ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 555

SOVEREIGN DEBT RENEGOTIATION UNDER ASYMMETRIC INFORMATION

Kenneth M. Kletzer

Yale University

June 1988

(Revised: October 1988)

Notes: The Centre for Economic Policy Research supported this research. This paper was presented at the Centre for Economic Policy Research Conference "Economic Aspects of International Security" November 20, 1987.

I thank T.N. Srinivasan for helpful conversations.

Sovereign Debt Renegotiation Under Asymmetric Information

Abstract

This paper analyzes equilibrium debt contracts under potential renegotiation in the presence of sovereign risk. A simple model of borrowing from abroad to smooth consumption with stochastic national income is studied. Borrowers can choose to repudiate their debt obligations but face sanctions for doing so. Under free entry in loan contracts, equilibrium debt renegotiations take the form of reductions in current debt-service obligations with a new equilibrium market debt-contract. Under symmetric information, net inflows of funds are never provided in a renegotiation to a recalcitrant debtor. This contradicts part of the rationale given by several authors for a strategy of "defensive lending" to problem debtors. Asymmetric information about some debtor characteristic is introduced, and renegotiation of existing debt-service obligations is shown to give rise to separating equilibria. Because of the presence of private information, new net inflows may occur along with significant increases in future debt obligations in the event of renegotiation. The implications of these results for the dynamics of debt-service obligations and several extensions of the simple model are discussed.

Sovereign Debt Renegotiation Under Asymmetric Information

Kenneth M. Kletzer

This paper presents a theoretical analysis of the negotiation of the terms of contracts between private lenders and sovereign debtors. Credit market equilibrium when debt contracts are subject to renegotiation is studied in a framework which emphasizes the ability of a sovereign to repudiate its debt obligations. Our objective is to examine the consequences of contract renegotiation for new capital inflows to a country and the growth of its external debt burden. Some of the results suggest that private negotiations lead to socially inefficient outcomes.

Several authors have discussed the potential of new loans to problem debtor nations for increasing the present value of existing external debt. Two issues need to be distinguished. The first of these, which is addressed in this paper, is the hypothesis that additional funds provided to a currently recalcitrant sovereign debtor form part of an optimal strategy for creditors as a whole. Cline (1983), Krugman (1985, 1987), Sachs (1984), and others argue that additional funds reduce the probability of default on outstanding debt. Cline, in particular, merely assumes that new loans reduce the likelihood of default, so that additional loans which taken by themselves achieve negative expected profits provide the benefit of raising total expected repayments of outstanding debt. Therefore, the total return on the incremental loans to all creditors exceeds their opportunity cost. The second issue is that private lenders do not provide additional funds which increase the expected present value of all existing debt because existing creditors may be unable to internalize all the benefits due to the public-goods aspect of the new loans.

The characteristics of equilibrium loan contracts subject to subsequent renegotiation are discussed in a simple model of borrowing from abroad to smooth consumption when national income is stochastic, following the approach of Eaton and Gersovitz (1981). Borrowers have the ability to repudiate their obligations, but face sanctions for doing so. Lenders are assumed to be risk neutral and there is free entry in loan contracts, so that new creditors will provide any debt contract which assures them non-negative expected profits given existing debt service obligations. When a debtor suffers a low income state, repudiation with consequent penalization can be superior to meeting debt service obligations as originally contracted and choosing a new debt contract that provides zero expected profits to lenders.

In this case, existing creditors have an incentive to reduce the repayment obligations and refrain from declaring a default. A breach of contract does not automatically lead to declaration of default, because this is a subsequent option available to creditors and need not be exercised.

Equilibrium contract renegotiations are first examined for the case in which the debtor's current state is common knowledge. Optimal second-best renegotiations for the creditor in the presence of free entry in new debt contracts are shown to result in a reduction in existing debt service with no concurrent net inflow of funds to debtors. Furthermore, simple relending of funds to cover existing debt service payments in order to obtain an option on even larger future repayments is not, in general, optimal behavior for a creditor. The rationality for lenders of such "defensive" lending and debt reschedulings (suggested, for example, by Cline (1983) and Krugman (1985 and 1987) is put into question by these results. The first section of this paper shows that when debtor prefers the penalties that accompany repudiation to repaying its current debt and taking a new loan contract that assures non-negative expected profits to a new lender, then the old creditors' best actions are equivalent to reducing current debt service payments followed by a new market debt contract. Lindert (1986) makes a similar argument in a model without repayment renegotiations, in which the debtor can choose only full repayment or repudiation.

The section concludes by comparing debt contracts with subsequent renegotiation to equilibrium complete state-contingent loan contracts, constrained by the possibility of repudiation. Under free entry, ex post renegotiation of standard debt contracts does not lead to the same outcome as lending with ex ante specification of state-contingent repayment schedules.

The next section of the paper introduces asymmetric information about the debtor's state; this motivates the use of debt contracts in place of state contingent claims. Equilibrium renegotiations in this extension of the simple model of sovereign borrowing may entail debt reschedulings and new capital inflows in order to satisfy a set of incentive compatibility constraints. These alternatives to debt write-downs will separate borrowers according to their current state, which is not observable directly by creditors. By inducing self-selection by borrowers, equilibrium debt renegotiation offers induce revelation of the debtor's private information. The introduction of private information qualifies the argument made in the perfect information model, but for very different reasons than those given by proponents of defensive lending. Debt-renegotiation in this model leads to a dynamic behavior of net capital flows and debt-service obligations that may be of some interest. Poor states of the world for debtors lead to large increases in debt burdens although the net inflow of capital is negative or small. The marginal rate of interest for rescheduled debt can become very large as a consequence of the asymmetry in information.

A natural extension of the analysis of these two sections is the introduction of bilateral bargaining ex post, to give debtors more market power than simply access to new lenders. The adoption of the noncooperative strategic approach to the Nash bargaining problem in the complete and perfect information model will not affect qualitatively the outcomes of debt renegotiation. Under incomplete information about debtor characteristics, separation of different types of borrowers occurs through strategic delay rather than choice over a number of simultaneous offers made by creditors. The third section presents an approach to extending the analysis under asymmetric information to a strategic Nash bargaining framework. Both separating and pooling equilibria are possible outcomes. The model outlined includes capital accumulation, so that depreciation of the per capita physical capital stock is part of the social cost of delaying agreement in debt renegotiations.

The fourth section briefly summarizes a multi-period contracting approach when the debtor has sovereign immunity and lenders can credibly enter into contractual obligations binding on them which are enforceable in creditor nation courts. An application of such contracts, which incorporate, explicitly or implicitly, the possibility of revision, is the self-enforcement of restrictions on debt-dilution and provisions for debt-seniority. Contracts providing access to future loans on favorable terms provide an incentive for performance contingent on future events; repayment terms for early periods compensate lenders for the expected loss on these future contract options.

An alternative to the two extreme information assumptions is also discussed. A more realistic assumption may be that the debtor's current state is observable by creditors, but that policies (for example, those affecting investment levels) chosen by debtors are unobserved by lenders. In this case, the pattern of capital flows over time induces debtors to choose certain policies, but contracts cannot be written or renegotiated contingent upon the choice of policy. Capital flows can, at best, depend only on the history of borrower income in a stochastic model. The principal-agent approach to the repeated moral hazard problem can be extended to the sovereign borrowing case under a number of restrictive assumptions to yield a characterization of constrained optimal capital flows contingent on income. With the violation of the validity of a major assumption a distinct possibility arises that a complementarity between the collective actions of lenders and borrower policy choices can lead to inferior outcomes. This is especially likely when the policy instruments available for transfering resources from the private sector to the government for debt service are distortionary. Equilibria can exist that involve periods of large capital outflows requiring policies creating significant deadweight losses. At the margin, no additional loan will achieve non-negative profits. However the burden of distortionary policy can lead to a fundamental non-convexity. A large reduction in the current trade surplus may lead to an adequate shift in the marginal productivity of new loans to support the lower current debt service requirement. Since this requires lending by creditors with concurrent domestic policy revision by debtors, coordination can be a significant problem.

The last section offers concluding remarks and discusses implications of the asymmetric information case for the dynamics of debt service obligations. These suggest a significant social inefficiency reflected in the onset of repayment difficulties.

I. Debt Renegotiation in a Principal-Agent Model

This section discusses the renegotiation of debt service obligations in a version of the familiar Eaton-Gersovitz (1981) model. The sovereign debtor always has the option to repudiate its obligations outright and suffer consequent sanctions. The reduction in social welfare for the debtor country that sanctions can cause is limited, so that the borrower has limited liability for debt obligations. We assume that the threat of penalization for repudiation is credible and that creditors receive nothing by imposing sanctions. The behavior of the borrower is derived by maximizing a discounted stream of felicity of current consumption subject to a set of constraints. This represents a decision maker's social welfare function. A single good is produced and consumed. For simplicity, we ignore investment, so that output is an exogenous random variable. Under the informational assumptions of this section and the next, investment plays no essential qualitative role.

If the debtor chooses to repudiate, it receives a level of utility, $\bar{V}$, which depends on the current realization of output, y, and possibly on the value of the outstanding debt service obligations, R. That is, the repudiation level of utility depends on the debtor's current state, (y,R). The borrower's felicity function, $U(c)$, is concave, displays positive marginal felicity of current consumption, c, and is continuous. In equilibrium, the borrower will face a set of debt-contract offers in the event it chooses to pay current debt service and another set of offers if it seeks to renegotiate current contractual obligations. Because we assume a stationary environment (output is identically and independently distributed each period), the borrower can always select the same debt contract each period by paying the interest obligation on the constant principal every period. Since the realized level of output is observed before current consumption and the new loan is chosen, the borrower will select different contracts (or repudiation), including a possible request to renegotiate, depending upon the current state, (y,R).

An important assumption is that there is free entry in debt contracts — any expected profitable contract will be offered by a pool of potential lenders. If a loan providing non-negative expected profits will be accepted by a borrower, then it will be offered by some creditor. When a debtor prefers repudiation to repayment and selection of a new debt contract from this pool of potential lenders, existing creditors have an incentive to offer combinations of net current payments and new debt service obligations that cannot be obtained from the market. We model such renegotiations in a setting in which current creditors make offers to their debtors who choose to accept or reject these offers, but do not make counteroffers. This corresponds to a principal-agent setting in which the market power of existing creditors is limited by the potential entry of new creditors.

The utility maximization problem for debtors is first described. This provides constraints for the creditor's maximization problem. A debt contract is a pair (ℓ_t, R_{t+1}), where ℓ_t is the principal provided at time t and R_{t+1} is the total debt service obligation due at time $t+1$, or, equivalently, the time $t+1$ present value of the contracted repayment obligations.

In the event of full repayment, the borrower's value is given by:

$$V^r(y_t,R_t) = \max\ [U(y_t + \ell_t - R_t) + \beta EV(y_{t+1},R_{t+1})], \tag{1}$$

with respect to ℓ_t and R_{t+1}

subject to $(\ell_t, R_{t+1}) \in S$,

where the set S is independent of (y_t, R_t). The expectation is taken with respect to y_{t+1}; $V(\cdot,\cdot)$ will be defined below. The set S is the equilibrium set of debt contracts providing non-negative expected profits. The difference, $(\ell_t - R_t)$, is the net inflow of funds at time t. The discount factor, β, is between 0 and 1.

Let the debtor's repudiation value under limited liability be given by $\bar{V}(y_t,R_t)$ which is increasing in y_t and non-increasing in R_t. In the event of renegotiation, the debtor will choose a contract from a set of debt contracts that depend on the information available to creditors. We assume that this always includes R_t and discuss the case in which y_t is common knowledge in this section. In the next section, it is debtor private information.

Define:

$$V^{re}(y_t, R_t) = \max\ [U(y_t + \ell_t - R_t) + \beta EV(y_{t+1}, R_{t+1})], \tag{2}$$

subject to $(\ell_t, R_{t+1}) \in S(y_t, R_t)$.

This latter set contains S and will include additional contracts if $V^r(y_t,R_t)$ is less than $\bar{V}(y_t,R_t)$. The value of the debtor's optimal program is just

$$V(y_t, R_t) = \max\ \{V^{re}(y_t, R_t), \bar{V}(y_t,R_t)\}, \tag{3}$$

since $V^{re}(y_t,R_t)$ is at least as great as $V^r(y_t,R_t)$.

The distribution for output is assumed to have compact support. The expectation of V is taken with respect to y_t. We use the shorter notation $EV(R)$ for the remainder.

Creditors are assumed to be risk neutral (therefore, expected profit maximizers) and face an opportunity cost of loans given by a discount factor, ρ. A one-period debt contract provides expected profits given by

$$E\pi(\ell_t, R_{t+1}) = -\ell_t + \rho E(R|R_{t+1}), \tag{4}$$

where the expectation, taken with respect to the distribution of output, is of the actual period $t+1$ present value of debt service payments conditional on the contractual obligation, R_{t+1}.

The legal status of existing debt service obligations within or between creditor nations will be crucial for determining the set of offered contracts. For example, while loan covenants binding on debtor behavior may not be credibly enforceable, seniority provisions binding on subsequent lenders may be enforceable in creditor nation courts. A senior creditor may be able to recover fully any payments made to successor lenders in its home country up to its contractual claim. On the other hand, if all claims have equal priority, creditors will share according to some proportions in actual settlements.

Suppose that the variable x denotes the surplus available for meeting debt service in an equilibrium settlement of obligations and that x is distributed according to the cumulative distribution function F(x). This distribution depends upon the distribution of y and is conditional on R, in the general case. With strict seniority, the senior creditor obtains expected profits

$$E\pi(\ell,R) = -\ell + \rho\ [\ \int_0^R x dF(x) + R \int_R^M dF(x)\ ], \tag{5}$$

where M is the maximum total settlement possible.

A second creditor will obtain

$$E\pi(\bar{\ell},\ \bar{R};\ R) = -\bar{\ell} + \rho\ [\ \int_R^{\bar{R}} (x-R) dF(x) + \bar{R} \int_{\bar{R}}^M dF\ ]$$

with contract $(\bar{\ell}, \bar{R})$ given prior commitments R. In such an instance, the set of new debt contracts available to a borrower will be identical for any number of concurrent loans taken. The debtor can do no better than to accept a zero expected profit contract from a single source.

If lenders share in payments according to the portion of their claims in total claims, then each lender attains expected profits

$$E\pi(\ell_i,R_i;R) = -\ell + \rho\ [(R_i/R) \int_0^R x\ dF(x) + R_i \int_R^M dF(x)\ ],$$

where $R = \sum_i R_i$.

In this case, in an equilibrium debt contract, each lender correctly anticipates subsequent contract offers so that expected profits for every creditor are non-negative. The set of total debt contracts that attain non-negative expected profits is the same whenever obligations to new lenders do not take precedence over existing debt, since the conditional distribution of x is unaffected. However, the equilibrium debt contract will not be the same. Under strict seniority, the choice of contract made in equation (1) will be the best zero-expected profit contract for the debtor (equivalent to the Nash equilibrium contract under observability defined in Kletzer (1984)). In the absence of seniority provisions (for example, the neutral case above), the equilibrium contract will be an interest-rate taking zero-profit contract, as defined in Kletzer (1984) (equivalently, in Gale and Hellwig (1985)). This type of contract is socially inefficient, in that it is dominated for the debtor by the strict seniority outcome. For now, we assume that seniority provisions enforceable between creditors in their home courts are credible.

The initial description of equilibrium debt renegotiations in this standard approach will be made assuming that the debtor always has the option to pay contractual debt service and select a new debt contract that will realize a non-negative expected profit. However, a new debt contract may not be offered if existing obligations are not met, because new creditors' claims are junior to existing claims. If new funds are offered when old debts are not being serviced, in the absence of a negotiated settlement, the debt service obligations on these new funds are at least as great as they would be for incremental funds taken in addition to the original contract (that is, the additional debt service that would be incurred to obtain a larger original contract). The additional debt service obligations will be even greater if the old creditors can claim additional interest from payments made to the new suppliers.

Free entry in debt contracts and the limited liability of debtors impose limitations on the outcomes attainable by creditors in debt service renegotiations. Constrained contract renegotiations for the lender can be described using a principal-agent framework in which the creditor offers contract revisions to the debtor. We first assume that the borrowers' current output, y, is common knowledge (throughout, we assume that the debtor's utility function is common knowledge). In this setting, a first-best contract may not be a standard debt contract with ex post renegotiation of debt service because additional risk sharing may be provided by state-contingent contracts. We first discuss renegotiation of debt contracts because this corresponds more closely to the framework in which the case for defensive lending has been argued. The structure of equilibrium state-contingent contracts in this approach is discussed at the end of this section.

Because the equilibrium set of debt contracts offered will be bounded from above in ℓ, there exist states such that the borrower prefers repudiation to full repayment. These states can be shown to occur with positive probability. Because creditors lose the entire opportunity cost of their loans in a repudiation, any settlement that provides some current repayment or net expected future payment will be preferred by the creditor. The borrower's alternative of choosing a zero-expected profit contract (but junior claim) from another lender without repaying will, at the worst, result in a loss to the current creditors of the opportunity interest on the maximum settlement they would obtain in the current state. If we assume, for simplicity, that no additional interest is attainable, then the debtor prefers to repudiate if

$$\bar{V}(y,R) > \max\{\max_{(\bar{\ell},\bar{R}+R)\in S} (U(y + \bar{\ell}) + \beta EV\ (R + \bar{R})),\ V^r(y,R)\}, \tag{6}$$

Modification for imperfectly enforceable seniority clauses or enforceable contracts specifying overdue interest charges is straightforward.

Whenever (6) obtains, creditors will select contracts that provide the debtor with utility at least equal to the repudiation level. These offers will depend only on the debtor's current state. If we make the simplification that $\bar{V}(y,R) = \bar{V}(y)$, then the equilibrium expected profits for debt contracts is given by (5), where $F(x)$ depends on the level of debt service obligations and the distribution of y. The set S is given by

$$S = \{\ (\ell,R)\ |\ E\pi\ (\ell,R) \geq 0\}$$

When only the lender makes offers that the debtor accepts or rejects (in the presense of free entry of new creditors under our seniority assumption), the equilibrium renegotiated offers satisfy

$$\max\ [\ R - \ell(y_t) + \rho \int_0^{R(y_t)} x dF(x) + \rho\ R(y_t) \int_{R(y_t)}^{M} dF(x)\ ] \tag{7}$$

with respect to $\ell(y_t)$, $R(y_t)$

s.t. $\bar{V}\ (y_t) \leq U(\ y_t + \ell(y_t) - R) + \beta EV(R(y_t))$.

Note that any solution cannot be contained in S since (6) holds. The solution to this problem is identical to the solution to the problem:

$$\max R$$

$$\text{s.t. } \bar{V}(y_t) \leq \max_{(\ell',\ R') \in S} \left[\ U(y_t + \ell' - R) + \beta EV(R')\ \right],$$

with respect to $(\ell',\ R') \in S$

The profit-maximizing lender will never choose to make an offer of a net flow of funds to or from a debtor that involves an incremental loan providing negative expected profits. Any creditor-optimal renegotiation is equivalent to a simple reduction in current debt service (in expected present value terms) plus a new loan attainable from any potential entrant. The creditor should be indifferent between offering a current net payment with a new debt service obligation and offering a reduction in the current debt service just enough that a new creditor will take over the debt and the borrower will not choose to repudiate. Because the debtor always has the option to repudiate, the expected value of continuation in (1)–(3) must be at least as great as the expected value under repudiation. Therefore, whenever (6) obtains, contractual performance must require a net outflow of output from the debtor. A solution to problem (7) never entails a new inflow of funds to the borrower.

A common argument (for example, Krugman (1985)) is that the relending of contractual debt service obligations when a debtor is unwilling to meet them currently is a preferred action for lenders because the option on higher future payments is obtained at no cost in new capital. The above discussion lends considerable doubt to this common sense proposition. Relending old debt service with interest and a zero current net flow of funds is not generally the optimal ex post contract for the creditor. The maximum expected present value of a renegotiated contract is attained by reducing debt obligations by just enough so that the debtor can achieve its repudiation level of utility by selecting its optimal new zero-expected profit debt contract. This debtor action (or its equivalent, in which current creditors are also the new providers of the zero-expected profit loan) extracts all the debtor's surplus; therefore, no other contract renegotiation is superior for the current creditors. The option on new debt repayments under the rollover scheme has a positive opportunity cost; starting at the creditor-optimal revised contract devised above, a negative expected profit loan must be made implicitly to provide the rescheduled loan that gives the debtor utility equal to its repudiation utility.

Figure 1 depicts the set of new debt contracts, S, and indifference curves for the debtor for the more general case of the repudiation utility depending on both y and R. The horizontal axis measures the net inflow of resources to the debtor. The set S is not bounded from above in contractual obligations in the presence of equilibrium renegotiations. The maximum expected present value of a renegotiation attainable by creditors can be seen to be the greatest value of $-R$ such that the set S intersects the repudiation indifference curve.

Because the debtor has limited liability for debt obligations in this framework, the utility attained with a current zero net flow of resources is always at least as great as the level of utility received by repudiating. Therefore, no new flows from lenders are required to avoid repudiation.

The presence of debt service obligations as a state variable introduces a simple history dependence in the expected utility of debtors and the renegotiation offers made available. Past realizations of income affect current choices of debtors and, in the event of renegotiation, existing creditors. In this simple Markov model, however, the set of new debt contracts, S, is unaffected. In equilibrium, only partial risk sharing between risk-neutral lenders and risk-averse borrowers is achieved through debt contracts with renegotiation. This occurs because creditors are limited in their abilities to obtain large payments in the best income states of nature.

An alternative approach to the problem of lending with potential repudiation under complete and perfect information is to derive the first-best type of contract (constrained by the threat of repudiation) that will be state contingent. The problem of characterizing such contracts in our setting can be posed as a simple static constrained maximization problem by recognizing

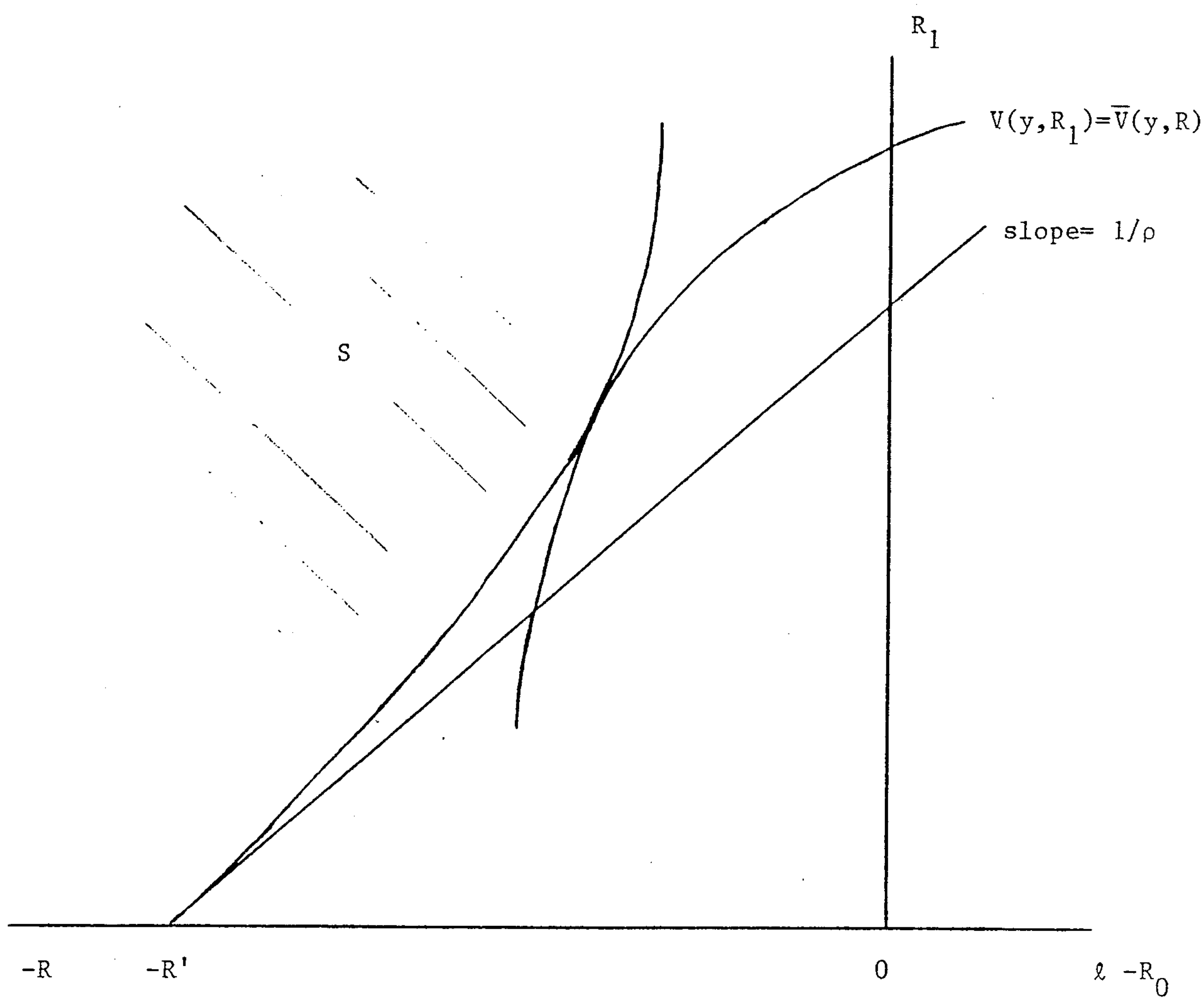

Figure 1

that the future utility of the debtor will be determined completely by the difference between income and debt service obligations, $(y_{t+1} - R(y_{t+1}))$, since repudiation will never occur (if creditors receive nothing in that event).

The first-best problem with free entry in loan contracts is given by (assuming $\bar{V}$ is only a function of y):

$$\max V(y_t - R_t) = U(y_t + \ell_t - R_t) + \beta EV(y_{t+1} - R(y_{t+1})),$$

with respect to ℓ_t and $R(y_{t+1})$,

subject to $\quad V(y_{t+1} - R(y_{t+1})) \geq \bar{V}(y_{t+1})$, for each y_{t+1}

and $\quad -\ell_t + \rho E(R(y_{t+1})) = 0.$

The solution to this problem is straightforward and entails constant consumption for a subset of the output states, if ρ equals β. The states for which utility may be held to the repudiation level will be the high output states, not the low ones. Consumption is imperfectly smoothed if repudiation is superior in some (high output) state to consumption of mean output. This contrasts sharply with the second-best renegotiation case. If ρ exceeds β (the borrower is more impatient than the lender), then a sequence of one-period loan contracts may smooth income for early periods, but after the repudiation constraint becomes binding in the best output state, consumption will continue to decline and will not be smoothed thereafter in the framework used above. Worrall (1987) adopts a trigger-strategy penalty mechanism for competitive lenders and shows that long-run consumption smoothing can result.

In the perfect information setting, first-best contracts are fully state contingent. Such contracts cannot be replicated by simple debt contracts with ex post renegotiation in the presence of free entry of new creditors. The latter type of contracts may arise as first-best outcomes when debtors' current states are observable by creditors only at a cost (for example, see Gale and Hellwig (1985)).

II. Private Information and Separating Equilibria

In this section, debtors are assumed to possess private information about the utility they receive by accepting various debt contracts. Therefore they have an incentive to report incorrectly their willingness to repudiate to obtain a reduction in debt service payments. Whenever lenders perceive a positive probability, given current debt obligations, that a borrower would prefer repudiation to the selection of a new debt contract with repayment, the borrower may be able to misrepresent its private information. If creditors are unable to observe the realized value of output, under the equilibrium renegotiation scheme of the previous section in every output state the debtor will claim willingness to repudiate. Some contract with debt service reduction chosen in a low output state will be preferred in a high output state to repayment. Creditors will seek to design the offers they make in debt renegotiations to induce correct revelation of the private information. Lenders will want to offer debt renegotiation packages which will be chosen over repudiation in poor events but which are inferior to repayment in favorable outcomes.

The private information possessed by debtors can be anything that affects the social welfare attained by choosing different debt contracts. For example, national leadership may be better informed about factors determining the social costs of achieving given levels of trade surplus than are foreign creditors. For expositional simplicity, let the realized value of output be unobservable by creditors, although we intend it to be a proxy for some measure of debtor country surplus. The distribution of output is assumed to be common knowledge, as are all other characteristics of the borrower. Also, suppose that output, y, can only take a finite number of values with positive probability. These are given by $y_1, y_2, \ldots, y_n$ in increasing order. The random variable, y, can be thought of as parameterizing a class of utility functions for the national leadership. Creditors do not know what type of decision-maker they face at each date. Each period a new type is drawn from the common distribution. In this interpretation, the period length is the time a particular type is in power. Again, the identification of y with output is not intended to be literal.

The creditors' problem is to choose a set of contracts to offer in the event of renegotiation requests such that their ex ante expected profit is maximized, when debtors ex post maximize utility over the set of contracts (including renegotiation packages) available. A contract renegotiation will be chosen only if it is the maximal contract in the realized state over the set of contracts offered for all states. The creditor's inability to observe output implies that debtor self-selection alone must be relied upon to assure the anticipated behavior in each output state. The creditor's problem is to design a contract set that induces truthful revelation. The equilibrium set of renegotiations offered will separate different output realizations through contract choice, so that ex post the private information is revealed.

The set of equilibrium offers under free entry in ex ante contracts, debtor-creditor relationships) and debtor limited liability is characterized again using a principal-agent framework. Because simple reductions in debt service will be chosen by the borrower in either low or high output states, offered revisions of debt repayments under asymmetric information about output realizations must observe a self-selection constraint. The contracts offered to assure non-repudiation in low output states must be inferior to other contracts available when the debtor realizes high output value. The addition of constraints assuring correct contract selection leads to a separating equilibrium. There will be n contracts available, with a different contract selected in each output realization. The contract intended to be selected in a particular state will provide the maximum utility to the debtor in that state over the set of offers. Some of these contracts will simply be the best choices over the set of new debt contracts available from any potential creditor. That is, the set of ex ante debt contracts will always be available with repayment of contractual debt service.

The set of ex ante debt contracts (those available from any new entrant creditor following repayment) will be found by first characterizing the set of ex post repayment revisions offered in equilibrium for a given current debt service obligation, R. Each member of the set of debt contracts offered by the current creditor will consist of a current net payment and a debt service obligation for the next period. These contracts will not be equivalent to the debt reductions derived in the previous section. Imposition of the self-selection contraints is found to result in lower ex post profit in each state than could be attained if the value of output were observed directly by the creditor. The equilibrium set of contracts involve higher levels of debt service for the next period for low output realizations than would arise with symmetric information.

The set of ex ante offers is derived using the solution to the creditor's ex post problem, as a perfect equilibrium. The set of initial non-negative expected profit contracts offered is a subset of what it would be without private information. Lenders are assured non-negative expected profits ex ante, so that ex ante debtor utility is lower than under symmetric information. In most states, however, debtors are better off ex post than if they could then report their output state before revised repayment offers are made. In states for which repudiation provides higher utility than full repayment, the debtor can receive higher utility under debt renegotiation than the repudiation level. Since under symmetric information, the debtor is always forced to either its repudiation utility level or its maximal utility over the set of new contracts with repayment (whichever is larger), direct reporting of the value of output before the choice of a contract ex post is incredible. Direct revelation only occurs with the selection of a separating equilibrium contract revision.

Given a level of existing debt service obligations, R, the existing creditor's problem is to find contracts, (ℓ_i, R_i), for each i, to maximize expected profits. The set of zero expected profit debt contracts, S, will be found implicitly; however, we assume that it is non-empty and define a loan offer, ℓ', for each next period debt service obligation, R_i. That is, $\ell'(R_i)$ is the size loan which repayment obligation R_i equals in expected present value for creditors. The present value loss to a creditor from offering the contract

(ℓ_i, R_i), is $(\ell_i - \ell'(R_i))$.

The existing creditor's problem is given by

$$\max \sum_{i=1}^{n} p_i(\ell'_i(R_i) - \ell_i) \tag{8}$$

with respect to $\{(\ell_i, R_i)\}$ for $i=1, \ldots, n$, subject to, for all i,

(a) $U(y_i + \ell_i - R) + \beta EV(R_i) \geq \bar{V}(y_i, R)$

(b) $U(y_i + \ell_i - R) + \beta EV(R_i) \geq V^r(y_i, R)$

(c) $U(y_i + \ell_i - R) + \beta EV(R_i) \geq U(y_i + \ell_j - R) + \beta EV(R_j)$,
for all $j \neq i$.

The probability of output y_i being realized is p_i. Constraint (a) is the restriction that repudiation is inferior to the debt contract offered for each value y_i, and (b) is the restriction on offers created by free entry in new contracts. The third is the self-selection constraint. The contract (ℓ_i, R_i) is at least as good for the debtor in state i as every other offer. We assume that indifference for the debtor is resolved in the lender's favor to assure a solution.

The solution to this problem yields a set of n offers ex post such that debt repudiation never occurs. The contracts offered to the debtor which are taken in some states for which repudiation is superior to repayment on contracted terms can provide greater utility than outright repudiation. Likewise, in some states for which selection of a new ex ante debt contract (with full repayment) is preferred to repudiation, the debtor will attain even higher utility by taking a contract offered by the current creditor but not by new entrants. The self-selection constraints produce these possibilities by creating trade-offs between expected profit in different states. The equilibrium contracts are interrelated.

The following proposition summarizes the properties of the equilibrium set of debt renegotiations. Define $V(x_i, R_i) \equiv U(y_i + \ell_i - R) + \beta EV(R_i)$, where $x_i = \ell_i - R$.

<u>Proposition</u>: Given current debt service obligations, the lender's most preferred debt renegotiations satisfy:

a) x_i and R_i are both non-increasing in i.

b) $V_i(x_i, R_i)$ is non-decreasing in i.

c) If $\bar{V}(y_1,R) < V^r(y_1,R)$, then $V_1(x_1,R_1) = \bar{V}(y_1,R)$.

(An analogous condition may hold for additional i)

d) Whenever $V_i(x_i,R_i) > max\{\bar{V}(y_i,R),\ V^r(y_i,R)\}$,

$V_i(x_i,R_i) = V_i(x_{i-1},R_{i-1})$ holds.

e) If $V_i(x_i,R_i) = V^r(y_i,R)$, then $(\ell_i,R_i) \in S$, $\ell_i = x_i + R$, and $(\ell_j,R_j) \in S$, for all $j > i$, so that $V_j(x_j,R_j) = V^r(y_j,\ R)$, also.

The proof of this proposition and additional hypotheses are contained in the appendix. Sappington (1983) presents similar results to part of the above for a simpler limited liability principal-agent problem.

In equilibrium, utility is nondecreasing and the net payment by the debtor is nondecreasing in output, while the next period debt service obligation is nonincreasing in output. The set of debt renegotiations offered forces the debtor in the lowest output state, if repudiation is ever preferred to repayment, to its repudiation level of utility. This may also be true for higher states.

The debtor may choose contracts from the ex ante zero expected profit set (contracts new entrants offer) in some high output states. The equilibrium ex post contract in these states may provide even higher utility. If the debtor attains just $V^r(y_i,R)$ in state y_i, then the existing creditor just offers the same set of debt contracts which any new entrant will offer, S. If the solution to the creditor's problem has the debtor choose repayment and a new zero expected profit contract in a state j, then the equilibrium choice in all higher states is also repayment as contracted. Result (d) states that the debtor is indifferent between the equilibrium debt contract for the realized state under renegotiation and the contract offered for the next lowest state, except, possibly, in two situations. The first occurs when the current state renegotiated debt contract provides just the repudiation level of utility for that state. The second occurs when the contract chosen in equilibrium involves full repayment for the present realization of output.

The continuous-state indifference property of result (d) and the above exceptions deserve explanation. If the debtor is offered a contract, (x_{i-1}, R_{i-1}), expected present value for the creditor in the next highest state can always be increased if the debtor's utility can be reduced in this next highest state. Therefore, unless utility cannot be reduced further in state i, the debtor is indifferent between the debt renegotiations for that state and for the next lower state. When the debtor achieves exactly the repudiation level of utility or the level assured by free entry in new debt contracts, this indifference may or may not hold. If the debtor chooses a new debt contract with full repayment in both the present state and next lower state, under concavity of felicity, this property does not hold.

Figure 2 shows a separating equilibrium set of debt renegotiations. The intertemporal marginal rate of substitution portrayed decreases with y for a given contract because $U(c)$ is strictly concave. Concavity is important for demonstrating the proposition; however, concavity of $U(c)$ does not imply that the derived indifference curves are convex everywhere. The relationship between expected value and contractual debt service obligations depends on the entire set of equilibrium debt contracts. The indifference curves are drawn smooth in Figure 2 for simplicity; with a finite number of states, they will each contain kinks.

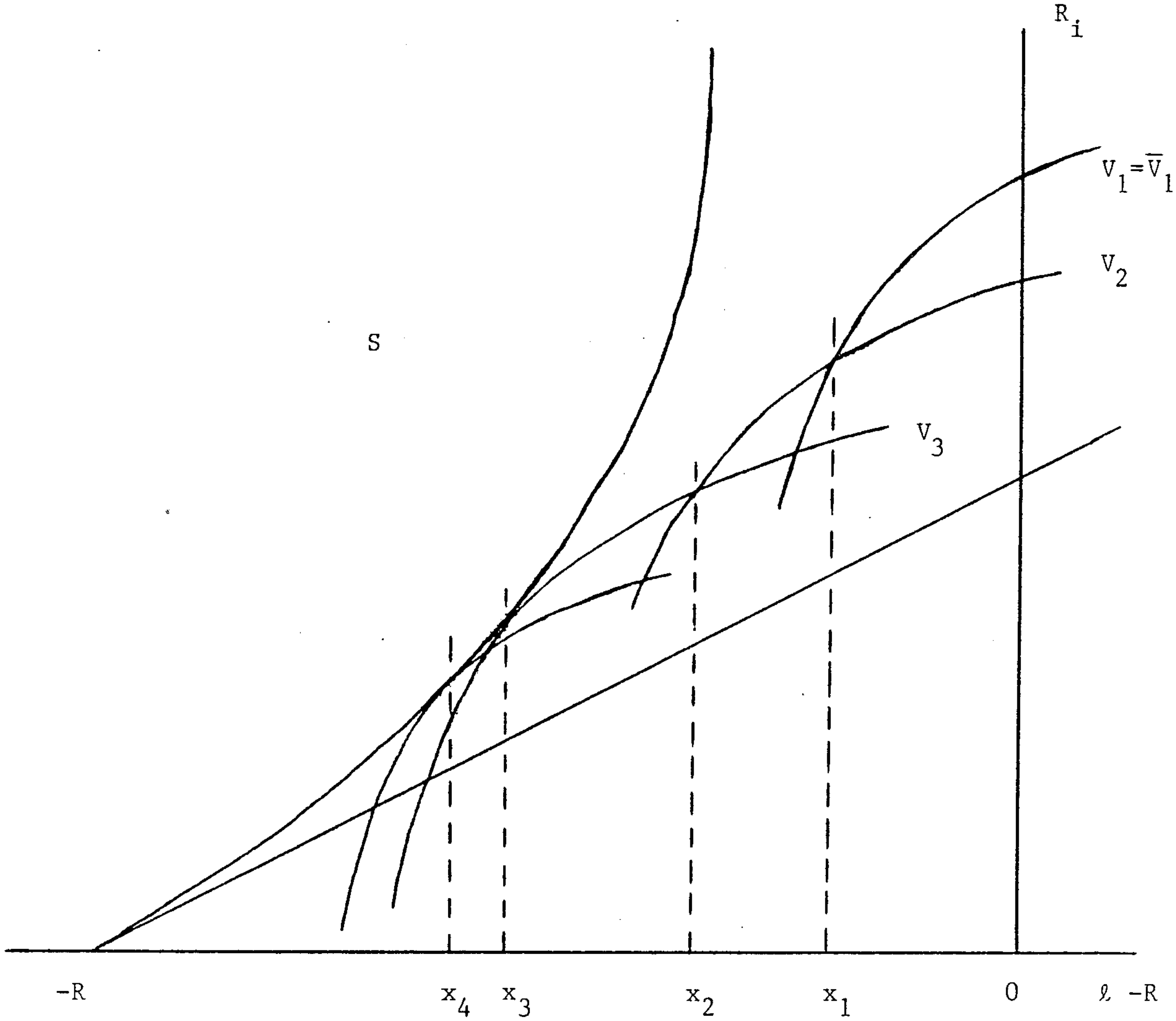

Figure 2

The equilibrium ex post contracts display a simple relationship between the intertemporal rate of substitution in contract terms along the boundary of S (zero expected profit contracts) and the intertemporal marginal rate of substitution. These are equal if full repayment occurs in equilibrium. If the debtor in state i is assigned contract (x_i,R_i), then the slope of the boundary of S at the contract $(\ell'(R_i),R_i)$ equals the intertemporal rate of substitution if the debtor is not indifferent in state $i+1$ between this contract and (x_{i+1},R_{i+1}). In the case of continuous state indifference, the rate of contract substitution equals a weighted sum of the marginal rate of substitution in state i and in state $i+1$. The weight on the state $i+1$ marginal rate of substitution is negative, but smaller in absolute value than the weight on the state i rate of substitution. This reflects the trade-off to ex post expected profit between lowering state i profit by revising R_i and x_i and increasing state $i+1$ profit by reducing utility in state $i+1$ (lowering x_{i+1}). The marginal rate of substitution of R_i for x_i in state i is less than the intertemporal rate of contract substitution. Therefore, state i profit alone is not maximized. The weights are implicitly given in the proof of the proposition; they depend upon the probability distribution of output and the marginal felicity of consumption in the two states.

Derivation of the set of initial loan contracts, S, remains. The ex ante expected profit is given by

$$E\pi = -\ell + \rho[R + \sum_{i=1}^{n} p_i(\ell'(R_i) - \ell_i)],$$

where (ℓ_i,R_i) are solutions to the creditor's ex post optimization problem. The last term (summand) is the expected present value of the reduction in debt service received. Even if ℓ_i exceeds $\ell'(R_i)$, the lender's return may exceed opportunity cost in some states. Maximization of expected profit will lead to a non-zero probability that the debtor is willing to repudiate. Risk neutrality of creditors allows risk-averse debtors to achieve some degree of insurance. As in the well-known principal-agent literature (for example, Holmstrom (1979), Harris and Raviv (1979)), risk sharing is incomplete due to the need for equilibrium debt renegotiation to observe the self-selection constraints. Maximization of ex ante expected profit gives the set of non-negative expected profit contracts offered by new entrants. We assume that the utility function for the debtor, possible output states, and lender's discount factor are adequate to assure that the set is non-empty and potential debtors choose to borrow initially.

An important consequence of the proposition is that the maximal ex ante contractual debt service obligation is at least as great as the resulting ex post debt service for the succeeding period in the lowest output state. Any increase in debt obligations beyond this level will never be met. A corollary to the proposition is that this level of debt obligations is the maximum amount such that ex post, the debtor repays in full and selects a new zero expected profit contract in the highest output state. Figure 3 portrays this equilibrium. The indifference curves are vertical beyond R_1, as increases in R_i have no effect on the debtor because such incremental repayment obligations are never repaid.

In a separating equilibrium, the net capital outflow from the debtor can be either positive or negative in a state for which repudiation dominates full repayment of existing debt service and choice of a new ex ante debt contract. This contrasts with the equilibrium outcome under symmetric information. The possibility that the lender provides additional inflows to a recalcitrant debtor arises when the repudiation level of utility depends upon the debt service obligations that are repudiated. Contracts that satisfy the necessary conditions for expected profit maximization in low states may involve positive values of x, because the intertemporal marginal rate of substitution is finite for the repudiation level of utility at contracts with zero net outflows (x equal to zero). This possibility does not arise if the cost of repudiation

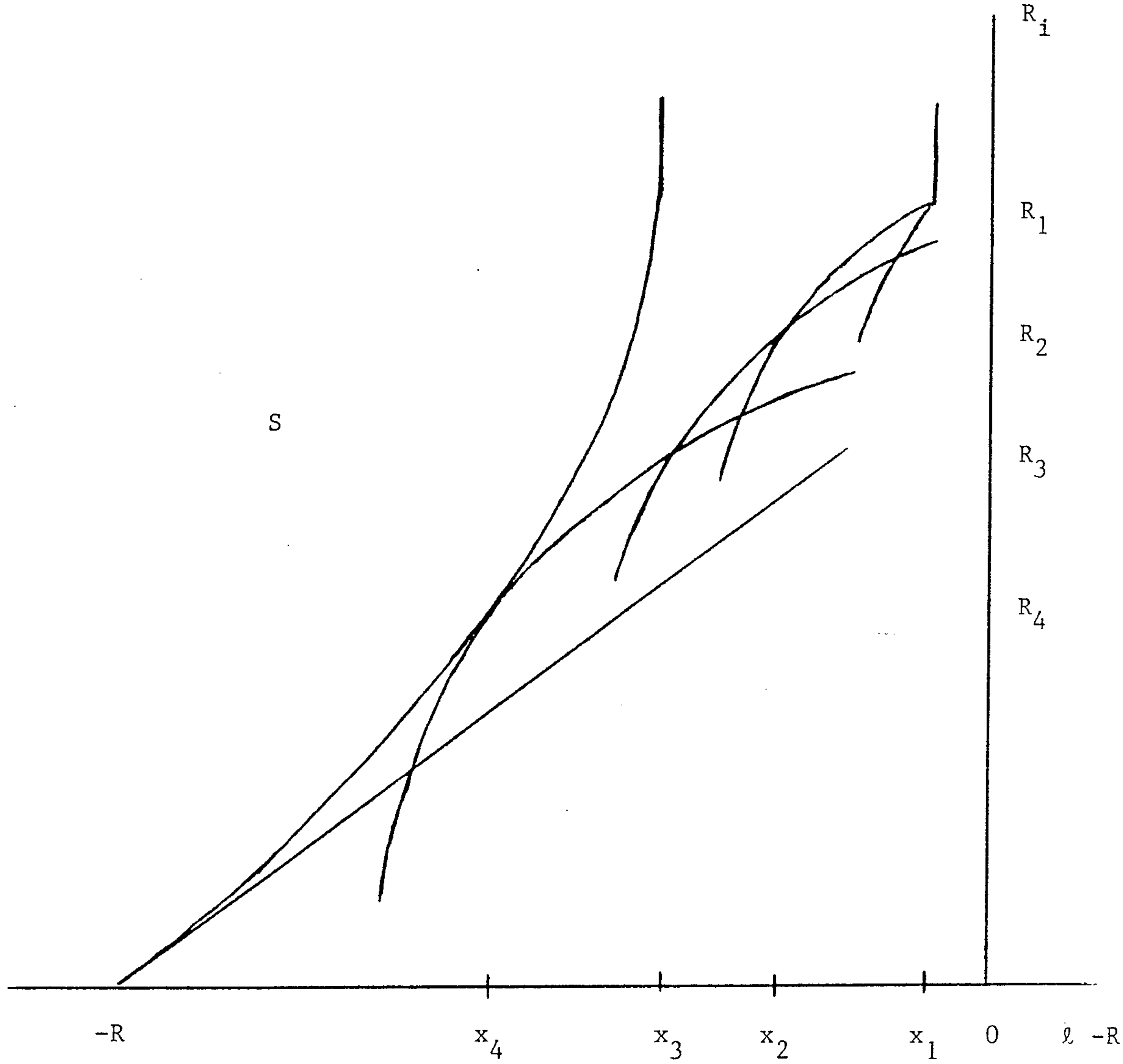

Figure 3

depends only on the current value of output. In this case, the debtor will always prefer a contract with zero net outflow to repudiation, regardless of the next-period repayment obligation. Therefore, in the lowest state, a net payment to the existing creditor is made under debt renegotiation.

Two properties of the solution to the creditors' two-stage optimization problem are notable. The first is that there may be many equilibria; nothing in this framework rules them out. Multiple equilibria are likely to occur when repudiation costs depend upon current debt service obligations. The second is that there is no reason that an ex ante state-contingent contract not be written. Such contracts can specify the same equilibrium set of contracts as derived in the revision problem. Ex post, the debtor reveals its private information by selecting the net payment and contract for the next period we characterized above. Because the sovereign debtor always can choose to accept none of the contracted payments and next period obligations and select a new ex ante contract if utility is higher by doing so, such state-contingent contracts will be solutions to an identical two-stage problem. The only difference is in interpretation. We have solved the problem of contract selection by creditors when the debtor has limited liability, and there is always free entry in new debtor-creditor relationships. That is, the equilibrium state-contingent contract will assure the debtor at least as much utility in the highest state as it attains by meeting the current obligation and choosing a new creditor.

While we have represented the solution as formed by an initial debt contract followed by debt renegotiation offers by the current creditor, when both sides place positive probabilities on all possible subsequent events, a more complex state-contingent contract would suffice. There is no need for renegotiation. If an event occurs upon which one of the parties to the contract placed zero probability, however, a contract renegotiation can be mutually beneficial. For example, if there is an unanticipated change in the world interest rate (creditors' discount rate), the creditor may wish to revise the set of ex post contracts offered. If a full state-contingent contract binding on the creditor is in force, then only changes favoring the debtor can be acceptable. With private information, both the existing separating contracts and any new offers can be selected by the debtor. If, instead, the ex ante contract specifies only the loan principal and debt service obligation, then the ex post contracts can be offered by the creditor after observation of the interest rate shift. The creditors can offer initial relationships that specify contracts for each output state in every possible realization of the world interest rate. Because the world interest rate is public information, such contingent debt contracts can provide welfare gains when the creditor is risk neutral and debtor risk averse. If the creditor's discount rate is private information (for example, depending upon the remainder of its loan portfolio), then debt contracts with subsequent renegotiation possibilities can arise in equilibrium. We can also appeal to the costs of writing or resolving disputes over complex state-contingent contracts to support the assumption of renegotiation and to rule out interest rate contingencies.

The effects of an interest rate shock (realization of ρ anticipated with zero probability) on the set of ex post separating contracts is ambiguous. An increase in creditors' discount rates causes a reduction in the set of ex ante contracts. In states for which repayment is preferred to repudiation (although the equilibrium debt renegotiation is not necessarily in S), there is a tendency for x_i to fall and R_i to increase (therefore, utility must decrease). For states in which repudiation is superior, however, the change in contract is ambiguous. The interest rate increase will lead to a reduction in debtor expected utility and a reduction in the present value of the contract for the creditor.

III. Separation through Costly Delay in Bargaining

The principal-agent framework used in the previous two sections provides insight into the nature of debt renegotiation outcomes when borrowers have more bargaining power than just the options to return to the loan market or repudiate. If we allow output in any given period to be storable for some positive length of time (however short, or with arbitrarily large but finite rate of depreciation), then the equilibria derived in both the perfect and imperfect information cases are equilibria for a strategic bargaining game in which the creditor makes all offers (see Sobel and Takahashi (1983)). For a strategic bargaining game, with alternating offers, debtors will achieve better outcomes ex post than were attained in the preceding solutions. Nevertheless, the ex ante contract offers will adjust to account for the ex post divisions of surplus in any subgame perfect equilibrium.

Bulow and Rogoff (1986) adopt the strategic approach to Nash bargaining games under complete information, due to Rubinstein (1982), to sovereign debt negotiations. The creditor who acquires the right to impose sanctions by making an initial loan sells a promise not to impose sanctions each period to the debtor. The amount paid in the subgame perfect equilibrium each period for this property right is just the debt service payment. The discounted stream of these prices is equal to the amount initially lent under perfect competition among lenders. The perfect equilibrium is unique if penalization benefits creditors an arbitrarily small amount. The complete information model presented earlier in this paper needs minor additional assumptions to fit the Rubinstein (1982) framework: let output be storable and let the debtor be risk neutral. In this setting, renegotiations will never result in new inflows unless a new creditor will also supply them. Under risk aversion, access to new credit in the presence of seniority provisions can become the object bargained over, but the characterization of renegotiations will not be affected. In the complete information bargaining approach, there is no particular reason why the initial contract does not simply specify the perfect equilibrium debt service payments. If it does, then no bargaining actually takes place.

The asymmetric information model can also be extended to a bargaining framework. Delays to agreement can lead to separation of debtors by type in an alternating offers bargaining game. Simultaneously offered contracts by the creditor no longer serve the purpose of inducing truthful revelation. Incomplete information can be introduced, as before, through asymmetric observability of output, or through private information about rates of time preference. Delaying agreement can arise strategically to separate borrowers with different realizations of privately observed random variables, or of different social preferences, which are unobserved by creditors. Delay can also arise because one or both parties find that waiting for publicly observed information to arrive is individually rational. This case may be important when creditors, as well as debtors, have limited liability and are therefore risk loving.

This section outlines an approach to modeling socially costly delays to a resolution of debt repayment problems. The impasse in the current repayments crisis and the consequent lack of funds to finance capital formation have been discussed widely. In noncooperative Nash bargaining models, equilibrium delay to agreement has been shown to arise in the presence of incomplete information by a large number of authors. We discuss one source of delay: strategic delay necessary to convey the debtor's private information.

Our approach is to adopt the bargaining model with one-sided incomplete information of Admati and Perry (1987), which has equilibrium paths displaying strategic delay to external borrowing by a growing economy. Following Bulow and Rogoff, we assume that by lending the creditor purchases a right to impose sanctions; the promise not to exercise this right is then sold to the debtor at the subgame-perfect price and time. Unlike their model, agreement need not occur immediately here. A major cost of delay to agreement will be the absence of new credit. New creditors may not provide additional funds to a growing debtor in the presence of unresolved existing claims. The reason is that the net inflow of resources will affect the bargaining game between old creditors and the debtor and therefore the investment undertaken by the borrower. The future flow of output following a given loan will, in general, be less if existing claims need to be resolved.

Several possible approaches can motivate the adoption of the strategic delay model. The debtor is assumed to have private information about the value it places on avoiding sanctions. Sanctions are assumed to lead to lower levels of per capita consumption than are attainable along an equilibrium path for the bargaining game, so that debt repudiation will never occur in equilibrium. Capital accumulation is possible, and either the labor force grows at a constant proportional rate or physical capital depreciates. Foreign borrowing can be motivated by assuming that either the planner's discount rate or the marginal productivity of capital exceed the world rate of interest. A simple model is one in which output, which depreciates in storage, is traded for capital goods which are noncompetitive imports. During an impasse, the per capita capital stock declines.

The private information of the debtor is about the surplus available to pay creditors. This can be the current realized value of output in a stochastic model, as in previous sections, or it can be the minimum level of per capita consumption politically acceptable in a renegotiation, or other debtor characteristics. Suppose that whenever per capita consumption falls below some level, $\bar{c}$, political leadership is replaced immediately (through either parliamentary or nondemocratic means). Then the surplus available to service debt obligations, that is, the value placed on purchasing the promise not to impose sanctions, is the amount of current resources exceeding those needed to sustain $\bar{c}$ along a perfect equilibrium path. The country's policymakers are likely to be more informed about $\bar{c}$, or, more generally, the social cost of generating given levels of trade surpluses (for example, the excess burden of indirect taxes).

We assume that output is produced using capital and labor according to a constant returns-to-scale technology. Output is storable (depreciation can occur, but need not) and is consumed or traded for investment goods, which are not produced at home. Let output be given by

$$y_t = f(k_t)$$

and let

$$\Delta k_t = k_{t+1} - k_t = i_t - nk_t$$

Storage is given by s_t, so that

$$y_t = c_t + (\gamma s_{t-1} - s_t) - R_t,$$

where γ is the rate of depreciation of stored output and R_t is output exported.

The trade surplus is just

$$R_t - i_t,$$

We ignore sanctions by assuming that repudiations lead to consumption equal to or less than the minimum politically acceptable in a negotiated settlement. If lenders benefit from imposing sanctions, by any arbitrarily small positive amount, then no subgame-perfect equilibrium involves repudiation without consequent penalization (see Bulow and Rogoff). We simply assume that penalization for repudiation is a credible threat.

The policymaker's social welfare function is just

$$U = \sum_{t=0}^{\infty} \beta^t c_t.$$

The value of the optimal capital accumulation program along a subgame-perfect equilibrium path can be defined directly. We first need to note that once the debtor's private information is revealed, a complete information bargaining subgame follows for the model described here. The creditor's lack of information about the value of sanctions to the debtor derives from potential differences in the type of debtor, rather than imperfect information about its current state. This assumption allows us to look at a single episode, but the generalization is a formal exercise.

The debtor's type is characterized by the maximum surplus it can transfer to creditors in exchange for suspension of the threat of sanctions at a given time. Time matters both because the social discount rate is positive and the per capita capital stock declines during delays to agreement.

Suppose that the low $\bar{c}$ type repays at time 0. Then the surplus (denoted h_0) in a given state, k_0, is defined by the problem

$$V(k_0 \mid h) = \max_{k_1} \{ c_0 + \beta V(k_1) \}$$

subject to

$$k_1 = k_0 + i - nk_0,$$

$$c_0 = f(k_0) - (h + i),$$

where $V(k_1)$ is the value of the debtor's utility along a subsequent equilibrium path. Let h_0 be the maximum value of h such that $c_0 \geq \bar{c}$.

We can derive the debtor's value in terms of the amount paid the creditor and the time at which settlement takes place by noting that if its type is revealed, then subsequent negotiations have the unique complete information bargaining solution, so that the value function is well defined. If a pooling equilibrium results (which is a possible outcome), then the game repeats. If the state variable, k_t, is observed by the creditor, however, the type can be inferred after one round with a pooling equilibrium outcome.

For given k_0, define the debtor's value of an agreement as

$S(h_t - R,\ t)$, for the low $\bar{c}$ type, and

$S(\ell_t - R,\ t)$, for the high $\bar{c}$ type,

where $\ell_t < h_t$ for an agreement which transfers an amount R at time t to creditors. $S(\cdot,\cdot)$ is increasing in the first argument and decreasing in the second. The approach of Admati and Perry (1987) can now be applied.

Suppose at time 0, the lender can make an offer to which the debtor replies at time 1. The debtor will never accept an offer that provides less value than the value of an offer it can make at time 1 that would be accepted by the lender. The discount factor for the lender is determined by the opportunity interest rate. The results of Admati and Perry can be directly applied to this model with algebraic modification. The high surplus type can refuse a current high offer and wait to receive an offer that the low surplus type would accept. In equilibrium, the low surplus debtor cannot offer at time 1 an amount which the high surplus type would prefer to wait and offer to taking the time 0 offer. The low surplus debtor must wait long enough to make a counteroffer to separate itself from the high surplus type when the creditor's first (time 0) offer is the equilibrium offer for the high type in the complete-information bargaining game.

Multiple equilibria emerge from this approach. Unique separating equilibria exist for large enough creditors' priors that the debtor is of the high surplus type. These involve offering the complete information game division for the high type at time 0. The low type offers its complete-information game equilibrium division after a time delay adequate to signal its type. Separation becomes costly by reducing the surplus obtained by the low value debtor and reducing through delay the available output that may be divided.

If the creditor's prior belief is that there is a low probability that the debtor is the high surplus type, both multiple separating and pooling equilibria are possible. For low priors, there exists only a unique pooling equilibrium in which no delay occurs. This latter equilibrium involves lenders offering the complete information equilibrium repayment for the low surplus type in time 0. Either type accepts this offer.

One consequence of introducing capital stock depreciation as a cost of delay is to increase the possibilities for pooling equilibria to arise. Another is that the cost of delay to the high surplus type can, in general, be lower than the cost for the low surplus type. Of course, the depreciation of the capital stock also increases the effective discount factor for the lenders. Resulting separating equilbria may entail even longer delays with capital decumulation when the cost of delay is lower for high surplus types. If there are many possible types of debtors (as noted above), a separating equilibrium (or mixed pooling and separating equilibrium) must entail a delay between counteroffers made by each possible type of debtor, in declining order of surplus. Because this type of delay does not disappear as the length of time between possible offers shrinks to zero, significant costly delays to agreement can arise.

IV. Possible Extensions

Multi-Period Contracting

In the simple stationary consumption-smoothing model with potential repudiation, multi-period debt contracts serve no additional purpose if seniority provisions are enforceable. If every creditor claims on an equal footing renegotiation proceeds, then multi-period contracts with renegotiation may arise in equilibrium. Creditors offering zero-expected profit loans recognize that an entrant will offer an additional loan on terms preferred by the debtor to those that would not reduce the value of earlier creditors' claims. A two-period contract may be profitable that reduces the debtor's incentive to borrow additional amounts. Such contracts can increase the ex ante utility of the debtor in equilibrium by moving the chosen contract away from the interest-rate-taking one toward the constrained first-best one

(Kletzer (1984)). Because renegotiation is possible, such a contract offers the debtor an option to choose a particular second-period loan that, in events in which it would be taken, new lenders would not offer.

An example of such contracts is one offering a loan that, taken by itself, is expected to be profitable for the first period. A clause is included which obligates the lender to provide a new loan during the second period, which entrants would not offer if performance criteria are met by the debtor. If these covenants are not fulfilled, the lender can choose to declare a default and not provide the second loan. A restriction on debt dilution in the first period is a potential covenant; this type of contract can be self-enforcing for the sovereign debtor. In the case of sovereign loans, creditors may be subject to third-party enforcement of their obligation if the debtor does not breach the contract, which can specify that disputes be brought to the home court of the creditor. The debtor will generally choose not to breach the contract through first-period debt dilution. Because the debtor can choose to exercise the second-period option or select another debt contract in the absence of renegotiation, the debtor's expected utility the second period is increased, inducing first-period performance (if output in the first-period is private information, then contract breach may occur in equilibrium). These two-period loans may provide access to debt contracts in the second period that the debtor desires in poor output states over market contracts and chooses not to accept in high output states. Because of the debtor's limited liability (and consequent market imperfection), these loans offer insurance possibilities that a sequence of one-period loans with renegotiation do not. In the event of a demanded second-period revision of debt service obligations (which may become less probable), the obligations of the creditor to supply a second loan can be voided by a contract clause.

Therefore, in the event of a renegotiation of debt service, the multi-period contracts have no effects.

The creditor's two-period lending problem is to maximize the expected two-period profit with respect to the choice of contract terms while deciding whether or not to declare a subsequent default in the event of contract breach subject to a series of constraints. These constraints include the debtor's choice of accepting the contract over other contracts available and the equilibrium choices in each output state at each of the two future dates of the debtor. That is, the creditor correctly values the repayment streams along each equilibrium path for the subsequent subgames. In the absence of creditor observability of the debtor's output, the incentive compatibility constraints employed in the previous section are imposed at each date.

If the opportunity cost to creditors is a random variable, then an additional motive arises for multi-period contracts. Since the set of offered contracts shrinks with an increase in the world rate of interest, the second-period loan option will provide desirable insurance opportunities to the debtor; if the lenders' opportunity cost of funds falls, then the second-period (or later) debt contract can be revised. In equilibrium, in these events the resulting debt contract will be the debtor's best contract from among those offered by other lenders. While risk-neutral lenders will offer multi-period contracts providing higher utility to borrowers than equilibrium single-period loans, interest rate increases benefit borrowers ex post and interest rate declines lead to contract revision ex post. Therefore, the length of multi-period contracts in equilibrium is limited by the ex ante expected profitability of debtor welfare-improving contracts. Such contracts exist at all because the limited liability of debtors leads to equilibrium contractual marginal rates of interest exceeding average rates of interest on their debts.

Unobservable Debtor Policy Choices

The supposition that debtor income is unobservable by creditors may strike readers as peculiarly unrealistic. The natural alternative is to suppose that income is publicly observable while policy choices by the debtor affecting the distribution of income are unobserved by creditors. In a stochastic environment, moral hazard in policy selection arises if policies enhancing the probability of favorable outcomes for creditors (that is, if they raise anticipated debt repayments) are costly to debtors. The choice between investment and current consumption is a standard example.

The first-best contracts for simple principal-agent problems have been characterized when output is publicly observable, while the agent's choice of an action affecting the distribtuion of output is known only to the agent (Holmstrom (1979) and Rogerson (1985)). These contracts specify divisions of output as functions of the observable quantity, output alone. In the repeated principal-agent problem, the first-best contract depends upon the entire past history of output, as well as current output. The extent of risk sharing between a risk-neutral principal and risk-averse agent is limited by the necessity that the output-contingent contract provide incentives for the agent to choose output-increasing actions.

In the model used in this paper, assume that debtor income is observed by lenders, but that the distribution of income realizations depends upon a set of current policies selected by the debtor, which cannot be observed directly by creditors. Let the distribution of income conditional on policy choice be stationary, and assume that current-period felicity depends positively on current consumption and negatively on some measure of policy choice (for example, investment).

Constrained first-best capital flows can be characterized under a number of special assumptions for the problem of maximizing debtor utility subject to the constraints that repudiation is never chosen in equilibrium, expected profits are zero in every period, and the contract is incentive compatible in the choice of policy. In a nontrivial step, this problem can be reduced to a static maximization problem using Bellman's equation when the incentive compatibility constraint can be written as a first-order condition (Spear and Srivastava (1987)). The pattern of capital flows between lenders and borrowers over time (as a function of the history of income) can be characterized if additional assumptions are made about the nature of the conditional distribution of income.

Suppose that the only policy instruments available to the debtor government for transferring resources from the private sector to service debt create distortions in the domestic economy (for example, commodity taxes). In this case, the contracts that satisfy the first-order incentive compatibility condition (that is, are locally maximal for lenders) will tend not to lead to the optimal pattern of capital flows (constrained by the asymmetry of information). In such a model, a serious coordination problem can arise between creditors and debtors because complementarities between policy choices and external capital flows can arise. Large net capital outflows may be compatible with distortionary policies that reduce the expected return to new loans. The possibility that unsatisfactory equilibria arise when the policies required to meet large debt service obligations are distortionary can create a significant international public policy problem. While the public goods problem of cooperation between lenders suggested by others (for example, Sachs

(1984), Krugman (1985, 1987)) may not be severe in light of the possibilities for coordination between creditors, a problem of coordination between debtor-government policy selection and creditor lending choices would tend to be particularly difficult to address.

The derivation of a first-best contract solution for the model with privately observed investment levels but publicly observed output is possible given strong assumptions. However, the introduction of explicit debt-contracts with ex post renegotiation into this framework makes for a very difficult theoretical problem. Second-best contracting with renegotiations should involve outcomes (new flows of capital) that depend upon the entire past history of output.

V. Conclusions

The principal-agent framework adopted in this paper has implications for evaluating the argument for "defensive" lending to recalcitrant debtors. Under perfect information, a debt renegotiation never entails new concurrent flows of funds to the debtor and always involves a contract equivalent to a debt write-down combined with a new zero-expected profit loan. The "rescheduling" of willingly unmet debt service obligations in the form of a new loan does not occur in equilibrium in the model of this paper. The present value of the option on potential future repayments is less than its opportunity cost to the creditor at the margin in the stationary stochastic environment.

In the presence of informational asymmetries, equilibrium for the creditor-debtor renegotiation problem is a separating type. In lower output states, smaller current payments are made with larger debt service obligations carried forward. A debtor unwilling to meet current debt service may obtain new net inflows in a constrained optimal response by creditors only in the version of the model in which the penalties for repudiation increase with the debt service repudiated. This follows because a debtor may prefer to repudiate now with R relatively low to simply consuming current output while incurring larger future debt service obligations with the consequent reduction in expected utility.

The separating nature of equilibria derived in the imperfect information case may have implications for the evaluation of the (stochastic) debt service burden. Subsequent poor output realizations may lead in only a few steps to the maximal level of debt service obligations possible with net outflows or only minor net inflows of capital along the way. This might be the most significant cost of the informational imperfection. Such an expansion of debt service burden does not occur in equilibrium under symmetric information. In the first model, the debt service obligations have a stationary unconditional long-run distribution; under asymmetric information, they follow a simple Markov process instead.

Our model stands in contrast to an important paper on indeterminacy in lending under possible bankruptcy by Hellwig (1977). In that paper, the creditor sets a credit limit, which is optimal ex post to relax when it is reached by the debtor. If it is not relaxed, bankruptcy occurs automatically and the lender receives nothing. Additional loans are expected to be profitable because they raise the value of existing loans; no new creditor will provide them, but an existing creditor should. This is exactly "defensive" lending. However, the interest schedule is given to the creditor, and the creditors' policies are restricted to setting limits on the stock of debt (so that time inconsistency arises). We have relaxed two constraints imposed by Hellwig: default need not be declared following a breach of contract, and the interest charged in a renegotiation of debt is a choice variable for the existing creditors. Current lenders have access to a richer set of policies. Hellwig uses a hazard process for income, while we adopt a stationary one. It is not clear if this is essential.

Appendix

Outline of proof of proposition:

To show that x_i is non-increasing in i, we use the self-selection constraint

$$U(y_i + x_i) + \beta EV(R_i) \geq U(y_i + x_j) + \beta EV(R_j).$$

Let $i>j$, then $U(y_j + x_i)+\beta EV(R_i) > U(y_j+x_j) + \beta EV(R_j)$. if $x_i > x_j$, because $U(c)$ is strictly concave. This violates the self-selection constraint for state j. Therefore, $x_i \leq x_j$. Monotonicity of $EV(R)$ in R implies that $R_i \leq R_j$, again using the state i self-selection constraint.

$V_i(x_i,R_i) \equiv U(y_i + x_i) + \beta EV(R_i)$ is non-decreasing in i by

$$U(y_i + x_i) + \beta EV(R_i) \geq U(y_i + x_j) + \beta EV(R_j)$$
$$> U(y_j + x_j) + \beta EV(R_j),$$

since $y_i > y_j$.

The Lagrangian for the creditor's optimization problem is

$$L = \sum_{i=1}^{n} p_i \ (\ell'(R_i) - \ell_i) + \sum_{i=1}^{n} \sum_{j\neq i} \alpha_{ij}(V_i(x_i,R_i) - V_i(x_j,R_j))$$
$$+ \sum_{i=1}^{n} \delta_i(V_i(x_i,R_i) - V^r(y_i,R))$$
$$+ \sum_{i=1}^{n} \gamma_i(V_i(x_i,R_i) - \bar{V}(y_i,R)).$$

Necessary conditions for a maximum are

$$p_i = ((\delta_i + \gamma_i) + \sum_{j\neq i} \alpha_{ij}) \ U'(y_i + x_i) - \sum_{j\neq i} \alpha_{ji} \ U'(y_j + x_i)$$

$$p_i \cdot (d\ell'/dR_i) = ((\delta_i + \gamma_i) + \sum_{j\neq i} (\alpha_{ij} - \alpha_{ji}))(-\beta EV'(R_i)).$$

Because the derivative of ℓ_i with respect to R_i may not be well defined for discrete values of y, (2) should be interpreted as the appropriate weak inequalities for right and left derivatives. The function $\ell'_i(R_i)$ can be shown to be continuous.

Following Sappington (1983), $\alpha_{ij} = 0$ for $j>i+1$ and for $j<i-1$. Using the fact that $x_j < x_{i+1}$ if $j>i+1$, suppose the converse. Then, the i self-selection constraint implies

$$U(y_i + x_j) + \beta EV(R_j) \geq U(y_i + x_{i+1}) + \beta EV(R_{i+1}).$$

Concavity of $U(c)$ implies

$$U(y_{i+1} + x_j) + \beta EV(R_j) > U(y_{i+1} + x_{i+1}) + \beta EV(R_{i+1}),$$

which contradicts the $(i+1)$ self-selection constraint. A similar argument holds for $j<i-1$. Therefore, only $\alpha_{i\ i+1}$, $\alpha_{i\ i-1}$ can be non-zero for any i. Further, note that if $\alpha_{i\ i-1} > 0$, then

$$U(y_i + x_i) + \beta EV(R_i) = U(y_i + x_{i-1}) + \beta EV(R_{i-1})$$ and

strict concavity of $U(c)$ and $x_i < x_{i-1}$ imply that

$$U(y_{i-1} + x_i) + \beta EV(R_i) < U(y_{i-1} + x_{i-1}) + \beta EV(R_{i-1}).$$

Therefore, if $\alpha_{i\ i-1} > 0$, $\alpha_{i-1\ i} = 0$, and conversely.

Similarly, for $\alpha_{i\ i+1}$ and $\alpha_{i+1\ i}$.

S is convex, since $R + \sum_{i=1}^{n} p_i(\ell'(R_i) - \ell_i)$ is non-decreasing in R. The following arguments assume that $d\ell'/dR_i$ is continuous in R_i.

Rewriting (1):

$$p_n = (\delta_n + \alpha_{n\ n-1})\ U'(y_n + x_n) - \alpha_{n-1\ n}\ U'(y_{n-1} + x_n)$$

If $\delta_n > 0$, then $\alpha_{n-1\ n}$ must be zero. Otherwise, either

$$V_n(x_n,R_n) < V^r(y_n,\ R),\ \text{or}$$

$$V_{n-1}(x_n,R_n) < V^r(y_{n-1},R).$$

This follows by simply increasing x_n by ϵ and R_n by δ such that expected profit remains zero. If $\delta_{n-1} > 0$, then $\alpha_{n\ n-1} = 0$ by the same argument.

Let k be the minimum value for i such that $\delta_k>0$. Note that $\delta_k>0$ implies that $\delta_{k+1}>0$, because $\alpha_{k+1\ k}$ and $\alpha_{k\ k+1}$ are both zero. Also, whenever $\gamma_i>0$, $V_i(x_i,R_i) = \bar{V}(y_i,R)$ which implies that $\bar{V}(y_i,R) \geq V^r(y_i,R)$.

In case of equality, $\gamma_i + \delta_i > 0$, and with inequality, $\delta_i = 0$. We can let $\delta_i=0$ whenever $\gamma_i>0$. Let ℓ be the maximum value of i such that $\gamma_i>0$. (1) implies:

$$
\begin{aligned}
p_n &= \delta_n U'(y_n + x_n) \\
&\vdots \\
p_{k+1} &= \delta_{k+1} U'(y_{k+1} + x_{k+1}) \\
p_k &= (\delta_k + \alpha_{k\ k-1})\, U'(y_k + x_k) \\
p_j &= (\gamma_j + \alpha_{j\ j-1} + \alpha_{j\ j+1})\, U'(y_j + x_j) \\
&\quad - \alpha_{j-1\ j}\, U'(y_{j-1} + x_j) \\
&\quad - \alpha_{j+1\ j}\, U'(y_{j+1} + x_j),
\end{aligned}
$$

for all j < k, and

$$p_1 = (\gamma_1 + \alpha_{1\ 2})\, U'(y_1 + x_1) - \alpha_{2\ 1}\, U'(y_2 + x_1)$$

Suppose γ_1 is zero, then $\alpha_{1\ 2} > 0$; using both (1) and (2), this implies that $(\ell'(R_i) - \ell_i)$ must increase if (x_1, R_1) is changed so that $V_1(x_1,R_1)$ falls until $\gamma_1>0$. If $\alpha_{1\ 2} > 0$, then the quotient of (1) and (2) for i=2 implies that reduction of (x_2,R_2) along V_2 constant increases expected profit. Therefore, $\alpha_{1\ 2} = 0$ and $\gamma_1 > 0$. Note, if $\ell'(R_i)$ has unequal right and left derivatives, then $\alpha_{k-1\ k}=0$ because S is convex, but $\alpha_{j\ j+1}$ need not be zero for $j \geq k$.

Summing (1) over all i gives

$$
\begin{aligned}
\sum_{i=1}^{n} p_i = 1 &= \sum_{i=k}^{n} \delta_i U'(y_i + x_i) + \sum_{i=2}^{k} \alpha_{i\ i-1}(U'(y_i+x_i) - U'(y_i+x_{i-1})) \\
&- \sum_{i=2}^{k} \alpha_{i-1\ i}(U'(y_{i-1}+x_i) - U'(y_{i-1}+x_{i-1})) + \sum_{i=1}^{\ell} \gamma_i\, U'(y_i + x_i).
\end{aligned}
$$

The arguments above can be used to imply that $\alpha_{i-1\ i} = 0$. Whenever $(\delta_{i-1} + \gamma_{i-1}) > 0$, $\alpha_{i\ i-1} = 0$ is possible, but not necessary. If $(\delta_{i-1} + \gamma_{i-1}) = 0$, then $\alpha_{i\ i-1} > 0$.

The above properties can be used recursively to derive values for each multiplier. The quotient of (1) and (2) when $\alpha_{i\ i-1} = 0$ yields

$$d\ell'(R_i)/dR_i = (-\beta EV'(R_i))/(U'(y_i + x_i)),$$

and if $\alpha_{i\ i-1} > 0$,

$$d\ell'(R_i)/dR_i = \frac{-\beta EV'(R_i)\ (\delta_i + \gamma_i + \alpha_{i\ i-1} - \alpha_{i+1\ i})}{(\delta_i+\gamma_i+\alpha_{i\ i-1})U'(y_i+x_i) - (\alpha_{i+1\ i})U'(y_{i+1}+x_i)}$$

$$< \frac{-\beta EV'(R_i)}{U'(y_i+x_i)}.$$