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YIELD RISK IN A DYNAMIC MODEL OF THE AGRICULTURAL HOUSEHOLD

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ABSTRACT

A dynamic model of the agricultural household is posited in which the household chooses goods, leisure, and land and labor to maximize expected utility over multiple periods. The effect of yield risk and household preferences on its production and consumption desicions are derived from the relationship between the household's direct utility function and a dynamic version of its indirect expected utility function. Similarities between the results derived from the standard agricultural household model and this model are shown. The model is, in general, nonseparable due to the absence of a contigent claims market. A special case is shown where a type of separability exists, although parameters and prices appearing in the indirect utility function determine the "risk parameter" in the supply and factor demand functions. In this special case, household demand is shown to depend on certainty-equivalent income. A numerical illustration of the model is also provided.

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by

Terry Roe and Theodore Graham-Tomasi[1]

1. Introduction

Numerous studies have found that farmers in developing countries prefer lower but certain levels of income to marginally higher uncertain income levels (Moscardi and de Janvry, 1977; Dillon and Scandizzo, 1978; and Binswanger, 1980). These studies have obtained estimates of farmers' aversion to risk ranging from a measurement of absolute risk aversion of .9 for Northeastern Brazil to partial risk aversion estimates of .316 to 1.74 for farmers in India. Since contingency markets are surely imperfect in developing economies, risk averse farmers tend, in an effort to reduce income uncertainty, to allocate resources to activities with lower expected marginal value products than they would in the absence of uncertainty[2].

The relationship between depressed income due to risk, and household consumption has not been studied in models of the agricultural household. An obvious implication of assuming the absence of risk when risk is present is that inferences drawn from these models may be misleading. The problem is to determine the nature of the misleading inferences that might otherwise be drawn. Moreover, failure to consider the affect of risk on household choices limits the insights that can be obtained into the welfare effects of market imperfections, such as those which inhibit households from allocating resources to off-farm activities, crop insurance or imperfections which provide limited access to production technologies and other risk reducing inputs.

In this chapter, we seek to incorporate production risk into a dynamic version of the agricultural household model. We investigate a fairly simple model in an effort to determine the impact of yield risk and the household's risk preferences on its production and consumption decisions. Our model yields the familiar result that consumption and production occurs along the locus of points formed by the tangency of marginal utilities and marginal products to their respective price ratios. An analogue of Roy's Identity is also found to hold which relates consumption and input demands to the derivatives of a dynamic version of the household's indirect utility function. At this point, the results depart from those of the traditional model. In general, separability between production and consumption decisions does not hold, although a special case is demonstrated where a type of separability exists. While relationships between the household's choices and increasing risk can be derived for this special case, parameters of the household's direct utility function and prices of the arguments appearing in this function are found to determine the "risk aversion parameter" appearing in the product supply and input demand functions. Also, for this special case, demand is found to be a function of certainty equivalent income. Hence, our findings suggest that parameter restrictions on estimating equations derived from models of the agricultural household which assume an absence of risk may be inappropriate if risk and risk aversion are important.

The paper is organized as follows. To provide perspective, a brief background on how the issues faced in this paper contrast to other models of uncertainty appears in the next section. Then, the basic model is specified and the dynamic programming approach to its solution is presented. A solution to the model is characterized in the fourth section. A specific problem is specified in the fifth section for which reduced form equations are derived and data from households in the Dominican Republic are used to illustrate various implications of the model. In the sixth section, we address some of the duality results and selected empirical questions. A final section is a discussion in which we point out some key failings of the model and possibilities for further research.

2. Background

The theory of the individual consumer provides some insight into the effect of increasing income uncertainty on consumption levels. However, results are not easy to obtain and generally depend on third derivative properties of the utility function. In the case of a single good, two period, utility function with uncertain income in the second period, the third derivative property implies the convexity of the marginal utility of the good consumed in the second period; this is compatible with decreasing absolute risk aversion. In the models considered by Leland (1968), Mirman (1971) and others[3], the third derivative property implies a decrease in first period consumption and/or an increase in savings as uncertainty increases. However, the problem faced by the agricultural household in our model is more complex than the problem studied in this literature in two ways.

First, income in these models is exogenous, and second, there is only one consumption good. Clearly, the essence of the agricultural household model as outlined in Chapter 2 is endogenous income and the existence of both a staple and a market good. Regarding the first issue, Block and Heineke (1973) study a static model with utility a function of income and labor.

They show that if $(-\partial^2U/\partial Y^2)/(\partial U/\partial Y)$, where Y is income, is decreasing in income for a given quantity of labor supply, then an increase in risk increases labor supply when there is additive income risk ($Y = wL + \tilde{Y}$). Thus, the individual "self-insures" against income risk by working more. With wage rate uncertainty ($Y = Y + \tilde{w}L$), Block and Heineke show that an increase in risk has an ambiguous effect. In a dynamic model, there is savings as well as labor effort and it is not clear that the Block and Heineke results will hold.

Regarding the existence of several goods, the definition and measurement of risk aversion in this situation has been studied by Kihlstrom and Mirman (1974; 1981). Stiglitz (1969) and Hanoch (1977) have investigated the implications of risk aversion for demands for commodities. All of these analyses take place in static models. The most important results for our purposes are those of Kihlstrom and Mirman, which indicate that with homothetic preferences and income risk, the risk preferences of the consumer are reflected by the indirect utility function considered as a function of income alone. This is similar to the dynamic, single-good models, which show that the value function in a dynamic programming approach to solving the problem embodies the curvature properties of the direct utility function (Miller, 1976).

Our efforts along these lines are complicated by the production activities of the agricultural household. Considering production decisions alone, the most relevant paper reports work by Pope and Kramer (1979), who study production uncertainty for a competitive firm.[4] They find that, if the production function is multiplicative in the random variable (a form we assume in this paper), then an increase in risk reduces output if absolute risk aversion is decreasing. Our research extends their model to a dynamic setting.

The introduction of risk into a dynamic model of the agricultural household has two significant implications: (1) in general, the model no longer is separable into independent consumption and production activities, although a special case is shown where a type of separability exists, and (2) restrictions on estimating equations derived from certainty theory are not appropriate when production is risky. These results hold even for our relatively simple special case in which utility is additively separable over time, input and output prices are known, risk enters the production function multiplicatively, and production shocks are distributed independently over time.

A basic reason for the lack of separability is that risk aversion in consumption induces risk aversion regarding profits.[5] Thus, the expected utility of profits must be maximized and the form of this function depends on the form of the consumption utility function and consumption decisions.

A more fundamental reason for the lack of separability is the absence of a market. As discussed in Chapter 2, separability of the static household production model obtains if a complete set of markets exists. This is extended to a two-period model by Igbal (Chapter 7) by introduction of a capital market. In this paper, with risk, separability does not hold because contingent claims markets do not exist; if contingent claims markets were introduced, separability would be restored. However, we feel that positing such perfect insurance markets is inappropriate.[6]

3. The Conceptual Framework

The household gains utility from a sequence of consumptions of goods X_t and leisure X_{lt} over its time horizon $t = 0,1,\ldots,T$ and from a bequest. The household's utility is given by an additively separable, time invariant utility function.

$$(1) \qquad U(\{X_t, X_{1t}, b_t\}_{t=0}^{t=T}) = \sum_{t=1}^{t=T} \alpha^t u(X_t, X_{1t}) + \alpha^{T+1}\delta(b_{T+1})$$

Here, as in the basic model presented in Chapter 2, there are two goods: an agricultural staple, X_{qt}, and a good purchased in a market, X_{mt}. The household is assumed to hold a single financial asset, b_t. The discount factor $\alpha = (1 + e)^{-1}$ where e is the rate of utility discount; we assume that $0 < e < 1$.

Farm production of the agricultural staple is given by the stochastic production function

$$(2) \qquad Q_{t+1} = Q(L_t, A_t; \tilde{\varepsilon}_{t+1}),$$

where L_t and A_t are labor and land inputs at t and $\tilde{\varepsilon}_t$ is a random variable. Note that both labor and land are variable here, and that there is a lag in production. L_t and A_t are the sum of allocations to production out of the household's endowments, plus net market purchases of labor and land.

In contrast to the basic model of Chapter 2, the model studied here is dynamic. The household consumes goods and leisure in period t from income generated by allocations of land and labor made in the previous period, t-1. We assume that Q_t is known when (X_{qt}, X_{mt}, X_{1t}) are chosen. In period t, the household also decides upon the resource allocation (L_t,A_t), which determines output in the next period, t+1. As with goods and leisure, we assume that Q_t is known when choices of A_t and L_t are made. Another departure from the basic model is the existence of a financial asset with rate of return, r.

As we shall see, this asset serves to smooth intertemporal household consumption by linking over time periods the household's marginal utility of income. Note that b_t represents beginning-of-period holdings of the asset.

Markets for commodities, land, and labor are assumed to exist. The market prices of X_{qt} and X_{mt} are P_{qt} and P_{mt}, respectively. The rental rate for land is a_t and the wage is w_t.

Full income in period t can be expressed as the value of the household's endowment of land and time plus interest income plus profits, i.e.,

$$(3) \qquad I_t = a_t\bar{A} + w_t\bar{L} + P_{qt}Q(L_{t-1}, A_{t-1}; \tilde{\varepsilon}_t) - a_tA_t - w_tL_t + (1+r)b_t$$
$$= \bar{\bar{W}}_t + \pi_t + (1+r)b_t,$$

where $\bar{A}$ and $\bar{L}$ represent endowments of land and labor, respectively. Expenditure on goods and leisure in period t is:

$$(4) \qquad C_t = P_{qt}X_{qt} + P_{mt}X_{mt} + w_tX_{lt}$$

Then, the holdings of the financial asset evolve according to

$$(5) \qquad b_{t+1} = I_t - C_t.$$

In summary, we have the following statement of the household's maximization problem.

$$\lambda:\ \max_{[z_t]} E \sum_t^T \alpha^t u(X_t,\ X_{1t}) + \alpha^{T+1}\delta(b_{T+1})$$

$$\text{s.t.}\quad Q_{t+1} = Q(L_t,\ A_t;\ \tilde{\varepsilon}_{t+1})$$

$$b_{t+1} = \bar{W}_t + \pi_t - C_t + (1+r)b_t,$$

where

$$z_t = (X_{qt},\ X_{mt},\ X_{1t},\ A_t,\ L_t).$$

Under an assumption that the stochastic process $[\varepsilon_t]$ is a stationary Markov process, the solution to λ can usefully be studied using a dynamic programming approach. A Markov process is a process such that the probability distribution on $\tilde{\varepsilon}_{t+1}$ is conditional only on ε_t and not on the entire history of the process. Thus, we write the conditional distribution on next period's realization of the random event (called the transition probability) as $\phi(\tilde{\varepsilon}_{t+1},\ \varepsilon_t)$.

Let $V^t(Q_t,\ b_t,\ \varepsilon_t)$ be the value function for the household's problem at date t. $V^t(.)$ gives the maximal expected present value of utility from date t to T+1, starting with "initial" condition $(Q_t,\ b_t,\ \varepsilon_t)$. Thus, V^o is the indirect objective function for the overall problem; it is the dynamic equivalent of the household's indirect utility function. The dynamic programming approach to characterizing a solution to the problem makes use of the recursive relationship

$$(6)\quad V^t(Q_t,b_t,\varepsilon_t) =$$

$$\sup_{\{z_t\}} [u(X_{qt},X_{mt},X_{1t}) + \alpha \int V^{t+1}(Q_{t+1},b_{t+1},\tilde{\varepsilon}_{t+1})d\phi(\tilde{\varepsilon}_{t+1},\varepsilon_t)$$

$$|Q_{t+1} = Q(L_t,A_t;\ \tilde{\varepsilon}_{t+1});\ b_{t+1} = \bar{W}_t + (1=r)b_t + \pi_t - C_t]$$

In the terminology of dynamic programming, the <u>state</u> of the system at t is the vector $(Q_t, b_t, \varepsilon_t)$. A <u>plan</u> is a map at each date giving the current action z_t as a function of the history of the state up until t, i.e., $z_t = z_t([Q_\tau, b_\tau, \varepsilon_\tau]_{\tau=0}^{\tau=t})$. An <u>optimal plan</u> is a solution to λ. An optimal plan, if one exists, solves the functional equation in (6) at each date.

Under some fairly mild assumptions, it is possible to show that an optimal plan for our problem exists, is continuous, and depends only on the current state and not the history of states. Furthermore, if the functions u(.) and Q(.) are strictly concave and p-times continuously differentiable, then if solutions are interior, it may be shown that the value function $V^t(.)$ is p-times differentiable, and that the optimal plan $z_t^*(Q_t, b_t, \varepsilon_t)$ is (p-1)-times differentiable. The optimal plan can be obtained by applying the Implicit Function Theorem to the first order necessary (and sufficient due to strict concavity) conditions for the problem

$$(7) \qquad \max_{z_t} u(X_{qt}, X_{mt}, X_{1t}) + \alpha E V^{t+1}(Q_{t+1}, b_{t+1}, \tilde{\varepsilon}_{t+1})$$

The statements in the previous two paragraphs are asserted without proof in this chapter since the proofs involve technical details which are not particularly interesting <u>per se</u>. A more formal analysis of a problem very similar to the one stated here is contained in another paper by the authors to which interested readers are referred for formal proofs of assertions in this paper (Graham-Tomasi and Roe, 1985).

4. Characterizing a Solution

We turn now to a special case of the problem λ in which the production function for the agricultural staple takes the form

$$(8) \qquad Q(L_t, A_t; \tilde{\varepsilon}_{t+1}) = f(L_t, A_t)\tilde{\varepsilon}_{t+1}$$

and where the process $\{\varepsilon_t\}$ is a sequence of independently and identically distributed random variales. For this special case, ε_t does not condition the distribution of $\tilde{\varepsilon}_{t+1}$. Thus, ε_t does not enter the value function directly as part of the state at t.

Let the price of goods and inputs be summarized by the vector $P_t = (P_{qt}, P_{mt}, w_t, a_t)$ and define

$$\begin{aligned} \delta^t(z_t; Q_t, b_t, P_t) &\equiv u(X_{qt}, X_{mt}, X_{1t}) + \alpha EV^{t+1}(Q_{t+1}, b_{t+1}) \\ &= u(X_{qt}, X_{mt}, X_{1t}) + \alpha EV^{t+1} (f(L_t, A_t)\, \varepsilon, \bar{W}_t + (1+r)b_t \\ &+ p_{qt}Q_t - w_tL_t - a_tA_t - P_{qt}X_{qt} - P_{mt}X_{mt} - w_tX_{1t}). \end{aligned}$$

Our discussion above indicates that z_t can be characterized by studying the first order necessary conditions

$$(9) \qquad 0 = \frac{\partial \delta^t}{\partial X_{qt}} = \frac{\partial u}{\partial X_{qt}} - \alpha P_{qt}\, E\left(\frac{\partial V^{t+1}}{\partial b_{t+1}}\right)$$

$$(10) \qquad 0 = \frac{\partial \delta^t}{\partial X_{mt}} = \frac{\partial u}{\partial X_{mt}} - \alpha P_{mt}\, E\left(\frac{\partial V^{t+1}}{\partial b_{t+1}}\right)$$

$$(11) \qquad 0 = \frac{\partial \delta^t}{\partial X_{1t}} = \frac{\partial u}{\partial X_{1t}} - \alpha w_t\, E\left(\frac{\partial V^{t+1}}{\partial b_{t+1}}\right)$$

$$(12) \qquad 0 = \frac{\partial \delta^t}{\partial L_t} = \alpha E\left(\frac{\partial V^{t+1}}{\partial Q_{t+1}} \frac{\partial f}{\partial L_t} \varepsilon\right) - w_t \alpha\, E\left(\frac{\partial V^{t+1}}{\partial b_{t+1}}\right)$$

$$(13) \qquad 0 = \frac{\partial \delta^t}{\partial A_t} = \alpha E\left(\frac{\partial V^{t+1}}{\partial Q_{t+1}} \frac{\partial f}{\partial A_t} \varepsilon\right) - a_t \alpha\, E\left(\frac{\partial V^{t+1}}{\partial b_{t+1}}\right)$$

We now offer some economic interpretations of these conditions. First, (9), (10) and (11) imply

$$(14)\qquad \frac{\partial u/\partial X_{qt}}{P_{qt}} = \frac{\partial u/\partial X_{mt}}{P_{mt}} = \frac{\partial u/\partial X_{1t}}{w_t}$$

This, of course, is the familiar result from static certainty theory that goods and leisure are consumed so as to equate marginal rates of substitution to price ratios. Thus, the household allocates the amount it decides to spend on consumption in accord with the usual efficiency principles.

Intertemporal allocations of goods can be characterized by considering $V^{t+1}(.)$. By definition,

$$(15)\qquad V^{t+1}(Q_{t+1}, b_{t+1}) \equiv \max_{z_{t+1}} \delta^{t+1}(.) = u(X_{qt+1}, X_{mqt+1}, X_{1t+1}) + \alpha E V^{t+2}(Q_{t+2}, b_{t+2})$$

As with choices of z_t, we have the following necessary condition for X_{qt+1}:

$$(16)\qquad 0 = \frac{\partial \delta^{t+1}}{\partial X_{qt+1}} = \frac{\partial u}{\partial X_{qt+1}} - \alpha P_{qt+1} E\left(\frac{\partial V^{t+2}}{\partial b_{t+2}}\right)$$

We also have from (15) that

$$(17)\qquad \frac{\partial V^{t+1}}{\partial b_{t+1}} = \alpha E\left(\frac{\partial V^{t+2}}{\partial b_{t+2}}\right)(1+r)$$

Substituting (17) into (16) yields

$$(18)\qquad \frac{\partial u}{\partial X_{qt+1}} \frac{(1+r)}{P_{qt+1}} = \frac{\partial V^{t+1}}{\partial b_{t+1}}$$

When X_{qt+1} is chosen, b_{t+1} is known. To compare this with the choice of X_{qt} to depict how the household plans to allocate consumption through time requires that we take the expectation of (18), conditional on information available at date t. Then, we substitute into (9) to obtain, after rearrangement,

$$\frac{\partial u/\partial X}{P_{qt}} = \frac{E(\partial u/\partial X_{qt+1})/(1+e)}{P_{qt+1}/(1+r)}$$

Thus, analogous to (14), the household equates the marginal rate of substitution between current consumption and the expected present value of future consumption (discounting at the utility discount rate) of a good to the ratio of current price to present value future price (discounting at the rate of return on the financial asset) of that good.

On the production side, our model can be given familiar interpretations as well. From (15), we have

$$(19) \quad \frac{\partial V^{t+1}}{\partial Q_{t+1}} = \alpha P_{qt+1} \; E(\frac{\partial V^{t+2}}{\partial b_{t+2}})$$

Substituting (17) into (19) taking expectations, and substituting the resulting expression into (13) yields

$$(20) \qquad E[\frac{\partial V^{t+1}}{\partial b_{t+1}} \left(\frac{P_{qt+1}}{(1+r)} \frac{\partial f}{\partial A_t}(.) \; \varepsilon_{t+1} - a_t\right)] = 0$$

This is a first order condition for a firm with risk preferences represented by the utility function $V^{t+1}(.)$ if it were to maximize the expected utility of profits. In our model, costs are incurred at date t and output sold at date t+1; hence, the output price is discounted.[7] Of course, a similar expression holds for the labor input.

It is possible to show that the usual static efficiency conditions concerning the choice of inputs holds in our framework. To see this, divide (12) by (13) to get

$$\frac{E(\frac{\partial V^{t+1}}{\partial Q_{t+1}} \frac{\partial f}{\partial L_t} \varepsilon)}{E(\frac{\partial V^{t+1}}{\partial Q_{t+1}} \frac{\partial f}{\partial A_t} \varepsilon)} = \frac{w_t}{a_t} .$$

But, when evaluated at optimal choices L_t^* and A_t^*, the derivatives $\partial f/\partial L_t$ and $\partial f/\partial A_t$ are constants. Thus, they can be taken out of the expectations operation to achieve

$$(21) \quad \frac{\partial f/\partial L_t}{\partial f/\partial A_t} = \frac{w_t}{a_t}$$

This is a direct consequence of our use of a multiplicative form for our production function as stated in (8). A similar result was derived by Pope and Kramer (1979) in a static model.

Returning to equation (20), we see why the model is not separable into consumption and production aspects of the household's problem. The function $V^t(.)$ is a value function and, therefore, depends on the maximized quantities of all choice variables, including consumption goods. The consumption goods enter $V^{t+1}(.)$ through the transition equation on assets. The risk preferences for solving the problem of maximizing the expected utility of profit must be derived from the household's preferences for income risk and ultimately from their preferences concerning consumption variability. Moreover, the results availble from the theory of the firm and the theory of the consumer under uncertainty do not, in general, carry over to our non-separable model.

5. Increases in Risk

It is apparent from the first order condition stated in (9) - (13) that general comparative statics results regarding changes in prices of goods and inputs, and changes in the interest rate can be obtained in the usual fashion. It also is apparent that, with as many choice variables and parameters as exist in our model, comparative statics results are going to be very tedious to obtain. To see the issues more clearly, we focus on a specific functional form of the general model presented above. In this case, unambiguous results can be obtained and problems of empirical application are more apparent.

This simplification permits the derivation of functional forms of the household's output supply and commodity and factor demand equations and a value function exhibiting constant absolute risk aversion (CARA); it is similar to the form of the indirect utility function derived by Stiglitz. This derivation also demonstrates a type of separability between the household's production and consumption decisions. An empirical example of the model is also presented. While the model was initialized to household data from the Dominican Republic, the empirical results are only intended to illustrate and provide further insights into the relationship between yield variance, risk aversion and the household's choices. And thus, by implication, to suggest some of the likely consequences of not accounting for this type of behavior in the more traditional-nonstochastic model of the agricultural household.

The specific form of the household's additively separable, time invariant utility function corresponding to (1) is

$$U(\{X_{qt}, X_{mt}, X_{1t}\}_{t=0}^{t=T}) = \sum_{t=0}^{t=T} (-\alpha^t \exp\{-\eta_i X_{it}^{\alpha i}\}) + \alpha^{T+1}(-\exp\{-K(b_{T+1})\})$$

where i = q,m,l and, as shown below, it is important to require that the coefficients α_i are positive and sum to unity. Hence, the direct utility function is a negative exponential where the exponential is a Cobb-Douglas (C-D) function, homogeneous of degree one.

No production is assumed to occur in the terminal period T+1 so, in terms of dynamic programming, the household's problem is to choose X_{qt}, X_{mt}, and X_{lt} to maximize terminal period utility subject to a given level of assets b_{T+1}. In this case, it is easily shown that the terminal period utility is given by

$$(22) \qquad -\exp\{-kb_{T+1}\} = -\exp\{-\pi_i \alpha_i^{\alpha_i} P_{iT+1}^{-\alpha_i} b_{T+1}\} = -\exp\{-K(b_{T+1})\}$$

for i = q,m,l. The exponent of b_{T+1} on the LHS of (22) is unity because of the assumption that the values of α_i sum to one.

To simplify the problem, we eliminate the production lag, and for convenience, let production be given by the C-D production function

$$Q_t = \bar{Q}_t \varepsilon = cL^{\gamma_1} A^{1-\gamma_1} \varepsilon, \quad 0 < \gamma_1 < 1,$$

where $\varepsilon \sim^{iid} N(1,V[\varepsilon])$.[8] The problem is further simplified by assuming that (a) prices remain unchanged and hence no time subscript appears on k in (22), and (b) we focus on only two periods. The two period assumption reduces the number of arguments in the t-th period value function but otherwise it does not alter the nature of the problem. The state variable b_{t+1} is given by

$$b_{t+1} = P_q Q_t - aA_t - wL_t + (1+r)b_t + a\bar{A} + w\bar{L} - P_q X_{qt} - P_m X_{mt} - wX_{lt}.$$

In light of the above assumptions, the two time period problem can be stated as:

$$\max_{\{z_t\}} -\exp\{-\pi_i X_{it}^{\alpha_i}\} + \alpha E[-\exp\{-kb_{t+1}\}]$$

or, from the moment generating function, it can be stated as:

$$\max_{\{z_t\}} -\exp\{-\pi_i X_{it}^{\alpha_i}\} - \alpha\exp\{-k(\bar{b}_{t+1} - .5k(P_q\bar{Q}_t)^2 V(\varepsilon))\}$$

where $\bar{b}_{t+1}$ is the mean of b_{t+1}.

From the first order necessary conditions, (14) implies the result, familiar to C-D forms, $X_{it} = (\alpha_i P_j/\alpha_j P_i)X_{jt}$ in the case of consumption while (21) implies the result $L_t = (\gamma_1 a/(1-\gamma_1)w)A_t$ in production. Moreover, the equivalent of (20) in this case is simply

$$P_q(\partial\bar{Q}_t/\partial L_t) - w - kP_q^2\,\bar{Q}_t\,(\partial\bar{Q}_t/L_t)V(\varepsilon) = 0$$

and similarly for A_t. This result is obtained because of the restrictions placed on the α_i. This result suggests a type of separability in the sense that production choices can be made independent of consumption choices. However, contrary to the traditional nonstochastic version of the household model, preferences over goods and leisure affect input choice through the parameters embodied in k. Furthermore, risk aversion, as determined by k, is also a function of prices P_q, P_m and w. Hence, contrary to most treatments of decision making under risk, the simple model illustrated here serves to reinforce the point made in the previous section that production depends on the properties of the direct utility function. Moreover, risk aversion (even in the case of constant absolute risk aversion) is not constant, but instead varies with changes in prices of the arguments appearing in the direct utility function.

The demand and supply functions are derived from the first order necessary conditions and the transversality condition. It can be verified that the household functions are:

$$(23) \qquad X_{it} = (\alpha_i/2P_i)(Y_t - (\log \alpha)/k), \; i = q,m,l$$

where Y_t is the utility certainty equivalent income given by

$$Y_t = P_q\bar{Q}_t - aA_t - wL_t + (1+r)b_t + a\bar{A} + w\bar{L} - .5k(P_t\bar{Q}_t)^2 \; V[\varepsilon].$$

The last term in (23) accounts for the substitution relationship between the utility the household obtains from current, relative to future, consumption. Since the discount term (α) is a fraction, its log is negative which serves to augment certainty equivalent income as preferences for current utility from current relative to future consumption increases. The "2" in the denominator "divides" certainty equivalent income between the current and the next period. Otherwise, (23) bears a close resemblence to the familiar demand functions derived from a direct utility funciton of the C-D form.

These results serve to show more explicitly the nature of the empirical biases that might result by omitting the influence of risk aversion on the household's consumption choices. The compensated price elasticity terms derived from the demand equations (assuming that they can be identified) are likely to be unaffected by risk attitudes. This result is also suggested by (14). However, the profit effect on consumption (equation 7, chapter 2) from a change in the price of a good (staple) produced by the household will likely be overestimated if risk is present in the form considered here. Namely, the income affect of a price change in a good the household produces is likely to be overestimated because the traditional model ignores the risk discount term which will increase in value (and thus decrease income) since

$\partial(.5k(P_qQ_t)^2V[\varepsilon])/\partial P_q > 0.$

The factor demand functions can be verified to be of the form:

(24)
$$L_t = (P_q - \gamma_1^{-\gamma_1}\gamma_2^{-\gamma_2}a^{\gamma_2}w^{\gamma_1}c^{-1})/kP_q^2V[\varepsilon]\gamma_1^{-\gamma_2}\gamma_2^{-\gamma_2}a^{-\gamma_2}w^{\gamma_2}c$$
and
$$A_t = (P_q - \gamma_1^{-\gamma_1}\gamma_2^{-\gamma_2}a^{\gamma_2}w^{\gamma_1}c^{-1})/kP_q^2V[\varepsilon]\gamma_1^{\gamma_1}\gamma_2^{-\gamma_1}a^{\gamma_1}w^{-\gamma_1}c$$

where $\gamma_2 = 1 - \gamma_1$. Hence, planned supply is

$$\bar{Q}_t = (P_qc - \gamma_1^{-\gamma_1}\gamma_2^{-\gamma_2}a^{\gamma_2}w^{\gamma_1})/ckP_q^2V[\varepsilon].$$

As already pointed out, these production relationships include k which contains the parameters and the prices of the arguments appearing in the direct utility function. This is an important departure from the literature where k is related to the Arrow-Pratt coefficient and, in the case of CARA, simply treated as a constant. In this sense, the problem is not separable. However, because of the restrictions placed on the parameters (α_i) of the direct utility function, the problem can be treated as though the household sought to maximize certainty equivalent income (Y_t) and then as though it sought to choose the levels of goods and leisure to consume subject to certainty equivalent income adjusted for the discount factor $\log(\alpha)/k$.

The biases in empirical estimates of the household's production choices from ignoring risk when it is present in the context of the model developed here is to overestimate the quantity of output and the resources allocated to production, and to underestimate the resources allocated to off-farm activities.

To provide some insights into the possible magnitudinal implications of risk aversion and yield variance on the household's choices, the model was initialized to farm household data from the Dominican Republic for the crop year 1975/76. Only those agricultural households reporting rice as their only cash crop were selected for the purposes of this illustration.

The utility function parameters chosen where $(\alpha_q, \alpha_m, \alpha_l) = (.01, .175, .815)$ and the production parameters were $(c, \gamma_1) = (180, .5)$. The other key data used to initialize the model appears in the third column of Table 1. The base solution, to which other solutions of the model are compared, is reported in the fourth column. The values reported in the remaining two columns are the results obtained from parametrically ranging yield variance by a -\+ 25 percent (denoted low and high risk respectively) of the yield variance assumed in the base solution.

As implied by (23), an increase in yield variance results in a decrease in current period consumption; in the case of a 25 percent increase in yield variance, the quantity of rice consumed decreased by about 19 percent. Condition (14), together with the homothenticity of the direct utility function, requires that the ratio of rice consumed to other goods consumed and to leisure remain unchanged to variations in yield variance. Thus the consumption of these items decreased accordingly.

An increase in yield variance also induces the household to decrease the quantity of rice produced by about 19 percent. Since the production function is homogeneous of degree 1, it follows from (21) that the labor-land ratio remains unchanged and rice yields remain unchanged. The increase in yield variance induces the household to increase the amount of land rented out and to decrease the amount of labor hired while, at the same time, reducing the amount of leisure consumed. In spite of the household's efforts to avoid the disutility of increases in the variance of yields (and hence income) assets transferred to the next period (b_{t+1}) decline.

It is clear from (23), (24) and the results reported in Table 1 that declining consumption and the transfer of resources to other activities is not a linear function of changes in yield variance. The empirical nature of this non-linearity for the illustrative problem considered here can be gleaned from Figure 1 where changes of the household's choices to numerous solutions of the model are charted. As yield variance increases, the welfare of the household becomes dependent on labor, land and asset markets. It is possible for the household to reach a point where it withdraws all of its land and labor resources from rice production.

Asset holdings, certainty equivalent income and the quantity of rice sold are charted in Figure 2. Rice sales decline as resources are withdrawn from rice production in spite of the household's decline in the quantity of rice consumed. At a sufficiently high yield variance, the household will become a deficit producer of rice. The level of asset holdings will also depend on the "riskless" alternatives the household faces in the asset, land and labor markets. Similarly, the level of certainty equivalent income will tend to converge, though at diminishing rates, to the income earned from the household's resources allocated to these markets.

6. Duality and Risk Aversion.

Duality results are very useful for providing restrictions on parameters in empirical investigations. For example, Hotelling's Lemma (Varian, 1978) and the symmetry of cross second derivatives (Young's Theorem) establishes the symmetry of derivatives of input demands with respect to factor prices.

The value function $V^t(.)$ is a dynamic indirect utility function. As such, one would expect that an analog of Roy's Identity would emerge relating goods demands and the derivatives of the value function.

Let

$$z^*(P_t) \quad (X^*_{qt}\ (P_t) - Q^*_t,\ X^*_{mt}\ (P_t),\ L^*_t\ (P_t) + X^*_{1t}\ (P_t) - \bar{L},\ A^*_t(P_t) - \bar{A}).$$

The first component of this optimal choice vector is the household's demand for the agricultural staple net of current supply. The third component is the household's net position in the labor market; i.e., it is purchases of market labor minus the hours the household works off the farm. Thus, it is net demand for labor. Of course, it may be negative and the household may be a net supplier of labor. Similarly, the last component is the household's net position in the rental market for land.

Differentiation of the value function and use of the envelope theorem constitutes a proof of the following analogy of Roy's Identity:

$$(25) \qquad z^*(P_t) = - \nabla_{pt} V^t(Q_t, b_t)/\alpha E(V^{t+1}(.)/\partial b_{t+1}),$$

where $\nabla_{pt} V^t$ is the gradient vector of partial derivatives of V^t with respect to prices. The denominator, the expected value of the marginal utility of wealth, plays the role of the derivative of the indirect utility function with respect to income in Roy's Identity.

Two points are worth noting. First, the market surplus and purchased good demand correspond to similar results obtained from applying the equivalent of (25) to the static model. The component for labor reflects net positions in the market as well as leisure decisions and the household's endowment.

Second, the net factor demand results correspond to the duality results obtained by Pope (1980, Eq. (8)) for the risk averse firm under price uncertainty. As shown by Pope (1978), no simple and general comparative static results are obtainable from the static model under uncertainty without additional restrictions on the form of the utility function. Thus, it is clear from (25) that no simple and general results can be derived from the general model. The efficiency in production results (21) suggests that properties of the cost function and the corresponding conditional factor demand functions are with one exception identical to those obtained from static efficiency theory. The exception is that the output ($\bar{Q}_t$) variable is planned (and hence not observable) and not realized output.

For the type of separability that exists in the specific model discussed in section 5, note that the first order conditions are identical to those obtained from maximizing certainty equivalent income, Y_t. In this case, the term $E\{\partial V^{t+1}(Q_{t+1},b_{t+1})/\partial b_{t+1}\}$ does not appear so that L_t^* and A_t^* follow directly from the envelope theorem. However, Q_t^* does not follow from the theorem because P_{qt} appears in the risk premium term of Y_t. From the Hessian of this problem, it can be shown that the sign of $\partial A_t^*/\partial a_t$ and $\partial L_t^*/\partial w_t$ cannot be established although symmetry of the cross partial derivatives holds.

It is important to point out that equation (25) is highly dependent on our assumptions that the production shocks are independently distributed and the prices are fixed and known. For, suppose to the contrary that production shocks form a Markov process and that they induce a Markov process on prices. Then, the current prices will condition the distribution of the future prices. In this circumstance, a derivative of $V^t(.)$ with respect to price has effects on both the choice variable directly and indirectly through an alteration in the household's subjective probability estimate of future prices (Taylor, 1984).

7. Discussion

In this brief chapter we have attempted to introduce production risk into a model of the agricultural household in as simple a manner as possible. Even then, we see that the analysis of the model becomes difficult. The main reason for this difficulty is that, in general, when risk is introduced separability of the model into independent consumption and production "sides" no longer obtains. This lack of separability severely complicates both the analysis of the theoretical model and the empirical estimation of the model's parameters.

Estimation of these parameters is of key importance, however, since many policy relevant results are ambiguous on a theoretical level and need to be determined empirically. Failure to account for risk and risk aversion when it is present can lead to misleading inferences. The specific model suggests that the income effect of an increase in the price of a staple might significantly overestimate the level of resources employed in its production. The consideration of risk also clearly establishes the importance of markets which permit households to self insure against increasing yield risk.

There are several issues raised by our analysis which are candidates for further research along these lines. Here we briefly discuss a few of these possibilities. First, and most obviously, it would be useful to have knowledge of what alternative functional forms for utility and production functions, in combination with distributions on the random variable, imply for behaviors toward risk and the effects of increases in risk. This would be beneficial for two reasons. First, it would clarify the results of our model under plausible representations of household activities. This may be of some policy relevance. For example, if increases in risk reduce consumption and production intensity, then institutions which allow more efficient risk sharing could increase output and consumption. Secondly, results establishing relationships between functional forms and comparative statics results (such as the type of separability found in the specific model) could guide researchers toward appropriate tests of the theory and away from imposing results by assumption.

A second avenue for further elaboration of the model concerns the form of the production function. This should be generalized in two ways. First, work by Pope and Kramer (1979) demonstrates that the assumption of multiplicative risk is quite special. In particular, it is this assumption which allows us to conclude that factors are used in accord with static efficiency principles (equation 21). As well, the multiplicative form implies that all inputs are risk increasing. More general formulations which are tractable yet allow risk-reducing inputs have been proposed.[9] Second, we have assumed that all inputs must be chosen before the realization of the random variable is known. The literature on the firm under price uncertainty informs us that the <u>timing</u> of the resolution of uncertainty relative to the timing of input choices is crucial to the effects of an increase in risk on production decisions.[10]

It would seem reasonable in our context to allow the household some ex-post flexibility. For example, while inputs associated with planting are fixed, irrigation decisions can be altered in response to the current realization of rainfall. The substitution possibilities between these ex-ante and ex-post inputs will be important in establishing the response of factor use to increase in risk. This would apply as well in a model with investment in durable capital.

A third possible generalization of the model would be to provide for multiple good production. This would allow an understanding of the choice of crop portfolios by households. One would conjecture that covariances of yields across crops would prove to be important. This would be on considerable policy relevance where some crops are grown for consumption and some for export.

It seems odd to write down a model with quantity risk but no price variability. This is especially problematical when production shocks are correlated across large numbers of households. Price variability can be incorporated into the model by considering joint distributions on prices and production shocks. It is possible to define and study increases in risk in this situation as well (see Epstein, 1978). However, as mentioned earlier, if prices are not independently distributed, then duality results become difficult to interpret. If all of the risk in prices is due to production risk, independence may be a reasonable assumption.

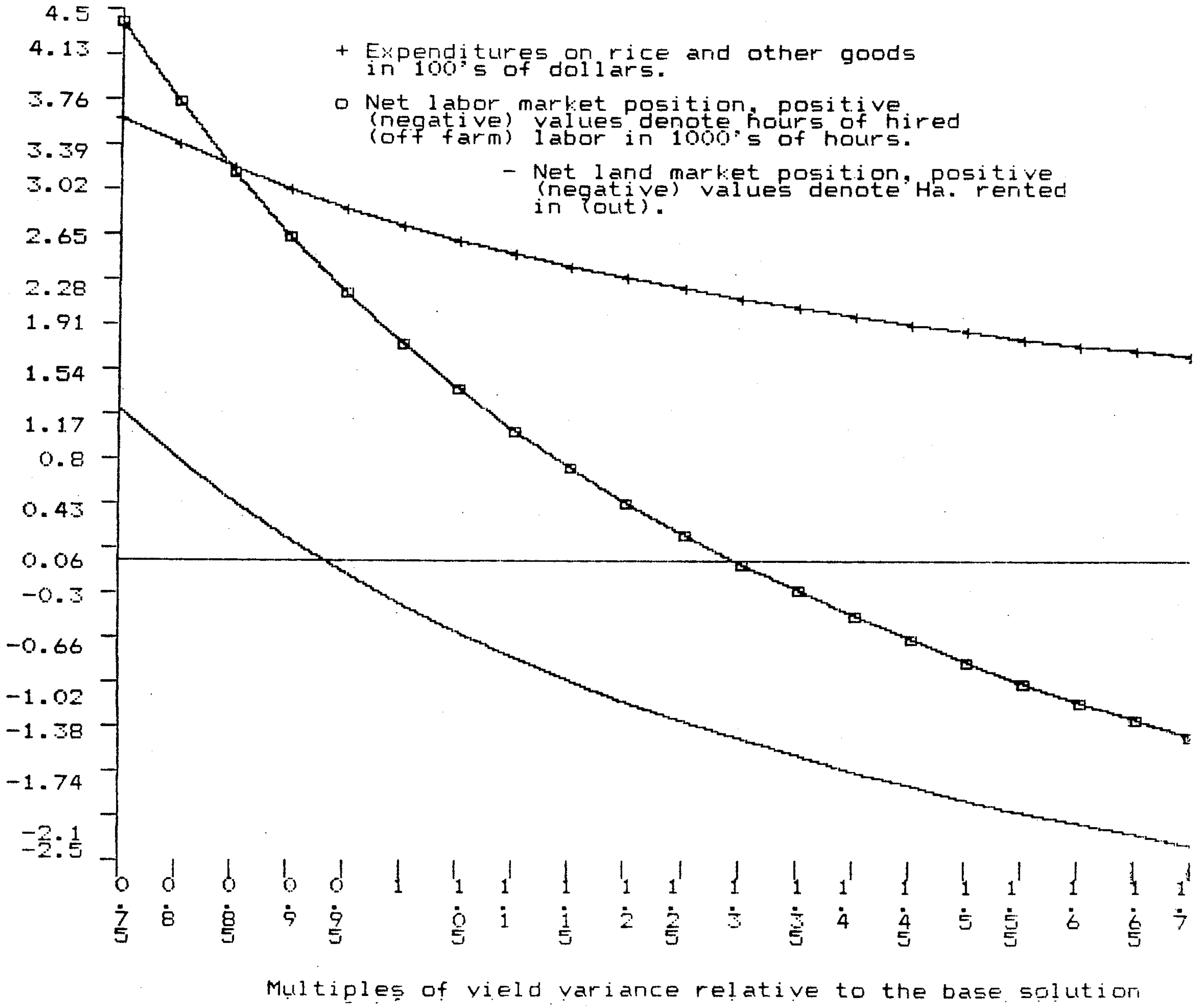
The consideration of a relationship between price risk and production risk points out one of the many issues involved in moving from a single-agent to a market model. In particular, and is well-known, some assessment or assumption of how agents form expectations is needed. This, of course, raises several issues of current debate in economics which must squarely be faced in future research.

Table 1

Selected Results to Illustrate the Effects of Yield Risk on Choice Variables

Item	Units	Values used to initialize model	Base Solution of model	Solutions obtained for two levels of yield risk measured relative to base solution[1] (Low risk)	(High risk)
Household Rice Consumption*	Kg./Household[2]	48.9	42.2	55.6	34.2
Rice price	$/kg.	0.352	unchanged		
Other goods consumption*	Index	91.2	173.4	228.4	140.4
Other goods Price	Index	1.5	unchanged		
Total expenditure*	$/Household	154.0	275.0	362.1	222.7
Production Labor input*	Hours/Ha.	1072.0	1066.6	1066.6	1066.6
Land in rice*	Ha.	5.2	4.8	6.4	3.9
Yield*	Kg./Ha.	3127.0	5879.0	5879.0	5879.0
Land rental rate	$/Ha.	448.0	unchanged		
Labor wage	$/Hour	0.42	unchanged		
Sales*	q/Yr.	162.1	282.8	377.1	226.3
Net labor allocation*	Hr./Yr.[3]	3231.5	1783.9	4411.3	207.5
Net land allocation*	Ha./Yr.[3]	na	-0.372	1.2	-1.3
State variable b(t+1)	100's $/Yr.	na	43.13	57.3	34.7

*Denotes choice variables.
[1]Yield risk of the base solution was augmented by the multiples .75 and 1.25 for the respective low and high yield risk solutions.
[2]Rice consumed is in terms of rough rice.
[3]Positive (negative) values denote quantities of hired (off farm) labor and similarly for land.


4.5
4.13
3.76
3.39
3.02
2.65
2.28
1.91
1.54
1.17
0.8
0.43
0.06
-0.3
-0.66
-1.02
-1.38
-1.74
-2.1
-2.5
+ Expenditures on rice and other goods in 100's of dollars.
o Net labor market position, positive (negative) values denote hours of hired (off farm) labor in 1000's of hours.
- Net land market position, positive (negative) values denote Ha. rented in (out).
0.75
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1.55
1.6
1.65
1.7
Multiples of yield variance relative to the base solution

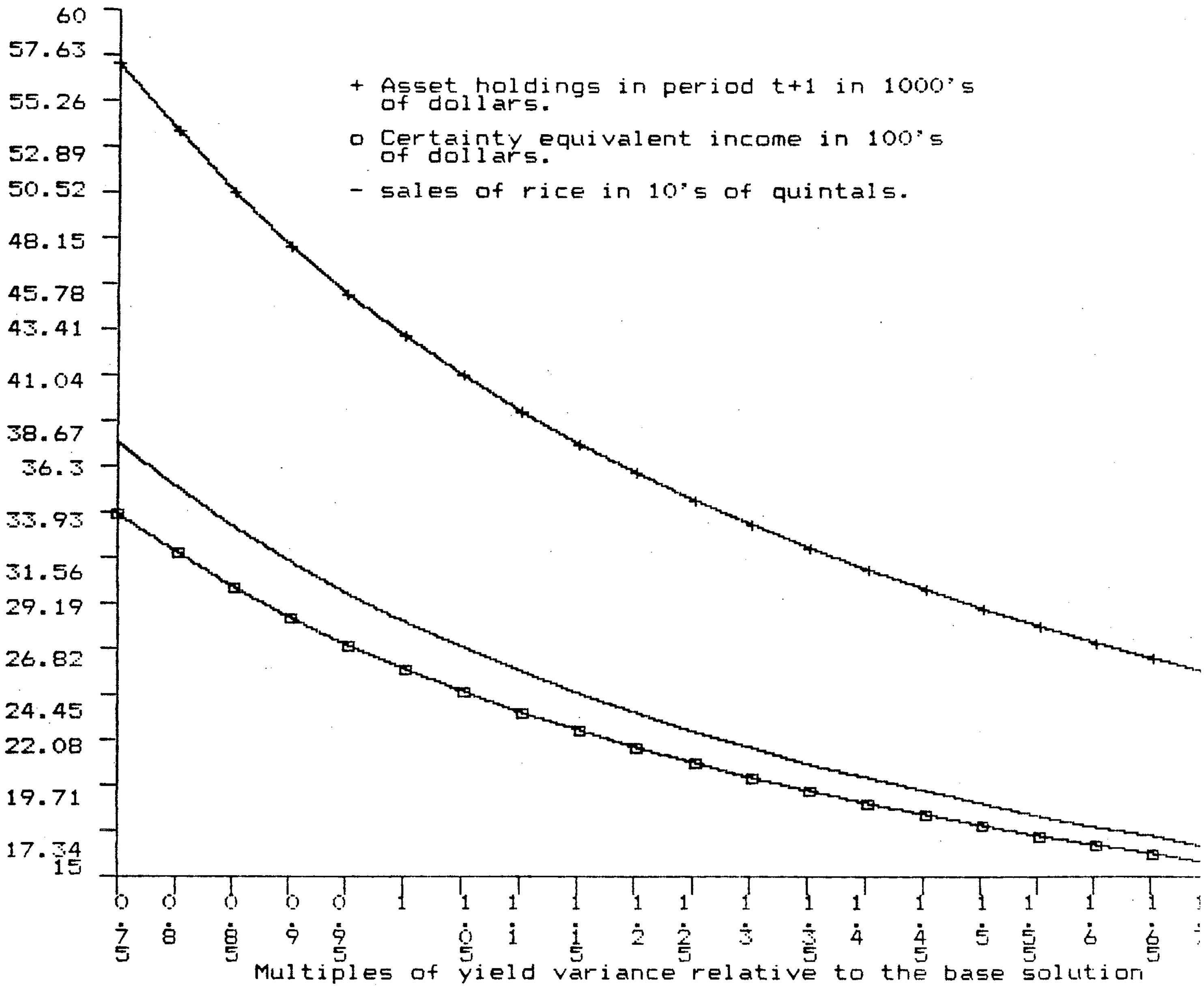

+ Asset holdings in period t+1 in 1000's of dollars.
o Certainty equivalent income in 100's of dollars.
- sales of rice in 10's of quintals.
60
57.63
55.26
52.89
50.52
48.15
45.78
43.41
41.04
38.67
36.3
33.93
31.56
29.19
26.82
24.45
22.08
19.71
17.34
15
Multiples of yield variance relative to the base solution

FIGURE 2

FOOTNOTES

[1]The authors thank John Strauss, Cliff Hildreth, Bob Myers, and participants in the Consumption Economics Workshop at the University of Minnesota for helpful comments on an earlier draft of this chapter.

[2]For a description of how farmers diversify crop production activities in order to lower the variation in their income associated with yield risk, see Walker and Jodha (1985).

[3]Leland (1968) considers income uncertainty in a two-period model with a utility function which is not additively separable over time, i.e., one of the form $U(C_1,C_2)$. He finds that if $(\partial^2U/\partial C_2^2)/(\partial U/\partial C_2)$ is increasing in C_1 and decreasing in C_2, then savings increases with increasing uncertainty. This result also is obtained by Sandmo (1970) for small risks. Mirman (1971) studies an additively separable utility funciton $U(C_1,C_2) = U^1(C_1) + U^2(C_2)$ in a two-period model. He shows that with rate of return uncertainty, period 1 savings increase (decrease) with an increase in uncertainty if $C_2dU^2(C_2)/dC_2$ is a convex (concave) function. Dreze and Modigliani (1972) provide a comprehensive exploration of the two-period model, including income and substitution effects of increasing uncertainty.

Phelps (1962) established, in an infinite horizon model with additively separable utility, that if the pre-period utility function exhibits decreasing absolute risk aversion, then an increase in income uncertainty increases savings. Miller (1976) generalizes the Phelps result somewhat. Miller demonstrates that, with an infinite horizon and additively separable utility function, consumption decreases when the sequence of incomes becomes more risky in the sense of Rothschild and Stiglitz (1970) if the marginal utility of consumption is convex. A similar result is obtained by Sibley

(1975) for a finite horizon model. A more complete review is provided by Lippman and McCall.

[4]There is a large literature on firms facing price uncertainty. A good summary and treatment is Epstein (1978).

[5]It is common to see analyses of firms under price uncertainty which posit some form of a utility function over profits (e.g., risk neutrality or risk aversion) with no discussion of where such a utility function comes from. A virtue of the household production model is that risk preferences concerning profit are deduced from risk preferences over consumption. That the introduction or risk may eliminate separability was pointed out by Barnum and Squire (1979, note 16, p. 39).

[6]Thus, general equilibrium models (with consumer incomes tied to firm profits) in which contingent claims markets do not exist and risk neutral behavior on the part of firms is posited may be inconsistent. A set of securities which spans the states of nature may replace contingent claims markets (Arrow, 1960).

[7]To see this, consider the problem

$$\max_{X} E\, U(\pi);\ \pi = pf(X) - w \,.\, x$$

First order conditions are

$$E\, [U'(\pi)\ (p\frac{\partial f}{\partial X_j} - w_j)] = 0 \text{ for all } j.$$

[8]The normality assumption implies the absurdity that a non-zero probability exists that negtive and extremely high yields might be observed. The alternative is to apply the formulas for the moments of a truncated normal distribution (see Johnson and kotz, pp. 81-83) or to maintain that the variance of ε is sufficiently small that our treatment leads to a good approximation of the actual distribution of yields. Another alternative is to permit to be distributed log normal. Levey shows that mean variance

analysis applied to a log normal distribution is a sufficient decision rule for all non-decreasing strictly concave utility function. In any case, the more rigorous approach of employing the formulas of a truncated normal distribution would seem to unnecessarily clutter the key purpose of the task at hand. Hence, we proceed with the normality assumption.

[9]Pope and Kramer suggest the form

$$F(A,L;\varepsilon) = f(A,L) + h(A,L)\varepsilon,$$

which admits risk reducing inputs depending on the shape of the function h(.). An input is risk reducing (increasing) if risk averse producers use more (less) of it than a risk neutral producer. One of the basic implications of our analysis is that risk neutrality does not make sense once consumers are added to the model explicitly, except under stringent assumptions.

[10]See, for example, Epstein (1978) for a review of earlier work in this area.

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