

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 42

THE RELEVANCE OF INTERNATIONAL LIQUIDITY TO DEVELOPED COUNTRIES

Richard N. Cooper

December 11, 1967

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

Preliminary
November 1967

International reserves are acceptable means of international payment held by national monetary authorities to be used to support the nation's exchange rate during periods in which total payments to foreigners exceed total receipts from foreigners. It is conventional to regard as international reserves gold, convertible currencies -- notably the dollar, the pound sterling, and (within the franc area) the French franc -- and increasingly the "reserve position" at the International Monetary Fund. International liquidity is a broader and vaguer concept, encompassing not only international reserves but also ready access by monetary authorities to acceptable means of international settlement, such as swap arrangements between central banks or conditional drawing rights on the IMF.

It is not necessary to repeat here the usual arguments concerning the need for improving the quality and the mechanisms for increasing the quantity of international liquidity -- the well-known shortage of gold at the prevailing price, the erratic element introduced into world reserve growth by linking it with a single nation's balance of payments, the perhaps over-emphasized risks of confidence crises arising from reliance on a national currency such as the dollar, and so on. Rather, I will focus on the relationship between liquidity and balance of payments adjustment under a regime of fixed exchange rates on the nature of disturbances to the balance of payments, on the reasons for differing opinions regarding the appropriate growth of liquidity, and on the consequences of inadequate liquidity. The frame of reference will be transactions among developed countries, countries that have convertible currencies and have agreed to eschew direct controls over trade and payments;

but the following remarks are not wholly inapplicable to other countries as well. It will be assumed that international payments are in long run equilibrium, achieved if necessary by occasional changes in exchange rate parities to correct any "fundamental disequilibrium" under the Bretton Woods rules of the game. We are therefore concerned with payments deficits which can be expected to disappear in the course of time.

To focus on the relationship between liquidity and adjustment presupposes that there is such a relationship, and it neglects other arguments for increasing international liquidity, of which two have been prominent. One relates a nation's required reserves to the total value of its international trade; the other relates it to the total value of liquid assets held domestically. The first of these arguments seems to represent either a confusion--there is no reason for monetary authorities to hold reserves for transactions purposes, for that demand will be well supplied by private sources--or a crude proxy for the precautionary demands discussed below. The second argument, promulgated especially by the Netherlands Bank, is apparently based on an assumption that the potential threat to a currency is proportional to the total liquid assets in the economy, and reserves represent precautionary holdings against a run on the currency by residents, much as a commercial bank holds some proportion of its total deposit liabilities in readily accessible form. In this sense international reserves would represent "backing" for the domestic currency, designed partly to instill confidence and partly to meet successfully a "run" by residents in the unlikely event it should arise. On this view reserves are valued chiefly for their balance-sheet role and in the best of worlds would not have to be used. In contrast, the view taken here is

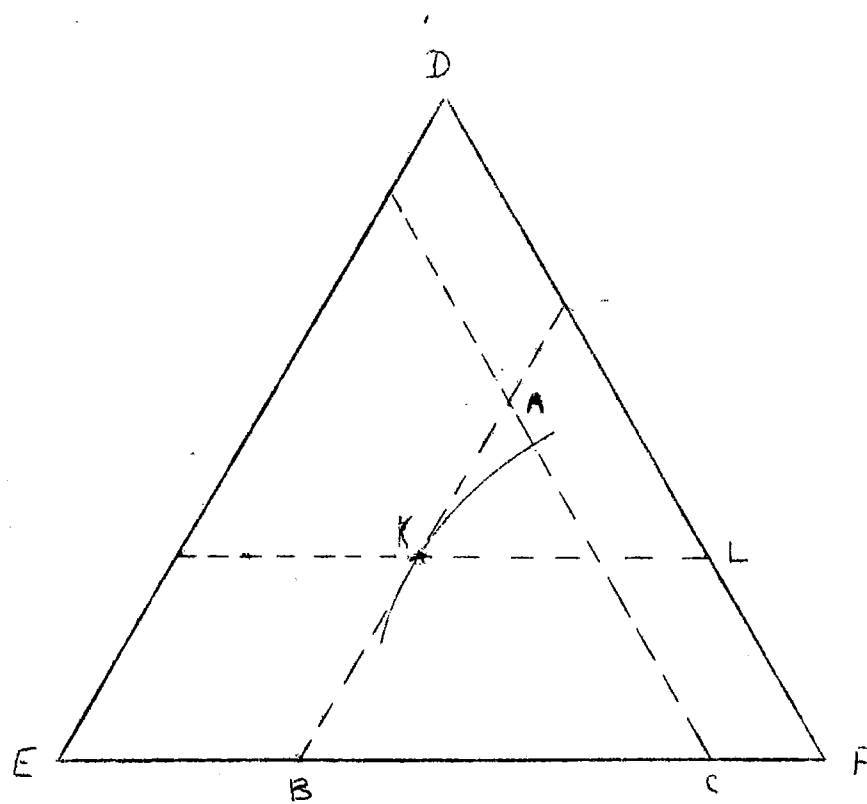
that international liquidity exists to finance payments deficits, with the aim of protecting both domestic and foreign policies against undue pressure from the balance of payments; it should not merely play the role of a finely-dressed mannequin placed in a show window to impress by-passers.

II

Any ex ante deficit of given size arising from some disturbance to a pre-existing equilibrium can either be financed or it can be eliminated through some offsetting action. Following H.G. Johnson, it is useful to classify policies for reducing current account deficits into two broad categories: expenditure-reducing policies and expenditure-switching policies. The first involve measures which improve the trade balance by reducing the level of aggregate demand; the second involve measures, such as import quotas or surcharges and export subsidies, which switch home and/or foreign demand from foreign to domestic products.

These three possible ways to cope with an ex ante deficit -- expenditure reduction (D), expenditure switching (E), and financing (F) -- can be illustrated with an equilateral triangle, the three vertices of which represent exclusive use of each of the three alternatives. The size of the triangle indicates the ex ante deficit, and any point in the triangle indicates the portion of the deficit which is handled by each of the three methods. Thus in Fig. 1 the point A indicates that of the total ex ante deficit EF, CF was eliminated by expenditure-switching policies, BC was eliminated by expenditure-reducing policies, and the remainder, EB, was financed. EB was the ex post, observed deficit. The same proportions could of course be read off any of the

Figure 1



sides, all of which are equal. The time period covered can be assumed to be the total period during which the ex ante deficit exists. This formulation requires specifying the "normal" level of aggregate demand (e.g. in terms of unemployment rate) and the normal degree of policy restraint on international transactions.

Looking at the problem in this way serves to remind us of two propositions. First, the ex post deficit is not typically the best measure of the size of the problem, since other objectives may have been sacrificed to reduce an even larger prospective deficit. Second, there is a three-way trade-off between financing and adjustment. Through expenditure reduction and adjustment through expenditure switching. The more we have of one, the less we need of the other two. Moreover, we cannot specify objectives with respect to the three methods independently, for example by ruling out restrictions on international transactions or subsidies, by setting high employment goals to be maintained at all times, and by keeping a tight rein on the possibilities for financing payments deficits. The three categories are meant to be exhaustive in a regime of fixed exchange rates; given the size of the deficit, some combination of them must be used, no matter what our feelings or rules about them. Thus for a given amount of usable liquidity indicated by the line BA, national preferences may yield a preferred mix¹ between expenditure

1. This preferred mix is likely to vary from country to country, not only on grounds of ideology--laissez-faire vs. economic management--but also in terms of the comparative costs of alternative policies. For a given improvement in the balance of payments, expenditure-reduction is more costly in terms of lost output, relative to expenditure switching, for a large and diversified economy with a small foreign trade sector than for a small and more specialized economy with a large foreign trade sector.

Drawing BA parallel to DE implies unconditional liquidity; but conditional liquidity--such as drawing rights on the IMF--may tilt BA relative to DE, indicating more liquidity will be made available if restrictions are avoided, for example.

reduction and expenditure-switching at K. But international rules and conventions may prevent greater use of switching policies than that indicated by the line CA. In fact case objectives are incompatible, and either the country must compromise its own national preferences (by moving to A) or it must violate the international rules. Both phenomena have been observed in recent years.

The analysis so far has really been oriented toward current account deficits. It must be modified to allow for private international capital movements. The distinction between expenditure-reducing policies and expenditure-switching policies is less relevant, since the dependence of international capital movements on the level of economic activity is far more ambiguous than is true of current transactions. But capital movements, like current account transactions, can be subjected to quota restrictions, taxes, and other restraints. They can also be influenced by monetary policy. It seems symmetrical to group the former with expenditure-switching policies, designating them together as "external measures," that is, measures designed to influence directly international transactions (E in Fig. 1); while monetary policy can be grouped with expenditure-reducing policies as "domestic measures," that is, measures designed to affect the balance of payments indirectly by operating on the level of domestic expenditure and asset holdings (D in Fig. 1).

To the extent that international capital movements are sensitive to interest rates, private capital movements could be used to "finance" any given prospective deficit, without relying on official financing. Monetary policy could be geared to induce the required capital inflow or outflow, and whatever unwanted depressing effect that might have on the level of economic activity could be offset by a more expansionary fiscal policy. This manipulation

of the monetary-fiscal mix does involve, however, the virtual abandonment of monetary policy for purposes other than the balance of payments; it requires a very supple fiscal policy, more flexible than that currently enjoyed in most countries; it presupposes that capital markets are well developed and creditors are able and willing to supply funds to finance even large deficits in response to interest incentives, whether or not they "approve" of the government in question; and it represents only a short-term solution -- quite appropriate in the context considered here -- and offers no substitute in the presence of fundamental disequilibrium for changes in exchange rates.

In a regime of fixed exchange rates the amount of international liquidity needed depends therefore in a crucial way on 1) the acceptable degree of flexibility in imposing restrictions¹ on international transactions, 2) the acceptable degree of flexibility in using monetary and fiscal policy to achieve correction in the balance of payments (this will encompass inflation in surplus countries as well as deflation in deficit countries), 3) the degree to which flows of private capital can be relied upon, and 4) not least, the size of the prospective imbalances requiring action. It is to this last point that we now turn.

III

As time goes on, the absolute size of payments imbalances (the size of the triangle) may be expected to grow. Britain's trade imbalances today exceed its total trade in Napoleon's day or even at the turn of the century,

1. "Restrictions" here is taken to include special taxes and surcharges and voluntary restraint programs, and even export subsidies. Policy "restraints and stimulants" would perhaps be a better term.

and the same is no doubt true of most other countries. But the precise relationship between growth in income and trade, on the one hand, and economic disturbances and payments imbalances, on the other, are highly complex. At best we can only sketch some considerations that bear on that relationship, with no pretense at precision.

First, what are the "economic disturbances" that give rise to payments imbalance? They include discovery of natural resources (gas under the North Sea), technological change leading to new products (commercial jet aircraft, stainless-steel razor blades) and new processes of production (basic oxygen steel, "float" glass), changes in consumer tastes (the waxing and waning of Beatle-mania); they also include changes in the total level of demand (investment booms or declines generated by changes in demand or technology or large shifts in government spending due to outbreaks of war or peace). Disturbances can also be of a more strictly financial type, involving changes in tastes or technology or policy (e.g. taxation of foreign income) with respect to the holding of securities and other financial assets.

We do not know much about the nature of these disturbances, since many of them can be observed only indirectly, through their effects on the level and composition of demand, yet these effects compound both the original disturbances and the reactions to them -- just as observed payments deficits typically reflect some adjustment to large prospective deficits. Probably the safest assumption to make is that they grow in proportion to total economic activity. It is true that one of the chief sources of disturbance has always been the vicissitudes of weather and pestilence working on food crops, and this becomes less important, relative to GNP, as total output grows and economies become more diversified. In a diversified economy, moreover, technological and other changes on the side of supply may be partially off-

setting as far as their effects on international payments are concerned. On the other hand, one might suppose some increase in demand disturbances relative to growing per capita GNP, on the grounds that the more total family consumption departs from expenditure on the basic necessities of life, the greater room there is for shifts in expenditure patterns. This would include expenditure by governments and investment spending by corporations. Moreover, the mutual interaction in expenditure patterns among families with increasingly similar consumption habits will prevent the law of large numbers to work in reducing variation.

However little we may be able to say about these underlying disturbances, however, we can be more definite about the transmission of disturbances from country to country; and that is what is important for the balance of payments. Industrial economies have unquestionably become more interdependent in recent years, as natural and artificial barriers to trade and capital movements have declined, as communications have improved, and as modern techniques of production have become more widely diffused. These developments will be reflected in larger marginal propensities to import, in higher interest (and tax) sensitivity of international capital movements, and probably in greater price competition, despite increasing product differentiation. These developments mean that a given disturbance taking place in one country will get translated into a larger imbalance in international payments. If domestic disturbances grow with GNP, and interdependence between national economies also increases, prospective payments imbalances will grow more rapidly than GNP. By itself, this would suggest that international liquidity needs to grow more rapidly than economic activity -- but not necessarily more rapidly than trade, which will also be growing more rapidly than total economic activity.

But the situation is more complicated than this conclusion implies. For the same growth in economic interdependence that enlarges prospective payments imbalances also tends to reduce the cost, in terms of deflation or inflation, for correcting a given payments imbalance. And in fact the growth in imbalances and the greater ease with which a given imbalance can be eliminated may be expected to offset one another exactly.¹ The same may be said for imbalances arising from the emergence of price differentials or from interest-sensitive capital movements; as interdependence grows, given differentials result in larger imbalances, but those differentials can be eliminated with no greater damage to other economic objectives.

In short, the same developments that transmit imbalances can also smooth adjustment. Thus to the extent that our views about the acceptable use of changes in aggregate demand or of restrictions for international adjustment are also relative to the size of the economy -- e.g., to the extent that we are concerned about the rate of unemployment, not the level of unemployment -- the requirements for international liquidity will increase in proportion to GNP if domestic disturbances do. Actually, our tolerance for variations in unemployment might be expected to diminish over time, as our ability to influence the level of aggregate demand improves; and this would require an increase in liquidity and/or in the use of restrictions on international transactions.

Growing interdependence among national economies has another implication for the need for international reserves, quite different from that just discussed. This need concerns the timing of macro-economic policies in major

1. This and the foregoing propositions are demonstrated more formally -- for a very simple two-country world -- in the appendix.

developed countries in relation to one another. Without closer cooperation among policy-makers, growing interdependence will result in a greater tendency to misjudge the appropriate choice and magnitude of policy measures. That choice depends not only on the initial disturbances and the response of the economy to those disturbances, but also on how policy-makers in other countries respond to the disturbances. Failure to take into account the response of others, and to allow for the impact of one's own actions on other countries, will put heavy demands on reserves during the process of restoring the economy to a desired state.¹ This is an avoidable increase in liquidity requirements, but avoidance requires much closer coordination of economic policies among industrial countries -- with an apparent loss of national sovereignty in the process.

The argument above assumes that countries are willing to use their international reserves when necessary to finance a payments deficit. But any given situation is always fraught with uncertainty: is this year's deficit really temporary, or does it mark the beginning of a persistent deficit? Even though this year's deficit may be due to clearly temporary causes, will it be followed by another "temporary" deficit next year -- and that by yet another? A sufficiently long period of bad luck can render any reserve level inadequate.

In view of these uncertainties, countries will make sequential decisions with respect to balance-of-payments policies, and they may be unprepared to tolerate reductions in reserves in any one period beyond a certain amount --

1. This is shown more formally, and by illustration, in my "Economic Policy Adjustment Among Interdependent Economies," Quarterly Journal of Economics, (forthcoming).

not because they prefer adjustment to financing, but because they are hedging against a future reduction in reserves that may force adjustment under less favorable circumstances. To the extent that this is so, changes in the variance of disturbances will also influence the change in demand for international liquidity. If the variance increases, the demand for liquidity (if unwanted and unnecessary adjustment is to be avoided) will grow more rapidly than for the reasons given above. Unfortunately, even less can be said about the evolution of the variance of disturbances than about the average size of disturbances.

IV

The foregoing admittedly has a rather abstract and indefinite flavor. But if it can be accepted as broadly correct, it suggests a growing need for international liquidity, with some presumption (in the absence of strong evidence or arguments to the contrary) that liquidity should grow roughly with the level of economic activity. Why then are there apparently sharp differences of opinion on the need for additional international liquidity? I venture to guess the differences arise from three sources.

The first, and most important, concerns differing judgments about the nature of the domestic disturbances. Some observers, especially in Europe, feel that disturbances to the balance of payments arise predominantly or largely as a result of mismanagement of monetary and fiscal policies by governments, rather than as a result of the factors discussed above. They argue moreover that the likelihood of mismanagement is closely related to the degree of international liquidity available and that balance of payments "discipline," i.e., the inability to finance deficits easily, is necessary to induce or compel

governments to adopt appropriate policies. On this view, the separation implicitly made above between "disturbance" and "adjustment" is a misleading one; on the contrary, there is thought to be a strong interaction between the two, with more liberal financing facilities leading to larger disturbances.

It is difficult to test this hypothesis empirically against the alternative one that inappropriate macro-economic policies are not the chief cause of imbalances in payments. This difficulty arises in part because the notion of "appropriate" policies is itself ambiguous. Appropriate with respect to what? Obviously not with respect to the balance of payments, for that would be tautological, although some of the arguments seem to have precisely this tautology in mind: payments deficits imply mismanagement of demand. Macro-economic policies could either be "neutral" with respect to the level of domestic activity (e.g. a constant full employment budget surplus or deficit), or they could be geared to counter ups and downs in private demand, relative to capacity (contra-cyclical full employment surplus or deficit). And policies could be appropriate for short-run stability in economic activity and still be inappropriate for long-run stability in the price level, if for example economic activity is stabilized around a level of unemployment which is "too low" for stability in labor costs.

Government acceptance of responsibility for maintaining economic stability with a reasonably low level of unemployment is widespread, and this might provide a workable definition of "appropriate" macro-economic policy. It would then be possible to calculate what national payments positions would have looked like if macro-economic policies had been geared successfully to

these ends. In other words, whether or not the timing or magnitude of fiscal or monetary measures "caused" variations in economic activity, we would hold macro-economic policy responsible for having failed to achieve domestic stability. This calculation would then give some idea, albeit an imperfect one, about the extent to which "inappropriate" macro-economic policies were in fact the source of imbalances in international payments.

Such a calculation for the United Kingdom, which is the only country I have tested, yields the interesting result that about half -- 53 percent to be precise -- of the year-to-year variations in Britain's balance on current and long-term capital account can be attributed to the failure of macro-economic policy to keep the rate of unemployment unchanged at 1.6 percent of the civilian labor force over the period 1955-1966,¹ The remaining half was due to other causes, some no doubt originating inside Britain and others originating outside. This result is somewhat surprising in view of the fact that Britain's "stop-go" economic policy was supposedly governed by balance-of-payments considerations, and one might have expected Britain's economic policy to be stabilizing with respect to the balance of payments rather than destabilizing, as it apparently was. Ironically, in this case the provision of more liquidity to Britain might actually have reduced the need for liquidity to finance payments deficits--provided the British authorities would have gotten the unemployment rate to 1.6 percent and kept it there.

1. This calculation is described, and the data reported, in my "Britain's Balance of Payments: Choices Among Conflicting Objectives," in Richard E. Caves (ed.), The British Economy: An American Appraisal, Brookings Institution, (forthcoming).

This one piece of evidence suggests a fifty-fifty division between the two hypotheses regarding the nature of disturbances to the balance of payments. The above calculation, however, implicitly imputes all variations in unemployment rates to "mismanagement" of macro-economic policy, and that undoubtedly places a greater burden on policy than even the strongest advocate of government management of aggregate demand would claim it could bear. To that extent, the balance of evidence shifts to the first hypothesis, attributing disturbances to factors other than government mismanagement. On the other hand, Britain's basic balance of payments position had a marked downward trend during the 1955-1966 period, even with unchanged unemployment rates, and some observers would attribute that to an ill-conceived attempt to run the economy at too high a pressure of demand, or average, throughout the period. One result was a rise in British export prices relative to those of its competitors. Whether this development represented "mismanagement" or correct management revealing a fundamental disequilibrium involves value judgments it is not necessary to make here. But if all of the secular deterioration is attributed to "mismanagement," then the year-to-year variation in Britain's payments position explained by mismanagement rises to 71 percent, leaving 29 percent to other factors. This calculation can be regarded as placing an upper limit on the mismanagement hypothesis, since it attributes the entire time trend in Britain's payments position to price-cost developments, with no allowance for the differential growth in Britain's overseas markets, changing supply conditions elsewhere, and other factors affecting the trend.

The second major reason for differences of view about the need for additional international liquidity involves differences in national interest with respect to distributing the "burden" of adjustment among countries. When there is one imbalance in payments there must be at least two. How should the required action be divided among countries (including, it should be noted, countries initially in equilibrium, for they will generally not escape the impact of measures taken elsewhere)? Adjustment, no matter what the means, is almost always painful, if only in political terms. (Where it is not, we do not observe imbalances for very long; those imbalances about which there is concern are perforce those for which there is some resistance to measures for eliminating them.) If some unpleasant action must be taken, countries in surplus would prefer the action to be taken by countries in deficit; and vice versa. This fact provides a powerful but unfortunate incentive to introduce moralizing into the debate, with each side attempting to attach moral blame to the other for the imbalance, instead of working out how it may be reduced at least cost to the community of nations.

Ample liquidity tends to shift the pressures for adjustment from deficit to surplus countries. Even if the surplus countries are content to accumulate reserves and take no positive steps to reduce the imbalance, they are forced to choose between accepting the internal inflationary consequences of a balance of payments surplus and taking positive action--raising taxes or reducing government expenditures or restricting domestic credit to an extent that they would otherwise not do--to offset the inflationary pressures. Limited supplies of international liquidity, on the other hand, tend to shift the pressure for adjustment from surplus to deficit countries. If the latter cannot finance a deficit, even one limited in duration, they must take steps

to reduce it either by reducing domestic expenditure or by restricting international payments.

This problem of distributing the burden of adjustment will always be present. A "world" welfare function is needed to resolve it. One such function would maximize utilization of productive capacity (subject to normal preferred operating levels) at all times; another would be to stabilize an index of world prices.¹ The problem of distribution may be resolved in part by recognition that neither surpluses nor deficits are perpetual, so that over the course of time all countries will find themselves on both sides; they therefore have an incentive to reach a reasonable balance of pressures. But in these matters governments have notoriously little time perspective.

Note that the distribution of burden at issue here is not the same as basic national differences in view over the choice between inflation and unemployment. The choice here, for the surplus country, is between inflation at home and unemployment abroad (mutatis mutandis for the deficit country); this is a very different choice than inflation versus unemployment at home, and it is quite consistent for surplus countries to be highly sensitive to domestic unemployment and still prefer a tight rein on international liquidity, which puts pressure on foreign employment.

The third reason for continuing differences in view on the need for additional international liquidity, trailing very much behind the first two,

1. Mundell argues that stabilizing a world price index would require countries, whether deficit or surplus, to adjust in inverse proportion to their economic size around a norm of price level stability. See Robert Mundell, "The Proper Division of the Burden of International Adjustment," National Banking Review, Vol. 3, September 1965, pp. 81-87.

concerns the fact, noted above, that for interdependent economies there is a trade-off between the need for international liquidity and economic cooperation among countries. True internationalists may want a tight rein on liquidity in order to induce and even to compel close cooperation in economic policies as a preferred alternative to other forms of adjustment. This view involves a gamble that the necessary cooperation will come forth before restrictions on international transactions, which would disrupt the move toward greater world unity. Coordination of policies cannot eliminate the need for international liquidity, but it can reduce the need substantially where countries have been pursuing policies at cross purposes, at least with respect to timing. The emphasis on more international liquidity accepts the sovereign nation as the appropriate unit for economic policy making.

V

The crucial test in whether or not there is a shortage of liquidity lies not in the growth of liquidity relative to our estimates of the growth in disturbances to international payments, but in how countries in fact behave with respect to the balance of payments. A shortage of liquidity, on the view taken here that liquidity is meant to be used to defend domestic and foreign policies, would express itself in the unnecessary use of restrictions over international payments or of deflation of domestic demand -- unnecessary in the sense that these policies had to be reversed in a relatively short time. Recent experience is not decisive on this point, but it has hardly been reassuring. Britain, Germany, Italy, and Japan -- four of the major industrial countries of the world -- all felt it necessary during the past decade to deflate at least in

part for balance-of-payments reasons, and all went further than they intended in doing so. It can be argued that this simply reflected "mismanagement" of demand, that some downward adjustment in aggregate demand was necessary in all cases and that greater skill in the management of demand would have avoided overshooting. But it could also be argued that greater liquidity -- or a greater willingness to use the liquidity available (for neither Germany nor Italy could be thought to have had a "shortage" of liquidity) -- would have permitted a more gradual reduction in demand, with less risk of going too far.

It is also true that after years of successful dismantlement of restrictions on international payments among industrial countries, such restrictions increased substantially in the mid-sixties. It is less clear that they were associated with "temporary" deficits. Canada and Britain both imposed, and then removed, surcharges on imports; but both countries also devalued their currencies before removing them or shortly thereafter. Many restrictions remain, mostly on capital movements out of Britain and the United States and on capital movements into Europe, but also on government purchases in many countries. The current and continuing relevance of international liquidity to developed countries hinges in large part¹ on whether the "mix" between liquidity and adjustment by use of restrictions is optimal for the world as

1. The mechanism for generating liquidity is also relevant for another problem, not discussed in this paper: the prospect that private demand for gold will increasingly outpace new supplies. Growing recognition of this prospect will stimulate speculative purchases of gold, but the intensity of such speculation is likely to depend very much on the apparent dependence of central banks on gold for international liquidity, i.e. on the alternative forms of liquidity that are available to them.

whole. If it is thought desirable to retain a world of fixed and rarely changed exchange rate parities, more liquidity will be necessary to avoid the inefficient methods of "partial devaluation" through restrictions as they have been used. Alternatively, if the risks of allowing more generous increases in liquidity, for the reasons given earlier, are thought to be too great, more effort should be devoted to considering what types of restrictions are least inefficient.

Appendix

$$\text{Let } Y = C + G + X - M + Z$$

$$C = C(Y)$$

$$M = M(Y)$$

$$X = X(Y') = M'$$

where Y is gross domestic product, C is consumption, X is exports of goods and services, M is imports of goods and services, all in constant prices. G is government expenditures used as the policy variable, and Z represents all autonomous expenditure, shifts in which create a "disturbance." Let a similar set of relationships apply for a second country, designated by primed variables.

Combining terms and differentiating completely yields the following system of simultaneous equations:

$$\begin{bmatrix} S + m & -m' \\ -m & S' + m' \end{bmatrix} \begin{bmatrix} dY' \\ dY' \end{bmatrix} = \begin{bmatrix} dZ + dG \\ dZ' + dG' \end{bmatrix}$$

where $S = 1 - \frac{\partial C}{\partial Y}$, $m = \frac{\partial M}{\partial Y}$, and similarly for the second country.

$$\text{Solving, } \begin{pmatrix} dY \\ dY' \end{pmatrix} = \frac{1}{\Delta} \begin{bmatrix} S' + m' & m' \\ m & S + m \end{bmatrix} \begin{pmatrix} dZ + dG \\ dZ' + dG' \end{pmatrix}$$

$$\text{where } \Delta = (S + m)(S' + m') - mm' = SS' + mS' + m'S$$

This gives the familiar Keynesian foreign trade multipliers with repercussions, in the absence of stabilizing fiscal policy.

Define $dB = dX - dM$. $dB' = -dB = 0$ initially. Suppose then $dZ < 0$, $dZ' = 0$, and the primed country takes fiscal action to hold its level of income unchanged. Then in the new equilibrium $dB' = \frac{m}{s+m} dZ$ from the simple foreign trade multiplier without repercussions. To eliminate its new deficit, the primed country will now have to contract government spending by an amount determined by $dB' = \frac{m's}{\Delta} dG'$, on the assumption the unprimed country takes no offsetting fiscal action. This will lead to a reduction in Y' by $dY' = \frac{s+m}{\Delta} dG' = \frac{m}{m's} dZ$.

Suppose now the economic interaction between the two countries is increased by a proportional increase k in m and m' . It follows then that

$$(1) \quad \frac{\partial}{\partial k} \left(\frac{dB'}{dZ} \right) = \frac{ms}{[s+(1+k)m]^2} > 0.$$

$$(2) \quad \frac{\partial}{\partial k} \left(\frac{dY'}{dZ} \right) \bigg|_{dB=0} = \frac{\partial}{\partial k} \left(\frac{m}{m's} \right) = 0.$$

In words: for given dZ the imbalance dB' increases with m , but the impact on income of eliminating this larger imbalance is unchanged if m' has grown by the same proportion.

What about the relationship between imbalances (dB') and trade (dM) for increasing k ? Suppose $M = a + mY + R(Y)$, a Taylor expansion with remainder R .

$$\dot{M} = \frac{dM}{M} = \frac{k(mY+R)}{a+mY+R} ;$$

$$\text{and } \dot{B} = \frac{d\left(\frac{dB'}{dZ}\right)}{\frac{dB'}{dZ}} = \frac{ks}{s+m} \quad \text{for small } k$$

Therefore

$$(3) \quad \dot{M} \gtrless \dot{B} \quad \text{as} \quad \frac{m}{s} \gtrless \frac{a}{mY+R} .$$

Thus $a \geq \frac{m}{s} (mY+R) > 0$ is necessary for $\dot{B} \geq \dot{M}$, so long as s and m are both positive. The average propensity to import must exceed the marginal propensity to import out of additional income. As income grows, this condition will at some point be violated, and for further increases in m and m' imbalance will grow less rapidly than imports.