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SEARCH, APPLICATIONS AND VACANCIES

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Section 1: Introduction

Over the last few years the efficiency of search equilibria has been examined by a number of authors. In a series of papers, Diamond [(1981), (1982a), (1982b), (1984)] has looked at this issue in depth. Diamond (1981) shows that in a market with a distribution of match specific mobility costs, an unemployment insurance program can improve the ex ante welfare of all workers by inducing each of them to forego opportunities with high mobility costs. Diamond (1982b) shows that in a market with no competition among agents, there are multiple equilibria, all of which are Pareto inefficient. The inefficiency

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occurs because no agent internalizes the value of his increased search activity to other searchers. Diamond and Maskin (1979), (1981), and Mortensen (1981), (1982a) (1982b) show in matching models that the characteristics of the inefficiencies in equilibrium depend upon the search technology. With the exception of Mortensen (1981), (1982a) none of these models allows for contemporaneous competition among searchers.[1]

Wilde (1977) has developed a model where the equilibrium price distribution is determined by the level of search intensity of consumers. When each consumer increases his intensity, all sellers lower their prices. This implies that the equilibrium search intensity is Pareto inefficient if only the welfare of consumers is considered; each consumer's welfare would increase if all consumers searched a little harder. Wilde's results depend crucially upon a lack of competition among consumers for the goods being sold. [See Stern (1985), Chapter IV for a further discussion].

Most labor markets are characterized by some degree of competition for a small number of job openings. This is especially true when the unemployment rate is high or there is a particularly attractive job opening. Firms may limit the number of job openings because of diminishing returns to scale in production and lags in the hiring process. [See Stern (1985), Chapter IV for a further discussion]. If the number of job openings is small relative to the number of workers searching for those openings, then the competition among the

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1 Matching problems without externalities have been examined as well. Jovanovic (1979) has examined markets where the productivity of a particular match is unknown ex ante. Crawford and Knoer (1981) have examined markets where the productivity of all possible matches is known ex ante.

workers will be a crucial aspect of the economic environment of the workers.[2]

The rivalry literature [see for example Kamien and Schwarz (1972), Loury (1979), Mortenson (1981) (1982a), (1982b), and Wright (1983)] has shown that when there is a common goal that a number of agents are striving to achieve, and when all of the benefits of achieving that goal go only to the first agent who is successful, then there is excessive rivalry among the agents. Each agent must choose an intensity with which to strive for the goal given the intensity of other agents. Marginal units of intensity are costly. An externality results because each agent ignores the effect his intensity has on the other agents' probability of achieving the goal first. The rivalry problem has been used mostly to examine the market for research and development.

This paper examines a generalized rivalry problem in the labor market. It employs a simple labor supply model as a framework to analyze labor markets characterized by search with competition among searchers. First, the labor market process for new hires is described. The searching worker's opportunities are determined by the market parameter which is the probability that an application will not generate a job offer. The model is closed by determining the value of the market parameter given the search strategy that each worker individually follows. There exists a nontrivial equilibrium, and it is Pareto inefficient when only the welfare of the workers is considered. To get rid of the inefficiency, a feasible, Pareto-improving government policy

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[2] Lucas and Prescott (1974) present an equilibrium search model with competition. However, the competition only affects the equilibrium wage because markets clear each period.

using taxes and unemployment insurance is introduced. This is done for an economy with search unemployment in both a static and a steady state dynamic framework.

Section 2: The Market Mechanism

There are B firms, each of which costlessly advertises n vacancies in the local want ads every period. A period is the length of time it takes for a firm to list an ad, receive applications, make offers, receive replies and hire those who accept. No deceptive advertising is allowed. There are also N identical unemployed workers every period who costlessly look through the want ads and determine the number of firms, m, to which they should apply. The cost of applying to m firms is C(m). This represents transportation costs, time costs, and any direct costs of informing firms of one's interest in a job. It is assumed that $C(0) = 0$, $C'(m) > 0$, $C''(m) \geq 0$ and C(B) is very large relative to the benefits of getting a job.

Each worker applies to firms without knowing exactly what other workers will do.[3] However, he knows or can derive the distribution function of the number of applicants at each firm. Once a firm has received applications for a period, if it receives at least n applications, it randomly offers n applicants jobs at a wage of w. If it receives fewer than n applications, it offers all applicants jobs at the same wage. Firms are not allowed to have waiting lists. A worker will accept any offer made to him unless he receives more than one

[3]Formally, this assumption means that asymmetric equilibria are ruled out since they require coordination among workers. A worker cannot announce where he plans to apply or discuss his decision with other workers. Therefore, workers must play symmetric roles at equilibrium.

offer in the same period. Then, since all offers have the same value, the worker randomly selects one of the offers. Once he has accepted an offer from a firm, he works for that firm forever receiving a wage w once a period. He receives an unemployment insurance payment (UI payment), u, once a period until he finds a job. It is assumed that u is less than w.

It is assumed that the equilibrium is a symmetric Nash equilibrium (which is sometimes called a 'supply side equilibrium' since all choices in the model are made by the suppliers of labor, i.e. the workers). This means that each unemployed worker treats the application strategies of other workers, and thus the probabilities of receiving job offers, as given, and that at equilibrium all workers adopt the same strategy. A worker prefers to apply to jobs with high probabilities of receiving offers over firms with low probabilities of receiving offers and randomly chooses among firms with the same probability of receiving an offer. Each worker forms expectations either through past experience in the labor market, through contact with other workers, or by computing where the Nash equilibrium will occur.[4]

The probability of being offered a job at a particular firm depends upon how many vacancies the firm advertises and the distribution function of the number of applicants it will receive. The explicit formula for this probability is derived later in the paper. For now, it is only important to recognize that in equilibrium, the probability of any worker receiving an offer from any firm must be the same for all firms. If, for any one worker, there were two firms with different probabilities of making offers, then the two firms would have different probabilities for everyone. Everyone applying to

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[4]Computation poses some problems when there is more than one Nash equilibrium.

the low probability firm would have incentive to apply to the high probability firm instead. But then the probability of receiving an offer at the low probability firm would be unity; it actually would be a high probability firm. Therefore, the application strategies could not be a Nash equilibrium. Thus, it must be true that in equilibrium, all firms have the same ex ante probability of offering a worker a job, and so a worker's decision is characterized by the number of firms to which he applies.

Section 3: The Worker's Problem

The first step in solving the supply side equilibrium is deriving the objective function that each unemployed worker maximizes. As in most of the search literature, it is assumed that a worker maximizes the expected value of search which equals the values of having a job and continued search, each weighted by the probability of being in that state, minus search costs. Let:

γ = probability of not being offered a job at a firm to which a worker applies.

If a worker applies to m firms, the probability of being offered at least one job is $(1-\gamma^m)$. Let β be each worker's discount factor. Let V(m) equal the value of applying to m firms. Then:

3.1) $$V(m) = u - C(m) + \beta(1 - \gamma^m)w/(1-\beta) + \beta\gamma^m V^*$$
$$= u - C(m) + \beta w/(1-\beta) - \gamma^m[\beta w/(1-\beta) - \beta V^*]$$

where V^* is the value of the optimal strategy that will be followed next period. Since the market is in a steady state, the optimal strategy will be the same every period.

The behavior of each worker can be derived by looking at the first order condition for equation (3.1):

$$3.2) \quad \partial V(m)/\partial m = -C'(m) - \gamma^m \ln\gamma[\beta w/(1-\beta) - \beta V^*] = 0.$$

The second order condition is:

$$3.3) \quad -C''(m) - \gamma^m (\ln\gamma)^2 [\beta w/(1-\beta) - \beta V^*] < 0.$$

Since $[\beta w/(1-\beta) - \beta V^*]$ is the difference in value between having a job and not having a job, it must be positive; otherwise there would be no search. Thus, the assumption that $C''(m) \geq 0$ implies that the second order condition holds globally. Therefore, the first order condition is a necessary and sufficient condition for a global maximum.

Equation (3.2) provides an implicit equation for m^*, the optimal level of applications. The necessary conditions for positive search can be derived by evaluating $\partial V/\partial m$ at $m = 0$:

$$3.4) \quad \partial V(0)/\partial m = -C'(0) - \ln\gamma[\beta w/(1-\beta) - \beta V^*(0)]$$

$$3.5) \quad = -C'(0) - \beta(w - u)\ln\gamma/(1-\beta) > 0$$

since if $m = 0$ is the optimal strategy today, it will also be the optimal strategy tomorrow. Thus, if the difference between w and u is high enough, γ is low enough, and $C'(0)$ is low enough, there will be positive search.[5]

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[5] This assumes that unemployed workers receive u whether or not they search. If u is paid to all unemployed workers, then workers only consider the difference between w and u in their search decision. If u is only paid to unemployed workers who search, then the sizes of w and u enter the search decision in a nonlinear way.

It can be shown by looking at the derivative of $\partial V/\partial m$ with respect to exogenous variables what the comparative statics for the workers are:

3.6) $\partial m^*/\partial\beta > 0$, $\partial m^*/\partial w > 0$, $\partial m^*/\partial u < 0$,[6] $\partial m^*/\partial C' < 0$,[7]

and $\partial m^*/\partial\gamma$ has the opposite sign of $(m^*\ln\gamma + 1)$. If w rises, then the difference in value between working and searching increases. This causes the worker to search more. If β rises, then the worker discounts the future less heavily, causing the value of the wage stream to rise more than the application costs. Thus, the number of applications rises. Similarly, higher marginal search costs cause the worker to apply less. Finally, if γ rises, then the incremental probability of getting a job by searching a little harder is $-\gamma^{m-1}(m\ln\gamma + 1)$ which can be either negative or positive.

To be more precise, m should be either an integer or a representation of a mixed strategy, and the first order analysis should be adjusted accordingly. However, as long as $m > 1$, the continuous approximation to the problem provides much insight with little loss of accuracy.

Section 4: Probability of Rejection

The only open parameter left to determine is γ, the probability of the worker not receiving an offer at a firm to which he applied. It is easier to

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[6] u is really the UI payment to be received the next period which increases V^*. The UI payment received this period has no effect on m since it is only a negative fixed cost of search.

[7] This is for a case where the marginal cost of applying rises by a constant amount for all m's. For example, if $C(m) = cm$, then $C'(m) = c$. More precisely, if $C_\alpha(m) = c(m) + \alpha m$, then $C_0(m) = C(m)$ and $C'_\alpha(m) = C'(m) + \alpha$. The assertion is that $dm^*/d\alpha < 0$.

compute γ by thinking of it as:

$$4.1)\qquad \sum_{a=0}^{N-1} [\Pr(\text{not offered job} \mid a + 1 \text{ applicants apply}) * \Pr(a \text{ other applicants apply})].$$

If there are n vacancies advertised:

$$4.2)\qquad \Pr(\text{not offered job} \mid a + 1 \text{ applicants apply}) = \max(0,\ 1 - n/(a + 1)).$$

Since there are N-1 workers other than the one evaluating γ and each of them applies to m^* of the B firms:

$$4.3)\qquad \Pr(a \text{ other applicants apply}) = \binom{N-1}{a}(m^*/B)^a(1 - m^*/B)^{N-1-a}.$$

This is a binomial random variable with parameters N-1 and m^*/B. It is assumed that m^* is less than B and that no worker applies to the same firm twice. It can be shown that the second assumption is optimal behavior because $m^* < B$ which holds by the assumption about costs. Therefore:

$$4.4)\qquad \gamma = \sum_{a=0}^{N-1} \max(0,\ 1 - n/(a+1))\binom{N-1}{a}(m^*/B)^a(1 - (m^*/B))^{N-1-a}.$$

Section 5: Equilibrium

It can be shown that there exists a Nash equilibrium to the supply side. An equilibrium is characterized by a pair, m^* and γ^*, such that when the probability of rejection is γ^*, each worker applies to m^* firms, and when each worker applies to m^* firms, the probability of rejection is γ^*. Since

$\partial^2V/\partial m^2 < 0$ for all values of m, there exists a unique solution to equation (3.2) for any given value of $\gamma > 0$ and V^*. Since V^* is the maximum value of V(m) for any level of γ, it is straightforward to show that V^* is a continuous, differentiable function of γ. Thus, there exists a unique solution to equation (3.2) for any value of γ. Denote this solution as $m = M(\gamma)$. Rewriting equation (3.2), $M(\gamma)$ satisfies:

5.1) $$\ln C'(M(\gamma)) - M(\gamma)\ln\gamma = g(\gamma)$$

if $M(\gamma) \geq 0$ where $g(\gamma) = \ln(-\ln\gamma) + \ln[\beta w/(1-\beta) - \beta V^*(\gamma)]$. Otherwise, $M(\gamma) = 0$. $g(\gamma)$ is a continuous function that is differentiable at all values of γ on the interval (0,1] except for one point.[8] The derivative, $M'(\gamma)$, exists for all values of $M(\gamma)$ where $M(\gamma)$ is positive. Thus, in equation (3.2), m can be written as a continuous function of γ on the half-open interval (0,1] that is differentiable at all points except for one. It can be shown that $\lim M(\gamma) = 0$ as $\gamma \to 0$. If M(0) is defined to be zero, then M is defined and continuous on the closed interval [0,1]. Also, from equation (4.4), γ can be written as:

5.2) $$\gamma^* = \Gamma(m^*).$$

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8 The one point is γ' where $M(\gamma') = 0$ and $M(\gamma) > 0$ for any $\gamma < \gamma'$. To the left of this point $dV^*/d\gamma < 0$, and to the right of this point $dV^*/d\gamma = 0$. This occurs because negative applications are not allowed. So if the solution to equation 5.1 is negative, then $M(\gamma)$ must be defined as equal to zero. The point where the nonnegativity constraint becomes binding is not differentiable, but it is still continuous.

Let:

5.3) $F(\gamma) = \Gamma(M(\gamma))$

$F(\gamma)$ is the probability of not being offered a job if everyone thought that the probability of not being offered a job was γ. If each worker thought that the probability of not being offered a job was γ, each would each apply to $m^* = M(\gamma)$ firms, and then the actual probability of not being offered a job would be $\Gamma(m^*)$. $F(\gamma)$ is continuous, and both its range and domain are the unit interval. Thus, by Brouwer's fixed point theorem, there exists a point γ^* where:

5.4) $F(\gamma^*) = \gamma^*$.

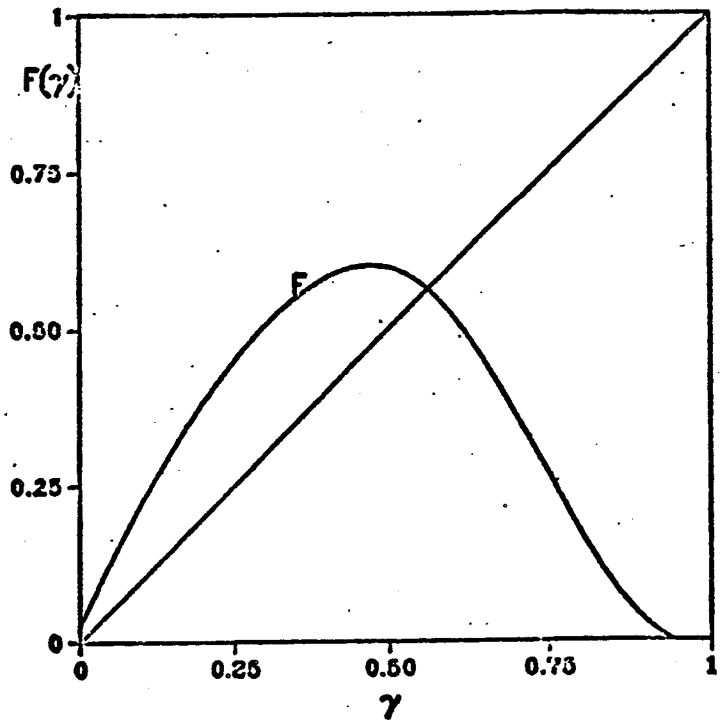
This point, γ^*, corresponds to a supply side equilibrium where $m^* = M(\gamma)$ is the symmetric Nash equilibrium strategy for each worker. Thus, there is at last one Nash equilibrium point.

The argument above only demonstrates the existence of an equilibrium. In fact the equilibrium may be at $\gamma = 0$. It can be shown that there is also at least one nontrivial equilibrium ($0 < \gamma < 1$). This is shown for the case where $C''(m) = 0$ although the result holds for the more general case, $C''(m) \geq 0$.

First, note that if $\gamma = 1$, then workers have no incentive to apply to any firm. But if no one applies at all, then equation (2.10) implies that $F(1) = 0$. If it can be shown that $\lim F(\gamma) > 0$ as $\gamma \to 0$ or that $\lim F'(\gamma) > 1$ as $\gamma \to 0$, then the result will have been shown. Since $F(\gamma)$ is below γ at unity, there must be a $0 < \gamma < 1$ where $F(\gamma) = \gamma$.

It can be shown that lim $F'(\gamma)$ as $\gamma \to 0$. Since lim $F(\gamma) \geq 0$ as $\gamma \to 0$, this is enough for the result.[9]

FIGURE 5.1 EQUILIBRIUM POINTS



[9] It is very difficult to determine how many equilibria there are since it is difficult to determine $F'(\gamma)$ at points other than $\gamma = 0$.

If there is only one equlibrium, then it will be stable. If there are more than one, then generically every other one will be stable. Assume that expectations about γ are adaptive, i.e.:

5.5a) $$\gamma_{t+1} = \gamma_t + \alpha[F(\gamma_t) - \gamma_t]$$

for some positive constant α. Then equilibria are stable if $F(\gamma)$ intersects γ from above. This occurs when $(\partial\Gamma/\partial m^*)(\partial M/\partial\gamma) < 1$.

Section 6: Welfare Results

It has already been noted that γ increases as m^* increases. But for any particular worker, γ is a function of all other workers' m^*'s, and any particular worker's m^* affects all other workers' γ's. Because the cost that a worker incurs in applying for a job includes only his search cost and no charge for the worker's effect on other workers' chances of getting a job when he submits extra applications, one might expect worker search at equilibrium to be inefficiently large. Actually, when there is unemployment insurance, a charge is implicitly levied for obtaining a job through the loss of unemployment insurance benefits.

This can be looked at more formally. The first order condition for each worker is described in equation (3.2). But this equation does not include a term for the effect of m^* on γ. On the other hand, if the workers were to form a coalition for one period, they would consider the effect of m^* on γ. Thus, the coalition's first order condition for the maximization problem described in equation (3.1) would be:

$$6.1)\qquad -C'(m) - [\gamma^m \ln\gamma + m\gamma^{m-1}((\partial\gamma/\gamma m)/(1 - (\partial\gamma/\partial m)(\partial m/\partial\gamma)))]^*$$

$$[\beta w/(1-\beta) - \beta V^*] = 0$$

which can be written as:

$$6.2)\qquad \partial V_I(m)/\partial m - m\gamma^{m-1}((\partial\gamma/\gamma m)/(1 - (\partial\gamma/\partial m)(\partial m/\partial\gamma)))^*$$

$$[\beta w/(1-\beta) - \beta V^*] = 0$$

where $\partial V_I(m)/\partial m$ is an individual's first order condition. Since the second term of equation (6.2) is positive and an individual would behave so that the first term was equal to zero, the supply side equilibrium cannot be Pareto optimal for workers, taking UI payments as given. A Pareto optimum would require $\partial V_I(m)/\partial m > 0$ which implies that the coalition's optimal choice of m is less than equilibrium m^*. Thus, there may be room for a social planner to intervene in order to decrease m^*. Such interventions could be UI payments or a tax on applications.

A social planner could maximize a representative worker's value of search by implementing a UI benefits program supported by a tax on the wages of workers once they were employed. The program could be built so that expected discounted UI benefit payments to each worker would be paid for by expected discounted wage tax revenues from that worker. Even though each worker's net balance would not equal zero, on average the program would be in discounted budget balance and the deviation from budget balance would be insignificant relative to the size of the program. The social planner would have to be aware of how each worker would react to both a UI benefit and a tax on wages. He would have to maximize a representative worker's value of search subject to the reaction function of workers to his program. A social planner's problem would be to solve:

6.3)
$$\max_{\tau_c,\tau_w,\,u} L = u - C(m) - \tau_c m + \beta w(1-\tau_w)/(1-\beta) - \gamma^m[\beta w(1-\tau_w)/(1-\beta) - \beta V^*]$$

$$\text{s.t. } -C'(m) - \tau_c m - \gamma^m \ln\gamma[\beta w(1-\tau_w)/(1-\beta) - \beta V^*] = 0$$

$$u - \tau_c m = [\beta w\tau_w(1-\gamma^m)]/(1-\beta)$$

where τ_w is a tax on wages, τ_c is a tax on applications, and u is a UI payment per period. The first constraint states that each individual maximizes his value of search using Nash expectations, and the second constraint states that expected discounted UI payments equal expected discounted tax revenues. The tax, τ_w, can be thought of as a steady state tax that started in the infinite past. If, instead, it is thought of as a tax that starts at some point in time, then those who are not searching initially should not be taxed; the government should borrow funds to pay for UI benefits and use later tax receipts to pay back the funds. In either of these ways the problem of an initial welfare transfer is avoided.

It is probably infeasible to have a tax on applications. There are too many ways that people actually apply for jobs, and many of them are difficult to monitor. Thus, τ_c is set equal to zero.

The optimal positive wage tax, τ_w, and UI payment, u, would be at a point where the derivative of the Lagrangian for equation (6.3) with respect to τ_w and u was equal to zero and the constraints were satisfied. The solution to this problem is too difficult to find analytically. But it can be shown that both τ_w and u should be positive.

In equation (6.3), substitute the government budget constraint into the Lagrangian for u with $\tau_c = 0$:

6.4) $$L = -C(m) + \beta w/(1-\beta) - \gamma^m[\beta w/(1-\beta) - \beta V^*].$$

Note that since there is a balanced budget, equation (6.4) contains no tax terms. Government intervention only affects welfare through its incentive effect on m. Now differentiate equation (6.4) with respect to τ_w at $\tau_w = 0$:

$$6.5)\qquad dL(0)/d\tau_w = [-C'(m^*) - \gamma^{m^*}\ln\gamma(\beta w/(1-\beta) - \beta V^*)]Dm^*/D\tau_w$$

$$- m^*\tau^{m^*-1}[\beta w/(1-\beta) - \beta V^*] *$$

$$((\partial\tau/\partial m^*)/(1 - (\partial\alpha/\partial m^*)(\partial m^*/\partial\alpha))(Dm^*/D\tau_w)$$

$$\text{where } Dm^*/D\tau_w = \partial m^*/\partial\tau_w + (\partial m^*/\partial u)(\partial u/\partial\tau_w) < 0$$

which is the total change in m^* at $\tau^w = 0$ when τ_w is changed, and m^* equals individuals' choice of m at $\tau_w = 0$. The first term of equation (6.5) is an individual's first order condition, and the second term is the effect of increases in m^* on γ^{m^*}. Since the first term equals zero:

$$6.6)\qquad dL(0)/d\tau_w = -m^*\gamma^{m^*-1}[\beta w/(1-\beta) - \beta V^*] *$$
$$((\partial\gamma/\partial m^*)/(1 - (\partial\gamma/\partial m^*)(\partial m^*/\partial\gamma)))(Dm^*/D\tau_w)$$

which is positive for stable equilibria (see footnote 9). The increase in L at $\tau_w = 0$ is the incremental reduction in not being offered a job by γ falling a little because m falls by $Dm^*/D\tau_w$. Thus welfare can be improved at $\tau_w = 0$ by increasing τ_w.

The government could alternately finance UI payments by a tax, τ_w, on employed workers high enough so that $u = x\tau_w w$ where x is the expected ratio of employed to unemployed workers. With this kind of budget balance, it is difficult to determine whether a positive tax is optimal since it depends upon steady state τ and x which in turn depend on the tax. A sufficient condition for the optimal tax to be positive is $x \geq \beta(1-\gamma^m)/(1-\beta)$.[10] It is easy to show that when x is relatively low and γ^m is relatively high (meaning there is a high unemployment rate), the optimal tax is not necessarily positive.

There is an externality in a market with this form of UI that is not present in the first form. In this form, when a worker accepts a job, he stops collecting UI benefits and starts paying taxes. The amount of taxes collected directly affects how much can be paid as UI benefits and vice versa because of the need for a balanced budget; the faster an average worker gets a job, the lower are taxes and the higher are UI benefits. Since any particular worker ignores his effect on the taxes other employed workers must pay and UI benefits other unemployed workers can receive, he is not searching fast enough. This externality is not present in the first form of a UI program because each worker's expected net receipts from the government are individually set equal to zero.

The first UI program analyzed in this paper only has social value because of its disincentive effect on search. Workers receive payments of u until they are employed, and then they pay premiums of $\tau_w w$ forever. The UI

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[10]This condition comes from each worker maximizing:

6.6a) $$V(m) = u - C(m) + \beta w(1-\tau_w)/(1-\beta) - \gamma^m[\beta w(1-\tau_w)/(1-\beta) - \beta V(m)]$$

and then the planner maximizing a representative worker's utility subject to a) each worker maximizing his own utility using Nash expectations and b) the budget constraint holding. Also, note that in equation (2.26) with $\tau_c = 0$, defining $x = \beta(1-\gamma^m)/(1-\beta)$ results in $u = w\tau_w x$.

program has a zero expected value. Workers are only ex ante better off with $\tau_w > 0$. Some workers, those who find jobs quickly, may be worse off than they would have been if τ_w had been zero.[11] To visualize how this program fits into the real world, we should think of all labor force participants as entering the market without jobs and also possibly later facing the risk of a spell of unemployment.

Section 7: Steady State Dynamics

There is one other externality which may exist but cannot easily be discussed within the framework of this model. When a worker searches a little harder he improves his chances of getting a job. Once he gets a job, he no longer searches. Therefore, his searching a little harder has the effect of reducing both the expected number of unemployed workers and the expected number of vacancies in the next period. To the extent this affects the next period's unemployed workers' chances of finding jobs, there is an interperiod externality.

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[11] If the planner preferred ex post equality over inequality, then the UI program would have some extra value. In the real world, social planners prefer equity. Since those who find jobs quickly benefit the least from the UI program, the program promotes ex post equality. But the UI program has no effect on ex ante equality since it has an ex ante expected value of zero. Thus, for this effect to be of any significance, the planner's ex ante social welfare function must be an expected value of a monotone function of a strictly concave combination of the workers' ex post utilities.

In order to discuss the importance of this externality with any more precision, both N, the total number of unemployed workers per period, and nB, the total number of vacancies per period, must become endogenous variables. Let E_N be the total, exogenous number of new entrants to the labor market each period and E_B be the total, exogenous number of new vacancies advertised. The economy can be described by four equations:

7.1) $$N_{t+1} = N_t\gamma_t^m + E_{Nt},$$

7.2) $$nB_{t+1} = nB_t\lambda_t + E_{Bt},$$

7.3) $$\gamma_t^m = \Gamma(N_t, B_t, m, n)^m,$$

7.4) $$\lambda_t = \Lambda(N_t, B_t, m, n)$$

where λ is the probability that a vacancy does not get filled. For example, if $n = 1$ and N_t and B_t are large, equations (7.3) and (7.4) are:

7.5) $$\gamma_t^m = [1 - (1 - e^{-\mu_t})/\mu_t]^m,$$

7.6) $$\lambda_t = 1 - (N_t/B_t)(1-\gamma_t^m).$$

where $\mu_t = mN_t/B_t$. The steady state levels of N and nB are:[12]

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[12]Only a steady state solution is meaningful here because m was derived under a steady state assumption. If the economy was not in a steady state, then m would not be constant.

7.7) $N = E_N/(1-\gamma^m)$,

$nB = E_B/(1-\lambda)$.

Note that equation (7.6) and (7.7) imply that steady states can only exist if $E_N = E_B$. This implies that the ratio, N/B, is not determined by the steady state equations, (7.7), alone. Instead, it can be shown that it is determined by the initial difference between N and B, by $E_N = E_B$, and by γ^m. Let $N - nB = A$. It can be shown that:

7.8) $N/nB = E_N/[E_N - A(1-\gamma^m)]$.

For simplicity, the case where n = 1 and where N and B approach infinity at the same rate is considered. In this case, the number of applications a firm receives approaches a Poisson distribution. So γ can be written as:

7.9)
$$\gamma = 1 - e^{-\mu} - (1 - e^{-\mu}(1+\mu))\mu$$
$$= 1 - (1 - e^{-\mu})/\mu$$

where $\mu = mN/B$. The probability of finding a job is $1-\gamma^m$. Substitution of the definition of γ from equation (7.9) and differentiation results in:

7.10) $d\gamma^m/dm = \gamma^m[\ln\gamma - (\mu(1-\gamma) + \gamma)/m] < 0$.

Increases in m increase workers chances of getting jobs even though γ rises.

The steady state effects of a small increase in m on γ can be determined. First of all, when m increases holding the number of workers and vacancies fixed, γ increases because of the mechanics discussed earlier (differentiate equation (7.9) with respect to m). But γ^m decreases which decreases the steady state level of N and B which has another effect on γ:

$$7.11)\quad d\gamma/dm = \partial\gamma/\partial m + [(\partial\gamma/\partial(N/B))(\partial(N/B)/\partial\gamma^m)(\partial\gamma^m/\partial m)/$$

$$(1 - (\partial(N/B)/\partial\gamma^m)(\partial\gamma^m/\partial(N/B)))].$$ [13]

$\partial g/\partial m$ is positive, $\partial\gamma/\partial(N/B)$ is positive, and $\partial\gamma^m/\partial m$ is negative. Only the sign of $\partial(N/B)/\partial\gamma^m$ is not determined.

If N = B, then the probability of a worker getting a job equals the probability of a firm filling a vacancy. Any reduction in N will result in a reduction in B of the same size. Changes in γ^m have no effect on the ratio, N/B; $\partial(N/B)/\partial\gamma^m = 0$. Therefore, when N = B, when the only unemployment is search unemployment, there is no interperiod effect. All of the results with exogenous levels of N and B still hold. An economy with only search unemployment is not Pareto efficient.

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[13]This equation is derived as follows: Write γ as a function of m and N/B using equation (7.9). Write N/B as a function of γ^m using equation (7.8). It can be shown that:

$$7.11a)\quad \partial(N/B)/\partial\gamma^m = (E_N/B)(1 - N/B)/(1-\gamma^m)$$

which has the same sign as 1 - N/B. Also, write γ^m as a function of m and N/B using equation (7.9). The result can be derived by differentiating each equation and then using Cramer's rule to solve the set of linear equations.

If N < B, then an increase in both N and B will increase N/B, making $\partial(N/B)/\partial\gamma^m$ positive. Whether the interperiod effect is negative or positive depends upon whether $(\partial\gamma^m/\partial(N/B))(\partial(N/B)/\partial\gamma^m)$ is greater than or less than one. If it is less than one, the interperiod effect is negative. In this case, it may even be possible for $d\gamma/dm$ to be negative. Otherwise the interperiod effect is positive. If N > B, then increases in N and B decrease N/B, increasing the size of the externality. The cases where $N \geq B$ are the most interesting cases to consider for both theoretical and empirical reasons. In these cases there is room for a UI program; the greater is N/B, the more beneficial is the program.

Section 8: Conclusions

A supply side equilibrium search model with no distribution of wage offers is presented in this paper. Workers search for job openings rather than high offers. Stern (1985), has empirically shown that this type of search is more prevalent than search for high offers. The existence of competition among workers for a limited number of job openings leads to an inefficiently high amount of search. However, an unemployment insurance program set up in the proper way can induce each worker to choose the socially optimal search intensity.

Many authors have discussed the effects of unemployment insurance on the behavior of workers looking for a job. Theoretical papers include Mortensen (1970) and Lippmann and McCall (1979). Empirical papers include Barron and Gilley (1979), Clark and Summers (1982), Classen (1977), Fields (1977), Hills (1982) and Holen (1977). The overwhelming consensus is that unemployment insurance decreases search intensity and increases the average spell of unemployment. The same result occurs in this paper. However, contrary to most other papers, this is found to have some positive value.

The proposed unemployment insurance program was quite different than the one that presently exisits in most western nations. In fact, the system that presently exisits was shown to possibly induce workers to decrease their search intensity too much. In the real world, unemployment insurance has other roles besides inducing workers to search optimally. Concerns for equity play a major role in the design of unemployment insurance. Thus, the optimal program must trade off equity against efficiency according to the preferences of society.

There is no mention of the demand side of the labor market. An analysis of the demand side is beyond the scope of this paper. See Stern (1985), Chapter IV for such a discussion. However, it should be noted that while the proposed unemployment insurance program improves the ex ante welfare of each worker, it potentially reduces the profits of the firms in the market or consumers buying the product being produced by the firms. This problem is common to the rivalry literature as well; no one has included the welfare of the product to be produced once R and D has been completed. A general equilibrium model of search with competition is still needed.

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