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THE U.S. STRATEGIC PETROLEUM RESERVE:
AN ANALYTIC FRAMEWORK

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Notes: Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.
1. Introduction

The U.S. Government pursues a number of policies that affect imports of oil. An excise tax is imposed on sales of gasoline and the U.S. Government maintains "strategic reserves" of oil in salt domes. There has been discussion of imposing a tariff on oil both to raise revenue and to improve the U.S. terms of trade.

Oil presents U.S. policy makers with a situation that is unusual in three respects. First, in most areas where a protectionist policy has been pursued by the U.S. government, the motivation has been primarily domestic, to maintain output and employment levels in different regions and sectors. The second-best nature of tariffs and quotas for these purposes is well known. In contrast, many of the existing and proposed policies toward oil have been justified partly on optimal tariff grounds; the United States is a large importer whose level of imports affects the world price. From a national perspective, restricting imports is a first-best policy. Indeed, the current level of protection may be too low.

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Second, oil is an exhaustible resource. Imports in any period affect in an essential way not only the international price today but the world equilibrium in all future periods. The static framework of most trade theoretic tariff analysis is inappropriate.

Third, the strategic behavior on the part of agents other than the U.S. government is important to the effects of policy. For one thing, OPEC constitutes a large supplier. For another, U.S. policies affect the storage and extraction behavior of private agents in the domestic and world economies. The interactions of these groups must be taken into account. Again, a static framework which assumes that all agents except the U.S. government are atomistic is inappropriate.

These three considerations make an analysis of optimal commercial policy in terms of traditional trade models difficult. Before an analysis of the welfare effects of the US Strategic Petroleum Reserve (SPR) can be attempted, an analytic framework identifying its effects on US welfare must be specified. Our purpose here is to develop such an analytic framework. The model we develop does not incorporate all aspects of the SPR that we believe to be important. Nevertheless, it suggests a set of considerations that necessarily arise in a strategic setting between a large importing country and a monopolistic supplier.

The remainder of the introduction provides a brief discussion of the background of the U.S. Strategic Petroleum Reserve and an outline of our analysis.
1.1 Background

The current pattern of general public concern about energy supplies is in sharp contrast to the pattern which prevailed prior to 1973. In the early years after World War II, the U.S. was essentially self-sufficient with respect to crude oil supplies. Concern, largely by members of the petroleum industry, was focused not on problems of shortage but on the price effects of abundance. As a result, the U.S. government imposed an oil import quota in 1959 of 9 percent of the estimated domestic demand. However, imports gradually increased over time and reached approximately 23 percent of total domestic demand by 1972. By April 1973, import quotas were discontinued and a tariff was briefly introduced. Despite the quota, considerable excess capacity for crude oil production developed during the sixties and regulatory federal and state agencies distributed production allocations to the various producers of crude petroleum. The real price of oil was continuously dropping during the sixties until October 1973.

As a response to the oil crisis of 1973 the International Energy Agency (IEA) of the OECD established the International Energy Program. Participants in the program pledged to establish reserves equal to 60 days consumption (to increase to 90 days in 1980). The Strategic Petroleum Reserve (SPR) program was created by the Energy Policy and Conservation Act of 1975 as the U.S. component of the program. Since 1977 (see Table 1) oil has been stored at five underground salt dome and salt mine sites in Louisiana and Texas. Purchases of oil proceeded at a rate of 21 thousand barrels per day during 1977 and 162 thousand barrels per day during 1978 (see Table 1). In late 1978, however, as a consequence of tight oil market conditions associated with the Iranian crisis, the Carter Administration postponed purchases of oil from the stockpile. At
that time seven stockpiling nations agreed to curtail stockpiling acquisitions if such acquisition would "result in any pressure on the world oil market." (Glatt, pp. 22-23). Consequently, purchases fell to a daily rate of 67 thousand barrels in 1979 and 44 thousand barrels in 1980. In 1980, however, oil market conditions slackened and purchases resumed. In that year Congress passed the Energy Security Act that required that the President acquire reserves at a minimum rate of 100 thousand barrels per day (Glatt, p. 11). In fact, during 1981 and 1982 the average acquisition rate has far exceeded that minimum. An issue for the management of the stockpile is whether or not acquisitions (or drawdowns) should respond to world oil market conditions (as the IEA agreement would suggest) or proceed independently of world market conditions (as implied to some extent by the Energy Security Act of 1980). Our analysis sheds some light on this issue.

As of March 1982 the Reserves contained 250 millions barrels of crude oil, while the current plan is to place 750 million barrels of oil in storage by the end of 1989. In most official documentation the SPR is viewed as protection against the consequences of a severe petroleum supply interruption. In somewhat different way Senator Henry Jackson, a strong supporter of the SPR, expressed his view that "...with a strategic petroleum reserve, we will have greater credibility, as I see it, in dealing with this problem (oil prices), and we'll help to stabilize the price situation, which otherwise could be one of great havoc."
Table 1

<table>
<thead>
<tr>
<th>Total World Production</th>
<th>World Minus USSR and China Production</th>
<th>OPEC Production</th>
<th>US Production</th>
<th>US Consumption</th>
<th>IEA² Consumption</th>
<th>SPR³</th>
<th>Million of Barrels Stocks</th>
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<tr>
<td>1973</td>
<td>55748</td>
<td>46193</td>
<td>30989</td>
<td>8208</td>
<td>17308</td>
<td>34150</td>
<td>1008</td>
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<tr>
<td>1974</td>
<td>55910</td>
<td>45595</td>
<td>30729</td>
<td>8774</td>
<td>16653</td>
<td>32960</td>
<td>1074</td>
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<tr>
<td>1975</td>
<td>52552</td>
<td>41837</td>
<td>27155</td>
<td>8375</td>
<td>16322</td>
<td>31870</td>
<td>1133</td>
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<tr>
<td>1976</td>
<td>57405</td>
<td>45592</td>
<td>30738</td>
<td>8132</td>
<td>17461</td>
<td>33770</td>
<td>1112</td>
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<td>1977</td>
<td>59795</td>
<td>47239</td>
<td>31278</td>
<td>8245</td>
<td>18431</td>
<td>34930</td>
<td>21</td>
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<tr>
<td>1978</td>
<td>60165</td>
<td>46898</td>
<td>29805</td>
<td>8707</td>
<td>18847</td>
<td>35880</td>
<td>162</td>
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<td>1979</td>
<td>62698</td>
<td>49116</td>
<td>30928</td>
<td>8552</td>
<td>18513</td>
<td>35900</td>
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<td>1980</td>
<td>59452</td>
<td>45568</td>
<td>26890</td>
<td>8597</td>
<td>17056</td>
<td>33000</td>
<td>44</td>
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<tr>
<td>1981</td>
<td>55710</td>
<td>41885</td>
<td>22665</td>
<td>8572</td>
<td>16058</td>
<td>31400</td>
<td>256</td>
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<tr>
<td>1982(March)</td>
<td>51800</td>
<td>37980</td>
<td>18415</td>
<td>8597</td>
<td>15560</td>
<td>31600</td>
<td>182</td>
</tr>
</tbody>
</table>

1. Petroleum Stocks include crude oil, unfinished oils, natural gas plant liquids and regined products.
2. The International Energy Agency includes 21 member nations (see details in the Monthly Energy Review).
3. Strategic Petroleum Reserves.

The U.S. Oil Industry requires about one billion barrels of crude oil as minimum operating stocks which are about 60 days of petroleum consumption. The current goal of the SPR would almost double the days consumption from the US stock (Table 1). The US is a large consumer of oil, it consumes about 36-40 percent of world oil production (excluding USSR and China). OPEC produces about 50-55 percent of the world production (excluding USSR and China). As such we suggest that the view that the world oil market consists of one large producer (OPEC) and one large consumer (the USA) is a reasonable first approximation. However, the effect of other (small) producers and consumers as well as the large local production of oil in the USA (about 60 percent of current US consumption) should be considered in extensions of this paper.

1.2 Outline

In section 2 we develop a simple two-period model of an oil importing country (the USA) and an oil exporter (OPEC). In section 3 we examine the competitive equilibrium of this model. We show that under certainty and in the presence of a full set of contingent commodity markets there is no role for inventories, not to mention government inventories, of any form. Introducing a "convenience yield" on inventories, on the basis of their use in facilitating production, provides a justification for holdings of inventories on the part of the private sector. In the absence of production externalities, however, there is no reason for the government to hold inventories. Introducing uncertainty by itself does not provide an argument in favor of US private inventories. Uncertainty combined with the absence of full contingent commodity markets or US property rights in OPEC
does imply a role for inventories as a form of portfolio diversification on the part of the USA. Private agents, however, have an incentive to hold inventories at the level that maximizes expected US national welfare. In the absence of externalities, then, we can find no argument in favor of US government inventories when all agents, including the US government, behave competitively.

Eckstein and Eichenbaum (1982) show that when oil suppliers are competitive and US imports have an effect on oil prices, there exists an optimal, time consistent tariff policy for the USA. However, there is no role for US government inventories. Eckstein and Eichenbaum conjectured that if there is a case for US government inventories it should stem from strategic considerations arising from the fact that oil prices decrease as US inventories rise.

In section 4 we turn to a strategic setting in which the US government and OPEC both have the potential to exercise market power. Imposing the optimal tariff each period (the strategy considered by Eckstein and Eichenbaum (1982)) provides the first-best means for the US government to exploit its market power. However, unless the US government sets its tariffs before OPEC establishes its price each period, the US government has no incentive to set a tariff at the ex ante optimal level at the time it makes its tariff decision.

In the absence of equity investment by OPEC in the USA the ex post optimal tariff is in fact zero. If OPEC has invested in US equity, however, the optimal ex post tariff is positive as long as oil and capital are complements in production. The tariff acts indirectly as a tax on OPEC's capital income. In anticipation of the tariff, OPEC sets a lower period
2 price. OPEC reduces its price so much that the US price is actually lower, despite the tariff. In addition, equity investment by OPEC acts directly to reduce OPEC's second period price. The reason is that OPEC takes into account the effect of its pricing decision on the rate of return on its investment in the USA. When capital and oil are substitutes, a higher oil price means a lower return. There are thus two channels whereby a high level of equity investment by OPEC in the USA acts to reduce the second period price of oil. Nevertheless, even when equity holdings are positive the US government would increase US welfare if it could credibly impose the tariff that is optimal from an ex ante perspective.

In this context inventories can act as a second-best substitute for a tariff. The US government can reduce the period 2 price by buying inventories in period 1 and selling them in period 2. In section 4 we show how, given the period 1 price, the US government has an incentive to buy inventories in period 1 and to sell them in period 2 in order to lower the period 2 price. No atomistic private agent has an incentive to pursue this policy since he will take the second period price as given. Whether or not the government’s ex post optimal inventory response actually raises US welfare vis-à-vis the no inventory situation cannot be ascertained in general. In fact it can go either way.

Nichols and Zeckhauser (1977) show how in the framework we consider here (in the absence of taxes or investment of any form), an inventory policy can raise US welfare as well as OPEC's. An inventory policy reduces the distortion due to OPEC's monopoly power. The USA and OPEC share the gain. We present their example in section 5. We find, however, that their result is very sensitive to their specification of the problem. We show in another example that if OPEC's utility function
is logarithmic rather than linear in each period's consumption, that a US inventory policy lowers US welfare relative to a no inventory situation. A lower US welfare is also obtained when OPEC and the US government set price and inventory simultaneously rather than with OPEC acting as a Stackelberg leader. In each case the positive impact of the anticipation of a US inventory on the first period price more than offsets its negative impact on second period price. When the US government chooses inventories the period 1 price is a bygone so that the US government nevertheless has an incentive to set inventories at a positive level. In this case the government's capacity to acquire a stockpile actually reduces US welfare. These results imply that if a government inventory policy is to raise US welfare, inventory purchases must respond to OPEC's prices, i.e. OPEC must act as a Stackelberg leader in setting price each period. Another example shows that, when the US government acts as a Stackelberg leader in setting inventories, the optimal level is zero.

Section 6 contains a discussion of some other work that considers the desirability of government inventories. Here we discuss papers by Maskin and Newbury (1978), Wright and Williams (1982) and Tolley and Wilman (1977). Finally, section 7 contains some concluding remarks.
2. The Technology

In this section we describe the main aspects of production and preferences of the environment that we consider in this paper. Our focus is on bilateral trade in an exhaustible resource, oil, that enters into production of a single consumed good. There are two nations: the oil consuming country - the "USA"; and the oil producing country - "OPEC". There are two periods for consumption and production. There is only one consumption good that if stored in the first period serves as capital in the second period. We define the following variables:

\[ C_i = \text{Consumption in the USA in period } i = 1, 2 \]
\[ C^*_i = \text{Consumption in OPEC in period } i = 1, 2 \]
\[ K_i = \text{Capital stock in the USA in period } i = 1, 2, \ (K_1 \text{ is given as an initial condition.}) \]
\[ \Delta K = K_2 - K_1 = \text{investment in capital in the USA in the first period} \]
\[ o_i = \text{consumption of oil in the USA in period } i = 1, 2 \]
\[ I = \text{inventories of oil in the USA at the end of the first period} \]
\[ M_i = \text{imports of oil in the USA in period } i = 1, 2 \]
\[ \theta_i = \text{one plus the import tax rate on oil in the USA in period } i = 1, \]
\[ \theta_k = \text{one plus the tax rate on foreign investment in the USA in period} \]
\[ P_i = \text{international price of oil in terms of the single consumption good in period } i = 1, 2. \]
\[ r \] = Interest payments on capital investment in the USA in the second period.

\[ R^* \] = Stock of oil in OPEC at the beginning of the first period

\[ Q_i = F(K_i, O_i) \] = output of the consumption good in the USA in period \( i = 1, 2 \). \( F(\cdot, \cdot) \) is strictly concave in both arguments.

\[ H \] = OPEC investment in the USA in period one

\[ D(I) = \text{Units of oil in the second period given an inventory of I units of oil in the first period. For all } I > 0, 0 \leq D(I) \leq I , \text{ and } D'(I) > 0 \text{ and } D''(I) > 0 \]

We assume that the production of the only consumption-capital good is done in the USA. There is no depreciation of capital and extraction costs of oil are zero. OPEC may invest some of its oil revenues in the first period in the USA and receive the interest payments in the second period. Finally, preferences of the representative consumer/producer in the USA and OPEC are given, respectively, by

\[
U(C_1, C_2) = U(C_1) + \beta U(C_2) \\
U^*(C_1^*, C_2^*) = U^*(C_1^*) + \beta^* U^*(C_2^*)
\]

where \( U(\cdot) \) and \( U^*(\cdot) \) are strictly concave and \( \beta \) and \( \beta^* \) are between zero and one. Obviously one may consider a much more complicated environment in which the total reserves of oil in OPEC, \( R^* \), are uncertain,
where the USA also has an exhaustable stock of oil, extraction of oil is costly, there are third countries, etc. We later consider some extensions along these lines, but we prefer first to present our model in its simplest form.

While this framework is very simple, we believe that it captures the essential relationships between the United States and the oil producing countries. First, it recognizes, albeit in a simple way, that the supply of oil depends fundamentally upon the intertemporal allocation of resources. Second, OPEC countries do receive a large share of their consumption goods from the OECD countries. Third, many OPEC countries have substantial investments in OECD countries. Our model allows their oil pricing decisions to affect their return on these investments.

After learning about the technological characteristics of this world and before observing the actual market structure, one might wonder: "Why is the government of the USA buying oil from OPEC and putting it in the Salt Domes in Louisiana? They call them Strategic Petroleum Reserves (SPR) - does it make sense?" In order to understand it we might first consider the competitive allocation of resources with no government intervention. The "second best" arguments in favor of the SPR are not considered, since we do not want to justify one policy instrument due to misuse of another policy instrument.
3. The Competitive Case

The perfect foresight optimal allocation can be characterized by solving the 'social planning' problem of the above economy. It is straightforward to show that this allocation is identical to the world competitive perfect foresight equilibrium.  

The social planning problem is to maximize

\[
(3.1) \quad \delta_1 [U(C_1) + \beta U(C_2)] + \delta_2 [U^*(C_1^*) + \beta U^*(C_2^*)]
\]

subject to

\[
(3.2) \quad C_1 + C_1^* + \Delta K \leq F(K_1, 0_1)
\]

\[
(3.3) \quad C_2 + C_2^* \leq F(K_1 + \Delta K, 0_2)
\]

\[
(3.4) \quad 0_1 + 0_2 + I - D(I) \leq R^*
\]

\[
\delta_1 \geq 0, \quad \delta_2 \geq 0
\]

by choice of \( C_1, C_2, C_1^*, C_2^*, \Delta K, 0_1, 0_2 \) and \( I \). Let \( \lambda_1, \lambda_2 \) and \( \mu \) be the Lagrangian multipliers of equations (3.2), (3.3) and (3.4), respectively. Equations (3.2) and (3.3) are the world budget constraints each period, Equation (3.4) states that world oil consumption across the two periods cannot exceed the total world supply, \( R \). Then, the first order condition with respect to inventories is

\[
(3.5) \quad -\mu(1 - D'(I)) \leq 0 \quad (= 0 \text{ if } I > 0)
\]

Given that \( \mu > 0 \), since we assume an economy in which oil is consumed each period (3.5) implies that \( I = 0 \) if \( 0 < D'(I) < 1 \). Given our assumption that
oil does not appreciate in storage, we conclude that in a perfect foresight equilibrium there will be no storage of oil. The reason is that the economy is better off holding the oil in the ground with zero inventory costs than above the ground incurring the cost $I - D(I)$. It is also obvious that the inclusion of linear extraction costs does not affect the above result.

It is of interest to see the characterization of the competitive equilibrium resulting from the above planning solution. Given that $P_1$ is the real price of oil in period $i, i = 1, 2$, we get that $P_1 = \frac{\mu}{\lambda_1} = P_2(K_1, 0_1)$, $i = 1, 2$, from the first order conditions with respect to $0_1$. Then, the equilibrium is characterized by the conditions

$$\frac{U'(C_1)}{\beta U'(C_2)} = \frac{U'^*(C_1^*)}{\beta U'^*(C_2^*)} = \frac{P_2}{P_1}$$

(3.6)

and

$$\frac{P_2}{P_1} = F_1(K_2, 0_2) = r$$

(3.7)

Equation (3.6) establishes that the marginal rate of substitution is equal to the marginal rate of transformation in both the USA and OPEC, and equation (3.7) is simply the Hotelling rule for extraction of an exhaustible resource. 5
3.1 Convenience Yield

Now we wonder not only about the government strategic petroleum reserves but also about the existence of private storage. In fact the private stocks of crude petroleum in the USA are as large as the level of monthly sales (about 300-350 million barrels) and their existence should be explained. The industry term for such stocks is 'operating stocks. They help to get the oil to the consumer. In the economic literature this is called the "convenience yield" (see Brennan, 1958) which can be analytically modeled using ad-hoc functional forms of costs of holding inventories. These typically yield an inventory rule that is a function of oil consumption or output production (see, e.g., Eckstein and Eichenbaum, 1982). Usually it is assumed that for an inventory below some given level, say \( I^* \), there are negative marginal costs of inventories where the level \( I^* \) is given exogenously. We could introduce a convenience yield into our example by considering a storage technology, \( D(I) \) that has the properties \( D(I) > I, D''(I) < 0 \) over some range \( I < I \). The competitive solution would then establish

\[
(3.8) \quad P_2D'(I) = P_1^{I^*}
\]

as the first-order condition for a maximum. Equation (3.6) and (3.7) would continue to characterize the optimum. Thus the competitive solution would be fully characterized by the conditions
The first three of these conditions would also characterize the planner's solution. If \( D(I) \) is increasing and differentiable the solution establishes \( D'(I^*) = 1 \) both for the social planner and for the competitive equilibrium. The "convenience yield" argument thus justifies private operating stocks but not any government SPR.

3.2 Uncertainty

Another popular reason for private and possibly public inventories is given by the existence of uncertainty about the oil supply or proven oil reserves. The argument is based on precautionary savings to smooth final consumption. In the presence of a full set of contingent commodity markets this argument seems without merit. Private agents could optimally insure by trading contingent claims. If storage is costly (i.e., if \( D(I) < I \)), then an allocation (supportable by a competitive equilibrium) without storage exists which is Pareto superior to any allocation with storage. This result would not extend to situation in which extraction costs are non-linear, however.

It is possible that a full set of contingent claim markets does not exist, however. A more fundamental problem might be the non-existence of property rights in an international context. Private agents in the USA cannot obtain property rights over oil that is in the ground in OPEC.
Americans may be prohibited from acquiring these rights or else they may not trust OPEC governments' willingness to enforce these rights. In this context an additional argument for storing oil emerges: as a form of insurance.

We illustrate this result in the competitive model by assuming that the total stock of OPEC reserves, \( R^* \), is not known until period 2.\(^6\) We assume there are no contingent commodity markets or futures markets. (In fact, there are no formal contingent markets, and futures markets are limited, none covering a period greater than one year.) All oil is sold on spot markets. The second period price, then, is established by equating second period supply, \((R^*(s) - O^{-1} - I + D(I))\), where \( R^*(s) \) denotes the oil supply in state of nature \( s \), to second period demand, \( O_2 \), determined by the condition

\[
(3.10) \quad F_2(K_1 + \Delta K, O_2) = P_2
\]

This condition implicitly defines a demand function

\[
(3.11) \quad O_2 = E(K_1 + \Delta K, P_2)
\]

which is increasing in \( K_1 + \Delta K \) and decreasing in \( P_2 \). Equilibrium price in state \( s \) is then established by the condition

\[
(3.12) \quad R^*(s) - O^{-1} - I + D(I) = E(K_1 + \Delta K, P_2(s))
\]

The interest payment on investment is given by

\[
(3.13) \quad F_1(K_1 + \Delta K, O_2(s)) = r(s).
\]
Consider now the inventory decision of a US agent in period 1. He chooses $\Delta K$, $I$ and $O_1$, taking $P_1$, $H$, and $r(s)$ as given, to maximize:

$$U[F(K_1, O_1) - \Delta K + H - P_1(O_1 + I)] +$$

$$\beta \sum_s \Pi(s) U[F(K_1 + \Delta K, E(K_1 + \Delta K, P_2(s))) - r(s) H$$

$$- P_2(s)[E(K_1 + \Delta K, P_2(s)) - D(I)]$$

Here $\Pi(s)$ denotes the probability with which $R^* = R(s)$. The first-order conditions for $\Delta K$ and $I$ are:

$$(3.14a) \quad U'(C_1) \geq \beta \sum_s \Pi(s) U'(C_2) r(s) \quad (= 0 \text{ if } \Delta K > 0)$$

$$(3.14b) \quad P_1 U'(C_1) \leq \beta \sum_s \Pi(s) U'(C_2) P_2(s) D'(I) \quad (= 0 \text{ if } I > 0)$$

If $\Delta K$ and $I$ are strictly positive, these conditions imply:

$$(3.15) \quad \sum_s \Pi(s) U'(C_2) r(s) = \sum_s \Pi(s) \frac{U'(C_2)}{P_1} \frac{P_2(s)}{P_1} D'(I)$$

The OPEC first order conditions with respect to $H$ and $M_1$ yield that
(3.16) \[ \sum_{s} \pi(s) U^{*'}(C_{2}) r(s) = \sum_{s} \pi(s) U^{*'}(C_{2}) \frac{P_{2}(s)}{P_{1}} \]

if \( H \) and \( M_{2} \) are positive. Under certainty (3.15) is inconsistent with (3.16), which yields the Hotelling rule, \( \frac{P_{2}}{P_{1}} = r \), since \( D'(I) < 1 \), (see (3.9), and the left-hand side of (3.15) is greater than the right-hand side). Hence, under certainty, \( I = 0 \). Under uncertainty, when \( U(C_{1}) \) is concave, then both (3.15) and (3.16) can hold as equalities.

Hence, there are equilibria in which \( I \) is positive. The reason is that under uncertainty \( U'(C_{2}) \) and \( P_{2}(s) \) are positively correlated when \( D(I) = 0 \). Via Shephard's lemma

(3.17) \[ \frac{dU'(C_{2})}{dP_{2}} = - U''(C_{2}) (O_{2} - D(I)). \]

The diminishing marginal utility of consumption implies that this expression is positive (assuming that some oil is imported in period 2). Thus when \( P_{2}(s) \) is high \( U'(C_{2}) \) will also be high: for oil importers, a high price of oil lowers consumption, raising the marginal utility of consumption.

The positive correlation between \( U'(C_{2}) \) and \( P_{2}(s) \) raises the term on the right hand side of equation (3.15). The expected return on inventories is greater because inventories serve as a hedge. This provides a justification for holding inventories.

Two comments about this result are in order. First, if US agents could buy oil in the futures market or obtain property rights over oil in the ground in OPEC, inventories would not be desirable as long as \( D(I) < I \). Second, this result, by itself, does not justify
the establishment of a government reserve unless the government has
a superior storage technology (i.e., for the government \( D(I) \) is larger.)

The simplest competitive case thus yields no justification for
inventories at all. A convenience yield, however, or uncertainty
but an incomplete set of contingent commodity markets and imperfect
cross-national property rights are reasons why oil stocks may benefit
the USA. In these cases the private sector holds a level of inventories
that maximizes social welfare as well. Therefore, one may still wonder
what scope there is for government holdings of inventories. Next we
find, however, that once strategic considerations in the relationship
between OPEC and the USA are introduced, an argument for a government
SPR emerges. An argument can also emerge, however, in favor of
divesting the US government of its capacity to store oil.
4. The Bilateral Monopoly Case: A Possible Justification for the SPR

The presence of national market power frequently yields situations in which government intervention can improve national welfare if not world welfare. The nationally optimal tariff is an example.

In fact in 1978 OPEC provided 65% of production in non-Communist countries while the U.S. accounted for 55 percent of consumption in main consuming countries. There is certainly a presumption of market power on the part of OPEC, to the extent that it can maintain its cohesive as a cartel. We assume here that it can. There seems to be a presumption of market power on the USA's part as well, although this is less strong. If we were to consider a potential oil-importing country cartel consisting of the OECD or the International Energy Agency (IEA), the assumption of a bilateral monopoly situation between sellers and buyers would certainly fit the facts closely. Even in the absence of a cartel arrangement among importers, the assumption of bilateral monopoly seems to capture much of the relationship between OPEC and the USA.

In this section we consider how the presence of a bilateral monopoly situation can create an incentive on the part of the US government to establish an SPR. To focus clearly on strategic considerations we ignore the convenience yield and uncertainty considerations raised earlier. In the next section we show, via example, that by pursuing an inventory policy the US government can raise US welfare. But it can also lower it. Because results are, in general, sensitive to the specification of behavior, we find it useful to discuss alternative "rules of the game" that we can choose among.
4.1 Rules of the Game

We now consider alternative rules of behavior in relationships between the USA and OPEC. We identify as OPEC's strategy variables the oil prices ($P_1, P_2$) and OPEC's levels of investment in the USA ($H$). The US government's strategy variables are the tariff rates on oil in periods 1 and 2 ($\theta_1 - 1$ and $\theta_2 - 1$), the tax rate on OPEC's investments ($\theta_k - 1$), and the level of government inventory holdings ($I$). US private agents, behaving atomistically, choose oil consumption in periods 1 and 2, ($O_1, O_2$), investment ($\Delta K$), and private inventories ($I^P$) to maximize discounted utility. We assume that US private agents correctly anticipate the policies that are actually pursued both by OPEC and by the US government but then take them parametrically.

4.1.1 Open Loop Policies

An open loop policy is one in which values of the strategy variables are set for the current and future periods as of the initial period. Within the class of open loop policies we can identify strategic variables that are chosen by one player prior to the choice of some other strategic variable by the other player (in which case the first player acts as a Stackelberg leader with respect to those variables, the first player taking into account the effect of his choice on the response of the second player) or the decisions are made simultaneously by the two players, in which case they act as non-cooperative Nash players with respect to those variables, each taking the level set by the other player as given in making his choice.

When the game is specified as open loop, the issue of time-consistency does not arise. The levels of the strategic variables set in the first period (whether in a Nash or Stackelberg fashion) are the or
actually implemented. A difficulty with this formulation is that the players may not have an incentive, in the second period, to follow the open loop solution. Because of this inconsistency, the open loop policy will not be credible. Open loop solutions therefore may not be able to explain the behavior that we observe. Nevertheless, the open loop solution provides an interesting benchmark against which to compare time consistent solutions.

4.1-2 Feedback Solutions and Perfect Equilibria

An alternative policy is one that maximizes the objectives of each player as of the period the policy is implemented, taking previous policy as given. The two players thus play a separate game each period. The policies that are pursued each period are the outcome of that period's game. Hence, the players' decisions are based upon feedback from the previous period. When players correctly take into account the effect of each period's decision on the outcome of subsequent games, then the solution to the set of games is described as "perfect." (See Selten, 1975, for a discussion of perfection and Kydland, 1977, for a discussion of the distinction between open loop and feedback solutions.) The advantage of a specification of this type is that the solution that emerges is based upon behavior that is in each player's interest at the time he acts.
Within the class of feedback solutions we can also distinguish between variables that are chosen in a Nash or Stackelberg fashion. This choice should be dictated by the underlying technology of the problem.

We do not consider all possibilities for structuring the game. We assume the following rules.

R1 (a) OPEC acts as a Stackelberg leader each period with respect to price (i.e., OPEC chooses $P_1$ before the US government chooses $Q_1$ and $I^E$; OPEC chooses $P_2$ before the USA chooses $Q_2$ and $Q_k$)

R1 (b) OPEC and the USA act as Nash players with respect to $P_1$, $Q_1$ and $I^E$ in period 1 and with respect to $P_2$, $Q_2$ and $Q_k$ in period 2.

R1 (c) The USA acts as a Stackelberg leader each period (i.e., the USA chooses $Q_1$ and $I^E$ before OPEC chooses $P_1$, the USA chooses $Q_2$ and $Q_k$ before OPEC chooses $P_2$).

R2 US private agents take the values of US government and OPEC strategic variables as parametric. Subject to these parameters they maximize utility.

R3 Both OPEC and the US government correctly anticipate the effect of their policy on US private agents' behavior.

R4 All agents have perfect foresight.
Rule 1(a) best captures the strategy implicit in the IEA's stockpiling procedures: purchases are made contingent upon the oil price that OPEC sets. Rules 1(b) and 1(c) reflect more accurately the stockpiling procedure embodied in the Energy Security Act: purchases proceed independently of OPEC's price.

4.2 The Solution

We now attempt to characterize the solution to the game. Since first period decisions affect outcomes in both periods while, in the second period, first period decisions and outcomes are a bygone, it is simplest to consider the second period first.

4.2-1 The Second Period

Profit maximizing firms in the US private sector choose \( O_2 \) to maximize profits. Given the US domestic price \( \theta_2 P_2 \) this behavior implies the first-order condition

\[
(4.1) \quad F_2(K_1^+ \Delta K, O_2) \leq \theta_2 P_2 \quad (= 0 \text{ if } O_2 > 0)
\]

which implicitly defines the second period oil demand function

\[
(4.2) \quad O_2 = E(\theta_2 P_2, K_1^+ \Delta K)
\]

where \( E_1 < 0, \ E_2 > 0 \) as oil and capital are substitutes or complements.
In the case of constant returns to scale (CRS) in capital and oil this function takes the form:

\[(4.2') \quad o_2 = e(\theta_2 p_2)(k_1 + \Delta k)\]

Substituting (4.2) into (4.1) gives second period output as a function of the capital stock and the second period oil price:

\[(4.3) \quad g(k_1 + \Delta k, \theta_2 p_2) \quad g_1 > 0, \quad g_2 < 0\]

In the case of CRS this function takes the form:

\[(4.3') g(\theta_2 p_2)(k_1 + \Delta k)\]

OPEC's investment in the USA pays an interest rate \(r\) equal, before tax, to the marginal product of capital

\[g_1(k_1 + \Delta k, \theta_2 p_2) (= g(\theta_2 p_2) \text{ under CRS}).\]

We assume that the US government's objective is to maximize the utility of US private agents. In period 2 first period consumption is, of course, a bygone and the policy in period 2 can only affect period 2 consumption. The US government therefore maximizes \(U(c_2)\) where

\[(4.4) \quad c_2 = g(k_1 + \Delta k, \theta_2 p_2) - \theta_k g_1 h' - p_2 [e(\theta_2 p_2, k_1 + \Delta k) - d(r^g) - d(r^f)]\]
Under rules R1 (a) and R1 (b) government policy involves choices of $\theta_2$ and $\theta_k$ that maximize $C_2$ taking $P_2$, as well as $\Delta K, I^K$ and $\theta_1$, as given. $C_2$ is strictly decreasing in $\theta_k$ and a maximum, therefore, involves establishing $\theta_k$ at its minimum level (zero), effectively confiscating OPEC investments. When $\theta_k = 0$ the first-order condition for a maximum with respect to $\theta_2$ is given by

\begin{equation}
F_2 - P_2 = 0
\end{equation}

which is satisfied at $\theta_2 = 1$, the zero tariff condition. Since the USA acts taking $P_2$ as given, the optimal tariff is zero.

An interesting case emerges when the US is constrained to set $\theta_k > 0$, i.e., not to confiscate fully OPEC investment. In this case the first-order condition for $\theta_2$ is

\begin{equation}
F_2 = P_2 + \theta_k F_{12} H
\end{equation}

Thus if capital and oil are complements ($F_{12} > 0$) then the tariff on oil should be positive (raising $F_2$ above $P_2$) and conversely if they are substitutes ($F_{12} < 0$). Intuitively, the tariff acts as an indirect tax on OPEC investments. If the US government is constrained not to tax these investment fully, then a tariff redistributes income away from OPEC to the USA. In the CRS case the formula for the optimal tariff is given by
\[ t^* = \frac{\Theta_k H}{K + \Delta K - Q_k H} \]

in which case the tariff is independent of \( P_2 \). When there is no OPEC investment in equity or when \( \Theta_k = 0 \) (confiscation of OPEC equity) the optimal tariff is zero.

Consider, now, OPEC's problem. In period 2 OPEC sets \( P_2 \) to maximize the utility of OPEC's period 2 consumption. As with the USA, period 1 consumption is at this point a bygone. OPEC therefore sets \( P_2 \) to maximize period 2 utility, \( U^*(C_2) \), where

\[ C_2^* = P_2(0_2 - D(I^E) - D(I^P)) + \Theta_k C_1 H. \]

Under rule R1(a) OPEC considers the effect of \( P_2 \) on \( \theta_2 \). The first-order condition with respect to \( P_2 \) is given by:

\[ 0_2 - D(I^E) - D(I^P) + \left( P_2 + \Theta_k C_1 H \right) \frac{d0_2}{d(\Theta_2 P_2)} (\theta_2 + \frac{d\theta_2}{dP_2} P_2) = 0 \]

subject to the constraint \( 0_2 - D(I^E) - D(I^P) \leq R^* - M_1 \).

Dividing (4.9) by \( 0_2 \) yields

\[ 1 - \frac{D(I^E) - D(I^P)}{0_2} - \lambda(\Theta_2 P_2)(1 + \tau)(1 + \frac{\Theta_k C_1 H}{P_2}) = 0 \]

where \( \lambda(\Theta_2 P_2) \equiv \frac{d0_2}{d(\Theta_2 P_2)} \frac{\Theta_2 P_2}{0_2} \), the elasticity of US oil demand
with respect to the US price \((\theta_2 P_2)\) and \(\xi = \frac{d\theta_2}{dP_2} \frac{P_2}{\theta_2}\), the elasticity of the US tariff with respect to \(P_2\). Note that under CRS, \(\xi = 0\); the US tariff is independent of \(P_2\).

Condition (4.9') implicitly defines \(P_2\) as a function of \(I^g\), \(I^p\), \(\theta_2\), \(\theta_k\), and \(H\). The most important point to note is that the \(P_2\) that solves (4.9') falls as \(I^g\) and \(I^p\) rise as a share of \(\theta_2\). In addition, when \(H = 0\), \(P_2\) falls as \(\theta_2\) rises to maintain a constant domestic price. If \(H > 0\) and \(G_{12} > 0\) (oil and capital are complements) an increase in \(\theta_2\) causes \(P_2\) to fall in greater proportion, lowering not only the world price but the domestic price as well.\(^{10}\)

This completes the characterization of second-period equilibrium under rule RL(a), with OPEC acting as a Stackelberg leader in setting \(P_2\). When the level of \(\theta_2\) implied by equation (4.6) is independent of \(P_2\), as in the case under CRS, then the solution under rule RL(b), with OPEC and the US acting as Nash players, is exactly the same as under rule RL(a). Under rule RL(c), with the US acting as a Stackelberg Leader in setting \(\theta_2\), the US can impose the traditional optimal tariff. From equation (4.9') \(\theta_2 P_2\) stays constant or falls as \(\theta_2\) rises if \(G_{12} H > 0\). In this case the optimal tariff rate is infinite. Introducing extraction
costs or other buyers would modify this result, but the point is that the US can exert its monopsony power via tariffs only if it is able to commit itself to a tariff rate before OPEC sets $P_2$.

4.2-2 The First Period. Taking the solutions to the second period choice variables, $\theta_2$ and $P_2$, as given depending upon $I^P, I^E, K_1 + \Delta K, H$, and $R^* - M_2$, we now consider how these magnitudes are determined in period 1. Here we assume $\theta_k = 1$ (no taxation of OPEC investment income). The US private sector takes OPEC and US government policy variables $(P_1, H, \theta_1, I_g)$ as given to maximize

$$U(C_1) + \beta U(C_2)$$

with respect to $\theta_1$, $\Delta K$ and $I^P$ where

$$C_1 = F(K_1, \theta_1) - \theta_1 P_1 (\theta_1 + I^P) - \Delta K + H - T_1$$

$$C_2 = G(K_1 + \Delta K, \theta_2 P_2) - \theta_2 P_2 (\theta_2 - I^P) - G_1 H - T_2$$

Here $T_1$ and $T_2$ denote taxes each period. We assume that they are imposed in a lump-sum fashion. The government budget constraint implies.

$$T_1 = (1 - \theta_1) P_1 (\theta_1 + I^P) + P_1 I^E$$

$$T_2 = (1 - \theta_2) P_2 (\theta_2 - D(I^P)) - P_2 D(I^E)$$
First-order conditions for a maximum are

\[(4.13a) \quad F_2(K_1, 0_1) - \theta_1 P_1 \leq 0 \quad (= 0 \text{ if } 0_1 > 0)\]

\[(4.13b) \quad -U'(C_1) + \beta U'(C_2) F_1 (K_1 + \Delta K, 0_2) \leq 0 \quad (= 0 \text{ if } \Delta K > 0)\]

\[(4.13c) \quad -U'(C_1) 0_1 P_1 + \beta U'(C_2) \theta_2 P_2 D'(I^p) \leq 0 \quad (= 0 \text{ if } I^p > 0)\]

These equations implicitly define functions for first period oil demand, investment demand, and private inventory demand.

Consider now the problem facing the US government under rules R1(a) and R1(b). Taking \( P_1 \) parametrically the US government chooses \( \theta_1 \) and \( I^g \) to maximize social welfare, given, as before, by expression (4.10).

The US government correctly anticipates the effect of its decisions this period on this period's private sector behavior (as determined by equations (4.13)) and on the second period outcome.

Consider the first-order equation for a maximum with respect to \( I^g \),

\[(4.14) \quad -U'(C_1) P_1 + \beta U'(C_2) [P_2 D'(I^g) + \frac{dP_2}{dI^g}(0_2 - D(I^g) - D(I^p))] \leq 0 \quad (= 0 \text{ if } I^g > 0)\]

From (4.9) \( \frac{dP_2}{dI^g} \) is positive. Comparing (4.14) with (4.13c) observe that the US government has an incentive to invest in inventories beyond that facing the private sector. The reason is that individuals in the
US private sector, taking both $\theta_1 P_1$ and $\theta_2 P_2$ as given, do not take into account the effect of their own inventory decision on lowering the second period price. The US government internalizes the effect of its own inventory decision on the second period price. The US government then, facing a given first period price, has an incentive to accumulate inventories even when the private sector does not.

Subsidizing first period imports, via setting $\theta_1 > 1$, provides an alternative method of lowering $P_2$ by raising private inventories. This approach subsidizes first-period oil consumption as well as inventory accumulation, however. A direct government investment in inventories does not suffer this difficulty. The private sector continues to establish $P_2 = P_1$ whether or not $I^g$ is positive. If the government has available a storage technology that is not, at the margin, inferior to that provided by the private sector then the optimal first period tariff is zero.

Consider now OPEC's decision. OPEC chooses $P_1$ and $H$ to maximize

$$U(C_1^*) + \beta(C_2^*)$$

where

$$C_1^* = P_1M_1 - H$$

$$C_2^* = P_2M_2 + P_1H$$
Under rule R1(a) OPEC acts anticipating the effect of its choice on $I^g$ and $q_1$, as well as on the second period equilibrium. Under rules R1(b) and R1(c) it treats $I^g$ and $q_1$ as given. US government inventories augment first-period demand. Under rules R1(b) and R1(c) $p_1$ is necessarily greater when $I^g > 0$. This result does not necessarily emerge when OPEC is a leader. If $I^g$ is very price elastic it is conceivable that a government inventory purchase could lower $p_1$. In any event OPEC will set $p_1$ at a higher level under rules R1(b) and R1(c), given any level of $I^g$.

Finally, under rule R1(c), the US government chooses $q_1$ and $I^g$ anticipating OPEC's response. Because an increase in $I^g$ now raises $p_1$, the US government has less incentive to implement a reserve policy. While releasing the inventory lowers the price in period 2 acquiring it raises $p_1$. Under rules R1(a) and R1(b) US policy takes the second into account but not the first, $p_1$ is a bygone when $I^g$ is established. Nevertheless, OPEC, in anticipating (under R1(a)) or observing (under R1(b)) a US government inventory, is likely to establish a higher $p_1$ as a consequence.

Calculating the overall welfare effects of optimal inventory policy under alternative rules of the game is difficult in a general setting. In the next section we use a simple quadratic case to consider these issues further.
5. An Uneasy Case for Government Inventories: A Quadratic Example

We now consider a special case of the game discussed in section 4, making specific assumptions about the functional forms that describe technology and preferences. Our first and fourth examples assume that the behavior of the U.S. government and OPEC is described by rule R1(a), OPEC acts as a Stackelberg leader each period. Our third example is one in which the U.S. acts first (rule R1(c)).

We consider the following production function for $Q_1$

\[
Q_1 = F(K_1, 0_1) = a_0K_1 - \frac{a_1}{2}K_1^2 + a_2K_10_1 + a_30_1 - \frac{a_4}{2}0_1^2
\]

Note that this function exhibits decreasing returns to scale in capital and oil.

5.1 The Second Period

We assume that the return on investment in US capital is the same for U.S. citizens and OPEC members and is equal to the marginal product of capital, i.e.,

\[
F_1(K_2, 0_2) = a_0 - a_1K_2 + a_20_2.
\]

That is, the U.S. government sets $Q_k = 0$. The private sector sets the demand for imports of oil in the second period by equating the marginal product of oil to the market price, i.e.

\[
F_2(K_2, 0_2) = 0_2P_2 \quad \text{and} \quad 0_2 = M_2 + D(I)
\]
where \( I = I^p + I^g \) = private inventories + public inventories. Then we get that

\[
(5.3) \quad M_2 + D(I) = \frac{a_3}{a_4} + \frac{a_2}{a_4} (K_1 + \Delta K) - \frac{\theta_2}{a_4} P_2
\]

We consider only a limited set of instruments for US government intervention. In the second period the only instrument available is the tariff on oil. The objective of the US government is to maximize second period utility by maximizing \( C_2 \), i.e.

\[
\max_{\theta_2} \quad F(K_2, O_2) - \frac{H}{F_1} - P_2 M_2
\]

subject to (5.1) - (5.3). The first order condition is:

\[
[F_2 - \frac{F_1}{P_2} H - P_2] \frac{\partial M_2}{\partial \theta_2} = 0
\]

and the optimal tax on imports is

\[
(5.4) \quad \theta_2^* = \frac{a_2 H}{P_2} + 1
\]

Thus the optimal tariff rate is zero in two cases: (i) OPEC does not invest in the first period in the USA \((H = 0)\), or (ii) oil and capital are separable in the production of the consumption good \((a_2 = 0)\).
Now we turn to OPEC's determination of the second period price by maximizing its second period consumption, i.e., it maximizes \( P_2 M_2 + F_1 H \) subject to (5.1) - (5.4) by choice of \( P_2 \). The optimal \( P_2 \) turns out to be

\[
(5.5) \quad P_2 = \frac{a_3}{2} + \frac{a_2}{2} [(K_1 + \Delta K)] - a_2 H - \frac{a_4}{2} D(I)
\]

Again we observe that if oil and capital are separable in production \((a_2 = 0)\), the capital stock does not affect the determination of oil prices in the second period. Furthermore, \( P_2 \) is a linear function of capital but OPEC has an incentive to decrease oil prices as its investment in the US is larger. This result suggests why different members of OPEC would have different incentives in setting oil prices conditional on their portfolio decisions. Finally, it is important to observe that \( P_2 \) decreases as US inventories go up. This result establishes a possible role for public inventories if the US government in the first period takes into account OPEC supply behavior in the second period, while US private agents take \( P_2 \) parametrically. That \( P_2 \) falls as \( I \) rises is independent of the fact that the US takes \( P_2 \) parametrically in period 2 while OPEC is assumed to act upon (5.4), i.e., that OPEC is a Stackelberg leader in setting \( P_2 \).

Under rule RL(b), in which OPEC takes \( \theta_2^* \) parameterically so that \( \theta_2 \) and \( P_2 \) are set simultaneously in a noncooperative Nash game, then the optimal \( P_2 \) turns out to be

\[
(5.6) \quad P_2 = \frac{a_3}{2 \theta_2^*} + \frac{a_2}{a \theta_2^*} (K_1 + \Delta K) - \frac{a_2}{2 \theta_2^*} H - \frac{a_4}{2 \theta_2^*} D(I)
\]
Note that if $H = 0$ the Nash solution and the solution with OPEC as the Stackelberg leader yield the same price. Otherwise, $P_2$ may move either way with $\theta_2^*$. $P_2$ moves negatively with $D(I)$ as long as $\theta_2^*$ is positive. Whichever game is played in the second period, the oil price is not affected by total capital $(K_1 + \Delta K)$ and by $H$ in the same degree. The results in the second period are independent of the utility function since the maximization of welfare is equivalent to the maximization of consumption.

The third logical possibility, of course, obtains when the US acts as a Stackelberg leader (Rule R1(c)). As we showed in section 4, in this case the US can impose the optimal tariff, driving the world price to zero (the marginal extraction cost for oil that we have assumed here).

5.2 The First Period and the Complete Solution

5.2-1 Example 1 (Nichols and Zeckhauser)

To solve the first period problem we have to postulate a utility function for both the USA and OPEC. We first assume that utility is linear and that $\beta = \beta^* = 1$. In this case inventories benefit the USA. We then compare the government inventory policy with a tax/subsidy scheme. In order to do so, we make the following assumptions

A1: $H = \Delta K = 0$, i.e., no investment

A2: $D(I) = I$ i.e., no costs of inventory of oil

A1 implies that $\theta_2 = 1$ and as a result we get the following equations for the second period problem.

\begin{align*}
(5.7) & \quad M_2 = \frac{a_3}{a_4} P_2 - \frac{a_2}{a_4} I \\
(5.8) & \quad \omega_2 = \frac{a_3}{2} - \frac{a_4}{2} I
\end{align*}
where $\hat{a}_3 = a_2K_1 + a_3$

Note that these solutions obtain either when the US and OPEC establish $\theta_2$ and $P_2$ as the outcome of a noncooperative Nash game or when OPEC acts as a Stackelberg leader.

Together (5.7) and (5.8) yield

$$(5.9) \quad M_2 = \frac{\hat{a}_3}{2a_4} - \frac{1}{2} I$$

Since capital is constant we can write the production of the single good at time $i$ as:

$$(5.10) \quad Q_i = F(K_i, O_i) = a + \hat{a}_3O_1 - \frac{a_4}{2}O_1^2, \quad i = 1, 2$$

where $a = a_0 - \frac{a_1}{2}K_1^2$.

We consider the economy under alternative US government policies.

Case (I): the US government chooses both $M_1$ and $I$ in the USA in the first period taking the structure of the period 2 problem as given. Given the linear utility functions the US government’s problem is to maximize

$$F(K_1, M_1 - I) - P_1M_1 + F(K_1, M_2 + I) - P_2M_2$$

subject to (5.8), (5.9) and (5.10) by choice of $M_1$ and $I$. The first order conditions with respect to $I$ and $M_1$, respectively, are:

$$(5.11) \quad \frac{\hat{a}_3}{2} - a_4\left(\frac{\hat{a}_3}{2a_4} + \frac{1}{2}\right) + \frac{a_4}{2}\left(\frac{\hat{a}_3}{2a_4} - \frac{1}{2}I\right)$$

$$+ \frac{1}{2}\left(\frac{\hat{a}_3}{2} - \frac{a_4}{2}I\right) - \hat{a}_3 + a_4(M_1 - I) = 0$$
\[(5.12) \quad \hat{a}_3 - a_4 (M_1 - I) - P_1 = 0\]

Solving for \(I\) and \(M_1\) as functions of \(P_1\) we get,

\[(5.13) \quad I = \frac{\hat{a}_3}{a_4} - \frac{4}{3} \frac{P_1}{a_4}\]

\[(5.14) \quad M_1 = 2 \frac{\hat{a}_3}{a_4} - \frac{7}{3} \frac{P_1}{a_4}\]

Given the above result with respect to US government decision rules

OPEC's problem is to maximize \(P_1 M_1 + P_2 M_2\) subject to (5.8), (5.9), (5.10), (5.13) and (5.14) by choosing \(P_1\). The result is

\[(5.15) \quad P_1 = \frac{9}{17} \hat{a}_3\]

Hence, we have the following allocation of resources in the two periods:

\[
\begin{align*}
0_1 &= \frac{8}{17} \frac{\hat{a}_3}{a_4}, & 0_2 &= \frac{11}{17} \frac{\hat{a}_3}{a_4} \\
0_2 &= \frac{6}{17} \frac{\hat{a}_3}{a_4}, & I &= \frac{5}{17} \frac{\hat{a}_3}{a_4}
\end{align*}
\]

Utility levels in the U.S. and OPEC are, respectively

\[(5.17a) \quad U = C_1 + C_2 = 2a_1 K_1 - a_1 K_1^2 + \frac{155}{578} \left(\frac{\hat{a}_3}{a_4}\right)^2\]

\[(5.17b) \quad U^* = C_1^* + C_2^* = \frac{9}{17} \left(\frac{\hat{a}_3}{a_4}\right)^2\]
Hence, the price of oil falls from period one to period two and inventories are \( \frac{5}{11} \) of oil consumption at the second period. We now turn to the case where there is no US government intervention.

**Case (ii):** US private agents choose both oil consumption and oil inventories. There is no government intervention. US private agents maximize profits by setting \( \theta_1 \) such that \( F_2(K_1, \theta_1) = P_1 \) and they set \( I^P = 0 \) if \( P_1 < P_2 \) and otherwise \( I^P = 0 \). The first order conditions with respect to \( \theta_1 \) imply that

\[
(5.18) \quad 0_1 = \mu_1 - \frac{\hat{a}_3 - P_1}{a_4}
\]

As a result, we can solve OPEC's problem assuming that \( I^P = 0 \) and then see whether the condition for zero inventories is satisfied. OPEC's problem is to maximize \( P_1M_1 + P_2M_2 \) subject to (5.8), (5.9), (5.17) and \( I^P = 0 \).

Hence, we get that \( P_1 = P_2 = \frac{1}{2} \hat{a}_3 \) and the condition for zero inventories is satisfied. Furthermore, we get that \( \mu_1 = M_2 = \frac{1}{2} \frac{\hat{a}_3}{a_4} = 0_1 = 0_2 \).

Hence, the two periods are completely symmetric and the model is equivalent to the case in which OPEC is a simple monopoly in both periods separately.

Utility levels in the U.S. and OPEC are, respectively,

\[
(5.19a) \quad U = C_1 + C_2 = 2a_0 K_1 - a_1 K_1^2 + \frac{\hat{a}_3^2}{4a_4^2}
\]

\[
(5.19b) \quad U^* = C_1^* + C_2^* = \frac{\hat{a}_3^2}{2a_4^2}
\]

**Case (iii):** US private agents choose \( \theta_1 \) while the US government chooses inventories. The allocation of \( \theta_1 \) is determined by (5.17) which is identical to (5.12), the first order condition with respect
to $M_1$ in case (i). Hence, the solution for US optimal inventories turns out to be identical to that of case (i) - (5.13), and the final allocation of case (iii) and (i) are identical and given by (5.5) and (5.16).

**Result** In the above example a monopolistic OPEC behaves as a Stackelberg leader in a time consistent game and optimal private inventories are zero. This is equivalent to the result of zero private (optimal) inventories in the case of competition (section 3). However, given the fact that the government can exploit the effect of inventories on oil prices in the second period, we find that the optimal US allocation is to have a positive level of inventories which raises the first period oil price and lowers the second period price. Hence, the US government has a real cost of holding inventories, $(P_1 - P_2)I$, but it creates a welfare gain from changing the terms of trade and reducing the monopoly power of OPEC in the second period.

We can present the result on a graph.
FIGURE 1

\[ P_1^* = \frac{9}{17} \hat{a}_3 \]

\[ P_2^* = \frac{6}{17} \hat{a}_3 \]

\[ P = \frac{1}{2} \hat{a}_3 \]

PERIOD 1

\[ \frac{3\hat{a}_3}{4} \]

\[ \frac{\hat{a}_3}{2\hat{a}_4} \]

PERIOD 2

\[ \frac{\hat{a}_3}{2\hat{a}_4} \]
Moving from no intervention in the USA to a government inventory policy, the demand and marginal revenue curves that OPEC faces are moving from the solid lines to the broken lines. The USA loses the area \( P_1^* P B A \) of consumer surplus in the first period while it gains the area \( P P_2^* C B \) of consumer surplus in the second period and here the difference is positive.

Given the sequence of decisions that we assume here, in case (i) we characterize the optimal allocation for the US government. We show in case (ii) that the private sector does not achieve the same allocation since it cannot exploit the negative effect of inventories on the period 2 oil price. If the only government instrument is a public inventory (case iii) the allocation is the same as in the first case.

Comparing (5.17) with (5.19) note that US welfare is in fact greater when the US government chooses inventories optimally. In addition, OPEC welfare is greater as well. US inventory policy is reducing a monopoly distortion in a way that benefits both OPEC and the USA.

Note that under the inventory policy imports over the two periods together are greater than when the US government does not use inventories.

Could an optimal level of inventories be sustained by other policies? The answer is yes, if the government can impose lump-sum and firm specific taxes and/or subsidies to make holding the optimal level of
inventories consistent with the firm's profit maximizing problem. This set of incentives must be specified in the first period. However, once \( P_2 \) is determined in the second period there will be no incentive for the government to fulfill its obligations. The previous time consistency argument applies to the tax incentive program for private inventories. Only by buying the inventories in period 1 itself can the government credibly commit itself to a policy of lowering the second period price through increased inventories.

5.2-2 Example 2

Now assume that rule RL(b) applies, the US government and OPEC set \( I^E \) and \( P_1 \) simultaneously as non-cooperative Nash players rather than sequentially, i.e., the US government chooses \( I^E \) taking \( P_1 \) as given, as before, and OPEC sets \( P_1 \) taking \( I^E \) as given. In the consequent equilibrium we get

\[
\begin{align*}
\bar{0}_1 &= \frac{2a_3}{5} \quad & \bar{0}_2 &= \frac{3a_3}{5} \\
\bar{P}_1 &= \frac{3a_3}{5} \quad & \bar{P}_2 &= \frac{2a_3}{5} \\
\hat{I}^E &= \frac{a_3}{5a_4}
\end{align*}
\]

while

\[
\begin{align*}
U &= C_1 + C_2 = 2a_0 K_1 - a_1 K_1^2 + \frac{11}{50} a_3 \\
U^* &= C_1^* + C_2^* = \frac{12}{25} \frac{a_3}{a_4}
\end{align*}
\]
Compared with a situation in which \( I^E = 0 \), the USA is again now worse off while OPEC is again better off.

Moving from a situation in which the USA acts entirely as a Stackelberg follower to one in which the US government and OPEC acts as Nash players reduces US welfare. The reason is that US inventory demand is price elastic. Given the structure of the problem in period 2, the US government's demand for inventories is given by

\[
I^E = \frac{a_3}{a_4} - \frac{4P_1}{3a_4}
\]  

(5.23)

When OPEC incorporates (5.23) into its decision-making it sets, ceteris paribus, a lower price. Taking \( I^E \) as given it perceives total demand as more inelastic and consequently sets a higher \( P_1 \).

This result is illustrated in Figure 2. While the US inventory demand shifts OPEC's demand curve rightward in a Nash game the slope of OPEC's perceived marginal revenue curve is unaffected by a US government inventory policy. When OPEC acts as a leader the optimal US government inventory policy makes the perceived MR curve flatter. OPEC consequently charges a lower price each period.

5.2-3. Example 3

Consider now the problem posed in example 1 for the case in which the US acts as a Stackelberg leader, i.e., rule R1(c) applies in period 1. We continue to assume that rules R1(a) or R1(b) apply in period 2, so that the structure of the second period game is unchanged. We assume 0 tariffs.

The US government now sets \( I^E \) taking the price response of OPEC,

\[
P_1 = \frac{a_3}{2} + \frac{a_4 I^E}{2},
\]

(5.24)
as given. It is straightforward to show that in this case the optimal
US government policy is to set $I^g = 0$. The same solution as that for
Example 1, case (ii), i.e., the competitive solution without government
intervention, obtains here. When the US government must precommit
itself to some level of inventories, it chooses a zero level. This
result obtains when OPEC has a Bernoulli utility function as well as
when OPEC's utility is linear.

5.2-4 Example 4.

We now show that a US government inventory policy is not necessarily
in the USA's interest even when OPEC acts as a Stackelberg leader. We make
the following small modification to example 1. Assume that instead of being
linear in consumption (as in equations (5.17b) and (5.19b)), OPEC's utility
function is Bernoulli:

\[ U^* = \log C^*_1 + \log C^*_2 \]

In this case the solution in the presence of a government inventory
(Cases (i) and (iii)) involves

\[ o_1 = \frac{5}{14} \frac{a_3}{a_4} \quad o_2 = \frac{4}{7} \frac{a_3}{a_4} \]

\[ p_1 = \frac{9}{14} \frac{a_3}{a_4} \quad p_2 = \frac{3}{7} \frac{a_3}{a_4} \]

In addition,

\[ U = C_1 + C_2 = 2a_0 K_1 - a_1 K_1^2 + \frac{11}{56} \frac{a_3^2}{a_4} \]

\[ U^* = \log C^*_1 + \log C^*_2 = \log \frac{9}{28} + 2\log \frac{3}{7} + 2\log \frac{a_3^2}{a_4} \]

When there is no government inventory (case ii) the solution
is exactly as that for example 1. The reason is that, in this case,
the choice of $P_1$ has no implications for intertemporal substitution in OPEC. Thus OPEC's utility is given by

\[(5.28) \quad U^* = 2\log \frac{1}{4} + 2\log \frac{x_2}{x_4}\]

while the USA's welfare continues to be given by (5.19a)

Again, comparing (5.27b) and (5.28), note that OPEC has benefitted because the US has pursued an inventory policy. The USA, however, has lost; (5.27a) is less than (5.19a). The reason is that, when OPEC has diminishing marginal utility of consumption in period 2, it is less willing to transfer consumption from period 2 to period 1 in response to a US inventory policy. It sets higher prices in both periods to maintain a higher consumption level in period 2. The US is consequently worse off. In terms of Figure 1, when OPEC has a Bernoulli objective function $P_1^*$ and $P_2^*$ are displaced upward relative to $P_1$. The loss in period 1 from having an inventory is consequently greater while the gain in period 2 is less. Note also that here total imports over the two periods have fallen because of the inventory policy.

Given that the USA is better off without a government inventory will it in fact set $I^E = 0$? If the US government does set $I^E$ taking $P_1$ as given it will set $I^E > 0$ for all $P_1 < \frac{3a_3}{4}$, given the structure of the remaining problem. As in example 2 once $P_1$ is set it is too late for the US to affect $P_1$ via its inventory policy.

Consider a situation in which the US government announced that it would establish $I^E = 0$. If OPEC believed this announcement it would establish $P_1 = \frac{a_3}{2}$. The US government would then have an incentive to establish $I = \frac{3a_3}{3a_4}$ and drive $P_2 = \frac{a_3}{3}$. Anticipating this behavior OPEC will in fact set $P_1$ higher. In example 1 the USA nevertheless benefitted from having a government inventory when OPEC adjusted $P_1$ in anticipation of period 1 inventory purchases. An implication of
this example and example 2 is that the US government can actually lower US welfare by developing the capacity to maintain inventories. The absence of such a capacity constitutes a credible commitment not to store oil before OPEC establishes P.

5.3 Conclusion

These examples suggest that, in a strategic setting, the ability of the US government to pursue an inventory policy can have both desirable and undesirable consequences, depending upon both the nature of OPEC's preferences and upon the structure of the process whereby OPEC sets prices and the US sets inventories.

Our results can be interpreted in light of Samuelson's (1972) analysis of the desirability of destabilizing speculation. Like Samuelson, we are considering a situation in which given demand and supply conditions persist for two periods. Samuelson showed that in a competitive setting, that is, one in which buyers and sellers behave as price takers, a destabilizing speculator would raise the welfare of both buyers and sellers. His own losses would exceed the gain of the other two groups combined, however. Hence, in our example, if the USA faced a competitive OPEC there would be no positive role for a US government inventory policy. The US government would be acting as a destabilizing speculator. The gain to the rest of the world, not just to US consumers, would fall short of the capital loss the US government would sustain in buying in period 1 to sell in period 2.
In facing a monopolistic seller, however, our examples indicate, first of all, that a US government inventory policy can raise not only US but world welfare. The reason is that the optimal US inventory rule makes US demand, on net, more elastic over the two periods. As a consequence the distortion due to monopoly is diminished and both sides can benefit. More oil is consumed overall, so the world is moved closer to the competitive equilibrium.

This result requires that OPEC set price incorporating the US government response into its decision. An implication is that to succeed at raising US welfare the US government inventory purchases should respond very closely to actual oil prices; i.e., the government should, according to our model, establish purchasing rules that are price contingent.

A second implication of our examples is that, unless the US government acts as a leader in setting \( I^G \) before OPEC sets \( P_1 \), it may have an incentive to establish a positive inventory even when US welfare is higher when there is no commitment to no inventories. The reason is that the loss to the USA from having an inventory is incorporated in the first period price. Once OPEC has established this price it is too late for the US government to avoid the undesirable consequences of having an inventory. From that point on the benefits exceed the costs.
6. Other Arguments for Government Inventories

Our analysis has focused on convenience yields, uncertainty, and strategic interactions to explain the existence of petroleum reserves. Only in the third case did we find an argument for government intervention. Other economists have analysed the case for a strategic reserve and we discuss their results here. Closest in spirit to our own analysis is the paper by Maskin and Newbury (1978) which examines the possible effect of US monopsony power on the optimal tariff response. Wright and Williams (1982) have argued that reserves may be justified as a second-best response to other (suboptimal) government policies, in particular, price controls. Finally, the stockpile has been justified as a means of reducing US vulnerability to the threat of an embargo. Tolley and Wilman (1977) discuss this issue.

6.1 U.S. Monopsony Power and Government Inventories

Maskin and Newbury (1978) develop a two-period model in which a monopsonistic U.S. faces a competitive set of oil producers and other buyers. The optimal open-loop policy is for the U.S. to establish a monopsony price (via an optimal tariff, for instance) that must be equal (in discounted terms) across the two periods to extract positive supplies in the two periods. The two prices must be equal because of Hotelling's formula. In the second period, however, the U.S. has an incentive to deviate from the period 2 price that is optimal from the open-loop perspective. The reason is that the effect of the period 2 price on oil producers' willingness to hold oil in the ground in period 1 is at this point a bygone. The price that is optimal from period 2's perspective can be higher or lower than that which was optimal ex ante. If oil producers and other buyers
believe the announced open loop rule in making their period 1 decisions about extraction the USA can benefit from reneging on the contract. If, however, the rest of the world anticipates the reneging the USA can lose from its monopsony position. If, say, the US government has an incentive to revise the price downward in period 2 and individuals correctly anticipate this revision, the period 1 price will be driven down as well (again via the Hotelling rule). The consequent equilibrium can be worse from the US perspective than one in which the USA has no monopsony power at all. The USA would be best off if it could precommit itself to its optimal open loop policy. If this is not possible it could benefit by somehow divesting itself of its monopsony power in the second period. Otherwise, the anticipation that the USA will exercise monopsony power in the second period leads to behavior by other agents in the first period that is detrimental to the USA.

In this context Maskin and Newbury show that the USA can benefit from government storage in period 1 as a means of precommitting itself to a course of action. By buying stocks of oil the US government can establish that it has an interest in maintaining the announced price of oil in the second period when, in the absence of storage, it would want to revise the second period price downward. Maskin and Newbury find that in a rational expectations equilibrium the USA cannot be hurt by a US government stockpile while in some circumstances the USA will strictly benefit. The argument here is again in favor of a government inventory. Private agents do not have an incentive to invest in inventories as a means of making the government's optimal tariff commitment credible.
6.2 Price Controls and Government Inventories

Wright and Williams (1982) develop a model in which agents anticipate that in some periods (e.g., when the price is high) the government will impose price controls on oil. The private rate of return on storing oil into these periods is consequently lower than the social rate of return. The private sector consequently stores too little. There is scope for additional government reserves. Government storage here is a second best response to other distortionary government policies. The government does not actually have to impose price controls for a justification for inventories to emerge. Private agents simply need to anticipate that controls will be applied with some probability. Wright and Williams do not attempt to model why the government would impose controls and, hence, why it cannot credibly commit itself never to impose controls.

6.3 Vulnerability and Government Inventories

The threat of a future embargo by OPEC can provide an additional justification for an inventory. In a competitive setting, of course, this issue does not arise. In the face of a monopolistic exporter, however, the supplier could decide to curtail supplies at some moment. A complete modelling of the embargo issue would require a specification of the supplier's motives in imposing an embargo. A real possibility is that a government inventory is a means of preventing an embargo.
Tolley and Wilman (1977) show that if a country is faced with an exogenous threat of an embargo that a justification for inventories emerges. There is scope for government intervention, however, only when the embargo generates external effects. Otherwise, individuals would have an incentive to maintain the socially optimal level of inventories themselves in the face of an embargo threat, as we showed in section 3. They derive the optimal level of the government inventory as a function of the externalities generated by the embargo and the exogenous likelihood and length of a potential embargo.

A more complete analysis would specify (1) the nature of the externalities and (2) the effect of the inventory policy itself on the likelihood and duration of an embargo. An analysis of this sort could be provided in a multiperiod game-theoretic framework. It remains an important topic for future research. Riesman and Aiyagari (1982) consider the desirability of the embargo policy to the sellers. They find that only in a very special case can this policy improve the seller's position from a purely economic perspective.

The oil price shocks of the last decade have spawned a large literature on policies toward oil. There exists a number of other articles that have considered aspects of policies toward oil and/or optimal stockpile behavior. Examples include Nordhaus (1974), Calvo and Findlay (1978), Gilbert (1978), Wright (1980), Teisberg (1981), Ulph and Folie (1981), Newbery (1981), and Epple, Hansen and Roberds (1982).
7. Conclusion

This paper investigates the desirability of US government oil inventories in a two period, two country model in which the world stock of oil is exhaustible. We show that in competitive markets under certainty or uncertainty there is no welfare improving role for public inventories and, leaving aside operating stocks, a precautionary demand for stocks of oil is due to the exclusion of international insurance markets or property rights.

We show that only under a limited set of strategic games between the USA and OPEC can one justify public strategic petroleum reserves. Even then their desirability depends upon the structure of preferences.

An inventory policy is inferior to one of imposing optimal tariffs in the two periods. But implementing the optimal tariff may not constitute a time consistent policy (see Kydland and Prescott, 1977): while the USA could bring US welfare to a higher level by imposing optimal tariffs in the two periods, the US may not have an incentive actually to impose the tariff in the period in which it acts. A threat to impose the tariff at the time OPEC sets price may therefore not be credible. An SPR, while not raising US welfare to a level equal to that when optimal tariffs are imposed, may nevertheless raise welfare above that attainable by any other time consistent policy. An inventory constitutes a second-best, but credible, alternative to an optimal tariff policy.

In all our examples the government inventory makes a loss. Consequently, private, atomistic agents, acting as price takers, have no incentive to hold any inventories at all. Inventories serve the purpose of driving down the price in the second period. The price is driven down for all second period users. Any non-altruistic individual considering investing
in an inventory will not take into account the effect of his own inventory holding on lowering the price for other US individuals. The case is one of a classic externality. A US government that maximizes US welfare will internalize this effect. Hence, in moving to a strategic setting, a justification for a government SPR can be made. As its name implies, strategic consideration seem to have motivated the establishment of the US SPR (see Senator Jackson's statement quoted in the introduction.)

Whether or not a US inventory enhances US welfare depends very much upon the structure of decision making in the US and OPEC, and upon the parameters of the system. We find three examples in which the presence of an SPR reduces US welfare relative to a situation of zero inventories. Nevertheless, once OPEC has acted, the USA may find it in its interest to pursue an inventory policy. Holding inventories may then constitute a time consistent policy that is inferior to a credible precommitment to hold zero inventories. Merely by developing the capacity to hold inventories the SPR can reduce US welfare.

Another aspect of our analysis is to show that if OPEC invests some of its first period income in US equities a credible, welfare enhancing tariff policy on the part of the US can emerge. We have not considered the interaction between OPEC investment and US government inventories here. We consider this avenue as a promising one for further research on the SPR. One possibility is that, since US inventories raise OPEC's first period income relative to its second period income, that an inventory policy will increase OPEC's equity investment in the USA. For the reasons we discuss in section 3 and 4 this investment acts further to reduce the second period price. There is a second channel, then, whereby a US government purchase of inventories in period one can reduce the price of oil in period two.
FOOTNOTES

1 Nichols and Zeckhauser show that a stockpile can reduce US welfare when the resource constraint is binding. In this context, however, OPEC is not exercising monopoly power by restricting total supply. In fact, even when the resource constraint is not binding the inventory can reduce US welfare, as we show.

2 For detailed description of the SPR see Glatt (1982). For a discussion of the quota system that prevailed during the period 1954-1971 see Dam (1971). Dam suggest that in 1969 tariff equivalent of the quota averaged about $1.25 per barrel.


4 See Varian [1978].

5 Here we assume that capital cannot be consumed and therefore that the interest rate is equal to the marginal product of capital.

6 This uncertainty could arise either from imperfect information about the physical quantity of OPEC's oil or from uncertainty about OPEC's desire to sell oil to the USA. The possibility of an embargo, for example, creates uncertainty about OPEC's supply of oil to the USA. To be consistent with the analysis here the embargo must be considered as a possibility that is exogenous to the USA's behavior. We discuss this issue of an endogenous embargo in section 6.

7 U.S. Imports that year equalled more than one-third of OPEC's Production. See Table 1.
8 Observe that $C_1 = F_1$ and so $G_{12} = F_{12}$.

9 See Marion and Svensson (1981) for a competitive model that deals with the relationship between the oil price and OPEC's investments.

10 This result is reminiscent of the well-known Metzler paradox. Here it arises because of the interaction between the price of oil and the return on capital.

11 Note that it is assumed here that $R^* > \frac{19}{17} \frac{a_3}{a_4}$.

12 This allocation (case (1)) is optimal subject to the particular rules of the game that we assumed for the US and OPEC.
REFERENCES


