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THE INVARIANCE OF R+D TO THE NUMBER OF FIRMS IN THE INDUSTRY: EQUILIBRIUM AND EFFICIENCY UNDER BERTRAND COMPETITION

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ABSTRACT

This paper presents certain results, of remarkable simplicity, concerning the market allocation of resources to research and development (R+D) and its comparison to socially efficient allocations. In contrast to many previous studies, we posit that a firm can undertake more than one project (at desired levels of intensity) aimed at the same innovation, if it is profitable to do so. The market is characterized by a Bertrand equilibrium in the product market. We show that the marginal private decisions are independent of the number of firms in the industry; as a result, the equilibrium R+D (that is, the number of projects undertaken in the market, and the level of effort spent on each project) is invariant to the number of firms. The equilibrium level of effort per project is also invariant to the magnitude of (appropriable) rents from successful innovation.

The number of firms affects the gains from innovation to consumers and firms; for any research program, a larger number of firms entails larger gains to consumers, smaller gains to firms, and larger aggregate social benefits. While the market equilibrium level of effort per project is shown to coincide with the socially efficient level, the market undertakes fewer projects than is desirable.

THE INVARIANCE OF R+D TO THE NUMBER OF FIRMS IN THE INDUSTRY: EQUILIBRIUM AND EFFICIENCY UNDER BERTRAND COMPETITION

Raaj Kumar Sah and Joseph E. Stiglitz*

A major concern of the recent research in the theory of innovation has been the effect of market structure on private marginal returns from innovation, and, thus, on the equilibrium level of market R+D. Recent work has also emphasized the relationship between marginal private returns and social returns which, in general, may not be the same.[1]

The present analysis is based on a model in which the product market is characterized by Bertrand equilibria. We establish four remarkably simple results.

(i) <u>The number of firms in the industry has no effect on the pace of innovation</u>. That is, the marginal decisions of a firm to undertake an additional research project, or to spend additional efforts on a project, are unaffected by the number of firms. The resulting invariance of the market equilibrium is in marked contrast with many previous studies which have found the number of firms in the industry to be a critical determinant of the market R+D. One of the consequences of our result is that policies aimed at altering the number of firms in the industry have no effect on market innovation.

(ii) <u>The intensity at which a research project is pursued in the market is invariant to the magnitude of (appropriable) rent from successful innovation</u>. <u>If the rent is larger, then the number of projects undertaken is larger</u>.

(iii) The number of firms in the industry affects the gains from innovation to firms and consumers and, thus, it affects aggregate social gains. Specifically: A larger number of firms raises consumers' gains, lowers firms' gains, and raises aggregate social gains from innovation. This result also differs from that of some earlier studies which have suggested that a larger number of firms may reduce the incentives for innovation to a degree that the social welfare is reduced.

(iv) Regardless of the number of firms in the industry, fewer projects are undertaken in the market than is socially optimal, though the intensity at which each project is pursued is optimal.

An important feature of our model is that a firm may undertake more than one research project aimed at the same innovation, if it is profitable to do so. This assumption, we believe, is more plausible from an economic viewpoint than the one underlying many previous models, in which a firm can undertake only one research project. Also, it is easy to understand why this difference in assumption has a significant effect on the analysis of R+D. Under our assumption, a firm has a larger set of instruments and thus, in general, its behavior is quite different from that when it is constrained to undertake a single project. The resulting market equilibrium in research is also, therefore, different. This argument holds regardless of the particular model one uses (for example, the particular assumptions one makes concerning the nature of the product market competition, and the strategic environment of firms); though the specific implications of our assumption would, of course, depend on the characteristics of the model.

THE MODEL AND RESULTS

A research project has a binary outcome: successful innovation or status quo. If e is the variable effort (expenditure) on a research project, then the probability of its success is $p(e)$, where $e \geq 0$, $1 \geq p \geq 0$, and $p_e > 0$.[2] The outcome of different projects is independent of one another. A firm can undertake as many projects as it desires, all of which are aimed at the same innovation. Thus, if e_{ij} denotes the effort by the i-th firm on its project j, and if this firm undertakes $j = 1, \ldots, k_i$ projects, then the probability that at least one of the projects undertaken by this firm is successful is given by $q_i = 1 - \prod_{j=1}^{k_i} (1 - p(e_{ij}))$.

The product market is characterized by Bertrand competition. Specifically, the rent gained by a firm is R if it innovates and if no other firm innovates.[3] If two or more firms innovate, then the benefits of innovation accrue solely to consumers. (The determination of R, and that of consumers' gains, is discussed later.) h_i denotes the probability that all firms, other than the i-th firm, are unable to innovate. That is, $h_i = \prod_{f \neq i} \prod_{j=1}^{k_f} (1 - p(e_{fi}))$, where $f = 1, \ldots, N$ denotes the firms. $N \geq 1$, and it is finite. Then, the (expected) profit of firm i is $\pi_i = Rh_i q_i - \sum_{j=1}^{k_i} (e_{ij} + a)$, where a is the fixed cost of undertaking a project.

We focus here on the symmetric interior Nash equilibrium in which each firm undertakes the same portfolio of projects and, further, if a firm undertakes more than one project, then it undertakes identical projects.[4] Therefore

(1) $q = 1 - (1 - p(e))^{k}$, and

(2) $h = (1 - p(e))^{Nk-k}$.

The first order conditions with respect to e and k , for a firm's optimum, are: $Rhq_e - k = 0$, and $Rhq_k - (e + a) = 0$, respectively. These equilibrium conditions can be restated, using (1) and (2), as[5]

(3) $R(1 - p)^{n-1}p_e - 1 = 0$, and

(4) $-R(1 - p)^{n}\ln(1 - p) - (e + a) = 0$,

where $n = Nk$ is the total number of projects undertaken in the market.

Note that the above expressions determine the investment, e , in each project, and the total number, n , of projects undertaken in the market. A change in N simply changes k , while keeping n and e unchanged. Thus, the only effect of N is on the number of projects a firm undertakes, which is $k = n/N$. In a duopoly, for instance, each of two firms undertakes half as many projects as a monopoly would have undertaken.

It follows then that the number of firms in the market has no impact on (a) the total number of research projects undertaken; (b) the intensity of each of the projects; and, therefore (c) the probability of a successful innovation. It is also apparent that public policies aimed at altering the number of firms do not influence the nature of research activities undertaken in the market.

The intuitive idea behind this result is as follows. Consider the marginal decision of a firm to undertake the last project (or to invest the last dollar on a project). This project (or dollar) yields a benefit only if the other projects undertaken by this firm fail, as well as if all of the projects undertaken by other firms fail. The marginal decisions are thus influenced by the total number of projects undertaken in the market; and not by how these projects are distributed between the firm making the decision and other firms. Thus, whether the marginal project yields a return, as well as the return from the marginal effort invested in a project are independent of the number of firms. Furthermore, it is easily verified that this independence holds in more general models as well; for instance, when a firm has a vector of control variables, e, and when the expected cost of a project is a general function of the control variables.

A still stronger result is obtained by solving (4) for $(1 - p)^n$ and substituting the resulting expression into (3). This yields

$$(5) \qquad -(e + a)p_e/(1 - p)\ln(1 - p) - 1 = 0 \ .$$

The above expression characterizes the optimal e, and it does not contain R or N. Thus, the optimal effort per project is independent not only of the number of firms in the industry, but also of the magnitude of rent from successful innovation. Further, by perturbing (3) with respect to R, and noting that e is invariant to this perturbation, we obtain

$$(6) \qquad dn/dR = -1/R\ln(1 - p) > 0 \ .$$

Thus, a larger number of projects is undertaken in the market if the rent from innovation is larger.

The above analysis also brings out clearly the difference between the consequences of our assumption that k is determined endogenously, and the standard assumption under which k is exogenously fixed at unity. In the latter case, it is apparent from (3) that the optimal effort per project (and hence the probability of a successful innovation in the market) depends, in general, on the number of firms.

Welfare Analysis: The neutrality results we have derived might give an impression that public policy (affecting the number of firms in the industry) has no role to play in the context of research and innovation. This is not correct because, as we shall see, the number of firms has a significant effect on the gains from innovation to firms and consumers.

First, consider the gains to consumers. Suppose the successful outcome of a research project leads to a reduction in the (fixed) unit cost from c_0 to c_2.[6] The current (competitive) price is c_0. If only one firm innovates then it effectively becomes a monopoly, and sets a price c_1, where $c_0 > c_1 > c_2$. The corresponding rent to the firm is R, and the consumers' gain is $S_1(c_0, c_1)$. If two or more firms innovate then, due to Bertrand competition, the price is reduced to c_2, and the consumers' gain is $S_2(c_0, c_2)$. Clearly, $S_2 > S_1$, and $S_2 - S_1 > R$, from the standard arguments based on consumer surplus.

Thus the (expected) gain to consumers is

$$(7) \qquad S = S_2 g + S_1 Nhq$$

where $g = \sum_{i=2}^{N} \binom{N}{i} q^i (1 - q)^{N-i}$ is the probability that two or more firms are able to innovate, and Nhq is the probability that only one firm is able to innovate. g can be expressed as

(8) $g = z - Nhq$, where

(9) $z = 1 - (1 - p)^n$.

Therefore, the gain to consumers can be rewritten as $S_2 z - (S_2 - S_1)Nhq$. Now, note from (9) that z does not depend on N . Further[7]

(10) $d(Nhq)/dN = h[k\ln(1 - p) + q] < 0$.

Thus, the gain to consumers is larger if the number of firms is larger. This is what we would expect, because if the same number of total projects is divided among a larger number of firms then the probability of two or more firms being able to innovate is higher and, hence, the gain to consumers is larger.

The above argument also suggests that a larger number of firms would lower the aggregate profit of firms. This can be ascertained as follows. The aggregate corporate profit is given by

(11) $N\pi = RNhq - Nk(e + a)$.

Now, note that the last term in the above right hand side does not depend on N , whereas, from (10), the first term is decreasing in N . Thus, $d(N\pi)/dN < 0$. Further, $d\pi/dN = [d(N\pi)/dN - \pi]/N < 0$, if a firm's profit is nonnegative (which we assume). Therefore, a larger number of firms lowers the profit for a single firm, as well as for the industry profit.

Since the number of firms has opposite effects on consumers and firms, we combine these two effects to study the societal implications. Our analysis here assigns equal weights to the gains of consumers and firms but, as we shall see, some of our results hold for asymmetric weights as well. The social gain is $B = S + N\pi$, which, from (7) and (11), can be expressed as

$$(12) \qquad B = S_2 z - (S_2 - S_1 - R)Nhq - Nk(e + a) \ .$$

Using the same arguments as before, and recalling that $S_2 - S_1 > R$, it follows that a larger number of firms yields a larger social gain from innovation.[8] This result has a simple interpretation. An increase in N increases the probability of consumers' gain by the same amount as it reduces the probability of firms being able to capture the rents, but the gains to consumers exceed the losses to firms.

The last result also provides some insights on public policy. If the government can alter the number of firms in a non-distortive manner, then the (optimal) number of firms should be set such that each firm undertakes a single project. An example of non-distortive instrument is an entry subsidy; provided the social weights on public revenue and corporate profits are the same. (This conclusion, obviously, does not extend to distortive instruments, such as investment tax credits.)[9]

<u>Social Optimum</u>: We finally consider the socially optimal resource allocation to R+D, and contrast it with the market allocation described above. Let n denote the number of projects undertaken by the planner. Then z, given in (9), is the probability that at least one project is successful; in which case consumers receive the full benefits of innovation. The expected social gain is: $S_2 z - n(e + a)$. The corresponding first order conditions, with respect to e and n, characterizing the internal optimum, can be expressed as

$$(13) \qquad S_2(1 - p)^{n-1} p_e - 1 = 0$$

$$(14) \qquad -S_2(1 - p)^n \ln(1 - p) - (e + a) = 0 \ .$$

Note the similarity between the social allocation described above, and the market equilibrium described by (3) and (4). The two sets of expressions are identical except that the gain from successful innovation is R for a firm, whereas it is S_2 for the planner. This similarity should not be surprising because, once again, the marginal decision of the planner (to undertake the last project, or to invest the last dollar on a project) depends on the total number of projects that have already been undertaken; just the way it did for a firm in the market. Now, recall that $S_2 > R$, $de/dR = 0$ and, from (6), $dn/dR > 0$. An immediate consequence of the above similarity, then, is that the market undertakes fewer projects than is socially desirable, but each project is undertaken at the socially efficient level.

CONCLUSIONS

The relationship between the market structure and the nature of market R+D, and that between the private and social marginal returns from innovative activity are, in general, complicated. This paper establishes two invariance results in the central case of Bertrand competition: the market R+D (that is, the number of research projects undertaken as well as the nature of individual projects) is invariant to the number of firms in the industry; and the nature of individual projects is also invariant to the magnitude of the rent that a firm gains from successful innovation.

Use of these invariance results has enabled us to compare the research undertaken in the market to socially efficient allocations. We have shown, for instance, that the market undertakes a smaller number of projects than is socially desirable, though each project is undertaken at the socially efficient level. We have also hinted at some policy conclusions; for example, the desirability of increasing the number of firms (which yields larger gains to consumers, smaller gains to firms, and larger social welfare gains). Similar relationships between social and private returns may not, however, obtain under other forms of competition (for example, Cournot). Our analysis thus suggests the need to compare the outcomes of policies aimed at encouraging price competition versus other forms of competition (for example, quantity competition). The means by which the government may affect a choice in the <u>modes</u> of competition is, however, a question beyond the scope of this short paper.

FOOTNOTES

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1. For instance, in patent races, where the private return is either zero, when the firm is not first to invent, or the total (appropriable) return when it is; while the social return is the increase in the present value from having the invention earlier than it otherwise would have been available. See Barzel (1968), Dasgupta and Stiglitz (1980), Kamian and Swartz (1982), Loury (1979), and Stiglitz (forthcoming).

2. Subscripts e and k denote partial derivatives with respect to these variables.

3. Here we abstract from issues concerning the timing and the scale of innovations; that is, by spending more resources, one can alter the date of innovation or the magnitude of rent.

4. There may not always exist a symmetric interior Nash equilibrium, because of the non-concavity of the relevant functions. At an interior equilibrium, $e > 0$, $k \geq 1$, and both e and k are finite.

5. For simplicity, we are treating k as a continuous variable. If k is treated as an integer, then the expression analogous to (4) is: $R(1 - p)^{n-1}p \geq (e + a) \geq R(1 - p)^{n}p$, with at least one strict inequality. This does not affect the invariance result derived below.

6. The present analysis can be easily extended to the case where the unit cost is not fixed.

7. The sign of the right hand side of (10) is obtained as follows. $q(k)$ is easily seen to be strictly concave in k. Thus $q(k) - q(k = 0) < q_k(k = 0)k$. Using (1), then, $k\ln(1 - p) + q(k) < 0$, Thus, (10) is negative.

8. This conclusion holds even if the social weight on consumers' gain is larger than that on firms' profits.

9. It should also be pointed out that certain instruments of policy may not be feasible due to informational problems. For example, it may be difficult to monitor the number of projects undertaken by a firm.

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