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A "GOLD STANDARD" ISN'T VIABLE UNLESS SUPPORTED BY SUFFICIENTLY FLEXIBLE MONETARY AND FISCAL POLICY

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A "Gold Standard" Isn't Viable Unless Supported by Sufficiently Flexible Monetary and Fiscal Policy

ABSTRACT

The paper studies an idealized gold standard in a two-country setting. Without flexible national domestic credit expansion (dce) policies which offset the effect of money demand shocks on international gold reserves, the gold standard collapses with certainty in finite time through a speculative selling attack against one of the currencies. Various policies for postponing a collapse are considered.

When a responsive dce policy eliminates the danger of a run on a country's reserves, the exogenous shocks disturbing the system which previously were reflected in reserve flows, now show up in the behaviour of the public debt. Unless the primary (non-interest) government deficit is permitted to respond to these shocks, the public debt is likely to rise (or fall) to unsustainable levels.

For the idealized gold standard analysed in the paper, viability can be achieved only through the active and flexible use of monetary and fiscal policy.

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1. Introduction

This paper is a theoretical study of the viability of a fixed exchange rate regime which for brevity I refer to as a gold standard. In a two-country world the monetary authority of each country guarantees the convertibility of its non-interest bearing fiat currency into gold at a fixed price. There is an exogenously given global stock of gold, which has no non-monetary uses. I therefore abstract from one feature of the historical gold standard, under which the international reserve asset had uses as a private consumption good and as an industrial intermediate good and raw material, and could itself be produced through an extractive production process. The essence of "gold" in this model is that it is an "outside" non-interest bearing fiduciary asset whose aggregate quantity is exogenous to each private and public agent in the global economic system and[1] to the system as a whole.

By viability of the gold standard I mean its probability of survival and the expected duration of its survival. Special attention is paid to the way in which particular monetary, financial and fiscal policy actions, or rules, affect the viability of the system.

The paper extends to a two-country setting the single country analyses of a collapsing managed exchange rate regime by Krugman [1979], Flood and Garber [1983, 1984a, b], Obstfeld [1984a, b, c] and others.

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[1] For a more descriptively realistic analysis of certain aspects of the historical Gold Standard see e.g. Barro [1979], Eichengreen [1984, 1985a,b], Barsky and Summers [1985], Bordo and Schwartz [1984].

These papers in turn were macroeconomic applications of the seminal paper by Salant and Henderson [1978] on collapsing commodity stockpiling schemes. My approach owes a lot to the work of Grilli [1986] who analysed buying and selling attacks on the same currency in a small country model. I use his idea of specifying two shadow floating exchange rates, one governing a buying attack and one governing a selling attack, although for the formal analysis in this paper I reduce these two to a single shadow floating exchange rate between two absorbing barriers. Flood and Garber [1983] use a log-linear continuous time stochastic two-country model to analyse a different class of problems (stochastic process switches) which could be formalized in terms of the behaviour of a stochastic process and a single absorbing barrier.

Section 2 outlines the model. Section 3 develops the characterization of the "shadow floating exchange rate" as a Wiener process, with or without drift, between two absorbing barriers. As long as the shadow floating exchange rate stays between the barriers (in the viable or safe range) the gold standard survives. When the shadow rate reaches either barrier, the gold standard collapses through a speculative selling attack on one of the two currencies.

Section 4 looks at the various ways in which policy actions can affect the shadow floating exchange rate process and thus the viability of the gold standard. It considers in detail those policy actions that, (leaving unchanged the nature of the shadow floating exchange rate process as a Wiener process with drift between two absorbing barriers and leaving unchanged the variance of that process), alter the width of the viable range, the drift paramer (the instantaneous mean of the process) and the initial value of the shadow rate. Section 5 considers feedback policies that turn the (non-stationary) Wiener process into a stationary stochastic process. Section 6 looks at policies that can alter directly the variance parameter of the original Wiener process.

2. The Model

The two country model to be analysed is a linearized version of the model given in equations (1) to (10).[2]

1) $\ell(i, y) = \frac{M}{P}$ $\qquad \ell_i = \ell_i^* < 0; \; \ell_y = \ell_{y*} > 0$

2) $\ell(i*, y*) = \frac{M*}{P*}$

3) $P = P^* S$

4) $i = i^* + E_t\left(\frac{\dot{S}}{S}\right)$

5) $\dot{M} + \dot{B} - S\dot{R} = \Delta + iB$

6) $\dot{M}^* + \dot{B}^* - \dot{R}^* = \Delta^* + i^* B^*$

7) $R + R^* = \bar{R} > 0$

8) $R > R_{MIN}$

9) $R^* > R^*_{MIN}$

10) $\bar{R} > R_{MIN} + R^*_{MIN}$

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[2] Because the model includes accounting identities as well as relative price levels, there is no convenient log-linear specification.

The two countries are assumed to have identical demand for money functions (equations (1) and (2)). This permits us to analyse exchange rate behaviour without having to specify a goods market equilibrium condition. Starred variables refer to foreign country variables. i is the nominal interest rate, y real output, M the nominal money stock, P the price level, B the stock of government bonds, R the stock of foreign exchange reserves, Δ the primary (i.e. non-interest) government deficit and S the spot exchange rate (the domestic currency price of foreign currency). We assume $i, i^* > 0$. Interest is not paid on reserves, a realistic assumption under a gold standard, when the domestic and foreign currency prices of gold are fixed.

There is a single traded good. (3) is the law of one price. Financial markets are efficient and there are risk-neutral speculators, so uncovered interest parity (UIP) holds, as given in (4). E_t is the expectation operator conditional on information available at time t.

The total stock of gold in the world economy is exogenously given. Consumption and production of gold are therefore ignored.

Each country maintains convertibility of its currency into gold as long as its stock of reserves exceeds an exogenously given critical minimum value (R_{MIN} for the home country, R^*_{MIN} for the foreign country). If either country's stock of reserves falls to its lower threshold, the fixed exchange rate regime collapses and a free float of indefinite duration begins. Equation (10) states that there are enough global reserves to satisfy the two countries' minimal requirements simultaneously.[3]

Linearizing the model and evaluating it at the fixed exchange rate $\underline{S}$ which, without loss of generality, can be set equal to unity, we obtain the following first order differential equation for the floating exchange rate:[4]

$$11) \quad E_t dS(t) = \alpha_s S(t)d(t) - \alpha_M[M(t) - M^*(t)]dt + Z(t)dt$$

where

$$11a) \quad \alpha_s = - \left(\ell_i^{-1} \frac{M}{P}\right)_0 > 0$$

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[3] Most of the analysis is concerned only with <u>differences</u> between the values of country-specific variables such as domestic credit expansion, money demand growth etc. It may be helpful to think of country-specific or global variables (real output, the world price level under a fixed exchange rate) as not having any long-run trend. This makes the standard convention in this literature of specifying reserve floors or ceilings in nominal terms (rather than in real terms or relative to some nominal scale variable such as nominal income, trade or financial wealth) less objectionable. The analytically most convenient way of specifying the world commodity market equilibrium condition required to solve for individual countries' price levels and nominal interest rates, is to postulate a fixed ex-ante real interest rate.

[4] For algebraic convenience we also assume that $M_o = S_o M_o^*$.

11b) $\alpha_M = -\left(\frac{\ell_i^{-1}}{p^*}\right)_0 > 0$

11c) $Z(t) = -(\ell_i^{-1}\ \ell_y)_0\ (y(t) - y^*(t))$

When a speculative selling attack against the home country's currency occurs and the fixed exchange rate regime collapses at $t = \tau$, the money stocks for $t \geq \tau$ are given by:

(12a) $M(t) = D(t) + R_{MIN}$

(12b) $M^*(t) = D^*(t) + \bar{R} - R_{MIN}$

D and D^* are the home and foreign stocks of domestic credit respectively, i.e. $dD = [\Delta + iB]\ dt - dB$

When a speculative selling attack against the foreign country's currency occurs and the fixed exchange rate regime collapses at $t = \tau$, the money stocks for $t \geq \tau$ are given by:

(13a) $M(t) = D(t) + \bar{R} - R^*_{MIN}$

(13b) $M^*(t) = D^*(t) + R^*_{MIN}$

Following Grilli [1986], equation (14a) describes the evolution of $\approx$ S, the shadow floating exchange rate that would prevail for $t \geq \tau$ if the fixed exchange rate regime collapses at $t = \tau$ as the result of a speculative selling attack against the home country's currency. Equation (14b) governs $\tilde{S}$, the shadow floating exchange rate that would prevail for $t \geq \tau$ if the fixed exchange rate collapses at $t = \tau$ as the result of a speculative selling attack against the foreign currency.

$$(14a)\quad E_t d\approx{S}(t) = \alpha_S \approx{S}(t)dt - \alpha_M (D(t)-D^*(t) - \bar{R} + 2R_{MIN})dt + Z(t)dt$$

$$(14b)\quad E_t d\tilde{S}(t) = \alpha_S \tilde{S}(t)dt - \alpha_M (D(t)-D^*(t) + \bar{R} - 2R^*_{MIN})dt + Z(t)dt$$

An important simplifying assumption is that $D(t)$ and $D^*(t)$ (and indeed $Z(t)$) are unaffected by the occurrence and timing of a collapse of either currency.

Choosing convergent forward-looking solutions for $\approx{S}$ and $\tilde{S}$ we find

$$(15a)\quad \approx{S}(t) = \int_t^\infty e^{\alpha_S(t-u)} E_t[\alpha_M(D(u) - D^*(u) - \bar{R} + 2R_{MIN}) - Z(u)]du$$

$$(15b)\quad \tilde{S}(t) = \int_t^\infty e^{\alpha_S(t-u)} E_t[\alpha_M(D(u) - D^*(u) + \bar{R} - 2R^*_{MIN}) - Z(u)]du$$

The fixed exchange rate regime collapses through a speculative selling attack on the home currency when the home currency shadow floating exchange rate equals the fixed rate for the first time, i.e.

when

(16a) $\approx{S}(t) = \underline{S}; \quad \approx{S}(t') < \underline{S}, \; t' < t$

It collapses as the result of a speculative selling attack on the foreign currency when the foreign currency shadow floating exchange rate equals the fixed rate for the first time, i.e. when

(16b) $\tilde{S}(t) = \underline{S}; \quad \tilde{S}(t') > \underline{S}, \; t' < t$

The fixed exchange rate regime survives as long as

(17) $\overset{\approx}{S} < \underline{S}$ and $\tilde{S} > \underline{S}$

Note from (15a, b) that

(18a) $\tilde{S}(t) = \overset{\approx}{S}(t) + K$

where

(18b) $K = \dfrac{2\alpha_M}{\alpha_S}\left[\bar{R} - (R_{MIN} + R^*_{MIN})\right]$

From the assumption of minimal global reserve adequacy (eqn(10)) it follows that $K > 0$ and therefore that

(18c) $\tilde{S}(t) > \approx{S}(t)$

We therefore rewrite condition (17) for viability of the fixed exchange rate system as:

$$\underline{S} < \tilde{S} < \underline{S} + K$$

Without loss of generality we can choose the lower bound to equal zero.[5] Formally, the exercise we are performing can therefore be seen as the analysis of a continuous time stochastic process $\tilde{S}$ between two absorbing barriers; the lower barrier 0 and the upper barrier $K > 0$.

(19) $0 < \tilde{S} < K$

This is illustrated in Figure 1. $S^0(t)$ represents a realization which is absorbed at the upper barrier at t_0. $S^1(t)$ is absorbed at the lower barrier at t_1.

Except in Section 5 we assume that dD, dD^* and dZ are each governed by mutually serially independent Brownian motion with drift t, i.e.

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[5] We can always replace expressions such as prob($\underline{S} < \tilde{S} < \underline{S} + K$) with the equivalent prob($0 < \tilde{S} - \underline{S} < K$), if we also change the initial condition of the stochastic process involved in an appropriate manner.

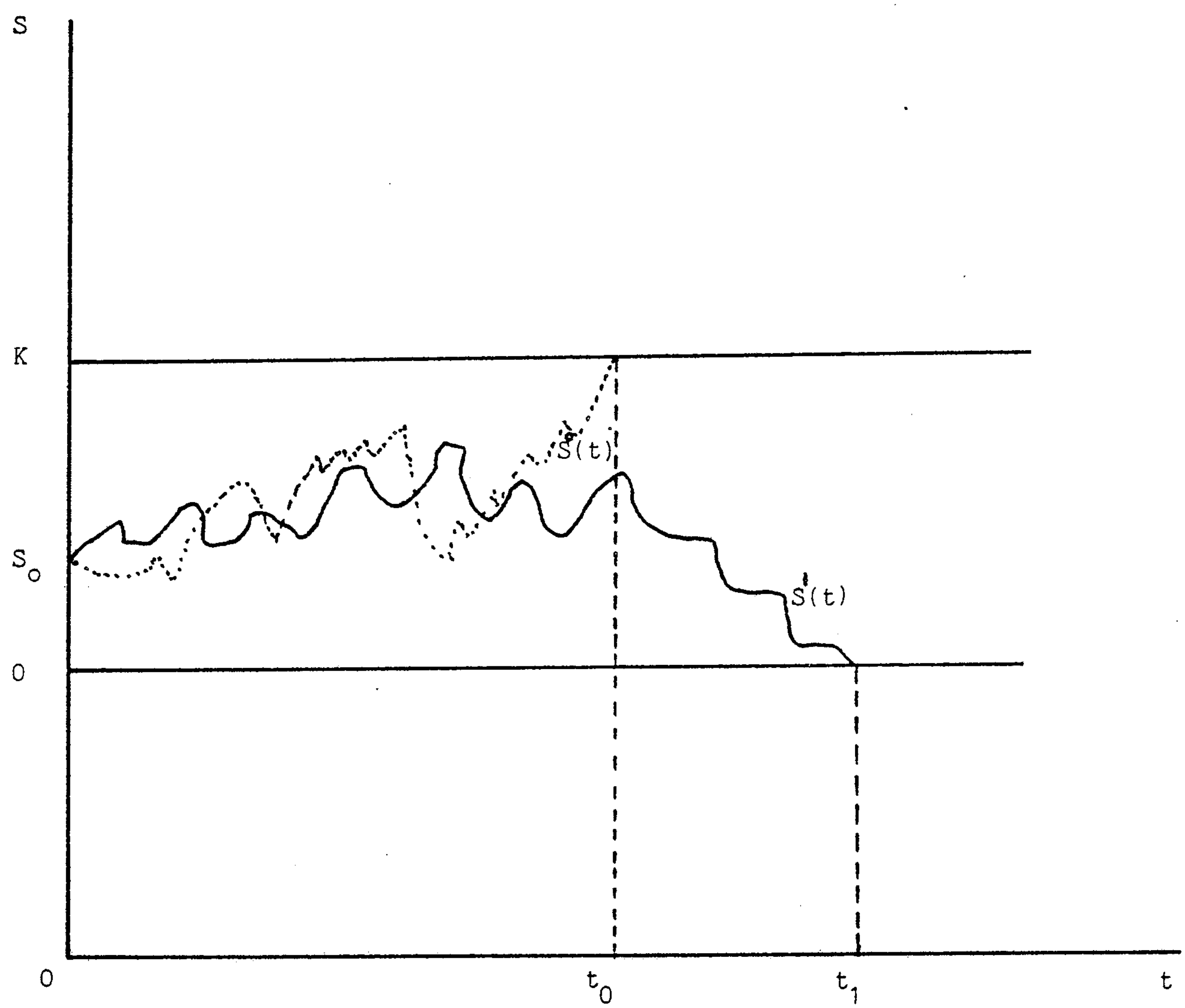

Figure 1

(20a) $D(t) = D(0) + \mu t + w(t)$

(20b) $D^*(t) = D^*(0) + \mu^* t + w^*(t)$

(20c) $Z(t) = Z(0) + \mu_Z t + w_Z(t)$

Given an initial condition, D, D^* and Z will therefore be continuous functions of time. The drift coefficients are μ, μ^* and μ_Z respectively. They can be positive, zero or negative.

$dw(t) \sim NIID(0, \sigma^2 dt)$
$dw^*(t) \sim NIID(0, \sigma^{*2} dt)$
$dw_Z(t) \sim NIID(0, \sigma_Z^2 dt)$ [6]

Define:

[6] The independent increments dw, dw^*, dZ are not assumed to be contemporaneously independent.

(21) $F(t) = \alpha_M(D(t) - D^*(t) + \bar{R} - 2R^*_{MIN}) - Z(t)$

F(t) is a Wiener process.[7] Its drift is

(22a) $\mu_F = \alpha_M(\mu - \mu^*) - \mu_Z$

Its variance parameter is

(22b) $\sigma^2_F = \alpha^2_M(\sigma^2_\mu + \sigma^2_{\mu*}) + \sigma^2_Z + 2\alpha_M(\sigma_{w*w_Z} - \sigma_{ww_Z} - \alpha_M\sigma_{ww*})$

Its initial value is

(22c) $F(0) = \alpha_M(D(0) - D^*(0) + \bar{R} - 2R^*_{MIN}) - Z(0)$

From (15a) and the definition of F(t) in (21) it follows that

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7 $dF = \left[\alpha_M(\mu - \mu^*) - \mu_Z\right] dt + dw_F$

where

$E(dw_F) = 0$

$Var(dw_F) = \sigma^2_F dt$

and

$Cov(dw_F(t), dw_F(s)) = 0, \; t \neq s$

$$(23)\quad \tilde{S}(t) = \frac{F(t)}{\alpha_S} + \frac{\mu_F}{\alpha_S^2}$$

Therefore, $\tilde{S}(t)$ is a Wiener process. Its drift is

$$(24a)\quad \mu_S = \frac{1}{\alpha_S}\,\mu_F = \frac{\alpha_M}{\alpha_S}(\mu - \mu^*) - \frac{\mu_Z}{\alpha_S}$$

Its variance parameter is

$$(24b)\quad \sigma_S^2 = \frac{1}{\alpha_S^2}\,\sigma_F^2 = \left[\frac{\alpha_M}{\alpha_S}\right]^2 \left[\sigma_\mu^2 + \sigma_{\mu*}^2\right] + \frac{1}{\alpha_S^2}\,\alpha_Z^2 + \frac{2\alpha_M}{\alpha_S^2}\,(\sigma_{w*w_Z} - \sigma_{ww_Z} - \alpha_M\sigma_{ww*})$$

Its initial value is

$$(24c)\quad \tilde{S}(0) = S_0 = \frac{F(0)}{\alpha_S} + \frac{\mu_F}{\alpha_S^2} = \frac{\alpha_M(D(0) - D^*(0) + \bar{R} - 2R^*_{MIN})}{\alpha_S} \; \frac{-Z(0)}{\alpha_S} + \frac{\alpha_M(\mu - \mu^*) - \mu_Z}{\alpha_S^2}$$

The expected value of the shadow floating exchange rate is therefore driven up by the excess of the home country trend rate of domestic credit expansion, μ, over the foreign trend rate of domestic credit expansion μ^* and by the excess of the foreign trend rate of change of money demand over the home country trend rate of change of money demand, $-\mu_Z$.

3. The Gold Standard as a Wiener Process With Two Absorbing Barriers.

From the characterization of $\tilde{S}$ as a Wiener process between two absorbing barriers, a number of propositions follow immediately.

Proposition 1. Under the conditions specified in Section 2, the gold standard collapses in finite time (with probability 1).

Proof. A Wiener process, with or without drift (i.e. with $\mu_S \lesseqgtr 0$), between two absorbing barriers is absorbed with probability 1 in finite time. See e.g. J. M. Harrison [1985, p. 43].

Note that the variance of the shadow floating exchange rate increases without bound as t increases. Any gold standard with a finite global stock of gold reserves will be non-viable, in the sense that a speculative attack will exhaust one country's reserves with probability one in finite time.

It is important to note that, at a point in time, the 'fundamental' shocks to domestic credit and money demand are infinitesimal, i.e. of measure zero compared to the stock of reserves at that instant. It always takes a finite amount of time to achieve a finite change in the stock of reserves through the cumulative effect of these shocks. The finite, stock-shift rundown in a country's stock of reserves at the moment a speculative attack occurs, reflects the endogenous behaviour of speculators, not the occurrence of an exogenous shock that is large relative to the existing stock of reserves.

An implication of this specification of the exogenous shocks is that neither currency will ever be at a forward discount or premium, as long as the gold standard survives. At the instant a speculative attack brings down the gold standard, a forward discount or premium consistent with UIP may of course emerge.

Let τ be the time at which the gold standard collapses, either because $\tilde{S}$ reaches $K > 0$ or because it reaches 0. Let $\tilde{S}(0) = S_0 \in [0, K]$ be the initial value of $\tilde{S}$ at $t = 0$ and let μ_S and σ_S^2 be the drift and variance parameter of $\tilde{S}$. $p(S_0, S; t)$ is the conditional probability that $\tilde{S}$ is at S at time $t > 0$ given that it starts at S_0 at time $t = 0$, when there are two absorbing barriers, at $K > 0$ and at 0, and with $0 < S_0 < K$. Then (see Cox and Miller [1965, p. 219-234])

$$(25a)\ p(S_0, S; t) = \left[\exp\left(\frac{\mu_S(S-S_0)}{\sigma_S^2}\right)\right]\sum_{n=1}^{\infty} a_n e^{-\lambda_n t} \sin\left(\frac{n\pi S}{K}\right)$$

where

(25b) $a_n = \frac{2}{K} \sin\left[\frac{n\pi S_0}{K}\right]$

and

(25c) $\lambda_n = \frac{1}{2}\left\{\frac{\mu_S^2}{\sigma_S^2} + \frac{n^2\pi^2\sigma_S^2}{K^2}\right\}$

G(t), the probability that $\tilde{S}$ is absorbed no later than time t (i.e. prob ($\tau < t$)) is given by

(26) $G(t) = 1 - \int_0^K p(S_0, S; t)\, ds$

Note that $g(t) \equiv G'(t) = -\frac{d}{dt}\int_0^K p(S_0, S; t)dS$ is the probability density function of τ.

The moment generating function of τ is

$$g^*(v) = \int_0^\infty e^{-vt} g(t)\, dt \qquad \text{for } v \geq 0.$$

The moments of τ (such as the first moment $E_0(\tau)$) are found by differentiating $g^*(v)$ w.r.t. v, since

$$(27)\quad E_0(\tau^n) = (-1)^n \frac{d^n}{dv^n} g^*(v)\Big|_{v=0}$$

For the Wiener process with drift μ_S, variance parameter σ_S^2, upper absorbing barrier K and lower absorbing barrier 0 < K, starting at S_0,

$$(28a)\quad g^*(v) = \frac{\left\{1-e^{K\theta_2(v)}\right\} e^{S_0\theta_1(v)} - \left\{1 - e^{K\theta_1(v)}\right\} e^{S_0\theta_2(v)}}{e^{K\theta_1(v)} - e^{K\theta_2(v)}}$$

where

$$(28b)\quad \theta_1(v),\ \theta_2(v) = \frac{-\mu_S \pm \sqrt{\mu_S^2 + 2\ v\ \sigma_S^2}}{\sigma_S^2}$$

From (27) and (28a, b) we derive the expected duration of the interval until the collapse given in (29, a, b).

$$(29a)\quad E_0(\tau) = \frac{S_0(K - S_0)}{\sigma_S^2} \text{ if } \mu_S = 0$$ [8]

[8] (29a) is derived from (29b) by applying L'Hôspital's rule twice.

(29b) $$E_0(\tau) = \frac{1}{\mu_S}\left\{-S_0 + K\,\frac{[1 - \exp(-2\mu_S S_0/\sigma_S^2)]}{[1 - \exp(-2\mu_S K/\sigma_S^2)]}\right\} \text{ if } \mu_S \neq 0$$

Finally (see e.g. Cox and Miller [1965, p. 233]), the probability of absorption at the upper barrier (i.e. the probability of the gold standard collapsing through a speculative selling attack on the home country currency) $\pi_+(S_0)$ is given by

(30a) $$\pi_+(S_0) = \frac{S_0}{K} \text{ if } \mu_S = 0$$

(30a') $$\pi_+(S_0) = \frac{\exp(-2\mu_S S_0/\sigma_S^2) - 1}{\exp(-2\mu_S K/\sigma_S^2) - 1} \text{ if } \mu_S \neq 0$$

The probability of absorption at the lower barrier $\pi_-(S_0)$, is then of course given by

(30b) $$\pi_-(S_0) = 1 - \pi_+(S_0).$$

4. Monetary, Financial and Fiscal Policy Actions and the Viability of the Gold Standard.

No attention is paid in this paper to strategic behaviour by the monetary and fiscal authorities of the two countries. It is probably easiest to think of the various policy actions as being implemented by mutual agreement or co-operatively; formally, we simply vary policy parameters in an exogenously given manner without worrying about conjectured and actual responses of other policy makers, time consistency of the actions or rules etc. Consideration of those very important issues is beyond the scope of this paper.

We distinguish 5 different kinds of policy actions.

a) Policies that change the initial value or starting value of the shadow floating exchange rate process at $t = 0$, S_0.

b) Policies that change the drift of the shadow floating exchange rate process, μ_S.

c) Policies that change the width of the "safe or viable band", K.

d) Policies that change the $\tilde{S}$ process from a Wiener process with drift μ_S and variance parameter σ_S^2 to a stationary first order linear stochastic process whose "forcing variable" is a Wiener process without drift, i.e. to

$$(31)\quad d\tilde{S} = \alpha_S'\tilde{S}dt + dw_S'$$
$$\alpha_S' < 0$$
$$dw_S' \sim NIID\ (0,\ \sigma_S'^2 dt)$$

e) Policies that change the variance parameter of the $\tilde{S}$ process, σ_S^2.

It will become clear that any given policy action or rule change may bring about several of these five consequences at the same time.

A number of results are immediately established.

$$(32a)\quad \frac{\partial \pi_+(S_0)}{\partial S_0} = \frac{1}{K} \text{ if } \mu_S = 0$$

$$(32b)\quad \frac{\partial \pi_+(S_0)}{\partial S_0} = \frac{(2\mu_S/\sigma_S^2)\ \exp(-2\mu_S S_0/\sigma_S^2)}{1 - \exp(-2\mu_S K/\sigma_S^2)} > 0 \text{ if } \mu_S \neq 0$$

Clearly, policies that raise S_0 increase the probability of the collapse of the gold standard occurring through a selling attack on the home currency.

$$(32b)\quad \frac{\partial \pi_+(S_0)}{\partial K} = -\frac{S_0}{K^2} \text{ if } \mu_S = 0$$

$$(32b')\quad \frac{\partial \pi_+(S_0)}{\partial K} = (-2\mu_S/\sigma_S^2)\exp(-2\mu_S K/\sigma_S^2)\ \frac{[1 - \exp(-2\mu_S S_0/\sigma_S^2)]}{[1 - \exp(-2\mu_S K/\sigma_S^2)]} < 0 \text{ if } \mu_S \neq 0$$

Clearly, policies that widen the viable band by raising the upper limit reduce the probability of the collapse of the gold standard occurring through a selling attack on the home currency.

Note from the definition of K in (18b) that an increase in K means an increase in the global stock of reserves and/or a reduction in the countries' critical reserve thresholds. Since the experiment holds S_0 constant (and μ_S), the increase in $\bar{R}$ or reduction in R_{MIN} or R^*_{MIN}, must be accompanied by a reduction in D(0) or an increase in $D^*(0)$ (see 24c). Such a combination of events obviously makes it more likely that the collapse, when it occurs, will occur through a run on the foreign currency. The increase in world reserves is allocated to the home country.

The effects of an increse in σ_S^2 and of an increase in μ_S on $\pi^*(S_0)$ are given in (32c, d) below

(32c) $$\frac{\partial \pi_+(S_0)}{\partial \sigma_S^2} = 0 \text{ if } \mu_S = 0$$

(32c') $$\frac{\partial \pi_+(S_0)}{\partial \sigma_S^2} = \frac{2\mu_S}{(\sigma_S^2)^2} A \gtrless 0 \text{ according as to } \mu_S \lessgtr 0 \text{ if } \mu_S \neq 0$$

(32d) $$\frac{\partial \pi_+(S_0)}{\partial \mu_S} = \frac{-2}{\sigma_S^2} A > 0$$

where

$$(32e)\ A = \frac{-S_0(1 - \exp(-2\mu_S K/\sigma_S^2))\exp(-2\mu_S S_0/\sigma_S^2) + K(1 - \exp(-2\mu_S S_0/\sigma_S^2))\exp(-2\mu_S K/\sigma_S^2)}{[1 - \exp(-2\mu_S K/\sigma_S^2)]^2}$$ [9]

$A < 0.$

The effect of policies that raise S_0, K, σ_S^2 and μ_S on the expected duration of the period for which the gold standard survives are given in equations (33 a, b, c, d).

$$(33a)\ \frac{\partial E_0(\tau)}{\partial S_0} = \frac{K - 2S_0}{\sigma_S^2} \quad \text{if } \mu_S = 0$$

$$(33a')\ \frac{\partial E_0(\tau)}{\partial S_0} = -\frac{1}{\mu_S} + \frac{2}{\sigma_S^2} K \frac{\exp(-2\mu_S S_0/\sigma_S^2)}{[1 - \exp(-2\mu_S K/\sigma_S^2)]} \quad \text{if } \mu_S \neq 0.$$

$$(33b)\ \frac{\partial E_0(\tau)}{\partial K} = \frac{S_0}{\sigma_S^2} > 0 \quad \text{if } \mu_S = 0$$

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[9] From (29a), $E_0(\tau) > 0 \Rightarrow -S_0(1 - \exp(-2\mu_S K/\sigma_S^2)) + K(1-\exp(-2\mu_S S_0/\sigma_S^2)) > 0$ for $\mu_S \lessgtr 0$. Let Ω denote the numerator of A in (32c). Then $\Omega/\exp(-2\mu_S(K + S_0)/\sigma_S^2) =$

$$-[-S_0(1 - \exp(2\mu_S K/\sigma_S^2)) + K(1 - \exp(2\mu_S S_0/\sigma_S^2))].$$

Therefore $\Omega < 0$ and $A < 0$.

$$(33b')\ \frac{\partial E_0(\tau)}{\partial K} = \frac{(1 - \exp(-2\mu_S S_0/\sigma_S^2))}{(1 - \exp(-2\mu_S K/\sigma_S^2))}\ \frac{1}{\mu_S}\ [1-[\exp(-2\mu_S K/\sigma_S^2)](1+(2\mu_S K/\sigma_S^2))] > 0 \text{ if } \mu_S \neq 0$$

$$(33c)\ \frac{\partial E_0(\tau)}{\partial \sigma_S^2} = \frac{-E_0(\tau)}{\sigma_S^2} < 0 \quad \text{if } \mu_S = 0$$

$$(33c')\ \frac{\partial E_0(\tau)}{\partial \sigma_S^2} = \frac{2K}{(\sigma_S^2)^2} A < 0 \text{ if } \mu_S \neq 0$$

$$(33d)\ \frac{\partial E_0(\tau)}{\partial \mu_S} = 0 \text{ if } \mu_S = 0$$

$$(33d')\ \frac{\partial E_0(\tau)}{\partial \mu_S} = -\frac{1}{\mu_S}\left[E_0(\tau) + \frac{2K}{\sigma_S^2} A\right]$$

Equation (33a) shows that when there is zero drift raising the starting value S_0 will raise (lower) $E_0(\tau)$ if S_0 is below (above) the halfway mark of the survival range, i.e. for $S_0 < \frac{K}{2}$ ($S_0 > \frac{K}{2}$).
If $\mu_S \neq 0$, (33a') tells us that the effect on $E_0(\tau)$ of an increase in S_0 is positive for $0 \leq S_0 < \hat{S}_0$, negative for $\hat{S}_0 < S_0 \leq K$, where $\hat{S}_0$ is defined by $\frac{\partial E_0(\tau)}{\partial S_0} = 0$. The first term on the RHS of (33a') can be interpreted as

the effect on $E_0(\tau)$ in the absence of any uncertainty[10]. The second term on the RHS of (33a') always has the opposite sign of the first term. It decreases with S_0, regardless of the sign of μ_S.

When $\mu_S = 0$, widening the width of the viable range by raising its upper limit, K, obviously lengthens the expected survival period, (33b). The same holds when $\mu_S \neq 0$ (33b').[11]

That a higher value of the variance parameter reduces the expected duration of the gold standard's survival is not surprising (33 c, c').

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[10] In that case τ is non-stochastic and given by $\tau = \begin{cases} \dfrac{K - S_0}{\mu_S} & \text{if } \mu_S > 0 \\ \dfrac{-S_0}{\mu_S} & \text{if } \mu_S < 0 \end{cases}$

[11] For the case $\mu_S \neq 0$, first consider $\mu_S > 0$. The sign of (33b') is always the sign of $1 - \exp(-2\mu_S K/\sigma_S^2)\,[1 + (2\mu_S K/\sigma_S^2)]$. We know that if $x \geq 0$ and m is any positive interger, then $e^{-x} \leq (1 + \frac{x}{m})^{-m}$. Applying this with $m = 1$ and $x = \dfrac{2\mu_S L}{\sigma_S^2}$ we find $\dfrac{\partial E_0(\tau)}{\partial K} > 0$. For the case $\mu_S < 0$ we know that if $x \geq 0$, m is any positive integer and $x < m$ then $e^x \leq (1 - \frac{x}{m})^{-m}$. Applying this with $m = 1$ and $x = \dfrac{-2\mu_S K}{\sigma_S^2}$, $0 < x < 1$, we find $e^x(1 - x) < 1$ and again $\dfrac{\partial E_0(\tau)}{\partial K} > 0$. For the case $\mu_S < 0$ and $1 + \dfrac{2\mu_S K}{\sigma_S^2} < 0$, the result that $\dfrac{\partial E_0(\tau)}{\partial K} > 0$ is obvious.

A higher value of the drift parameter μ_S appears to have an ambiguous effect on $E_0(\tau)$. Without uncertainty, an increase in μ_S will raise $E_0(\tau)$ if μ_S is negative, lower $E_0(\tau)$ if μ_S is positive. With uncertainty, the two terms in the square brackets on the RHS of (33d) have opposite signs and more information is needed to sign $\frac{\partial E_0(\tau)}{\partial \mu_S}$.

We now consider alternative policy actions at t = 0, that maximize the expectation of the survival period of the gold standard.

Policies that Increase K.

From (18b) it is apparent that, given the values of the parameters α_m and α_S, the viable range of $\tilde{S}$ can be increased only by an increase in the exogenous global stock of reserves $\bar{R}$ or a lowering of the critical reserve thresholds R_{min} and R^*_{min}. From (24c) we note that an increase in $\bar{R}$ or a reduction in R^*_{min} [12] will raise S_0 by half as much as K. S(0) is kept constant when $\bar{R}$ is increased and $D(0) - D^*(0)$ [or $\frac{1}{\alpha_S}\mu_S$] is reduced by half the amount of the increase in $\bar{R}$. If the lower bound of the viable range were L < K rather than zero, we could represent a "symmetric" increase in the viable range by an equal increase in the upper bound K and reduction in the lower bound L. Such a policy would leave S_0 unchanged for given values of D, D^* and μ_S.

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[12] It was assumed, in (24c) that $R_{min} = R^*_{min}$.

In our model, an increase in K accompanied by an increase in S_0 of half the magnitude of the rise in K can be achieved either by equal reductions in R_{min} and R^*_{min} or by an increase in $\bar{R}$ that is distributed equally between the two countries. An increase in K with constant S_0 and constant μ_S requires an increase in $D^*(0)$ and/or reduction in $D(0)$. Take the example of an increase in $D^*(0)$ which neutralizes the effect of an increase in $\bar{R}$ on S_0. Barring helicopter manoeuvres, such a stock-shift increase in D^* can only be brought about through an open market purchase of bonds by the foreign government, since $dD^* = [\Delta^* + i^* B^*] dt - dB^*$.

As a smaller stock of foreign government bonds is now outstanding, government debt service will be less as a result of the open market purchase than it would otherwise have been (by $i^*_0 dB^*$ approximately). Unless the government now continues (flow) purchases of its debt or increases its primary deficit Δ^* by the amount of the reduction in debt service, its expected rate of domestic credit expansion $\mu^* dt$ will be lowered, resulting in increases in S_0 and μ_S. Indefinite government bond purchases (or sales) are not feasible. An increase in K with constant S_0 and μ_S, therefore, represents a combination of an increase in world reserves (or reductions in reserve thresholds), an open market sale by the home country government (or an open market purchase by the foreign government) and a reduction in the home government's primary deficit (or an increase in the foreign government's primary deficit). (See Buiter [1976] for a discussion of these issues in a small country setting).

We can summarize the main findings of the discussion of increases in K as follows.

Proposition 2. A larger global stock of gold reserves raises the length of the expected survival period of the gold standard. Collapse in finite time remains a certainty, however, with any finite stock of reserves.

Policies That Change S_0.

I now consider policies that change S_0 without causing changes in K, μ_S or σ_S^2. It is easily seen from (24c), (18b) and (24a) and (24b) that such policies are redictributions of the given global stock of gold. An increase in $D(0) - D^*(0)$ redistributes reserves from the home country to the foreign country and raises S_0. Such stock-shift changes in $D(0)$ or $D^*(0)$ are brought about by open market sales or purchases. To prevent changes in μ_S from resulting from such open market sales or purchases, fundamental fiscal corrections, i.e. changes in the primary deficit must be brought out, as discussed in the preceding subsection. Given K, μ_S and σ_S^2 there exists a unique value of S_0, $\hat{S}_0$ (and therefore a unique value of $D(0) - D^*(0)$) that maximizes $E_0(\tau)$. This is obtained from (33a').[13]

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[13] The second order conditions are satisfied. For $\mu_S > 0$ and $\mu_S < 0$

$$\frac{\partial^2 E_0(\tau)}{\partial S_0^2} = \frac{-4\mu_S}{\sigma_S^4} \frac{\exp(-2\mu_S S_0/\sigma_S^2)}{[1 - \exp(-2\mu_S K/\sigma_S^2)]} < 0$$

For $\mu_S = 0$, $\dfrac{\partial^2 E_0(\tau)}{\partial S_0^2} = \dfrac{-2}{\sigma_S^2} < 0$

(34) $\hat{S}_0 = \frac{1}{2}K$ if $\mu_S = 0$

(34') $\frac{1}{\mu_S} = \frac{2}{\sigma_S^2} K \frac{\exp(-2\mu_S \hat{S}_0/\sigma_S^2)}{[1 - \exp(-2\mu_S K/\sigma_S^2)]}$ if $\mu_S \neq 0$

The problem with such redistributions of world reserves is that even when they raise $E_0(\tau)$ (something both countries can agree on), the probability of the collapse of the gold standard, when it occurs, occurring through a speculative selling attack on currency X always increases (decreases) when country X gives up (acquires) reserves through the open market sales and purchases outlined above. If there is any opprobrium attached to succombing to a speculative selling attack, countries that have excessive reserves (from the point of view of the survival of the gold standard) may yet be loath to give them up. This leads to Proposition 3.

<u>Proposition 3</u>. There exists an $E_0(\tau)$ maximizing initial distribution of reserves defined by (34, 34'), (24c) and $R(0) = \frac{D^*(0) - D(0) + \bar{R}}{2}$.[14] Since any reserve redistribution increases

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[14] With identical money demand functions, identical initial income levels, a fixed exchange rate $\underline{S} = 1$ and UIP, it follows that $M(0) = M^*(0)$. Since $M = D + R$ and $M^* = D^* + \bar{R} - R$, Proposition 3 follows.

the likelihood of the collapse of the gold standard, when it occurs, occurring through a selling attack on the currency of the country that has given up reserves, co-operative redistributions of reserves may be difficult to achieve without a mechanism for compensating the loser.

Policies that Change μ_S.

From (24a) it is apparent that an increase in the difference between the expected rates of change of the home country's and the foreign country's stocks of domestic credit, increases μ_S. An increase in $\mu - \mu^*$, which raises μ_S would, however, also raise S_0 (see (24c)), which is an increasing function of the "present discounted value" of all future expected dce differentials.

To raise μ_S without also raising S_0. the increase in $\mu - \mu^*$ must be accompanied by a reduction in $D(0) - D^*(0)$, i.e.

$$(35)\quad d(D-D^*) = -\frac{1}{\alpha_S} d(\mu-\mu^*)$$

Note that an open market sale by the home government (say) which is not accompanied by a reduction in the primary deficit and is not followed by further borrowing, lowers D but, through the increased debt service requirements, raises μ by $d\mu \approx -i_0 dD$. A once-off open market sale by the home government therefore raises μ_S and will raise (lower) S_0 if $i_0 > a_S$ ($i_0 < a_S$).

It does not appear to be correct that $\mu_S = 0$ always maximizes $E_0(\tau)$ for given values of S_0 (and K and σ_S^2). E.g. with a value of S_0

very close to the lower end of the viable range, 0, a positive value of μ_S might well raise $E_0(\tau)$. I conjecture, but have not been able to prove, that when <u>both</u> μ_S and S_0 are the subject of policy choice, however, $E_0(\tau)$ is maximized when $\mu_S = 0$ and $S_0 = \frac{1}{2}K$.[15]

Note that this requires

$$\mu - \mu^* = \sigma_M^{-1} \mu_Z$$

15

$$\frac{\partial^2 E_0(\tau)}{\partial \mu_S^2} = 0 \text{ if } \mu_S = 0; \quad \frac{\partial^2 E_0(\tau)}{\partial \mu_S^2} = \frac{2}{\mu_S^2}[E_0(\tau) + (2KA/\sigma_S^2)] - (2K/\mu_S \sigma_S^2)\frac{\partial A}{\partial \mu_S}$$

if $\mu_S \neq 0$. $\frac{\partial E_0(\tau)}{\partial \mu_S} = 0$ if $\mu_S = 0$; $\frac{\partial E_0(\tau)}{\partial \mu_S} = -\frac{1}{\mu_S}[E_0(\tau) + (2KA/\sigma_S^2)]$ if $\mu_S \neq 0$. Therefore, at an interior extremum, if $\mu_S \neq 0$,

$\frac{\partial^2 E_0)\tau)}{\partial \mu_S^2} = -(2K/\mu_S \sigma_S^2)\frac{\partial A}{\partial \mu_S}$. I have been unable to sign $\frac{\partial A}{\partial \mu_S}$. Also,

$\frac{\partial^2 E_0(\tau)}{\partial S_0 \partial \mu_S} = 0$ if $\mu_S = 0$ and

$$\frac{\partial^2 E_0(\tau)}{\partial S_0 \partial \mu_S} = \frac{1}{\mu_S^2} - \frac{4K\,[(K - S_0)\exp(-2\mu_S(K + S_0)/\sigma_S^2) + S_0 \exp(-2\mu_S S_0/\sigma_S^2)]}{\sigma_S^2\,[1 - \exp(-2\mu_S K/\sigma_S^2 0]^2}$$

which seems ambiguous. Only $\frac{\partial^2 E_0(\tau)}{\partial S_0^2}$ given in fn. 13 can be signed unambiguously. $\mu_S = 0$ and $S_0 = \frac{1}{2}K$ do satisfy the first order conditions for an interior extremum.

and

$$D(0) - D^*(0) = \frac{K}{2}\frac{\alpha_S}{\alpha_M} + \frac{Z(0)}{\alpha_M} - \bar{R} + 2R^*_{MIN}$$

To achieve these combined targets for the initial stocks of domestic credit $D(0) - D^*(0)$ and for subsequent trend or expected dce's $\mu - \mu^*$, will in general require using both once-off open market purchases or sales and a choice of (relative) primary deficits $\Delta - \Delta^*$.

Therefore:

Proposition 4. The $E_0(\tau)$ maximizing selection of μ_S and S_0 can be achieved only by using both monetary and fiscal policy instruments.

Note that only if $\mu_S = \sigma_S^2 = 0$ will the gold standard not run into reserve exhaustion problems. In that case the value of S_0 is immaterial as long as it is in the interior of the viable range.

5. Policies That Achieve a Stationary Shadow Exchange Rate Process

Thus far the stochastic process governing the forcing variables D, D^* and Z (given in 20a, b, c) and the stochastic process governing the shadow floating exchange rate $\tilde{S}$ (given in (23)) have been Wiener processes with drift. Such processes are non-stationary. Specifically, with a constant variance parameter σ_S^2 and drift μ_S, $\tilde{S}(t) = \tilde{S}(0) + \mu_S t + w_S(t)$ where $dw_S(t) \sim NIID(0, \sigma_S^2 dt)$.

The variance of $\tilde{S}(t)$ therefore increases linearly with t and $\tilde{S}$ is nonstationary even if $\mu_S = 0$.

We maintain the assumption that Z(t) is governed by the Wiener process with drift given in (20c), but permit the policy instruments to be governed by linear feedback rules that relate dce to the state variable $\tilde{S}$. For simplicity we assume that the feedback rules are exact or non-stochastic, i.e. that the only sources of noise in the system are the money demands in the two countries, Z(t). D(t) and $D^*(t)$ can be managed in such a way that $F(t) \equiv \alpha_M(D(t)-D^*(t) + \bar{R} - 2R^*_{MIN}) - Z(t)$ is governed by

(36) $dF(t) = \alpha_F \tilde{S}(t)dt + dw_F$[16]

where $dw_F \sim NIID(0, \sigma_Z^2 dt)$.

The first order representation of the shadow exchange rate equation (14b) and the feedback rule (36) is given in (37)

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[16] Note that in general, $dF(t) = \alpha_m d(D(t) - D^*(t)) - dZ(t) = \alpha_m d(D(t) - D^*(t)) - \mu_Z dt - dw_Z$. The non-stochastic linear feedback rule $d(D - D^*) = \frac{\alpha_F}{\alpha_m}\tilde{S}(t)dt + \frac{\mu_Z}{\alpha_m}dt$ achieves (36), where $w_F = -w_Z$.

$$(37)\quad \begin{bmatrix} E_t d\tilde{S}(t) \\ \\ dF(t) \end{bmatrix} = \begin{bmatrix} \alpha_S & 1 \\ \\ \alpha_F & 0 \end{bmatrix} \begin{bmatrix} \tilde{S}(t)dt \\ \\ F(t)dt \end{bmatrix} + \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} dw_F(t)$$

The characteristic equation of this dynamic system has one unstable root ($\rho_2 > 0$) and one stable root ($\rho_1 < 0$) i.f.f. $\alpha_F > 0$. Assuming this condition to be satisfied, the solution for the non-predetermined variable $\tilde{S}$ and the predetermined variable F(t) can be found to be (see e.g. Buiter [1984]).

$$(38)\quad \tilde{S}(t) = (\rho_1 - \alpha_S)\ F(t)^{17}$$

Therefore

$$(39)\quad d\tilde{S}(t) = \alpha_S'\tilde{S}(t)dt + dw_S'$$

with

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17 $\rho_{1,2} = \dfrac{\alpha_S \pm \sqrt{\alpha_S^2 + 4\alpha_F}}{2}$

The second term one might expect on the R.H.S. of (38),

$-E_t \int_t^{\infty} e^{\rho_2(t - \tau)} dw_F(\tau)d\tau = 0$ because w_F is white noise.

$$\alpha_S' = (\rho_1 - \alpha_S)\alpha_F < 0$$

and

$$dw_S' = (\rho_1 - \alpha_S)dw_F \sim NIID(0,(\rho_1 - \alpha_S)^2\sigma_Z^2 dt)$$

It is therefore rather straightforward (technically) to design dce feedback rules that turn the original non-stationary shadow exchange rate process into the stationary process given in (39). The fact that (39) is stationary does not, however, resolve the problem of non-viability of the gold standard.

Proposition 5.

The gold standard whose shadow exchange rate is governed by the stationary process in (39) will collapse in finite time with probability 1.

Proof: The first order linear stochastic differential equation whose forcing variable is a Wiener process is known as the Ornstein-Uhlenbeck (O.U.) process. In Cox and Miller [1965, p. 2334] it is shown that the stationary O.U. process is recurrent, i.e. any state is reached from any other state in finite time with probability 1. Hence given any initial state S_0, $0 < S_0 < K$, the barriers will be reached in finite time with probability one. This proposition can be extended to higher order stationary linear stochastic processes with constant coefficients.

6. Flexible dce Rules and the Viability of a Gold Standard

From equation (15b) it is apparent that there exist policies for D and D^* that ensure that the shadow floating exchange rate will never break through its upper or lower barrier. Policies that fix $\tilde{S}$ at any value between 0 and K preclude a speculative run on either currency. To achieve this, both the drift μ_S (given in (24a) and the variance parameter σ_S^2 (given in (24b)) must be set equal to zero. To set the drift equal to zero does not require any special technical ability of the policy makers. Any values of μ and μ^* satisfying $\mu - \mu^* = \frac{\mu_Z}{\alpha_m}$ will do, and the policy can be specified in an open-loop or non-contingent manner.

To reduce the variance parameter to zero, however, requires a highly contingent or conditional policy rule. It is e.g. not sufficient for the authorities merely to refrain from adding additional "noise" to the system, by following a non-stochastic open-loop rule with $\sigma_\mu^2 = \sigma_{\mu*}^2 = \sigma_{ww_Z} = \sigma_{w*w_Z} = \sigma_{ww*} = 0$.

From (24b) this would leave the variance of the shadow exchange rate process at $\frac{1}{\alpha_S^2} \sigma_Z^2 > 0$ if, as seems certain, there are stochastic shocks to relative money demand growth. For σ_S^2 to equal zero, D(t) and/or $D^*(t)$ must be stochastic, (σ_μ^2 and/or $\sigma_{\mu*}^2 > 0$) with one or more of covariance parameters σ_{w*w_Z}, σ_{ww_Z}, and σ_{ww*} chosen to satisfy $\sigma_S^2 = 0$ in (24b). Equivalently, instantaneous feedback rules must be designed which ensure that the forcing variable in the shadow exchange rate process is constant over time, i.e. that $dF(t) = 0$ or

(40) $d(D(t) - D^*(t)) = \frac{1}{\alpha_m} dZ(t)$ for all t.

This perfect automatic stabilizer instantaneously matches any (random) excess of home country money demand growth over foreign country money demand growth with an equal excess of home country domestic credit expansion over foreign country domestic credit expansion. There never is any net movement of reserves between the two monetary authorities.

Fiscal Aspects of DCE Rules Consistent With the Survival of a Gold Standard

From the two government budget identities (5) and (6), it follows that, as long as the fixed exchange rate regime survives and $i = i^*$,

$$d(D - D^*) = (\Delta - \Delta^*)dt + i(B - B^*)dt - d(B - B^*)$$

Under the dce rule given in (40), this means that

(41) $d(B - B^*) = [(\Delta - \Delta^*) + i(B - B^*)]dt - \frac{1}{\alpha_m} dZ(t)$

Even if the authorities follow the very conservative fiscal policy of matching (differences in) debt service $i(B - B^*)$ with (differences in) primary surpluses $(-(\Delta - \Delta^*))$, relative stocks of public sector debt will still "get out of hand," since in that case

$$(42)\quad d(B - B^*) = -\frac{1}{\alpha_m} dZ$$

i.e. relative public debts is a Wiener process (without drift). The stabilization of the shadow exchange rate appears to be achieved only at the expense of the destabilization of the public debts of the two countries.

We assume that each country has an upper bound on its public debt, $\bar{B}$ for the home country and $\bar{B}^*$ for the foreign country.[18] $B > \bar{B}$ means a home country default on its public debt; $B^* > \bar{B}^*$ means insolvency of the foreign government.

Thus far we have characterized, in (40) and (42), the behaviour of $D - D^*$ and $B - B^*$. A benchmark "symmetric" specification of the behaviour of each of the national authorities which is consistent with (40) is that each country's dce equals the growth in the demand for that country's money due to real output growth, i.e.

$$(43a)\quad dD(t) = -\frac{1}{\alpha_m}(\ell_i^{-1}\ell_y)_0\, dy(t)$$

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18 Such an upper bound exists when there is no Ricardian debt neutrality, the nominal interest rate exceeds the growth rate of nominal taxable capacity and there are limits on the magnitude of the future primary surpluses and the future flow of seigniorage revenues that can be achieved. A more general model would specify these limits in terms of e.g. public debt - national income ratios, but nothing crucial is lost if we interpret our model as one characterized by zero long run real growth and inflation [see also fn 3].

(43b) $dD^{*}(t) = -\frac{1}{\alpha_m}(\ell_i^{-1}\ \ell_y)_0\ dy^{*}(t)$ [19]

Where y(t) and $Y^{*}(t)$ are governed by Brownian motion.

Assume each country follows the fiscal policy rule of equating its primary surplus and the interest cost of servicing its public debt, i.e. a strict balanced budget rule;

(44a) $\Delta + iB = 0$

(44b) $\Delta^{*} + iB^{*} = 0$

Given (43a, b) and (44a, b) each country's real output-related money demand shocks will be reflected in open market purchases or sales of public debt, i.e.

(45a) $dB = \frac{1}{\alpha_m}(\ell_i^{-1}\ \ell_y)_0\ dy$

and

(45b) $dB* = \frac{1}{\alpha_m}(\ell_i^{-1}\ \ell_y)_0\ dy*$

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[19] Note, from (11c) that $Z(t) = -(\ell_i^{-1}\ \ell_y)_0\ (y(t) - y^{*}(t))$.

A policy of accommodating shocks to money demand due to variations in real output (thus avoiding any net flow of international reserves and fixing $\tilde{S}$) and of balancing the budget (44 a,b) will therefore result in the national debt following a Wiener process if real output follows a Wiener process.

If the Wiener process for y (y^*) has no drift, then B (B^*) will exceed any finite upper limit with probability 1 in finite time, and the authorities will have to default on their debt. If y (y^*) has negative drift, it is true a fortiori that any finite upper bound on B (B^*) will be reached in finite time with probability 1. If y (y^*) has positive drift $\mu_y > 0$ ($\mu_{y^*} > 0$) then the probability that, starting from B_0 (B_0^*) the upper bound $\bar{B}$ ($\bar{B}^*$) will be reached in finite time is

$$\exp\left[\frac{-2\mu_B(B_0 - \bar{B})}{\sigma_B^2}\right] \quad \left[\exp\left[\frac{-2\mu_{B^*}(B^*_0 - \bar{B}^*)}{\sigma^2_{B^*}}\right]\right] \quad \text{where } \mu_B = \frac{1}{\alpha_m}(\ell_i^{-1}\ell_y)_0\,\mu_y < 0$$

and $\sigma_B^2 = \left[\frac{1}{\alpha_m}(\ell_i^{-1}\ell_y)_0\right]^2 \sigma_y^2$. (See J. M. Harrison [1985, p. 43]). While this probability is less than 1, the probability that B (B^*) will reach an arbitrarily low (even negative) value in finite time is 1 when $\mu_y > 0$ ($\mu_{y*} > 0$). This possibility of the government becoming an arbitrarily large creditor to the private sector is certainly an unusual one.

If the government were to run a budget deficit (surplus) equal to the trend growth in money demand due to real output growth, (44 a, b) would be replaced by

(44a') $\Delta + iB = \frac{-1}{\alpha_m} (\ell_i^{-1} \ell_y)_0 \mu_y$

(44b') $\Delta^* + iB^* = \frac{-1}{\alpha_m} (\ell_i^{-1} \ell_y)_0 \mu_{y*}$

Such a financing policy (a balanced budget policy corrected for non-inflationary seigniorage) takes the drift out of the national debt processes (45 a, b) which now become Wiener processes without drift. As was pointed out already, this "conservative" open-loop rule entails government default in finite time with probability one. The risk of a foreign exchange crisis is eliminated, but an eventual government solvency crisis has become a certainty.

To ensure that the critical gold reserve thresholds will not be violated and that debt default is ruled out, the primary deficit Δ (Δ^*) will have to be varied in response to exogenous shocks. The simplest policy that guarantees survival of the gold standard and government solvency is one where the primary deficit adjusts continuously to accommodate all real income-related changes in money demand, i.e.

(44a") $(\Delta + iB)dt = - \frac{1}{\alpha_m} (\ell_i^{-1} \ell_y)_0 \, dy$

and

(44b") $(\Delta^* + iB^*)dt = - \frac{1}{\alpha_m} (\ell_i^{-1} \ell_y)_0 \, dy^*$

Under this set of policy rules (43 a, b); (44 a", b") $dR \equiv -dR^* = 0$ and $dB = dB^* = 0$. International liquidity and public sector solvency are made certain by adopting a set of highly contingent or conditional dce and primary deficit rules.

While the rule given here is not the only one consistent with a viable gold standard and government solvency, any viable rule must be capable of eliminating the possibility that independent shocks will cumulate in ways that threaten lower or upper bounds on certain asset stocks. Another policy which rules out the possibility of running out of gold reserves and which does not require the instantaneous matching of dce and money demand changes, is to let reserves decline freely to some given level above the minimum threshold level ($R_{MIN} + r$, $r > 0$ for the home country, say) and to engage in a stock-shift open market sale of government bonds whenever $R_{MIN} + r$ is reached. The open market-sale would restore reserves to $R_{MIN} + r'$, $r' > r$, say. If both countries were to pursue such a policy, the process governing the shadow floating exchange rate would become a Wiener process between reflecting barriers. To prevent these discrete open market sales from cumulating into unsustainable public debt growth, the primary deficit will have to respond to variations in debt service. This means that all viable rules are feedback, contingent or conditional rules. I summarize this as:

Proposition 6.

In order to rule out both the collapse of the gold standard and a public sector solvency crisis, both monetary policy (dce or stock-shift open market operations) and fiscal policy (variations in the primary deficit) will have to be specified through contingent or conditional rules.

Conclusion

The idealized two-country gold standard studied in this paper turns out to lack long-run viability unless monetary and fiscal policy are used very flexibly to offset the effects of independent exogenous shocks on international reserves and the public debt.

Absent a flexible dce policy which offsets the effect of money demand shocks on the stock of reserves, the gold standard collapses in finite time with probability 1. This collapse occurs through a speculative selling attack against one of the two countries' currencies, which brings that country's stock of gold reserves to its critical minimal threshold level. When this happens the monetary authority ends convertibility of the currency into gold at a fixed parity and a period of free floating commences.

Even when an eventual collapse is certain, there are once-off monetary and fiscal policy actions that can raise the expected duration of the "life" of the gold standard. An increase in the exogenous stock of international reserves (through a gold discovery or through the issuing of "paper gold" (S.D.R.'s) by a supranational monetary authority) raises the expected survival period of the gold standard. For any given global stock of reserves, there exists an initial distribution of reserves that maximizes the gold standard's expected lifetime. This distribution can be achieved through (stock-shift) open market operations and adjustments in public sector primary non-interest deficits. Expected dce (the "drift" of the domestic credit stock process) can be adjusted by the monetary authorities to alter the "drift" of international reserves and of the "shadow floating exchange rate" whose behaviour determines the timing of the collapse and the currency that will be the subject of a selling attack.

The behaviour of international reserves in the "viable range" where each country's stock of reserves exceeds its critical threshold value, can in general be mapped into the behaviour of a shadow floating exchange rate between two absorbing barriers. The results summarized thus far hold for a world in which the exogenous variables (domestic credit and money demand) follow Wiener processes with or without drift. When the exogenous variables are governed by Wiener processes, the shadow exchange rate is also a Wiener process. Since Wiener processes are non-stationary, it may be thought that the certainty of collapse in finite time is due to that specific feature of the model. This is not the case. Fairly simple dce feedback rules relating national dce differences to the level of the shadow floating exchange rate can transform the shadow floating exchange rate process into a stationary stochastic process. The proposition that a critical reserve threshold will be breached in finite time with probability 1 remains valid even for stationary shadow exchange rate processes.

Finally, I consider flexible dce rules which permit domestic credit expansion to respond instantaneously to real income-related money demand shocks. While this stops the movement of reserves, it will create public debt problems. If e.g. the authorities follow a balanced budget policy, money demand shocks will, when matched by variations in domestic credit, be reflected one-for-one in the public debt. If there is an upper bound on the level of public debt consistent with government solvency, this upper bound is likely to be breached eventually. If the authorities follow a budgetary policy of running a deficit or surplus equal to the trend growth of money demand (i.e. equal to the trend non-inflationary seigniorage that can be raised) a solvency crisis is certain in finite time.

To prevent both reserve thresholds and public debt ceilings from being breached, flexible dce policy and a flexible use of the primary (non-interest) public sector deficit are required in this model. Viability of the idealized gold standard analysed here requires the active and flexible support of monetary and fiscal policy. Even *prima facie* 'sound' unconditional monetary and fiscal rules such as : no sterilization of balance-of payments deficits or surpluses and a balanced budget, are inconsistent with the gold standard's long-run survival. This vulnerability of the gold standard should not come as a surprise of those who have studied the theory and history of commodity stabilization schemes which attempt to stabilize the price of some commodity by purchasing for or selling from a buffer stock. Formally, the gold standard is essentially an extreme version of such a commodity stabilization scheme, as it aims not merely to stabilize but to fix the price of a commodity. The same laws of probability that cause the eventual collapse of commodity stabilization schemes, most recently in the case of tin, jeopardize the long-run viability of a gold standard.

An issue which remains to be investigated is the extent to which the conclusions of this paper carry over to the case where international reserves consist (perhaps in part) of the liabilities of one or more of the national monetary authorities and carry a market-determined rate of interest. When reserves can be borrowed with little or no financial penalty, the distinction between liquidity crises and solvency crises becomes blurred, and we should only expect to see a run on a nation's currency as one aspect of a default crisis affecting the whole of that nation's public debt.

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