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PRODUCT LINE RIVALRY

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Abstract:

This paper examines product selection by multi-product firms, taking explicit account of the sequential nature of real decisions: firms choose product lines before the quantity or price rivalry with other firms is resolved. Unlike most previous work on multi-product firms, we focus on demand side strategic considerations rather than on the cost side as a determinant of product offerings. We find that close substitutes being produced by the same firms (segmentation) is a natural outcome, in contrast to conventional wisdom. The influence of entry deterrence on product selection and the possible policy of restricting firms' product offering are also considered.

Product Line Rivalry

1. Introduction

Most firms offer entire product lines rather than single products. There is, however, only a relatively small body of literature devoted to product line selection by multiproduct firms.[1]. What analysis does exist has focussed almost entirely on cost considerations. Multi-product firms are seen to emerge as the consequence of economies of scope in production. A recent survey of the literature on multiproduct firms (Bailey and Friedlander, 1982) addresses only cost-side considerations. Our view is that interactions between the demands for different products, as well as interactions between their supplies, is an important determinant of the range of products that a single firm will produce.

How is it that product lines are determined? Certainly we see several different patterns in the real world. Probably the most common pattern is that each firm produces a wide range of varieties within a product group, and a number of firms produce very similar, and sometimes virtually identical, products. Ford and General Motors produce closely competing product lines, as do Nikon, Canon, and Minolta in the camera industry, and so on.

Occasionally, however, one firm manages to gain almost exclusive control over a well-defined part of the product spectrum, and does not venture into other parts. An obvious example is Rolls Royce in the automobile industry, and a trip through a department store will yield a number of other examples. Naturally, more complex versions of this basic pattern arise. A single firm may have several areas of control in a product group, or two or three firms may dominate one part of the product spectrum, while other firms produce less closely substitutable product lines. In addition, a fairly common historical pattern is for firms to expand the scope of their product offerings and compete more directly with each other as the market grows.

Spence(1976), in a paper concerned mainly with single product firms, suggests the result that, in the multiproduct case, close substitutes will be produced by different firms. The reason is fairly straightforward: if firm A produces product 1 and product 2 is a close substitute, then production of product 2 is likely to appear more attractive to firm B than to firm A because B will not be concerned about the consequent reduction in demand for product 1.

However, one wonders, mightn't firm A recognize that if it doesn't produce product 2, firm B will, and therefore try to pre-empt B. Strategic pre-emption requires a two-stage (or more) decision process. In choosing to produce a particular product firms must anticipate that this will have some effect on later competition. But surely this is precisely the way product selection occurs. As argued by Prescott and Visscher(1977), product selections, once decided upon, are not easily changed. Product selection is a commitment[2] which, to a first approximation, is taken as given in the following output or price rivalry.

In this paper we make a series of straightforward but, we feel, significant points about product selection by multiproduct firms. In particular, using a very simple structure, we find that sequential decisions on product type and output naturally give rise to equilibria in which a single firm monopolizes close substitutes. Such outcomes only hold for certain levels of demand and, therefore, might be observed only over some portion of the life cycle of the industry.

Section 2 sets out the basic model, section 3 briefly considers monopoly, and section 4 derives the main results on product line rivalry.

Section 5 presents a result on product line choice and entry deterrence, section 6 addresses the issue of whether restricting firms to one product each can reduce market variety, and section 7 contains concluding remarks.

2. The Model

To examine product line rivalry we consider a constellation of four possible products. Two products are close substitutes for each other and more distant substitutes for the other pair, which are in turn close substitutes for each other. In particular, commodity pairs (1,2) and (3,4) are close substitutes, while pairs (1,3), (1,4), (2,3) and (2,4) are more distant substitutes. This is about the simplest structure in which the question of whether competing multiproduct firms produce close or distant substitutes can be addressed. Demand for the four products arises from a utility function that is approximated by the form

$$U(X,m) = u(x^1,x^2,x^3,x^4) + m \tag{1}$$

where x^i is consumption of product i, X denotes a vector of x^is, and m is expenditure on other goods. The inverse demand for product i is then just the derivative of u with respect to x^i:

$$p^i(X) = u_i(x^1,x^2,x^3,x^4) \tag{2}$$

where p^i is the inverse demand (or price) for product i, and where subscripts denote partial derivatives. To focus as clearly as possible on the essential issues we assume that the demand structure is perfectly symmetric except for the differences in substitutability already described. To say that goods 1 and 2 are closer substitutes than 1 and 3 means that the response of p^1 to a change in the output of good 2 is greater in absolute value than the response to a change in x^3. Since these goods are substitutes the cross price effects are negative, and we have

$$p^i{}_j(X) < p^i{}_k(X) \tag{3}$$

where i and j are close substitutes and i and k are more distant substitutes. We could imagine that the degree of substitutability might vary with X and that goods that were close substitutes in some ranges might be distant substitutes in others. To make the questions we wish to address well-defined without complication, (3) is assumed to hold uniformly: goods 1 and 2, for example, are always closer substitutes than 1 and 3.

One example of a computationally simple demand structure with the properties described here arises from quadratic utility: $u = aX + X^TBX$, where b_{12} and b_{34} are equal and exceed other off-diagonal elements of this symmetric matrix B, which are themselves equal. We will use this functional form in some illustrations of our results.

A firm that produces any of the four products must have made three decisions: (i) how many products to produce (the scope decision) (ii) which particular products to produce (the line decision) and (iii) which quantity to produce (or which price to charge) for each product. As logical possibilities we might imagine that these decisions could be made sequentially, or that the first or second two, or even all three, could be made simultaneously. Which assumption is appropriate in any particular case depends on actual technological considerations. In most of our analysis we treat the product line decision as strictly prior to the final price or quantity choice. In other words, firms establish prices or quantities taking their own and their rivals line and scope decisions as given.

The final stage may, as indicated, be either a price decision or a quantity decision.[3] For concreteness, in this paper, we take quantity as the third stage decision variable. We have examined a number of our results when price is the final decision variable, and the central insights of our analysis were not affected.

The overall equilibrium concept we work with is the subgame perfect equilibrium. This equilibrium concept incorporates two important ideas: first, the equilibrium is non-cooperative so that at each stage equilibrium occurs when each firm is maximizing profit, given the current and previous strategy variable levels chosen by its rival, and second, each firm understands at any stage how future stages will be affected by current decisions.[4]

Our assumptions about technology are very simple. Each firm incurs a sunk cost K for each product it plans to produce at the time of the scope decision. (Indeed, it may be this sunk cost which contributes to making the scope decision credible.) A constant marginal cost c is incurred at the time quantity decisions are made. K and c are assumed to be the same for all four products. Note that, while there is interaction among demands for the four commodities, we assume that their cost structures are independent: in particular, there are no economies or diseconomies of scope.

3. Monopoly

Although we are principally concerned with the rivalry between firms, there is one important insight to be established for the monopoly case. Specifically, if a monopolist chooses to produce only two products, it will choose two distant substitutes rather than two close substitutes. The reasoning involved is fairly straightforward, but it is worth being precise. Imagine that the monopolist is producing products 1 and 2, which are close substitutes, at the profit-maximizing levels. By symmetry $x^1=x^2(=x)$. Holding the output level of product 1 fixed, imagine replacing production of x^2 with an equal amount of x^3. Let $X' = (x,x,0,0)$ and let $X'' = (x,0,x,0)$. The effect on the price of good 1 is as follows.

$$\Delta p^1 = p^1(X") - p^1(X')$$
$$= (p^1_3(X^*) - p^1_2(X^*))x \qquad (4)$$

for some X* on the line segment joining X' and X". (This is an example of the mean value theorem[5].) From (3), this price change must be positive and, by symmetry, the other price must also rise. Therefore, even without optimal readjustment of quantities, prices and profits must rise. Therefore, a monopolist who plans to produce just two goods, and who is unconcerned about entry, will produce two distant substitutes. This result serves as a useful base for comparison with the two firm case.

4. Three-Stage Duopoly

4.1 The output decision

We now discuss the duopoly equilibrium when firms make the scope, line, and output decisions in sequence. We examine the decisions in reverse order, starting with the output decisions of firms already committed to particular line and scope decisions. The third stage is modelled as a Nash quantity (or Cournot) game, taking line and scope decisions of both firms as given. There are many possible configurations of scope and line with which firms might enter the quantity stage. Each of the four products might be produced by firm A, by firm B, by both firms, or by neither firm, giving rise to 256 possibilities. Many configurations are, however, isomorphic to one another, and many are relatively uninteresting in that they do not bear on the questions of interest, as, for example, when one firm produces all four products and the other nothing.

The most interestiong situations for our analysis are those in which each firm produces two products. Firm A might produce one pair of close substitutes while firm B produces the other pair. We refer to this case as market segmentation. An alternative, market interlacing, occurs when each firm produces two less closely related products, as, for example, if firm A produces goods 1 and 3 while firm B produces goods 2 and 4. Note that segmentation or interlacing is determined in the line stage, and is taken as given when final output levels are being determined.

Consider the firms' profits in the two cases. Firm A's profit under segmentation is

$$\pi^s = p^1x^1 + p^2x^2 - c(x^1 + x^2) - 2K \tag{5}$$

where, for concreteness, firm A is assumed to produce goods 1 and 2. The superscript s denotes segmentation. The first order condition associated with product 1 can be written

$$\pi_1^s = MR^1 + x^2p_1^2 - c = 0 \tag{6}$$

where $MR^1 = x^1p^1{}_1 + p^1$, is own marginal revenue. Second order conditions are

$$\pi_{ii}^s < 0 \tag{7}$$

$$\pi_{11}^s\pi_{22}^s - \pi_{12}^s\pi_{21}^s > 0 \tag{8}$$

The first order condition for product 2 is similar to (6). Firm B producing products 3 and 4 has symmetric first and second order conditions. Each first order condition shows, implicitly, the profit-maximizing choice for one product, given the output levels of the others. The solution of these four reaction functions is the (noncooperative) Nash equilibrium in quantities. By symmetry there is an equilibrium in which all quantities are equal (and all prices are equal), and we assume that demand is sufficiently regular that this equilibrium is unique. We assume also that own marginal revenue declines when the output of any other good rises.

$$MR^i_j < 0 \tag{9}$$

This is a natural condition which holds for most (but not all) plausible demand structures. (See our discussion in Section 6).

Under interlacing, firm A produces, let us say, products 1 and 3. (One can substitute i and k to achieve generality.) This leads to the first order condition

$$\pi^t_1 = MR^1 + x^3 p^3_1 - c = 0 \tag{10}$$

and to similar second order conditions as before. The subscript t denotes interlacing. The fundamental comparative property of segmentation and interlacing is expressed in Proposition 1.

Proposition 1

The segmented structure gives rise to higher prices and profits than the interlaced structure.

Proof: By symmetry all products sell for the same price, denoted p^s under segmentation and p^t under interlacing. There are three possibilities: $p^t = p^s$, $p^t > p^s$, or $p^t < p^s$. The first two lead to contradictions. Consider first the equality case. If prices are equal then quantities must also be equal in the two regimes, and so must MR^1. However, ${p^2}_1 < {p^3}_1$ so (6) and (10) cannot both be satisfied. Therefore p^t cannot equal p^s.

Now consider $p^t > p^s$. This implies $x^t < x^s$ and, by (9) that MR^1 is larger under interlacing. Since $p_1^2 < p_1^3 < 0$, it follows once again that (6) and (10) could not both be satisfied. Therefore $p^t < p^s$ as was to be shown.

That profits are higher when price is higher and output lower follows from the observation that both structures have output levels above the joint-maximizing level, and that profit declines monotonically with symmetric increases in output beyond the joint-maximizing level.

* * *

These interlacing and segmentation structures are only two of many possibilities. Even confining attention to the scope structure of two products per firm we might imagine that the same product or products could be produced by both firms, leaving one or two products unproduced. It is clear that profits in such cases would normally fall short of profits even in the interlaced case, given the symmetric structure of demand. There is also a series of cases in which each product is produced by at least one firm, with some overlapping in the sense that some products are produced by both firms. We defer consideration of these and other cases for the present so as to move on to consideration of the line decision.

4.2 The line decision

When making the line decision firms take the scope decision of how many products to produce as fixed, and have only to decide upon which products to produce. The case for separation of the line and scope decisions is not as compelling as that for separation of line and output decisions. Nevertheless, it seems to capture an important flavour of real product selection in that firms often commit themselves to a particular market, especially to new or anticipated markets, well before actual product types are decided upon. Separation of line and scope is not crucial to the analysis in any case, but it is our feeling that the full three stage model brings out the logical structure of the argument most clearly.

When making the product line decision firms are aware that they will be involved in a noncooperative output game in the future and take this into account in the line stage. Consider first the case in which each firm is committed, from the scope stage, to producing two products. There are several possibilities we wish to examine. Firms may make product choices simultaneously, in which case the equilibrium is just the usual noncooperative reaction function equilibrium where reactions are product selections. Alternatively, firms may move sequentially in choosing product type. In addition, we introduce a simultaneous Stackelberg equilibrium concept.

The Nash product selection reaction functions are quite simple. Given the two products chosen by a rival, one's "Nash reaction" is the optimal choice for one's own two products. With symmetric demand, we restrict attention to the case in which, given the choice of one's rival, the best response is to choose the other two products.[6] Proposition 2 follows directly.

<u>Proposition 2</u>

Both market segmentation and market interlacing are Nash equilibria.

* * *

The reason that both segmented and interlaced structures are observed may simply be that both are Nash equilibria at the line stage and can therefore be part of a subgame perfect equilibrium structure for the entire game. This simple insight itself seems a worthwhile addition to the Spence(1976) discussion. When the real sequencing of product selection and output rivalry are taken into account it is quite possible that close substitutes will be produced by a single firm.

In fact the case for segmentation is much stronger than this. Imagine now that the line decision process is slightly asymmetric so that one firm is able to choose its two products first. This may occur because of random factors in the product selection process or for some other exogenous reason. Formally the game becomes a four stage game. As part of the perfect equilibrium structure, the first firm to choose knows what its rival will do in the next stage; in particular it knows that whichever products it chooses, its rival will choose the other two[7]. Effectively, then, the first firm is in a position to choose either market segmentation or market interlacing as industry structures. Since market segmentation leads to higher profits for each firm, Proposition 3 is immediate.

<u>Proposition 3</u>

If firms enter the line stage sequentially rather than simultaneously, market segmentation is the equilibrium.

* * *

There is some disagreement over whether exogenous asymmetries of opportunity of the sort underlying Proposition 3 are appropriate. We remain agnostic on this issue, but we now present an equilibrium concept that does not rely on asymmetries but leads unambiguously to market segmentation: the Stackelberg leader-leader equilibrium.

We define a Stackelberg strategy as one which involves taking into account the contemporaneous noncooperative reaction of one's rival in setting one's own strategy.[8] In the output case, if both players follow Stackelberg strategies, the outcome is not an equilibrium because both firms choose an output other than what the other expects. However, a Stackelberg strategy at the line stage leads a firm to assume that if it chooses two products, its rival will choose the other two, leading it to choose two close substitutes. The other firm, also following a Stackelberg strategy, will be doing the same thing. The Stackelberg equilibrium arises when the firms choose different pairs.

<u>Proposition 4</u>

The product line game has a joint Stackelberg equilibrium, and this equilibrium is characterized by market segmentation.

* * *

Note that this joint Stackelberg solution, because it coincides with a Nash equilibrium, is self-enforcing from a noncooperative point of view, and is therefore credible to the firms in earlier stages, and is, as a result, admissible as part of a subgame perfect equilibrium structure.

Stepping back for a moment from the formal equilibrium concept, let us consider the nature of the firms we are considering. These firms are not naive. They understand the incentive structure in which they operate and they perceive that other firms are very much like themselves. They go to the Nash output equilibrium, not because of a naive adjustment mechanism based on expectations that are continuously falsified, but because they accept that the Nash equilibrium is an individually rational solution. Firms would like to collude, but in the absence of clearly specified enforcement mechanisms that would make collusion individually rational, cannot expect to achieve the collusive outcome. What they can do is make decisions at earlier stages that affect the outcome of the Nash game. If an earlier decision yields a Nash equilibrium that is strictly better for both firms, then surely this is the decision one would expect the firms to make. This is the nature of the joint Stackelberg equilibrium, and leads to a strong presumption in favour of the segmented solution, given the initial scope decision of two products each.

4.3 The Scope Decision

Why should the firms settle on two products each? Consider the Nash reaction functions for the scope decision. Demand may be sufficiently low that if one firm commits to only one product, the other firm would prefer not to enter at all. On the other hand, demand may be so great that even if one firm committed itself to all four products, the optimal response of its rival would be to produce all four products also: complete overlapping.[9] Only if demand is in that intermediate range where the optimal response to a scope decision of two is also two can the equilibrium structure described in the previous subsection emerge. Nevertheless, the point remains that there are ranges of demand for which market segmentation is the full subgame perfect equilibrium. However, as growth occurred in the market and the game were repeated, market segmentation would be replaced by market overlapping.

The overlapping market is similar in essence to the Spence(1976) argument that similar products will be produced by different firms. With overlapping, identical products are produced by different firms. The reason is as follows. Suppose firm A has committed itself to all four products, and firm B is considering entering. Firm B recognizes that in the final output stage it will not take into account effects of its production on the revenue of firm A: it knows it will have an incentive to expand output beyond what firm A would choose for itself and therefore knows it will earn some variable profit. If demand is sufficiently high so that this variable profit will exceed sunk costs, firm B will enter. Firm A cannot deter entry because it can make no credible threat; the only thing firm B believes about firm A is that it will always follow its current individual best interest. If, as in Dixit(1980), Spence(1977,1979) or Friedman(1977), we introduced a capital decision which could affect final stage marginal cost, Firm A could undertake additional strategic behaviour, but it would not even then necessarily find it profitable to deter entry by firm B.

We have presented a very simple model which we believe throws some light on product line rivalry between firms. We find that market segmentation is a very reasonable outcome once the multi-stage structure of market rivalry is explicitly recognized, although many other configurations are possible, including overlapping of firms. In the next section we discuss how the threat of further entry can lead to a situaion in which interlacing rather than segmentation is the more likely outcome.

5. Competition as Entry Deterrence

Our analysis thus far has assumed that at the time firms make their line decisions there is no possibility of further entry. Relaxing this assumption leads to the possibility that the outcome of sequential product line choice or a joint Stackelberg equilibrium is one of market interlacing: firms that have already entered and made a scope decision deliberately choose an interlaced structure to make the market more competitive, reducing the profitability of further entry.

Consider again the constellation of four products of equation (1) and the product line decisions of two firms, firms A and B, each having a scope of two products. Propositions 2 and 3 established the presumption in favour of segmentation in the absence of a threat of entry. Assume, however, that further entry is possible. Consider, for simplicity, the case of a single firm enterring and establishing production of just one of the four products. Because of the symmetry of our specification, it does not matter which one, so assume that it is product 1. If firm A has committed itself to products 1 and 2 and firm B to products 3 and 4 (the segmented case), then the profits of the three firms will be given by

$$\pi^{ASE} = p^1(X)\ x_A^1 + p^2(X)\ x^2 - c(x_A^1 + x^2) - 2K \tag{11}$$

$$\pi^{BSE} = p^3(X)\ x^3 + p^4(X)\ x^4 - c(x^3 + x^4) - 2K \tag{12}$$

$$\pi^{ESE} = p^1(X)\ x_E^1 - cx_E^1 - K \tag{13}$$

where $X = (x_A^1 + x_E^1, x^2, x^3, x^4)$

and the outputs are at their Cournot equilibrium values.
Here x_A^1 denotes the output of commodity 1 produced by firm A and x_E^1 that produced by the entrant; π^{ASE}, π^{BSE} and π^{ESE} denote equilibrium profits of firm A, firm B, and the entrant, respectively, under market segmentation with entry.

Under market interlacing, with firm A committed to producing product 1 and 3 and firm B to 2 and 4, with entry profits will be given by

$$\pi^{AIE} = p^1(X)\ x_A^1 + p^3(X)\ x^3 - c(x_A^1 + x^3) - 2K \tag{15}$$

$$\pi^{BIE} = p^2(X)\ x^2 + p^4(X)\ x^4 - c(x^2+x^4) - 2K \tag{16}$$

$$\pi^{EIE} = p^1(X)\, x_E^1 - cx_E^1 - K \tag{17}$$

where now π^{AIE}, π^{BIE} and π^{EIE} denote the equilibrium levels of the three firms' profits when product lines are interlaced. X continues to be defined by (14), and the outputs assume their Cournot values under interlacing.

Finally, under market intelacing <u>without entry</u> firms A and B will earn π^{AI} and π^{BI} given by

$$\pi^{AI} = p^1(X)\, x^1 + p^3(X)\, x^3 - c(x^1 + x^3) - 2K \tag{18}$$

$$\pi^{BI} = p^2(X)\, x^2 + p^4(X)\, x^4 - c(x^2 + x^4) - 2K \tag{19}$$

$$X = (x^1, x^2, x^3, x^4) \tag{20}$$

We now state:

<u>Proposition 5</u>

With the threat of further entry the interlaced stucture can give rise to higher prices and profits than the segmented structure.

<u>Proof</u>: This result obtains if : (1) under segmentation entry is profitable (π^{ESE} 0), (2) under interlacing it is not ($\pi^{EIE} < 0$) and (3) firms A and B earn higher profits with an interlaced structure and no entry than with a segmented structure with entry ($\pi^{AI} > \pi^{ASE}$ and $\pi^{BI} > \pi^{BSE}$). To establish that these three conditions can be satisfied simulataneously we present an example using the linear demand structure introduced in Section 2, $u = aX + X^TBX$. (The calculations are long and tedious, and were done on a

computer). When $b_{ii} = 1$, $i = 1, 2, 3, 4$; $b_{12} = b_{34} = .3$ $b_{13} = b_{14} = b_{23} = b_{24} = .1$, $a = 5$, $c = 2$, and $K = .4$ we obtain, under segmentation with entry:

$$P = (2.93, 3.26, 3.37, 3.37)$$
$$\pi^{ASE} = 0.16$$
$$\pi^{BSE} = 0.64$$
$$\pi^{ESE} = 0.03$$

Under interlacing with entry:

$$P = (2.85, 3.20, 3.23, 3.25)$$
$$\pi^{AIE} = 0.23$$
$$\pi^{BIE} = 0.56$$
$$\pi^{EIE} = -0.04$$

Finally, under interlacing without entry:

$$P = (3.27, 3.27, 3.27, 3.27)$$
$$\pi^{AI} = 0.66$$
$$\pi^{BI} = 0.66$$

At these values market segmentation permits entry while interlacing does not. The initial two entrants earn higher profits under interlacing without entry than under a segmented structure with entry.

* * *

When profits are higher under an interlaced structure, Proposition 2 continues to hold; both market configurations constitute Nash equilibria. Proposition 3 and 4 are changed, however. Sequential entry leads to market interlacing, which is also the outcome of a joint Stackelberg equilibrium.

It is important to note that we are considering situations in which a third firm commits itself to entry after firms A and B have made their product line decisions. If the entrant had committed itself beforehand, a threat by firms A and B to interlace is not credible. The subgame perfect solution will in our example again be one of segmentation.

6. On the Possibility of Destructive Competition

Our analysis thus far has considered how competition between firms leads to choice of product line. An interesting, closely related issue concerns how market structure can affect the degree of product variety. In this section we ask whether limiting each firm to one product can reduce product variety. Limiting product offering by any given firm does not seem to be a major regulatory objective, but it sometimes is used as a pro-competitive policy, as, for example, in granting of local radio licenses: each station is allowed at most one frequency.

We examine the simplest possible case. There are two possible closely substitutable products. For some levels of demand the market will not support two rival firms, but a single firm could survive, producing a single product, and the other firm, being rational, would not enter. It is also possible that a single firm producing both products could earn nonnegative profits, while two noncooperative firms with one product each could not.

This suggests the possibility that restricting an incumbent two-product monopolist to a single product might serve no purpose other than to reduce variety. A foresighted rival might still choose not to enter. The chief finding of this section is, somewhat surprisingly, that under fairly weak restrictions on preferences this result is impossible. If two Nash-Cournot duopolists, each producing one product type, would sustain losses, then a monopolist would never choose to produce both products. A regulation confining each firm to a single product will not, in itself, reduce product variety.

Consider a situation in which there are two products, each with inverse demand functions given by

$$p^1 = f^1(x^1, x^2) \tag{21}$$

$$p^2 = f^2(x^1, x^2) \tag{22}$$

where

$$f^1(x^1, x^2) = f^2(x^2, x^1)$$

These demand functions have the properties that

$$f^i_i < f^i_j < 0 \qquad i \neq j \tag{23}$$

where $f^i_j \equiv \partial f^i/\partial x^j$. If $f^i_j = 0$ there is no substitutability between the two commodities while $f^i_j = f^i_i$ if they are perfect substitutes.

Let the combined revenue from selling x^1 and x^2 be given by the function $R(x^1, x^2)$ where

$$R(x^1, x^2) = x^1 f^1(x^1, x^2) + x^2 f^2(x^1, x^2) \tag{24}$$

and let $R^i(x^1, x^2)$ represent the revenue from selling x^i given x^j, $j \neq i$. Thus

$$R^i(x^1, x^2) = x^i f^i(x^1, x^2) \tag{25}$$

We call $R(x^1, x^2)$ the total revenue function and $R^i(x^1, x^2)$ the own revenue function. Symmetry of f^i guarantees that $R^1(x^1, x^2) = R^2(x^2, x^1)$.

For simplicity, we assume marginal cost is zero; therefore, the profit of a two-product monopolist is

$$\pi^{2m} = \max_{x^1, x^2} [R(x^1, x^2) - 2K] \tag{26}$$

Our symmetry assumption along with second-order conditions for a maximum guarantee that the two-product monopolist will establish $x^1 = x^2$. Denote the common value of x_i that attains π^{2m} as x^{2m}. A two-product monopolist therefore earns a profit

$$\pi^{2m} = R(x^{2m}, x^{2m}) - 2K = 2R^1(x^{2m}, x^{2m}) - 2K \tag{27}$$

A one-product monopolist can attain a profit level

$$\pi^{1m} = \max_{x^1} [R(x^1, 0) - K] \tag{28}$$

Denote the value of x_1 that attains π^{1m} as x^{1m}. A one product monopolist can therefore earn a profit

$$\pi^{1m} = R(x^{1m}, 0) - K = R^1(x^{1m}, 0) - K \tag{29}$$

(Here we have assumed, with no loss of generality, that the one-product monopolist produces product 1).

Finally, consider the case of Cournot duopoly. The firm producing product line 1 will choose x_1 to attain

$$\pi = \max_{x^1} [R^1(x^1, x^2) - K] \tag{30}$$

Similarly for the firm producing line 2. Our assumption of symmetry and the Routh-Hurwicz stability conditions for a Cournot duopoly insure that the two firms establish the same level of output, which we denote x^d. The profit of each duopolist is therefore

$$\pi = R(x^d,x^d)/2 \; - \; K = R^1(x^d,x^d) - K \tag{31}$$

Stability of the Cournot duopoly equilibrium and satsfaction of the second-order condition for a maximum for a two product monopolist imply the Routh-Hurwicz conditions on the total revenue function:

$$R_{ii} = x f^i_{ii} + f^i_i < 0 \qquad i=1,2 \tag{32a}$$

$$R_{11}R_{22} - R_{12}R_{21} < 0 \tag{32b}$$

We also impose the following restrictions on the marginal revenue functions:

$$R^i_{ij} = f^i_j + x f^i_{ij} < 0 \qquad i \neq j \tag{33a}$$

$$R^i_{jj} = x_i f^i_{jj} < 0 \qquad i \neq j \tag{33b}$$

Conditions (33) imply restrictions on preferences in addition to those implied by the Routh-Hurwicz stability conditions and require some comment.

First, condition (33a) is equivalent to the restriction that each Cournot duopolist's reaction function in x_1, x_2 space is negatively sloped, since the slope of the reaction functions have the sign of R_{ij}. This condition is satisfied in the linear case and whenever demand for either product line becomes less price elastic as output of the other product rises. In order to be violated the demand for either product line must become significantly more elastic as the output of the other product line increases. Condition (33a) is always satisfied in the two extreme cases in which (i) there is no substitutability between the two product lines (when $R_{ij} = 0$) and (ii) the two product lines are perfect substitutes (when $R^i_{ij} = R^i_{ii} < 0$). Second condition (33b) states that the reduction in revenue from each product line due to a one unit increase in the output of the other product line diminishes as output of the other product line rises. Like condition (33a), this condition is satisfied in the case of linear demand functions. Also like condition (33a), condition (33b) is always satisfied in the extreme case in which there is no substitutability between the two product lines (when

$R^i_{jj} = 0$)

Proposition 6

Under conditions (33a) (negatively-sloped reaction function) and (33b) (diminishing marginal cross-product effects on revenue) two products with independent production technologies will not be produced by a monopolist when two duopolists would not produce them.

Proof: See Appendix

Thus a one product per firm regulation will not lower variety. Furthermore, since such a regulation would cause replacement of a two product monopoly by a noncooperative duopoly, for certain levels of demand, such a regulation could actually improve welfare.

7. Concluding Remarks

This paper has focussed on the (in our view) much neglected subject of product line selection by multiproduct firms. We have restricted attention to "demand side" influences on product selection. It is fairly clear that "cost-side" considerations are very important. In this paper products are independent on the cost side, but if there were, for example, economies of scope between particular products, there would, rather obviously, be, other things equal, a stronger incentive for one firm to produce these products. Oil refineries produce a spectrum of different fuels, from heavy oil to light fuels like kerosene, because they are all byproducts of each other: a fairly strong form of economies of scope.

There is a substantial recent literature on economies of scope and multiproduct firms culminating in the 1982 book by Baumol, Panzar and Willig (BPW). One other important difference between this work and the present paper, aside from the role of the cost side in the analysis, is the assumption concerning the expectations of firms. In BPW, before a firm enters, it takes the current price of each product as given, which produces, not surprisingly, an outcome with some resemblance to perfect competition. Our assumption is rather different: firms understand, before anything is actually produced, how the noncooperative game will work out.

Our basic result is that recognizing the real sequential nature of decision making is important in understanding product line rivalry. Market segmentation, in which each firms controls a certain part of the product spectrum, is an equilibrium outcome, although it will only be observed over some fraction of the life cycle of the industry. An interesting extension suggests itself if we consider the possibility of further entry beyond the first two firms in an industry. An interlaced structure, in which close substitutes are produced by different firms is a more competitive structure than segmentation. More to the point, it is a commitment to greater competition from the point of view of an additional potential entrant. the entrant might therefore be deterred from entry in an interlaced market when it would enter a segmented market: competition as entry deterrence.

We have also considered the possible effect of restricting the number of products a firm may produce. In the simple case of two products we found that such a restriction would not lower variety. If independent Cournot duopolists could not make nonnegative profits producing one product each, a two firm monopolist would not choose to produce two products either.

Our results have implications for the research and development activity of firms that is aimed at introducing new products. Our theory suggests that a firm that is guaranteed a monopoly over a range of potential products will seek to develop those products that are most distant substitutes for what it is currently producing. Production of these products will reduce demand for the monopolist's current products least. When production of a range of potential products is limited to a group of competing firms that are established in the market, firms will seek to develop products that are close substitutes for what they currently produce, since joint production of these products will lead to less intense price and output competition at a later stage. Finally, if there is threat of entry by firms currently outside the market, firms may seek to develop products that are more distant substitutes *because* the consequent competition may be so intense as to deter entry.

Appendix

Proof of Proposition 6:

We need to establish that $\pi^{d} < 0$ (that each duopolist sustains a loss) implies $\pi^{1m} > \pi^{2m}$ (that a monopolist would choose to produce only one product line). In terms of the contrapositive, we need to show that $\pi^{1m} - \pi^{2m} < 0$ implies that $\pi^{d} > 0$, or (with zero marginal cost) that

$$2R^{1}(x^{2m},x^{2m}) - R^{1}(x^{1m},0) > K \tag{A1}$$

implies

$$R^{1}(x^{d},x^{d}) > K \tag{A2}$$

Expression (A1) implies (A2) if the following condition holds:

$$2R^{1}(x^{2m}, x^{2m}) - R^{1}(x^{1m}, 0) - R^{1}(x^{d}, x^{d}) < 0 \tag{A3}$$

i.e. if the revenue of a two product monopolist is less than that of a duopolist plus that of a one-product monopolist.

The rest of the proof is devoted to showing that (A3) holds.

By the definition of a maximum

$$R^{1}(x^{2m}, 0) < R^{1}(x^{1m}, 0) \tag{A4}$$

$$R^{1}(x^{2m}, x^{d}) < R^{1}(x^{d}, x^{d}) \tag{A5}$$

Condition (A3) is therefore implied by

$$2R^{1}(x^{2m}, x^{2m}) - R^{1}(x^{2m}, 0) - R^{1}(x^{2m}, x^{d}) < 0 \tag{A6}$$

Condition (33a) (negatively-sloped reaction functions) implies that

$$x^{d} < x^{1m} \tag{A7}$$

since x^{d} is the value of x^{1} that maximizes $R^{1}(x^{1}, x^{d})$ while x^{1m} maximizes $R^{1}(x^{1}, 0)$.

The Routh-Hurwicz stability condition guarantees that

$$x^{1m} < 2x^{2m} \tag{A8}$$

This result can be shown by defining the function

$$g(\lambda) = R_1[x^{2m} + \lambda x^{2m}, (1-\lambda)x^{2m}] \tag{A9}$$

where R is the total revenue function as defined on page 19.

Taking the derivative of g yields

$$g'(\lambda) = (R_{11} - R_{12})x^{2m} \tag{A10}$$

which is negative if the stability condition is satisfied. We have $g(0) = R_1(x^{2m}, x^{2m}) = 0$, by the first order condition for the monopolist. Then since $g(1) = R_1(2x^{2m}, 0)$ it follows that $R_1(2x^{2m}, 0) < 0$. This implies that $x^{1m} < 2x^{2m}$ since $R_1(x^{1m}, 0) = 0$ and $R_{11} < 0$.

Conditions (A7) and (A8) together imply

$$X^d < 2x^{2m} \tag{A11}$$

The final step is to note that the following condition holds

$$2R^1(x^{2m}, x^{2m}) - R^1(x^{2m}, 0) - R^1(x^{2m}, 2x^{2m}) < 0 \tag{A12}$$

This follows because $R^1(x^1, x^2)$ is a convex function of x^2, given x^1 (by (33b)). Since (x^{2m}, x^{2m}) is the midpoint between $(x^{2m}, 0)$ and $(x^{2m}, 2x^{2m})$, then $R^1(x^{2m}, x^{2m})$ is less than the average of $R^1(x^{2m}, 0)$ and $R^1(x^{2m}, 2x^{2m})$, which leads directly to (A12).

Then (A12) and (A11) imply (A6), which implies (A3), which completes the proof.

* * *

Footnotes

1. The two classic approaches to product selection derive from the work of Hotelling (1929) and Chamberlin (1933). Most of recent work in these traditions, including the widely cited work of Lancaster (1979), Dixit and Stiglitz (1977), Spence (1976) and Salop (1979) focus on the one product per firm case.
2. The term commitment (or "credible threat") refers to the important idea that in strategic interaction, a firm (or player) might reasonably be expected to believe that a rival will only pursue actions that are in the rival's best interests. Threats that can only be carried out through suboptimal behaviour (as in the Sylos-Labini limit output model) are not credible. This idea goes back at least as far as Schelling (1956), but has only received attention recently. Recent work includes Spence (1977, 1979), Friedman (1979), Dixit (1980) and Eaton and Lipsey (1980, 1981).
3. Whether price or quantity Nash equilibria are appropriate depends on the nature of production. Indeed, it may be useful to think of quantity and price as occurring sequentially. If a quantity decision is a credible commitment, due perhaps to practical irrevesibilities in production and high storage costs, and price later clears the market, then quantity should be regarded as the third stage decision variable. Alternatively, if a price announcement is a credible threat, with quantity being the residual variable that clears the market, the third stage should be modelled as a price game. (This interpretation is in Friedman (1980). For some other comments on the issue of price vs.

quantity Nash equilibria see Bresnahan (1981).)

4. The concept of subgame perfection is associated with Selten (1975) and has been the focus of considerable recent attention in the literature. One use of the concept similar to ours is Shaked and Sutton (1982).

5. This is the "mean value for several variables" as described in, for example, Rosenlicht (1968). This kind of use of the mean value theorem must be well known but does not seem to have as widely used in economics as it might have. One similar use is in Spencer(1979).

6. It might seem obvious that with symmetric demand the best response to a two-product choice by a rival is to choose the other two products. (Certainly this is true for the linear case). However, the result is not completely general, but we restrict attention to cases for which it does hold.

7. This market structure is similar to the one examined by Prescott and Visscher (1977), who considered sequential entry by firms precommitted to a single product. Here we consider sequential entry by firms precommitted to two products. Prescott and Visscher also assume that the entry and line decisions are simultaneous rather than sequential as we assume here.

8. The term Stackelberg is sometimes used to mean that one player acts before another. Stackelberg's original model can be interpreted in either way, and usage seems to be divided.

9. This last result depends upon our assumption that competition at the final stage is a Nash game in outputs (a Cournot-Nash game). Were it a Nash game in prices (a Bertrand-Nash game) then no <u>more</u> than one firm would ever produce the same product. (Apart from this, our conclusions here are relatively insensitive to the precise nature of

competition at the output stage.)

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