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AN OVERVIEW OF AGRICULTURAL HOUSEHOLD MODELS: THEORY

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Abstract

This paper is one of two introductory, survey chapters for a book, Agricultural Household Models: Extensions, Applications and Policy, being coedited by Inderjit Singh, Lyn Squire and John Strauss for the World Bank. The paper compares and contrasts differing theoretical models of an agricultural household, which have heretofore been presented as alternative, unrelated models. The essential characteristics of an agricultural household is the interlinkage of its consumption and production decisions. How these decisions are related depends crucially on how markets are modeled. In particular whether a household's production and consumption decisions are separable becomes very important in this context. This is spelled out, and models corresponding to several assumptions are examined. These include models with no labor market, Z-goods models, models with certain types of ccommodity heterogeneity, as well as models with perfect markets. In the analysis their relationships to each other are stressed.

An Overview of Agricultural Household Models: Theory*

I. Introduction

The study of agricultural households is important for understanding the effects of various types of public interventions at both the household and aggregate levels. Agricultural, or farm, households are different from traditional economic households because they produce some of the commodities that they consume, and they supply some or all of the labor used on the farm.[1] Thus the concept covers a continuum of households, ranging from those which are purely subsistence, consuming virtually all their output and not buying or selling labor, to those which are commercial, selling all their output, but which use family (and perhaps hired) labor to produce it.

These households are a major form of economic organization in developing countries. Roughly seventy percent of the labor force in low income developing countries was employed in the agricultural sector in 1980, while roughly forty-five percent of the middle income developing countries' labor force was so employed (Table 1). While not all the agricultural labor force is comprised of farm household members, some are landless laborers, Table 1 clearly suggests that such households are very numerous. Consequently it is important to account for their behavior as economic actors when analyzing government interventions into the economy.

Governments in developing countries impose interventions which affect and are affected by farm households. Policies affecting prices of agricultural commodities and inputs are pervasive (for sample, see T. W. Schultz, 1978). Such policies may be designed to influence production, marketing, consumption or trade, and may be designed to provide revenue, encourage industrialization, mitigate other price distortions, and so on. Perhaps the most common of these policies are trade policies designed to promote

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industrialization and generate revenues by giving heavy protection, often using quantitative restrictions. Typically such policies lower the domestic price of exportables relative to both importables and non traded goods. When combined with export taxes and government marketing boards, there can be a very large difference between the price agricultural households receive for their export crops and the price they would have recieved without market interventions. As the recent report Accelerated Development in Sub-Saharan Africa pointed out, this pattern is common for sub-Saharan African economies, but it is not exclusive to them.

Analyzing such policies using traditional economic tools such as consumers' and producers' surplus can be quite useful as a start, although for many questions such tools may be misleading. For example, if output supply is elastic for an exported cash crop, because there exists substitution possibilities withimported food crops, an increase in the export tax would be expected to decrease government revenues. However, if there is an imported food which is heavily subsidized this need not be so since less of the food might be imported. In order to trace out the effects of a higher export tax on imports of foods it is necessary to analyze the change in the marketed surplus of foods, not simply in production. To do this requires predicting the changes in farm household consumption as well as production. The basic idea of farm household modeling is that the two are linked. In the example, the increase in the export tax should depress farm profits, which will lead the household to consume less food, thus releasing to the market even more than the increment in food production, and in turn making a reduction in government revenues less likely.

As another example we can examine the effect of government interventions in the area of agricultural technology. What, for example, might be the effect of introducing a new chemical-biological technology on the demand for landless laborers? This has been debated (see for instance Krishna, 1975), but focus has been on the demand for total labor -- both family and hired. To predict the effects of new technologies on the demand for hired labor requires examining what happens to family labor supply not just the total demand for labor. If the package increases farm incomes, it will lead families to supply less labor. If hired and family labor are perfect substitutes, demand for hired labor should then increase (decrease) by more (less) than the demand for total labor.

In general any analysis examining the consumption or labor supply of agricultural households has to account for the interdependency of household production and consumption. Agricultural household modeling combines these two fundamental units of microeconomic analysis -- the household and the firm. The two units are linked since farm enterprise activities contribute to household income, and therefore effect household consumption. It turns out that this more general model of household production and consumption can lead to results which contradict the orthodox demand theory. For example, a rise in output price may lead to higher, not lower, consumption, of a good which the household both consumes and produces. This may occur because demand for the good responds positively to income, since a rise in output price will raise farm profits and hence family income, some of which will be spent on the good whose price has risen. If consumption rose enough, marketed surplus of the good could even fall as its price increases.

Under certain circumstances the only interdependence between the household and firm activities of an agricultural household comes through income. In this case the production activities of the household can be analyzed separately from the consumption activities, the model becoming split into profit maximizing and utility maximizing components. The traditional analysis of farm output supply and input demand using the theory of the firm is then valid. Empirical analysis of both household consumption and production becomes considerably more tractable, and as a result most of the empirical analyses to date have used such separable models.

In a static model, the key assumption needed to obtain separation of the household's production and consumption decisions is that the household be a price taker for every commodity, including family labor, which is both consumed and produced. This means that perfectly competitive markets must exist for each such good. Intuitively, the household can make its consumption and production decisions separately, since any difference can be bought or sold on the market at a fixed price. There exist a variety of reasons why separability might not hold. Market power is one reason, but is not typically thought to be important in farm household modeling.[2] Absence of a market, e.g. for labor, would violate the price taker assumption. More realistically, family and hired labor might be imperfect substitutes in production, while no family labor is sold out. In either of these latter two cases, we can think of a virtual, or shadow, price, as the price which would just equate the household demand for total (or family) labor with its supply. Such a price will depend on all the variables which the household takes as given, those affecting either consumption or production. Since this virtual price of labor will in turn affect both sets of decisions, there will exist another source, in addition to income, of interdependency. Now, farm output supply and input demands will depend on household preferences so that the traditional theory of the firm will be inappropriate to analyze them. Analytical results which contradict predictions from the theory of the firm can be obtained. For example, a rise in output price may cause a fall in production, if family labor supply is sufficiently lowered by the income increasing effect of the price rise. Holding the virtual price of family labor constant, the increase in output price raises the demand for family labor, and lowers the supply. Thus the virtual price has to rise, just enough to reequate the demand and supply of family labor. That rise in the virtual price of family labor, an input into production, will cause a reduction in output, which if greater than the initial increase in output will give us the counterintuitive result. To contrast, had the household sold some labor on the market, in addition to working on its own farm, the fixed market wage would have been the appropriate price to use in the analysis -- the farm household model would be separable. Then output could never decrease when its price rose, holding other prices constant, so long as the production function was well-behaved.

There does not exist a single prototype agricultural household model, rather there are many such models depending on what issues are being examined. The implications of each model depends very much on the assumptions used, especially assumptions about markets and prices.

In the rest of the chapter several theoretical models are compared and contrasted. In Section 2 a model is presented which assumes fixed market prices. The separability property and comparative statics are derived. The concept of a virtual price is then explicitly defined, and it is shown how the response of the virtual price to exogenous variables can be obtained. It will turn out that with a minimum number of assumptions this response can be signed. These results are then used to examine the comparative statics of various farm household models, when the household faces virtual rather than market prices. In doing so the difference in the comparative statics between separable and nonseparable models becomes clear. Section three outlines a model in which the market for labor is absent. In Section 4 models which incorporate Z-goods are discussed, while Section 5 treats models with certain types of commodity heterogeneity. Finally conditions under which agricultural household models are separable are summarized.

2. A General Approach

A. A Basic Model: The Household as Price Taker.

In this section we develop a static model in which all prices are taken as exogenous. Assume the household maximizes its utility subject to its constraints. Three constraints are specified at first: a production function constraint, and time and budget constraints. To date agricultural household models have not been used to address issues of intrafamily distribution, so in that spirit a household utility function is assumed to exist. Let

$$U(X_1, \ldots, X_L) \qquad (1)$$

be the utility function, which is well behaved: quasi-concave with positive partial derivatives. The arguments are household consumption of commodity i, with X_L denoting total leisure time.[3] Utility is maximized subject to a budget constraint

$$Y = \sum_{i=1}^{L} p_i X_i \qquad (2)$$

where Y is the household's full income (see equation (3)), and the p_i's are commodity prices (p_L being the wage rate) which may or may not be exogenous to the household. For now assume that these prices are taken exogenously by the household, but this assumption will be relaxed shortly. If full income can be taken as predetermined then the household's decision is the standard consumption-leisure decision.

Full income of an agricultural household equals the value of its time endowment, plus the value of the household's production less the value of variable inputs required for production of outputs, plus any non-wage, non-household production income such as remittances.

$$Y = p_L T + \sum_{j=1}^{M} q_j Q_j - \sum_{i=1}^{N} r_i R_i - p_L R_L + E \qquad (3)$$

where
$T \equiv$ time endowment
$Q_j \equiv$ output, for $j = 1,\ldots, M$
$R_i \equiv$ non-labor variable inputs, for $i = 1,\ldots, N$
$R_L \equiv$ labor demand
$q_j \equiv$ price of Q_j
$r_i \equiv$ price of R_i
$E \equiv$ exogenous income

For the moment it is assumed that R_L is total labor demanded by the household, both family and hired, which are assumed to be perfect substitutes, an assumption we will relax in Section D. Outputs and inputs are related by an implicit production function

$$G(Q_1,\ldots,Q_M, R_1,\ldots,R_N, R_L,K_1,\ldots,K_O) = 0 \qquad (4)$$

where K_i's are fixed inputs. This is a general specification which allows for separate production functions for different outputs, or for joint production. G is assumed to satisfy the usual properties for production functions: it is quasi-convex, increasing in outputs and decreasing in inputs. If the household maximizes utility (1) subject to its budget (2 and 3) and production function (4) constraints and to prices (p,q,r) being fixed, then the household's choices can be modeled as two separate decisions, even though the decisions are simultaneous in time (Nakajima, 1969; Jorgenson and Lau, 1969). The household behaves as though it maximizes the revenue side of its full income, equation (3), subject to its production function constraint, and then maximizes utility subject to its full income constraint, equation (2). Since neither the value of endowed time nor exogenous income are household choice variables, maximizing full income is equivalent to maximizing the value of outputs less variable inputs, or profits.

To see that the model is separable between revenue and expenditure the comparative statics are examined. Let the household consume three commodities:[4] leisure, X_L; a good which is purchased on the market, X_m; and a good, X_c, produced by the household. The household uses labor, R_L, another variable input, R_v, and a fixed input K to produce both Q_c and another crop, Q_s. All of Q_s is sold on the market (a cash crop). The Lagrange function can be written as

$$\pounds = U(X_L,X_m,X_c) + \lambda(p_L T + (q_s Q_s + p_c Q_c - p_L R_L - r_v R_v) + E - p_L X_L - p_m X_m - p_c X_c) + \mu G(Q_s,Q_c,R_L,R_v,K) \quad (5)$$

Assuming interior solutions, the first order conditions are;

$$\frac{\partial \mathcal{L}}{\partial X_L} = U_L - \lambda p_L = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial X_m} = U_m - \lambda p_m = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_c} = U_c - \lambda p_c = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = p_L(T-X_L-R_L) + q_s Q_s + p_c(Q_c-X_c) - r_v R_v - p_m X_m + E = 0$$

$$\frac{1}{\lambda}\frac{\partial \mathcal{L}}{\partial Q_s} = q_s + \frac{\mu}{\lambda} G_s = 0$$

$$\frac{1}{\lambda}\frac{\partial \mathcal{L}}{\partial Q_c} = p_c + \frac{\mu}{\lambda} G_c = 0$$

$$\frac{1}{\lambda}\frac{\partial \mathcal{L}}{\partial R_L} = -p_L + \frac{\mu}{\lambda} G_L = 0$$

$$\frac{1}{\lambda}\frac{\partial \mathcal{L}}{\partial R_v} = -r_v + \frac{\mu}{\lambda} G_v = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = G(Q_s, Q_c, R_L, R_v, K) = 0$$

Totally differentiating (6)

$$\begin{bmatrix} U_{LL} & U_{Lm} & U_{Lc} & -p_L & 0 & 0 & 0 & 0 & 0 \\ U_{mL} & U_{mm} & U_{mc} & -p_m & 0 & 0 & 0 & 0 & 0 \\ U_{cL} & U_{cm} & U_{cc} & -p_c & 0 & 0 & 0 & 0 & 0 \\ -p_L & -p_m & -p_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda} G_{ss} & \frac{\mu}{\lambda} G_{sc} & \frac{\mu}{\lambda} G_{sL} & \frac{\mu}{\lambda} G_{sv} & G_s \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda} G_{cs} & \frac{\mu}{\lambda} G_{cc} & \frac{\mu}{\lambda} G_{cL} & \frac{\mu}{\lambda} G_{cv} & G_c \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda} G_{Ls} & \frac{\mu}{\lambda} G_{Lc} & \frac{\mu}{\lambda} G_{LL} & \frac{\mu}{\lambda} G_{LV} & G_L \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda} G_{vs} & \frac{\mu}{\lambda} G_{vc} & \frac{\mu}{\lambda} G_{vL} & \frac{\mu}{\lambda} G_{vv} & G_v \\ 0 & 0 & 0 & 0 & G_s & G_c & G_L & G_v & 0 \end{bmatrix} \begin{bmatrix} dX_L \\ dX_m \\ dX_c \\ d\lambda \\ dQ_s \\ dQ_c \\ dR_L \\ dR_v \\ d(\frac{\mu}{\lambda}) \end{bmatrix} = \begin{bmatrix} \lambda dp_L \\ \lambda dp_m \\ \lambda dp_c \\ \psi \\ -dq_s \\ -dq_c \\ dp_L \\ dr_v \\ 0 \end{bmatrix} \qquad (7)$$

Is obtained,[5]

where $\psi = -(T-X_L-R_L)dp_L + X_m dp_m - (Q_c-X_c)dp_c - dE - Q_s dq_s + R_v dr_v - \frac{\mu}{\lambda} G_k dk.$

This system of equations is of the form $\begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ which can be solved as $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} A_{11}^{-1} & C_1 \\ A_{22}^{-1} & C_2 \end{pmatrix}$. The first set of equations, corresponding to the upper left block of the bordered Hessian matrix, gives the solution for commodity demands and the marginal utility of full income. The second set of equations gives the solution for output supplies, variable input demands and the associated multiplier. The assumptions concerning the utility and production functions insures that second order conditions are met. Hence the two decision-problems can indeed be solved for separately, despite their simultaneity in time. This characteristic of separability has been an essential characteristic of most empirical studies to date, because of the computational tractability it affords (see Chapter 2).

Separation notwithstanding equation (7) demonstrates the principal message of the farm household literature, that farm technology, quantities of fixed inputs, and prices of variable inputs and of outputs do affect consumption decisions. Given separation, however, the reverse is not true. Preferences, prices of consumption commodities, and income do not affect production decisions. Output supply responds positively to own price at all times due to the quasi-convexity assumption on the production function, $\frac{\partial Q_s}{\partial q_s} = \frac{-\Omega_{11}}{\Omega} > 0$, where Ω is the determinant of the bordered Hessian matrix corresponding to A_{22} and Ω_{11} is the cofactor of the first row and column. Changes in the price of the cash crop, q_s, will affect consumption of the purchased commodity, X_m, through changed income. From equation (7) it can be seen that $\frac{\partial X_m}{\partial Q_s} = Q_s \frac{\partial X_m}{\partial E}$. Likewise changes in quantities of fixed inputs, K, will affect income, hence consumption of X_{mi}: $\frac{\partial X_m}{\partial K} = \frac{\mu}{\lambda} G_k \frac{\partial X_m}{\partial E}$. Assuming X_m is a normal commodity, increments to fixed inputs and to output prices of cash crops will induce higher consumption of X_m. For commodities which are also produced by the household, own price effects are

$$\frac{\partial X_c}{\partial p_c} = \left.\frac{\partial X_c}{\partial p_c}\right|_u + (Q_c - X_c)\frac{\partial X_c}{\partial E} \qquad (8)$$

Thus a change in price of X_c has the usual negative substitution effect, and an income effect which is weighted by net sales (or marketed surplus) of X_c, not consumption of X_c. The income effect is positive for a net seller and negative for a net buyer. In consequence, for net sellers, consumption of X_c might respond positively to changes in its own price even though X_c is a normal good.

The income effect for a farm household has an extra term, $Q_c \frac{\partial X_c}{\partial E}$, as compared to the pure consuming household. This extra effect results from the profits component of full income being raised, hence can be referred to as a "profits effect". To see this note that from equation (3) $dY = Tdp_L + d\pi + dE$ where $\pi \equiv$ profits, the value of outputs less the value of variable inputs. Also from equation (3) and the first order conditions $d\pi = Q_s dq_s + Q_c dp_c - R_L dp_L - R_v dr_v + \frac{\mu}{\lambda} G_k dK$ (see footnote 5.). Substituting into equation (7) the fourth element is derived as $\psi = -(T - X_L)dp_L + X_m dp_m + X_c dp_{\bar{c}} - d\pi - dE$ in the right hand vector. Thus $X_c(p_L, p_m, p_c, q_s, r_v, K, E) = X_c(p_L, p_m, p_c, \pi, E)$ and

$$\left.\frac{\partial X_c}{\partial p_c}\right|_\pi = \left.\frac{\partial X_c}{\partial p_c}\right|_u - X_c \frac{\partial X_c}{\partial Y} \qquad (8a)$$

which is identical to the pure consumer case, while

$$\frac{\partial X_c}{\partial p_c} = \left.\frac{\partial X_c}{\partial p_c}\right|_u - X_c \frac{\partial X_c}{\partial Y} + \frac{\partial X_c}{\partial Y} \frac{\partial \pi}{\partial p_c} \qquad (8b)$$

Since $\frac{\partial \pi}{\partial p_c} = Q_c$, from above, the extra effect does indeed come through charging farm profits. The comparative status for leisure are similar,

$$\frac{\partial X_L}{\partial p_L} = \left.\frac{\partial X_L}{\partial p_L}\right|_u + (T-X_L-R_L) \frac{\partial X_L}{\partial E} \qquad (9)$$

The income effect is weighted by household labor supply minus labor demand (marketed surplus of labor), not by household labor supply. Assuming that leisure is a normal good this makes a backward bending supply curve less likely than if the household were solely a supplier of labor.

B. Deriving Virtual (Shadow) Prices

To explore the consequences of making prices endogenous to the household it will be convenient to use duality results to express the equilibrium of the household. We can define the full income function as the maximization of equation (3) with respect to outputs and variable inputs subject to the production function, (4), and can write

$$Y = \Lambda(q_s, p_c, p_L, r_v, K, T, E) = p_L T + \pi(q_s, p_c, p_L, r_v, K) + E \qquad (10)$$

Clearly the full income function can be written as the sum of the value of endowed time, a restricted (or short run) profits function and exogenous income. The profits function has the usual properties such as being convex in all prices. For the expenditure side of full income we can define an expenditure function as the minimum expenditure (equation 2) required to meet a specified level of utility, $e(p_L, p_m, p_c, \overline{U})$. It obeys the usual properties, in particular it is concave in prices, and the partial derivatives with respect to price are the Hicksian (compensated) demand functions.

Now we are in a position to relax our assumption that prices are fixed market prices. The household's equilibrium is characterized by equality between the household's full income function, Λ, and its expenditure function, e, where the expenditure function is evaluated at the utility level achieved at the household's optimum. This condition will hold whether or not households face given market prices. Now suppose that a household is constrained to equate consumption with production for some commodity(ies). One possible reason for this would be nonexistence of a market. Consequently the household's equilibrium will be characterized by a set of additional conditions -- equality of household demand and household supply for each commodity for which there is no market (Dixit and Norman, 1980).[6] This second set of equilibrium conditions implicitly defines a set of virtual prices --or shadow prices, see footnote 15--(Neary and Roberts, 1980; Deaton and Muellbauer, 1980, Chapter 4.3; Sicular, this volume), which if they existed would induce the household to equate supply and demand for these commodities.

These virtual prices are not fixed for the household as market prices are assumed to be. Rather they are determined by the household's choices. From the household's equilibrium it can be seen that they will be a function of market prices, time endowment, fixed inputs, and either exogenous income or utility.[7] Consequently these prices depend on both the household's preferences and its production technology. Changes in market prices will now affect behavior directly, as before, and indirectly through changes in the virtual prices. Some mechanism of identifying the consequences of this additional effect is therefore needed in order to illuminate the significance of one's assumptions regarding price formation. That mechanism will be the comparative statics of the virtual price, which will now be developed.

To be specific, suppose, for the moment arbitrarily, that there exists no market for labor. The household equilibrium is characterized by

$$\begin{aligned} e(\bar{p}_L^*, p_m, p_c, \bar{U}) &= \bar{p}_L^* T + \pi(q_s, p_c, \bar{p}_L^*, r_v, K) + E \\ e_L(\bar{p}_L^*, p_m, p_c, \bar{U}) &= T + \pi_L(q_s, p_c, \bar{p}_L^*, r_v, K) \end{aligned} \tag{11}$$

where $e_L = \frac{\partial e}{\partial p_L^*}$ and likewise $\pi_L = \frac{\partial \pi}{\partial p_L^*}$.

The second equation gives the Hicksian leisure demand on the left hand side and time endowment minus labor demand on the right. From this equation $\bar{p}_L^*$, the compensated virtual price, can be solved for as

$$\bar{p}_L^* = \bar{p}_L^*(p_m, p_c, q_s, r_v, K, U) \tag{12}$$

Note that the utility level is being held constant, and not exogenous income. Alternatively, the Marshallian leisure demand, $X_L(p_L^*, p_m, p_c, p_L^*T+\pi+E)$, can be set equal to time minus labor demand, and a solution obtained:

$$p_L^* = p_L^*(p_m, p_c, q_s, r_v, K, E) \tag{13}$$

To relate the functions $\overline{p}_L^*$ and p_L^* a somewhat different "expenditure" function is needed. Let

$$e'(p_L, p_m, p_c, q_s, r_v, K, T, \overline{U}) = \min_{\substack{X_L, X_m, X_c \\ Q_s, Q_c, R_L, R_v}} p_L X_L + p_m X_m + p_c X_c - p_L T - q_s Q_s - p_c Q_c + p_L R_L + r_v R_v \tag{14}$$

$$\text{st } U(.) = \overline{U} \text{ and } G(.) = 0$$

This represents the minimum exogenous income, E, necessary to achieve utility level $\overline{U}$, given the production function and prices. It is clear that e' meets all the conditions which a regular expenditure function does, and that

$$e'(p_L, p_m, p_c, q_s, r_v, K, T, \overline{U}) = e(p_L, p_m, p_c, \overline{U}) - p_L T - \pi(q_s, p_c, p_L, r_v, K) \tag{15}$$

In equation (13), if exogenous income E is evaluated at e' (hence full income, Y, at e) then Marshallian leisure demand equals the Hicksian demand and $p_L^* = \overline{p}_L^*$. Using this equality

$$\frac{\partial \overline{p}_L^*}{\partial Z} \quad \left.\frac{\partial p_L^*}{\partial Z}\right|_E + \frac{\partial p_L^*}{\partial E}\frac{\partial e'}{\partial Z} \qquad Z = p_m, p_c, q_s, r_v, K \tag{16}$$

With utility constant the response of the virtual price can be expressed in terms of second partial derivatives of the expenditure and profit functions. Using the implicit function rule and equation (11)

$$\frac{\partial \bar{p}_L^*}{\partial Z} = -(e_{LZ} - \pi_{LZ})/(e_{LL} - \pi_{LL}) \ , \ Z = p_m, p_c, q_s, r_v, K \qquad (17)$$

The denominator is unambiguously negative due to the concavity of the expenditure function and the convexity of the profits function. The numerator can be either sign, but often the sign will be determinate if one is willing to assume that commodities are substitutes or complements in consumption or production. for instance if $Z = p_m$ the numerator is $-e_{Lm}$, which is negative if leisure and X_m are substitutes. If $X = p_c$ the numerator is $\pi_{Lc} - e_{Lc}$. The first term is the response of output of X_c to wage, which should be negative. The second term is negative if leisure and X_c are substitutes. For an input price, r_v, the numerator is π_{Lv} which can be positive or negative depending on whether labor and input R_v are gross substitutes or complements.

Equation (17) is a basic result which will be repeatedly used in subsequent discussion to illuminate the effects of totally or partly absent markets. It allows one to sign the partial derivatives of the compensated virtual price, making this device of use in looking at the comparative statics. Moreover it allows one to directly compare models which make differing assumptions concerning the nature of prices which the household faces.

The sign of the response of the compensated virtual price, $\bar{p}_L^*$, to exogenous variables can be given a very intuitive interpretation.

If for instance the price of the cash crop rises, the demand schedule for labor should shift upwards. Given that other market prices, fixed inputs, and utility are constant, the virtual wage has to rise in order to reequate compensated labor supply with demand. Such a rise will lower labor demand along the new schedule, while raising compensated, or Hicksian, labor supply.

As should be clear, whether prices are exogenous for commodities which are both consumed and produced by the household affects the type of interdependency between the household's consumption and production choices. For such commodities the virtual prices are functions of both household preferences and production technology. Because these prices help to determine both consumption and production choices--they belong in both the expenditure and the full income functions--the household commodity demands will depend on production technology both through the virtual price and through full income. Output supplies and input demands will depend on preferences through the virtual price. If, however, the household faces only market prices, or if it faces a virtual price for a commodity which is consumed but not produced (or vice versa), then production choices will not depend on household preferences, but consumption choices will depend on production technology through full income. The model is then separable.

3. Models With Absent Markets: Labor

In the historical development of agricultural household models partially autarkic behavior has been very important. One of the earliest models can be traced to the Russian economist A. V. Chayanov (1925).[8] He was concerned with explaining Russian peasant households' allocation of labor between work and leisure given his observation that virtually no hired labor was used in farm production activities. He recognized that such households were not simply maximizing profits as in the theory of the firm, rather they had a "subjective equilibrium" in which they equated the marginal utility of household consumption with the marginal utility of leisure. His analysis was embellished by a group of Japanese economists, including Tanaka (1951) and Nakajima (1957), during the 1950's and 60's. Nakajima (1969) in particular gave the model currency among English-speaking economists. He gave a mathematical formulation to Chayanov's model, and proposed some additional ones as well. Nakajima's model of a pure commercial family farm without a labor market (Nakajima, 1969) assumed that households sold all of their output, and purchased commodities from the market, while producing the output with family labor and a fixed amount of land. In this paper's notation he assumed $X_c = Q_c = R_v = 0$ and $Q_L = T - R_L$. He also allowed for the possibility of a minimum subsistence consumption requirement as well as a target income. In a different version (his semi-subsistence family farm) he allows the family to consume some of its output, and in another version introduces two outputs. Similar models of "peasant" households were advanced by Mellor (1963) and Sen (1966) and by economic anthropologists such as Fisk and Shand (1969). These models are thus special cases of the general form of the agricultural household model developed in Section 2.

One major use of these models in which the family supplied all of its labor was to explore the effects on labor supply (hence on labor demand, and on output since labor was assumed to be the only variable input) of changes in different variables. The effect of output price was of particular interest because of the seemingly perverse possibility that output might respond negatively to output price. This might occur if the income effect, resulting in more leisure demand, were large enough. Nakajima showed that an exogenous increase in land input might also reduce output, because it too would have an income effect on leisure. Nakajima separated the response of labor supply to output price into substitution and income effects, showing that the income compensated response of labor supply to output price was positive. Sen showed the possibility of a negative output response to output price, as well as the possibility of no output response to the withdrawal of family workers. The latter might occur if the remaining family laborers worked sufficiently hard to offset the reduced number of hours worked as workers were withdrawn. This in turn required that the virtual wage (or its ratio to output price, Sen's real cost of labor) be constant, which would be the case in Sen's model if the marginal utilities of both income and leisure were roughly constant.

The possibility of a negative response of labor demand (and of output supply) to output price at the household level is dependent on the constrained equality of labor demand and labor supply.[9] If markets exist for all commodities then the model is separable and labor demand will respond positively to output price so long as it is not an inferior input. Nakajima noted this when discussing his model with a labor market and a cash crop. Both Jorgenson and Lau (1969) and Krishna (1964, 1969) proposed separable semi-subsistence models in which labor is marketed and output is partially consumed at home. Jorgenson and Lau's paper has been particularly influential, forming the basis on which most of the empirical work to date has been conducted.

The difference which absence of a labor market makes to the comparative statics of leisure and commodity demand can easily be seen by using the notion of a virtual wage. Write the Marshallian demand as $X_i(p_L^*, p_m, p_c, p_L^*T + \pi(q_s, p_c, p_L^*, r_v, K) + E)$. Differentiate this with respect to q_s to obtain

$$\frac{\partial X_i}{\partial q_s} = \frac{\partial X_i}{\partial p_L^*}\frac{\partial p_L^*}{\partial q_s} + Q_s \frac{\partial X_i}{\partial Y} \qquad i = L, M \tag{18}$$

Output price has two effects on the demand for leisure or for the market purchased good: it has an income effect by changing profits (the second term), and it changes the virtual price for labor. Clearly when the household is a price taker in the labor market the latter effect is zero.

Equation (18) can be decomposed into substitution and income effects, which will help in signing the uncompensated changes in the demand for leisure and the market purchased commodity. First, it can be shown that the uncompensated effect with respect to the virtual wage equals the compensated effect. To do this it will be useful to equate Marshallian and Hicksian demands by evaluating full income, Y, at e and the virtual wage at $\bar{p}_L^*$ (i.e. both holding utility constant).

$$X_i(\bar{p}_L^*, p_m, p_c, e(\bar{p}_L^*, p_m, p_c, U)) = X_i^c(\bar{p}_L^*, p_m, p_c, U) \quad i = L,M \tag{19}$$

Differentiating both sides of (19) with respect to the cash crop price, q_s, and using $\frac{\partial e}{\partial \bar{p}_L^*} = X_L$ results in

$$\frac{\partial X_i}{\partial p_L^*}\Big|_Y \frac{\partial \bar{p}_L^*}{\partial q_s} + X_L \frac{\partial X_i}{\partial Y} \frac{\partial \bar{p}_L^*}{\partial q_s} = \frac{\partial X_i^c}{\partial p_L^*} \frac{\partial \bar{p}_L^*}{\partial q_s} \qquad i=L,M \tag{20}$$

Since

$$\frac{\partial X_i}{\partial p_L^*} = \frac{\partial X_i}{\partial p_L^*}\Big|_Y + (T - R_L) \frac{\partial X_i}{\partial Y} \tag{21}$$

and since labor supply equals labor demand, so that $X_L = T - R_L$, it can be shown (using equation (20)) that $\frac{\partial X_i}{\partial p_L^*} = \frac{\partial X_i^c}{\partial p_L^*}$. Thus the income effect of a change in the virtual wage equals zero, which is intuitive since the net marketed surplus is zero when no labor market exists.

The term $\frac{\partial p_L^*}{\partial q_s}$ in equation (18) can be made more transparent by noting from (16) that $\frac{\partial p_L^*}{\partial q_s} = \frac{\partial \bar{p}_L^*}{\partial q_s} + Q_s \frac{\partial p_L^*}{\partial E}$, (recall that $\frac{\partial e'}{\partial q_s} = -Q_s$). When this is substituted into (18) one obtains

$$\frac{\partial X_i}{\partial q_s} = \frac{\partial X_i^c}{\partial p_L^*} \frac{\partial \bar{p}_L^*}{\partial q_s} + Q_s \left(\frac{\partial X_i}{\partial Y} + \frac{\partial X_i}{\partial p_L^*} \frac{\partial p_L^*}{\partial E} \right) \qquad i = L,M \tag{22}$$

$$= \frac{\partial X_i^c}{\partial p_L^*} \frac{\partial \bar{p}_L^*}{\partial q_s} + Q_s \frac{\partial X_i}{\partial E} \qquad i = L,M \tag{22a}$$

Equations (22) and (22a) show the decomposed income and substitution effects. They also clarify the significance of one's view regarding the labor market. If the labor market does exist then the household faces market prices so the substitution effect (the first term in (22a)) is zero and the entire effect of the change in output price is captured by the income effect $(Q_s \frac{\partial X_i}{\partial Y})$. This is positive providing leisure or the purchased commodity are normal goods. When the labor market is absent there is a substitution effect caused by the change in the income compensated virtual wage. Using equation (17) we can rewrite this substitution effect as,

$$\frac{\partial x_i^c}{\partial p_L^*}\frac{\partial \overline{p}_L^*}{\partial q_s} = e_{Li}\pi_{Ls}/(e_{LL} - \pi_{LL}) \quad i = L,M \tag{23}$$

If the compensated virtual wage rises (that is if in equation (23), $\pi_{LS} < 0$), then there is a substitution away from leisure or towards the purchased commodity (if it is a substitute for leisure). The income effect comes in two parts, first a traditional looking income effect and second a substitution-type effect due to an induced change in the uncompensated virtual wage, p_L^*.[10] From equation (22) we can see that when leisure is normal, $\frac{\partial p_L^*}{\partial E} > 0$, the income effect is smaller for leisure and larger for purchased goods (assuming substituteability with leisure) when the labor market does not exist than when it does exist. An increase in exogenous income raises the uncompensated virtual wage which induces a substitution away from leisure or towards the purchased commodity.

Presuming that the entire income effect is positive, the net effect of a rise in output price q_s on leisure is indeterminant, while it will be positive for the purchased commodity. This is the same result, of course, as is obtained by both Nakajima (1969) and Sen (1966). Some analysts (e.g. Barnum and Squire, 1980) have argued that since the income effect is weighted by output, Q_s, it ought to outweigh the substitution effect, so that leisure (labor supply) should respond positively (negatively) to changes in output price. Of course this is an empirical question. Using similar reasoning would imply that landless households should possess backward bending supply curves since the income effect in that case is weighted by labor supply. Clearly such an assertion is an overgeneralization, and depends on empirical parameters. The point is there is little guidance to the size of the substitution effect.

A. Output Response

If labor is the only variable input then the sign of output response to output price must be the opposite to the leisure response. More generally we can write output supply Q_s as $Q_s = \frac{\partial \pi}{\partial q_s}(q_s, p_c, p_L^*, r_v, K)$ consequently

$$\frac{\partial Q_s}{\partial q_s} = \pi_{ss} + \pi_{sL} \frac{\partial p_L^*}{\partial q_s} \tag{24}$$

The first term is the output supply response when the virtual wage is fixed, and is positive. The second term is negative assuming that output responds negatively to the virtual wage ($\pi_{sL}<0$), so that the sign of the entire expression is indeterminant. It is possible to show that holding household utility constant the response is positive.[11] Substituting for $\frac{\partial p_L^*}{\partial q_s}$ from equation (16)

$$\frac{\partial Q_s}{\partial q_s} = (\pi_{ss} + \pi_{sL} \frac{\partial \bar{p}_L^*}{\partial q_s}) + Q_s \pi_{sL} \frac{\partial p_L}{\partial q_s} \tag{25}$$

The first two terms are the response of output supply holding utility constant. The third term is an income effect, which is negative if π_{sL} is. The second term equals $\pi_{sL}^2/(e_{LL} - \pi_{LL})$ so it is negative. However summing it with π_{ss} results in a non-negative quantity because the function e' (equation (15)) is concave in prices, so that $\frac{\partial^2 e'}{\partial q^2} \frac{\partial^2 e'}{\partial p_L^{*2}} - \left[\frac{\partial^2 e'}{\partial q_s \partial p_L^*}\right]^2 \geq 0$. Straightforward algebra shows that this expression is simply the first two terms in equation (25) multiplied by $-\frac{\partial^2 e'}{\partial p_L^{*2}}$. The magnitude of π_{sL}, and consequently the likelihood of a negative output response, will be influenced by the number of variable inputs and the partial elasticity of substitution between labor and these other inputs. Presumably the more inputs and the more substitutable they are, the less negative π_{sL} will be and the more likely will be a positive response to output price. Clearly when the virtual wage is exogenous to the household, output response will be positive, and greater than when virtual wage is endogenous.

If the household consumes some of the output whose price is changing, Q_c, the comparative statics have an additional substitution effect, and the income effect is weighted by net output sold (marketed surplus) and not by total output.

$$\frac{\partial X_i^c}{\partial p_L} = \frac{\partial X_i^c}{\partial p_c}\Big|_{\bar{p}_L^*} + \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_c}, \quad i = L,M,C \tag{26}$$

Again using equation (17), $\frac{\partial \bar{p}_L^*}{\partial p_c} = (\pi_{cL} - e_{cL})/(e_{LL} - \pi_{LL})$ which is positive if Q_c and leisure are substitutes. Deriving the comparative statics as before one finds

$$\frac{\partial X_i}{\partial p_c} = \left[\frac{\partial X_i^c}{\partial p_c}\Big|_{p_L^*} + \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_c}\right] + (Q_c - X_c)\left[\frac{\partial X_i^c}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} + \frac{\partial X_i}{\partial Y}\right] \quad i = L,M,C \tag{27}$$

$$\frac{\partial X_i}{\partial p_c} = \frac{\partial X_i^c}{\partial p_c} + (Q_c - X_c)\frac{\partial X_i}{\partial E} \quad i=L,M,C \tag{27a}$$

The substitution effect for leisure demand can be of either sign. It is not necessarily positive, even if X_c and leisure are substitutes holding the virtual wage constant. The income compensated response of X_c can also be of either sign when the wage is virtual, since an increase in the price, p_c, will increase the compensated virtual wage leading to a substitution toward X_c. Clearly the substitution effect for X_c will be less negative than when the labor market exists, analogous to the result obtained by Neary and Roberts (1980) for the pure rationing case. The income effect has an extra term, which for X_c and X_m is positive if leisure is a substitute and is negative for leisure demand.

D. Marketed Surplus Response

If we examine the response of marketed surplus of X_c, $Q_c - X_c$, to changes in p_c we obtain from (25), (16) and (27)

$$\frac{\partial(Q_c - X_c)}{\partial p_c} = \left[\frac{\partial Q_c}{\partial p_c}\bigg|_{\overline{p_L^*}} + \frac{\partial Q_c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_c} - \frac{\partial X_c^c}{\partial p_c}\bigg|_{\overline{p_L^*}} - \frac{\partial X_c^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_c} \right]$$

$$+ (Q_c - X_c)\left(\frac{\partial Q_c}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} - \frac{\partial X_c^c}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} - \frac{\partial X_c}{\partial Y} \right) \quad (28)$$

The first four terms (in brackets) hold utility constant, and therefore comprise the substitution effect. It is straightforward to see that this effect equals

$$-\frac{\partial^2 e'}{\partial p_L^{*2}}\left[\frac{\partial^2 e'}{\partial p_c^2}\frac{\partial^2 e'}{\partial p_L^{*2}} - \left(\frac{\partial^2 e'}{\partial p_c \partial p_L^*} \right)^2 \right]$$ and consequently is nonnegative

(remember that e' is concave in prices). The last term equals

$$(Q_c - X_c)\left(\frac{\partial Q_c}{\partial E} - \frac{\partial X_c}{\partial E} \right)$$ and so is the income effect, which should

be negative if marketed surplus is positive and X_c is a normal good. Consequently marketed surplus of X_c might respond positively or negatively to increases in its own price. Comparing this result with that when the labor market exists, one can see that the extra substitution effects will be negative if X_c and leisure are substitutes since the compensated virtual wage will then rise. The extra income effects should also be negative, so that a greater possibility exists of obtaining a negative own price response of marketed surplus of X_c.

The comparative statics with respect to changes in p_m, r_v, K and T are very similar to equation (22), the response of the compensated virtual wage being different as is the term weighting the income effect. Specific formulae are left for the interested reader to derive.

4. Models With Absent Markets: Z-Goods

It should be clear that which market one assumes not to exist does not affect the foregoing argument. Hence the existence of a labor market is a necessary but not sufficient condition for an agricultural household model to be separable. All markets must exist for separability (though this is not a sufficient condition -- see section 5). It happens that historically it was the labor market that economists thought was least likely to exist for peasant farms. That view has been changing, however. Active rural labor markets have been found to exist according to several recent studies (Squire, 1981; Bardhan, 1979; Rosenzweig, 1978; Spencer and Byerlee, 1977; Binswanger and Rosenzweig, forthcoming) although not necessarily perfectly competitive ones. More recently there has been focus on the nonexistence of a market for so-called Z-goods. This was first formalized by Hymer and Resnick (1969) who refer to Z-goods as non-agricultural, non-leisure activities. In general the commodities Hymer and Resnick refer to, such as food processing and metal working, are commodities for which small scale rural industries have been found to exist by recent investigators

(Anderson and Leiserson, 1980; and Liedholm and Chuta, 1976). However Z-goods equally as well refer to nontraded outputs of household production activities such as the number and quality of children, home maintenance or food preparation. In this way the household production models of Becker (1965) and Gronau (1973, 1977) can be incorporated into agricultural household models.

Hymer and Resnick were concerned with the increasing specialization of agricultural household activities which they saw as occuring over time, resulting in an increasing marketed surplus from agricultural households. Rather than focus on the leisure-labor tradeoff they focused on the Z-goods-food tradeoff. In terms of the general model in section 2, households produce foods, Q_c, which they consume, and sell the surplus in exchange for manufactured commodities, X_m. They produce Z-goods, our R_L, which they consume entirely at home, $R_L = X_L$. Labor supply did not enter their model, but implicitly it is assumed to be fixed in amount and equal to labor demand, thus it is not a choice variable. In terms of this model labor is one of the fixed inputs, K, and it does not appear in the utility function.[12] There are no other variable inputs, $R_v = 0$, nor does there exist a cash crop, $Q_s = 0$. These assumptions imply that the product transformation curve between foods and Z-goods has the usual downward-sloping, concave shape. Consequently to find the sign of the effect of a change in the price of foods, p_c, on output of foods only the effect on demand (hence supply) of Z-goods needs to be considered. That is $\frac{\partial X_L}{\partial p_c}$ is wanted, which is given by our equation (27). The substitution effect can be of either sign. If Z-goods and foods are substitutes a rise in food prices will increase Z-goods consumption, holding the compensated virtual price of Z-goods constant. However, this will force up the virtual price leading to a substitution away from Z-goods consumption. The income effect is weighted by the marketed surplus of foods, presumed to be positive. Hymer and Resnick assume that Z-goods consumption is inferior and that the combined substitution effect is small so that the net effect of a rise in foods price will be a fall in Z-goods consumption (and production), hence a rise in food production. Of course if foods are consumed by the household the food consumption response to food price needs to be examined before what happens to marketed surplus of foods can be judged. As seen from equation (28) marketed surplus of food can either rise or fall in response to an increase in food price, provided the household has a positive marketed surplus and Z-goods are normal (so $\frac{\partial p_L^*}{\partial E} > 0$). However, if Z-goods are inferior then its virtual price falls when exogenous income rises so that production of foods rises and compensated consumption of foods falls (provided foods and Z-goods are substitutes), making it more likely that the response of marketed surplus is positive.

The Hymer and Resnick assumption that leisure and labor demand are not choice variables can be relaxed. Let the production of Z-goods be Q_z, which equals consumption, X_z. Leisure is again denoted by X_L and labor demand R_L and $Q_s = 0$. If it is assumed that no labor market exists then two virtual prices exist, one for labor and one for Z-goods. There are thus two equality constraints on supply and demand rather than one. Using the implicit function rule and the fact that e' (equation (15)) is a concave function of prices it can be shown that the compensated virtual prices of both Z-goods and labor will rise in response to a rise in food price, p_c, provided foods, leisure and Z-goods are all Hicks-substitutes, and provided labor demand rises and Z-goods output falls when food price rises, holding virtual prices constant. A rise in p_c holding the two virtual prices constant raises compensated consumption of both leisure and Z-goods, raising labor demand and lowering Z-goods output. A combination of a rise in the price of Z-goods and labor can restore equilibrium in both markets. Considering changes in the marketed surplus of food when food price changes there are now two extra substitution effects and two extra income effects when labor is a choice variable. Under the current assumptions a rise in food price leads to a rise in the compensated virtual wage which lowers food production and raises food consumption, thereby lowering marketed surplus.[13] The extra income effects come through higher income raising the uncompensated virtual wage (since leisure is normal) which again should lower food production and raise food consumption. In particular the income effect of raising the virtual wage counters the income effect of lowering the virtual Z-goods price (assuming again that Z-goods are inferior) as Barnum and Squire (1979, p. 36) argue.

Finally the labor market can be allowed to exist, but not the Z-goods market. The response of marketed food surplus to changes in food price is given again by equation (28), interpreting p_L^* as the Z-goods virtual price. Now, however, the price of labor, not its quantity, is being held constant in the expenditure and full income functions. Thus the magnitude of the terms will be different from the fixed labor situation. In particular the cross price terms are likely to be smaller as is the change in the compensated virtual price, $\frac{\partial p_L^*}{\partial p_c}$. For example Z-goods production need not go down by as much (or at all) when food price increases since demand for labor and other variable inputs will likely rise.[14] This should occur since labor demand should rise as food production rises (holding the virtual price constant), and it will rise further in response to the rise in the compensated virtual price of Z-goods. The income effect on the virtual price of Z-goods would further add to labor demand if Z-goods were normal, however, under the inferiority assumption labor demand would be dampened.

Of course if both Z-goods and labor markets exist so that virtual prices are fixed, food output will respond positively to food price. Then food marketed surplus will only respond negatively to price if food consumption responds more positively (because of a large income effect) than does production.

As an alternative to the Hymer and Resnick interpretation Z-goods might be interpreted as being synonymous with household production activities. The original work of Becker (1965), Lancaster (1966), and Muth (1966) emphasizes that the commodities which yield household utility are produced within the household by goods purchased in the market and by labor. In terms of this general model X_c is a vector of commodities consumed and produced in the home. Market purchased inputs are denoted by R_v ($X_m = 0$), and labor demand, R_L, is a vector of time allocated to the production of each commodity. Leisure usually is not considered so total time is the sum of time spent in household production, plus market work.[15]

One of the major uses to which the household production approach has been put is to model the demand for the quantity (and quality) of children. Rosenzweig (1977) has applied such a model to agricultural households. The primary interest in these models is in explaining the comparative statics of child demand and education. The effect of agricultural production (or market work opportunities) is to reduce the shadow price of children, since they work on the farm and possibly on the market (this ignores possible quantity-quality tradeoffs, e.g., Rosenzweig, 1982). Changes in male and female wages now affect the shadow price of children through substitution among farm production inputs as well as through the child production function.

An elaboration of the household production framework by Gronau (1973) provides results almost identical to the model of Hymer and Resnick. Gronau's model amounts to relabeling food consumption as leisure and food production as labor demand. He too has a market purchased and a home produced (Z) commodity, with home production using labor and purchased inputs. Like the Hymer and Resnick model, a virtual price exists for the home produced (Z) good. If no labor is supplied to the market there will exist a virtual (shadow) wage as well, and the analysis is comparable to the Hymer and Resnick model when labor is a choice variable but no market for it exists. In a later paper Gronau (1977) assumes that the market purchased and the household produced commodities are perfect substitutes in consumption and so may be added. So long as market purchases are positive and labor is sold on the market this model is separable. If labor is not sold on the market a virtual (shadow) price for labor exists and if market purchases of the home produced commodity are zero a virtual price for it exists.[16] Huffman and Lange (1982) have a slightly different version of Gronau's model in which the household is explicitly an agricultural household. The household jointly produces a farm and a household commodity (X_s and X_c), selling the former and consuming the latter. Labor is sold on the market, but the only market purchases are for production inputs. A virtual price exists for the household commodity and the model is not separable. If, however, the farm and household commodities had separate production functions and fixed inputs could only be allocated to one enterprise, the model would be separable between farm production decisions and the rest.

5. Partly Absent Markets: Commodity Heterogeneity

Even if all markets exist households may face a virtual price which depends on both production technology and household preferences, so that again an agricultural household model would not be separable. This can occur because markets are partly absent or because of institutionally imposed constraints (see Sicular, this volume, for an analysis of such constraints imposed on a production team in the Peoples Republic of China). In particular a household may be able to sell a commodity but not buy it, or vice versa. If this commodity is both consumed and produced by the household then the household's optimum may be at a corner at which consumption equals production. Such corner solutions are especially likely to occur when commodities are heterogeneous. For example, hired and family labor may be imperfect substitutes because of extra monitoring or search costs of hired labor. On-farm and off-farm labor may give different levels of disutility (see Lopez, this volume). Alternatively a commodity consumed out of home production may have a different quality than the same commodity purchased on the market, resulting in differing sales and purchase prices.

Households can sell and consume family labor or home production, but they can not purchase them. This raises the possibility that at the market price supply might be less than demand, which is not possible. For such corner solutions the commodity in question has a virtual price which would equate supply and demand. The virtual price will be higher than the market price provided that compensated marketed surplus responds positively to price.

If households have preferences between on-farm and off-farm labor, then even if hired and family labor are perfect substitutes in production there may exist excess supply of on-farm labor at the market wage, in which case the virtual wage will be lower.

It should be clear that the comparative statics for these equilibria are identical to those considered earlier for the cases in which no market exists. Also, if these corner solutions are not birding then the model is separable, the market prices being the opportunity costs. This will complicate empirical work since, if such heterogeneity exists, a sample is likely to include both households at corners and households at interior solutions.

6. Summary

This chapter has reviewed some basic, static agricultural household models. A key modeling issue is under what circumstances a model is separable. This is very important for applied empirical work since it makes the problem far more tractable (see Chapter 2). It has been shown that a sufficient condition for separability is that all markets exist for commodities which are both produced and consumed, with the household being a price taker in each one, and that such commodities be homogeneous. So long as households can buy or sell as much as they want at given prices, production and consumption decisions can be treated as if they were sequential, production decisions being made first, even though they may be made simultaneously. Such strong conditions are not necessary, however. In particular, the homogeneity assumption can be dropped. However, in this case the agricultural household model remains separable only if the household does not choose to be at a corner for a commodity which it both produces and consumes (for example, consuming all of its output). If a corner solution is chosen, then a virtual price exists, which is a function of both preferences and technology, so that the household's decision is no longer separable. Note that even in the case of heterogeneity, it is still necessary to assume that all markets exist and that prices be given to households to achieve separability. If even one market does not exist (for a commodity which is consumed and produced), then separability between consumption and production decisions breaks down.[17]

Historically, nonseparable agricultural household models were thought to be relevant, primarily because labor markets were presumed not to exist. As more has been learned about rural labor markets in developing countries this assumption has become increasingly questioned. This does not mean that empirically relevant models have to be separable, but the reasons for nonseparability need to be clearly spelled out. It very well may be that reasons having to do with commodity heterogeneity are more important empirically than complete absence of markets.

Footnotes

[1]This definition can be traced to Krishna, 1969.

[2]This reveals a gap between the farm household literature and the literature on share tenancy and market interlinkages, in which market power may play a role.

[3]Clearly the X_i's can be a vector of commodity consumption for different members of the family as well. For instance we might want X_L to include male, female or children's leisure time separately. We could also allow household characteristics such as number of members to enter the utility function separately. So long as these are viewed as fixed this will not change the analysis.

[4]Obviously all these scalars could just as well be vectors.

[5]When differentiaitng the budget constraint we have substituted $-\frac{\mu}{\lambda}(G_s dQ_s + G_c dQ_c + G_L dR_L + G_v dR_v)$ for $q_s dQ_s + p_c dC_c - p_L dR_L - r_v dR_v$. This equals $\frac{\mu}{\lambda} G_k dK$ since $G(\cdot)=0$.

[6]Dixit and Norman use these conditions to characterize an economy under autarky.

[7]They will also be a function of fixed household characteristics if these are introduced into the model.

[8]See J. Millar (1970) for a reinterpretation.

[9]At the market level labor demand might respond negatively to output price if wage is bid up sufficiently (see Barnum and Squire, 1980).

[10]This two part income effect is identical to equation (24) of Neary and Roberts, once their equation (19) has been substituted in.

[11]See Lopez (1980) for a somewhat different demonstration of this.

[12]Alternatively leisure can enter the utility as a fixed factor, similar to other fixed household characteristics such as household size and age distribution. In this case the expenditure function will include leisure as a conditioning variable just as a short run cost or profit function includes fixed inputs.

[13]Other terms in the substitution effect such as $\frac{\partial X_c^c}{\partial p_L^*}$ and $\frac{\partial Q_c}{\partial p_L^*}$ will be changing magnitude compared to the situation of fixed labor supply.

[14]This assumes $\frac{\partial \bar{p}_L^*}{\partial p_c}$ is positive, which it would not be if $\pi_{Lc} - e_{Lc} > 0$.

[15]It is often assumed that Z-goods production is not joint and that it exhibits constant returns to scale. If there exist no fixed inputs the supply (and profit) functions will be ill-defined so that shadow (or implicit) prices cannot be defined in terms of equality between household supply and demands. Rather they are defined implicitly by the partial derivatives of the cost functions with respect to output (Pollak and Wachter, 1975). However if fixed inputs do exist, or the production functions are strictly convex, shadow (or virtual) prices can be implicitly defined from the equality of household demand and supply functions.

[16]If the household could sell its home produced commodity on the market as well as buy it then the market price would be the shadow price (assuming that quality adjusted sales and purchase prices were identical) even if purchases were zero. Likewise if the household could hire labor which was a perfect substitute for its own labor then the shadow wage would be the market wage even if market supply were zero.

[17] With multiple outputs it is possible for a subset of production decisions to be separable from other production and consumption decisions. This could occur if the production functions were nonjoint (separate), and if there was no fixed factor which had to be allocated to the different production activities. An example might be household and farm production. With no market for the household good, household production and consumption decisions are not separable, but they might be jointly separable from farm production.

Table 1

Percentage of Labor Force in Agriculture, 1980[a]
Selected Developing Countries

All Low Income Economies	70
Bangladesh	74
China	69
Malawi	86
India	69
Sierra Leone	65
Haiti	74
All Middle Income Economies	44
Egypt	50
Dominican Republic	49
Nigeria	54
Indonesia	55
Philippines	46
Korea	34
Malaysia	50

[a]From Table 21, World Bank, World Development Report 1983. Low income economies are those with a 1981 per capita income of less than US$410. Middle income economies are those with a 1981 per capita income over U.S.$410.

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