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THE EFFECTS OF PRICE UNCERTAINTY ON THE FACTOR CHOICES

OF THE COMPETITIVE FIRM

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Abstract

If the price of a variable factor or of output is uncertain when the quasi-fixed factor must be chosen by a competitive firm, the profit-maximizing factor allocations differ from those made when prices are fixed at their expected values. This paper shows that the responses of each factor to input or output price uncertainty are related to observable characteristics of the input demand and output supply functions. The effects on expected relative factor intensity are also investigated; they are related to the elasticity of substitution in the case of the CES production function.

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It is well known that if the price of a variable factor or of
output is uncertain when the level of a quasi-fixed factor must be
chosen by a competitive firm, the profit-maximizing allocative choices
differ from those made when all prices are fixed at their expected
values. The expected responses of input levels to price uncertainty
depend on the characteristics of the production function, but even
their signs are difficult to determine empirically, as they depend
on, among other parameters, the third derivatives of the production
function, which are difficult to identify. This paper shows that the
response of the quasi-fixed factor, and the expected short-run
response of the variable factor are directly related to observable
characteristics of the input demand and output supply functions.
But concentration on the responses of individual inputs provides only
a partial view of the allocative effects of price uncertainty. When
the full effects on both factors are considered together, both types
of price uncertainty induce different biases in expected factor pro-
portions, relative to choices under certainty. The directions of these
biases cannot be inferred from results regarding the sign of the response
of either factor viewed in isolation.

Previous studies in this field include Sandmo [11], Batra and Ullah
[1], and Holthausen [4], who showed that non-neutral
attitudes toward risk can affect the level of output and the amount
of a quasi-fixed input chosen by a firm faced with product price

uncertainty.   Stewart [12] showed a similar result for factor price uncertainty. The above studies all assume explicitly or implicitly [6; 4] that there is no reduction in price uncertainty between the time (if any) separating decisions on different inputs allowed. Thus the usual distinction between short-run and long-run allocative decisions is ruled out.

Turnovsky   [13]   showed that this assumption is crucial, using a model which maintained an alternative assumption that firms could modify their planned output at additional cost after the selling price is known. The effect of output price uncertainty on planned output (which is long-run cost minimizing output, to be distinguished from expected output) is ambiguous, even under risk neutrality [13, 412].

More recently Hartman [3] and Epstein [2] have followed Turnovsky's approach of allowing some flexibility in technology after prices are known. Hartman concentrates on the effect of increasing output price uncertainty on the level of the "quasi-fixed" factor which must be chosen before prices are known, using a two-factor model where the other factor can be allocated after output price is known (See also Perrakis [8]).   Epstein adopts a generalized version of this model. Under risk neutrality these studies show that the effect on the quasi-fixed factor(s) depends on the convexity of the marginal productivity function of the quasi-fixed factor with respect to output price.

The model outlined in the next section similarly recognizes the importance of the distinction between quasi-fixed and variable factors of production. Risk neutrality is assumed throughout.   Section II

addresses the issue which has attracted the greatest recent interest, the effects of input and output price uncertainty on the quasi-fixed factor. It shows that the direction of both these effects can be directly inferred from observable characteristics of the factor demand and/or output supply curves.

In Section III, I consider the net effect of both types of price uncertainty on the expected input of the variable factor. The short-run effects of input and output price uncertainty depend on characteristics of the input demand and output supply functions, respectively. The long-run effects, which also include the indirect effect through the response of the quasi-fixed factor, are also examined.

The individual factor responses of Section II and III are brought together in Section IV, which considers the effects of price un-certainty on expected factor proportions. Several interesting results are obtained for marginal price uncertainty. The effects of marginal mean-preserving spreads in input and output price are shown to be quite distinct, and depend in different ways on the specification of the production function. Straightforward results are obtained for some popular production function specifications. The conclusions are contained in Section V.

I. The Model

The competitive price-taking firm under consideration produces an output Y using two factors H and J. The production function is

(1) $Y = F(H, J)$

Throughout this paper partial derivatives will be denoted by numeric subscripts, except where clarity demands more formal notation. Thus partial derivatives of F with respect to H and J are F_1, F_2 respectively. The following conditions are assumed:

$$(2) \qquad F_1 > 0, \quad F_2 > 0, \quad F_{11} < 0, \quad F_{22} < 0.$$

For the size of the firm to be determinate, it is also necessary to assume diminishing returns to scale.

$$(3) \qquad F_{11} F_{22} - F_{12}^2 > 0.$$

Production is envisaged as taking place in discrete time periods. Within each period the price j of input J and the price p of output Y are constant, and are generated by stationary stochastic processes with independent cumulative frequency distributions with finite variances, U(j) and V(p), respectively, which are known by all agents. Technology is such that for production in any time period the input of factor H must be chosen, at price h, in the previous period (when the relevant output price, p, and the price j of the other input, is not known). In this sense H is a quasi-fixed factor. J is chosen in the production period, when all the relevant prices are known, so factor J is a variable factor. It is assumed that H, J, h and j are strictly positive.

The choice of H is determined by maximization of the long-run expected profit function given its own price h. At that time p, j and J for production in the next period are unknown. But it is known that they will be related by the maximization of short-run

profits, Π :

(4)
$$\Pi = p \, F \, (H, \, J) - jJ$$

The first order condition is:

(5)
$$pF_2(H, \, J) = j$$

This can be solved to obtain the short-run derived demand for J:

(6)
$$J^* = J^* \, (H, \, p, \, j)$$

Substitute (6) in (4) to obtain the short-run profit function

(7)
$$g(H, \, p, \, j) = pF(H, \, J^*(H, \, p, \, j)) - jJ^*(H, \, p, \, j)$$

By the envelope theorem

(8)
$$g_1 = pF_1(H, \, J^*(H, \, p, j))$$

(9)
$$g_3 = -J^*(H, \, p, \, j)$$

where J^* is the short-run derived demand for the variable input, and

(10)
$$g_2 = F(H, \, J^*(H, \, p, \, j))$$
$$= S(H, \, p, \, j)$$

where $S(H, \, p, \, j)$ is the short-run supply function for Y.

II. The Response of the Quasi-Fixed Factor

The response of H to increased variability in the prices of J and Y is derived as follows. Expected long run profits are given by

(11)
$$E(\Pi) - hH = E[pF(H, \, J^*(H, \, p, \, j)) - jJ^*(H, \, p, \, j) - hH]$$
$$= E[g(H, \, p, \, j) - hH]$$

where E denotes the conditional expectation given the information

set available in the period prior to the production period.

The first order condition for optimum H is

(12) $\qquad E[g_1 - h] = 0$

If g_1 is convex (concave) in the price in question, over the

entire range of prices considered, increasing price variability

defined as a mean-preserving spread (m.p.s.) in price will increase

(decrease) $E[g_1]$ $\qquad$ [9; 10].[2] The

expected-profit-maximizing producer will change H in the same

direction. Using these conditions, direct and general inferences

regarding the direction of change in H can be drawn from inspection

of the factor demand and output supply functions of the firm. To

see this, consider first the response to input price uncertainty.

Differentiating (9),

(13) $\qquad g_{33} = -J_3^*$,

and it follows that

(14) $\qquad g_{331} = -J_{31}^*$

Now if g_{331} is positive (negative), an m.p.s. in j will increase

(decrease) the profit-maximizing level of the quasi-fixed input H.

Further, as shown in Appendix I,

(15) $\qquad g_{221} = F_2^2 \, g_{331}$

Since g_{221} has the same sign as g_{331}, an m.p.s. in p will move

H in the same direction as an m.p.s in j. Therefore a sufficient

condition for an increase (decrease) in H in response

to an m.p.s. in the distribution of input or output price

is that at higher levels of the quasi-fixed factor H,

the slope of the short-run derived demand curve for J

is less (more) steep at every input price j, given p.

Examples of the alternate possibilities are illustrated in Figures

1a and 1b, in which J^* denotes the short-run demand curve, output

price p is fixed at P_o, and $H_2 > H_1$.

The effects of price uncertainty on H can also be inferred

by inspection of the short-run supply curve. Differentiating (10),

(16) $$g_{22} = F_2 J_2^* = S_2$$

Therefore

(17) $$g_{221} = S_{21} .$$

Thus if the slope of the short-run supply curve at any price p

is lower (higher) given j, at higher levels of H, an m.p.s. in the

distribution of p or, (using (15)), of j will increase (decrease) H. Examples of

these alternate cases are shown in Figures 2a and 2b, in which the

short-run supply curve is $S(H,P,j)$, j is fixed at j_o and $H_2 > H_1$.

Therefore if <u>either</u> the short-run demand curve for the variable

input, or the short-run output supply curve, can be estimated at

different levels of H, the effect of H on the slope

indicates the effect of price uncertainty on H.[3/] The intuition

behind these results is that H responds to price uncertainty in

the direction which makes production "more flexible" in the short run.

Note in addition that equations (13) and (16), and Appendix 1 equations

(A.1) and (A.2) show that g_{22} and g_{33} are both positive. Therefore the

insight of Oi [7] that risk-neutral producers prefer output price
uncertainty to certainty is confirmed, and in addition producers gain
from input price uncertainty in this model.

The above conditions are much more intuitive, and empirically
relevant, than conditions derived directly from the production function,
which have been the focus of the studies on the response of the quasi-
fixed factor mentioned in the introduction. However, for the sake of
completeness, expressions for g_{221} and g_{33}, in terms of the derivatives

Appendix I

of the production function, are derived in Appendix 1. It is obvious
that general conclusions are not forthcoming.

One problem is that if the production function is not homothetic,
one component of the effect of an m.p.s. in price p or j might be a pure
scale effect on expected factor <u>proportions</u> due to a change in the scale
of the firm's planned output, induced by the rise in expected profit-
ability indicated by the convexity of g in j and p (see Appendix I,
equations (A.3) and (A.4)). This possibility is eliminated if the
production function is homogeneous of some degree μ; $0 < \mu < 1.$ ($\mu \geq 1$
is ruled out by the assumption of a determinate equilibrium for a
profit-maximizing price-taking firm.) Accordingly homogeneity is assumed
in the following results regarding the response of the quasi-fixed factor,
expressed in terms of the parameters of the production function, which

Appendix II

are derived in Appendix II.

1) If $F_{222} \leq 0$, and F is homogeneous of degree μ , $0 < \mu < 1$,
the quasi-fixed factor increases in response to an m.p.s. in input or output price

2) If F_{222} is positive, the response depends on the values of the
parameters F_2, F_{22}, F_{222} and μ.

As Perrakis   [8]   has shown, the response of H is positive for the
general form F(W(H), J) which is linear homogeneous in (W,J) where W(H) is
any increasing and concave function, including the Cobb-Douglas with decreasing
returns to scale as a special case. For the CES function

(18)
$$Y = G(H,J) = [aH^{-\beta} + bJ^{-\beta}]^{-\frac{\mu}{\beta}}$$

where a, b > 0, 0 < μ < 1, β > -1, β ≠ 1, the sign of the response

of H to an m.p.s. in output price is the same as the sign of G_{HJ}, or of

(μ + β) as Hartman [3, 678] . has shown. As indicated above, the

response of H to an m.p.s. in the price of the variable input is in

the same direction as the response to an m.p.s. in output price.

III . The Response of the Variable Factor

The choice of the variable input J occurs after the prices

p and j are known. As for the response of H considered above, the

short-run response of J, holding H constant, can be directly inferred from

observation of the supply and factor demand curves of the firm.

The sign of the short-run response of the expected value of J to an m.p.s.

in j is positive (negative) as the derived demand curve J^* is convex

(concave) in j. If, as is generally the case, demand curves are

assumed to be convex or linear in j, an m.p.s. in j would not

be expected to decrease the short-run demand for J. But distinguishing

empirically between the linear case, in which a certainty equivalence

result holds, and, for example, a convex constant elasticity

specification, may be a difficult task. The curvature

of the short-run supply curve determines the direct response to an

m.p.s. in p. From (10)

(19)
$$S_2 = F_2 \, J_2^*$$

Therefore

(20)
$$J_2^* = S_2/F_2$$

and

$$(21) \qquad J_{22}^* = (F_2 S_{22} - S_2 F_{22} J_2^*)/F_2^2$$

By implicit differentiation of (5),

$$(22) \qquad J_2^* = -F_2/(pF_{22})$$

Substituting in (21),

$$(23) \qquad J_{22}^* = (S_{22} + S_2/p)/F_2$$

Thus

$$(24) \qquad J_{22}^* \gtreqless 0 \text{ as } pS_{22}/S_2 \gtreqless -1$$

The term pS_{22}/S_2 is a measure of curvature analogous to the Arrow-Pratt measure of relative risk aversion. Only if the short-run supply function is concave in p, and has sufficient curvature so that the value of this measure is less than minus one, will an m.p.s. in output price reduce the expected input of J, given H.

If, to take two common empirical specifications, the supply curve is linear, or has constant positive elasticity with non-negative slope, $(S = aP^b$, a, b > 0) then output price uncertainty will always increase the expected demand for the variable input J, given H. If, however, a relatively elastic supply curve turns sharply upwards at some quantity, as might be true, for example, in a production function in which there is an absolute upper bound on short-run output, then a mean-preserving spread in price in the

region of the up-turn might decrease the expected use of the variable factor J.

The sign of the effects of price uncertainty on short-run demand for J can also be related to the parameters of the production function. Take the partial derivative of equation (A.1) in Appendix I with respect to j:

$$(25) \quad J^*_{33} = - F_{222}/p^2 F^3_{22}$$

Similarly, from (A.2),

$$(26) \quad J^*_{22} = (2F_2 F_{22} - F^2_2 F_{222}/F_{22})/(p^2 F^2_{22})$$

Thus the direct effect of an m.p.s. in input price j on the expected value of J has the same sign as F_{222}, while $F_{222} \leq 0$ is a sufficient condition for the direct effect of an m.p.s. in p to be a decrease in the expected value of the variable input.

Till now we have considered only the short-run response of J, given H. But as shown in Section II above, the level of H also responds to changes in the dispersion of prices p and j. By implicit differentiation of (5),

$$(27) \quad J^*_1 = - F_{12}/F_{22}$$

This equation shows that, given a realized output price p, the response

of J^* to greater ex ante uncertainty in p will depend on the signs of the response of H and of F_{12}, as noted by Hartman [3]. The same is true of the response to greater uncertainty in j. In the case of the CES production function (18), the response is clearly non-negative in both cases.

The net long-run effect of price uncertainty on the expected input of the variable factor is the sum of the direct effects considered above, and the indirect effect through H. To aggregate the two types of effect, it is necessary to restrict our attention to analysis of a marginal mean-preserving spread of price, defined by the following application to the input price j.

For an initially deterministic input price $\bar{j}$, substitute $\bar{j} + a_3 \psi$ where ψ is white noise with zero mean and unit variance, and a_3 is a scalar. [4] Use of this substitution in implicit differentiation of (12) shows that, at $a_3 = 0$,

$$(28) \qquad \partial H / \partial a_3 = 0 .$$

Differentiating once more we find that

$$(29) \qquad \partial^2 H / \partial a_3^2 = - g_{133}/g_{11} .$$

Similarly, substituting $\bar{p} + a_2 \psi$ for p, we can derive

$$(30) \qquad \partial^2 H / \partial a_2^2 = - g_{122}/g_{11} .$$

Remembering that (5) holds throughout,

$$(31) \qquad g_{11} = p(F_{11} + F_{12} J_1^*) = p(F_{11} - F_{12}^2/F_{22}) \ .$$

The total (direct and indirect) effects of marginal mean-preserving spreads in j and p are given by

$$(32) \qquad \partial^2 J^*/\partial a_k^2 = J_{kk}^* + J_1^* \ \partial^2 H/\partial a_k^2$$

$$= J_{kk}^* + F_{12} g_{kk1}/(F_{22} g_{11}) \ , \qquad k = 1, 2$$

The direction of these responses is indeterminate in the absence of further restrictions. In the Cobb-Douglas case, both the direct and indirect responses are positive in (32), so that the net response of the variable input to a marginal mean preserving spread (m.m.p.s) in input or output price is greater than the direct response taking H as given. But if, for example, the production function is homogeneous and $F_{222} < 0$, the direct and indirect effects have opposite sign, and the sign of the net effect depends on the specific parameter values.

IV. The Effect of Price Uncertainty on Expected Factor Proportions

The two preceding sections have shown that input and output price uncertainty affects the levels of both the quasi-fixed and the variable factor, even if the decision maker is a risk-neutral profit-maximizer.

Although the analysis of the variable factor response goes beyond previous studies in considering both the direct and indirect effects of marginal price uncertainty, up to now the study has followed the usual approach of considering the response of each factor separately. This approach has the disadvantage of confounding the effects of changes in relative factor intensities with the implications of changes in the absolute level of inputs induced by the expected profit increase due to increased price variability, shown in Section II above. In this section the analysis is extended to consider the effects of marginal price uncertainty on relative demands, by combining the results of the previous sections in assessing the effects of an m.m.p.s in input or output price on the expected ratio of the variable to the quasi-fixed factor.

The effect on expected factor intensities of marginal uncertainty in p or j is given by

$$\partial^2 E[J/H]/\partial a_k^2 = H^{-2}\left[HJ_{kk}^* + HJ_1^* \ \partial^2 H/\partial a_k^2 - J^* \ \partial^2 H/\partial a_k^2 \right], \ k= 2,3 \qquad (33)$$

where the three terms on the right hand side represent the direct response of J, the indirect response of J, and the response of H, respectively, and the derivative is evaluated at $a_k = 0$. Similarly, using (27),

$$\partial^2 E[J/H]/ \ \partial a_k^2 = H^{-2}\left\{ J_{kk}^* H + \ g_{kk1}/g_{11}\left[HF_{12}/F_{22} + J^* \right] \right\} \quad k = 2,3 \qquad (34)$$

If the production function is homogeneous of some degree μ, $0 < \mu < 1$, one can substitute (A.10) from Appendix II for F_{12} in (34):

$$\partial^2 E[J^*/H]/\partial a_k^2 = [F_{22}g_{11}J_{kk}^*H + g_{kk1}(\mu-1)F_2]/(F_{22}g_{11}H^2) \qquad k = 2, 3 \qquad (35)$$

If $F_{222} < 0$, J_{22}^* and J_{33}^* are negative, and $g_{3\hat{3}1}$ and g_{221} are positive. Therefore $F_{222} < 0$ implies that (35) is negative, in which case marginal input or output price uncertainty reduces the expected relative intensity of the variable factor. If $F_{222} > 0$, further restrictions on the production function are necessary to determine the signs of (35) for $k = j$ and $k = p$. The CES function presented in (18) above provides an instructive example. After some rather tedious manipulation,[5'] (35) can be rewritten for $k = j$, as

$$\partial^2 E[J^*/H]/\partial a_3^2 = -ab^2\mu^3(1-\mu)(\beta+1)H^{-\beta-1}J^{-2\beta-5}M^{-(3\mu/\beta+5)} \cdot$$

$$[2(\mu+\beta)bJ^{-\beta}-(\beta+2)M][\mu+\beta)bJ^{-\beta}-(\beta+1)M]/(pF_{22}^4g_{11}H^2) \qquad (36)$$

where $M \equiv aH^{-\beta} + bJ^{-\beta}$, and the derivative is evaluated at $a_3 = 0$.

Since the last term in square brackets in the numerator is negative, and $g_{11} < 0$, the sign of (36) is the opposite of the sign of $[2(\mu+\beta)bJ^{-\beta}-(\beta+2)M]$. A sufficient condition that the response is positive is $\beta < 2(1-\mu)$, or $\sigma > 1/(3-2\mu)$. Since a necessary condition for determinate firm size in a competitive setting is $\mu < 1$, for $\sigma \geq 1$ the response of the expected factor intensity $E[J^*/H]$ to marginal uncertainty in the price of the variable input will always be positive.

Using (25) and (26), the condition for the effect of a marginal mean-preserving spread in output price may be expressed as:

$$\partial^2 E(J/H)/\partial a_2^2 = 2F_2/P^2 H F_{22} + F_2^2 \left[(F_{22}g_{11}J_{33}^* H + g_{331}(\mu-1)F_2)/G_{11}F_{22}H^2 \right] \tag{37}$$

The term in square brackets is identical to the right hand side of (35). Using this fact, we obtain

$$\partial^2 E(J/H)/\partial a_2^2 = F_2 \beta(\beta+1)\mu^4(1-\mu)ab^3 H^{-\beta-1}J^{-3} {}^{-6}_M {}^{-(4\mu/\beta+5)} \cdot$$

$$[1+\beta)aH^{-\beta}+(1-\mu)bJ^{-\beta}]/(pF_{22}^4 g_{11}H^2) \tag{38}$$

The sign of this expression is opposite to the sign of β .

If β is negative, σ is greater than unity, and the effect of marginal output price uncertainty is to increase the expected relative intensity of the variable factor. The cases where the response of H is negative belong in this category. If β is positive (σ is less than unity), the effect of marginal output price uncertainty is to decrease the expected relative intensity of the variable factor. For the Cobb-Douglas function the expected relative intensity is unaffected by output price uncertainty. As the previous sections showed, the expected input of the variable factor J, and the input of the quasi-fixed factor H both increase when the technology is Cobb-Douglas, but the expected net increase induced in the variable factor is exactly proportional to the adjustment of the quasi-fixed factor. The constancy of factor shares of the Cobb-Douglas function is preserved in expectation even in this stochastic two-stage decision making environment.

Table I   Table I summarizes the responses derived for the CES case.

The change in H, emphasized in previous studies, is obviously no useful guide to the expected relative factor intensity responses to either input or output price uncertainty.

V. Conclusions and Implications

In a competitive industry with one quasi-fixed and one variable factor, price uncertainty affects the factor input decisions of rational producers, even if they are risk-neutral. Intuitive and practical criteria for the direction of change of the quasi-fixed factor in response to a change in either input or output price dispersion are the signs of the changes in the slopes of either the short-run supply curve or the short-run demand curve for the variable factor, as the quasi-fixed factor increases. In all cases, H shifts in the direction which induces greater short-run flexibility in responding to price fluctuations.

The short-run effects of input or output price variation on the variable factor depend on the curvature of the short-run derived factor demand curve or of the short-run output supply curve, respectively. But the net long-run response also includes the effect induced by the shift in the quasi-fixed factor discussed in the preceding paragraph.

To find the factor bias of price uncertainty it is necessary to derive the effects on expected factor proportions. Relative factor intensities can increase or decrease in response to mean-preserving spreads in the price of output or of the variable input. For the popular homogeneous C.E.S. production function, rather

straightforward conclusions emerge, as shown in Table 1. More generally price uncertainty induces a bias in expected relative factor intensity, the direction of which cannot be inferred from the response of either the fixed factor, or the expected response of the variable factor, taken in isolation.

These results may be important for empirical methodology. Studies of technological choice or production efficiency of firms are mis-specified if, as is usually the case, they recognize a distinction between quasi-fixed and variable factors but ignore the existence of the price uncertainty which helps make this distinction meaningful. For even if, as here, risk neutrality is assumed, the rational decision maker will not consider only the mean values of the distributions of the expected prices of output and variable inputs in his technological choices; higher moments of the price distributions will also affect the profit-maximizing factor choices and relative factor intensities. Price uncertainty in general affects the appropriate choice of technology.

[1]Only papers which, like this study, assume that output is a decision variable, and that price is exogenous, are listed here. It would be more satisfactory to model the fundamental source of uncertainty and let price reflect the endogenous competitive market response to it, as in Wright [14], but the short-run flexibility assumed here would considerably complicate the analysis. For similar reasons I, like previous authors, avoid explicit examination of the means of changing price dispersion and its dynamics. The case of storage is discussed in Wright and Williams [15; 16].

[2]Throughout this paper the distribution of prices is assumed to be such that short-run profit is maximized at a positive level of output.

[3]Of course the effect of H must be the same sign for all prices, and levels of H, in the relevant ranges of both variables, for the sufficient conditions mentioned above to hold.

[4]The approach used here is similar to that of Epstein [2, 256].

[5]For this CES case, $g_{11} = ab\mu^2(\beta+1)(1-\mu)H^{-(\beta+2)}J^{-(2\mu/\beta+2)}F_{22}^{-1} < 0.$

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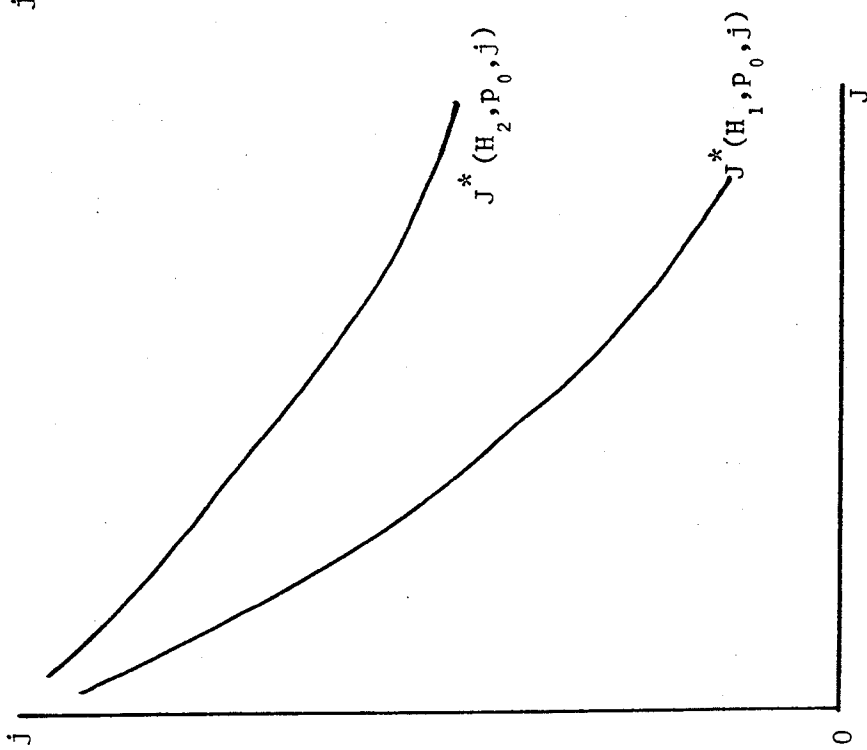
$J^*(H_2, P_0, j)$

$J^*(H_1, P_0, j)$

Figure 1b

H Decreases with Input or Output Price
Uncertainty

$J^*(H_2, P_0, j)$

$J^*(H_1, P_0, j)$

Figure 1a

H Increases with Input or Output Price
Uncertainty

p

$S(H_1, p, j_0)$

$S(H_2, p, j_0)$

0

Y
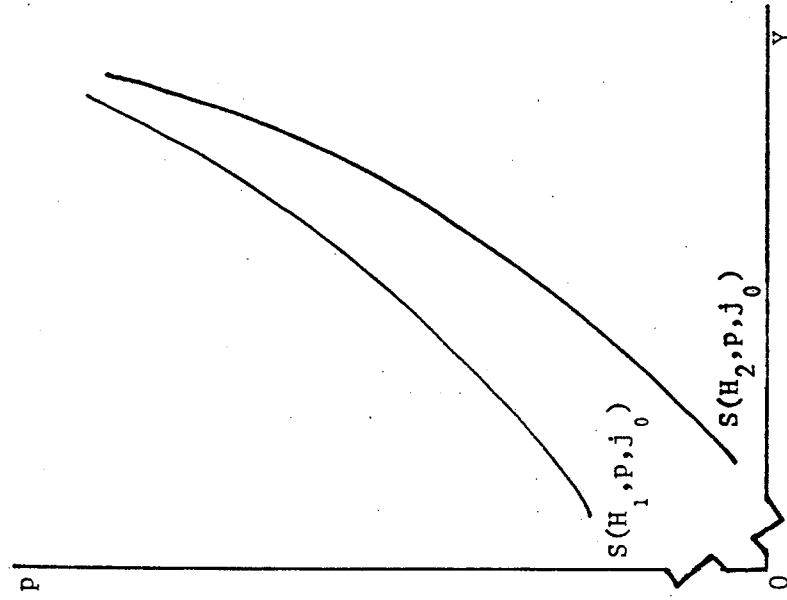
Figure 2a

H Increases with Input or Output Price
Uncertainty

p

$S(H_1, p, j_0)$

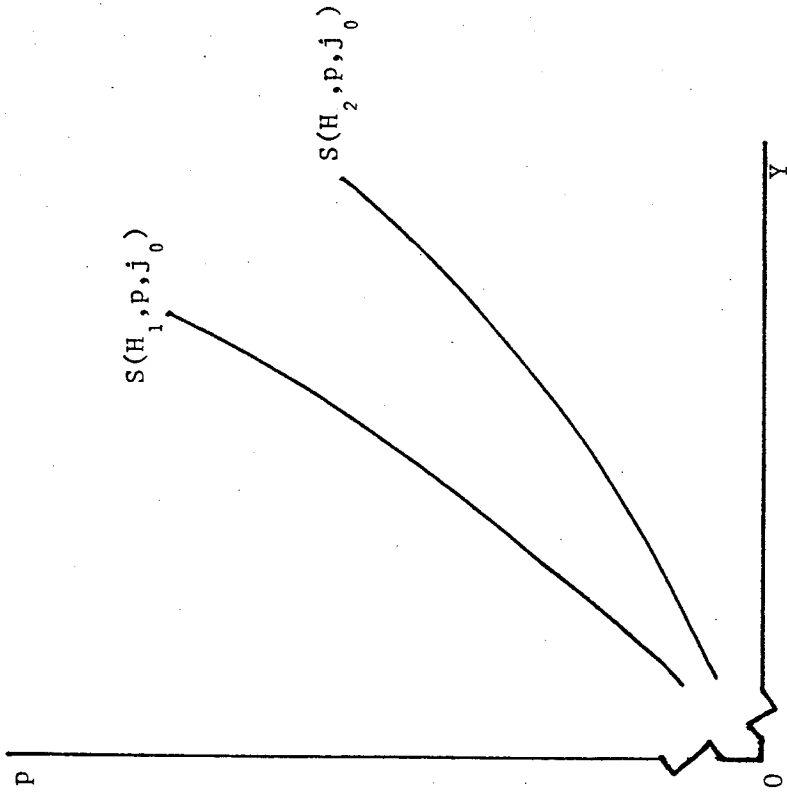$S(H_2, P, j_0)$

0

Y

Figure 2b

H Decreases with Input or Output Price
Uncertainty

Table I

Responses to Price Uncertainty Under CES Technology

Cases[a]	Direction of Response to Uncertainty in:			
	Input Price		Output Price	
	Quasi-Fixed Factor	Expected Relative Factor Intensity[b]	Quasi-Fixed Factor	Expected Relative Factor Intensity[b]
	H	E[J/H]	H	E[J/H]
(1) $0 < \sigma < 1/(3 - 2\mu)$	+	?	+	−
(2) $1/(3 - 2\mu) < \sigma < 1$	+	+	+	−
(3) $\sigma = 1$(Cobb-Douglas)	+	+	+	0
(4) $1 < \sigma < \dfrac{1}{1-\mu}$	+	+	+	+
(5) $\sigma = \dfrac{1}{1-\mu}$	0	+	0	+
(6) $\dfrac{1}{1-\mu} < \sigma$	−	+	−	+

[a]The degree of homogeneity is μ, $0 < \mu < 1$, and σ is the elasticity of substitution.

[b]The results for expected relative factor intensity are strictly valid only for marginal mean-preserving spreads in price.

Appendix I

The sign of the response of H to an m.p.s. in input or output price is given by the sign of g_{331} and g_{221}, respectively, which are similar, as the following derivations show:

By implicit differentiation of (5):

(A.1) $J_3^* = 1/(pF_{22})$

and

(A.2) $J_2^* = - F_2/(pF_{22})$

Therefore

(A.3) $g_{33} = -1/(pF_{22})$

and

(A.4) $g_{22} = - F_2^2/(pF_{22})$

Take the partial derivative of (A.3) with respect to H:

(A.5) $g_{331} = (F_{221} + F_{222}J_1^*)/pF_{22}^2$

By implicit differentiation of (5) in the text:

(A.6) $J_1^* = - F_{12}/F_{22}$

Substitute (A.6) in (A.5):

(A.7) $g_{331} = (F_{22}F_{221} - F_{21}F_{222})/(pF_{22}^3)$

Appendix I continued:

Now consider the effect of a mean-preserving spread (m.p.s.) in p.

Differentiate (A.4)

$$g_{221} = -[2F_2F_{21}F_{22} + 2F_2F_{22}^2J_1^* - F_2^2F_{221} - F_2^2F_{222}J_1^*]/(pF_{22}^2) \qquad (A.8)$$

Substitute (A.5) in (A.8):

$$g_{221} = F_2^2(F_{22}F_{221} - F_{21}F_{222})/(pF_{22}^3) = F_2^2g_{331} \qquad (A.9)$$

Thus sign (g_{221}) = sign (g_{331}). This sign determines the effect on H of a mean-preserving spread in p or j.

Appendix II

Assume the production function $F(H,J)$ is homogeneous of degree μ, $0 < \mu < 1$. Then the first and second partial derivatives F_2 and F_{22} are homogeneous of degree $(\mu - 1)$ and $(\mu - 2)$ respectively. Hence by Euler's theorem:

$$F_{21} = [(\mu-1)F_2 - JF_{22}]/H \tag{A.10}$$

and

$$F_{221} = [(\mu-2)F_{22} - JF_{222}]/H \tag{A.11}$$

Substituting F_{21} from (A.10) and F_{221} from (A.11) in (A.7),

$$g_{331} = [(\mu-2)F_{22}^2 - (\mu-1)F_2 F_{222}]/(HpF_{22}^3) \tag{A.12}$$

Hence

$$g_{331} \gtreqless 0 \quad \text{as} \quad \mu-2/\mu-1 \gtreqless F_2 F_{222}/F_{22}^2 \tag{A.13}$$

Since $0 < \mu < 1$, and given (2), a sufficient condition that g_1 be convex in the input price j is

$$F_{222} \leq 0 \quad . \tag{A.14}$$

Since, by (A.9), g_{221} has the same sign as g_{331}, (A.13) and (A.14) can also be used to determine the convexity or concavity of g_1 in p.