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OPTIMAL FOREIGN BORROWING OF CAPITAL IN A TWO-SECTOR MODEL

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1. Introduction

This paper investigates the motives of a small country for borrowing in order to purchase capital equipment on international markets. The country produces capital and a non-traded consumption good, but may wish to borrow or lend capital to achieve a higher level of welfare. This analysis can be distinguished from most examinations of optimal international indebtedness, which focus on a consumption motive for borrowing.[1] Although ultimately our country would like to enjoy high levels of consumption, it cannot borrow consumption goods directly.

We are particularly interested in the change in borrowing patterns as the time horizon of individuals in the small country changes. A move toward giving more weight to the future might be interpreted as a desire to forego current felicity in order to develop and achieve higher long-run levels of well-being. Under the assumptions in this paper, such a shift would lead to higher steady-state consumption at the expense of current consumption. In a model where borrowing is for consumption (e.g., Obstfeld (1982)) such a cut would lead immediately to less borrowing or more international lending by the small-country. In our model, in order to cut current consumption, the capital stock employed in the consumption goods industry must be reduced. If the consumer goods industry is capital-intensive the country would reduce its capital stock immediately by lending abroad. However, if consumption goods are labor-intensive then the overall capital stock of the country must rise, which requires international borrowing. This latter case offers a possible explanation for why a country that becomes more future-oriented and wants to increase its long-run wealth may wish to increase its foreign indebtedness.

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A second issue deals with the literature on stages in the balance of payment. Fischer and Frenkel (1974) discuss a traditional argument which asserts that a capital-poor country will go through stages in its balance of payments as it grows. Initially the country borrows from the rest of the world, but as it matures it becomes a lender. It may reach a stage where it has accumulated claims on other countries and finances a trade balance deficit with earnings from those claims. Fischer and Frenkel demonstrate such a phenomenon in a two-sector model in which consumption goods only are traded. They assume a saving function which depends only on wealth. Bazdarich (1978), however, shows that in the same context, but with optimizing consumers, there are no stages of borrowing. This conclusion is reversed when borrowing occurs to obtain capital. We show that optimizing agents in a capital poor country may indeed initially borrow to increase their capital stock, but then run down their debts over time. They could eventually become net owners of claims on the rest of the world.

The two industries, capital goods and consumption goods, have constant returns to scale technology and use labor and capital in the production process. Both factors are freely mobile between industries. Capital can be borrowed internationally at a given interest rate. Identical individuals maximize discounted utility of consumption. The discount rate is determined endogenously, and is parameterized as in Uzawa (1968). The endogeneity of the discount rate is essential in allowing this economy to achieve steady-state. With a constant discount rate, a small country with an infinite time horizon that borrows at a given world interest rate will not have a convergent consumption path toward a steady state (see Bardhan (1967)). The type of preferences described by Uzawa allow convergence. In the international context, Findlay (1978) and Obstfeld (1981, 1982) have successfully modelled optimal foreign borrowing under these assumptions.

The availability of capital at an exogenously determined world interest rate fixes the shadow wage and rate of return to capital. When time preference changes and the country wants to cut current consumption, its pattern of borrowing is determined by the Rybczynski theorem.[2] If the consumption goods industry is capital intensive, a reduction in the country's capital stock will shrink consumption and expand capital output. If capital goods production is capital intensive, an expansion of the capital stock through borrowing will reduce consumption output and increase manufacture of capital goods.

It is also interesting to examine how the steady state rate of capital imports is affected by a shift in time preference. In the long run, new production of investment goods is used to replace capital that is depreciating in both industries. If new investment is not sufficient to cover depreciation, capital must be imported from abroad. When the country moves to a higher level of steady-state consumption, steady-state production of investment goods falls. This tends to raise the rate of capital flows from abroad. Furthermore, if the consumption goods industry is capital intensive, its expansion requires an expansion in the country's steady-state capital stock. This, in turn, increases the amount of new capital required in order to offset the depreciation of the capital stock, thus reinforcing the need for more inflows of capital. On the other hand, if the consumption goods industry is labor intensive, steady-state capital, and hence steady-state depreciation, must fall. This reduces the need for imports of capital but this effect will not outweigh the effect of reduced production of new investment goods. Capital imports in the steady state must rise.

The rest of this paper is organized as follows. Section 2 presents the model and derives conditions for optimality. Section 3 examines the steady state. The dynamics of foreign borrowing and capital accumulation are studied in Section 4. Concluding remarks are in Section 5.

2. The Model

Identical individuals maximize discounted infinite horizon utility. Utility is derived from a consumption good which is produced domestically and is not traded. Capital and consumption goods are produced domestically using capital and labor as inputs. Both factors are freely mobile between industries. Each good has a constant returns to scale production function. Capital may be borrowed internationally. The country is small, so that it takes the interest rate as given.

An individual's utility u_t at each moment in time is related to consumption c_t:

$$u_t = u(c_t) \ .$$

We assume

$$u'(c_t) > 0 \quad \text{and} \quad u''(c_t) < 0 \ . \tag{1}$$

To avoid corner solutions, we postulate

$$\lim_{c \to 0} \ u'(c_t) = \infty \ . \tag{2}$$

The individual maximizes the integral

$$V = \int_0^\infty u_t e^{-\Delta_t} dt \tag{3}$$

where

$$\Delta_t = \int_0^t \delta_s ds \tag{4}$$

and δ_s is the instantaneous subjective discount rate at time s. Following Uzawa, we take δ_s to be a function of utility at time s:

$$\delta_s = \delta(u_s) \ . \tag{5}$$

We also assume

$$\delta > 0 \ , \quad \delta' > 0 \ , \quad \delta - \delta' u > 0 \ , \quad \delta'' > 0 \ , \tag{6}$$

as in Uzawa.

For simplicity the total labor force is 1, and v is the amount of labor employed in the sector that produces capital goods. The total amount of capital in the economy is k, and in each sector k_1 and k_2, where the subscript 1 is for the investment goods industry and 2 for the consumption good. So,

$$k_1 + k_2 = k \ . \tag{7}$$

We assume output in the capital goods industry is a function of the quantity of labor and capital goods employed in that industry,

$$Y_1 = F(k_1, v) ,$$

and likewise for the consumption good

$$Y_2 = G(k_2, 1-v) .$$

The production functions are homogeneous of degree one, and satisfy the usual convexity conditions and Inada conditions.

The homogeneity of the production functions allows us to define

$$f(v/k_1) \equiv F(1, v/k_1) = \frac{1}{k_1} F(k_1, v)$$

and

$$g((1-v)/k_2) \equiv G(1, (1-v)/k_2) = \frac{1}{k_2} G(k_2, 1-v) .$$

It is convenient notationally to define

$$\ell_1 \equiv v/k_1 \quad \text{and} \quad \ell_2 \equiv (1-v)/k_2 .$$

We assume that capital in both industries depreciates at a rate n. The total capital stock for the economy evolves according to

$$\dot{k} = k_1 f(\ell_1) - nk + \tau . \tag{8}$$

In this equation τ represents the rate at which capital is imported from abroad. If the country lends abroad, it acquires an asset, b. The rate of return on lending is assumed to be exogenously given to the small country as r-n. Therefore, the path of b is determined by

$$\dot{b} = (r-n)b - \tau \,. \tag{9}$$

Equation (9) gives the current account for this country.

We must impose the constraints

$$c \geq 0, \quad k \geq 0, \quad k_2 \geq 0, \quad v \leq 1, \quad k_1 \geq 0, \quad v \geq 0 \,.$$

Given assumption (2), the first four constraints are never binding. The last two may be binding. There could be specialization of production in the consumption good. This possibility will be ignored in the text, and taken up in Appendix 2.

Individuals also face the lifetime constraint:[3]

$$b_t - \int_t^\infty \tau_s e^{-(r-n)(s-t)} ds \geq 0 \,. \tag{10}$$

This constraint says that the amount of the small country's debt ($-b_t$) must be less than or equal to the sum of the discounted amount the country plans to pay back each period ($-\tau_s$). Without such a constraint, with the infinite horizon planning problem an arbitrarily high level of utility could be achieved by borrowing an arbitrary amount each period and meeting interest payments through further borrowing. Given (11) we have

$$\int_t^\infty \tau_s e^{-(r-n)(s-t)}ds = b_t - \lim_{s\to\infty} b_s e^{-(r-n)s} .$$

So, the constraint (10) may be written as

$$\lim_{s\to\infty} b_s e^{-(r-n)s} \geq 0 . \tag{11}$$

It will be shown that along the optimal trajectory described in this section and section 4, condition (11) is satisfied.

From (4) we have the fact that

$$d\Delta = \delta(u(c_t))dt \tag{12}$$

so that (3) may be rewritten as

$$V = \int_0^\infty \frac{u(c_t)}{\delta(u(c_t))} e^{-\Delta} d\Delta . \tag{13}$$

Using (12) and (13) necessary conditions for an optimum can be found by choosing c, τ, k_1, and v to maximize the Hamiltonian:[4]

$$\begin{aligned} H(c, \tau, k_1, v, k, b, q_1, q_2, \lambda) &= \frac{1}{\delta(u(c))} [u(c) \\ &+ q_1(k_1 f(v/k_1) - nk + \tau) \qquad (14) \\ &+ q_2((r-n)b - \tau)] + \lambda[(k-k_1)g(\frac{1-v}{k-k_1}) - c] . \end{aligned}$$

The first-order conditions for a maximum are given by

$$\lambda = \frac{\delta u' - \delta' u'[u + q_1(k_1 f(\ell_1) - nk + \tau) + q_2((r-n)b - \tau)]}{\delta^2} \tag{15}$$

$$c = k_2 g(\ell_2) \tag{16}$$

$$q_1 = q_2 \tag{17}$$

$$f(\ell_1) - \ell_1 f'(\ell_1) = \frac{\lambda\delta}{q_1}[g(\ell_2) - \ell_2 g'(\ell_2)] \tag{18}$$

$$f'(\ell_1) = \frac{\lambda\delta}{q_1} g'(\ell_2) . \tag{19}$$

Equations (15) - (19) maximize the Hamiltonian given the fact that $u(c)/\delta$ is concave (which is assured by conditions (6)). The motion of the system is described by (8), (9) and

$$\dot{q}_1 = (\delta + n)q_1 - \lambda\delta[g(\ell_2) - \ell_2 g'(\ell_2)] , \tag{20}$$

$$\dot{q}_2 = (\delta + n - r)q_2 . \tag{21}$$

Equation (15) defines the shadow value of consumption. Equation (16) states that no consumption goods are wasted. The condition that new capital have an equal shadow value if it is used at home or lent abroad is given by (17). Equality of the values of the marginal product of capital between industries and of the values of the marginal product of labor between industries is guaranteed by (18) and (19) respectively, where $\lambda\delta/q_1$ represents the implicit price of consumption goods relative to capital goods.

Relation (15) can be simplified using (17) to give

$$\lambda = \frac{\delta u' - \delta' u'[u + q_1(k_1 f(\ell_1) - nk + (r-n)b)]}{\delta^2} \tag{22}$$

Relations (17), (18), (20) and (21) can be solved to give

$$r = f(\ell_1) - \ell_1 f'(\ell_1) , \tag{23}$$

or that the marginal product of capital in industry 1 equals r. Equation (23) defines ℓ_1 implicitly as a function of r. With the value of r given exogenously to the small country,

$$\ell_1 = \bar{\ell}_1 . \tag{24}$$

Next, using (18), (19) and (24) we can write

$$\frac{g(\ell_2)}{g'(\ell_2)} - \ell_2 = \frac{f(\bar{\ell}_1)}{f'(\bar{\ell}_1)} - \bar{\ell}_1 . \tag{25}$$

This implies

$$\ell_2 = \bar{\ell}_2 \ . \tag{26}$$

We see that the availability of foreign capital at a fixed interest rate nails down the labor-capital ratios in both industries. As new capital is injected into (or drained out of) the economy, labor adjusts between the two industries so as to maintain the labor-capital ratios at $\bar{\ell}_1$ and $\bar{\ell}_2$.

It is now a straightforward exercise to solve for the control variables, c, v, τ and k_1 in terms of the state variables k and b.

Since

$$k_1\bar{\ell}_1 + k_2\bar{\ell}_2 = 1 \ , \tag{27}$$

we have

$$k_1 = \frac{1 - k\bar{\ell}_2}{\bar{\ell}_1 - \bar{\ell}_2} \ . \tag{28}$$

Using (7)

$$k_2 = \frac{k\bar{\ell}_1 - 1}{\bar{\ell}_1 - \bar{\ell}_2} \ . \tag{29}$$

So, it follows from (16) that

$$c = \frac{k\bar{\ell}_1 - 1}{\bar{\ell}_1 - \bar{\ell}_2}\, g(\bar{\ell}_2) \ . \tag{30}$$

The amount of labor in industry 1 is simply

$$v = k_1\overline{\ell}_1 = \left(\frac{1 - k\overline{\ell}_2}{\overline{\ell}_1 - \overline{\ell}_2}\right)\overline{\ell}_1 . \tag{31}$$

The shadow prices can also be solved as functions of the state variables. Equations (17) and (19) imply

$$q_1 = q_2 = \lambda\delta \frac{g'(\overline{\ell}_2)}{f'(\overline{\ell}_1)} . \tag{32}$$

Equations (22) and (32) can be solved simultaneously to give λ, q_1, and q_2 as functions of b and k.

The rate of borrowing from abroad is given from (8) as

$$\tau = \dot{k} + nk - \left(\frac{1 - k\overline{\ell}_2}{\overline{\ell}_1 - \overline{\ell}_2}\right)f(\overline{\ell}_1) . \tag{33}$$

In order to express τ as a function of the state variables alone, $\dot{k}$ must be solved for the state variables. The constancy of the marginal products of labor and capital imply that the (shadow) price of capital goods relative to consumption goods must remain constant. By equation (32) we have

$$\dot{q}_1/q_1 = (\dot{\lambda\delta})/\lambda\delta ,$$

or, using equations (17) and (21),

$$\delta + n - r = (\dot{\lambda\delta})/\lambda\delta . \tag{34}$$

Taking the time derivative of equation (22) yields

$$(\dot{\lambda\delta})/\lambda\delta = \psi_1(\dot{k}, \dot{b}, k, b) \tag{35}$$

where ψ_1 is a very non-linear function. Equations (8) and (9) give us

$$\dot{b} = (r-n)b - \dot{k} - nk + k_1 f(\bar{\ell}_1) . \tag{36}$$

So, we can write

$$(\dot{\lambda\delta})/\lambda\delta = \psi_2(\dot{k}, k, b) . \tag{37}$$

Recognizing from (30) that c is a function of k alone, (34) and (36) imply

$$\dot{k} = \psi(k, b) . \tag{38}$$

(See Appendix 1 for the derivation of (38)). Plugging (38) into (33) solves τ in terms of k and b:

$$\tau = \psi(k, b) + nk - \left(\frac{1 - k\bar{\ell}_2}{\bar{\ell}_1 - \bar{\ell}_2}\right) f(\bar{\ell}_1) . \tag{39}$$

Equation (39) says that the excess of the country's growth in capital stock plus depreciation over its new production of capital must be made up by borrowing from abroad.

It is now possible to obtain a value for the objective function that depends only on k_t and b_t. Substituting (15), (28), (30), (31), (32) and (39) into (14) gives the Hamiltonian at time t as an expression in k_t and b_t:

$$H_t(c, \tau, k_1, v, k, b, q_1, q_2, \lambda) = H_t(k_t, b_t) .$$

The next section examines the steady state of the model. The effect on the steady state of a change in time preference is studied. Section 4 looks at the dynamic evolution of the system given by (9), (38) and (39). In both of those sections (as in this section) attention will be focussed on the economy when there is incomplete specialization in production.

3. The Steady State

It will be shown in the next section that the economy converges to a steady state in which $\dot{k}$, $\dot{b}$ and $\dot{q}$ are all zero. It is useful to examine the properties of the steady state because the economy will, in the absence of shocks, spend only a finite time outside a neighborhood of the steady state.

The condition that the costate variables be stationary in the long run tells us from (21):

$$\delta(u(c^*)) = r - n . \tag{40}$$

Given the monotonicity of δ and u, there is a unique level of consumption associated with each real interest rate in the long run. Since consumption is a function only of the capital stock, equation (40) also pins down the long-run total domestic capital. More specifically, we have from (40) and (30) that

$$k^* = \frac{\bar{\ell}_1 - \bar{\ell}_2}{\bar{\ell}_1} \left[\frac{u^{-1}(\delta^{-1}(r - n))}{g(\bar{\ell}_2)}\right] - \frac{1}{\bar{\ell}_1} \quad . \tag{41}$$

We will assume that k^* lies between $1/\bar{\ell}_1$ and $1/\bar{\ell}_2$. The case when k^* lies outside these bounds is taken up in Appendix 2. (If the consumption goods sector is capital intensive it can be shown that $k^* < 1/\bar{\ell}_1$ and if the consumption goods sector is labor intensive $k^* > 1/\bar{\ell}_1$. This follows because assumption (2) rules out specialization in the investment good.)

The steady state rate of capital imports can be gotten directly from (39) as

$$\tau^* = nk^* - \left(\frac{1 - k^*\bar{\ell}_2}{\bar{\ell}_1 - \bar{\ell}_2}\right) f(\bar{\ell}_1) \quad . \tag{42}$$

This country imports capital in the steady state if the rate of depreciation exceeds the rate of production of new capital.

The steady state holdings of foreign bonds is given by

$$b^* = \frac{1}{r-n}\,\tau^* = \frac{1}{r-n}\left[nk^* - \left(\frac{1 - k^*\bar{\ell}_2}{\bar{\ell}_1 - \bar{\ell}_2}\right) f(\bar{\ell}_1)\right] \quad . \tag{43}$$

If there are capital imports from abroad in the steady state, then we must hold a positive stock of foreign assets. The interest payments on these bonds equals the amount of capital imports.

In section 2 the values for the control variables and the shadow prices were determined as functions of the state variables. Their long run values are therefore determined as the values of the functions evaluated at the steady state levels of b and k.

We can now ask how steady state levels of consumption and capital imports change as time preferences change. We will consider a shift in the δ function so that the instantaneous discount rate δ_t is lower for every level of consumption. This is a movement toward giving more weight to the future. For example, if a planner changed his behavior so as to value future consumption more relative to current consumption, his discount rate would have fallen. We might then say that this shift in the δ function represents a policy shift toward "development" - if by development is meant higher long run levels of well-being.

It follows immediately from (40) and the assumptions made on $\delta(u(c))$ in (6) that

$$dc^*/d\overline{\delta} < 0$$

where $\overline{\delta}$ is a shift parameter in the δ function. A shift down in the discount rate must lead to higher steady state levels of consumption if relation (40) is to hold.

An increase in steady state consumption requires from (16) an increase in k_2 given the constancy of $\overline{\ell}_2$. However, an increase in the capital used in the consumption goods industry may require either an increase or decrease in the economy's total capital stock. Figure 1 illustrates this point. The graph shows the determination of k_1 and k_2 as the intersection of the lines of equations (7) and (27). The top figure, in which the consumption goods industry is capital intensive, shows that an increase in the total capital stock k yields an increase in k_2. However, when the consumption goods industry is labor intensive, as in the bottom figure, k_2 can increase only if k falls. This is simply Rybczynski's theorem. Formally, from (29) we have

$$\frac{dk^*}{dk_2^*} = \frac{\bar{\ell}_1 - \bar{\ell}_2}{\bar{\ell}_1} \gtrless 0 \quad \text{as} \quad \bar{\ell}_1 \gtrless \bar{\ell}_2 . \tag{44}$$

The steady state level of capital imports is determined by the size of the total capital stock and by the amount of capital devoted to the investment goods industry. The shift down in $\delta(u(c))$ reduces the size of the investment goods sector, and thus increases the need for imports of capital to maintain the nation's capital stock. If, in addition, the consumption goods sector is capital intensive, then, from above, the total steady-state capital stock will be larger. This implies even more imports to maintain the depreciating capital stock. Thus, if the capital goods sector is labor intensive, increasing patience will unambiguously raise steady state capital imports. By equation (43) it will also increase the holdings of foreign bonds required in the long run to maintain a zero current account.

If the consumption goods sector is labor intensive, then the increase in the weight given to future consumption lowers the steady-state capital stock. While this tends to reduce proportionally the need for capital imports (in order to offset depreciation), the output of the local investment goods industry shrinks more than proportionally, by the Rybcznski theorem. So, steady-state capital imports still increase. The change in steady-state capital imports for a given change in k_2 (which is monotonically related to the change in the level of consumption) is given by

$$d\tau^*/dk_2^* = \frac{1}{\bar{\ell}_1}\,[n\bar{\ell}_1 - \bar{\ell}_2(n - f(\bar{\ell}_1))] \tag{45}$$

$$= \frac{1}{\bar{\ell}_1}\,[\ell_2(r - n) + \ell_1(n + \ell_2 f')] > 0 \;.$$

4. Dynamics

In this section we examine the dynamics of the system outside the steady-state. We are particularly interested in how the system adjusts in response to a shift towards more patience. We saw in section 2 that the system could be solved in terms of the two state variables b and k. It is convenient in this section to define the total wealth of the small country, $w = b + k$.

The rate of change of k is given by equation (38). To find $\dot{w}$, we add equations (8) and (9) and use (28) to obtain

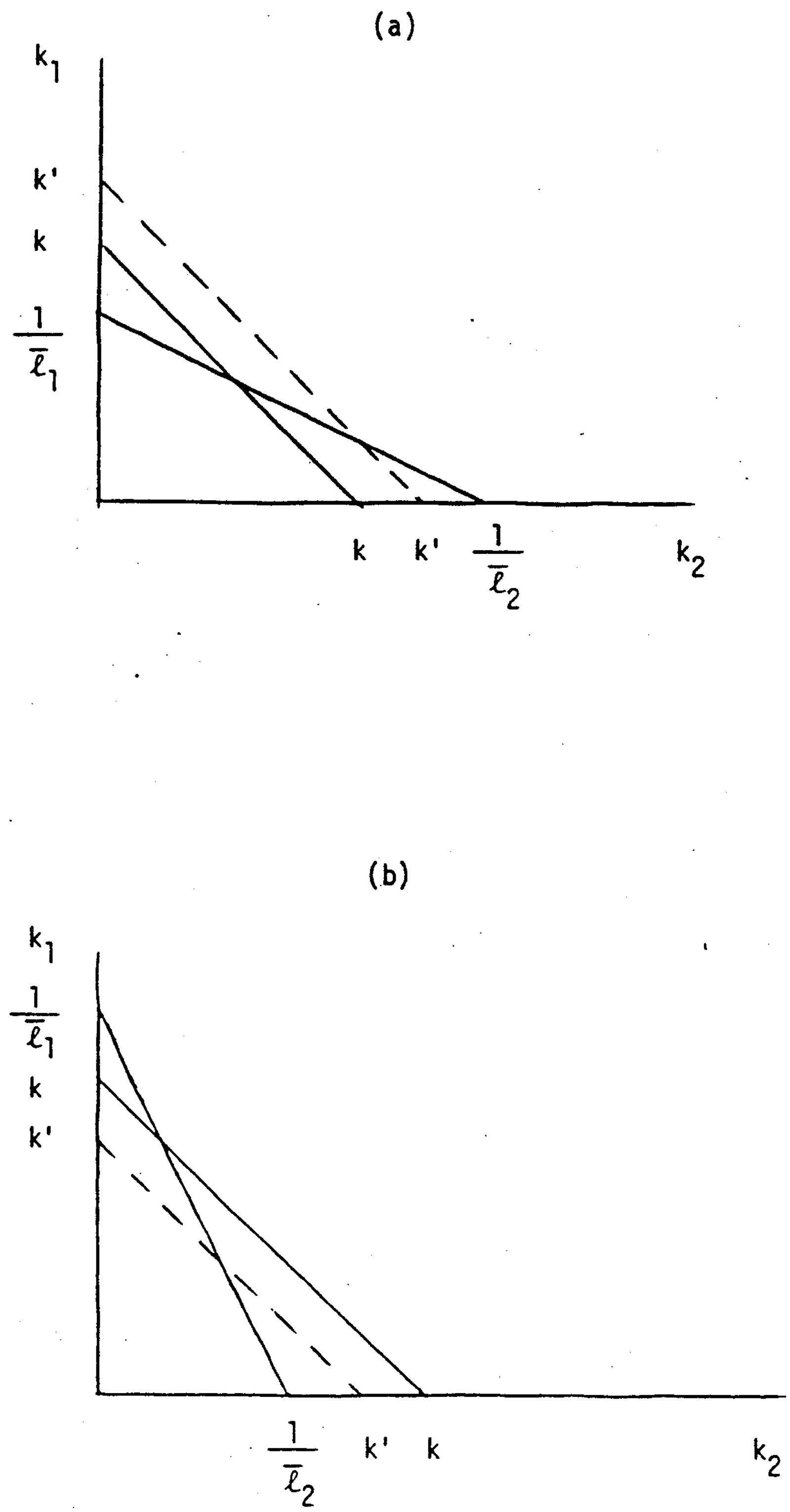
$$\dot{b} + \dot{k} = (n + \frac{\bar{\ell}_2 f(\bar{\ell}_1)}{\bar{\ell}_1 - \bar{\ell}_2})\;(\hat{k} - k) + (r - n)\;b \;, \tag{46}$$

where

$$\hat{k} \equiv \frac{f(\bar{\ell}_1)}{n(\bar{\ell}_1 - \bar{\ell}_2) + \bar{\ell}_2 f(\bar{\ell}_1)} \;.$$

The $\dot{w} = 0$ line is linear in b, k space. We have from (46) that

Figure 1


(a)
k_1
k'
k
$\frac{1}{\bar{\ell}_1}$
k
k'
$\frac{1}{\bar{\ell}_2}$
k_2
(b)
k_1
$\frac{1}{\bar{\ell}_1}$
k
k'
$\frac{1}{\bar{\ell}_2}$
k'
k
k_2

$$db/dk\big|_{\dot{w}=0} > 0\ , \qquad \text{if } \bar{\ell}_1 > \bar{\ell}_2$$

and

$$db/dk\big|_{\dot{w}=0} < -1 \qquad \text{if } \bar{\ell}_1 < \bar{\ell}_2\ .$$

The latter inequality follows from (23) which implies

$$\frac{\bar{\ell}_2 f(\bar{\ell}_1)}{\bar{\ell}_2 - \bar{\ell}_1} > r, \qquad \text{if } \bar{\ell}_1 < \bar{\ell}_2\ .$$

The stability properties of this system near the steady state can be obtained by linearizing (38) and (46): (see Appendix 1)

$$\begin{bmatrix} \dot{k} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{(\bar{\ell}_1-\bar{\ell}_2)\delta' u'(r-n)g'(\bar{\ell}_2)}{D\bar{\ell}_1 g(\bar{\ell}_2)f'(\bar{\ell}_1)} \\ -\dfrac{\bar{\ell}_1 f'(\bar{\ell}_1)g(\bar{\ell}_2)}{(\bar{\ell}_1-\bar{\ell}_2)g'(\bar{\ell}_2)} & r-n \end{bmatrix} \begin{bmatrix} k-k^* \\ w-w^* \end{bmatrix}, \qquad (47)$$

where

$$D \equiv \frac{u''(\delta - \delta' u) - \delta'' u u'^2}{u'(\delta - \delta' u)} < 0\ .$$

The products of the eigenvalues, λ_1 and λ_2, are given by the determinant of the above matrix:

$$\lambda_1\lambda_2 = \frac{\delta'u'}{D}(r - n) < 0 .$$

Hence, one eigenvalue, say λ_1, is positive and the other is negative. Since

$$\lambda_1 + \lambda_2 = r - n ,$$

we have

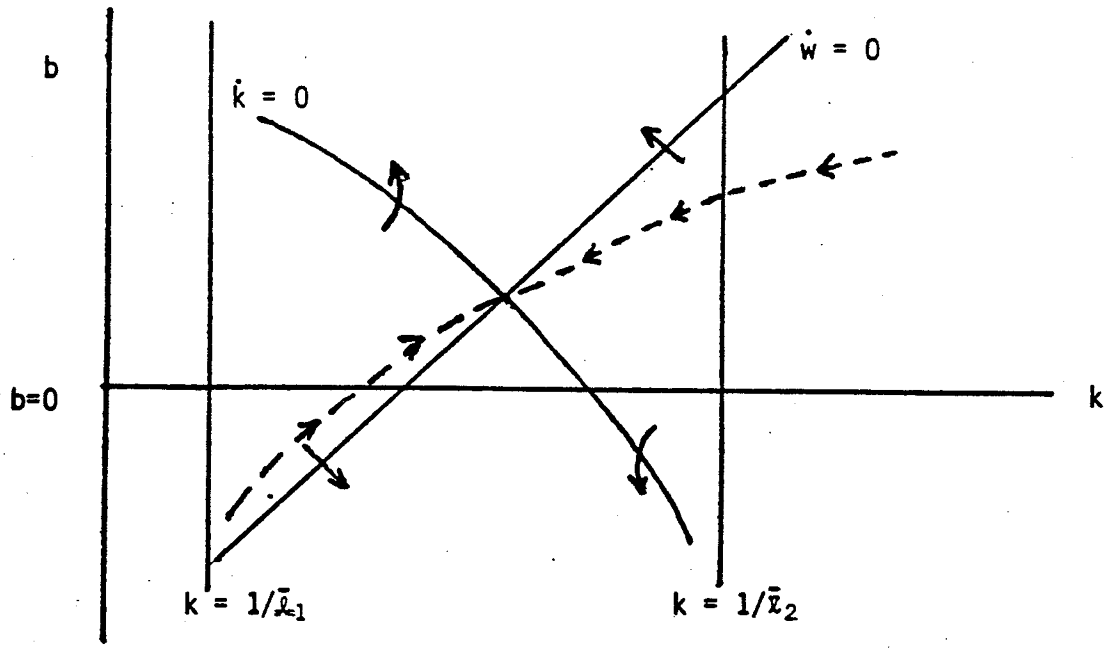
$$\lambda_1 > r - n .$$

There is a single stable saddle path given by

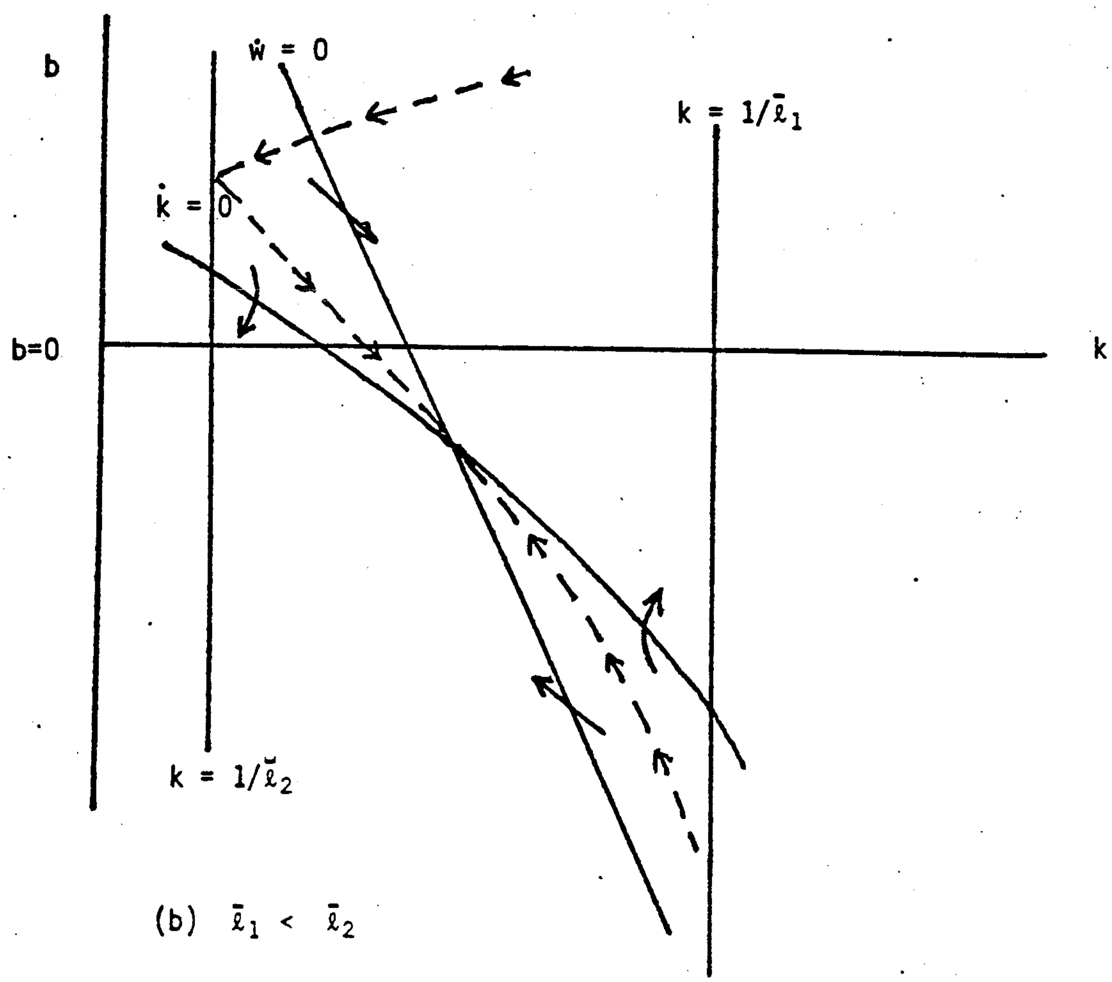
$$w - w^* \approx \frac{\bar{\ell}_1 f'(\bar{\ell}_1)g(\bar{\ell}_2)}{\lambda_1(\bar{\ell}_1 - \bar{\ell}_2)g'(\bar{\ell}_2)}(k - k^*) . \tag{48}$$

Possible phase planes are depicted in Figure 2. (The $\dot{k} = 0$ line is drawn for the case in which δ is concave in c, although none of the analysis depends on this. Also, note that the steady-state level of b can be either positive or negative in both figures 2(a) and 2(b).) The slope of the saddle path near the steady-state can be shown to be positive if the investment goods sector is labor-intensive and negative if it is capital intensive. Integrating backwards in time from the steady-state as k approaches $1/\bar{\ell}_1$ (k increasing or decreasing as the investment goods sector is capital or

Figure 2



(a) $\bar{\ell}_1 > \bar{\ell}_2$



(b) $\bar{\ell}_1 < \bar{\ell}_2$

labor intensive), bond holdings, b, decline so that for all non-negative levels of wealth, production in the consumption goods sector is positive.[5] This follows because the marginal felicity of consumption is unbounded as consumption falls to zero. Integrating backwards in the opposite direction, consumption rises until the entire labor force and capital stock are employed in the consumption goods sector ($k = 1/\bar{\ell}_2$). Thereafter, the economy is completely specialized in the production of the consumption good and both consumption and the capital stock rise backwards along an optimal path for complete specialization in the consumption goods sector. The equations of motion for k and b under complete specialization are given in Appendix 2.

Sufficient conditions for a path of k and b to be optimal are given by[6]

$$\lim_{\Delta\to\infty} e^{-\Delta} q_1 k_\Delta = 0 , \quad \lim_{\Delta\to\infty} e^{-\Delta} q_2 b_\Delta = 0 . \tag{49}$$

The path that leads to the steady state is an optimal path. Thus, we will concentrate on the dynamics along the saddle path. Notice that along the path the intertemporal budget constraint expressed in (11) is satisfied. If initial conditions are given which are not on the saddle path, then there will be a jump in the state variables b and k, such that wealth remains constant.[7]

We now consider the effects of a change in time preference on the capital stock and foreign bond holdings from an initial position of steady-state equilibrium. There will be a jump in the state variables b and k at the time of the changes in the δ function, but total wealth, w, cannot jump immediately. There may be at first a trade of some capital for some foreign bonds (leaving wealth unchanged) or vice-versa. After this, state variables will adjust smoothly.

We know that the long-run effect of increasing patience is to raise the steady state capital stock if consumption is capital intensive and lower the steady state capital stock if consumption goods are labor intensive. To get the effect on the initial changes in the capital stock (and hence the initial borrowing) we simply differentiate (48) holding w_t constant. Recognizing that

$$w^* = \frac{\bar{\ell}_1 f'(\bar{\ell}_1) g(\bar{\ell}_2)}{(r-n)(\bar{\ell}_1 - \bar{\ell}_2) q'(\bar{\ell}_2)} k^* - \frac{f(\bar{\ell}_1)}{(r-n)(\bar{\ell}_1 - \bar{\ell}_2)} ,$$

we have that

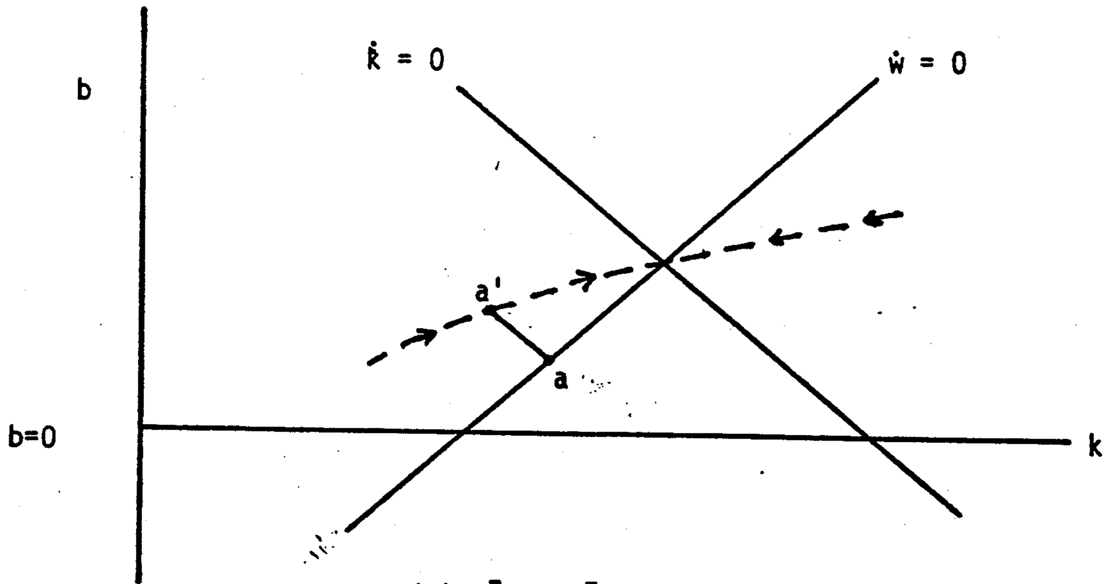
$$\frac{dk}{dk^*} = 1 - \frac{\lambda_1}{r - n} < 0 .$$

Thus the initial change in the capital stock is always in the opposite direction of the eventual change in the steady state capital stock. This implies that initially there is a drop in consumption.

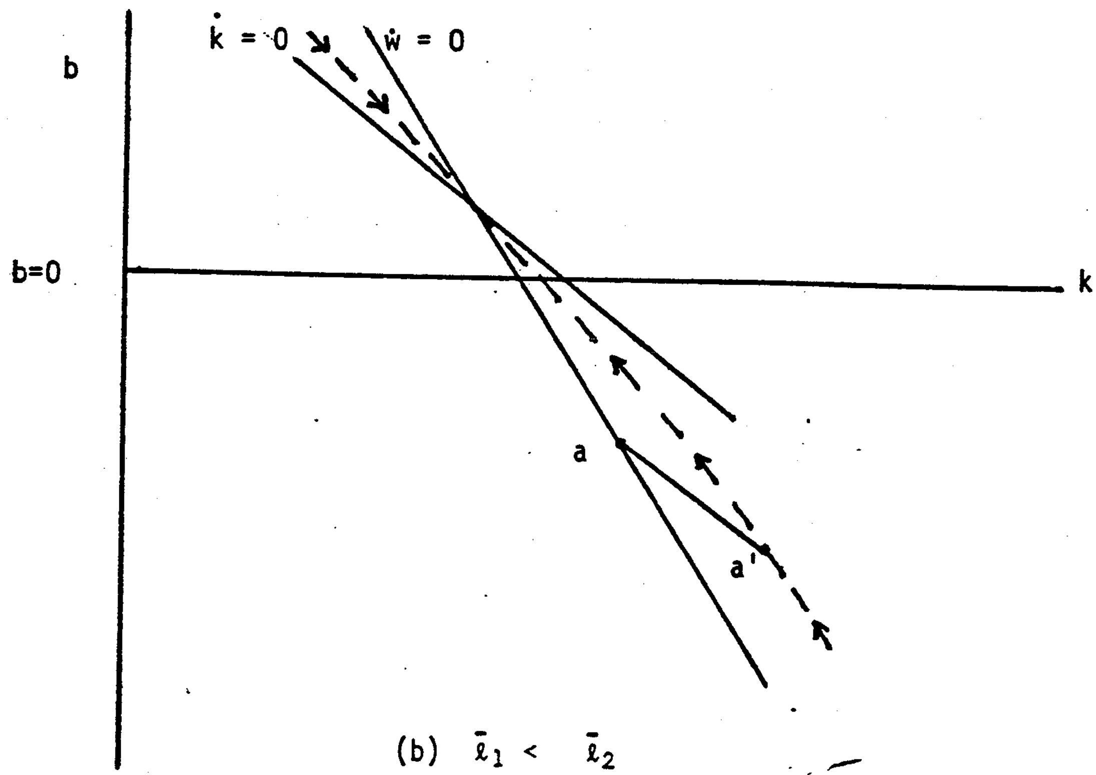
If the consumption goods industry is labor intensive, there must be an initial increase in the capital stock that expands investment goods production and contracts consumption goods production. In this case, the shift down in the discount function (increased concern for the future) leads to foreign borrowing initially. In the case in which consumption is capital intensive, there will be foreign lending on impact.

These results can be understood more completely with the help of the phase diagrams. Figure 3(a) is for the case in which consumption goods are capital intensive. Here, the $\dot{w} = 0$ line must slope upwards, and the shift down in $\delta(u(c))$ must lead to a higher steady state capital stock. The initial

Figure 3



(a) $\bar{\ell}_1 > \bar{\ell}_2$



(b) $\bar{\ell}_1 < \bar{\ell}_2$

steady state was at a point such as a. The change in time preference moves the economy up the $\dot{w} = 0$ line to a new steady state at point b. (The $\dot{w} = 0$ line does not shift with the change in the δ function.) In order to get onto the new saddle path from position a, this country must lend abroad the amount of capital given by the distance aa'. It then accumulates capital and foreign bonds along the path to the steady state.

When consumption goods are labor intensive Figure 3(b) applies. The $\dot{w} = 0$ line slopes down, and the change in time preference leads to a lower steady state capital. However, initially there must be some foreign borrowing that allows the capital stock to jump from a to a'. This is the case in which a more future oriented economy would resort at first to some international borrowing. After the jump to a', capital stocks fall, but foreign bonds and wealth accumulate along the path to steady state.

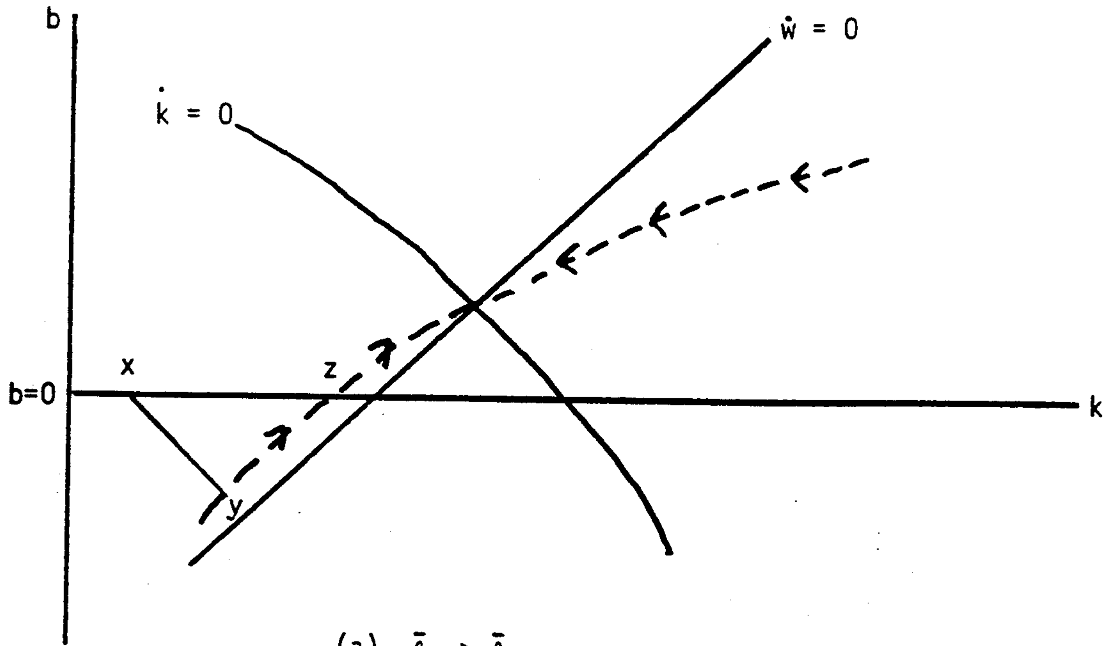
Next, consider a capital-poor country which initially has no external debt or foreign bond holdings and begins to exchange freely capital goods and bonds with the rest of the world. The country should initially issue debt and import capital until it is on the saddle path with wealth equal to its initial capital stock. It will then have a capital stock which is greater (less) than the steady-state capital stock if the investment goods sector is capital-intensive (labor-intensive). Adjustment towards the steady-state occurs as the country runs a current account surplus and reduces its debt. In the steady-state, the country has a balanced current account and can be either a net debtor or creditor. In the latter case, it will have a trade account deficit and service account surplus. If the country were capital rich, the dynamics would follow a pattern opposite to the one described above. However, the country may completely specialize in the production of the consumption good until its capital stock falls sufficiently through depreciation so that the value of the marginal product of caital is equal across sectors.

Figure 4 depicts a sequence of events that is consistent with Fischer and Frenkel's description of stages of the balance of payments. The country is initially at a point such as x when its markets open up to free trade. It immediately runs a current account deficit and acquires capital as it moves from x to y. This stage would correpsond to the young debtor stage described by Fischer and Frenkel. As the country moves from y to z, it reduces its debt to the rest of the world. It is a mature debtor in this stage. From z to the steady-state, the country is a young creditor. It is a net creditor and is increasing its claims on the rest of the world. At the steady-state the country finances a deficit on trade account with interest payments on its holdings of foreign debt. It is a mature creditor with a balanced current account.

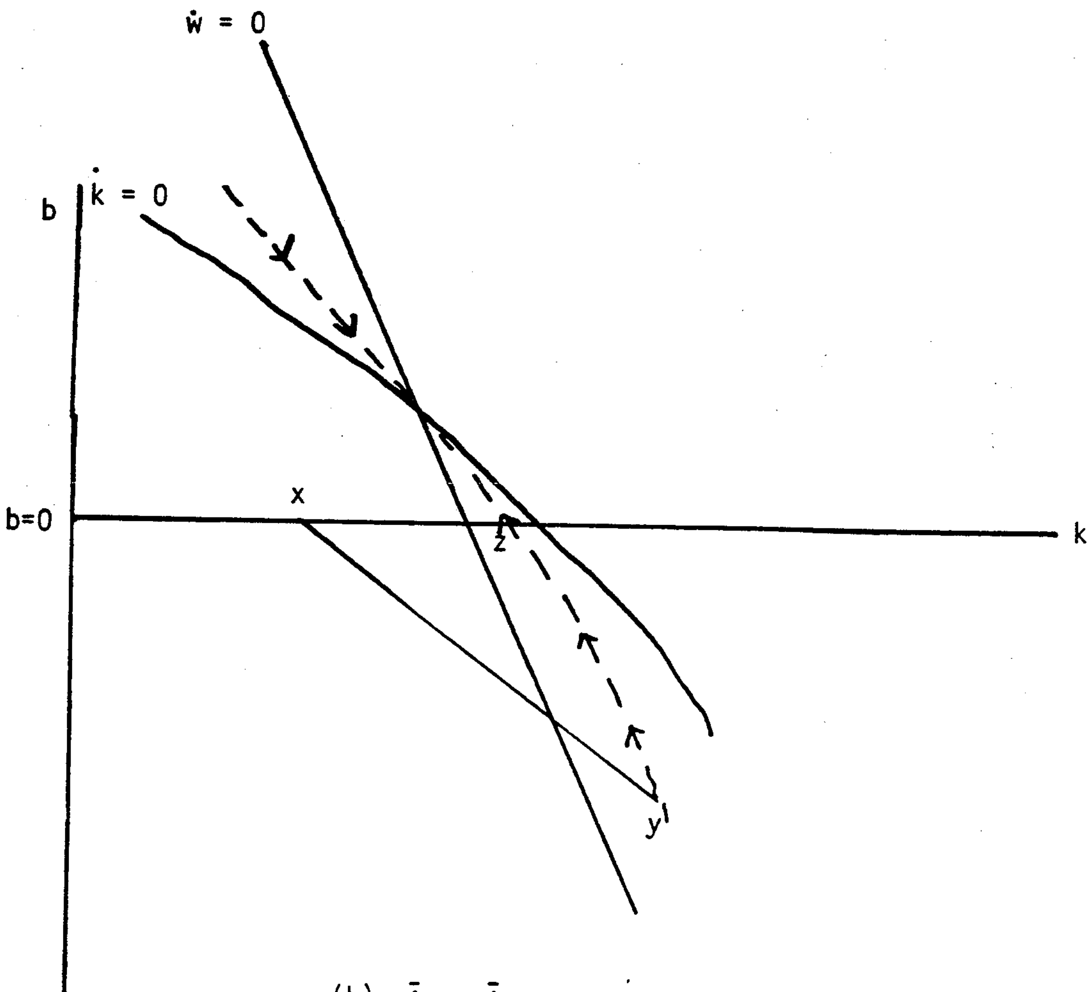
5. Conclusion

This paper builds a simple framework in which factors are mobile within a country, and only capital can be traded abroad in return for promises to repay more capital in the future. In this set-up we found that a shift in time preference toward a more future orientation leads to a cut-back in current consumption levels as a means of obtaining higher long-run levels of consumption. In a model in which only consumption goods can be lent or borrowed this would imply an increase in lending abroad. But in our model the drop in the level of activity in the consumption sector is accompanied by an increase in domestic production of investment goods. In the case in which the

Figure 4



(a) $\bar{\ell}_1 > \bar{\ell}_2$



(b) $\bar{\ell}_2 > \bar{\ell}_1$

amount of capital that can profitably be reallocated from the consumption sector is insufficient (the case in which capital goods production is capital intensive) borrowing of machines from abroad must occur.

This model makes several special assumptions which could be relaxed. Neary (1978) has criticized the assumption of perfect inter-sectoral factor mobility in this type of model. Short-run specificity of capital might lead to an adjustment motive for borrowing. It might also be useful to test the robustness of the results of this paper by considering models with richer trading opportunities.

Appendix 1

Let

$$w = k + b$$

Then

$$\dot{w} = k_1 f - nk + (r - n)b$$

$$= k_1 f - rk + (r - n)w$$

$$= k_1(f - r) - rk_2 + (r - n)w$$

$$= k_1 \ell_1 f - rk_2 + (r - n)w$$

$$= (f'/g')(k_1 \ell_1 g' - gk_2 + k_2 \ell_2 g') + (r - n)w$$

$$= f' - (f'/g')gk_2 + (r - n)w$$

$$= f' + (r - n)w - (f'/g')c \ .$$

This says saving is equal to labor income plus interest income less consumption.

Using (22) and (32)

$$\lambda\delta = u' - (\delta' u'/\delta)[u + \lambda\delta(g' - c + (g'/f')(r - n)w] \ .$$

It is convenient to write

$$s = g' - c + (g'/f')(r - n)w$$

where s is saving in terms of good 2.

We then have

$$\lambda\delta = [(u'/\delta)(\delta - \delta'u)]/\Delta$$

where

$$\Delta \equiv 1 + (\delta'u'/\delta)s \ .$$

Then,

$$(\dot{\lambda\delta})/\lambda\delta = \tilde{D}\dot{c} - \dot{\Delta}/\Delta$$

where

$$\tilde{D} \equiv [(\delta u'' - \delta'u'^2)(\delta - \delta'u) - \delta\delta''uu'^2]/\delta u'(\delta - \delta'u) \ .$$

$$\dot{\Delta}/\Delta = [(\tilde{A}s - \delta'u'/\delta)/\Delta]\dot{c} + [(\delta'u'/\delta)(r - n)/\Delta]s \ ,$$

where

$$\tilde{A} = (\delta\delta''u'^2 + \delta\delta'u'' - \delta'^2u'^2)/\delta^2 \ .$$

Thus

$$\delta + n - r = [(\tilde{D}\Delta + \delta'u'/\delta - \tilde{A}s)/\Delta]\dot{c} - [(\delta'u'/\delta)(r - n)/\Delta]s .$$

So

$$(\delta + n - r)\Delta + (\delta'u'/\delta)(r - n)s = (\tilde{D}\Delta + \delta'u'/\delta - \tilde{A}s)\dot{c} .$$

$$(\delta + n - r)(1 + (\delta'u'/\delta)s) + (\delta'u'/\delta)(r - n)s$$

$$= [\tilde{D}(1 + (\delta'u'/\delta)s) + \delta'u'/\delta - \tilde{A}s]\dot{c}$$

$$\delta + n - r + \delta'u's = [(\tilde{D} + \delta'u'/\delta) + (\tilde{D}(\delta'u'/\delta) - \tilde{A})s]\dot{c}$$

Then, we have

$$\dot{c} = (\delta + n - r + \delta'u's)/(D + As)$$

where

$$D = \tilde{D} + \delta'u'/\delta = [u''(\delta - \delta'u) - \delta''uu'^2]/u'(\delta - \delta'u)$$

and

$$A = D(\delta'u'/\delta) - \tilde{A} = -\delta''u'^2/(\delta - \delta'u) .$$

Since

$$\dot{c} = (g\ell_1/(\ell_1 - \ell_2))\dot{k}$$

it follows that

$$\dot{k} = (\ell_1 - \ell_2)(\delta + n - r + \delta' u' s)/g\ell_1(D + As) \text{ .}$$

Since s is a function of k and b only, and c is a function only of k, this is equation (38) of the text.

To derive the linearization of $\dot{k}$ at the point where $\dot{k} = 0$ and s = 0, we use the fact that

$$\left.\frac{dw}{dk}\right|_{\dot{k}=0} = \frac{-(\overline{\ell}_1 - \overline{\ell}_2)(\delta'' u'^2 + \delta' u'')\dot{w}}{g(\overline{\ell}_2)\overline{\ell}_1 \delta' u'(r - n)} \text{ .}$$

Near $\dot{w} = 0$, this derivative equals zero. Hence, the $\dot{k} = 0$ line has a slope of -1 in b, k space near equilibrium.

Appendix 2

Here we study the behavior of the system when the economy completely specializes in production of the consumption good in steady state.

The first-order conditions in this case are

$$\dot{k} = \tau - nk$$

$$\dot{b} = (r - n)b - \tau$$

$$\lambda\delta = (u'/\delta)(\delta - \delta'u) - (\delta'u'/\delta)q((r - n)b - nk)$$

$$c = kg(1/k)$$

$$\dot{q} = (\delta + n)q - \lambda\delta[g(1/k) - (1/k)g'(1/k)]$$

$$\dot{q} = (\delta + n - r)q \ .$$

The first-order conditions differ from the case of non-specialized production in that the marginal products of labor and capital in the consumption goods industry exceed those in the (non-existent) investment goods industry. The marginal products, the labor/capital ratios in each industry and the relative shadow price of the goods were all constant in the non-specialized case, but all vary over time here.

Steady-state consumption is determined by

$$\delta(u(c^*)) = r - n \ .$$

Steady-state capital is given by

$$c^* = k^*g(1/k^*) \ .$$

Finally,

$$nk^* = (r - n)b^*$$

gives us steady-state bond holdings.

As in the non-specialized case, dynamics can be studied by focusing on the equations for $\dot{k}$ and $\dot{w}$. We have immediately

$$\dot{w} = \dot{k} + \dot{b} = (r - n)w - rk \ .$$

The equation for $\dot{k}$ will require more work. Much of the derivation parallels that in Appendix 1. The definitions of some of the symbols used here, and some missing steps in the derivation are supplied in Appendix 1.

First, note that

$$rq = \lambda\delta(g - g'/k) \ .$$

Therefore

$$\dot{q}/q = [d(\lambda\delta)dt]/\lambda\delta + [d(g - g'/k)/dt]/(g - g'/k) \ .$$

But

$$\dot{q}/q = \delta(u(c(k))) + n - r \ .$$

So, we have from the first-order conditions that $\lambda\delta$ can be solved as a function of k and w. This means the time derivative of $\lambda\delta$ is a function of $\dot{k}$, $\dot{w}$, k and w. But, we have already seen that $\dot{w}$ is a function of k and w. So, the equation

$$\delta + n - r = [d(\lambda\delta)/dt]/\lambda\delta + [d(g - g'/k)/dt]/(g - g'/k)$$

gives $\dot{k}$ implicitly as a function of k and w.

We have

$$[d(g - g'/k)/dt]/(g - g'/k) = [(g''/k^3)/(g - g'/k)]\dot{k} \ .$$

Next

$$\lambda\delta = [(u'/\delta)(\delta - \delta'u)]/[1 + (\delta'u'/\delta)s]$$

where

$$s = (g - g'/k)[((r - n)/r)w - k] \ .$$

Then

$$[d(\lambda\delta)/dt]/\lambda\delta = \dot{\tilde{D}c} - \dot{\Delta}/\Delta$$

as in Appendix 1. In the case of specialized production we have

$$\dot{c} = (g - g'/k)\dot{k} .$$

Some calculations can show

$$\dot{\Delta}/\Delta = (1/\Delta)[\tilde{A}s(g - g'/k)\dot{k} - (\delta'u'/\delta)(g - g'/k)\dot{k}$$

$$+ (\delta'u'/\delta)(r - n)s + (\delta'u'/\delta)s((g''/k^3)/(g - g'/k))\dot{k}] .$$

All of this can be combined to give

$$(\delta + n - r)\Delta = \tilde{D}\Delta(g - g'/k)\dot{k} - \tilde{A}s(g - g'/k)\dot{k} + (\delta'u'/\delta)(g - g'/k)\dot{k}$$

$$- (\delta'u'/\delta)(r - n)s - (\delta'u'/\delta)s[(g''/k^3)/(g - g'/k)]\dot{k}$$

$$+ \Delta[(g''/k^3)/(g - g'/k)]\dot{k}$$

or

$$\delta + n - r + \delta'u's = (\tilde{D} + \delta'u'/\delta)(g - g'/k)\dot{k}$$

$$+ (\tilde{D}(\delta'u'/\delta) - \tilde{A})s(g - g'/k)\dot{k} + [(g''/k^3)/(g - g'/k)]\dot{k} .$$

So, solving for $\dot{k}$

$$\dot{k} = (\delta + n - r + \delta'u's)/[(D + As)(g - g'/k) + (g''/k^3)/(g - g'/k)] .$$

This can be linearized around the steady state as

$$\dot{k} \approx [\delta' u'((r - n)/r)/(D + (g''/k)/(kg - g')^2)](w - \bar{w}) .$$

The coefficient on $w - \bar{w}$ is negative. Thus, the $\dot{k}$ and $\dot{w}$ equations yield a phase diagram qualitatively identical to that of Figure 2(a) - the case in which the consumption good is capital intensive. Here, moving backwards in time from the steady-state, there may be a period in which both the consumption good and capital good are produced.

As in Figure 3(a), an increase in concern for the future (a shift down in the δ function) leads to an immediate drop in the domestic capital stock as capital is lent abroad.

Footnotes

[1]See, for example, Bardhan (1967, 1970), Obstfeld (1981, 1982) or Dornbusch (1983). There are many similarities between the model in this paper and the one in Obstfeld (1982). Obstfeld considers an economy with a constant flow of non-produced income. His model is equivalent to a model in which a schmoo good (can be used as consumption or capital) is produced. If capital can be traded freely, output is constant at the level where the marginal product of capital equals the world interest rate. Borrowing only occurs for consumption purposes once output is fixed.

A previous paper that has investigated international borrowing in a context in which capital goods are traded is Fischer and Frenkel (1972). They point out that if the capital stock is costlessly adjustable, and there is free trade in consumption, capital and securities, that a country's capital stock is indeterminate. They introduce an investment function, which suffices to determine the stock of capital at any time. Here, we avoid the indeterminacy because the consumption good is non-traded. Fischer and Frenkel study the dynamics of capital accumulation with the assumption of a constant saving rate. This paper investigates international borrowing under the assumption that agents maximize utility of consumption.

[2]Rybczynski's (1955) theorem states that at constant prices, an increase in one factor endowment will increase the output of the good intensive in that factor more than proportionally and will reduce the output of the other good.

[3]See Arrow and Kurz (1970) and Obstfeld (1981, 1982) for discussions of similar conditions.

[4]See Arrow and Kurz (1970). Equation (17) follows from the Kuhn-Tucker conditions if borrowing is finite.

[5]Along the $\dot{w} = 0$ line, b can easily be shown to be less than -k when $k = 1/\bar{\ell}_1$, as drawn in Figure 2.

[6]See Arrow and Kurz (1970).

[7]See Arrow and Kurz (1970).

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