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THE ARCHITECTURE OF ECONOMIC SYSTEMS:

HIERARCHIES AND POLYARCHIES

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ABSTRACT

We present some new ways of looking at economic systems. The aspect of human behavior which we emphasize is that individuals' judgments entail errors (they sometimes reject good projects and accept bad projects). The aspect of an economic system on which we focus is its architecture; that is, how the decision making units are organized together within a system, who gathers what information, and who communicates what with whom. The architecture of a system affects its performance not only because it influences the nature of errors which individuals make within the system, but also because it has a critical effect on the aggregation of individuals' errors. We analyze and compare the performance of two polar architectures, with decentralized (polyarchical) versus centralized (hierarchical) decision making authorities. Also, we discuss several extensions of our analysis.

THE ARCHITECTURE OF ECONOMIC SYSTEMS:

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By Raaj Kumar Sah and Joseph E. Stiglitz*

There is a widespread belief that the performance of an economic system is influenced by its internal organization. Yet, there is very little in traditional economic analysis which investigates such a relationship.¹ In this paper, we present some new ways of looking at economic organizations. Although we motivate our discussion in the context of a comparison of economies, our analysis has implications for the internal structure of large business organizations as well.

The thesis of this paper is that central to an understanding of the performance of an economic system is an understanding of its architecture. The architecture (like that of a computer or electrical system) describes, among other things, how the constituent decision making units are arranged together in a system, how the decision making authority is distributed within a system, who gathers what information, and who communicates what with whom.

The feature of human behavior which plays a central role in our analysis is that all decision makers make errors of judgment. For concreteness we focus on decisions which involve accepting or rejecting certain projects. Individuals (or the constituents of economic systems) make these decisions based on the information available to them, and errors in judgment arise since information is almost never complete. As a result, some projects which get accepted should have been rejected, and some projects which are rejected should have been accepted. Using an analogy from the classical

theory of statistical inference, these errors correspond to Type-II and Type-I errors.

The two specific architectures which we study in this paper are what we call polyarchies and hierarchies. We think of a polyarchy as a system in which there are several (and possibly competing) decision makers who can undertake projects (or ideas) independently of one another. In contrast, decision making authority is more concentrated in a hierarchy in the sense that only a few individuals (or, only one individual) can undertake projects while others provide support in decision making. Clearly, these architectures are suggestive of a market-oriented economy and a bureaucracy-oriented economy.²

The issue of the relative merits of these two polar forms of economic organization has long been debated, but there is a surprising lacuna of analytical research. Of the standard arguments in favor of decentralization, there are three about which we will have nothing to say in the present paper: the alleged link between economic and political decentralization (and the corresponding view that polyarchical forms of economic organization are a necessary concomitant of a democratic society); the view that a decentralized economic structure provides an efficient method for solving a once-and-for-all allocation problem;³ and the view that better incentive mechanisms can often be designed within decentralized economic systems.⁴

Our analysis will, however, cast light on several other aspects of the debate concerning the relative merits of polyarchies versus hierarchies: advocates of polyarchies point out that a good project has many opportunities (chances) of being accepted in their system, while critics contend that polyarchies fail to provide adequate checks against incompetent decision making. Critics of hierarchical structures claim that there is a high

cost to providing these checks; there are direct costs of additional evaluations, and there are indirect costs because good projects get rejected in the process of ensuring that bad projects do not get undertaken. But then the critics of polyarchies object to the extra costs involved in duplicative evaluation of the same project by different individuals (or organizations) and, more generally, to the consequences of failing to coordinate in the presence of externalities. Finally, advocates of polyarchies point to its virtues in economies of communication and its stimulative effects on innovation.⁵ There is a grain of truth in each of these views. The question is, under what circumstances does a polyarchy perform better, or worse, than a hierarchy. The model we present in this paper provides a framework for answering this.

A key consequence of how individuals are arranged together is that the aggregation of their errors is different under different economic systems. The aggregation of errors, in turn, influences the performance of a system. For example, in a market economy, if one firm rejects a profitable idea (say, for a new product), then there is a possibility that some other firm might accept it. In contrast, if a single bureau makes such decisions and this bureau rejects the idea, then the idea must remain unused. The same, however, is also true for those ideas which are unprofitable. As a result, one would expect a greater incidence of Type-II error in a polyarchy, and a greater incidence of Type-I error in a hierarchy.

The architecture of a system influences not only the aggregation of individuals' errors but also the nature of errors which different individuals make. We posit that individuals use rational decision rules to accept or reject projects, based on the information available to them. The information available to a person consists of what he collects directly and what

he receives from others, each entailing costs and benefits. In general, therefore, the information available to individuals, and the (imperfect) decision rules based on them which individuals employ, will differ across economic systems.

The costs of acquiring and communicating information (leading to misjudgments by individuals) are thus central features of the technology underlying our analysis. These costs include the direct costs (time and resources) and the indirect costs which result from the inevitable contamination that occurs in the process of information communication. Communication, like decision making, is always imperfect. No individual ever fully communicates perfectly what he knows to another.

Another important feature is the limited capacities of individuals to gather, absorb, and process information within a limited amount of time. This is why organizations, groups of individuals, may be able to do more (make better decisions) than any single individual. But the fact that communication is costly and imperfect means that an organization with two individuals, each of whom can process a given amount of information, in say a month, is not the same as a single individual who has the capacity of processing twice that amount of information within the same time period. It also means that how people are arranged within an organization, who they communicate with under what circumstances, has important implications for the performance of the organization.

The paper is organized as follows. We begin, in Section I, by presenting a simple model of the decision structure within a polyarchy and a hierarchy. In Section II, we assume that the nature of individuals' errors and the mix of available projects is exogenously specified, and analyze how changes in these exogenous features influence the performance of the

two systems. We also compare the performance of the two systems assuming that the exogenous parameters faced by the two systems are similar. The difference in the performance of the two systems is thus attributable to their differences in architecture.

In Section III, we discuss the collection and processing of information in the two systems. In particular, we show how (Bayesian) screening rules are determined which, in turn, provide an endogenous characterization of the nature of errors made by individuals in the two systems. Several extensions of our approach are outlined in Section IV. We comment on certain organizational forms other than hierarchies and polyarchies, and also on those forms in which the decisions to undertake projects are based on vetoes rather than on acceptances. In addition, we discuss certain economic factors, other than the lack of complete information, which may affect the performance of organizations. Concluding remarks are presented at the end of the paper.

I. THE MODEL

In the following model, a polyarchy consists of two firms, and a hierarchy consists of two bureaus. The task of a bureau or a firm is to screen projects. Each project has a scalar (net) benefit, which can be positive, negative, or zero. A screen (that is, a bureau or a firm) evaluates every project and accepts or rejects the project.⁶

 Insert Figures 1 and 2 about here

The decision process in a polyarchy and a hierarchy are depicted in Figure 1 and 2 respectively. In a polyarchy, the two firms screen the

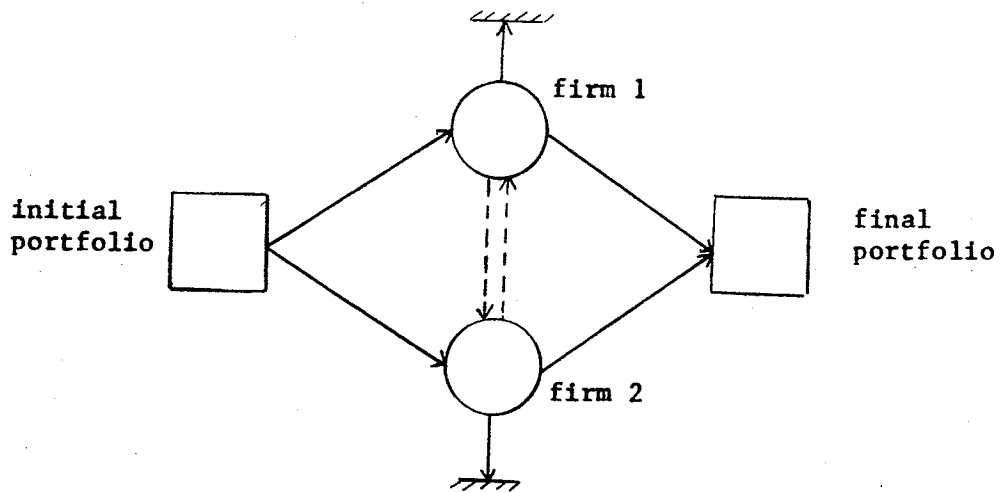


Figure 1: Polyarchy

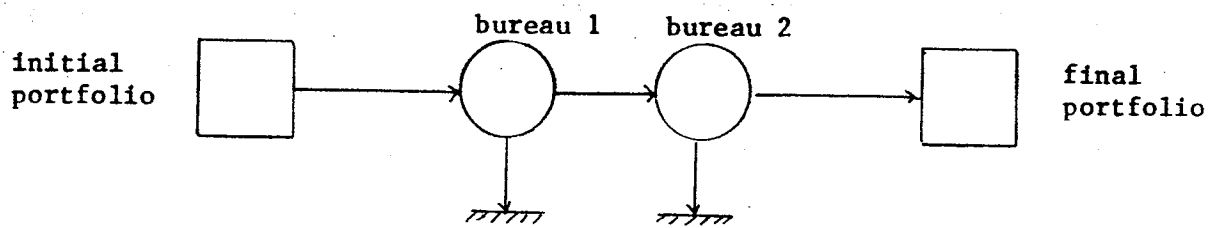


Figure 2: Hierarchy

projects independently. For specificity, one may think of projects arriving randomly (with probability one-half) at one of the two firms. If a particular project is accepted by a firm, then it is no longer available to the other firm. If the project is rejected, then it goes to the other firm where, once again, it can be accepted or rejected. Neither firm screens the same project twice, so that a project can not cycle back and forth between firms.⁷ The portfolio of projects selected in a polyarchy therefore consists of the projects accepted separately by each of the two firms.

In contrast, in a hierarchy, all projects are first evaluated by the lower bureau (bureau 1); those which are accepted are forwarded to the higher bureau (bureau 2) and others are discarded. The projects selected by the system then are those which are selected by the higher bureau. Drawing an analogy from the design of relay circuits, the screens are placed in series in a hierarchy whereas they are placed in parallel in a polyarchy.

The superscripts P and H represent a polyarchy and a hierarchy respectively. For brevity, we use the superscript s, where s = P or H. x denotes the expected net benefit (profit) from a project,⁸ and $p^s(x)$ denotes the probability that this project will pass through a screen in the system. We refer to $p^s(x)$ as the screening function and assume, at present, that the screening functions corresponding to the two screens within a system are identical. Then the probabilities that the project x will be accepted in the system s, denoted by $f^s(x)$, are given by⁹

$$(1) \quad f^P \equiv p^P(2 - p^P)$$

$$(2) \quad f^H \equiv (p^H)^2.$$

We refer to the portfolio of projects available to an economic system

as the initial portfolio and the portfolio that it selects as the final portfolio. For system s , N^s denotes the number of projects in the initial portfolio, and $g^s(x)$ denotes its probability density function. The initial portfolio contains both profitable and unprofitable projects, that is, there are projects with positive as well as negative x 's.¹⁰

The final portfolios represent how the economic systems have performed. These can be characterized using many different summary statistics. For example, the fraction of initial projects selected by the systems, denoted by n^s , is given by

$$(3) \quad n^s = E[f^s],$$

in which E denotes the expectation operator; the expectation is calculated with respect to the density function for the system. On the other hand, if we are interested in studying the profitability of alternative systems, then an important statistic is the (expected) profit. We denote this by Y^s , which is¹¹

$$(4) \quad Y^s = N^s E[xf^s].$$

Screening function: The screening function $p(x)$ denotes the probabilities that projects with different levels of profit have of being accepted by a bureau or a firm. It is a reduced form representation of the error making properties of a screen. It can take any form, provided

$$(5) \quad 1 \geq p(x) \geq 0,$$

for all x , and the strict inequalities hold for at least some x .¹²

Two properties of the screening function are of special interest. The

first is its slope, $p_x(x)$.¹³ If $p_x(x)$ is positive then a project with higher profit has a higher local probability of being accepted by a screen. If $p_x(x) = 0$, then the screening is indiscriminate, since it does not distinguish between a better and a worse project. While it is possible that the sign of p_x may change over the range of projects, we consider here only those screens which have at least some, but not complete, discriminating ability throughout the range of projects. That is, $p_x > 0$, for all x . Further, if p and p^1 represent two screens, and if $p_x(z) > p_x^1(z)$, then we refer to the former screen as locally more discriminating at $x = z$. The second important property of screens is the level of $p(x)$. If $p(z) > p^1(z)$ then we call the former screen locally slacker, and the latter locally tighter, at $x = z$.

In some cases examined in this paper, we employ linear screening functions. The corresponding conclusions hold approximately for those screens for which the curvature of $p(x)$ is small. In such cases, $p(x)$ is expressed as

$$(6) \quad p(x) = \bar{p} + p_x(x - \mu),$$

where $\bar{p} = p(\mu)$, and μ is the mean of the initial portfolio, that is, $\mu = E[x]$. Thus, $\bar{p}$ is the probability that the average project will pass through a screen. Clearly, a higher $\bar{p}$ and p_x imply globally higher slackness and discriminating capability.

If screening were perfect, then all projects with $x > 0$ would be accepted and those with $x < 0$ would be rejected; that is, $p(x) = 1$ if $x > 0$, and $p(x) = 0$ if $x \leq 0$. In this case, the performances of a polyarchy and a hierarchy are identical in every respect, as can be easily verified from (1) and (2). Thus, the architecture of economic systems might

cease to be a relevant issue, if one believes that human decision making is absolutely faultless. Even a casual observation of actual functioning of business and public organizations, however, makes it abundantly clear that errors of judgment are an integral part of human decision making. This realization not only makes it necessary to recast the traditional view of organizations, but it also provides a potential basis for understanding certain hitherto unrecognized differences among different types of organizational systems.

II. PORTFOLIOS SELECTED IN ALTERNATIVE SYSTEMS

We investigate two questions in this section. First, how is the portfolio selected in each of the two systems affected by the exogenous parameters representing the initial portfolios and the characteristics of the screens? Second, what is the relative performance of the final portfolios in the two systems, and how is this influenced by the exogenous parameters? To examine the second question, we assume that the two systems have the same initial portfolios and screening functions; the difference in their performance is thus solely attributable to the difference in their architecture. This assumption is not required for examining the first question since it does not involve any comparison across the systems. The two statistics for evaluating the final portfolio which we employ are the proportion of original projects selected, and the profit from the final portfolio.

A. The Size of Final Portfolios

The proportion of the initial portfolio selected by the two systems, n^S , is given in (3). Denoting the difference in these proportions by Δn , we find that

$$(7) \quad \Delta n = n^P - n^H > 0, \text{ since}$$

$$(8) \quad f^P - f^H = 2p(x)\{1 - p(x)\} \geq 0,$$

from (5), and it is strictly positive for some x . Therefore: A polyarchy selects a larger proportion of initial projects than a hierarchy.

The reason behind this result is intuitive. Consider a hypothetical situation in which the second firm in a polyarchy does not exist, and the higher bureau in a hierarchy does not exist. The proportion of projects accepted in the two systems, would then be the same, namely, $E[p(x)]$. Since the second firm accepts at least some projects, and since the higher bureau rejects at least some projects, the actual proportion of projects accepted in a polyarchy must exceed that in a hierarchy. It is also obvious that this result holds for good as well as bad projects. Further, the result does not depend on how one defines good versus bad projects, provided there is some probability that a screen will accept at least some good and some bad projects.

Thus: A polyarchy accepts a larger proportion of good as well as bad projects compared to a hierarchy, no matter how one defines good and bad projects. Therefore, the incidence of Type-I error is relatively higher in a hierarchy, whereas the incidence of Type-II error is relatively higher in a polyarchy.

The above result suggests that there may be circumstances in which a polyarchy performs better than a hierarchy (when it is more important to avoid Type-I errors) and other circumstances in which a hierarchy performs better than a polyarchy (when it is important to avoid Type-II errors). We attempt to identify each of these situations in the next subsection.

To determine the impact of initial portfolios on the size of final

portfolios, note from (1) and (2) that $f^S(x)$ is increasing in x . Additionally, $f^P(x)$ is concave and $f^H(x)$ is convex in x , if the screening function is linear. Therefore, the standard properties of statistical dominance (under an assumption that the end points of the projects' distribution are fixed) yield the following results.

A worsening in the initial portfolio in the sense of first-order stochastic dominance leads to a smaller proportion of initial projects being selected in both systems. With linear screening function, a mean preserving spread in the initial portfolio leads to a smaller proportion of initial projects being selected in a polyarchy, and a larger proportion being selected in a hierarchy.

Insert Figure 3 about here

These results can be seen in Figure 3. In this figure, f^P and f^H are concave and convex in x , since p is linear. n^S is the area above the x -axis bounded by the product of f^S and g . Naturally, this area corresponding to f^P is larger than that to f^H ; and this area enlarges, for both a polyarchy and a hierarchy, if the density weight $g(x)$ shifts from lower x to higher x . Also, if the density weight shifts from the mean to the two sides, due to a mean preserving spread, then the area representing n^S decreases in a polyarchy and it increases in a hierarchy.

Explicit expressions for n^S are derived in Appendix I(a), where the screening function is linear. From these expressions, one can ascertain how n^S is influenced by changes in the screening function. We find that: With linear screening function, a higher slackness in screening raises the proportion of projects selected in both systems.¹⁴ And, a higher discrimin-

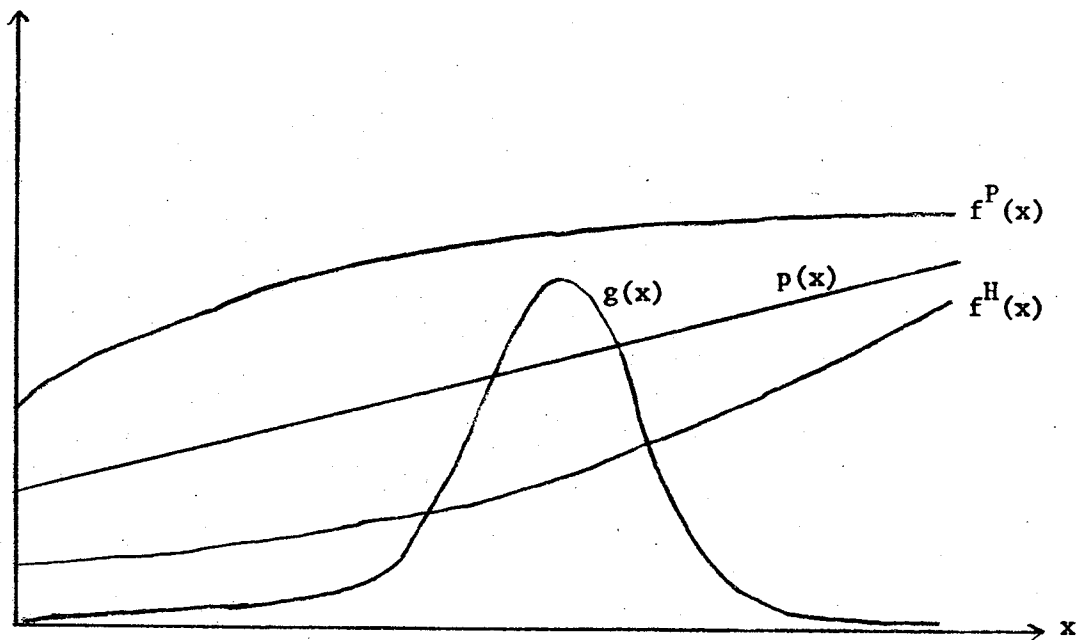


Figure 3: Probabilities of Acceptance in Alternative Systems

ating ability in screening lowers the proportion selected in a polyarchy, whereas it raises the proportion selected in a hierarchy.

B. Profit in Alternative Systems

Two Types of Projects: Consider an initial portfolio consisting of two types of projects, good and bad, with respective net profits z_1 and $-z_2$, where z 's are positive. The respective probabilities of approval by a screen are denoted by parameters p_1 and p_2 ; that is, $p_1 = p(z_1)$, and $p_2 = p(-z_2)$. The initial portfolio contains a fraction α of good projects. $\Delta Y = Y^P - Y^H$ denotes the difference in the profit levels of the two systems. Then from (4)¹⁵

$$(9) \quad \Delta Y = 2[p_1(1 - p_1)\alpha z_1 - p_2(1 - p_2)(1 - \alpha)z_2] .$$

An improvement in the initial portfolio in the present model is represented by a larger α or a larger z_1/z_2 . From (9), therefore: A better initial portfolio implies that the relative performance of a polyarchy, compared to a hierarchy, is better.¹⁶

The above expression also allows us to demarcate the parameters' space into two regions: one in which a polyarchy has a higher profit than a hierarchy, and the other in which the reverse holds. To see this in its simplest form, first assume that $z_1 = z_2 = z$; that is, a good and a bad project have symmetric gain and loss. Then the parameters which determine the sign of (9) are p_1 , p_2 , and α .

Insert Figure 4 about here

Figure 4 summarizes the results. We are concerned only with the area

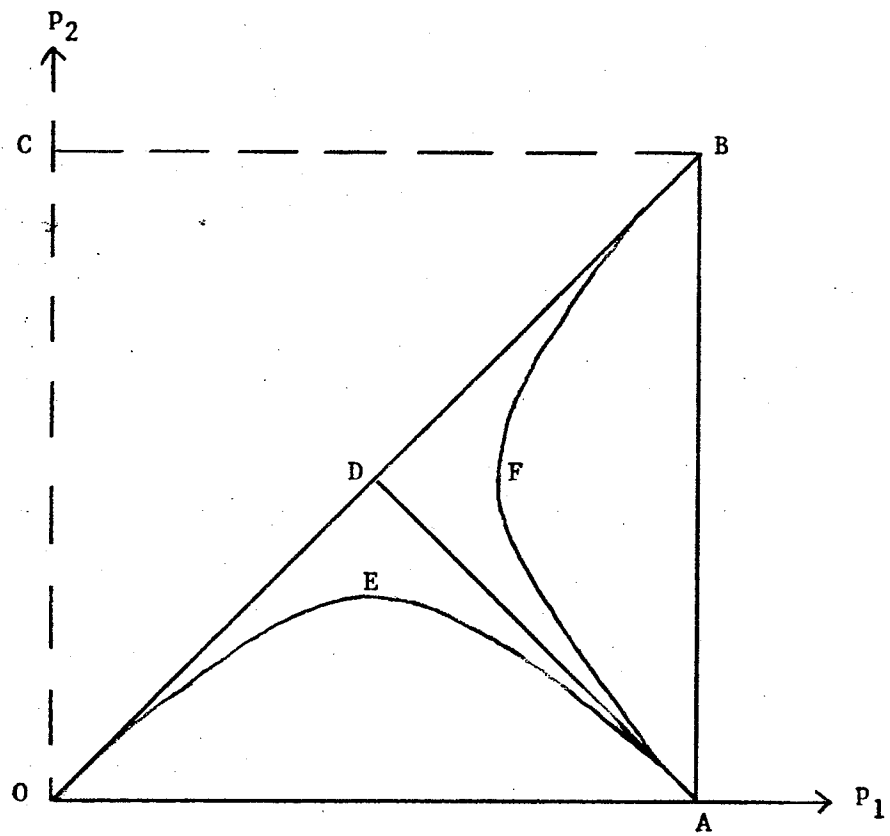


Figure 4: Comparison of a Polyarchy and a Hierarchy

below the 45-degree line, since screens have some discriminating capability; that is, $p_1 > p_2$. Now consider, for a moment, the case in which the initial portfolio contains good and bad projects in equal proportions, that is, $\alpha = 1/2$. Then a polyarchy has a higher profit in the region ODA, while the reverse holds in the region ADB. That is, polyarchy has a higher profit if

$$(10) \quad 1 - p_1 > p_2,$$

and the reverse holds otherwise.

This result has a simple explanation. Recall that $(1 - p_1)$ is a screen's Type-I error, that is, the probability of rejecting a good project; and p_2 is a screen's Type-II error, that is, the probability of accepting a bad project. Now, if a screen is more likely to reject a good project than to accept a bad project, that is if (10) holds, then it must be the case that a polyarchy (which gives a second chance to the rejected projects) would do better.

If the initial portfolio contains a smaller proportion of good projects, that is if $\alpha < 1/2$, then from (9), we find that the parameter space is separated by a hyperbola like OEA, which is inside the region ODA. A polyarchy has a higher profit within the region OEA, and the reverse holds outside of it. The region OEA shrinks if the proportion of good projects in the initial portfolio is smaller, and it coincides with the line OA if $\alpha \rightarrow 0$. The opposite case, in which the initial portfolio contains a greater proportion of good project has a parallel implication. A polyarchy then has a higher profit outside of the region AFB, and the reverse holds inside it. Not surprisingly, the two regions OEA and AFB coincide with the triangles ODA and ADB respectively, as α tends to one-half.

The same figure allows us to visualize the case in which good and bad projects have different gain and loss, that is when $z_1 \neq z_2$. As we have already noted z_1/z_2 plays a role analogous to α . Specifically, the line AD separates the two relevant regions in Figure 4 if $\alpha = z_2/(z_1 + z_2)$. A hyperbola like OEA is the boundary if either α or z_1/z_2 is smaller than what would satisfy the last equation. If α or z_1/z_2 is larger, on the other hand, then a hyperbola like AFB is the relevant boundary.¹⁷

There is another way in which the results can be seen intuitively. Suppose that we subjected each project to two screenings.¹⁸ Clearly, if both screens indicated that the project was bad, the project should be rejected, and if both indicated that the project was good, it should be accepted. A trade off arises in those cases where there is a mixed review. Whether a project with a mixed review should be undertaken depends on the profit from such a project. The probability of a good project getting a mixed review is $2p_1(1 - p_1)$, while the probability of a bad project getting a mixed review is $2p_2(1 - p_2)$. Hence the expected profit from projects with mixed reviews is the same as (9).

Now, if it turns out that the expression (9) is positive, it means that projects with mixed reviews should be accepted; this is precisely what polyarchy ensures. Similarly, if it turns out that (9) is negative, it means that the projects with mixed reviews should be rejected, and this is precisely what hierarchy ensures.

A General Project Portfolio: Before concluding this section, we briefly consider an initial portfolio consisting of a continuum of projects. Recall from (8) that $f^P - f^H = 2p(1 - p)$. Then, $\Delta Y = 2E[\psi]$, where $\psi = xp(1 - p)$. To determine the effect of a change in the initial portfolio (once again,

in the sense of first and second order stochastic dominance), we need the following derivatives of ψ

$$(11) \quad \psi_x = p(1 - p) + x(1 - 2p)p_x, \text{ and}$$

$$(12) \quad \psi_{xx} = 2(1 - 2p)p_x - 2xp_x^2 + x(1 - 2p)p_{xx}.$$

It is apparent that the above expressions can be positive or negative and, thus, the effect of a change in the initial portfolio is ambiguous in general.¹⁹ However, if the parameter values are restricted within certain ranges, then it is possible to predict certain effects. The simplest example is when the projects have values close to zero (that is, good projects have small positive profits and bad projects have small negative profits). Then from (11), ψ is increasing in x and, thus: A worsening in the initial portfolio in the sense of first-order stochastic dominance lowers the relative performance of a polyarchy. Also, if $p(x) < 1/2$ then, from (12), ψ is convex in x . In this case: A mean preserving spread in the initial portfolio improves the relative performance of a polyarchy.

Another example of a restricted set of exogenous parameters is when the screening function is linear. The corresponding expression for ΔY is presented in Appendix I(b). The relevant aspects of the initial portfolio are now represented by its mean, variance and skewness.²⁰

We find that when the initial portfolio is symmetric and has zero mean, a polyarchy performs better or worse than a hierarchy depending simply on whether $\bar{p}$ is less than or more than one-half, that is, whether the screening is tight or slack. A higher mean or a greater negative skewness, on the other hand, improves the relative performance of a polyarchy. The effect of a change in the mean is, of course, what we would have expected from

our earlier discussion of the case in which there are two types of projects. Further, if the initial portfolio has nonpositive skewness and mean,²¹ and if the screening is tight (that is, if $\bar{p} < 1/2$) then:

(i) a polyarchy's relative performance improves if the variance in the initial portfolio is higher and, at a sufficiently high variance, a polyarchy's profit is greater than that of a hierarchy, and (ii) a polyarchy's relative performance improves if the screens have greater discriminating ability.

III. SCREENING RULES AND THE ACQUISITION OF INFORMATION

In the preceding section, the performance of decision makers (screens) was represented by exogenous screening functions, $p(x)$. In fact, these functions may be endogenous, and may differ markedly across organizations. It is thus conceptually possible that some of our earlier results may be altered when we take this endogeneity into account. In this section, we show how optimal screening rules can be derived for decision making with imperfect information concerning the value of projects. Other possible determinants of the performance of individuals are discussed later.

We posit that the information on projects is contaminated by random errors, so that a perfect inference is not always possible. A firm or a bureau collects information on projects, and the nature and the accuracy of information collected is influenced by the costs and the technologies of information acquisition and communication. The projects are then selected based on optimal (expected profit maximizing) rules concerning when to accept or reject a project. Though in some simple situations, it is clearly possible that the screening rules used within the two systems are identical;²²

in general, we would expect the screening rules to differ across the systems. The following comments clarify some of the sources of such differences.

First, as remarked earlier, an important property of polyarchies is the independence (in decision making) of their constituent units. Firms determine what is optimal for them (for example, what information to collect and what screening rules to use) independently of one another. This independence, which we think is a central feature of market-like systems, also has its costs; in the sense that (in contrast to a hierarchy) 'no one' is maximizing the system's aggregate profit in a polyarchy.

Second, the architecture of the economic system itself conveys some information to its constituents, which they will use in setting decision rules. For example, consider a polyarchy in which the firms do not share any information with one another (either because it is prohibited, or because it is too expensive). Even in this case, each firm knows that some of the projects it receives are those rejected by the other firm and, consequently, the portfolio of projects faced by a firm is not an exact replica of the initial portfolio, but it has been modified by the other firm. This implicit information will certainly be used by the firms.

Third, the screening rules are influenced by the nature of information sharing among the constituents of a system. For example, if firms in a polyarchy do not share any information with one another, then a firm can not distinguish between the projects which are being evaluated for the first time from those projects which were rejected by the other firm.²³ A firm, therefore, must use the same screening rule for all projects that it receives. Now, consider a simple form of information sharing in which firms label rejected projects. In this case, a firm can use different

screening rules for the two categories of projects. At the same time, however, firms must also pay the cost of labelling.

Similarly, in a hierarchy, if the lower bureau provides its observation to the higher bureau (at some cost), then the latter would screen projects using the observations made by both bureaus. If there is no information sharing between the bureaus, the upper bureau's screening will be based on its own observation, and on the knowledge of how the portfolio has been modified by the lower bureau, but not on what value was observed by the lower bureau for a particular project. Note also that a labelling of rejected projects has no economic value in a hierarchy since these projects are not reevaluated.

Though the extent of information sharing is likely to vary under different circumstances, it is clear from our earlier discussion that there is never perfect and complete information communication. The assumption of limited communication plays a critical role in the subsequent analysis; we model here the polar case in which the firms in a polyarchy can not communicate at all, and the bureaus in a hierarchy communicate only binary signals (whether they think a project is good or bad); they cannot communicate their actual information concerning the characteristics of projects.²⁴

We assume that a project evaluator observes the mean of the project with an error. He observes

$$(13) \quad y = x + \theta$$

where $E[\theta] = 0$, and the distribution of θ is independent across projects and observations. Project evaluators use reservation levels for screening; a project is accepted if its observed profit is above the reservation level, R , and it is rejected otherwise.²⁵ Denote the distribution function of

θ by $L(\theta)$. The screening function, then, is given by

$$(14) \quad p(x, R) \equiv \text{Prob}[y > R] = 1 - L(R - x) .$$

The above expression yields $p_x > 0$, and $p_R < 0$.²⁶ In other words, the probability that a project is accepted by a screen is increasing in the quality of the project, and it is decreasing in the reservation level.

This is what we would normally expect.

In a polyarchy, denote the two firms by superscripts i and j . If R^i is the reservation level for firm i , then $p^i \equiv p(x, R^i)$ denotes the corresponding screening function. For firm i , then, the probability that a given project will arrive to be evaluated for the first time is $1/2$, and the probability that the same project will arrive after being rejected from firm j is $(1 - p^j)/2$. Since these two kinds of arrivals can not be distinguished, the probability that a given project will arrive at firm i is $\frac{1}{2}(2 - p^j)$, and the probability that it will be selected is $\frac{1}{2}p^i(2 - p^j)$. Thus, the profit of firm i is

$$(15) \quad Y^{iP} = \frac{1}{2}E[xp^i(2 - p^j)] , \text{ and}$$

$$(16) \quad Y^P = Y^{1P} + Y^{2P} .$$

From (15), it is clear that there is an externality associated with the reservation levels in a polyarchy: the more projects a firm rejects, the poorer the mix of projects reviewed by the other firm.²⁷ In the analysis below, we restrict ourselves to the symmetric Nash optimum for the two firms in a polyarchy; the firm i maximizes (15) with respect to R^i , taking R^j as given. The resulting optimal reservation level is denoted by R^P .

In a hierarchy, R^i denotes the reservation level for the bureau i . The profit is

$$(17) \quad Y^H = E[xp^1 p^2] .$$

The optimal reservation levels are denoted by R^{iH} . We can compare the screening policies of the two systems under assumptions that the optimal reservation levels are internal for both systems, and that the optimal reservation level is the same for the two bureaus in a hierarchy²⁸ That is, $R^{1H} = R^{2H} = R^H$. In this case, R^P is characterized by the first order condition of (15); that is, by

$$(18) \quad 2E[xp_R] = E[xpp_R] .$$

It is easily verified that both sides of the above expression are negative numbers.²⁹ Thus, $\partial Y^H / \partial R = 2E[xpp_R]$, evaluated at R^P , is negative. Now, $\partial Y^H / \partial R = 0$ at R^H , and Y^H is assumed strictly concave in R . It follows that $R^P > R^H$. Thus: A polyarchy is more conservative in screening than a hierarchy.

This result should not be surprising: while in a hierarchy, the lower bureau knows that any decision it makes will be rechecked at the upper bureau; and the upper bureau knows that all projects it receives have been checked at the lower bureau; in a polyarchy, each firm knows that its decision will not be rechecked; and to make matters worse, it knows that the set of projects which it is examining includes many that have already been examined elsewhere, and have been rejected.³⁰

Since a polyarchy uses more conservative screening rules than a hierarchy, we can no longer be sure that a polyarchy accepts more good projects, or that a hierarchy rejects more bad projects. This may, in turn, have a

critical effect on the relative performance of the two systems, as we shall see below.

Two Types of Projects: Denote a screen's observations on good and bad projects by y_1 and y_2 . A useful property of the reservation levels is seen as follows. Suppose for a moment that the reservation level in a screen is set at the highest observation that a bad project can yield; that is $R = \max y_2$. Then, the bad projects are completely blocked, although many good projects might be rejected as a consequence. It is obvious that there is no gain in setting the reservation level at a level higher than $\max y_2$, since by doing so one loses additional good projects, without affecting bad projects (which are not being selected in any case). A parallel argument shows that there is no gain in setting R below the lowest observation that a good project can yield, at which level all good projects are accepted.³¹ Thus

$$(19) \quad \max y_2 \geq R \geq \min y_1$$

For brevity, we refer to the upper and lower limit in (19) as the highest and the lowest reservation levels.

In Appendix II, we have derived explicit solutions for symmetric projects, that is, $z_1 = z_2 = z$, when the observation errors are uniformly distributed with mean zero. The qualitative results are summarized below:

(i) A worse initial portfolio corresponds to a tighter screening in both systems. This makes intuitive sense since a tighter screening implies a lower probability of acceptance for both good and bad projects, and this is more desirable if there is a smaller proportion of good projects in the initial portfolio.³²

(ii) A polyarchy has a higher (lower) profit than a hierarchy if the proportion of good projects in the initial portfolio is less (more) than one-half. Obviously, if one hypothesizes that unprofitable ideas typically outnumber the profitable ones in a portfolio, then the present example suggests that a polyarchy is a superior institutional arrangement. This result reverses, in a sense, our earlier conclusion that (with exogenous screening functions) the relative performance of polyarchy is worse if the initial portfolio is worse. This is because the screens in both systems now adjust their screening rules to take into account the quality of the initial portfolio they are facing. When these adjustments are made then we find that, if faced with a bad portfolio, decision makers in both systems protect themselves by raising their reservation levels. But at the same time, firms in a polyarchy derive additional advantage from being able to give a second chance to the rejected projects. This improves their relative performance.

Cost of Information Acquisition: Thus far, the accuracy of information available to screens has been exogenously specified, and we have abstracted from the costs of information gathering. Organizations must also decide, however, on how much resources to spend on gathering information. We represent information gathering as follows. The accuracy of a screen's observation can be influenced, to some extent, by affecting the distribution of errors associated with its observations. Higher accuracy costs more but it increases the probability of acceptance of good projects and lowers the probability of acceptance of bad projects. The distribution function of θ is now represented by $L(\theta, \delta)$, where a higher δ implies a greater accuracy. The corresponding screening function is

$$p(x, R, \delta) = 1 - L(R - x, \delta) .$$

Denote the cost per observation by $C(\delta)$.³³

Then (15) and (17) are modified as

$$(20) \quad Y^{iP} = \frac{1}{2}E[(2 - p^j)\{xp^i - C(\delta^i)\}] , \text{ and}$$

$$(21) \quad Y^H = E[p^1\{xp^2 - C(\delta^2)\}] - C(\delta^1) .$$

Note that the cost of information now depends not only on the accuracy of observation, but also on how many projects are evaluated by a screen.

Expression (20) shows that the firms in a polyarchy face an externality in information gathering; this externality is parallel to the one we discussed earlier concerning reservation levels. An improvement in the accuracy of screening by firm j affects firm i 's profit; some of the gains to firm j represent the good projects that otherwise would have been undertaken by firm i ; moreover, whenever firm j rejects a project, it increases i 's screening costs; as a result, firms 'overestimate' the return to screening.

A hierarchy, on the other hand, faces a trade off whether to do a more accurate screening at the lower bureau or at the higher bureau. A higher quality screening at the lower bureau improves the portfolio to be evaluated by the higher bureau, but it also costs more because a larger number of projects is evaluated by the lower bureau. A lower quality screening at the lower bureau, on the other hand, imposes a potential cost because too many good projects might be thrown out of the system in this process.

A general comparison of the accuracy of information in the two systems, however, is not possible; at the same reservation levels, the lower quality of the portfolio being reviewed by the firms in a polyarchy³⁴ might lead to a higher expenditure on screening. Also, the overestimation of the returns to screening may induce firms to spend more on screening. But since the reservation levels of firms may be higher, it is unclear whether the

marginal information has less (or more) value for firms at these higher reservation levels.

IV. EXTENSIONS

The objective of this paper has been to set out a basic framework within which the performance of alternative economic systems can be evaluated. The analysis can be extended in several ways.

(a) Organizational Forms: We have examined here only two polar architectures, hierarchies and polyarchies. Another polar architecture which we have analyzed elsewhere is represented by committees which may differ both in their size and in their rules for decision making; for example, majority rule, two-third majority.³⁵ We view these polar architectures as building blocks for typical economic organizations which are complex mixtures of hierarchies, polyarchies, and committees. In market economies, for example, large firms often interact with one another in a polyarchical manner, while each of them itself is akin to a hierarchy. Also, firms and bureaus often have internal structures in which different types of information is gathered and different types of decisions are made in different stages. For example, one often observes that projects are first evaluated on technical grounds, then on financial and economic grounds, and finally, on strategic and political grounds.

(b) Initial Portfolio: Our comparison of alternative systems has been based on an assumption that the initial portfolio (described by $g(x)$ and N) is the same for the two systems. Since the probabilities of acceptance of projects of different types differ across organizations, any given reward structure (for acceptance of projects) will provide different incentives for project inventors in different organizations. For instance, recall that

if the screening function is exogenous, then the probability of having a project accepted by a hierarchy is always smaller. Thus, a hierarchy would have to provide greater rewards for accepted projects to induce the same effort level by inventors.

(c) Alternative Decision Rules: We have followed the natural presumption that a project is not undertaken unless it is approved by the organization: within a hierarchy, by both bureaus; within a polyarchy, by at least one firm. We could, of course, imagine quite different organization of decision making: for instance, a hierarchy in which all projects are accepted except those which get vetoed by both bureaus and a polyarchy in which a project is accepted unless vetoed by one of the units.

We refer to organizations operating according to the veto rule as a veto hierarchy and polyarchy, in contrast to those we have analyzed earlier, which we refer to as an acceptance hierarchy and polyarchy. It is easy to show that, in the absence of costs of coordination: An acceptance polyarchy (hierarchy) is equivalent to a veto hierarchy (polyarchy).³⁶

Note, however, that the coordination requirements may be markedly different depending on the architecture and the nature of decision rules. For instance, an acceptance polyarchy does not require any informational coordination among its constituent units; a firm does not need to send information to other firms concerning which projects it has rejected. This is an important aspect of the independence of firms within market-like systems, which we stressed earlier. In comparison, in a veto polyarchy (in which one unit can veto a project from being undertaken), a unit must inform other units which projects it has rejected. Similarly, in a veto hierarchy the lower bureau must inform the higher bureau which projects it has rejected. If there are significant costs to informational coordination,

then it is clear that an acceptance polyarchy may have an advantage over other organizational forms.

(d) Selecting a Fixed Number of Projects: The organizations described above face no limit to the number of projects which could be undertaken; the only problem they face is to decide on screening rules such that the marginal project undertaken has a positive expected value. We now briefly consider what happens when there are a fixed number of projects to be undertaken.³⁷ Clearly, the projects undertaken will be those with the highest value of y , that is, those which appear to be the best. If the same initial portfolio is faced by alternative organizations, and if the number of projects to be selected is large (out of an even larger portfolio) then this problem is roughly equivalent to the one in which organizations choose reservation levels so that a fixed proportion of the initial portfolio is selected. Denote this proportion by n . Now, recall from (7) that if both systems have the same screening rules (reservation levels), then the fraction of projects selected by a polyarchy is higher than that by a hierarchy. It follows then that in the present context: A polyarchy employs more conservative acceptance rules than a hierarchy.

To see how the performance of the two systems can be compared, consider the case in which there are two kinds of projects. Then the reservation levels in the two systems, R^P and R^H , are obtained from

$$(22) \quad n = \alpha f_1^S + (1 - \alpha) f_2^S$$

where, as defined earlier, $f^P = p(x, R^P) \{2 - p(x, R^P)\}$, and $f^H = p^2(x, R^H)$. A system's profit is $Y^S = \alpha z_1 f_1^S - (1 - \alpha) z_2 f_2^S$. The profit can be rewritten, using (22), as $Y^S = -n z_2 + \alpha (z_1 + z_2) f_1^S$. It follows that

$$(23) \quad \Delta Y \geq 0, \text{ if } f_1^P - f_1^H \geq 0.$$

The above expression has a clear intuition. Since both systems must select the same number of projects, their relative performance depends simply on which one of the two systems has a higher probability of accepting good projects.

(e) Performance of Screens: In Section III, we argued that the performance of screens within an organization is affected by the information implicit in the organization's architecture as well as by the resources spent on information gathering. Here we point out some other important aspects which may influence the performance of screens and, therefore, the performance of organizations.

For instance, different individuals have different abilities; some individuals are better at screening (or make different types of errors) than do others. Different organizational architectures may also differ in (i) how sensitive their performance is to the assignment of individuals with different capabilities to different positions within the organization and (ii) how well they do in selecting individuals for different positions. The performance of organizations concerning the choice of decision makers can be studied in a manner parallel to our analysis of the choice of projects. It is obvious that the choice of decision makers within an organization is influenced by its architecture. This provides a basis for analyzing the 'rules of succession,' and what we call (in our 1985c paper) the self-perpetuating aspects of economic systems.

Another characteristic of economic systems which affects their performance is the nature of rewards and punishments meted out to the decision makers. This has, of course, been the question around which much of the

recent literature on incentives has focussed. In the present paper we have not emphasized this important aspect as much as it deserves.³⁸ This is because we think that some of the incentive problems are rather well understood in what is already a vast literature, whereas the new aspects upon which we focus here have received insufficient attention.

On the other hand, we should point out that the architecture of a system may be critical in determining what reward systems are feasible and desirable in different systems. For example, in a hierarchical structure, promotions often constitute an important part of the rewards, a kind of reward which may or may not be desirable in a polyarchy. On the other hand, in polyarchical structures several parallel units perform similar functions, and it is possible to devise reward structures based on relative performance. These reward structures have been shown to have many desirable properties concerning incentive, risk, and flexibility (see footnote 4), and these may not be feasible in a hierarchical system. Different architectures may also differ in the degree of individual accountability which is feasible within them. One criticism of modern bureaucracies is, for example, that collective decision making makes it difficult to reward and punish bureaucrats individually.

The architecture of an economic system affects the behavior of the organization in other ways as well, some of which have been extensively studied outside economics. Social psychologists have emphasized, for example, that individuals' behavior may differ if they have participated in a decision making process compared to when they have been ordered to undertake a particular task. Though these aspects of human behavior have not traditionally been incorporated into economic analysis, if they are important determinants of economic behavior, for example, of the effort exerted

by individuals or of the quality of their decision making, then they should be.

V. CONCLUDING REMARKS

This paper has attempted to develop a new framework within which we can characterize and compare the performance of alternative ways in which the activities of society are organized. The particular aspect of human behavior on which we have focussed on is that much of human decision making entails errors of judgment. We have shown that the architecture of an organization influences what kind of error individuals make within the organization, and how individuals' errors add up to affect the organization's performance. Specifically, we have analyzed and compared the performance of two polar architectures (polyarchies and hierarchies) which have features of decentralized versus centralized decision making authority.

Our approach in this paper has been positive and comparative. We have modelled specific economic systems and analyzed their performance, individually and comparatively, to ascertain the set of circumstances under which one system performs better or worse than the other.³⁹ Though the problem of finding the optimum economic system, given a set of alternative systems and a set of external circumstances, is conceptually no different, we have refrained from emphasizing the normative aspects. This is in part because there is little reason to believe that societies choose their organizational form optimally. Indeed, given our arguments concerning the errors in making investment decisions, why should we expect an absence of errors in grand decisions, such as how to organize the societal structure itself.

Moreover, the problems of the design of economic systems are so complex that it might not be reasonable to expect that one would find the best of all possible systems. The analogy to computer architecture is then suggestive: the standard question in this case is not to find the "best" architecture, since it is nearly impossible to find it, but to analyze the properties of alternative structures, with a view to identify potential improvements.

Although we have motivated the present analysis in the context of alternative economic systems, our approach has implications for the internal organization of large corporations as well. There are two main differences. First, certain kinds of externalities (such as the externality in screening and information gathering in polyarchies, which we pointed out earlier) might be internalized by a corporation in setting its internal rules. Second, if one internal architecture is better than another (based on whatever the corporate criterion might be), then it might be more reasonable (in contrast to a societal situation) to expect a corporation to adopt the better architecture.⁴⁰

We have argued that there is more to the comparison of economic systems than is adequately reflected in the Lange-Lerner-Taylor analysis; there is more at stake than simply a comparison of alternative algorithms for arriving at a once-and-for-all allocation of society's resources. We hope we have convinced the reader that it is possible to construct simple models which reflect some important but hitherto unrecognized aspects of the comparison between alternative systems. In particular, we have suggested a basic parallelism between the sequence of decision making and the architecture of economic systems. Also, we have indicated how our analysis may be extended in a variety of ways. But, we would be the first to argue that our

present analysis is far from exhaustive. In our future work, therefore, we hope to study other important aspects of organizational architecture.

APPENDIX I

(a) Substitution of (6) into (1) and (2) yields $f^S(x)$ for the linear screening function. From (3), then, we obtain

$$(A1) \quad n^P = \bar{p}(2 - \bar{p}) - p_x^2 V, \quad \text{and} \quad n^H = \bar{p}^2 + p_x^2 V$$

where $V = E[(x - \mu)^2]$ is the variance of the initial portfolio. Therefore: $\partial n^S / \partial \bar{p} > 0$, $\partial n^P / \partial p_x < 0$, and $\partial n^H / \partial p_x > 0$. Also, note here that the parameters ($\bar{p}$, p_x and V) can not take arbitrary values, because they must satisfy certain consistency requirements. For instance, from (7), we know that $n^P > n^H$. Thus, from (A1)

$$(A2) \quad V < \bar{p}(1 - \bar{p}) / p_x^2.$$

(b) Recall that $\Delta Y = Y^P - Y^H = E[x(f^P - f^H)]$. We obtain $(f^P - f^H)$, once again, by substituting (6) into (1) and (2). This yields

$$(A3) \quad \Delta Y = 2\mu[\bar{p}(1 - \bar{p}) - p_x^2 V] + 2[(1 - 2\bar{p})p_x V - p_x^2 \eta V^{3/2}]$$

where $\eta = E[(x - \mu)^3] / V^{3/2}$ is the coefficient of skewness and, from (A2), the expression within the first square bracket of (A3) is positive. Therefore: $\partial(\Delta Y) / \partial \mu > 0$, and $\partial(\Delta Y) / \partial \eta < 0$. Also, if $\bar{p} < 1/2$, $\mu \leq 0$, and $\eta \leq 0$, then (A3) yields: $\partial(\Delta Y) / \partial V > 0$, $\partial(\Delta Y) / \partial \bar{p} < 0$, and $\partial(\Delta Y) / \partial p_x > 0$. Further, under the same set of conditions, (A3) shows that: $\Delta Y > 0$, if $V > -\mu\bar{p}(1 - \bar{p}) / [(1 - 2\bar{p})p_x - \mu p_x^2]$. Thus $\Delta Y > 0$ if V is sufficiently high. It is easily verified that this conclusion is unaffected by the restriction (A2) on how large V can be.

APPENDIX II

The distribution function of θ is: $L(\theta) = (\theta + u)/2u$ if $u \geq \theta \geq -u$, $L(\theta) = 0$ if $\theta < -u$, and $L(\theta) = 1$ if $\theta > u$. Expression (19) yields

$$(A4) \quad -z + u \geq R \geq z - u.$$

Define a parameter $\delta = z/u$; a smaller δ means larger errors in observation. Clearly, $\delta > 0$ for finite errors. Also, $1 > \delta$, otherwise a perfect inference between good and bad projects is possible. Thus, $1 > \delta > 0$.

Next, recall that p_1 and p_2 denote the probabilities of acceptance (by a screen) of good and bad projects. From the above definition of $L(\theta)$, and from (14), we obtain $p_1 = (u + z - R)/2u$, and $p_2 = (u - z - R)/2u$, if R is within the range (A4). It follows then that

$$(A5) \quad p_2 = p_1 - \delta.$$

The above expression allows a considerable simplification because we can use p_1 as the control variable to determine the optimal screening rules. From (A4), the range of p_1 is: $1 \geq p_1 \geq \delta$. If $p_1 = 1$, then the lowest reservation level is being chosen; if $p_1 = \delta$, then the highest reservation level is being chosen. For other values of p_1 , the corresponding reservation level can be recovered from the definition of p_1 provided above.

Using (A5), we can express (15) and (17) as

$$(A6) \quad Y^{iP} = \frac{z}{2} [\alpha(2 - p_1^j) p_1^i - (1 - \alpha)(2 - p_1^j + \delta)(p_1^i - \delta)]$$

$$(A7) \quad Y^H = z[\alpha p_1^1 p_1^2 - (1 - \alpha)(p_1^1 - \delta)(p_1^2 - \delta)] .$$

We focus on the symmetric optimum in a polyarchy. The optimal value of p_1 in this case is denoted by p_1^P . The optimal p_1 's for a hierarchy are denoted by p_1^{1H} and p_1^{2H} . The optimum in a polyarchy is characterized by

$$(A8) \quad p_1^P = 1, \text{ and } Y^P = z\alpha - z(1 - \alpha)(1 - \delta^2), \text{ if } \alpha \geq (1 + \delta)/(2 + \delta),$$

$$(A9) \quad p_1^P = 2 - \delta(1 - \alpha)/(2\alpha - 1), \text{ and } Y^P = z\delta^2\alpha(1 - \alpha)/(2\alpha - 1),$$

if $(1 + \delta)/(2 + \delta) \geq \alpha \geq 2/(4 - \delta)$, and

$$(A10) \quad p_1^P = \delta, \text{ and } Y^P = z\delta\alpha(2 - \delta), \text{ otherwise.}$$

In a hierarchy, direct computations using (A7) show that an internal solution is always dominated by one of the corner solutions. A comparison among the corner solutions yields

$$(A11) \quad p_1^{1H} = p_1^{2H} = 1, \text{ and } Y^H = z\alpha - z(1 - \alpha)(1 - \delta)^2, \text{ if } \alpha > (1 - \delta)/(2 - \delta),$$

and

$$(A12) \quad p_1^{1H} = 1, \quad p_1^{2H} = \delta, \text{ and } Y^H = z\alpha\delta, \text{ otherwise.}$$

Interchanging p_1 's in (A12) provides an alternative (but equivalent) optimal screening rule, under which a hierarchy rejects all bad projects.

Note the following results. Both systems adopt the highest (lowest) reservation level at low (high) values of α . But, the level of α below (above) which a hierarchy adopts this rule is lower than the corresponding

level of α in a polyarchy. Second, the reservation level is higher in both systems (either continuously, or in steps) if α is lower.

The relative profit level is calculated as $\Delta Y = Y^P - Y^H$. First, take the range: $0 < \alpha < (1 - \delta)/(2 - \delta)$. In this range, expressions (A10) and (A12) apply, and $\Delta Y = z\alpha\delta(1 - \delta) > 0$. Second, take the range: $2/(4 - \delta) > \alpha > (1 - \delta)/(2 - \delta)$. In this range, (A10) and (A11) apply, and $\Delta Y = z(1 - 2\alpha)(1 - \delta)^2$. Thus, $\Delta Y \geq 0$, if $1/2 \geq \alpha$. Third, take the range $(1 + \delta)/(2 + \delta) > \alpha > 2/(4 - \delta)$. In this range, (A9) and (A11) apply, and $\partial(\Delta Y)/\partial\alpha < 0$. Evaluating ΔY at $\alpha = 2/(4 - \delta)$, we find that $\Delta Y < 0$. Thus, $\Delta Y < 0$ within this range is α . Finally, take the range $1 > \alpha > (1 + \delta)/(2 + \delta)$. Expressions (A8) and (A11) apply in this case, and $\Delta Y = 2z(1 - \alpha)\delta(\delta - 1) < 0$. Therefore, the result: $\Delta Y \geq 0$ if $1/2 \geq \alpha$ applies to the entire range of α .

FOOTNOTES

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1. In fact, quite the contrary view has sometimes been put forward. The Lange-Lerner-Taylor equivalence theorem, for example, argued that a market economy and a bureaucratic economy using a price system behave in an essentially identical manner. This equivalence theorem, however, is based on models of both socialist and market economies which fail to capture the salient characteristics of either form of economic organization. See Frederick von Hayek (1935), Oskar Lange and Fred Taylor (1964), Ludwig von Mises (1935), among others, on the market socialism debate.
2. Of course, there are other ways in which market- and bureaucracy-oriented economies may differ. Among those advanced in the literature are: prices versus quantity as tools of allocation [see Martin Weitzman's (1974) classic paper], and private versus public (collective) ownership of resources.

3. This is essentially the view embedded in the standard Arrow-Debreu model of the economy. In their analysis, firms do not engage in the collection and analysis of information; firms have a simple once-and-for-all decision to make; they just find the book of blueprints describing the optimal production plan corresponding to various sets of relative prices.
4. This is, for instance, the view that has been emphasized by Nalebuff and Stiglitz (1983). It should be noted that we ignore incentive issues in this paper not because they are unimportant, but because we feel that they have been discussed extensively elsewhere, while the particular issues upon which we focus attention here have not.
5. Further advantage cited by those who view themselves to be innovative, non-bureaucratic people is that in polyarchies such people do better. The differences in the types of people who are successful within different organizational forms, who obtain decision making positions, may play an important part in the differences in organizational performance. We shall comment on this briefly below.
6. A screen may, at present, be thought of as a black box which flashes a light when it deems a project to be good. More detailed interpretations of the screening process are described below.
7. We assume here that a firm can not distinguish between the projects which are being evaluated for the first time from those which have been rejected by the other firm. This can be visualized as follows. An identical initial portfolio of projects arrives in each period, it takes one full period to examine projects, and firms do not share any

information with one another. A firm in a particular period, then, evaluates a portfolio consisting of the new projects which it received in the beginning of the period, and the projects which were rejected by the other firm in the previous period, without being able to distinguish between the two categories of projects.

8. This scalar valuation includes all relevant benefits and costs. Also, we are assuming that the inter-project externalities are not significant (that is, the profit from one project does not depend significantly on whether some other projects are undertaken or not), and that there is no restriction on the number of projects that can be undertaken. Of course, this is not the only formulation which one can use for comparing alternative economic systems. We later discuss another formulation in which only a fixed number of projects can be undertaken.
9. In a polyarchy, the probability that a project which goes to the first firm is approved is $p(x)$. It gets rejected with probability $1 - p(x)$; the probability that it then gets approved by the second firm is again $p(x)$. Hence the total probability of acceptance is $p(x) + (1 - p(x))p(x) = p(x)(2 - p(x))$. Similarly, in a hierarchy, the probability that a project is approved by the lower bureau is $p(x)$. The probability that the same project given to the higher bureau is approved is again $p(x)$. Hence, the probability of a project being approved is $p^2(x)$. This analysis assumes that the decisions (judgments) are independent across screens and projects.

10. Portfolios containing only profitable projects, or only unprofitable projects are not worth considering since, if project evaluators know what kind of portfolio it is, then the issue of screening disappears altogether.
11. We assume at present that the screening costs depend only on the number of screens. Then the costs can be suppressed because they do not affect our conclusions. Other formulations of screening costs are discussed later. Also, we assume that if there is uncertainty in the outcome of projects, then the probability of acceptance of a project by a screen, $p(x)$, depends only on the expected profit, x , of the project.
12. The last condition merely rules out those uninteresting cases in which all projects are either accepted, or rejected.
13. The subscripts of p 's denote the variable with respect to which a partial derivative is being taken.
14. This can be seen directly in Figure 3.
15. We suppress the number of projects, N , in the expressions for Y^P , Y^H , and ΔY , throughout the rest of the paper.
16. This is simply because the relative advantage of a hierarchy is in rejecting bad projects, whereas the relative advantage of a polyarchy is in accepting good projects. If the initial portfolio worsens, that is if there are more bad projects, then the former advantage becomes increasingly more important and the relative performance of a hierarchy improves. However, we must emphasize that, in general, the probability

that a project is accepted or rejected by a screen (that is, the rules for project acceptance and rejection) will be affected by the mix of available projects. One might suspect, for instance, that if there were a large proportion of bad projects, screening would become relatively tighter in a polyarchy, and this might improve its relative performance. Thus, the kind of comparative statics conducted here, where only one parameter is changed at a time, must be treated with caution. Endogenous screening functions are discussed later.

17. In fact, the same approach can be used even when the initial portfolios are entirely different in the two systems. Take the general case in which, for $i = 1$ and 2 , N_i^S is the number of projects, and z_i^S is the positive number denoting the profit or loss from a project of type i in the system s . Then the center of hyperbolas is given by $p_i = N_i^P z_i^P / [N_i^P z_i^P + N_i^H z_i^H]$. The slopes of the asymptotes are $\pm [(N_1^P z_1^P + N_1^H z_1^H) / (N_2^P z_2^P + N_2^H z_2^H)]^{1/2}$. Other relevant details, as well as various special cases, can be easily worked out.
18. This, of course, begs the question of the optimal number of screens. We have analyzed this question in our 1985a paper.
19. In fact, under such a situation, there exist parent distributions, $g(x)$, for which a particular mean preserving spread will increase ΔY , whereas some other mean preserving spread will decrease ΔY . See Rothschild and Stiglitz (1970).
20. More generally, if $p(x)$ is a polynomial of degree m then, from (4), Y^S is a function of up to $(2m + 1)$ moments of the distribution of the initial portfolio.

21. The mean is likely to be negative because, typically, unprofitable ideas outnumber profitable ones.
22. For example, if an impartial outsider provides binary information on projects (a project is good or bad), and if there is no information sharing between screens, then the profit maximizing decision rule (that is, whether to act according to, or contrary to, the outsider's advice) gives rise to identical screening functions in the two systems in all circumstances except one. The exception arises when a hierarchy's optimal decision rule is such that one bureau acts according to the outsider's advice, while the other acts contrary to it.
23. We assume, however, that a firm has the information whether a project was evaluated earlier by them or not, and the firm uses this information to prevent a project from being evaluated more than once.
24. In our model, an individual observes a variable y , which is a signal concerning the mean profit x . Clearly, in this simple case, the individual could, in principle, communicate all that he knows. But our objective is to provide some insights into the more general situation, where individuals observe a multi-dimensional set of variables, which they integrate with their priors, to form a judgment; they can neither communicate all of what they have observed concerning the project, nor can they fully articulate their priors.
25. Note that optimal policies can be characterized in terms of a single reservation level only if some mild regularity conditions are satisfied by the nature of the error terms. An example where these conditions are not satisfied is as follows. If there are two types of projects,

and if the variance of observation errors on the bad project is much larger than that on the good project, it may be the case that a very high value of y is more likely to correspond to a bad project than is a somewhat lower value of y . Such anomalies are common in statistical decision theory.

26. Provided $(R - x)$ is between the two end points of the distribution of θ .
27. This is an illustration of our earlier remark that the independence of firms within a polyarchy imposes some costs on the system. A polyarchy's profit Y^P in the present case is, in general, smaller than in the hypothetical case in which there is no independence and, consequently, the reservation levels for both firms can be set at levels which maximize Y^P . Also, note that the above externality is quite different from the one in which the return from a project depends on whether that project (or some other set of projects) is undertaken by other firms. The latter is the more familiar externality which provides the usual argument for coordinated decision making.
28. Clearly, R^{1H} may not always equal R^{2H} . But the symmetry of p^1 and p^2 in (17) shows that if the optimum is unique and internal then R^{1H} must equal R^{2H} . Later, when we make the accuracy of screening a control variable, we shall see that Y^H is not symmetric in p^1 and p^2 .
29. If $p(0)$ is the probability of acceptance for a project with zero profit, then the right hand side of (18) is $p(0)E[xp_R] + E[x\{p(x) - p(0)\}p_R]$. The last term is negative because

$p_R < 0$, and $p(x) \geq p(0)$ if $x \geq 0$. Using this, (18) yields:
 $\{2 - p(0)\}E[xp_R] < 0$ or, equivalently, $E[xp_R] < 0$. Hence both sides
of (18) must be negative.

30. This conservatism is reflected in market economies by firms insisting on a high expected return in order to undertake a project. For example, firms often have a decision rule that only projects with an expected return in excess of 20 percent be undertaken, but the actual average returns are considerably smaller. Firms know that to attain the required return, they have to set high reservation levels.
31. If y_1 and y_2 do not overlap, that is, if $\min y_1 > \max y_2$, then the decision problem disappears because perfect selection is possible by setting R in the non-overlapping region. We do not concern ourselves with these situations.
32. In addition, the result we noted earlier, that a polyarchy screens more conservatively than a hierarchy, can be confirmed for this example, even though the optimum here entails corner solutions, and the reservation levels for the two bureaus in a hierarchy are not always the same. Specifically, both systems employ reservation levels which lead to the rejection of all bad projects when the mix of projects is very bad. However, the proportion of good projects below which a hierarchy adopts such a policy is lower than the corresponding proportion in a polyarchy. Similarly, when the mix of projects is very good, both systems employ rules under which all good projects are accepted; but the proportion of good projects above which a hierarchy adopts this rule is lower than the corresponding proportion in a polyarchy.

33. In the present formulation, although a screen makes only one observation, the accuracy of its observation is a choice variable. A more accurate observation has an increasingly higher cost; that is, C is increasing and convex in δ . An alternative formulation is the one in which the accuracy of observation is fixed and the cost per observation is also fixed, but a screen can make as many observations as it wants. In this case, the optimal information gathering will be characterized by a sequential sample plan.
34. Because each firm in the polyarchy has all the rejected projects of the other firm, it faces a greater proportion of bad projects than even the lower bureau in a hierarchy.
35. Note that a hierarchy can be viewed as a special case of a committee in which approval requires unanimity. A polyarchy, on the other hand, can be viewed as a committee in which approval requires only a single vote. Such a view, however, abstracts from important aspects of the sequence of decision making. See our 1985a paper for an extensive analysis of the performance of committees.
36. The probability of acceptance of a project in an acceptance hierarchy is p^2 . The probability of being rejected by the first unit in a veto polyarchy is $1 - p$, and that of being rejected by the second unit (given that the project has not been rejected by the first unit) is $1 - p$. Thus, the total probability of being rejected is $1 - p + p(1 - p) = 1 - p^2$. Hence, the probability of acceptance is just p^2 . Also, the number of evaluations is the same in these two systems.

37. We can think of there being a fixed number of dam sites. The question then is, which projects to place at various sites. In the present analysis, the opportunity cost of dam sites is endogenous; in the rest of the paper, the opportunity cost of dam sites is taken as exogenous.
38. For example, our model for determining screening rules can be extended to include the possible effect of individuals' efforts on errors, and the effort-reward trade off which is optimal in each of the two systems. Also, there may be incentive problems associated with whether it is in the interest of individuals to share information. Note, however, that the incentive structure may not always be critical in determining individuals' performance; it may not take much more effort to make a good decision than a bad one, but it may take much more ability.
39. It might be useful here to comment on a particular definition of hierarchy which sometimes arises in economic discussions. According to this definition, a 'hierarchy' is a supra-organizational entity which can adopt whatever organizational form it desires. Not surprisingly, a hierarchy--so defined--is always a better system since it has extra degrees of control (in choosing its organizational form) compared to any specific organizational form. This definition is not very useful, since all it says (tautologically) is that a society which can choose among various organizational forms is better than the one which can't. In any case, if one uses the above definition, then our analysis can be viewed as delineating conditions under which this 'hierarchy' should organize itself as a polyarchy or a hierarchy (as we have defined it).
40. The comparative historical study by Alfred Chandler (1962), for instance, points to such corporate responses.

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