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IMPORT DEMAND FUNCTIONS RERVISITED

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Abstract

This paper explores the properties of a model where neither imported and domestic products are perfect substitutes nor are their demands separable. A restatement of the Marshall-Lerner condition is then proposed. The model is also used for the study of international trade flows and for the analysis of quantitative restrictions in international trade. Finally, an empirical application to the case of Colombia is presented.

The treatment of imports in most models is made under one of the following two assumptions. Either imported and domestic products are perfect substitutes or demand for imported products is separable from demand for domestic products.

In the case of perfect substitutability, all the effort is devoted to modelling the demand for domestic products, the demand for imports is then obtained by difference between domestic demand and domestic supply. It has however been observed that, even when considering a very disaggregated level, we still have, within the same category of products, coexistence of imported and domestic products (and of imports and exports). This has suggested that, in most cases, they are imperfect substitutes.

On the other hand, when import demand functions are only specified as a function of income and of prices of imported products, excluding the price of domestic products, the implicit assumption is that demand for imported products is separable from domestic demand for domestic products. However, this assumption introduces several restrictions on the substitution between domestic and imported products.

The purpose of this paper is to explore the properties of a model between this two polar cases, where neither imported and domestic products are perfect substitutes nor are their demands separable. This means that instead of modelling only the demand for domestic products as in the first case or only the demand for imports as in the second case, we need to treat them together.

Section I contains an introduction. Section II gives the general formulation of the model which is then used for the generalization of the Marshall-Lerner condition to the case of several sources of imports. It is also shown how this model can be used for the study of international trade flows. In this section, I also show the convenience of this approach for the study of quantitative restrictions in international trade. Section III gives the specification of the model for the empirical estimation and a presentation of the results. Finally, in Section IV, I give a summary of the main conclusions.

I. Introduction

A review of the literature of import demand functions has been established by Leamer and Stern (1970), Magee (1975) and Goldstein and Khan (1982). The purpose of this section is to give a general overview of the studies that have motivated this paper.

As indicated before, the fact that the demand for imports and for domestic products are not separable was taken into consideration by including among the explanatory variables of import demand functions the price of domestic tradable products. Therefore the demand for imports can be written

$$(1) \qquad m = f(E, pm, p)$$

where m is the quantity of imports, E is an activity variable, for example income, pm is the price of imported products and p the price of domestic tradable products. Variable p permits obtaining the elasticity of substitution between imported and tradable domestic products.

For the study of import competition and welfare analysis of international trade, the interest was focused on the estimation of this elasticity of substitution and different variants of the system formed by equations (1) and (2) were estimated. That is

$$(2) \qquad d = f(E, pm, p)$$

where d stands for tradable domestic products, see for instance Mutti (1977).

The increasing interest in international trade and the construction of world trade models has also moved researchers to make the distinction between different sources of imports. After the seminal article of Armington (1969), different specifications of the following model were estimated

$$(3) \qquad m_j = f_j(E, pm_1, \ldots, pm_k, \ldots, pm_n) \qquad j=1, 2, \ldots, n$$

where m_j is the quantity of imports coming from country j and pm_k is the price of products imported from country k. Some applications of (3) are Theil and Clemens (1978) and Snella (1979). As in the case of only one source of imports, the underlying assumption of (3) is that imported and domestic products are separable.

Among the consequences of separability as listed by Winters (1982) are, first, that the marginal rate of substitution between imported products from different sources is independent of domestic products. Second, pure substitution between domestic and imported products is restricted by the following relation

$$\partial d^c/\partial pm_j = u(\partial d/\partial E)(\partial m_j/\partial E) \qquad j=1, 2, \ldots, n$$

where the superscript c indicates the compensated demand and the term u is common to all sources of imports.[1] Hence, the only difference in cross-price effects is given by the income effect of imports.

Similarly to the case of one source of imports, the complete model in the case of n sources of imports is given by the following system of equations

$$(4) \qquad \begin{cases} m_j = f_j(E, pm_1, \ldots, pm_k, \ldots, pm_n, p) & j=1,2,\ldots, n \\ d = f(E, pm_1, \ldots, pm_k, \ldots, pm_n, p) & \end{cases}$$

Typically system of equations (4) is derived from the minimization of a cost function under the constraint of a production function or as a maximization of a utility function under the budget constraint as in section III.

Different specifications of this complete system of equations have been estimated. Corado and de Melo (1983) estimated Portuguese imports from the European Economic Community, Requena (1983) estimated Colombian imports from Latin-american countries and Winters (1983) British imports form the European Economic Community. Using the AIDS system, Winters (1982) tested and rejected the hypothesis of separability between British imports from EEC and domestic products.

II. The Model and some applications

Let us write the system of equations (4) in vector notation

$$m = m(E, pm, p) \tag{5}$$

$$d = d(E, pm, p) \tag{6}$$

where d is the domestic demand for domestic tradables, m is the vector of imported products, p is the price of domestic tradables, pm is the vector of prices of imported products and E is the nominal expenditure.

In order to explore some of the consequences of adding the domestic demand for domestic tradables, let us assume that all domestic products are tradables and consider the model formed by equations (5), (6) and

$$x_j = x_j(y_j^*, p, pm_j) \qquad j = 1, 2, \ldots, n \tag{7}$$

In (7) exports to country j (x_j) are a function of income of country j (y_j), the price of exports (p) and the competing price of our exports to country j (pm_j).

Marshall-Lerner conditions

For the derivation of the Marshall-Lerner conditions let us write the trade balance

$$T = \sum_i px_i - \sum_i pm_i m_i$$

The partial differential with respect to prices is given by

$$\delta T = \Big[\sum_i x_i + \sum_i p\frac{\partial x_i}{\partial p} - \sum_i pm_i\frac{\partial m_i}{\partial p}\Big]\delta p + \sum_j \Big[p\frac{\partial x_j}{\partial pm_j} - \sum_i pm_i\frac{\partial m_i}{\partial pm_j} - m_j\Big]\delta pm_i$$

Using the standard assumption that at the initial point all trade balances are in equilibrium, that is $px_i = pm_i m_i$ for all i, and therefore nominal income is equal to nominal expenditure or py=E, we can write the above equation in terms of elasticities

$$(8)\quad \frac{\delta T}{p\sum_i x_i} = \Big[1 + \sum_i w_i \eta x_{io} - \sum w_i \eta m_{io}\Big]\dot{p} + \sum_j \Big[w_j \eta x_{jj} - \sum_i w_i \eta m_{ij} - w_j\Big]\dot{pm}_j$$

where

$w_j = pm_j m_j / \sum_k pm_k m_k$

Share of imported products from country j in total imports.

$nx_{jj} = El(x_j/pm_j)$ Price elasticity of the demand for exports in country j with respect to pm_j (and nx_{jo} is the elasticity with respect to p)

$nm_{ij} = El(m_i/pm_j)$ Price elasticity of the demand for imports from country i with respect to pm_j (and nm_{io} is the elasticity with respect to p)

A dot over a variable, i.e. $\dot{y}$, means dy/y.

From the first expression in the right hand side of (8) we have that the condition for a deterioration of the trade balance, due to an increase in domestic prices is given by

$$\sum_j w_j nm_{jo} - \sum_j w_j nx_{jo} > 1$$

In words, the sum of the absolute values of the average elasticity of imports with respect to domestic prices and the average elasticity of exports, with respect to domestic prices must exceed unity. In a similar way, if the price of imported products m_j increases, trade balance will improve if

$$w_j nx_{jj} - \sum_i w_i nm_{ij} > w_j$$

which is, as it can be seen in table 2, an obvious generalization of the M-L condition.

Adding (6) to the import demand functions, we can write (8) as a function of the price elasticities of demand for domestic products. Let us first see that the adding up relation is given by

$$pd + \sum_i pm_i m_i = E$$

then by Cournot aggregation

$$w_o nm_{oj} + \sum_i w_i nm_{ij} + w_j = 0$$

and
$$w_o nm_{oo} + \sum_i w_i nm_{io} + w_o = 0$$

Replacing these two expressions in (8) we have

$$(9) \quad \frac{\delta T}{p\sum_i x_i} = \left[1+w_o+\sum_i w_i nx_{io}+ w_o nm_{oo}\right]\dot{p} + \sum_j\left[w_j nx_{jj} + w_o nm_{oj} + w_j - w_i\right]\dot{pm}_j$$

Therefore, the M-L conditions can be restated in the following way. From the first expression in brackets in the right hand side of equation (9), an increase in domestic prices will deteriorate the trade balance if

$$1 + w_o + \sum_i w_i nx_{io} + w_o nm_{oo} < 0$$

Changing the weights $w_i = pm_i m_i / \sum_k pm_k m_k$ by $w_i^* = pm_i m_i / pd + \sum_k pm_k m_k$, that is, taking in the denominator not only the demand for imported products but also the demand for domestic products, we can rewrite[2] the above inequality

$$\sum_i w_i^* nx_{io} + w_o^* nm_{oo} < -1$$

or more simply, using an obvious change of notation

$$\overline{nd} < -1$$

Thus, an increase in domestic prices will deteriorate the trade balance if the average of the elasticities of domestic demand for domestic products and of foreign demand for domestic products (our exports) with respect to domestic prices is elastic.

On the other hand, from the second expression in brackets in the right hand side of equation (9), an increase in the price of imports from country j will improve trade balance if the weighted sum of the elasticity of the demand for our exports and the elasticity of the demand for domestic products with respect to pm_j is positive. This means that when imports coming from country j are complementary to domestic products (nm_{oj} is negative), the result could be reversed and an increase in the price of imports could lead to a deterioration of trade balance. The reason is that, because of complementarity, the increase in prices causes a decrease in the demand for domestic products, and an increase in imports from other sources, more than an increase in our exports giving a net deterioration in trade balance.

The results are summarized in table 1 where the first column gives the standard Marshall-Lerner conditions, the second column gives the generalization to the case of many sources of imports and, finally, in the third column the same results are expressed in terms of the price elasticities of the foreign demand for our exports and of the price elasticities of the domestic demand for domestic products.

Table 1 : Different expressions for the Marshall-Lerner conditions

One source of imports	Many sources of imports. In terms of elasticities of demand for imports and for exports	In terms of elasticities of domestic demand for domestic products and of demand for exports
$nm_{1o}-nx_{1o} > 1$	$\overline{nm}_{.o}-\overline{nx}_{.o} > 1$	$-\overline{nd}_{.o} > 1$
$-nm_{11}+nx_{11} > 1$	$-\overline{nm}_{.j}+w_j nx_{jj} > w_j$	$w_j nx_{jj}+w_o nm_{oj} > 0$

<u>International Trade Flows</u>

If we consider the whole trade matrix, our exports become the imports of our partners and the model can be written as follows

$$(10) \qquad m_k = m_k(E_k,\ p) \qquad\qquad k = 1,\ \ldots,\ n$$

$$(11) \qquad E_k = E_k[y_k,\ P_k(p)] \qquad\qquad k = 1,\ \ldots,\ n$$

$$(12) \qquad y_k = \sum_j m_{kj} \qquad\qquad k = 1,\ \ldots,\ n$$

Where relations (5), (6) are expressed in vector notation in equation (10), m_k is now the vector of imported <u>and</u> domestic products of country k, m_{ik} (the ith element of vector m_k) are the imports of country k from country i, $m_{kk} = d_k$ (the kth element of vector m_k) is the domestic demand for domestic tradables in country k. In equation (11), nominal expenditure in country k (E_k) is a function of real income of country k (y_k) and of a price index (P_k). This price index is a homogeneous function of degree one in all prices (p). In (12) income is determined by the sum of domestic demand for domestic products and the total amount of exports.

As in the previous case we have the adding-up condition

$$p'm_k = \sum_i p_i m_{ik} = E_k$$

Let us write (12) in vector notation

(13) $$y = M\xi$$

where y is the (nx1) vector of y_k, M is a (nxn) matrix composed by m_{ik} and ξ is a (nx1) vector composed by ones. Differentiation of (13) with respect to y gives

(14) $$\dot{y} = M^*\dot{y} + \acute{a}$$

where $\acute{a}$ is the vector of the proportional increase in autonomous spending in each country[3] and the elements of M^* aré

$$m^*_{ij} = El(m_{ij}/E_j)El(E_j/y_j)m_{ij}/y_i$$

The solution of (14) is given by

(15) $$\dot{y} = (I-M^*)^{-1}\acute{a}$$

Which gives the increase in income due to an increase in autonomous spending.

Before showing a similar result in the case of a change in prices let us see under what conditions an increase in prices will leave the trade balance unchanged. The trade balance of country i is given by

$$T_i = p_i \sum_{j \neq i} m_{ij} - \sum_{k \neq i} p_k m_{ki}$$

adding and subtracting $p_i m_{ii}$ ($= p_i d_i$) we obtain

$$T_i = p_i \sum_j m_{ij} - \sum_j p_j m_{ji}$$

Partial differentiation with respect to p_k gives

$$\frac{\delta T_i}{\delta p_k} = \delta_{ik} \sum_j m_{ij} + p_i \sum_j \frac{\partial m_{ij}}{\partial E_j} \frac{\partial E_j}{\partial P_j} \frac{\partial P_j}{\partial p_k} + p_i \sum_j \frac{\partial m_{ij}}{\partial p_k}$$
$$- \left[\sum_j p_j \frac{m_{ji}}{\partial E_i}\right] \frac{\partial E_i}{\partial P_i} \frac{\partial P_i}{\partial p_k} - \sum_j p_j \frac{\partial m_{ji}}{\partial p_k} - m_{ki} = 0$$

Due to Cournot aggregation, the sum of the two last terms of the right hand side expression is zero. Also due to Engel aggregation, the term in brackets in the fourth term of the right hand side expression is equal to 1. It then follows

$$\text{(16)} \qquad p_i \sum_j \frac{\partial m_{ij}}{\partial E_j} \frac{\partial E_j}{\partial P_j} \frac{\partial P_j}{\partial p_k} + p_i \sum_j \frac{\partial m_{ij}}{\partial p_k} = - \delta_{ik} \sum_j m_{ij} + \frac{\partial E_i}{\partial P_i} \frac{\partial P_i}{\partial p_k}$$
$$= - \delta_{ik} y_i + \frac{\partial E_i}{\partial P_i} \frac{\partial P_i}{\partial p_k}$$

where for the second equality we have used (12) and δ_{ik} is the delta of Kronecker. We can now differentiate (12) to obtain

$$(17) \qquad dy_i = \sum_j \frac{\partial m_{ij}}{\partial E_j}\frac{\partial E_j}{\partial y_j} dy_j + \sum_k \Big[\sum_j \frac{\partial m_{ij}}{\partial E_j}\frac{\partial E_j}{\partial P_j}\frac{\partial P_j}{\partial p_k} + \sum_j \frac{\partial m_{ij}}{\partial p_k} \Big] dp_k$$

Using (16) to replace the term in brackets, we find the increase in income due to an increase in prices such that the trade balance remains constant

$$dy_i\Big|_{T=\text{const.}} = \sum_j \frac{\partial m_{ij}}{\partial E_j}\frac{\partial E_j}{\partial y_j} dy_j - y_i\frac{dp_i}{p_i} + \frac{\partial E_i}{\partial P_i}\Big[\sum_k \frac{\partial P_i}{\partial p_k} dp_k\Big]\frac{1}{p_i}$$

Which can be written

$$\dot{y}_i\Big|_{T=\text{const.}} = \sum_j m^*_{ij}\dot{y}_j - \dot{p}_i + h_i\sum_k n_{ik}\dot{p}_k$$

Where m^*_{ij} is the same as before.

$h_i = El(E_i/P_i) = d\ln E_i/d\ln P_i$

$n_{ik} = El(P_i/p_k) = d\ln P_i/d\ln p_k$

And in matrix notation

$$\dot{y}_i\Big|_{T=\text{const.}} = M^*\dot{y} - (I-\hat{H}N)\dot{p}$$

With $\hat{H}$ a diagonal matrix whose elements are h_i. The solution is given by

$$(18) \qquad \dot{y}\Big|_{T=\text{const.}} = -(I - M^*)^{-1}(I - \hat{H}N)\dot{p}$$

Equation (18) indicates that even if there is a large increase in the price of nominal income ($\dot{p}$), what is going to have an influence on the movement of nominal income ($\dot{y}$) is the difference between the movement in nominal price of income and the movement in the nominal price of expenditure ($\dot{p}-\widehat{H}N\dot{p}$).

The system of equations (18) is useful for the characterization of some optimal policies. We can write the singular value decomposition of the matrix $I-M^*$ as

$$(19) \qquad I - M^* = GSD' = \sum_i s_i g_i d_i'$$

where s_i is the ith singular value, g_i is the ith left-singular vector and d_i is the ith right-singular vector.[4]

Then (18) can be written as

$$(20) \qquad \dot{y}\,\Big|_{T=\text{const.}} = -DS^{-1}G'(I - \widehat{H}N)\dot{p}$$

Let us imagine that due to an increase in the price of the product of one country, all countries proceed to a change in their prices, through changes in their exchange rates. We can then examine the movement in $\dot{p}$ that will keep all trade balances constant and at the same time will minimize the reduction in income. This can be expressed by[5]

$$\min_{(\dot{p})} \dot{y}'\dot{y}\,\Big|_{T=\text{const.}} \quad \text{s.t.} \quad \dot{p}'(I-\widehat{H}N)'(I-\widehat{H}N)\dot{p} = 1$$

The normalization constraint is assumed for convenience and could be relaxed without changing the nature of the solution.[6] Using the singular value decomposition in (19) we can find the perturbation of prices that will minimize the perturbation in income. The solution is $\dot{p}=g_{max}$ the left-singular vector associated with the maximum singular value. The value at the optimum is the inverse of the square of the maximum singular value $\dot{y}'\dot{y} = 1/s_{max}^2$. The change in income is given by $\dot{y} = -(1/s_{max})d_{max}$, where d_{max} is the right singular vector associated to the maximum singular value.

Rationing

An increasingly common practice in international trade is the use of quantitative restrictions of imports. The approach that we have adopted is appropriate for the study of this type of problem. Since in our case the demands for domestic and for imported products are related, it is then possible to find the relationships between the constrained demand functions.

In this section we are going to introduce rationing in the import demand functions; then the multipliers under rationing will be computed and compared to the unconstrained multipliers.

Another way of writing (14), in a two-country model, is

$$(21) \qquad \begin{vmatrix} 1 - \dfrac{\partial m_{11}}{\partial E_1}\dfrac{\partial E_1}{\partial y_1} & -\dfrac{\partial m_{12}}{\partial E_2}\dfrac{\partial E_2}{\partial y_2} \\ -\dfrac{\partial m_{21}}{\partial E_1}\dfrac{\partial E_1}{\partial y_1} & 1 - \dfrac{\partial m_{22}}{\partial E_2}\dfrac{\partial E_2}{\partial y_2} \end{vmatrix} \begin{vmatrix} dy_1 \\ \\ dy_2 \end{vmatrix} = \begin{vmatrix} da_1 \\ \\ da_2 \end{vmatrix}$$

or in matrix notation $\quad Ady = da.$

Now, let us assume that imports of country 1 from country 2 are rationed, i.e., $m_{21}=\bar{m}_{21}$. For this purpose, the government of country 1 sells import licences by auction to importers at competitive prices $\bar{p}_2$. Furthermore, we are going to assume that all the revenue collected by the government $(\bar{p}_2-p_2)\bar{m}_{21}$ is distributed among its citizens as a compensation for the loss due to the increase in the price of imported products in the domestic market. We can now use the result obtained by Neary and Roberts (1980) in a different context. That is

$$\frac{\partial \bar{m}_{21}}{\partial E_1} = 0$$

$$\frac{\partial \bar{m}_{11}}{\partial E_1} = \frac{\partial m_{11}}{\partial E_1} - \frac{\partial m_{11}^c}{\partial \bar{p}_2}\left[\frac{\partial m_{21}^c}{\partial \bar{p}_2}\right]^{-1}\frac{\partial m_{21}}{\partial E_1}$$

where the superscript c stands for the compensated import demand function, the symbol $^-$ stands for the constrained demand function and $\bar{p}_2$ is the price at which the unconstrained demand for imports of country 1 from country 2 would be equal to $\bar{m}_{21}$.

The first equation above shows that an increase in income in country 1 has, of course, no effect on the level of (rationed) imports. The second equation indicates that an increase in income will raise the demand for domestic products for two reasons. Firstly, as usual, through the income effect and secondly, because the increase in income will tend to increase imports which is not possible, the quantitative restriction will become tighter and the final effect will be an increase in the demand for domestic products as long as they are (pure) substitutes for imported products.

With this correspondence between the constrained and the unconstrained demand functions, it is now possible to write the relation (21) under rationing

$$
(22)\quad \begin{vmatrix} 1 - \dfrac{\partial m_{11}}{\partial E_1}\dfrac{\partial E_1}{\partial y_1} + \dfrac{\partial m^c_{11}}{\partial \bar{p}_2}\left[\dfrac{\partial m^c_{21}}{\partial \bar{p}_2}\right]^{-1}\dfrac{\partial m_{21}}{\partial E_1}\dfrac{\partial E_1}{\partial y_1} & -\dfrac{\partial m_{12}}{\partial E_2}\dfrac{\partial E_2}{\partial y_2} \\ 0 & 1 - \dfrac{\partial m_{22}}{\partial E_2}\dfrac{\partial E_2}{\partial y_2} \end{vmatrix} \begin{vmatrix} d\bar{y}_1 \\ d\bar{y}_2 \end{vmatrix} = \begin{vmatrix} da_1 \\ da_2 \end{vmatrix}
$$

or in matrix notation $\bar{A}dy = da$.

Let us adopt the following notation

$$
\eta = \frac{\partial m^c_{11}}{\partial \bar{p}_2}\left[\frac{\partial m^c_{21}}{\partial \bar{p}_2}\right]^{-1}
$$

$$
\alpha_{ij} = \left[\frac{\partial m_{ij}}{\partial E_j}\frac{\partial E_j}{\partial y_j}\right]\left[1 - \frac{\partial m_{jj}}{\partial E_j}\frac{\partial E_j}{\partial y_j}\right]^{-1}
$$

and $0 < \alpha_{ij} < 1$ for all $i \neq j$

Then, it can be shown that

$$
\frac{\det(\bar{A})}{\det(A)} = \frac{1 + \eta\alpha_{21}}{1-\alpha_{12}\alpha_{21}}
$$

From (21) we have the effect of autonomous spending on y_1

$$
dy_1 = \left\{\left[1 - \frac{\partial m_{22}}{\partial E_2}\frac{\partial E_2}{\partial y_2}\right] da_1 + \left[\frac{\partial m_{12}}{\partial E_2}\frac{\partial E_2}{\partial y_2}\right] da_2\right\}\frac{1}{\det(A)}
$$

The effect of autonomous spending on y_1, under rationing of imports, is obtained from (22)

$$d\bar{y}_1 = \left\{ \left[1 - \frac{\partial m_{22}}{\partial E_2}\frac{\partial E_2}{\partial y_2}\right] da_1 + \left[\frac{\partial m_{12}}{\partial E_2}\frac{\partial E_2}{\partial y_2}\right] da_2 \right\} \frac{1}{\det(\bar{A})}$$

Therefore, the inequality $d\bar{y}_1 < dy_1$ is equivalent to $\det(A) < \det(\bar{A})$ or to $\alpha_{12} > -\eta$. Since α_{12} is less than one but very close to one, this condition is almost equivalent to $-\eta < 1$, which means that approximately the following inequality must hold

$$\frac{\partial m^c_{11}}{\partial \bar{p}_2} < -\frac{\partial m^c_{21}}{\partial \bar{p}_2}$$

That is, the compensated cross-price effect of domestic products is, in absolute value, less than the compensated own-price effect of imports. Under this condition, rationing of imports ($\bar{m}_{21}$) decreases the effect of autonomous spending on (y_1).

The effects of autonomous spending on income of country 2 (y_2) are given in a similar way by

$$dy_2 = \left\{ \left[\frac{\partial m_{21}}{\partial E_1}\frac{\partial E_1}{\partial y_1}\right] da_1 + \left[1 - \frac{\partial m_{11}}{\partial E_2}\frac{\partial E_2}{\partial y_2}\right] da_2 \right\} \frac{1}{\det(A)}$$

and, under rationing of m_{21}, by

$$d\bar{y}_2 = \left\{ 1 - \frac{\partial m_{11}}{\partial E_1}\frac{\partial E_1}{\partial y_1} + \frac{\partial m^c_{11}}{\partial p_2}\left[\frac{\partial m^c_{21}}{\partial p_2}\right]^{-1}\frac{\partial m_{21}}{\partial E_1}\frac{\partial E_1}{\partial y_1} \right\} \frac{da_2}{\det(\bar{A})}$$

This last expression shows the obvious result that under rationing an autonomous spending in country 1 has no effect on income in country 2.

On the other hand, if there is an increase in autonomous spending only in country 2, the inequality $d\bar{y}_2 < dy_2$ is equivalent to

$$1+\eta\alpha_{21} < \det(\bar{A})/\det(A) = (1+\eta\alpha_{21})/(1-\alpha_{12}\alpha_{21})$$

or finally to $\alpha_{12}\alpha_{21}>0$ which is always true because α_{ij} is positive for all i,j. Therefore, rationing of imports of country 1 form country 2 will always decrease the effects, on income of country 2, of an autonomous spending in country 2.

Finally, in the case of autonomous spending in both countries, it can be shown (making the simplifying assumption that $da_1=da_2$) that $d\bar{y}_2 < dy_2$ is equivalent to $\alpha_{12}>-1$ and, since α_{12} is positive, this condition is always satisfied. Therefore, in this case too, rationing of imports of country 1 from country 2 ($\bar{m}_{21}$) will always have a negative effect on income of country 2.

The results are summarized in table 2 where it is shown that quantitative restrictions on imports from country 2 to country 1, will always decrease income in country 2. Also, under the fairly general condition that the compensated cross-price effect of domestic products is, in absolute value, smaller than the compensated own-price effect of imports, quantitative restrictions will also reduce income in country 1.

Table 2 : Conditions for a less important increase in income induced by an increase in autonomous spending under quantitative rationing of imports

An increase in \ induces	$d\bar{y}_1 < dy_1$	$d\bar{y}_2 < dy_2$
da_1	$\alpha_{12} > -\eta$	always
da_2	$\alpha_{12} > -\eta$	$\alpha_{12}\alpha_{21} > 0$ always
$da_1=da_2$	$\alpha_{12} > -\eta$	$\alpha_{12} > -1$ always

III. <u>Specification of the Model</u>

Let us assume that consumers maximize the following indirect utility function

$$v = \sum_i \frac{a_i}{b_i} \left[\frac{E-p'c}{p_i} \right]^{b_i}$$

where, as before, E is the nominal expenditure, p is the vector of all prices, a_i, b_i and c_i are parameters. We are going to adopt the convention that $m_1 = d_1$ (the demand for domestic products).

Using Roy's identity

$$m_k = -(\partial v/\partial p_k)/(\partial v/\partial E)$$

we obtain the (Marshallian) demand functions

$$(23) \qquad m_k = c_k + \frac{a_k\,(E-p'c)^{b_k}\,p_k^{-(b_k+1)}}{\sum_i a_i\,(E-p'c)^{b_i-1}\,p_i^{-b_i}} \qquad k=1,2,\ldots,n$$

The system of equations (23) satisfies the adding-up condition $p'm = E$ and it contains as particular cases two well-known systems. If all the parameters c_k are equal to zero we have the Indirect Addilog[8] and if all the parameters b_k are equal to zero we find the Linear Expenditure System.

For purposes of testing it can be useful to adopt the following specification

$$(24) \qquad m_k = c_k + \frac{a_k\,(E-p'c)^{b_k}\,p_k^{-(d_k+1)}}{\sum_i a_i\,(E-p'c)^{b_i-1}\,p_i^{-d_i}} \qquad k=1,2,\ldots,n$$

We have the following set of possible tests :

(i) $b_k = d_k$, for symmetry of the Slutsky matrix and homogeneity[9]

(ii) $c_k = 0$ and (i) for all k, for Indirect Addilog.

(iii) $b_k = 0$ and (i) for all k, for LES.

The main advantage of (24) is that it has a relatively low number of parameters. The restriction is that, imposing a specific functional form, it does not have the flexibility of other systems (for example AIDS) which are more flexible but have a large number of parameters. The other restriction is that the non-linearities are important and, depending on the sample data, the estimation procedure can be difficult.

The following econometric specification of the model was estimated [10]

$$(25) \qquad \pi_{kt} m_{kt} = \frac{e^{a_k} \varepsilon_t^{b_k+1} \pi_{kt}^{-d_k}}{\sum_j e^{a_j} \varepsilon_t^{b_j} \pi_{jt}^{-d_j}} + u_{kt} \qquad k=1,2,\ldots,n$$

Where all the variables are evaluated at time t, ε_t is the real domestic expenditure, π_{kt} is the real price of m_{kt} and u_{kt} is a stochastic error. This model corresponds to model (24) with all parameters c_k equal to zero and evaluated at real prices, that is, all prices have been deflated by the price index of products from all origins.[11]

If we note m_t the vector of imports, π_t the vector of real prices (and π_t the diagonal matrix whose elements are those of π_t), u_t the vector of stochastic errors and z the vector of all parameters a_k, b_k and c_k, the functions (25) can be written in vectorial notation

$$\pi_t m_t = f(\pi_t, \varepsilon_t, z) + u_t$$

with the following conditional expectations of the errors

$$E(u_t \mid \pi_t, \varepsilon_t) = 0$$

$$E(u_t u_\tau' \mid \pi_t, \varepsilon_t) = \delta_{t\tau}\Omega$$

where the symbol δ with two subscripts is the delta of Kronecker and Ω the covariance matrix.

The adding-up condition implies that

$$\xi'\pi_t m_t = \xi' f(\pi_t, \varepsilon_t, z) = \varepsilon_t \quad \text{and} \quad \xi' u_t = 0$$

That is, the sum of real imports and real demand for domestic products must be equal to real expenditure. Also, the sum of stochastic errors is zero (as before, ξ is the vector whose elements are equal to one). Since $\xi'\Omega = 0'$, the rank of the covariance matrix is no greater than n-1. This restriction has to be taken into consideration during the estimation procedure. The estimation of the vector of parameters z was obtained by the method of maximum likelihood.[12]

A complete description of the data can be found in Requena (1983). The data correspond to Colombia for the period 1963-1975. The sources of supply are 1) domestic, 2) Latin American countries and 3) the rest of the world. Imports are evaluated at CIF prices plus import taxes. As usual in most empirical international trade studies, prices are based on unit values, in this case they are Paasche price indexes taking unit values at the level of 3 digit of the SITC classification.

Table 3 : Regression results (1)

		Intermed.	Food	Consumer non-durables	Consumer durables	Transport equipment
Colombia	a_1	-10.2278 (2.425)	26.4657 (10.288)	-5.1382 (2.123)	-6.6469 (2.648)	-8.3591 (3.839)
	b_1	0.8294 (0.168)	-1.5726 (0.723)	0.5839 (0.165)	0.6222 (0.204)	0.5868 (0.288)
	d_1	-25.4529 (5.821)	-84.8203 (25.631)	-24.2416 (2.950)	-8.0389 (7.094)	-3.7329 (1.574)
	R^2	0.996	0.999	0.998	0.993	0.931
Latin America	a_2	-7.8440 (3.884)	0.1176 (9.577)	-6.9162 (4.197)	-7.4801 (6.299)	-50.5156 (12.075)
	b_2	0.3796 (0.268)	-0.1359 (0.673)	0.3412 (0.331)	0.3926 (0.481)	3.4325 (0.886)
	d_2	0.4227 (0.109)	-0.8000 -	-0.1652 (0.882)	-0.8500 -	6.6938 (2.000)
	R^2	0.895	0.932	0.783	0.467	0.820
Rest of the World	a_3	0.0 -	0.0 -	0.0 -	0.0 -	0.0 -
	b_3	0.0 -	0.0 -	0.0 -	0.0 -	0.0 -
	d_3	3.3141 (1.616)	1.8884 (0.573)	1.3000 -	1.5753 (1.974)	5.9799 (2.924)
	R^2	0.823	0.786	0.667	0.680	0.808
Total R^2		0.993	0.999	0.995	0.988	0.890

(1) In parenthesis the asymptotic standard errors.

It can be seen in equation (25) that the parameters a_k and b_k can be arbitrarily changed by any constant without changing the value of the function. Therefore, a normalization rule is needed i.e., $a_3=b_3=0$.[13]

The results are presented in table 3, they show high R^2 in most cases and the model seems to explain relatively well the behavior of imports from different sources.

The best results are those of intermediate products (about 30 percent of total imports) and equipment and transport products (about 40 percent of total imports) with low asymptotic errors for all the parameters (first and fifth columns in table 3). One possible explanation of this result is that the factors that have not been considered (quotas, prior licenses) play an important role in the explanation of imports in the case of products where imports are a small share of domestic market (food, consumer non-durables and consumer durables). On the other hand, in the case of products where imports are a large share of the domestic market (intermediate, transport and equipment products), expenditure and prices seem to be good explanatory variables.

Nominal expenditure elasticities and nominal price elasticities have been evaluated at the average point of the observations and they are presented in table 4. Expenditure elasticity of the demand for Colombian products (fourth column, row m_1 in every block) is close to unity for most products and relatively elastic for transport and equipment, intermediate and consumer durable products. Expenditure elasticities of the demand for products coming from Latin-american

Table 4 : Nominal elasticities

$El(m_k/p_j)$		Price elasticities p_1	p_2	p_3	Expenditure Elasticity
Intermediate	m_1	-0.870	-0.058	-0.200	1.127
	m_2	-3.201	-1.124	3.647	0.677
	m_3	-0.452	0.343	-0.189	0.298
Food	m_1	-0.972	-0.006	0.006	0.972
	m_2	-3.690	0.008	1.274	2.409
	m_3	-1.184	0.214	-1.575	2.545
Consumer non durables	m_1	-0.914	-0.021	-0.121	1.056
	m_2	-2.276	-0.672	2.134	0.813
	m_3	-0.651	0.177	0.002	0.472
Consumer durables	m_1	-0.994	-0.048	-0.075	1.117
	m_2	-2.090	-0.073	1.276	0.887
	m_3	0.167	0.124	-0.786	0.495
Transport equipment	m_1	-1.139	0.004	-0.196	1.326
	m_2	-0.945	-7.613	4.387	4.171
	m_3	0.105	0.108	-0.952	0.739

countries (fourth column, row m_2 in every block) are in general higher than those of the demand for products coming from the rest of the world (fourth column, row m_3 in every block).

The price elasticities show that the demand for Colombian products are independent of prices of products coming from Latin America and from the rest of the world (columns p_2 and p_3, row m_1 in every block). On the other hand, Colombian prices appear to have a complementarity effect on the demand for Latin-american products (high values for column p_1, row m_2 in every block).

Cross-price elasticities indicate a substitution effect between products coming from Latin America and those coming from the rest of the world (positive values in column p_3, rows m_2 and in column p_2 rows m_3 respectively). Moreover, the increase in the demand for Latin-american products due to an increase in prices in the rest of the world (column p_3, rows m_2) is greater than the increase in the demand for products coming from the rest of the world due to an increase in the price of Latin-american products (column p_2, rows m_3).

One of the consequences of estimating the model in real prices is the assumption of homogeneity of degree zero in prices, therefore the following relation among price and expenditure elasticities holds

$$\sum_j El(m_i/p_j) + El(m_i/E) = 0$$

This relation can be a strong restriction when there are only a few sources of imports. The high expenditure elasticity of the demand for Latin-american transport and equipment products and the high cross price elasticity $El(m_2/p_3)$ imply, by the above relation, the abnormally high own price elasticity $El(m_2/p_2)$. The two obvious ways of avoiding this drawback, and therefore making this relation less restrictive, are considering more sources of imports and/or estimating the model in nominal prices which means dropping the homogeneity (no money illusion) condition.

IV Conclusions

In this paper it has been argued that the study of the complete model of demand for domestic and imported products is useful for the analysis of many of the current problems in international trade. Three examples were considered. First, the formulation of the Marshall-Lerner condition in terms of the price elasticity of domestic demand for domestic products and of price elasticities of demand for our exports. This formulation focuses the attention on the fact that a change in relative prices of imports has some effects in the domestic demand for domestic products that are important in the determination of income. Second, the effects on international trade flows of changes in autonomous spending and in prices, where it has been shown that some optimal policies are related to the singular value decomposition of the matrix of income elasticities of the demand for domestic and for imported products. Third, when rationing of imports is taken into consideration, the effects on income of an increase in autonomous spending depend on the substitution between imported and domestic products but in most cases the final result will be a decrease in income in both countries. In these three cases the relationships implied by the complete system can be used for the simplification of the analysis.

Furthermore, the empirical implementation of this approach to the case of Colombian imports appears to give interesting results.

Footnotes

[1]For a discussion of this point in the general case see Deaton and Muellbauer (1980) p. 128.

[2]Note that we can multiply the inequality by a positive fraction

$$\frac{\sum_k pm_k m_k}{pd+\sum_k pm_k m_k}\left[1 + w_o\right] + \frac{\sum_k pm_k m_k}{pd+\sum_k pm_k m_k}\left[\sum w_i nx_{io} + w_o nm_{oo}\right] < 0$$

which reduces to

$$1 + \sum_i w_i^* nx_{io} + w_o^* nm_{oo} < 0$$

[3]Where the elements of vector a are given by

$\dot{a}_i = da_i/y_i$.

[4]Columns of matrix G are left-singular vectors of $(I-M^*)$, columns of D are right-singular vectors and S is a diagonal matrix whose elements are the singular values. Furthermore, by orthogonality of the singular vectors, $G'G = D'D = I$.

[5]The generalization to the case $\dot{y}'A\dot{y}$ with any symmetric matrix A is straightforward.

[6]Under the constraint $\dot{p}'\dot{p}=1$, the solution will be related to the singular value decomposition of the matrix $(I-M^{*})^{-1}(I-\hat{H}N)$.

[7]The result is direct. It relies on the fact that $G'G = D'D = I$.

[8]See Houthakker (1960).

[9]It is possible to impose homogeneity without imposing the symmetry of the Slutsky matrix. This is done by allowing b_k and d_k to be different but estimating the model in real prices.

[10]Specification (25) was first proposed by Carlevaro (1977) for the study of consumption demand functions.

[11]That is, by the following price index $P_t = \left[\sum_k p_{kt}m_{ko}\right]/\left[\sum_k p_{ko}m_{ko}\right]$

[12]The program used was GCM elaborated by Snella (1978).

[13]Furthermore, due probably to the small size of our sample, some of the parameters were ill behaved. They were then fixed to a value such as the elasticity of the price associated to this parameter was zero.

References

Armington, P. (1969), A Theory of Demand for Products Distinguished by Place of Production, IMF Staff Papers, vol. XVI, no. 1.

Carlevaro, F. (1977), Rapport de mission aupres de l'OCDE, mimeo.

Corado, C. and J. de Melo (1983), An ex-ante Model for Estimating the Impact on Trade Flows of a Country's Accession to a Custom Union, mimeo.

Deaton, A. and J. Muellbauer (1980), Economics of Consumer Behavior, Cambridge, Cambridge University Press.

Goldstein, M. and M. S. Khan (1982), Income and Price Effects in Foreign Trade, IMF, mimeo.

Houthakker, H. S. (1960), Additive Preferences, Econometrica, vol. 28, pp. 244-257

Leamer, E. E. and R. M. Stern (1970), Quantitative International Economics, Boston, Allyn and Bacon.

Magee, S. P. (1975), Prices, Income and Foreign Trade, in : International Trade and Finance : Frontiers for Research, P. B. Kenen (ed.),Cambridge, Cambridge University Press.

Mutti, J. (1977), The Specification of Demand Equations for Imports and Domestic Substitutes, Southern Economic Journal, vol. 44, no. 1, pp. 68-73.

Neary, J. P. and K. W. S. Roberts (1980), The Theory of Household Behaviour Under Rationing, European Economic Review, vol. 13, pp.25-42.

Requena, J. (1983), <u>Influence des échanges intra-latinoaméricains sur l'économie colombienne : un modèle multisectoriel</u>, Thèse de doctorat, Université de Genève.

Snella, J.-J. (1978), A Program for Non Linear Multivariate Regression GCM, <u>Econometrica</u>, vol. 46, no.2.

Snella, J.-J. (1979). <u>An Econometric Model of International Trade</u>, mimeo, Département d'économétrie, Université de Genève.

Theil, H. and K. Clemens (1978), A Differential Approach to U.S. Import Demand, <u>Economic Letters</u>, vol. 1 pp. 249-252.

Winters, L. A. (1982), <u>Separability and the Specification of Foreign Trade Functions</u>, University of Bristol, Discussion Paper no. 82/126.

Winters, L. A (1983), <u>British Imports of Manufactures and the Common Market</u>, mimeo, Department of Economics, University of Bristol.