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DEATH, POPULATION GROWTH, PRODUCTIVITY GROWTH AND DEBT NEUTRALITY

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Abstract

Debt neutrality is said to occur if, given a program for public spending on current goods and services over time, the real equilibrium of the economy (private consumption, investment, relative prices, etc.) is independent of the pattern of government borrowing and lump-sum taxation over time. The paper brings together work of Blanchard on individual uncertain lifetimes and debt neutrality and Weil on population growth and debt neutrality. It is shown that there will be debt neutrality if and only if the sum of the rate of growth of population and the individual probability of death equals zero. If this condition holds, non-zero rates of growth of labor productivity will not destroy debt neutrality.

I. Introduction

This paper reconsiders the necessary and sufficient conditions for debt neutrality. There is debt neutrality if, given a program for public expenditure on current goods and services over time, the real equilibrium of the economy is not affected by a change in the pattern over time of lump-sum taxes. If there is debt neutrality for instance, the substitution of borrowing today for lump-sum taxation today (followed by such further changes in the time path of future lump-sum taxes as are required for maintaining government solvency) does not affect the current and future behavior of private consumption and capital formation.

I consider this issue in a simple closed economy growth model. There is a single produced commodity which can be consumed privately, consumed publicly or used in private capital formation. Population and labor supply grow at the constant exogenous instantaneous proportional rate n. Labor-augmenting technical change occurs at the constant exogenous instantaneous proportional rate π. Private consumption behavior is modeled following the Yaari-Blanchard uncertain lifetimes approach (Yaari [1965], Blanchard [1984, 1985]). The constant instantaneous probability of death of each individual is $\lambda \geq 0$.

The paper combines the results of Blanchard [1984, 1985] about debt neutrality and uncertain lifetimes and of Weil [1985] about debt neutrality and population growth and completes the triad by considering the implications of productivity growth for debt neutrality.

Blanchard showed that uncertain lifetimes $(\lambda > 0)$ are sufficient for absence of debt neutrality. They drive a wedge between effective (risk-adjusted) private sector discount rates and government discount rates. The future flow of resources expected to be available to those private agents currently alive grows at the exponential rate $\pi-\lambda$. Governments can tax the resources not only of those private agents currently alive, but also of those yet to be born. Their resource base grows at the exponential rate $\pi+n$. In Blanchard's model, the size of the total population is non-stochastic and constant. Weil showed that even with infinite-lived consumers, population growth alone $(n > 0)$ would, again by expanding the intertemporal resource base of the government beyond that available to those households currently alive, destroy debt neutrality. For debt neutrality, intergenerational linkages are necessary (say through an operative bequest motive). Infinite horizons for "isolated" individual consumers are insufficient if $n \neq 0$.

In this paper I show that $\lambda + n = 0$ is necessary and sufficient for debt neutrality. It follows that, as long as $\lambda + n = 0$, non-zero productivity growth $(\pi \neq 0)$ does not destroy neutrality. The intuition is that productivity growth, with $\lambda + n = 0$, augments equally the future resource bases of the individual consumer and the government.

I also show that, even though the probability of death λ and population growth enter additively in the criterion for debt neutrality, <u>changes</u> in λ will have different effects on the economy from changes in n (and changes in π).

Section II develops the model. Section III gives the conditions for debt neutrality in a rather general way, for any pattern of lump-sum taxation over time that is consistent with government solvency. Section IV gives a more detailed analysis of a specific kind of tax policy: a short-run cut in lump-sum taxes which, over time, is transformed into a long-run increase in lump-sum taxes. Section V uses this example to illustrate the different effects of changes in λ, n and π on the behavior of the economic system.

II. The Model

A. The Individual's Consumption Behavior

I shall use the simplest version of the Yaari-Blanchard model of consumer behavior (Yaari [1985]), Blanchard [1984, 1985]). The only novelty is in the consideration of population growth and productivity growth in the subsection on aggregation.

At each instant t, a consumer born at time $s \leq t$ solves the following problem.

$$(1) \qquad \max_{\{\bar{c}(s,v)\}} W(s,t) = \max_{\{\bar{c}(s,v)\}} E_t \int_t^\infty e^{-\delta(v-t)} \ln \bar{c}(s,v)dv \qquad \delta > 0$$

E_t is the expectation operator conditional on period t information; $\bar{c}$ is individual consumption of the single good; δ is the pure rate of time preference. During his or her lifetime each consumer faces a common and constant instantaneous probability of death (or probability of dynastic extinction through childlessness) $\lambda \geq 0$. The probability at time t of surviving until time $v \geq t$ is therefore given by $e^{-\lambda(v-t)}$. Equation (1) can therefore be rewritten as

$$(2) \qquad \max_{\{\bar{c}(s,v)\}} \int_t^\infty e^{-(\delta+\lambda)(v-t)} \ln \bar{c}(s,v)dv$$

The consumer's instantaneous flow budget identity is given by

$$(3) \qquad \frac{d}{dt} \bar{a}(s,t) \equiv (r(t)+\lambda)\, \bar{a}(s,t) + \bar{w}(s,t) - \bar{\tau}(s,t) - \bar{c}(s,t).$$

$\bar{a}$ is the consumer's financial or non-human wealth. r is the instantaneous real interest rate, $\bar{w}$ the real wage and $\bar{\tau}$ lump-sum taxes net of transfers.

The term $\lambda\bar{a}$ on the r.h.s of (3) reflects the operation of efficient life insurance or annuities markets. Each consumer makes the following contract with an insurance company: as long as he (she) lives, (s)he receives a rate of return ρ on his (her) total financial asset holdings at each instant. When (s)he dies, the entire estate accrues to the insurance company. (If $\bar{a}$ is negative, the consumer pays the insurance company a premium rate ρ, with his (her) debt cancelled when (s)he dies). The insurance industry is competitive with free entry. There is a large number of people (or 'cohort') born at each instant, and λ is both the instantaneous probability of death for an individual and the fraction of each cohort (and therefore of the total population) which dies at each instant. The competitive (zero expected) profit) rate of return paid by or to the insurance company is therefore $\rho = \lambda$. (Note, <u>not</u> $\rho = n + \lambda$, where n is the instantaneous proportional population growth rate. A fraction λ of each cohort dies each instant, so a fraction λ of the economy's non-human wealth accrues to the insurance companies each instant. It is this that gets paid out by the insurance companies to the surviving agents).

Integrating (3) forward in time and imposing the terminal boundary condition (4), we obtain the individual household's intertemporal budget constraint or solvency constraint given in (5a,b)

$$\text{(4)} \qquad \lim_{\ell \to \infty} \bar{a}(s,\ell)\, e^{-\int_t^{\ell}(r(u)+\lambda)du} = 0$$

$$\text{(5a)} \qquad \int_t^{\infty} \bar{c}(s,v)\, e^{-\int_t^{v}(r(u)+\lambda)du}\, dv \equiv \bar{a}(s,t) + \bar{h}(s,t)$$

$$(5b)\qquad \bar{h}(s,t) \equiv \int_t^{\infty} \left(\bar{w}(s,v) - \bar{\tau}(s,v)\right) e^{-\int_t^v (r(u)+\lambda)\, du}\, dv$$

$\bar{h}$ is the consumer's human capital, the present discounted value (using the "risk-adjusted" discount rate $r + \lambda$) of expected future ater-tax labor income. Note that (5b) implies:

$$(5b')\qquad \frac{d}{dt}\, \bar{h}(s,t) \equiv (r(t)+\lambda)\, \bar{h}\,(s,t) - \left(\bar{w}(s,t) - \bar{\tau}(s,t)\right)$$

The consumption function generated by this maximization program is well-known to be:

$$(6)\qquad \bar{c}(s,t) = (\delta + \lambda)\left(\bar{a}(s,t) + \bar{h}(s,t)\right)$$

Equations (3), (5b') and (6) imply

$$(6')\qquad \frac{d}{dt}\, \bar{c}(s,t) = (r(t) - \delta)\, \bar{c}(s,t)$$

B. Aggregation

At each instant a new age cohort composed of many agents is born. The size of the cohort born at time t is $(n + \lambda)e^{nt}$, $n \geq 0$. Since λ, the (constant) instantaneous probability of death of an agent, is also the fraction of agents in each cohort which die at each instant, the size of the surviving cohort at time t which was born at time $s \leq t$ is $(n+\lambda)\, e^{ns}\, e^{-\lambda(t-s)}$. Total population at any instant t is given by $(n+\lambda)\, e^{-\lambda t} \int_{\infty}^{t} e^{(\lambda+n)s} ds = e^{nt}$

For any individual agent's stock or flow variable $\bar{v}(s,t)$ we define the corresponding population aggregate $V(t)$ to be

(7) $$V(t) \equiv (n+\lambda)\, e^{-\lambda t} \int_{\infty}^{t} \bar{v}(s,t)\, e^{(n+\lambda)s}\, ds$$

Each agent, regardless of age, earns the same wage income and pays the same taxes, i.e.

(8a) $$\bar{w}(s,t) = \bar{w}(t)$$

(8b) $$\bar{\tau}(s,t) = \bar{\tau}(t)$$

It follows that each surviving agent has the same human capital.

(8c) $$\bar{h}(s,t) = \bar{h}(t),$$

By straightforward direct computation, and using the notational convention given in (7), aggregate consumption is given by:

(9a) $$C(t) = (\delta+\lambda)\, \big(A(t) + H(t)\big)$$

(9b) $$\dot{A}(t) \equiv r(t)\, A(t) + W(t) - T(t) - C(t)$$ 1/

(9c) $$\dot{H}(t) \equiv (r+\lambda+n)\, H(t) + T(t) - W(t)$$ 2/

1/ We use the fact that $\bar{a}(t,t) = 0$, i.e. consumers are born without financial assets or liabilities.

2/ We use $\bar{h}(t,t)e^{nt} = \bar{h}(t)e^{nt} = H(t)$.

The absence of a λA term in (9b), unlike in (3), reflects the fact that the insurance companies' activities involve a transfer from those who die to those who survive, which does not alter the rate of return on aggregate non-human wealth. The presence of the nH term in (9c) reflects the fact that all surviving agents, even the newborn, have the same human capital.

There is a constant instantaneous proportional rate of growth of productivity π. Technical change is labor-augmenting or Harrod-neutral. By choice of units, the level of productivity at $t = 0$ is set equal to unity.

For each population aggregate stock or flow variable V, the corresponding quantity "per unit of labor measured in efficiency units," v, is defined by:

$$(10) \qquad v(t) \equiv V(t)e^{-(n+\pi)t}$$

Using this notational convention, consumption per unit of efficiency labor is governed by:

$$(11a) \qquad c = (\delta+\lambda)(a+h)$$

$$(11b) \qquad \dot{a} \equiv (r-(n+\pi))a + w - \tau - c$$

$$(11c) \qquad \dot{h} \equiv (r+\lambda-\pi)h + \tau - w$$

These last three equations imply:

$$(12) \qquad \dot{c} = (r - (\delta+\pi+\lambda))c - (\delta+\lambda)na + (\delta+\lambda)\lambda h$$

or

(12') $\dot{c} = (r-(\delta+\pi))c - (\delta+\lambda)(n+\lambda)a$

C. Production, the public sector and market equilibrium

Production is governed by a smooth twice-continuously differentiable neoclassical constant returns to scale production function. Capital and efficiency units of labor are the two inputs. Let y denote output per unit of efficiency labor and k capital per unit of efficiency labor, then:

(13a) $y = f(k);\ f'>0;\ f''>0;\ f(0)=0;\ \lim_{k\to 0} f'=\infty;\ \lim_{k\to\infty} f'=0.$

Competitive labor and financial markets ensure that:

(13b) $r = f'(k)$

(13c) $w = f(k) - kf'(k).$

Note that w is the wage rate per unit of efficiency labor. When w is constant, each worker's wage grows at the proportional rate π.

The government spends on goods and services g, levies lump-sum taxes τ and borrows by issuing government debt. (g, τ and b are per unit of efficiency labor) 1/

1/ I am assuming that government spending on goods and services is neither useful as public sector capital formation nor as public consumption in the private utility function. g could be entered additively into the intantaneous private utility function without affecting any of the results (except of course the welfare economics of variations in g). For the issue of debt neutrality, the role of g is not relevant.

The government's instantaneous budget identity is:

$$(14) \qquad \dot{b} \equiv g - \tau + (r - (n+\pi))b$$

Integrating the government's budget identity forward in time and imposing the terminal boundary condition given in (15) we obtain the familiar government intertemporal or present value budget constraint, or its solvency constraint, given in (16).

$$(15) \qquad \lim_{\ell\to\infty} b(\ell)e^{-\int_t^\ell (r(u)-(n+\pi)du} = 0$$

$$(16) \qquad b(t) \equiv \int_t^\infty (\tau(v)-g(v))e^{-\int_t^v (r(u)-(n+\pi))du} dv$$

Equilibrium in the goods market requires that:

$$(17) \qquad \dot{k} = y - c - g - (n+\pi)k$$

Since there are only two non-human assets, capital and government debt, it follows that:

$$(18) \qquad a = k + b$$

III. Debt (non-) Neutrality: A General Statement

It is evident from equations (11) to (18) that, given a path of g(t), variations in the government's paths or rules for lump-sum taxes, τ, can only affect current and/or future values of c, k, y, w and r by influencing private consumption. The conditions for debt neutrality are therefore simply the conditions for c to be independent of the current and future values of τ, as long as the path of g is left unchanged. In what follows, the analysis is restricted to paths or rules for τ consistent with government solvency, as defined in (16): the present discounted value of future primary (i.e. net of interest) government surpluses should be equal to (and therefore sufficient to service) the initial debt. The relevant discount rate is the real interest rate net of the rate of growth of labor in efficiency units $r-(n+\pi)$. Population growth and productivity growth both expand the future resource base on which the government can levy taxes to serve the debt.

Integrating (11c) forward in time and imposing the terminal boundary condition (19), we obtain human capital per unit of labor measured in efficiency units, h, as given in (20).

$$(19) \qquad \lim_{\ell\to\infty} h(\ell)e^{-\int_t^{\ell}(r(u)+\lambda-\pi)du} = 0$$

$$(20) \qquad h(t) = \int_t^{\infty}(w(v)-\tau(v))e^{-\int_t^{v}(r(u)+\lambda-\pi)du}dv$$

Substitute for h(t) in the consumption function (11a) using (20) and for a(t) using (18). Then add and subtract the term

$$(\delta+\lambda)\int_t^{\infty}g(v)e^{-\int_t^{v}(r(u)+\lambda-\pi)du}dv$$

and rearrange. This yields:

$$(21) \qquad c(t) = (\delta+\lambda)\left(k(t) + \int_t^{\infty}w(v)e^{-\int_t^{v}(r(u)+\lambda-\pi)du}dv\right)$$

$$-(\delta+\lambda)\int_t^{\infty}g(v)e^{-\int_t^{v}(r(u)+\lambda-\pi)du}dv$$

$$+ (\delta+\lambda) \left[b(t) - \int_t^\infty(\tau(v)-g(v))\right]e^{-\int_t^v(r(u)+\lambda-\pi)du}\, dv$$

The last term on the r.h.s. of (21) is the crucial one for debt neutrality. Comparing it with the government solvency constraint (16) shows that this last term on the r.h.s. of (21) will vanish i.f.f. $\lambda + n = 0$. If $\lambda + n \neq 0$, i.e. in practice (ignoring the case of negative population growth) if $\lambda + n > 0$, debt neutrality will not hold. This is the most general statement of the conditions for debt neutrality. What follows becomes more specific by putting some restrictions on the paths of taxes.

Consider two economies identical in all respects except for the initial stock of debt, which is greater in economy I, and for current and future lump-sum taxes which differ between the two economies in such a way as to ensure government solvency for both economy I and economy II, in spite of the larger initial stock of debt in economy I. I.e. $\delta^I = \delta^{II} = \delta$; $\lambda^I = \lambda^{II} = \lambda$; $\pi^I = \pi^{II} = \pi$; $k^I(t) = k^{II}(t) = k(t)$; $w^I(v) = w^{II}(v) = w(v)$, $r^I(v) = r^{II}(v) = r(v)$, $g^I(v) = g^{II}(v) = g(v)$ for all $v \geq t$. To maintain government solvency with $b^I(t) > b^{II}(t)$ we require, from (16) that

$$(22) \qquad b^I(t) - b^{II}(t) = \int_t^\infty\left(\tau^I(v)-\tau^{II}(v)\right)e^{-\int_t^v[r(u)-(n+\pi)]du}\, dv > 0$$

Adding and subtracting the term $\int_t^\infty\left(\tau^I(v)-\tau^{II}(v)\right)e^{-\int_t^v(r(u)+\lambda-\pi)du}\, dv$ in (22) and rearranging yields:

$$(23) \qquad b^I(t) - b^{II}(t) = \int_t^\infty(\tau^I(v)-\tau^{II}(v))e^{-\int_t^v(r(u)+\lambda-\pi)du}\, dv$$

$$+ \int_t^\infty(\tau^I(v)-\tau^{II}(v))e^{-\int_t^v[r(u)-(n+\pi)]du}\left(1-e^{-(\lambda+n)(v-t)}\right)dv$$

It is clear that the higher initial debt in economy I could be serviced by tax policies that have $\tau^{I}(v) \geq \tau^{II}(v)$ for all $v \geq t$ and $\tau^{I}(v) > \tau^{II}(v)$ for at least one finite interval of time beyond t. For all such policies, the second term on the r.h.s. of (23) is strictly positive for $\lambda + n > 0$. It equals zero for $\lambda + n = 0$.

Let us call this term $\Omega(t)$, i.e.

$$(24) \qquad \Omega(t) = \int_t^{\infty}\left(\tau^{I}(v)-\tau^{II}(v)\right)e^{-\int_t^v[r(u)-(n+\pi)]du}\left(1-e^{-(\lambda+n)(v-t)}\right)dv$$

It is the excess of the present discounted value of the differences in future taxes using the government's effective discount rate $r-(n+\pi)$ over the present discounted value of the differences in future taxes using the private sector's effective discount rate $r+\lambda-\pi$.

The difference in private consumption between the two economies is given by

$$(25) \qquad c^{I}(t) - c^{II}(t) = (\delta+\lambda)\,\Omega(t).$$

For the strictly higher path of taxes in economy I (i.e. with $\tau^{I}(v) \geq \tau^{II}(v)$ for all v and $\tau^{I}(v) > \tau^{II}(v)$ for some finite interval), $\Omega(t)$ is strictly positive if and only if $\lambda + n > 0$, because in that case the household sector discounts a positive stream of differences using a higher effective discount rate than the government. 1/

1/ This result will also hold for many policies for which $\tau^{I}(v) < \tau^{II}(v)$ for some finite interval(s), but the proofs become very case-specific. The behavior of taxes in the model studied in Section IV is characterized by $\tau^{I}(v) < \tau^{II}(v)$ for small v and $\tau^{I}(v) > \tau^{II}(v)$ for large v.

To establish absence of debt neutrality, we only have to show that $c^{I}(t) \neq c^{II}(t)$ if $b^{I}(t) \neq b^{II}(t)$ and only lump-sum taxes differ between economies I and II to maintain government solvency. In fact we have shown more, by establishing a strong presumption of "financial crowding out": $b^{I}(t) > b^{II}(t)$ was seen to imply $c^{I}(t) > c^{II}(t)$ if and only if $\lambda + n > 0$ for the class of tax policies considered. How this incipient increase in private consumption is translated into actual behavior is very model-specific, as it depends on the behavior of current and future expected interest rates and wage rates. Some degree of financial crowding out seems likely, however, and the closed economy example solved in the next section confirms this. In a small open economy with an exogenous interest rate, the crowding out would take the form of public debt displacing net foreign assets rather than real capital (see Blanchard (1985) and Buiter (1986a)).

The findings of this section can be summarized as follows:

<u>Proposition</u>: $\lambda + n = 0$ is necessary and sufficient for debt neutrality

<u>Corollary</u>: if $\lambda + n = 0$, $\pi \neq 0$ does not invalidate debt neutrality.

Finally note that Blanchard's measure of fiscal stance F(t) becomes (see Blanchard (1985)).

$$(26) \qquad F(t) \equiv g(t) - (\delta + \lambda) \int_t^\infty g(v) e^{-\int_t^v (r(u)+\lambda-\pi)du} dv$$

$$+ (\delta+\lambda)(b(t) - \int_t^\infty (\tau(v)-g(v)) e^{-\int_t^v r(u)+\lambda-\pi)du} dv)$$

We have already discussed the third term on the r.h.s. of (26), the financing term. The first and second term given the effect of public spending on aggregate (private plus public) consumption demand, at given current and expected future interest rates and wage rates. Demand is boosted by public consumption spending to the extent that its current value exceeds the "permanent" value defined by the second term on the r.h.s. of (26).

IV. Financial Crowding Out and Fiscal Policy: An Example

In this section, I complete the model of Section II by adding a behavioral relationship for taxes which has the following properties: (1) it almost certainly stabilizes the public debt process; (2) it pins down very transparently the change in the long-run level of taxes and (3) a long-run increase in taxation is preceded by a short-run cut in taxes and vice versa. As shown in (27) τ feeds back from the deficit.

$$\tau = \tau_0 + \theta\dot{b} \qquad \theta < -1 \tag{27}$$

Under this rule, the debt dynamics is governed by:

$$\dot{b} = (1 + \theta)^{-1} (g - \tau_0) + (1 + \theta)^{-1} (r - (n + \pi))\, b \tag{28}$$

In the long run ($\dot{b} = 0$), taxes are given by τ_0. An increase in τ_0, however, implies in the short run a reduction in τ which disappears gradually and changes into an eventual increase:

$$\tau = \frac{1}{1+\theta} \tau_0 + \frac{\theta}{1+\theta} g + \frac{\theta}{1+\theta} (r - (n + \pi))b \tag{29}$$

I have assumed, as I shall in what follows, that $r > n + \pi$, i.e. that the "intrinsic" debt-deficit dynamics is explosive, because the real interest rate r exceeds the long-run growth rate of the tax base, $n + \pi$. Assigning the value -2 to θ as was done in Buiter (1986a) results in the debt-deficit process becoming the exact mirror image of what it would be under exogenous taxes ($\theta = 0$) since with $\theta = -2$ we have

(29') $\tau = -\tau_0 + 2g + 2(r-(n+\pi))b$

and

(28') $\dot{b} = -(g-\tau_0) - (r-(n+\pi))b$

The state-space representation of the model with equation (27) added involves three state variables. One possible representation is given below in equations (30a, b, c). The linearization of the system around a stationary equilibrium k_0, h_0 and b_0 is given in (31).

(30a) $\dot{k} = f(k) - (\delta+\lambda)\ (b+k+h) - g - (n+\pi)k$

(30b) $\dot{h} = (f'(k)+\lambda-\pi)\ h + \frac{1}{1-\theta}\ \tau_0 + \frac{\theta}{1-\theta}\ g + \frac{\theta}{1-\theta}\ (f'(k)-(n+\pi))b$

(30c) $\dot{b} = (1+\theta)^{-1}\ (g-\tau_0) + (1+\theta)^{-1}\ \left[f'(k)-(n+\pi)\right]b$

(31) $$\begin{bmatrix} \dot{h} \\ \dot{b} \\ \dot{h} \end{bmatrix} \approx \begin{bmatrix} r-(\delta+\lambda+n+\pi) & -(\delta+\lambda) & -(\delta+\lambda) \\ bf''(1+\theta)^{-1} & \left[r-(n+\pi)\right](1+\theta)^{-1} & 0 \\ (h+k+\frac{\theta}{1+\theta}b)f'' & \frac{\theta}{1+\theta}\left[r-(n+\pi)\right] & r+\lambda-\pi \end{bmatrix} \begin{bmatrix} k-k_0 \\ b-b_0 \\ h-h_0 \end{bmatrix}$$

The characteristic equation of the state matrix S in (31) is

(32a) $$\rho^3 + b_2\rho^2 + b_1\rho + b_0 = 0$$

with

(32b) $$b_2 = -\left[(3+2\theta)(r-\pi) - (1+\theta)\delta - (2+\theta)n\right](1+\theta)^{-1}$$

(32c) $$b_1 = \left[r-(\delta+\lambda+n+\pi)\right](r+\lambda-\pi) + \left[r-(n+\pi)\right]\left[r-(\delta+\pi)+r-(\pi-n)\right](1+\theta)^{-1}$$
$$+ (\delta+\lambda)(h+k+b)f''$$

(32d) $$b_0 = (1+\theta)^{-1}\{\left[r-(\delta+\lambda+n+\pi)\right](r+\lambda-\pi)\left[r-(n+\pi)\right]$$
$$+ (\delta+\lambda)f''\left[(h+k)(r-(n+\pi)) + b(r+\lambda-\pi)\right]\}$$

The following relations hold between the three roots ρ_1, ρ_2, ρ_3 and the coefficients of the polynomial b_1, b_2, b_3:

(33a) $$b_2 = \rho_1 + \rho_2 + \rho_3 = \text{Trace}(S)$$

(33b) $$b_1 = \rho_1\rho_2 + \rho_1\rho_3 + \rho_2\rho_3$$

(33c) $$b_0 = \rho_1\rho_2\rho_3 = \det(S).$$ 1/

The dynamic system in (31) has two predetermined state variables (k and b) and one non-predetermined state variable (h). For there to be a

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1/ det(S) means determinant of the matrix S.

(locally) unique continuously convergent solution to (21), the characteristic equation (32a) should have two stable characteristic roots, ρ_1 and ρ_2 say, with negative real parts and one unstable (positive) characteristic root, ρ_3 say.

A necessary condition for there to be the desired saddlepoint configuration is $b_0 \equiv \det(S) > 0$. Since $\theta < -1$, the term inside the curly brackets in (32d) should be negative. In open economy versions of this model with perfect capital mobility, r and therefore k is fixed exogenously and the second term inside the curly brackets of (32d) is absent. In these models the saddlepoint condition becomes (see Blanchard (1985) and Buiter (1986a)):

(34) $$\left[r-(\delta+\lambda+n+\pi)\right](r+\lambda-\pi)\left[r-(n+\pi)\right] < 0$$

This will be satisfied if

(35a) $$r > n+\pi$$

and

(35b) $$r < \delta + \lambda + n + \pi$$

With $n \geq 0$ and $\lambda \geq 0$, (35a) implies $r + \lambda - \pi > 0$. I shall assume that (34a, b) hold. 1/

A positive value of $b_0 \equiv \det(S)$ could have been generated by three unstable roots rather than two stable and one unstable root. Given $\det(S) > 0$, either $b_2 \equiv \mathrm{Trace}(S) \leq 0$ or $b_1 \leq 0$ is sufficient (but not necessary) for

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1/ (34) could also hold if all three of $r-(\delta+\lambda+n+\pi)$, $r+\lambda-\pi$ and $r-(n+\pi)$ were negative.

the desired saddlepoint configuration. It is clear from (32c) and (35a, b) that $r \geq \delta+\pi$ is sufficient for $b_1 < 0$. It can also be checked easily that with $\theta = -2$, the first two terms on the r.h.s. of (32c) sum to $-[\lambda(\lambda+\delta+n)+n\delta+(r-(n+\pi))^2]$. This will be negative if $r > n+\pi$. Since the last term on the r.h.s. of (32c) is also negative, the conditions given in (35a,b) are sufficient for the desired saddlepoint configuration if $\theta = -2$.

In long-run steady-state equilibrium, $\dot{a} = \dot{b} = \dot{k} = 0$ and

$$c = (\delta+\lambda)(a+h)$$

$$c = (r-(n+\pi))a + w-\tau_0$$

$$b = (\tau_0-g)\ (r-(n+\pi))^{-1}$$

$$a = b + k$$

$$r = f'(k)$$

$$w = f(k) - kf'(k)$$

These seven equations determine the long-run equilibrium values of c, a, h, b, k, w and r as functions of g, τ_0, δ, λ, n and π. The key long-run values of k and c can be solved conveniently from (36a, b).

(36a) $$c = (\delta+\lambda)\left((\tau_0-g)(f'(k)-(n+\pi))^{-1}+k+(f(k)-kf'(k)-\tau_0)\ (f'(k)+\lambda-\pi)^{-1}\right)$$

(36b) $$c = f(k) -g-(n+\pi)k.$$

Solving this for k as a function of τ_0, g, δ, λ, n and π we get

(37a) $k = \psi(\tau_0, g, \delta, \lambda, n, \pi)$

with

(37b) $\psi_{\tau_0} = N^{-1} \dfrac{(\delta+\lambda)(n+\lambda)}{(r-(n+\pi))(r+\lambda-\pi)}$

(37c) $\psi_g = N^{-1} \dfrac{(r-(n+\lambda+\pi+\delta))}{r-(n+\pi)}$

(37d) $N = r-(\delta+\lambda+n+\pi) + (\delta+\lambda)f'' \left(\dfrac{h+k}{r+\lambda-\pi} + \dfrac{b}{r-(n+\pi)}\right)$

From (35a, b) it follows that $N < 0$.

The remaining long-run multipliers will be discussed in the next section.

Since the assumption that $r > n+\pi$ and $r < n+\lambda+\pi+\delta$ implies $N < 0$, it follows that a higher long-run level of lump-sum taxes is associated with a lower long-run capital stock ($\psi_{\tau_0} < 0$) unless $n + \lambda = 0$ in which case debt neutrality prevails and the long-run capital stock is unaffected. 1/

A higher long-run level of public consumption is associated with a higher long-run capital stock when $\lambda + n > 0$. Consider the case where $\lambda+n=0$. From equation (11') it follows that stationary equilibria with a non-zero value of c are characterized by $r=\delta+\pi$. In that case, changes in g will not be associated with any long-run changes in k but will simply displace an equal amount of private consumption (see equation 36b). Whether these are short-run

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1/ Indeed the capital stock and private consumption at each instant are unaffected if $\lambda+n=0$.

effects on capital formation from an increase in g when $\lambda+n=0$ depends on whether the current change in public spending is equal to or differs from the "permanent" level $\delta\int_t^{\infty} g(v)e^{-\int_t^v (r(u)-\pi)du}dv$ (see equation 26).

Note that (37b) confirms our Proposition and its Corollary: debt neutrality ($\psi_{\tau_0} = 0$) requires $n+\lambda=0$; if $n+\lambda=0$, $\pi\neq0$ does not destroy debt neutrality. It is the difference between the public sector's future tax base (the resources of individuals alive today or yet to be born) and the future tax base of the individuals that are alive today (the resources owned by those individuals only, and not the resources of individuals yet to be born) that accounts for the non-neutrality of variations over time in the pattern of lump-sum taxation. The individual's expected future flow of resources grows at a rate $\pi-\lambda$. The government's expected future flow of resources grows at a rate $\pi+n$. Unless an individual is linked, through intergenerational gift and bequest motives to all those born after himself (herself), the resources of these future generations are not integrated into his (her) intertemporal budget constraint. An infinite lifetime ($\lambda = 0$) is not the same as intergenerational concern, nor does it imply the ability to effect the desired intergenerational transfers of resources. Productivity growth, when $n+\lambda=0$, augments the individual's resources over time in the same way as it augments the government's tax base.

The specific "crowding out" story associated with an increase in τ_0 in our model is some intrinsic interest. Take for concreteness the case where $\theta=-2$. From (29') it is clear that, since $r=f'(k)$ is given at a point in time, an increase in long-run lump-sum taxes τ_0 implies an equal and oppositive reduction in taxes at the initial date t_0. From (28') this generates a government deficit which is financed by borrowing. As the debt increases taxes are raised until they exceed their initial value and rise beyond it to the new higher level of τ_0. Capital will be decumulated in the process, which will raise the interest rate. Taxes, however, respond to such debt-service increases (see 29'). The higher taxes in the long run are required to service the increased stock of debt due to the early deficits associated with the early tax cut. Since the stable roots may be complex-conjugate, the approach to the steady state may involve oscillatory behavior. The details of the dynamic adjustment will depend on whether and when the change in τ_0 was anticipated.

The higher volume of long-run debt is associated with a long-run capital stock ($\psi_{\tau_0} < 0$ in (37b)) and thus a higher interest rate. That the long-run stock of debt is indeed higher can be seen from

$$\frac{db}{d\tau_0} = \frac{1}{r-(n+\pi)} - \frac{b}{r-(n+\pi)} f''\psi_{\tau_0}$$

i.e.

$$\frac{db}{d\tau_0} = \frac{1}{r-(n+\pi)} \cdot$$
$$\left[1- \frac{b(\delta+\lambda)\ (n+\lambda)\ f''}{b(\delta+\lambda)\ (n+\lambda)f''+(r-(n+\pi))(r+\lambda-\pi)(r-(\delta+\lambda+n+\pi))+(\delta+\lambda)(r-(n+\pi))f''(h+k+b)}\right]$$

This is positive given (35a, b).

Human capital falls in the long run: r is higher, w is lower and τ_0 is higher. Consumption obviously declines since

$$\frac{dc}{d\tau_0} = (r-(n+\pi))\ \psi_{\tau_0} < 0$$

Financial wealth is affected in an ambiguous manner. In the long run

$$a = \frac{(\delta+\pi-r)(w-\tau_0)}{(r+\lambda-\pi)(r-(n+\pi+\delta+\lambda))}$$

At given interest rates, a higher value of τ_0 will raise a if $\delta+\pi>r$, lower a otherwise. The decline in w as k declines reinforces this. The endogeneity of r does, however, leave the total effect ambiguous:

$$\frac{da}{d\tau_0} = - \left[\frac{\delta+\pi-r+\psi_{\tau_0} f''(w-\tau_0+(r-(\pi+\delta))b+(r-(\pi+n))a}{(r+\lambda-\pi)\ (r-(n+\pi+\delta+\lambda))}\right]$$

The dynamic story for the increase in g is also quite intuitive. Spending is raised at $t=t_0$ and is kept at its new higher level. From (29'), however, taxes are increased immediately by twice the amount of the increase in g. A budget surplus results and debt is retired. As debt is retired, taxes gradually (possibly in an oscillating manner) go back to their initial value τ_0. The lower debt and lower debt service (note that since k increases in the long run, r falls) permit the higher long-run level of public spending with unchanged long-run taxes. The exact time pattern of consumption and capital accumulation will of course depend on whether or not the increase in g was anticipated, when it was anticipated etc.

V. The Long-Run Comparative Statics of the "Deep Structural" Private Sector Parameters

Even though the population growth rate n an the probability of death λ enter the criterion for debt neutrality symmetrically, i.e. as $n+\lambda$, a change in n will not affect any endogenous variable of the system in the same way as a change in λ, unless (1) these changes are evaluated at $\lambda=n=0$ (and therefore at a stationary equilibrium with $r=\delta+\pi$) and (2) only a subset of the endogenous variables (k, r and w but not c, h, b and a) are considered. This can be shown by solving for the remaining long-run reduced form derivatives of equation (37a), reproduced below:

$$\dot{k} = \psi\,(\tau_0, g, \delta, \lambda, n, \pi).$$

(37e) $\psi_\delta = N^{-1}(b+k+h) < 0$

(37f) $\psi_\lambda = N^{-1}(b+k+\frac{r-(\pi+\delta)}{r+\lambda-\pi}\,h)$

(37g) $\psi_n = N^{-1}\left(\frac{(\delta+\lambda)}{r-(n+\pi)}b+k\right) < 0$

(37h) $\psi_\pi = N^{-1}\left(\frac{(\delta+\lambda)}{r-(n+\pi)}\,b + \frac{(\delta+\lambda)}{r+\lambda-\pi}h+k\right) < 0$

N, defined in (37d) is negative.

Not surprisingly, an increase in the rate of time preference, an increase in the population growth rate and an increase in the rate of labor-augmenting technical change all reduce the long-run capital-labor ratio (measured in efficiency units). An increase in the probability of death, i.e. a reduction in life expectancy will also reduce long-run k unless r is very much below $\pi+\delta$.

Since $\psi_\lambda-\psi_n = N^{-1}\left(\frac{(r-(n+\pi+\delta+\lambda))b}{r-(n+\pi)} + \frac{(r-(\pi+\delta))h}{r+\lambda-\pi}\right)$, ψ_λ will be larger than ψ_n (i.e. ψ_λ will be smaller numerically) if $r < \pi + \delta$.
If $r > \pi + \delta$ and $b = 0$, ψ_λ will be smaller (numerically larger) than ψ_n.

If $r > \pi + \delta$ and $b > 0$, the sign of $\psi_\lambda - \psi_n$ depends on the specific values of the parameters.

When $\lambda = n = 0$ and $r = \pi + \delta$ (37e to h) become

(37e') $\psi_\delta = N^{-1}(b+k+h)$

(37f') $\psi_\lambda = N^{-1}(b+k)$

(37g') $\psi_n = N^{-1}(b+k)$

(37h) $\psi_\pi = N^{-1}(b+k+h)$

Thus when there is debt neutrality, a small increase in λ or in n will have the same effect on k. A small increase in δ will have the same effect on k as small increase in $\bar{\pi}$. The effects on c of small changes in λ and n around zero will of course be quite different from each other since in that case

$$\frac{dc}{d\lambda} = \delta N^{-1}(b+k) > \frac{dc}{dn} = \delta N^{-1}(b+k) - k$$

VI. Conclusion

The Yaari-Blanchard model of consumer behavior has been generalized to allow for population growth and productivity growth. Blanchard's finding, in models without population growth and productivity growth, that uncertain lifetimes destroy debt neutrality and Weil's finding that, in a model without uncertain lifetimes and productivity growth, population growth alone destroys debt neutrality, are special cases of the general model. If and only if the sum of the population growth rate and the individual's probability of death is zero will there be debt neutrality. Non-zero productivity growth by itself does not destroy debt neutrality.

Note that debt neutrality, when $\lambda+n=0$, occurs because the government satisfies its intertemporal present value budget constraint, i.e. because the government is solvent in the sense defined by equation (16). It is therefore not correct to say, if $\lambda+n=0$, that debt neutrality implies that the government's tax program doesn't matter. The correct statement is that any tax program that maintains solvency doesn't matter. If solvency is threatened, i.e. if the terminal condition that the present discounted value (using $r-(n+\pi)$ to discount) of the debt burden (debt per unit of efficiency labor or debt-GDP ratio) goes to zero does not hold, there will not be debt neutrality, regardless of the value of $n+\lambda$.

The analysis has been deliberately restricted to the case of lump-sum, non-distortionary taxes. Non lump-sum taxes have (dis)incentive effects that will destroy debt neutrality even when $n+\lambda=0$ and the government remains solvent. Here toc, however, the Yaari-Blanchard model contributes something new. As shown in Buiter (1986b), when there is a single "conventional" distortion such as a non-lump-sum tax, changes in the distortionary tax rate may have first-order income effects even when they are evaluated at a zero value of the distortionary tax rate. This result occurs when $r\neq\delta+\pi$, which can be the case in well-behaved stationary equilibria of the Yaari-Blanchard model if $n+\lambda\neq0$. The discrepancy between the interest rate and the pure rate of time preference plus the rate of labor augmentation acts like a second, "intrinsic" distortion and lands us in the realm of second-best even when there is but one conventional distortion.

Finally, the Yaari-Blanchard model 1/ may well become the workhorse of the late eighties for analytical macroeconomic research and teaching, because of its simplicity and flexibility.

1/ Especially in its more complex but more general version with instantaneous utility represented by a constant relative risk aversion function

$$\frac{1}{\mu} \bar{c}^{\mu}; \; \mu < 1.$$

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