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OPTIMAL AND TIME CONSISTENT EXCHANGE RATE MANAGEMENT IN AN OVERLAPPING GENERATIONS ECONOMY

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Abstract

This paper analyses exchange rate management in a simple overlapping generations model. This framework is used to evaluate alternative policies in terms of their implications for the welfare of individuals in the economy. The analysis identifies two objectives of monetary policy, providing a desirable store of value and collecting seigniorage. When the chief concern is to provide a desirable store of value (as when the monetary authority's major constituency consists of the asset holders of the economy), a policy of fixing the exchange rate does better when shocks are primarily of foreign origin while floating becomes more desirable when domestic shocks predominate. When seigniorage concerns are paramount (as when the authority's constituency is the young generation) flexible rates do better. When seigniorage concerns are paramount and when the monetary authority cannot establish a reputation for conducting monetary policy in a way that makes the currency a desirable store of value, a national currency may not be viable in the absence of exchange controls. Such controls may be justified in this situation.

1. Introduction

The design of monetary policy and exchange rate management is a topic that has received considerable attention in the last five years. Papers by Buiter (1979), Turnovsky (1982), Buiter and Eaton (1980), Flood and Marion (1982), Eaton and Turnovsky (1980) and Frenkel and Aizenman (1981) are examples of models that consider the optimality of alternative stabilization rules. The models used to evaluate alternative monetary and exchange rate policies and to derive optimal policies suffer from four deficiencies that this paper attempts to address.

First, models are constructed on the basis of a number of aggregate macroeconomic relationships that are not derived from underlying preferences and technologies. Their usefulness for policy evaluation is therefore questionable for the reason given by Lucas (1976): changes in policy may lead to changes in these aggregate relationships. Output supply and asset demands are examples of functional relationships that may be sensitive to policy changes.[1]

Second, policy evaluation is based upon ad hoc objective functions of the government rather than upon a comparison of the welfare of individuals in the economy under alternative policies. Assuming that the government is responsive to the welfare of its citizens, its policy objectives should derive from the preferences of individuals in the economy. When individual objective functions are specified alternative policies can then be evaluated in terms of their effects on individuals' utilities.

Third, discussion of optimal monetary policy in open economies has typically ignored the role of national monies in generating seigniorage for their governments. Fischer (1982) has recently discussed seigniorage as an objective of

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[1]This point is raised by Flood and Marion (1982), although they do not specify a model derived from the optimizing behavior of individuals.

monetary policy but provides no formal analysis. In fact, in some countries seigniorage constitutes a major source of revenue, possibly because it constitutes the administratively least expensive and least distortionary form of taxation.

Fourth, optimal policies are typically derived from the class of closed loop policies. As Kydland and Prescott (1977) have emphasized, such policies may not be time consistent. For many of the models discussed above the optimal closed loop and optimal feedback policies coincide.[2] Once seigniorage considerations are introduced, however, they diverge.

This paper develops a model of an open economy derivative of the Samuelson (1958) pure consumption loan model. Individuals have available to them as a store of value a foreign currency which depreciates in value at a stochastic rate that is exogenous to the economy under consideration. The population growth rate of this economy is also an exogenous stochastic process.

The government of this economy has the ability to provide its own currency as a store of value. Individuals choose their first period consumption and allocate their savings between the two currencies to maximize the expected utility of consumption over two periods. There is no individual bequest motive. New money issue is used to finance government expenditure which is assumed to benefit only the younger generation.

Bringing the economy toward the Golden Rule and generating seigniorage constitute two major goals of monetary policy. Providing a relatively riskless store of value and a stable source of seigniorage are two additional goals. The objectives of monetary policy to provide a desirable store of value and to generate a stable source of revenue for public expenditure are in sharp conflict in the short run but may be more compatible in the long run.

When the primary objective of the monetary authority is to provide a desirable store of value, one insight of the previous literature

[2]See Kydland (1977) for a discussion of the distinction between closed loop and feedback policies. See Calvo (1978) for a discussion of optimal and time consistent monetary policies in a non-stochastic closed economy context.

reemerges: a policy of predetermining the exchange rate each period tends to yield higher welfare when the domestic price level is stable relative to the domestic growth rate and conversely. A policy of having no national currency at all, relying solely upon the foreign currency as a store of value, can in some circumstances dominate a policy of having a currency fixed in value in terms of the foreign currency or freely floating against it. When the primary function of the monetary authority is to generate seigniorage, however, a policy of pure floating always dominates a fixed exchange rate policy or a policy of having no national currency.

When seigniorage considerations are present, a government that attempts to maximize the welfare only of the current young generation cannot sustain a currency. The only time consistent policy leads to a nonmonetary economy. Introducing the expected utility of future generations as a public good can reverse this result, however. Alternatively, when earning seigniorage is not an objective, time consistent policy can correspond to the optimal closed loop policy.

Section 2 derives the optimal savings and portfolio behavior of each atomistic individual in the economy under consideration. Section 3 imbeds this behavior in a simple, aggregate model to derive the behavior of the domestic price level and the exchange rate as functions of exogenous variables and policy parameters. The expected welfare of each generation in a nonmonetary economy is derived in section 4, and is compared with expected welfare under fixed and flexible exchange rate regimes for a monetary economy. Section 5 considers optimal feedback policies under alternative social welfare and individual utility functions. Section 6 discusses the role of reputation as a means of enforcing a monetary economy and the optimal closed loop policy. Some concluding remarks appear in section 7.

2. Optimal Consumption and Portfolio Behavior

Consider an economy of the Samuelson (1958) pure consumption loan variety. Individuals live two periods, earning an amount $\bar{Y}$ in the first period of their lives and nothing in the second period. An individual i entering the labor force in period t seeks to maximize a utility function of the form:

$$\log C_t^{iy} + \beta \log C_{t+1}^{io} + \omega G_t \tag{2.1}$$

where C_t^{iy} denotes i's consumption in the working period, C_{t+1}^{io} in the retirement period, and G_t denotes per worker government spending in the working period. Individuals are assumed not to derive utility from government spending in their retirement period. The parameter ω indicates the weight placed on government spending relative to private consumption.

Individuals have available to them as stores of value a domestic money and a foreign money. There is a single traded good the price of which, in period t, is P_t in terms of domestic money and P_t^* in terms of foreign money. Exact purchasing power parity (PPP) obtains so that

$$P_t = E_t P_t^* \tag{2.2}$$

where E_t denotes the domestic currency price of foreign currency. The country under consideration is small in the sense that domestic actions do not affect P_t^*. The role of inflation in terms of the foreign price level is Π_t^* so that

$$P_t^* = (1 + \Pi_t^*) P_{t-1}^* \tag{2.3}$$

where

$$\Pi_t^* = \Pi^* + u_t^{P*}$$

Here Π^* is a constant and u_t^{P*} a Gaussian white noise process with variance σ_{P*}^2. The individual also takes as given the domestic price level P_t which evolves according to the process

$$P_t = (1 + \Pi_t)\, P_{t-1} \tag{2.4}$$

where

$$\Pi_t = \Pi + u_t^P$$

Here Π is a constant and u_t^P a Gaussian white noise process with variance σ_p^2. The next section derives this process for the domestic price from the underlying macroeconomic equilibrium of the economy.

No voluntary intergenerational transfers take place. The two monies provide the only assets to transfer income from the working period to the retirement period. During the working period individual i thus chooses C_t^{iy} and divides his wealth between the two monies to maximize expected lifetime utility. Let λ_t^i denote individual i's share of wealth allocated toward the foreign money. Thus

$$\frac{M_t^i}{P_t} = (1 - \lambda_t^i)\, (\bar{Y} - C_t^{iy}) \tag{2.5}$$

$$\frac{E_t M_t^{*i}}{P_t} = \frac{M_t^{*i}}{P_t^*} = \lambda_t^i\, (\bar{Y} - C_t^{iy}) \tag{2.6}$$

where M_t^i and M_t^{*i} denote individual i's holdings of domestic and foreign money respectively. We assume that neither money can be held in negative amounts. Thus $Y - C_t^{iy}$, λ_t^i and $1 - \lambda_t^i$ must be nonnegative.

Consumption in retirement, C_{t+1}^{io}, is given by

$$C_{t+1}^{io} = \frac{M_t^i + E_{t+1} M_t^{*i}}{P_{t+1}} (\bar{Y} - C_t^{iy})$$

$$= [\lambda_t^i (P_t/P_{t+1}) + (1 - \lambda_t^i)\, (P_t^*/P_{t+1}^*)]\, (\bar{Y} - C_t^{iy}) \tag{2.7}$$

By assuming that the parameters Π, Π^*, $\sigma_p^{\ 2}$ and $\sigma_p^{\ 2}$ are sufficiently small to ignore the products of any two of them the processes (2.3) and (2.4) may be approximated by

$$P_t/P_{t+1} \cong 1 - \Pi - u_t^P \tag{2.8}$$

$$P_t^*/P_{t+1}^* \cong 1 - \Pi^* - u_t^{P*} \tag{2.9}$$

Combining (2.8) and (2.9) with the PPP relationship (2.2) gives

$$E_t/E_{t+1} \cong 1 + \Pi - \Pi^* + u_t^P - u_t^{P*} \equiv 1 + \Pi - \Pi^* + u_t^e \tag{2.10}$$

Here u_t^e is the implied error term in the change in E_t. A second-order Taylor series approximation of the expectation of log C_{t+1}^{io} around log $(\bar{Y} - C_t^{iy})$, using (2.8), (2.9) and (2.10) is

$$E\left[\log C_{t+1}^{io}\right] \cong \log(\bar{Y} - C_t^{iy}) + [-\Pi + \lambda_t^i(\Pi - \Pi^*)] - [\sigma_p^{\ 2} + (\lambda_t^i)^2\sigma_e^{\ 2} - 2\lambda_t^i\sigma_{ep}]/2 \tag{2.11}$$

where $\sigma_e^{\ 2}$ is the variance of u_t^e and σ_{ep} the covariance between u_t^e and u_t^p.

Substituting (2.11) into (2.1) and maximizing the resulting expression with respect to the choice variables C_t^{iy} and λ_t^i yields, as expected utility maximizing values:

$$\hat{C}_t^{iy} = \Delta\,\bar{Y} \qquad \Delta \equiv (1 + \beta)^{-1} \tag{2.12}$$

$$\hat{\lambda}_t^i = \min\left\{\max\left[0, \frac{\Pi - \Pi^* + \sigma_{ep}}{\sigma_e^{\ 2}}\right], 1\right\} \tag{2.13}$$

At these values individual i attains an expected level of utility.

$$U^i = \rho + (1 + \beta)\,\bar{y} + \begin{cases} -\Pi - 1/2\,\sigma_p^2 & \text{if } \lambda_t^i = 0 \\ -\Pi - 1/2\,\sigma_p^{\ 2} + \dfrac{(\Pi - \Pi^* + \sigma_{ep})^2}{2\sigma_e^{\ 2}} & \text{if } \lambda_t^i = \dfrac{\Pi - \Pi^* + \sigma_{ep}}{\sigma_e^{\ 2}} \\ -\Pi^* - 1/2\,\sigma_{p*}^2 & \text{if } \lambda_t^i = 1 \end{cases} + \omega G_t$$

where

$$\rho \equiv \beta \log \beta - (1 + \beta) \log (1 + \beta)$$

$$\bar{y} \equiv \log \bar{Y}$$

This section has characterized the optimal consumption and portfolio behavior of a single individual facing a given level of government spending and distributions of the foreign and domestic price levels and the exchange rate. The next section derives the level of government spending and the behavior of the domestic price level and exchange rate from aggregate characteristics of the economy and from government policy.

3. The Aggregate Economy

Since all individuals earn the same income and face the same distributions of prices, the aggregate share of foreign money in total savings, λ_t, is

$$\lambda_t = \hat{\lambda}_t^i \tag{3.1}$$

while the aggregate consumption of the working generation C_t^y is

$$C_t^y = \Delta\bar{Y}L_t \tag{3.2}$$

where L_t is the number of workers entering the labor force in period t.

The number of workers entering the labor force in period t is $(1 + n_t)$ times the number that entered the previous period, i.e.,

$$L_t = (1 + n_t) L_{t-1}$$

where

$$n_t = n + u_t^n \tag{3.3}$$

Here n is a constant and u_t^n a Gaussian white noise process with variance σ_n^2. Assume that u_t^n and u_t^{P*} are uncorrelated.

The nominal supply of domestic money, denoted M_t, grows at a rate m_t so that

$$M_t = (1 + m_t) M_{t-1} \tag{3.4}$$

New monetary issue is used to finance government spending. There are no other sources of government revenue.[3] Thus

$$G_t = \frac{M_t - M_{t-1}}{P_t L_t} = m_t \frac{M_{t-1}}{P_{t-1}} [(1 + \Pi_t) L_t]^{-1} \tag{3.5}$$

G_t thus constitutes total seigniorage gleaned in period t.

Following much of the previous literature on exchange rate intervention this model assumes that the only contemporaneous variable that the government observes is the exchange rate. (See, e.g., Buiter (1979), Turnovsky (1982), Buiter and Eaton (1980), Eaton and Turnovsky (1980), Frenkel and Aizenman (1981)). On the basis of information available at the end of the previous period the government sets a monetary growth rate g_t^M that is subject to revision in response to new information embodied in the exchange rate. The actual money growth rate is therefore

$$m_t = g_t^M + a_t u_t^e \tag{3.6}$$

where u_t^e, recall, is the unanticipated component of the exchange rate and a_t a policy parameter. Setting $a_t = 0$ corresponds to a regime of pure floating while the exchange rate is fixed within the period when $a_t = -\infty$. It is assumed that g_t^M is bounded from below by zero; the expected level of government spending cannot

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[3] An equivalent assumption for the purposes of this analysis is that other revenue sources are inelastically supplied in some amount $\bar{T}$ and that the utility function (2.1) is of the form:

$$\log C_t^{1y} + \beta \log C_{t+1}^{1o} + \omega (\bar{T} + G_t)$$

Alternatively, the utility function could incorporate deadweight losses from other sources of tax revenue as well as the benefits from government spending.

be negative.

Domestic money market equilibrium obtains when

$$\frac{M_t}{P_t} = (1 - \lambda_t)\ (1 - \Delta)\ \bar{Y}L_t \tag{3.7}$$

Taking the first difference of the logarithm of this expression, assuming stationarity or that $\lambda_t = \lambda_{t-1}$, yields

$$\frac{P_t - P_{t-1}}{P_{t-1}} = m_t - n_t = g_t^M - n + a_t u_t^e - u_t^n \tag{3.8}$$

The left hand side of (3.8) approximates the domestic inflation rate Π_t while PPP implies

$$u_t^e = u_t^P - u_t^{P*} \tag{3.9}$$

Assuming that individuals know the parameters of the actual inflation process equations (3.8) and (3.9) imply that

$$\Pi = g_t^M - n \tag{3.10}$$

$$u_t^P = -(u_t^n + a_t u_t^{P*})\ (1 - a_t)^{-1} \tag{3.11}$$

$$u_t^e = -(u_t^n + u_t^{P*})\ (1 - a_t)^{-1} \tag{3.12}$$

Therefore, as of period t

$$\sigma_p^2 = (\sigma_n^2 + a_{t+1}^2 \sigma_p{*}^2)\ (1 - a_{t+1})^{-2} \tag{3.13}$$

$$\sigma_e^2 = (\sigma_n^2 + \sigma_{p*}^2)\ (1 - a_{t+1})^{-2}$$

$$\sigma_{ep} = (\sigma_n^2 + a_{t+1}\sigma_{p*}^2)\ (1 - a_{t+1})^{-2}$$

Assuming that $\overset{*}{\Pi}$, n, g_t^M, σ_{p*}^2 and σ_n^2 are sufficiently small to treat the product of any two of them as zero, expected seigniorage per worker can be approximated as

$$E\left[G_t\right] = (1 - \lambda_{t-1})\ W\ [g_t^M - a_t^2\ (1 - a_t)^{-2}\ (\sigma_n^2 + \sigma_{p*}^2)] \tag{3.14}$$

where $W \equiv (1 - \Delta)\ \bar{Y}$, per capita savings.

Substituting equations (3.10), (3.13) and (3.14) into equation (2.14) gives an expression for the expected utility of a worker in generation t as of period $t-1$ as a function of exogenous and policy parameters.

The next section derives policy parameters that maximize the expected utility of the average generation. Sections 5 and 6 consider the dynamic consistency of these policies.

4. Fixed vs. Flexible Exchange Rates

This sections considers a monetary response that is repeated each period, i.e., one such that

$$g_t^M = g^M \quad \forall t \tag{4.1}$$

$$a_t = a \quad \forall t \tag{4.2}$$

to derive policies that maximize the expected utility of each generation. When policy is of the form (4.1) and (4.2) the model under consideration is stationary.

The share of foreign currency in total money balances is, from (2.13), (3.10), and (3.13).

$$\lambda = \min\left\{ \max\left[0, \frac{(g^M - n - \Pi^*)(1 - a)^2 + (\sigma_n^2 + a\sigma_p{*}^2)}{\sigma_n^2 + \sigma_{p*}^2} \right], 1 \right\} \tag{4.3}$$

The expected utility of seigniorage-financed government expenditure is

$$E[\omega G_t] = \omega(1 - \lambda)W[g^M - a^2(1 - a)^{-2}(\sigma_n^2 + \sigma_{p*}^2)] \tag{4.4}$$

The expected utility of a member of any generation, where the expectation is taken as of any period before entry into the labor force, is, therefore

$$U = \rho + (1 + \beta)\bar{y} - [\Pi^* + (1 - \lambda)(g^M - n - \Pi^*)]$$
$$- [(1 - \lambda)^2 \sigma_n^2 + (a - \lambda)^2 \sigma_p{*}^2](1 - a)^{-2}/2$$
$$+ \omega(1 - \lambda)W[g^M - a^2(1 - a)^{-2}(\sigma_n^2 + \sigma_{p*}^2)] \tag{4.5}$$

The values of g^M and a that maximize this expression, incorporating λ as defined by expression (4.3), constitute the optimal closed loop monetary response. Analytic solutions for the general case were not obtained.

It is nevertheless useful to consider instead three special cases: the non-monetary economy, the monetary economy with fixed exchange rates within the period, and a monetary economy with perfectly flexible rates.

4.1 A Nonmonetary Economy

From expression (4.3) observe that if

$$g^M > n + \Pi^* + \sigma_{p^*}^2/(1 - a) \tag{4.6}$$

then $\lambda = 1$, i.e., domestic currency is not held at all. Seigniorage from domestic money creation is zero and foreign money is the only store of value. Expected utility is

$$U^N = \rho + (1 + \beta)\,\bar{y} - \Pi^* - 1/2\,\sigma_{p^*}^2 \tag{4.7}$$

4.2 Fixed Exchange Rates

When $a = -\infty$ individuals will hold only domestic currency if $g^M - n < \Pi^*$ and only foreign currency if $g^M - n > \Pi^*$. The second case yields the nonmonetary economy. If $g^M - n = \Pi^*$ individuals are indifferent between the two currencies. For concreteness, assume that $\lambda = 0$ in this case. When $g^M \leq n + \Pi^*$, then, expected utility is given by

$$U^P = \rho + (1 + \beta)\bar{y} - (g^M - n) - 1/2\,\sigma_{p^*}^2 + \omega W[g^M - (\sigma_n^2 + \sigma_{p^*}^2)] \tag{4.8}$$

and when $g^M > n + \Pi^*$ by U^N. Therefore if $\omega W > 1$ it is optimal to set $g^M = \Pi^* + n$ while if $\omega W < 1$, to set $g^M = 0$. In the first case expected utility is

$$U^{P'} = \rho + (1 + \beta)\bar{y} - \Pi^* - \sigma_{p^*}^2/2 + \omega W[\Pi^* + n - (\sigma_n^2 + \sigma_{p^*}^2)] \tag{4.8'}$$

while in the second it is

$$U^{P'} = \rho + (1 + \beta)\,\bar{y} + n - \sigma_{p^*}^2/2 - \omega W(\sigma_n^2 + \sigma_{p^*}^2) \tag{4.8''}$$

Comparing (4.8') and (4.8") with (4.7), expected utility in a nonmonetary economy, note that a nonmonetary economy dominates a monetary economy with fixed exchange rates if and only if

1. $\omega W > 1$ <u>and</u>

$$\sigma_n^2 + \sigma_{p*}^2 > \Pi^* + n \tag{4.9}$$

or

2. $\omega W < 1$ <u>and</u>

$$\omega W(\sigma_n^2 + \sigma_{p*}^2) > \Pi^* + n$$

The only cost to establishing a currency with fixed rates relative to a no currency situation is the variability in seigniorage. This cost increases with the variance of the foreign price level and the domestic growth rate. The benefit is the ability either to earn seigniorage on domestic currency or else to establish the Golden Rule interest rate. Either benefit increases with the term $\Pi^* + n$, or the difference between the world interest rate ($-\Pi^*$) and the domestic growth rate.

4.3 Flexible Exchange Rates

When a = 0 individuals will hold foreign currency in proportion

$$\lambda = \min\left\{\max\left[0, \frac{(g^M - n - \Pi^*) + \sigma_n^2}{\sigma_n^2 + \sigma_{p*}^2}\right], 1\right\} \tag{4.10}$$

Maintaining a monetary economy requires that

$$g^M < n + \Pi^* + \sigma_{p*}^2 \tag{4.11}$$

while if

$$g^M < n + \Pi^* - \sigma_n^2. \tag{4.12}$$

no foreign currency will be held.

Assuming an interior solution for λ, average utility is

$$U^F = \rho + (1 + \beta)\bar{y} - (g^M - n) - 1/2\, \sigma_n^2$$

$$+ \frac{(g^M - n - \Pi^* + \sigma_n^2)^2}{2(\sigma_n^2 + \sigma_{p*}^2)} + \omega W g^M \; \frac{n + \Pi^* - g^M + \sigma_{p*}^2}{\sigma_n^2 + \sigma_{p*}^2} \tag{4.13}$$

The optimal monetary growth rate maximizes expression (4.13) subject to the constraints (4.11), (4.12) and $g^M \geq 0$. The second-order condition for a maximum is satisfied, however, if and only if $\omega W > 1$.

In view of this complication it is considerably simpler to focus on two particular special cases, one in which seigniorage effects are negligible ($\omega W=0$), and one in which they are paramount ($\omega W=\infty$). Each is treated in turn.

4.2.1 Absence of Seigniorage Effects

When no social welfare derives from seigniorage maximum average utility obtains when $g^M = 0$ and, if $n + \Pi^* \geq \sigma_n^2$, $\lambda = 0$, while if $n + \Pi^* < \sigma_n^2$ $\lambda = \frac{\sigma_n^2 - n - \Pi^*}{\sigma_n^2 + \sigma_{p*}^2}$. In the first case average utility is

$$U^{F'} = \rho + (1 + \beta)\bar{y} + n - 1/2\, \sigma_n^2 \tag{4.13'}$$

while in the second it is

$$U^{F''} = \rho + (1 + \beta)\bar{y} + n - 1/2\, \sigma_n^2 + \frac{(\sigma_n^2 - n - \Pi^*)^2}{2(\sigma_n^2 + \sigma_p^{*2})} \tag{4.13''}$$

Comparing expressions (4.13') and (4.13") with (4.7) note that a monetary economy with pure floating dominates a nonmonetary economy when $n + \Pi^*$ and σ_p^{*2} are large relative to σ_n^2.

The greater the domestic growth rate relative to the foreign inflation rate the higher is the increase in the rate of return from establishing a domestic currency. When $\sigma_{p^*}^2$ is large relative to σ_n^2 the return on domestic money under floating rates will be relatively less risky.

The desirability of fixing the exchange rate or allowing it to float can be determined by comparing expressions (4.13') and (4.13") with expression (4.8") evaluated at $\omega = 0$. The condition for fixed rates to dominate is:

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\begin{aligned}
&1.\quad n + \Pi^* > \sigma_n^2 \ \underline{\text{and}} \\
&\qquad \sigma_n^2 > \sigma_p^2 \\
&\text{or} \\
&2.\quad n + \Pi^* < \sigma_n^2 \ \underline{\text{and}} \\
&\qquad (n + \Pi^*)^2 - 2\sigma_n^2 (n + \Pi^*) + \sigma_p^4 < 0
\end{aligned}
\tag{4.14}
$$

When $\sigma_n^2 = 0$ floating rates necessarily dominate while if $\sigma_{p^*}^2 = 0$ fixed rates do. When $\sigma_n^2 = \sigma_p^2 = \sigma^2$ the choice is a matter of indifference if $n + \Pi^* > \sigma$ but if $n + \Pi^* < \sigma^2$ flexible rates are preferable. The reason is that, in this second case, the portfolio is diversified under flexible rates, and flexible rates allow a reduction in risk.

4.2.2 Dominance of Seigniorage Effects

To analyse the situation in which earning seigniorage is of paramount concern set $\omega = \infty$. From expression (4.13) observe that seigniorage is at a maximum when

$$g^M = (n + \Pi^* + \sigma_{p^*}^2)/2 \tag{4.15}$$

if $\lambda > 0$ at this value which requires, in turn, that

$$(n + \Pi^* - \sigma_{p^*}^2)/2 < \sigma_n^2 \tag{4.16}$$

In this case earnings from seigniorage, denoted S, are

$$S = \frac{(n + \Pi^* + \sigma_{p*}^2)^2}{4(\sigma_n^2 + \sigma_p*^2)} W \tag{4.17}$$

If $\lambda = 0$ at the value of g^M given by (4.15) then the monetary authority can set

$$g^M = n + \Pi^* - \sigma_n^2 \tag{4.18}$$

and earn

$$S = (n + \Pi^* - \sigma_n^2) W \tag{4.19}$$

Since both (4.17) and (4.19) are positive, while in a nonmonetary economy seigniorage is zero, when seigniorage dominates the welfare function a monetary policy with flexible rates always yields higher expected utility than a non-monetary economy.

Under fixed rates seigniorage earnings are stochastic because of the need to intervene in the foreign exchange market to stabilize the currency. The appropriate comparison then, is between expected seigniorage under the two regimes. Normalizing (4.8) by dividing through by ω note that maximum expected seigniorage under fixed rates as ω goes to infinity is

$$E(S) = [\Pi^* + n - (\sigma_n^2 + \sigma_{p*}^2)]W \tag{4.20}$$

By comparing (4.20) with (4.17) and (4.19) it can be shown that maximum expected seigniorage is necessarily higher under perfectly flexible exchange rates. When rates are perfectly flexible disturbances in the foreign price level affect neither the domestic price level nor the amount of money creation. They consequently do not affect real per capita seigniorage. Disturbances in the domestic population growth rate create domestic price level disturbances in the opposite direction. The two cancel each other to the point where, as a first-order approximation real seignorage per capita is non-stochastic. When exchange rates are fixed, however, variation in the domestic price level and population growth rate are no longer perfectly negatively correlated. As a consequence of Jensen's inequality, expected real seignorage per capita is lower. This effect is not offset by the fact that under fixed rates more money can be created, on average, without leading to substitution into the foreign currency. Under fixed rates $\lambda = 0$ whenever $g^M \leq n + \Pi^*$ while under flexible rates $\lambda = 0$ requires $g^M \leq n + \Pi^* - \sigma_n^2$.

5. Optimal Feedback and Closed Loop Policies

The last section compared expected utility in a situation in which the monetary authority pegs the exchange rate each period with one in which it sets the money supply independently of the exchange rate. It was assumed that the monetary authority could precommit itself to a monetary response that maximizes the expected utility of each generation.

The monetary authority may, however, respond only to the wishes of generations present at the time the monetary policy is implemented. At this point the money holdings of the old generation are a bygone, while the demand for money of the young generation depends upon its expectations of policy in a later period. If current policy has no effect on expectations of future policy the monetary authority will establish a level of monetary growth each period taking as given monetary policies in other periods and existing asset holdings.

In period t, then, the authority selects g_t^M and a_t to maximize a weighted average of the old and young generation's utility. Let α denote the weight assigned to the young generation's utility and $1-\alpha$ the weight to the old generation's utility.

The component of the expected utility of the old generation that is a function of policy in period t is

$$U_t^o = -[\Pi^* + (1 - \lambda_{t-1}) (g_t^M - n - \Pi^*)] - [(1 - \lambda_{t-1})^2 \sigma_n^2 + (a_t - \lambda_{t-1})^2 \sigma_{p*}^2] (1 - a_t)^{-2}/2 \tag{5.1}$$

while that of the young generation is

$$U_t^y = \omega(1 - \lambda_{t-1}) W[g_t^M - a_t^2(1 - a_t)^{-2} (\sigma_n^2 + \sigma_{p*}^2)] \tag{5.2}$$

A time consistent policy is a choice of g_t^M and a_t that maximizes $\alpha U_t^y + (1 - \alpha)U_t^o$.

When seigniorage earnings do not affect utility ($\omega = 0$) the young generation is unaffected by current policy. Situations in which seigniorage effects are absent ($\omega = 0$) and in which the old generation dominates the social welfare function ($\alpha = 0$) thus imply equivalent welfare criteria. Similarly, equivalent welfare criteria emerge when seigniorage effects are paramount ($\omega = \infty$) and when the young generation dominates the social welfare function ($\alpha = 1$).

One result of this section is that when the young generation dominates the social welfare function or when seigniorage dominates the individual utility function (i.e., when $\alpha = 1$ when or $\omega = \infty$) then time consistent (or optimal feedback) policy cannot sustain a monetary economy. The consequent equilibrium is in general inferior to the optimum that would emerge if the monetary authority could precommit itself to an alternative policy (i.e., to choosing the optimal closed loop policy). A second result is that when the old generation dominates the social welfare function or when seigniorage does not appear in the individual utility function (i.e., when $\alpha = 0$ or $\omega = 0$) then time consistent policy may also yield a nonmonetary economy or it may yield the optimal closed loop policy.

5.1 Dominance of Seigniorage Effects or Young Generation Dominant

For the case in which $\alpha = 1$ or $\omega = \infty$ this result is straightforward. U_t^y is maximized when $g_t^M = \infty$ and $a_t = 0$. Given λ_{t-1}, the higher the monetary growth rate the more revenue from seigniorage while exchange market intervention reduces expected seigniorage revenue.

When the policy parameters assume these values the rate of return on domestic currency is minus infinity. Wealthholders, anticipating in the previous period that these policies will be pursued will set $\lambda_{t-1} = 1$. Hence, in a rational expectations equilibrium, no seigniorage is collected. The economy degenerates to a nonmonetary economy with expected utility per generation U^N given by expression (4.7).

5.2 Absence of Seigniorage Effect or Old Generation Dominant

Somewhat more surprisingly, this same result can emerge when $\alpha = 1$ or $\omega = 0$. U_t^o is maximized when

$$g_t^M = 0 \tag{5.3}$$

$$a_t = \lambda_{t-1} - (1 - \lambda_{t-1})\,\sigma_n^2/\sigma_{p*}^2 \tag{5.4}$$

Individuals, anticipating that this policy will be implemented in period t when selecting their portfolios in period t - 1, will, from equation (4.3), choose

$$\lambda_{t-1} = \min\left\{\max\left[0,\ \frac{-(n + \Pi^*)\ (1 - \lambda_{t-1})^2(1 + \sigma_n^2/\sigma_{p*}^2)^2 + \lambda_{t-1}}{\sigma_n^2 + \sigma_{p*}^2}\right],1\right\} \tag{5.5}$$

This equation has two solutions, $\lambda_{t-1} = 1$, in which case $a_t = 1$, and $\lambda_{t-1} = 0$, in which case $a_t = -\sigma_n^2/\sigma_{p*}^2$

The first equilibrium, once again, constitutes a degeneration to a nonmonetary equilibrium with expected average utility U^N. The second implies an expected average utility.

$$U = \rho + (1 + \beta)\bar{y} + n - \sigma_n^2\sigma_{p*}^2(\sigma_n^2 + \sigma_{p*}^2)^{-1}/2$$

The policy of setting

$$g^M = 0 \tag{5.6}$$

$$a = -\sigma_n^2/\sigma_{p*}^2 \tag{5.7}$$

also constitutes an optimal closed loop policy when seigniorage effects are absent ($\omega = 0$). To see this observe that the derivative of expected average utility with respect to a, evaluated at the point $g^M = 0$, $a = -\sigma_n^2/\sigma_{p*}^2$, $\omega = 0$, is, from expression (4.5),

$$\frac{dU}{da} = -(n + \Pi^*)\frac{d\lambda}{da} \tag{5.8}$$

Since at this point $\lambda = 0$, $\frac{d\lambda}{da} = 0$. Hence the first-order condition for a time consistent policy also corresponds to the first-order condition for an optimal policy when $\omega = 0$. Since the second-order conditions for a maximum are satisfied, this time consistent equilibrium corresponds to the equilibrium that emerges when $\omega = 0$ and the optimal closed loop policy is pursued.

An intuitive explanation for the optimality of this equilibrium is that the larger the share of domestic currency in portfolios, the closer are the expected domestic interest rate facing consumers and the expected growth rate. A lower value of λ thus brings the economy closer to the Golden Rule, thereby raising welfare. The optimal value of a should therefore be chosen to minimize λ. In fact, when policy takes the form of (5.6) and (5.7), $\lambda = 0$, its minimum possible value. Optimal exchange rate management by the monetary authority thus provides a perfect substitute for currency diversification by private individuals as a means of minimizing risk. In other words, when the exchange rate is managed optimally private individuals have no incentive to hold foreign currency to reduce risk. The incentive that the monetary authorities have to minimize the risk associated with domestic currency each period leads them to stabilize the exchange rate in an optimal way.

5.3 Extension to the General Case

The conclusions of sections 5.1 and 5.2 suggest some results that would emerge if α and ω assume intermediate values.

First, for values of α and ω sufficiently low, a monetary economy can be sustained by time consistent policies. Second, the level of monetary growth will be higher and the amount of intervention lower than in the case when $\alpha = \omega = 0$. The reason is that to earn seigniorage it is necessary to set $g^M > 0$. Reducing a_t, the amount of intervention, will also raise expected revenue from seigniorage.

6. The Role of Reputation in Enforcing a Monetary Economy

A conclusion of the previous section is that when earning seigniorage is the predominant concern of the monetary authority, or that when the utility of the young generation dominates the social welfare function, then a time consistent policy cannot sustain a monetary equilibrium. It was assumed that the only objective of the monetary authority is to maximize a weighted average of the expected utilities of generations currently present.

An alternative objective is to maximize a weighted average of the expected utilities of current and all future generations. A reason for the monetary authority to take into account the welfare of future generations is that their welfare constitutes a public good to current generations, i.e., the utility of future generations as a group affects the welfare of the current generations, but no atomistic member of the current generation has an incentive to provide a bequest to any member of the subsequent generation. In this context the monetary authority's incentive to maintain the reputation of its currency to allow future generations to earn seigniorage can lead to the time consistency of a monetary economy.

Let the expected utility from seigniorage of a generation born at time t be given by

$$U_t = \max \{(1 - \lambda_{t-1})W[g_t^M - a_t^2(1 - a_t)^{-2} (\sigma_n^2 + \sigma_{p*}^2)],\bar{U}\} \qquad (6.1)$$

An upper bound $\bar{U}$ is placed on the welfare that can be generated from seigniorage to insure boundedness of the overall objective function.

Let the objective function of the monetary authority at time t be

$$W_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} U_\tau \qquad 0 \leq \delta < 1 \qquad (6.2)$$

where δ constitutes a discount factor. Taking future policies g_τ^M, a_τ as given, $\tau > t$, W_t is at a maximum when $a_t = 0$ and

$$g_t^M \geq \bar{U}/[(1 - \lambda_{t-1})W] \equiv g_t^* \qquad (6.3)$$

Denote this policy as the optimal one-period policy.

At some initial period t_o the monetary authority announces the policy response henceforth, denoted by the paramers $\tilde{g}_t$, $\tilde{a}_t$, $t \geq t_o$. If the monetary authority deviates from the policy it announces in some period t, individuals will anticipate that for $\forall \tau > t$, $g_\tau^M = g_\tau^*$, $a_\tau = 0$, i.e., that the optimal one period policy will be pursued. Let $\bar{U}$ be sufficiently high to imply

$$\bar{U}/W > n + \Pi^* + \sigma_{p*}^2 \qquad (6.4)$$

which from expression (4.3), insures that when individuals anticipate the optimal one-period policy in period τ they will select $\lambda_{\tau - 1} = 1$, i.e., hold no domestic currency.

If the authority deviates from its announced policy in period t, assuming that $\lambda_{t-1} < 1$, it can attain a level of its objective function $\bar{U}$ in that period and zero subsequently, since henceforth $\lambda = 1$. The economy degenerates to a nonmonetary economy. Thus the value of deviating from the announcement in any period t is simply $\bar{U}$.

By sticking with its announced policy in period t, assuming that this policy was anticipated in period t-1 and that the announced policy will be adhered to subsequently, the authority can attain a level of its objective function

$$W_t^a = \sum_{\tau=t}^{\infty} \delta^{\tau-t}[(1 - \lambda_{\tau-1}) W\tilde{g}_\tau^M - \tilde{a}_\tau^2(1 - \tilde{a}_\tau)^{-2} (\sigma_n^2 + \sigma_{p*}^2)] \qquad (6.5)$$

where λ_{t-1} is predetermined and $\lambda_{\tau-1}$, $\tau > t$, is given by

$$\lambda_{\tau-1} = \min\left\{\max\left[0, \frac{(\tilde{g}^{M}_{\tau} - n - \Pi^*)(1 - \tilde{a}_{\tau})^2 + \sigma_n^2 + \tilde{a}_{\tau}\sigma_{p^*}^2}{\sigma_n^2 + \sigma_{p^*}^2}\right], 1\right\} \quad (6.6)$$

The time consistencyof the announced policy requires that $W^a_t \geq \bar{U} \forall t \geq t_o$; that is, the value to the monetary authority of adhering to the announced policy, and thereby maintaining the expectation that it will continue to adhere to this policy,must exceed the maximum value of reverting to the optimal one-period policy.

The optimal credible policy is a choice of $\tilde{g}_t$, $\tilde{a}_t$ in the initial period t_o that maximizes $W^a_{t_o}$ subject to the constraints $W^a_t \geq \bar{U} \forall t \geq t_o$. For $t > t_o$ the first-order conditions for a maximum of W^a_t are the same as those for the unconstrained optimal closed loop policy. Since these first-order conditions are independent of t they imply a stationary solution for $t > t_o$. Thus let $\tilde{g}_t = \tilde{g}$, $\tilde{a}_t = \tilde{a}$ for t t_o. Denote

$t > t_o$. Denote

$$\bar{W} \equiv \max_{\tilde{g},\tilde{a}} W^a_t, \; t > t_o \quad (6.7)$$

If $\bar{W} > (1 - \delta)\,\bar{U}$ then the optimal credible policy in period t_o and the unconstrained optimal closed loop policy coincide. If $\bar{W} < (1 - \delta)\,\bar{U}$ the unconstrained optimal closed loop policy is not sustainable by a time consistent policy. For $t > t_o$ the economy degenerates to a nonmonetary economy.

7. Conclusion

This paper has analysed exchange rate management into very simple overlapping generations model. The purpose has been to evaluate monetary policy in an open economy on the basis of its implications for the welfare of individuals in the economy. While this paper has introduced a microeconomic framework for analysing monetary policy, it has done so at the expense of omitting a number of important features of open economies that have received attention elsewhere. For example, in this model government debt provides the only store of value. There is no productive capital and no distinction between assets that are held as stores of value and for transactions purposes. The implications of policy for output and employment are not considered. The lack of a consensus about the microeconomic causes of these phenomena makes their incorporation into an analysis of this sort difficult.

The analysis in this paper identifies two objectives of monetary policy: to provide a desirable store of value, i.e., one with a high and stable rate of return, and to collect a high and stable amount of seigniorage. Despite the difference in approach between this and other studies some similar results emerge. In particular, a policy of pure floating is likely to be more desirable when domestic supply is highly variable relative to the foreign price level and converesly. In addition, the benefits of having a national currency at all diminish when the foreign inflation rate is low and stable.

While having a national currency may be desirable from a national welfare perspective, time consistent policy on the part of the monetary authority may be unable to sustain a currency. This result is most likely to emerge when the primary concern of the monetary authority is the extraction of seigniorage (as when its major constituency is the young generation) and when it is unable to develop a reputation (as when the monetary authority is not perceived as a continuous, infinitely-lived organization). The fact that seigniorage provides a major source of revenue in some countries suggests why these are also countries that must institute exchange controls: the police power of the state is used to maintain the viability of domestic currency faced with competition from foreign currencies. When seigniorage provides the least distortionary source of government revenue at the margin, such policies may be optimal.

This paper has considered government liabilities that take the form of currency. Introducing a coupon on this liability would not affect the analysis. Hence the model applies to government borrowing generally rather than simply to monetary issue. Introducing a distinction between monetary and non-monetary debt would require introducing a transactions motive for holding money. This aspect of the microfoundations of exchange rate management has been explored by Stockman (1980) and Helpman (1981). An integration of the portfolio considerations examined here and the transactions motives treated in this other literature constitutes an important topic for future research.

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