to take into account the contemporaneous correlation between the U's.

Since it is not possible to partition the variance of Z into pieces accounted for by each innovation, it is appealing to apply an orthogonalization transformation for U, to obtain $e_t = TU_t$, where T is a matrix chosen to make the variance-covariance matrix of e_t the identity matrix. There are many ways one could choose T. Choosing T's of triangular form preserves the connection of the elements of e with the corresponding variables in Z in the sense that, if T is lower triangular,[5] e_{jt} is the normalized error in forecasting $Z_{i,t}$ for $i < j$. We can rewrite (B.5) as

(B.6) $$Z_t = \sum_{s=0}^{\infty} B_s e_{t-s}$$

[5] A lower triangular matrix has zero elements in the right hand side (above the diagonal elements) of the matrix.

Now the interpretation above for the components of the MAR can be applied to the components of the matrix function $B_s T^{-1}$, since the elements of ε are uncorrelated. In particular, the sum of squares from s = 0 to s = k of the i, j'th component of $B_s T^{-1}$ represents the part of error variance in the k + 1 (step-ahead) forecast of Z_i which is accounted for by the innovation in Z_j at s = 0.

Applying this type of orthogonalization is equivalent to restricting the system such that a "shock" in Z_1 has a contemporaneous effect on all n + m - 1 variables, Z_2 on all n + m - 2 variables,..., and Z_{n+m} only on itself. Hence, each triangularization imposes a particular block recursive system with respect to the contemporaneous relations among the variables. It is important to test this procedure by changing the ordering of the variables, to see whether there are important changes in the results. Note that in our model the following assumption

$$E[U_t U_t'] = \begin{pmatrix} \Sigma_v & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} = \Sigma$$

has been imposed and all the discussion is with respect to the correlation in Σ_v and Σ_ε. We report results that assume that both Σ_v and Σ_ε are diagonal. We review the results of other assumptions with respect to orthogonolizations of the covariance matrix. **When significant differences are observed, they are noted in the text.**

Once the A's in the VAR has been estimated, the matrix $B_s T^{-1}$ for s = 0, 1, a, ..., k, ... can be computed. Letting the i, j'th component, $\hat{b}_{ij}(s)$, of $\tilde{B}_s T^{-1}$, be the response of Z_i to an innovation or exogenous shock of one standard deviation in Z_j, then

$$\rho_{ij}^2(k) = \frac{\sum_{s=0}^{k} \tilde{b}_{ij}^2(s)}{\sum_{j=1}^{m+n} \sum_{s=0}^{k} \tilde{b}_{ij}^2(s)}$$

is the proportion of forecast error variance in Z_i, k periods ahead, produced by an innovation in Z_j. The vector of $\rho_{\cdot j}^2$ (k) for large k is called the variance decomposition of the variable Z_i. Under the condition that Σ is time invariant, stationarity of the VAR is equivalent to the condition that

$$\lim_{s\to\infty} b_{ij}(s) = 0 , \qquad \text{for all } i \text{ and } j .$$

Under that condition, $\rho_{ij}^2(k) \xrightarrow[k\to\infty]{} \rho_{ij}^2$, and ρ_{ij}^2 is the overall variance proportion of Z_i due to a one standard deviation shock in Z_j.[6]

The main objective of the estimation is to produce values of the A's that seem consistent with a theoretical model. The A's in the VAR are assumed to be related to the objective functions of people as well as to the parameters of given technical relationships and constraints imposed by the exogenous variables. Without an explicit model that gives rise to equations such as are represented in the VAR, we cannot say anything about the underlying economic system by looking merely at the magnitude of the coefficients of the A's.

---

6 The condition that $\lim_{s\to\infty} b_{ij}(s) = 0$ for all i, j, is equivalent to the λ's that solve $|\Lambda(\lambda)| = 0$ being outside the unit circle. **If this condition is violated in the estimated equations the interpretation of the variance decomposition of Z_i may be misleading.**

The VAR permits us to test formally the hypothesis of significant change in the parameters of the demographic variables equations during the second half of the last century. We test structural change by splitting the sample period into two parts at the time when the structural change is hypothesized to have occurred. Then we test whether there is a significant difference in parameters between the two parts of the sample. The test statistic is a modified likelihood ratio statistic, and it is distributed asymptotically as χ^2. We estimate the deterministic part, constant and trend, of the model jointly with the autoregressive part and test whether the trends (calendar time and its square) in our case are jointly statistically different from zero in each equation.

Using the information gleaned from tests of Granger causality (1969), from the variance decomposition, and from tests of structural change, we construct a VAR with zero restrictions on the A's and Σ. These zero restrictions are jointly tested system-wide and equation by equation. The dynamics of the restricted model are summarized by the MAR of the restricted VAR.

Appendix C: Statistical Tests of Model Specification

Appendix C

Table C.1

Test of Lag Length of the Subsystem of Endogenous Variables[a]

Test of lag length	$\log\|V_u\|$ (1)	$\log\|V_R\|$ (2)	$(195-58)[(2)-(1)] = \chi^2$ (3)	d.f (4)	Marginal Significance Level (5)
5 vs 4	-25.00	-24.60	54.8	50	.30
5 vs 3	-25.00	-24.08	126	100	.01
5 vs 2	-25.00	-23.62	190	150	.01

[a]There are 195 observations and 58 variables in each unrestricted equation with 5 lags. V_U and V_R correspond to the 5x5 matrices of the estimated variance-covariance of the innovations of the unrestricted and the restricted systems, respectively.

Table C.2

Tests of Lag Length: Period I

Test of lag length	$\log\|V_R\|$	likelihood ratio = χ^{2*} χ^2_1	χ^2_2	d.f.	marginal significance level for χ^2_1	for χ^2_2
5 vs 4	-25.11	57.65	123.17	50	.21	.0
5 vs 3	-24.33	97.36	207.3	100	.55	.0
5 vs 2	-23.60	134.67	287.8	150	.81	.0
5 vs 1	-22.95	167.84	358.7	200	.94	.0

*Sims (1980) modified: $\chi^2_1 = (T - k)[\log|V_R| - \log|V_U|]$

where k is the number of explanatory variables in one equation of the unrestricted model = 58, and
T is the number of annual observations = 109
χ^2_1 is the modified likelihood ratio test statistic

Conventional: $\chi^2_2 = T[\log|V_R| - \log|V_U|]$

$\log|V_U| = -26.24$

Table C.3

The Exogenous Variables Equations: 1756-1869

	Dependent Variables				
	WNTEMP	SPTEMP	SUTEMP	AUTEMP	RAIN
Constant	5.64	4.13	4.71	5.39	4.34
Lag 1	.058	-.008	.148	.061	.219
Lag 2	-.005	.139	-.046	.053	-.070
Lag 3	-.010	.064	.183	-.096	.071
Lag 4	-.010	.137	-.023	.063	.076
R^2	.007	.049	.053	.019	.062
Significance Level	.617	.960	.975	.985	.935

Table C.4*

The Variance-Covariance (Correlation) of the Residuals of the Equations

	CBR	IDR	NIDR	CROP	RWAGE	WNTEMP	SPTEMP	SUTEMP	AUTEMP	RAIN
CBR	.77 (1)	-.50	-1.1	-.87	.18	0	0	0	0	0
IDR	(-.31)	.33 (1)	.39	.25	-.11	0	0	0	0	0
NIDR	(-.42)	(.73)	.89 (1)	.25	-.09	0	0	0	0	0
CROP	(-.65)	(.09)	(.05)	.23 (1)	.01	0	0	0	0	0
RWAGE	(.07)	(-.21)	(-.10)	(.39)	.009 (1)	0	0	0	0	0
WNTEMP	(0)	(0)	(0)	(0)	(0)	.27 (1)	.15	.07	.11	-.13
SPTEMP	(0)	(0)	(0)	(0)	(0)	(.21)	.19 (1)	.05	.03	.02
SUTEMP	(0)	(0)	(0)	(0)	(0)	(.11)	(.35)	.13 (1)	.03	-.10
AUTEMP	(0)	(0)	(0)	(0)	(0)	(.09)	(.10)	(.13)	.49 (1)	.02
PAIN	(0)	(0)	(0)	(0)	(0)	(-.06)	(.04)	(-.20)	(.025)	.019 (1)

*The upper triangular component of the matrix shows the residual variance-covariance terms while the lower triangular component shows the residual correlations.

Table C.5

F-Test for Exclusion of Weather Variables

from Endogenous Variables Equations

	WNTEMP	SPTEMP	SUTEMP	AUTEMP	APREC
CBR	.007	.92	.28	.10	.30
IDR	.008	.64	.65	.18	.80
NIDR	.006	.77	.66	.20	.60
CROP	.61	.94	.80	.03	.41
RWAGE	.40	.54	.10	.57	.10

TABLE C.6

Two Block Recursive Orthogonalizations or Temporal Orderings of Model Decomposition of Variance: Percentage of Forecast Error Variance 25 Years Ahead Produced by Each Innovation

Innovation in:*

Response in	CBR	IDR	NIDR	CROP	RWAGE	WNTEMP	SPTEMP	SUTEMP	AUTEMP	RAIN
CBR	28	1	1	6	13	15	17	3	9	6
IDR	7	41	1	3	4	17	3	8	9	5
NIDR	9	14	16	8	5	25	7	6	6	5
CROP	7	1	1	46	9	9	9	4	9	5
RWAGE	5	2	2	8	31	11	4	4	18	14

	CBR	NIDR	IDR	RWAGE	CROP	WNTEMP	SPTEMP	SUTEMP	AUTEMP	RAIN
CBR	27	2	1	16	3	14	17	3	9	6
NIDR	9	28	2	7	6	25	7	6	6	5
IDR	7	20	22	5	3	17	3	8	9	5
RWAGE	5	1	3	37	2	11	4	4	18	14
CROP	7	1	1	12	43	9	9	4	9	5

*The triangularized innovation is according to the order of variables.

REFERENCES

Bengtsson, Tommy, "Short Run Changes in the Economy and Demographic Response, Southern Sweden 1751-1859", presented at international symposium on population economics in Lund, Sweden,(June 22-26, 1981).

Bernhardt, Eva, Trends and Variations in Swedish Fertility: A Cohort Study, URVAL Number 5, Stockholm,(1971).

Blayo, Y. "La Mortalite en France de 1740 a 1829," Population,Vol. 30, Special Issue,(November 1975),pp. 123-142.

Galbraith, V.L. and D.S. Thomas, "Birth Rates and Interwar Business Cycles", Journal of American Statistical Association, Vol 36, No. 216, (December 1941),pp. 465-476.

Gille, H. "The Demographic History of the Northern European Countries in the 18th Century", Population Studies, Vol. 3, No. 1,(June 1949),pp. 3-65.

Granger,C.W.J., "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica, 37:3 (July 1969), pp. 424-438.

Granger, C.W.J. and Newbold, "Forecasting Economic Time Series", (Academic Press: New York), 1977.

Hansen, L.P. and T.J. Sargent, "Frequency Domain Methods for Specifying and Estimating Rational Expectations Arbitrage Models", unpublished manuscript, (May 1980).

Heckscher, E.F., An Economic History of Sweden, Harvard University Press, Cambridge, Mass. (1954).

Hofsten, E. and H. Lundström, Swedish Population History URVAL, Number 8, Statistiska Central-byrån, Stockhokm, (1976).

Hyrenius, Hannes, "The Relation Between Birth Rates and Economic Activity in Sweden 1920-1944", Bulletin of the Oxford Institute of Statistics, Vol 8, (January 1946), pp. 14-21.

Jörberg, L. A History of Prices in Sweden, 1732-1919, Vol I and II, Lund; CWK Gleerup, (1972).

Jungenfelt, K.G., Löneandelen och den ekonomiska utvecklingen, Stockholm: Almquist and Wiksell, (1966).

Karlstrom, Urban, "Urbanization and Industrialization: Modeling Swedish Denoeconomic Development from 1870-1914", International Institute for Applied Systems Analysis, Working Paper 80-54, (April 1980).

Lee, R.D., "Forecasting Births in Post Transition Populations: Stochastic Renewal with Serially Correlated Fertility", Journal of The American Statistical Association,Vol. 69, No. 347, (Sept., 1974a), pp. 607-617.

________, "The Formal Dynamics of Controlled Populations and the Echo, The Boom and The First", Demography, Vol 11, No. 4, (November 1974b), pp. 563-585.

________., "Methods and Models for Analyzing Historical Series of Births, Deaths, and Marriages", in Population Patterns in the Past, (ed.) R.D. Lee, New York, Academic Press, (1977), pp. 337-370.

Lee, R.D., "Shortrun Fluctuations in Vital Rates, Prices and Weather", in Population Trends and Early Modern England, (eds.) E.A. Wrigley and R. Schofield, Harvard University Press, Cambridge, Mass. (1981).

LeRoy Ladurie, E., Times of Feast, Times of Famine , New York, Doubleday,(1971)

Mosher, W.D.,"Demographic Responses and Demographic Transitions: A Case Study of Sweden", Demography, Vol. 17, No. 4,(November 1980), pp. 395-412.

Mosk, Carl, Patriarchy and Fertility: The Evolution of Mortality in Japan and Sweden, 1880-1960, (mimeo) University of California at Berkeley, (1981).

Phelps-Brown, E.H., A Century of Pay, New York, St. Martins Press (1968).

Richards, Toni, "Weather, Nutrition and the Economy: The Analysis of Short Run Fluctuations in Births, Deaths, and Marriages, France 1740-1909", mimeo National Academy of Sciences, 1980.

__________, "A Technique for the Analysis of Long Swings in Births with Application to Early Modern France, Sweden and England," Office of Population Research, Princeton University, (October 1981a).

__________, "Long Cycles in Births: Early Modern France, Sweden and England", (Mimeo), Office of Population Research, Princeton University, (October 1981b).

Schultz, T. P., "Interrelationships Between Mortality and Fertility", (ed.) R.G. Ridker, Population and Development, Baltimore, MD : Johns Hopkins University Press (1976).

Sargent, T.J., "Estimating Vector Regressions Using Methods Not Based on Explicit Economic Theories," Federal Reserve Bank of Minneapolis Quarterly Review, Minneapolis, MN, (Summer 1979).

______, "Interpreting Economic Time Series," Journal of Political Economy, Vol. 89, No. 2 (1981), pp. 213-248.

Shryock, H.S. et al, The Methods and Materials of Demography, U.S. Department of Commerce, Bureau of the Census, Wash. D.C., Vol 2, (1971), p. 414.

Sims, Christopher A., "Money, Income and Causality," American Economic Review, 62: 4, 540-552, (Sept. 1972).

______,"Small Econometric Models of the U.S. and West Germany without Prior Restrictions," Center for Economic Research, University of Minnesota, Discussion Paper 105 (December 1978).

______, "Macroeconomics and Reality," Econometrica, 48:1, (1980), pp. 1-48.

Sundbärg, Gustav, Bevolkerungsstatistik Schwedens 1750-1900, P.A. Norstedt and Söner, Stockholm, (1907).

Sweden, Statistika Centralbyrån, Historical Statistics of Sweden, Vol. I, Population Stockholm, (1955).

______, Statistika Centralbyrian, Historical Statistics of Sweden Vol. II, Climate, Land Surveying, Agriculture, Forestry Fisheries, Stockholm (1959).

_________, Historisk Statistisk for Suerige Del I: Vefolkning Andraupplagan 1720-1967, 2nd Edition, Stockholm, (1969).

Thomas, D.S., Social and Economic Aspects of Swedish Population Movements 1750-1933, New York: MacMillan Company, (1941).

Trussell, J., J. Menken and S. Watkins, "The Nutrition-Fertility Link", Journal of Interdisciplinary History, (Winter 1980).

United Nations Statistical Office, Demographic Yearbook, Historical Supplement, New York, (1979).

Utterström, Gustav, "Some Population Problems in Pre-Industrial Sweden", Scand. Econ. Hist. Review, (1954), pp. 103-165.

_________, Population and Agriculture in Sweden, Circa 1700-1830", Scand. Econ. Hist. Review, Vol 9, No. 1, (1961), pp. 176-194.

_________, "Labour Policy and Population though in 18th Century Sweden", Scand. Econ. Hist. Review, Vol. 10, No. 2, (1962), pp. 262-279.

Weir, D.R., "Contraception, Infant Mortality and Breastfeeding in Rural France, 1740-1830", Stanford Project on the History of Fertility Control, Stanford University, Stanford California, (October 1981).

Wrigley, E.A. and R.S. Schofield, The Population History of England, 1541-1871, Cambridge, MA: Harvard University Press (1981).

Yule, G.U., "On the Changes in the Marriage and Birth Rates in England and Wales during the Past Half Century", Journal of the Royal Statistical Society, Vol 68, (1906), pp. 18-132.