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THE RELEVANCE OF ILLYRIA FOR LESS DEVELOPED COUNTRIES

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Introduction

In recent years, the unqualified assumption of profit maximization has been criticized by leading economists and a variety of alternatives postulated.¹ The debate has largely had a positive cast. In Professor Machlup's words,..."(the theory of the firm) is designed to explain and predict changes in observed prices (quoted, paid, received) as effects of particular changes in conditions (wage rates, interest rates, import duties, excise taxes, technology, etc.)."² The normative question, what should the firm maximize (or "satisfy"), is not a part of the debate. While the operations research literature is normative in emphasis, it is more concerned with how to maximize profits rather than whether to maximize profits. In this paper we provide a theoretical analysis of growth and efficiency in a business organization that does not maximize profits, but instead maximizes profits per worker. The need for the analysis arises because enterprise laws similar to those in Yugoslavia can lead firms to adopt this maximand. These same laws provide attractive inducements to political stability and generate a high measure of economic equity. Consequently, we argue that such a form of enterprise organization is apt to be of considerable interest to planners in mixed economies, if the long run production and growth of such firms can be shown to be economically efficient. As we note below, the analysis also

¹For a recent survey of the literature concerning the "marginalist controversy," see Professor Fritz Machlup's "Theories of the Firm: Marginalist, Behavioral, Managerial," American Economic Review, 57 (1967), pp. 1-33.

²Ibid., p. 9.

has relevance for the "marginalist controversy" and the "socialist controversy".

It is not surprising that socialist economists have been more interested in exploring alternative maximands than have their capitalist counterparts. Socialization of the means of production certainly brings the demise of the capitalist enterprise, but its organizational successor may take many forms, all of which invite an appraisal. The formal level of much of this literature, however, is often low. Since minor variations in enterprise organization may imply gross differences in economic behavior, a rigorous examination of alternatives is needed. Professor Benjamin Ward makes an important contribution to this problem in his recent book, The Socialist Economy.¹

Using the degree of decentralization to classify socialist enterprises, Ward's analysis samples from the middle and both ends of this spectrum. Idealized forms of institutions in the USSR, China and Yugoslavia are treated as representing particularly interesting examples of moderate centralization, extreme centralization and extreme decentralization respectively. This paper extends Ward's analysis of the decentralized, Illyrian,² case. Most impor-

¹ Benjamin Ward, The Socialist Economy: A Study of Organizational Alternatives, (New York: Random House, 1967).

² "Illyria" was the Roman province in the Balkans that roughly corresponds to today's Yugoslavia. The form of enterprise organization we postulate for Illyria is an idealized version of Workers Self-management as defined by Yugoslav law. Since other features of the Yugoslav system, particularly the Communist party and ideology, are not assumed, it cannot be too strongly employed that our predictions are not necessarily applicable to the typical Yugoslav enterprise.

tantly, we investigate the properties of the model for the Marshallian long run.¹ The introduction of technological progress and outward shifting demand permit us to draw additional conclusions concerning the characteristics and efficiency of the Illyrian firm's growth path. Also, the tools developed to deal with the long run case, yield a better understanding of some of the surprising results obtained by Ward for the short run. The next few introductory paragraphs describe the Illyrian firm and offer a few additional reasons why it is deserving of study.

Illyria is an idealized economy in which worker-managed plants produce in a totally decentralized, free market environment. By Illyrian law, enterprises are under the control of a Workers Council which is elected by the employees of the firm on a one-man, one-vote basis. This Council, in turn, selects a manager who operates the firm in free competition with all other firms. Inputs, outputs, techniques and price are as completely under the control of the Illyrian firm as they are of the capitalistic firm. There are two principal differences, however: first, the government retains ownership of all productive assets and charges a fee for their use; and second, although the employees decide what portion of earnings is to be paid out as personal income and what part is to be retained, the employee does not obtain any transferable title to funds that are plowed back. In both regards the Illyrian em-

¹The formal analysis contained in the Appendix to Chapter Eight of The Socialist Economy applies only to the Marshallian short run. This is also true of Ward's earlier article, "The Firm in Illyria - Market Syndicalism," American Economic Review, 48 (1958), pp. 566-89, and of Evsey Domar's "The Soviet Collective Farm as a Producer Cooperative," American Economic Review, 61 (1966), pp. 734-57. Both Ward and Domar restrict their analysis to the case where the total capital charge, "R," is a constant. We define R as the per unit capital charge so that the total capital charge, RK, is a variable. In Chapter Nine of The Socialist Economy, Ward briefly considers one long run aspect of Illyrian economics that is also central to our analysis -- the capital intensity of investment. Our conclusions on this matter (see page 14 below) are the same as his.

ployee qua owner differs from the capitalist stockholder. Throughout Illyria it is assumed that workers are completely self seeking, and management decisions reflect the collective consensus of the workers rather than any alternative goals. Under these conditions it is reasonable to postulate that the workers council will instruct management to maximize profits per worker. In any event, since the analytical core of this study abstracts from institutional details, the analysis is applicable to any firm which does maximize profits per worker, regardless of the exact institutional setting.

To the protagonists of the "socialist controversy",¹ the behavior of Illyrian firms is of obvious interest. It is one of the simplest organizational forms satisfying the requirements of a Lange-Lerner decentralized socialist state. A theoretical analysis of the efficiency of such organizations reflects on the feasibility of such a state. This evidence gains in importance because Illyria abstracts important features of the Yugoslav system. And Yugoslavia is the leading example of a decentralized, socialist state in operation.

This correspondence between the laws of Yugoslavia and the assumptions of Illyria returns us to the "marginalist-behaviorist" controversy.² Decentralized socialism in Yugoslavia provides a new arena for testing whether gross simplifications such as "firms behave as though they maximize profits per worker" can be used to deduce operational hypotheses concerning the variables described by Professor Machlup. Before this is possible, however, a marginalist theory for Illyria must be articulated, and that is the principal concern of our work.

¹This literature is reviewed by Ward, The Socialist Economy, Chapter 2.

²Although Ward carefully protects Illyrian theory from overly facile applications to Yugoslav reality, he weakens at a few points and confronts the model with facts. For example, see pages 584-5 of "The Firm in Illyria".

"Socialist controversy," "marginalist-behaviorist controversy," both phrases bespeak the maturity of the debate. As mentioned at the outset a more significant cause of the current interest in Illyria is from the point of view of economic development. Abstracted from its communist setting in Yugoslavia, Workers Self-management offers special attraction for use in the public sector of mixed economies. A brief discussion of some of the broader political and economic implications of Illyrian syndicalism is worthwhile. Afterwhich we turn to our main theme, the efficiency of Illyria.

The principal allure of Illyrian syndicalism is not so much the incentives it gives workers to maximize their personal incomes by maximizing enterprise profits, for there are many bonus systems which can achieve this. Rather, the attraction is its equitable solution to the problem of the rights of ownership, and its favorable implications for the evolution of a democratic society. The problem of obtaining an equitable distribution of ownership must bedevil the most wise and ambitious governments of less developed countries. Certainly, the more ambitious the development program, the more critical is the ownership question. For, as government forces higher rates of investment it concomitantly causes a transfer of assets toward the expanding sectors. This transfer of investible funds has its donor as well as its beneficiary, but the donation is seldom voluntary. The fortuitous solution where a landed gentry directly or indirectly volunteers funds to industry and trade has not occurred in most LDC's, and the levers necessary to manufacture such an outcome are not known. Consequently, ambitious development programs must typically coerce funds often from the broadest strata of society, the peasantry, and this is seldom done without a loss of political popularity with

this strata. One solution is to force the transfer but let ownership benefits rest with the donor. This, however, is seldom possible.

The alternative offered by Workers Self-management is to pass the prerogatives of ownership¹ forward to the employees of the expanding sectors. This offers a broadly based, comparatively equitable, redistribution of ownership among the users of capital. It is particularly well designed to kindle in the newly formed industrial work force an interest and enthusiasm for the government's development program.

The capitalist solution of making grants or loans to existing entrepreneurs channels the investment returns paid for by the many into the hands of the few. It also alienates both the peasant and the urban worker from a direct interest in the development program. In contrast, the centrally planned solution which in theory places the ownership rights of investible funds with all the people, offers a highly satisfactory outcome with respect to equity; and by making all workers in the modern sector directly dependent on the government for employment, it also tends to increase the power and stability of that government. Against these advantages must be balanced the questionable efficiency of the centrally planned economies. The trend of institutional reform in the Soviet Union and Eastern Europe towards greater decentralization suggests that complete centralization is a step in the development sequence that might profitably be skipped.

What we hope to resolve in this paper is whether the decentralized Illyrian firm can combine some of the equity attractions of the centrally planned firm with the efficiency commonly attributed to the capitalist firm. Special attention is given to the case of monopoly because this market structure,

¹"Prerogatives of ownership", in this case, refers to the right to distribute current but not past profits.

rather than perfect competition, more aptly describes the conditions under which the Illyrian firm would most likely be incorporated into a mixed economy. We first discuss monopoly, then incorporate perfect competition as a limiting case, and finally consider the growth path of a firm subject to expanding demand and technological progress.

The Comparative Statics of Illyrian Monopoly

The Illyrian firm is presumed to maximize personal income per worker. This is obtained as the difference between net sales per worker (after the deduction of all materials costs) and "interest" payments per worker. The interest charge is levied against all productive assets and is paid to the state. The maximand of the firm, M , may therefore be expressed as:

$$(1) \quad M = \frac{PY - RK}{L}$$

where Y , K and L are output, capital and labor, and P and R are the prices of output and capital.^{1,2}

In the general monopoly case the firm's demand curve is negatively sloped

¹ R , the price of capital, is referred to as the "interest rate", although the "rental rate" might be more meaningful.

²It is assumed that the supply of K is infinitely elastic at the price R , and that workers can be freely hired or fired by management. Against this last condition, the objection has been raised that the workers, fearing for the safety of their individual jobs, may instruct management to maximize M subject to the constraint (i) that no one be fired, or (ii) that the contraction of the work force be restricted to natural decrements due to retirements and job switching. It is our feeling that consideration of these alternative, constrained models is most easily performed as a special case of the unconstrained model presented here. In most cases, the modifications of the basic theory implied by these restrictions is obvious.

and has a price elasticity, η , that declines as the quantity sold increases. The production function is well behaved with symmetric assumptions concerning the effects of labor and capital on output including positive first derivatives and negative second derivatives. Returns to scale, ϵ , and the labor and capital output elasticities, α and β , are functions of the input vector (L, K) . More restrictively, we assume that ϵ is greater than unity at the origin and declines monotonically along any positively sloped isocline in the (L, K) space. That is, returns to scale decline whenever we increase one output without decreasing the other.¹ The conditions so far placed on the elasticity of demand and the scale coefficient may be stated mathematically as:

$$\frac{\partial \eta}{\partial Y} < 0 \quad \frac{\partial \epsilon}{\partial L} < 0 \quad \frac{\partial \epsilon}{\partial K} < 0$$

At this point it is convenient to state three deductions from the above assumptions that will be needed:

- i) $\epsilon = \alpha + \beta$; and less obviously
- ii) $\frac{\partial \alpha}{\partial L} + \frac{\partial \beta}{\partial L} < 0$, and
- iii) $\frac{\partial \beta}{\partial K} + \frac{\partial \alpha}{\partial K} < 0$.

The first result is well known and needs no comment. The second and third results state that the sum of the direct effect of each input on its own output elasticity and the indirect effect of each input on the other inputs' output elasticity are negative. For labor, this means that the negative effect of an increase in L on its own elasticity, α , is always sufficient to swamp any possible positive effect which the increase in L may have on the capital coefficient, β . Although the last two results are not explicitly used in the geometric analysis, they are implicit in many of the comparative

¹ This is the regular ultra passum law of Ragnar Frisch. See The Theory of Production, Rand McNally & Co., Chicago, 1965, p. 120.

static conclusions.

The first order conditions for an Illyrian long run optimum are:

$$(2) \quad \begin{aligned} L^2 \frac{\partial M}{\partial L} &= RK - PY + \alpha PY \left(1 - \frac{1}{\eta}\right) = 0, \text{ and} \\ LK \frac{\partial M}{\partial K} &= PY\beta \left(1 - \frac{1}{\eta}\right) - RK = 0. \end{aligned}$$

These two equations imply, respectively,

$$\alpha = \frac{PY - RK}{PY \left(1 - \frac{1}{\eta}\right)} \quad \text{and} \quad \beta = \frac{RK}{PY \left(1 - \frac{1}{\eta}\right)}$$

Defining $\gamma = 1 - \frac{1}{\eta}$, we arrive at the deceptively simple equilibrium condition

$$(3) \quad \epsilon = \frac{1}{\gamma} \equiv \gamma^{-1}$$

Since Illyrian behavior is often so contrary to conventional economic intuition, a few words of interpretation may be helpful. Consider not equation 3, but its reciprocal: $\epsilon^{-1} = \gamma$. It is easily verified that γ may also be defined as the elasticity of total revenue with respect to output.¹ What we now need is a correspondingly simple interpretation of ϵ^{-1} . For this we turn again to Frisch.² He shows that if the price of both inputs are fixed, then ϵ^{-1} is equal to the capitalist elasticity of total cost with respect to output. In some general sense then, the Illyrian firm equates the percentage increase in costs to the percentage increase in revenue; but this statement needs to be clarified and made more precise.

By assumption the price of capital, R , is fixed; however, the price of labor, M , is our maximand and is obviously not fixed. This raises the question of whether it is permissible to apply Frisch's conclusion that ϵ^{-1} is

¹ $\gamma = 1 - \frac{1}{\eta} = \frac{\partial PY}{\partial Y} \frac{Y}{PY}$

² Ibid., p. 167.

the elasticity of cost. It is permissible, but only at the optimal point where \hat{M} is a maximum. At this point $\partial M/\partial Y = 0$ and the condition of fixed input prices holds. We must also provide an Illyrian equivalent of "capitalist cost." Since ML and RK always exhaust total revenue, PY (there are no residual profits), it is more meaningful to talk about total disbursements rather than total cost. Therefore, ϵ^{-1} may be described as the elasticity of total disbursements for fixed M . In equilibrium, the percentage change in revenue must equal the percentage change in disbursements (M fixed).¹ If output is less than optimal, an increase will raise revenues faster than disbursements (at the suboptimal value of M) and permit an increase in M ; whereas if output is greater than optimal, a decrease will decrease revenues less than disbursements and also permit an increase in M . Consequently, \hat{M} is a true maximum if and only if the elasticity of disbursements (for the fixed value \hat{M}) equals the elasticity of total revenue. We return now to a further consideration of equation (3).

Equation (3) has some surprising implications. Under typical simplifying assumptions used in Econometric work -- ϵ and η constants -- a long run optimum does not exist.² More foreboding is the fact that if the initial value of ϵ is less than unity, then no equilibrium exists, even though all the rest of our assumptions are met. Or, in more familiar terms, no solution exists whenever the firm has a monotonically rising long run average cost³

¹Since total revenue and total disbursements are identical, so the elasticity of total revenue is always identically equal to the elasticity of total disbursements (M variable).

²Except under the unlikely condition that the two constants ϵ and γ are fortunately equal. In that case the solution is not determinate.

³Since we do not define cost curves for the Illyrian firm, reference here is to the conditions which would exist for a capitalist firm with identical technology, and a competitive labor market with wage rate W . In Illyria as well as capitalism, W is assumed to be the minimum acceptable rate.

curve. The determination of the optimal output level is illustrated in Figure 1.

The equilibrium defined by equation (3) exists in Figure 1 whenever the $\epsilon(Y)$ curve intersects the $\gamma(Y)^{-1}$ curve from above. This is obviously not possible if $\bar{\alpha} = \alpha(Y)$ and $\bar{\epsilon} = \epsilon(1)$ are unequal constants. It is also not possible if $\epsilon(Y)$ is everywhere less than one. This is because $\gamma(Y)^{-1}$ is positively sloped and has an initial value greater than one.¹ Consequently there is no determinant long run solution for the Illyrian monopolist if decreasing returns to scale are everywhere present.

Before investigating the comparative static implications of our model, it is useful to contrast the Illyrian and capitalist equilibrium output levels. The capitalist condition corresponding to equation (3) is,

$$\epsilon(Y) = \rho(Y) \gamma(Y)^{-1}$$

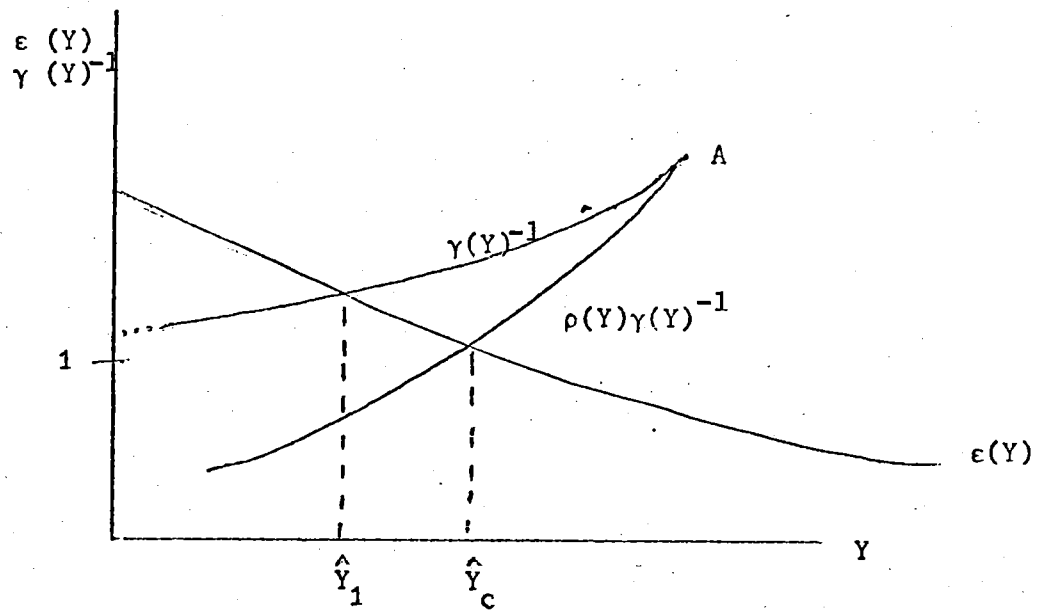
where $\rho(Y)$ is defined as the share of costs in net revenue ($\rho(Y) = \frac{RK + WL}{PY}$).

It is easily seen that $\rho < 1$ and $\frac{\partial \rho}{\partial Y} > 0$ in the vicinity of equilibrium.

Thus the function $\rho(Y) \gamma(Y)^{-1}$ is monotonically increasing and converges on $\gamma(Y)^{-1}$ at Point A as is shown in Figure 1. The convergence at A occurs when $RK + WL = PY$, so that $\rho(Y) = 1$. This is the zero profit solution for the Capitalist firm and is the only case where the Capitalist and Illyrian solutions are the same. In all other cases, the function $\rho(Y)$ causes Capitalist

¹The slope, $\frac{\partial \gamma(Y)^{-1}}{\partial Y} = -(\frac{1}{\eta-1})^{-1} \frac{\partial \eta}{\partial Y}$, is positive whenever η is greater than one. Values of η which are less than one can never be maximums since they involve values of $\gamma(Y)^{-1}$ and consequently $\epsilon(Y)$ which are negative. The latter occurs when output is so large that further increases in inputs cause decreases in output.

FIGURE 1.



output to be larger than Illyrian, and permits the existence of an equilibrium under capitalism regardless of whether the demand and scale coefficients are constant, or whether returns to scale are decreasing.

To analyze the comparative statics of the model, we must first look more closely at the behavior of ϵ over the space (L, K) . Just as we define isoquants in on the input space, we may also define isoscale contours. The slope of these contours is given by the equation⁸

$$\left. \frac{dK}{dL} \right|_{\epsilon} = - \frac{\frac{\partial \epsilon}{\partial L}}{\frac{\partial \epsilon}{\partial K}} = - \frac{\frac{\partial \alpha}{\partial L} + \frac{\partial \beta}{\partial L}}{\frac{\partial \beta}{\partial K} + \frac{\partial \alpha}{\partial K}}$$

As described earlier, both $\partial \epsilon / \partial L$ and $\partial \epsilon / \partial K$ are negative, so this also is true of $\left. \frac{dK}{dL} \right|_{\epsilon}$. From the existing assumptions it is not possible to deduce the curvature of the isoscale curves. However, under the rubric of "well behaved technology" we may reasonably assume that each isoscale curve is tangent to one and only one isoquant. It is sufficient for this result that the curvature of the isoscale curves be everywhere either less than or more than that of the isoquants. Assuming this latter condition and defining the locus of the tangencies as the Illyrian Crest we find that all points on one side of the Crest have isoscales cutting isoquants from below (above), while all points on the other side have isoscales cutting isoquants from above (below).

Figure 2 illustrates the location of the Illyrian Crest for the case where the curvature of the isoscales is less than that of the isoquants. Note that because we assume ϵ is decreasing in K and L , it decreases as we

⁸These contours are described by Frisch, ibid., page 122.

move outward along the Crest. The outer limit of the Illyrian economic region is defined by the isoscale curve for $\epsilon = 1$.¹ We may now derive output expansion paths for different interest rates and levels of demand.

The disbursement equation used to generate an Illyrian expansion path is

$$(4) \quad K = -\frac{M}{R}L + \frac{PY}{R}.$$

To derive the expansion path for a fixed value of R , first select any value of Y , next compute for different levels of output the term PY/R , and finally draw a tangent from the K axis intercept, PY/R , to the isoquant for Y . The slope of this tangent gives the maximum attainable value of M/R (and consequently M) which can be obtained by producing Y units of output with a fixed interest rate of R . The locus of such points for different output levels and the same fixed R we call the expansion path for R . A family of such paths is generated by changing the single parameter R . Three properties of these paths need to be noted.

First, if W is the capitalist competitive wage and if Illyrian workers refuse to work for less than this ($\min M = W$), then for any R the Illyrian expansion path is more capital intensive than its capitalist counterpart.² That is, in the space (L, K) the Illyrian path lies above the capitalist path. This is because the Illyrian worker-manager earns not only the competitive wage but also a share of the profits so that $\frac{M}{R} \geq \frac{W}{R}$. The two paths coincide only at the terminal point A corresponding to the zero profits solution in

¹The optimality properties of this particular contour are further discussed on page 20. The isoscale curve for $\epsilon = 1$ is denoted $\hat{\epsilon}$.

²This conclusion is also reached by Ward. See page 212 of The Socialist Economy.

Figure 1. Second, for both the Illyrian and Capitalist firms, the expansion paths associated with higher interest rates lie beneath those for lower interest rates. Third, selection of the optimum output along any given expansion path (R fixed) is shown geometrically as the point where the slope M/R of equation (4) is maximized. If an interior maximum is to exist M/R must initially be increasing near the origin, reach a maximum, and then decrease. This pattern is shown in Figure 3. The maximum value \hat{M}_0/R_0 along the expansion path E_0E_0 is obtained at the output level Y_0 . The other expansion path E_1E_1 lies beneath E_0E_0 and is therefore drawn for an interest rate R_1 which is greater than R_0 . Using these properties we can now analyze the comparative static effects of a change in the interest rate.

Consider an increase in the interest rate from R_0 to R_1 . Since a change in R has no effect on the $\gamma(Y)^{-1}$ curve of Figure 1, the new solution for Y will only involve a shift of the $\epsilon(Y)$ curve, either up, in which case output increases, or down, in which case output decreases. In either event, there will be a positive association between the change in Y and in ϵ . To determine in which direction $\epsilon(Y)$ will shift we must first determine the change in L and K . We assert that an increase in the interest rate will increase labor and decrease capital, moving the firm to the shaded area of Figure 3. The proof is by contradiction.

Since an increase in the interest rate moves the firm from E_0E_0 to E_1E_1 , and since E_1E_1 lies everywhere beneath E_0E_0 , we know that a movement in the opposite northwesterly direction where capital increases and labor decreases is not possible. Suppose, however, we move to the unshaded area beneath E_0

characterized by a decrease in both K and L. In this area output is smaller and the scale coefficient larger than at Y_0 , but this is a contradiction of the above conclusion that changes in ϵ and Y must be positively related. The converse argument holds for increases in both K and L. Therefore, we move to the shaded area as asserted.

The effect of an increase in the interest rate on output can now be solved by asking "What is the value of the scale coefficient when we optimally produce an output of Y_0 with an interest rate of R_1 ?" That is, what is the value of ϵ at the intersection of the expansion path E_1E_1 and the isoquant Y_0 . If ϵ is greater at this point than it is at Y_0 , then the $\epsilon(Y)$ curve is shifted up and output increases. If ϵ is less, then the $\epsilon(Y)$ curve is shifted downward and output decreases. But the sign of the change in ϵ is determined by whether or not the isoscale curves cut the isoquants from below. Therefore, the sign of the change in output depends upon which side of the Illyrian Crest we are: the location of the Crest has a critical importance not found in the Capitalist economy. We may formally summarize our results for changes in the interest rate as follows:

$$(5) \quad \frac{\partial K}{\partial R} < 0, \frac{\partial L}{\partial R} > 0 \text{ and} \\ \frac{\partial Y}{\partial R} \begin{matrix} > \\ < \end{matrix} 0 \text{ as } - \frac{dK}{dL} \bigg|_{\bar{Y}} \begin{matrix} > \\ < \end{matrix} - \frac{dK}{dL} \bigg|_{\bar{\epsilon}}.$$

Ambiguity concerning the sign of $\partial Y/\partial R$ in Illyria is not so surprising when we recall that similar ambiguities also exist in capitalist economics. Not with respect to $\partial Y/\partial R$, but with respect to $\partial L/\partial R$. That is, an increase in R causes an increase in capitalist employment due to a substitution of labor for capital, but a decrease in employment due to a contraction of out-

put. The resulting sign of $\partial L/\partial R$ is uncertain, although $\partial K/\partial R$ and $\partial Y/\partial R$ are both definitely negative. In Illyria, $\partial K/\partial R$ and $\partial L/\partial R$ are respectively negative and positive, but $\partial Y/\partial R$ is uncertain. The important role of the Illyrian Crest is easily understood if we break the effect of an increase in R down into two movements; first, a movement southeast along the isoquant Y_0 until the new expansion path E_1E_1 is encountered; and second, a movement either forward or backward along E_1E_1 until a new optimal output Y_1 is encountered. If during the first leg of this movement (output held constant) ϵ is increasing, then the second leg must involve an increase in Y in order to reduce $\epsilon(Y)$ towards its new intersection with $\gamma(Y)^{-1}$. The opposite is true if ϵ is decreasing during the first leg. Thus, the Illyrian Crest simply partitions (L,K) into two sections according to whether the first leg described above involves an increase or a decrease in ϵ . It is less surprising that the sign of $\partial Y/\partial R$ is ambiguous, than it is that the ambiguity can be resolved by such a simple concept as the Illyrian Crest.

Two additional results of interest concern the extreme cases when either the scale coefficient is a constant, or when the elasticity of demand (for a fixed demand curve) is a constant. In Figure 1, if $\epsilon(Y) = \bar{\epsilon}$, then no change

in output can occur in response to changes in R. Thus,

$$\left. \frac{dK}{dL} \right|_{\bar{\epsilon}, dR} = \left. \frac{dK}{dL} \right|_{\bar{Y}}$$

That is, an increase in the interest rate causes a movement downward along an isoquant. On the other hand, if $\bar{\gamma}(Y) = \bar{\gamma}$, then Y may change but ϵ may not.

Thus,

$$\left. \frac{dK}{dL} \right|_{\bar{\gamma}, dR} = \left. \frac{dK}{dL} \right|_{\bar{\epsilon}}$$

That is, an increase in the interest rate causes a movement downward along an isoscale curve. Using the tools developed above it is quite easy now to determine the effect of a shift in demand.

An arbitrary shift in the demand curve will disturb the equilibrium pictured in Figure 1 by shifting both $\gamma(Y)$ and $\epsilon(Y)$. As a temporary expedient we simplify the analysis by assuming that demand shifts in such a way that the price elasticity of demand for a given value of output is not affected by the shift. Consequently, under this strong assumption a shift in demand does not alter the curve $\gamma(Y)$ in Figure 1. This greatly simplifies the analysis since adjustment must now occur via the scale coefficient alone. We proceed by deriving a set of expansion paths for different values of P.

Since the expansion paths in Figure 3 are derived for constant values of both P and R the same family of curves exists for analyzing price changes as for interest rate changes. An essential difference, however, is that the expansion paths for higher prices lie above those of lower prices which is just the opposite from what we found for the interest rate. To prove this we

need only note that a shift in demand raises P , thereby increasing the intercept term, PY_0/R_0 , in equation (4). As a result the new tangency to the isoquant Y_0 must lie above the original one. Thus, outward shifts in demand cause a movement upward from path E_0E_0 to path E_2E_2 . It must then follow that increases in demand lead to increases in the capital stock, but decreases in employment. The net effect of this is to increase output if the isoscale curves cut the isoquant from above at the point of equilibrium, but to decrease output if the isoscale curve cuts the isoquant from below. The latter case generates a negatively sloped supply curve for output. We may summarize our results by:

$$(6) \quad \frac{\partial K}{\partial P} > 0 \quad \frac{\partial L}{\partial P} < 0 \quad \text{and}$$

$$\frac{\partial Y}{\partial P} \begin{matrix} < \\ > \end{matrix} 0 \quad \text{as} \quad - \frac{dK}{dL} \bigg|_{\bar{Y}} \begin{matrix} > \\ < \end{matrix} - \frac{dK}{dL} \bigg|_{\bar{\epsilon}}$$

The two extremes, a constant elasticity of demand and a constant scale coefficient, lead to results which are identical to those obtained for changes in the interest rate:

$$\begin{aligned} \frac{dK}{dL} \bigg|_{\bar{\epsilon}, dP} &= \frac{dK}{dL} \bigg|_{\bar{\epsilon}, dR} = \frac{dK}{dL} \bigg|_{\bar{Y}} \quad \text{and} \\ \frac{dK}{dL} \bigg|_{\bar{Y}, dP} &= \frac{dK}{dL} \bigg|_{\bar{Y}, dR} = \frac{dK}{dL} \bigg|_{\bar{\epsilon}} \end{aligned}$$

Also, from the appendix we find the following close association between the comparative static effects of changes in the interest rate and shifts in demand which keep η constant.

$$\frac{\partial K}{\partial R} = - \left(\epsilon_{\frac{\partial K}{\partial Y}} \right) \frac{\partial K}{\partial P}, \quad \frac{\partial L}{\partial P} = - \left(\epsilon_{\frac{\partial K}{\partial Y}} \right) \frac{\partial L}{\partial P}$$

$$\text{and finally, } \left. \frac{dY}{dR} \right|_{\bar{P}} = - \left(\epsilon_{\frac{\partial K}{\partial Y}} \right) \left. \frac{dY}{dP} \right|_{\bar{R}}$$

If it were true that demand shifts maintained a constant elasticity, the above symmetry would provide an empirically useful rule for using the government-controlled interest rate to bring about desired changes in output. Unfortunately, we have little information concerning the pattern of demand shifts in the developed or less developed countries.

The Comparative Statics of Illyrian Perfect Competition

Perfect competition is most easily treated as a limiting case of the general monopoly analysis. Setting the elasticity of demand equal to infinity causes the $\gamma(Y)$ curve in Figure 1 to be horizontal at unity. Therefore, the competitive equilibrium, if it exists, will be somewhere on the isoscale contour for $\bar{\epsilon} = 1$. Following Frisch, we call this contour the curve of technically optimum scale¹ and denote it by $\hat{\epsilon}$. The terminology is appropriate since this locus is the set of all input combinations which correspond to minimum points on the long run average cost curves of capitalist firms (different points on $\hat{\epsilon}$ correspond to different values of W/R). The conclusion is then that if perfect competition exists, the level of output will be technically optimum regardless of what value M takes.

¹Ibid., p. 122.

FIGURE 2.

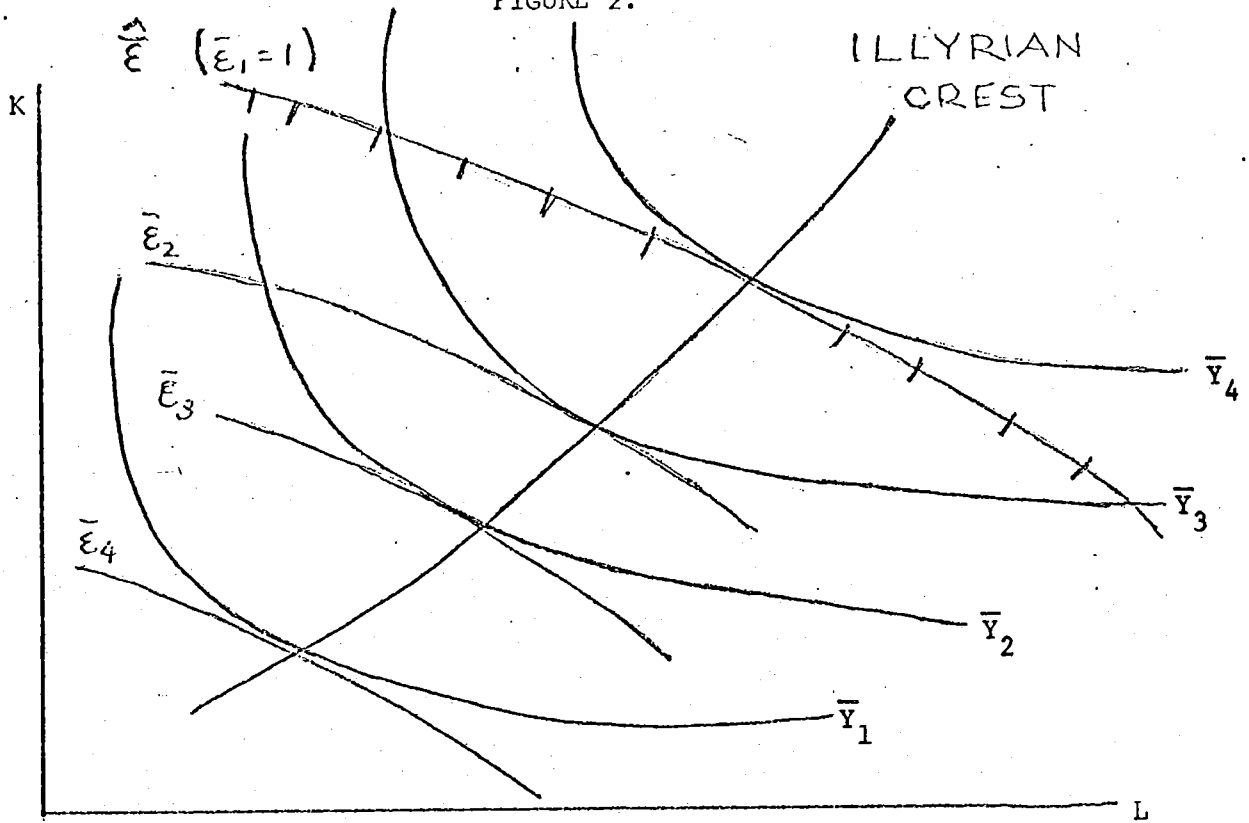
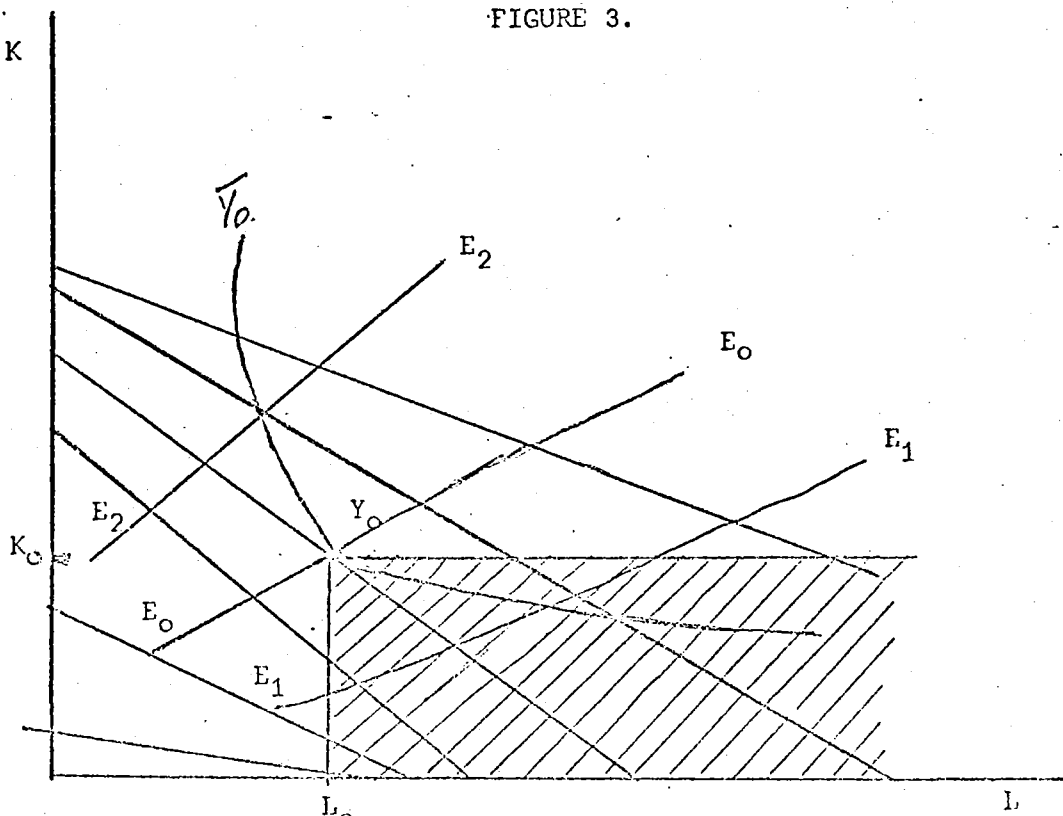


FIGURE 3.



The comparative static effects of changes in the interest rate and demand can be taken directly from the monopoly analysis under the particular condition $\bar{\gamma}(Y)^{-1} = 1$.

$$(7) \quad \frac{\partial K}{\partial R} < 0, \quad \frac{\partial L}{\partial R} > 0$$

$$\frac{\partial K}{\partial P} > 0, \quad \frac{\partial L}{\partial P} < 0$$

$$\frac{\partial Y}{\partial R} > 0$$

$$\frac{\partial Y}{\partial P} < 0$$

$$\text{as } - \frac{dK}{dL} \bigg|_{\bar{\epsilon}} < - \frac{dK}{dL} \bigg|_{\bar{Y}}$$

If the isoscale contours cut the isoquants from below, then $\frac{\partial Y}{\partial P} < 0$ and the supply curve for each firm is negatively sloped. Thus, there exists the possibility of an unstable intersection of industry supply and demand.

Further analysis of this case is worthwhile. If for all firms in the industry the curvature of $\bar{\epsilon}$ is everywhere less than that of \bar{Y} , and if industry supply and demand have an unstable intersection, then barring new entry any displacement of price above the equilibrium level will tend to generate a sequence of increases in per-worker income and decreases in output that continues until there exist only one man firms producing with highly capital intensive techniques. This movement is always northwest along the isoscale contour $\hat{\epsilon}$. Since we assume that the length of the work week is fixed, the one man firm puts a lower bound on output, albeit a contrived one.

Consider the other possibility -- a downward displacement of price from

the original equilibrium. A sequence of falling prices and increasing outputs occurs. New workers are hired and capital is decreased as we move downward along $\hat{\epsilon}$. This sequence, however, may be broken in two ways. First, earnings per worker may decline to the point that the existing firms cannot hire the additional workers needed to maintain increases in industry output. Assuming a perfect labor market, this will occur when each firm has gone downward along ϵ to the point where this isoscale curve intersects the Capitalist expansion path. Further decreases in price cannot be met by increases in output which means we have arrived at a vertical section of the supply curve. Second, if in moving downward along $\hat{\epsilon}$, the firm encounters the Illyrian Crest before it encounters the Capitalist expansion path, then

$$-\left. \frac{dK}{dL} \right|_{\hat{\epsilon}} \text{ becomes greater than } -\left. \frac{dK}{dL} \right|_{\bar{Y}}$$

and the slope of the supply curve becomes positive. Thus, whenever an unstable intersection of industry supply and demand occurs, it is bounded above and below by two other intersections, both of which are stable. When the existence of heterogeneous firms is allowed for the possibility of an unstable equilibrium becomes even less probable. What emerges from our analysis is that the competitive Illyrian long run industry supply curve is more inelastic than its Capitalist counterpart, but nevertheless is positively sloped and is composed of firms producing at the minimum point on their LRAC curves.

A brief description of short run competitive adjustment is needed as a background for constructing scenarios of long run adjustment. Figure 5 shows the short run response of an Illyrian firm which is displaced from a competi-

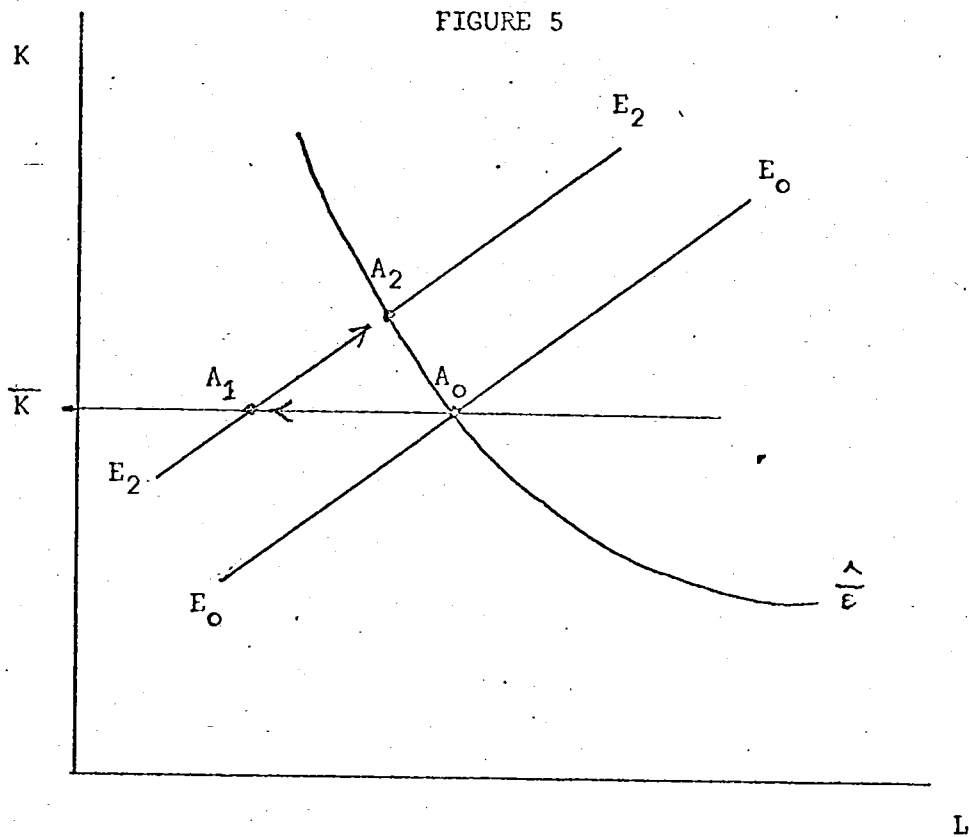
tive equilibrium by an increase in price. At point A_0 on expansion path E_0E_0 the firm produces an optimal output \hat{Y}_0 conditional on a fixed capital stock of \bar{K}_0 and an interest rate R_0 and price P_0 . If price is raised to P_1 , the firm moves backward along \bar{K}_0 to the expansion path E_2E_2 which is defined for the parameter pair (P_1, R_0) . The short run equilibrium at A_1 must be characterized by an output smaller than Y_0 . Consequently, the short run competitive supply curve is always negatively sloped. In the long run, however, even without the entry of new firms, existing enterprises move outward along E_2E_2 until they again reach the isoscale contour \hat{c} . Output at A_2 will be greater if the isoscale contours cut the isoquants from above at A_0 and smaller if vice versa. An analogous pattern, with signs reversed, may be derived for changes in the interest rate.

Comparison of this sequence of events with the typical Capitalist sequence clarifies why we obtain such eccentric predictions for Illyrian behavior. These eccentricities derive from the fact that the principal effect of changes in price is the composition of factor inputs and rewards rather than upon the level of output. The effect of changes in price on production are more closely akin to changes in the Capitalist wage rate than the Capitalist price of output. The same close association for Capitalist economies is found between price and profits so it is not surprising that the returns of Illyrian workers qua owners would have a similar property. The principal consequence of fusing ownership with labor input is to make the supply response of existing firms less flexible than it is under Capitalism.

In the long run, assuming perfect markets and free entry one might

expect that the two systems would move to identical solutions concerning the number of firms, output levels and factor proportions. This is not typically the case. The Illyrian solution will differ from the capitalist solution if the efficiency of firms is not identical, which, of course, is generally true. Because of this the Illyrian competitive solution cannot be pareto optimal. Our reasoning is as follows. Suppose that there are two firms, both of which have identical technologies in the sense that they have the same isoscale contours, but differ in that one is more efficient so that its output isoquants are uniformly higher. The curve of technically optimum scale is the same for the two firms, but the more efficient firm produces on this curve to the northeast of the less efficient firm. The existence of different marginal rates of substitution among firms is in violation of the pareto criterion.

FIGURE 5



Efficiency and the Growth of the Illyrian Firm

Having derived the comparative static effects of a shift in demand, we may move closer to our paramount interest -- How useful is the Illyrian form of organization in development programs? In particular, we take up how efficiently market syndicalism responds first to increases in demand and second to improvements in technology.

Up to this point, we have attempted to keep the assumptions of the analysis as general as possible. The principal restrictions were a well-behaved production function obeying the regular ultra passum law, and a downward sloping demand curve with declining price elasticity. In order to generate more meaningful and operational hypotheses concerning the behavior of Illyrian firms over time, it is necessary to strengthen the postulates by ruling out, hopefully, less likely states of the world. First, the regular ultra passum law itself. A stronger alternative, adopted in the following sections, is that the curvature of the isoscale contours is strictly less than that of the isoquants. This is equivalent to assuming the two joint conditions that the scale coefficient does not increase whenever output (not one of the inputs) is increased, and also does not increase along an isoquant whenever we move away from the Illyrian Crest. Although we lack empirical evidence defining changes in ϵ in terms of output rather than inputs seems to be a restrictive but not unreasonable assumption.

If all existing firms and potential entrants have identical technologies, Illyrian perfect competition is pareto optimal. Free entry, in this case, generates a horizontal long run industry supply curve at the same level, and

using the same factor proportions as for competitive capitalism. However, if new entrants are successively less efficient as defined on page 25, then in addition to the static inefficiency caused by differing marginal rates of substitution, we also have the dynamic problem that the most efficient firms wither away. The argument is that as industry demand expands, prices rise and intra-marginal firms begin to earn a "pure profit" for their worker-managers. This causes the intra-marginal firms to move upwards along the optional technical efficiency locus, \hat{e} , towards more capital intensive techniques. But, since they must ultimately pass over the Illyrian Crest, at this point, output for the efficient intra-marginal firms begins an unending decline. Thus, the prediction of a withering away of firms is validated so that the most efficient firms produce the least output. Clearly, industries with rising costs of supply are not good candidates for the establishment of market syndicalist enterprises. This condition is sufficiently common that it raises serious questions about the efficiency of Illyrian organizations whenever the number of sellers is large.

The market structure of greatest interest however is for one, or a few, sellers. When market syndicalist firms originate under governmental sponsorship they will typically be for key projects that establish new industries. The performance of Illyrian type organizations in the limiting monopoly case is critical. To increase the relevance of the analysis we turn our technology assumption a notch tighter, but loosen the demand assumption. We assume that the monopolist possesses a production function that generates an "L" shaped long run average cost curve under capitalism. This means that the scale co-

efficient must be initially greater than one, decline until it equals one, and remain at one for all larger outputs. While a similar conclusion can be reached without this assumption, its addition puts certain types of Illyrian behavior in clearest focus. There is, of course, good empirical evidence to support this variant. On the other hand, we no longer require demand shifts to leave the elasticity of demand invariant. This enables us to consider the more probable case where outward shifts in demand increase the price elasticity.

Figures 6 and 7 present the output and input solutions using this assumption. As demand shifts outward we presume that the elasticity of demand is increased so that $\gamma(Y)^{-1}$ shifts downward from γ_0^{-1} to γ_1^{-1} , $\epsilon(Y)$ shifts upwards from ϵ_0 to ϵ_1 , and output increases from \hat{Y}_0 to \hat{Y}_1 . Further outward shifts in demand, however, can never increase \hat{Y} above Y_{\max} . This is more clearly seen in Figure 7. If the initial Y_0 is below the Illyrian Crest, an increase in demand moves the firm upwards towards $\hat{\epsilon}$. Labor input may initially increase, but must ultimately decrease as further price rises cause the firm to move upwards along $\hat{\epsilon}$. When the firm passes over the Illyrian Crest it has an output strictly less than Y_{\max} , and a fortiori this is true as it moves northward and asymptotically approaches $\hat{\epsilon}$.

Our conclusion concerning the efficiency of market syndicalism for monopoly is similar to those for perfect competition. As demand increases, output of the firm is blocked by the isoscale contour $\hat{\epsilon}$. This places an absolute limit on the output of the monopolist that cannot be breached regardless of the position of the demand curve. Even in the case where both the Illyrian and Capitalist monopolists both operate below Y_{\max} , the Illyrian

FIGURE 6

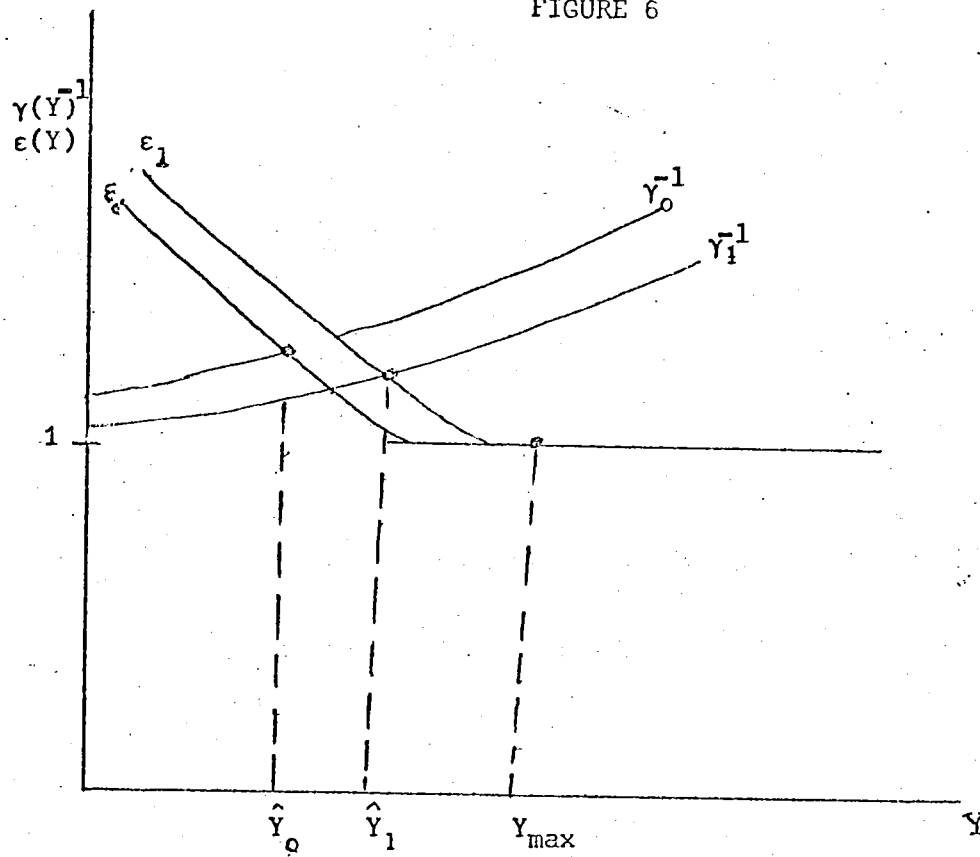
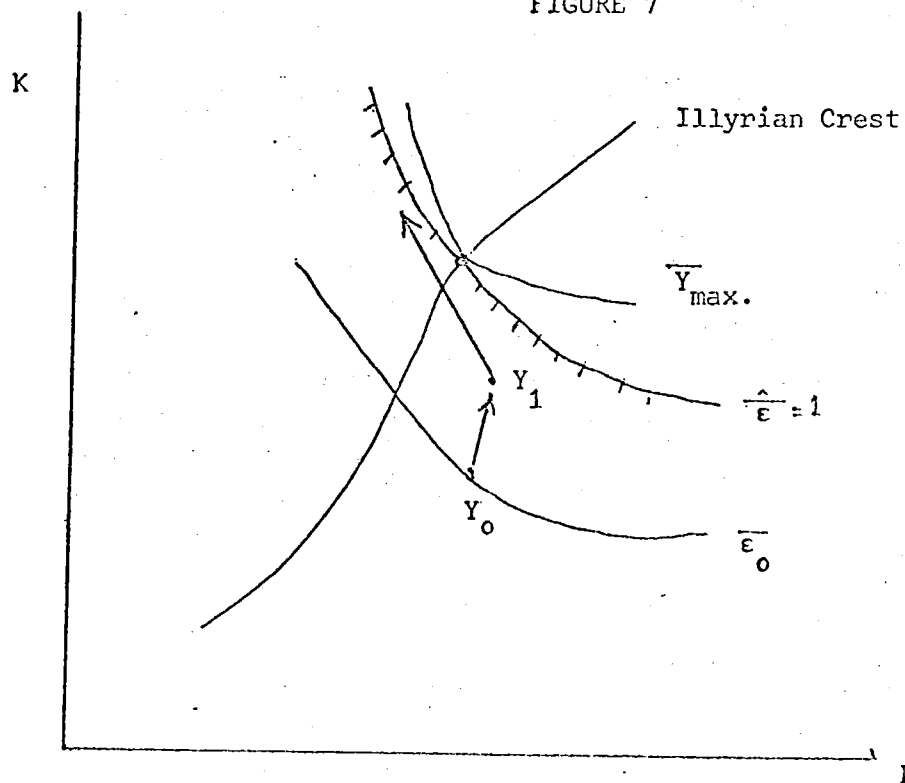


FIGURE 7



firm has smaller output than its Capitalist counterpart and is therefore more destructive to pareto optimality. If free entry is possible at rising costs, we are again faced with the problem of intra-marginal firms having unequal marginal rates of substitution between capital and labor. Whatever the market variant adopted for purposes of analysis, the efficient expansion of an Illyrian economy demands completely free and rapid entry of new firms. We turn next to a consideration of technological progress.

The analysis of technological change is similar to that employed by Murray Brown.¹ Technical advance is divided into two types -- neutral and non-neutral -- each of which may be further subdivided into two classes. Neutral technological progress involves alterations either in the scale of units which relate output and inputs (referred to by Brown as changes in efficiency), or alterations in returns to scale as measured by ϵ . Non-neutral technological progress involves alterations either in the marginal rate of substitution between inputs, or alterations in the elasticity of substitution.

Consider first a neutral technological advance that improves efficiency but does not affect returns to scale. For example, the slope of the output surface in any direction is multiplied at all points by a scalar greater than one. This is illustrated in Figure 2 by an upward labeling of all isoquants, but no change in the isoscale contours. The location of the Illyrian Crest is not affected but the labeling of the expansion paths for specific price levels is. Returning to equation (4), if Y_0 is the old output at (L_0, K_0) and Y_1 is the new output for this input combination, then the expansion path through (L_0, K_0) initially derived for P_0 is now the one

¹Brown, Murray, On the Theory and Measurement of Technological Change, (Cambridge University Press, London, 1966), especially Chapter II.

derived for $P_1 = P_0 \frac{Y_0}{Y_1}$. This follows from the symmetric manner in which P and Y enter equation (4). Since P_1 is less than P_0 , the new expansion path for P_0 lies northeast of the one passing through (L_0, K_0) . The implications of a neutral improvement in efficiency of the type described above are completely analagous to those for an upward shift in the demand curve when the elasticity of demand (and therefore the γ curve) is kept constant. Consequently, labor is decreased, capital is increased and the change in output depends on the location of the Illyrian Crest.

Next consider a neutral technological advance that improves efficiency and also raises the scale coefficient at all levels of output. For example, multiply all slopes by a scalar factor that itself increases with output. In this case, the isoquants are relabeled upwards and the isoscale contours are relabeled downwards. The former effect is described above, the latter effect involves an upward shift in the curve $\epsilon(Y)$.

The long run consequences of this reaction to neutral technological progress are quite negative. Improvements in the efficiency of production will lead to reductions in the output and labor inputs of all firms which are northwest of the Illyrian Crest. Moreover, even if a firm is initially southeast of the Crest, the improvements in efficiency cause the accumulation of capital and laying off of labor until ultimately all firms lie northwest of the Crest where output is decreasing. The only qualification of this result is that there may be an offsetting effect from an outward shift of $\epsilon(Y)$. Although this latter case might be of importance in some industries, it is not the typical case. Thus, the dangers of a backward bending supply curve, which Ward pointed out for short run perfect competition, also exist with respect to

technological progress. As firms become more efficient, their supply curves shift inward so that prices rise and quantity supplied declines. It therefore becomes essential that new firms steadily enter the market. But even this will not achieve optimality if firms are heterogeneous.

The effects of non-neutral change is more difficult to assay. Consider first, changes in the marginal rate of substitution that do not affect the elasticity of substitution. Suppose that at an initial point of equilibrium (L_0, K_0) the marginal product of labor is raised and the marginal product of capital is lowered in such a way that ϵ_0 and Y_0 are not altered.¹ The reader can easily verify that such an increase in the slope of the isoquant \bar{Y}_0 must lead to a substitution of labor for capital if the quantity Y_0 is to be produced.

However, if we are northwest of the Illyrian Crest, this substitution along Y_0 causes the optimal isoscale curve to lie above the new input point, and this induces an increase in output. If we are southeast of the Illyrian Crest, a parallel argument deduces a decrease in output. We conclude that an increase in the marginal product of labor relative to the marginal product of capital must increase labor, decrease capital and have an ambiguous affect upon output. Under these same assumptions a relative increase in the marginal product of capital has the opposite affect. That is, capital is increased and labor decreased. It is obviously important that government policy be directed towards raising the marginal product of labor relatively faster than the marginal product of capital.

¹See the appendix page ____ for a mathematical derivation of this case.

Technical change which affects only the elasticity of substitution is particularly relevant to an analysis of enterprise expansion. As Professor Stigler emphasizes, there is not one but rather many different "long runs."¹ Increases in the time duration of the analysis is apt to increase the elasticity of substitution by permitting the firm to overcome bottlenecks through research and the adoption of more complicated existing technologies. For the capitalist firm, therefore, an increase in the period of analysis, increases the elasticity of substitution and thereby the growth of the firm and the economy.² Is this also true of Illyria?

In the absence of any change in the curvature of the isoscale contours, a flattening of Illyrian isoquants tends to make the supply curve for output more vertical than it otherwise would be. (The extreme case of perfectly inelastic supply occurs when the isoscale contours and the isoquants have identical curvatures.) We might expect, however, that factors which make isoquants more linear would have the same affect on isoscale contours. If so, we cannot predict the affect on the elasticity of supply; but we can see that movements along a flatter \hat{c} will cause a more rapid substitution of capital for labor as demand expands or efficiency improves. Our conclusion must be that a consideration of longer time spans, when capital and labor become more perfectly substitutable, leads to even less satisfactory comparisons with the capitalist firm than does a comparison based upon shorter time spans.

¹George Stigler, "Production and Distribution in the Short Run," The Journal of Political Economy, Vol. 47, No. 3, (June 1939), p. 311.

²Murray Brown, op. cit., p. 27.

A truly dynamic theory of the expansion of the firm would describe the time path of capital, labor and output. Such a theory is beyond the scope of this paper. However, a comment on the investment decision and "plow-back" is called for. One might expect, that if there were no alternative outlets for investible funds other than plowing them back into the enterprise, and if the collective time preferences of workers were positive but less than the lending rate R , the workers might vote to reinvest some of their earnings. The benefit would be increased production next period. This is not correct, however. The fact that retained earnings are charged the full rate R makes it irrational to invest own funds in order to reduce the internal rate of return to the internal rate of time preference. Thus, all profits in Illyria will be paid out in wages and none will be reinvested in the enterprise.

Summary

The essential relationship for analyzing the long run efficiency and growth of the Illyrian firm is given by equation (3) and illustrated in Figure 1. It states that profit per worker will be maximized when the scale coefficient equals the inverse of the elasticity of total revenue with respect to output. This is equivalent to the intuitively more plausible condition that the elasticity of cost be equal to the elasticity of total revenue. It is immediately seen that solutions to this optimality problem will not exist in

certain economically important cases: most notably, when the scale coefficient is everywhere less than or equal to unity. A comparison with the Capitalist solution to the same problem indicates that monopoly output in Illyria will always be no greater than Capitalist output and will be equal only in the zero profits case.

The comparative static effects of increases in the interest rate and shifts in demand (which maintains a constant price elasticity at any given level of output) are inversely related by the formula on pages 16, 18, and 19. A rise in the interest rate leads to an increase in labor and a decrease in capital. An upward shift in demand leads to a decrease in labor and an increase in capital. The net effect of these conflicting changes in factor inputs on output depends upon the curvature of the isoscale contours. At the point of equilibrium, if the isoscale contour cuts the isoquant from below, a rise in the interest rate leads to an increase in output; and an upward shift in demand leads to a decrease in output. If the isoscale contour cuts the isoquant from above, the opposite is true. The importance of the curvature of the isoscale contours is a notable feature of Illyrian economics.

Perfect competition is treated as a limiting case where the elasticity of total revenue with respect to output is unity. We affirm Ward's conclusion that in the short run a rise in the interest rate increases output, while a rise in price decreases output. In the long run, however, it is argued that any negatively sloped segment of the firm's supply curve will be bounded above and below by inelastic or positively sloped segments. This, together with the aggregation of heterogeneous firms to obtain the industry supply curve, makes it quite unlikely that an unstable intersection of industry supply and demand

will occur. It does mean, however, that Ward is correct in asserting that long run industry supply will be highly inelastic.

A serious failure to meet pareto optimality standards is encountered in Illyrian competition (or, for that matter, whenever an industry structure with heterogeneous firms is postulated). The competitive firm always produces at a point of technically efficient scale, which is to say, it is always located on the isoscale contour for ϵ equal to unity, or in our notation the contour $\hat{\epsilon}$. While this is pareto optimal if all firms have the same efficiency, this is not true if heterogeneous efficiency characteristics cause firms to have different values of income per worker. In this case, the more efficient firms produce in the northwest portion of $\hat{\epsilon}$ where the marginal rate of substitution between capital and labor is higher. These interfirm differences in the marginal rates of substitution between capital and labor violate the paretian criterion.

The dynamic behavior of the Illyrian firm is analyzed with respect to continuing outward shifts in demand and technological progress. Assumptions are altered in this section so that the scale coefficient is a nonincreasing function of output rather than the inputs (or equivalently, the isoscale contours do not cut the isoquants from above). For perfect competition, this means that outward shifts in demand or improvements in efficiency due to neutral, technological progress will cause firms to move northwest along $\hat{\epsilon}$. Since both labor and output decrease in this direction, the firms with the most efficient technologies will have the smallest labor inputs and smallest output.

The expansion of the Illyrian monopolist with respect to demand shifts and neutral technological progress is similar to that of the Illyrian competitor in that they both move "along" $\hat{\epsilon}$. However, while the competitor is exactly on $\hat{\epsilon}$ the monopolist moves along but inside it. The distance inside is determined by the extent to which the elasticity of total revenue is greater than unity. This means that for the empirically important case of an "L" shaped capitalist LRAC curve, the Illyrian monopolist will never achieve even a minimally efficient scale. This conclusion is independent of the type of demand shift that is assumed. The logical extreme implied by continuing outward shifts in demand and neutral improvements in technological efficiency is a one-man firm producing a small output with a large amount of capital.

In judging the potential value of such an organizational form in the context of development programs, is it really true that Illyria will score as poorly by efficiency criteria as the last few paragraphs imply? No, not if the rapid entry of new, equally efficient firms can be assured. Toward this end the government must collect and make available to potential entrants relatively elaborate data on rates of return to investment, as well as assure easy access to capital. However, the constant need to generate new firms as old ones whither is an outcome that could appeal to only the most perverse planning bureaucracies.

Our theoretical testing of the Illyrian model yields agreement with Professor Ward's conclusion that "... (it) provides a strong measure of industrial democracy,"¹ and we might add political stability and economic equity.

¹Ward, The Socialist Economy, p. 254.

However, we differ importantly from his conclusion "...that a pareto optimal equilibrium can be sustained by (the) Illyrian organization, given appropriate and apparently not overwhelmingly difficult action by the state with respect to macroeconomic policy."¹ This is true only in the artificial case where we have completely free and effective entry of homogeneous firms. Heterogeneity destroys the optimality of Illyria and monopoly power has an even more restrictive effect on output in Illyria than in capitalism. While one can achieve optimal results by having a special interest rate for every firm, both the administrative morass and the reduction of incentives are apt to reduce productivity importantly.

¹Ibid.

MATHEMATICAL APPENDIX¹

We wish to maximize

$$M = \frac{PY - RK}{L}$$

subject to a production function, $Y = f(L, K)$, which has the properties:

$$Y_L, Y_K > 0$$

$$Y_{LL}, Y_{KK} < 0$$

$$Y_{LK} > 0$$

$$Y_{LL} Y_{KK} - Y_{LK}^2 > 0$$

and $\epsilon_L, \epsilon_K < 0;$

$$0 < \alpha, \beta < 1$$

and subject to a demand function, $P = h(Y, \xi)$ (where ξ is a shift parameter), which has the properties

$$P_Y < 0, P_\xi < 0, \text{ and } \eta_Y < 0.$$

The first order conditions for a maximum are

$$M_L = \frac{1}{L^2} [RK - PY + \alpha PY(1 - \frac{1}{\eta})] = 0, \text{ and}$$

$$M_K = \frac{1}{LK} [\beta PY(1 - \frac{1}{\eta}) - RK] = 0.$$

Combining these two and using $\epsilon = \alpha + \beta$ we obtain

$$\epsilon = \frac{1}{1 - \frac{1}{\eta}} = \frac{1}{\frac{\partial PY}{\partial Y} \frac{Y}{PY}} = \frac{1}{\gamma}.$$

Since values of η less than unity are not associated with maximums, we have

$\hat{\epsilon} \geq 1$; but since ϵ is assumed no greater than 2, this means $1 \leq \hat{\epsilon} \leq 2$.

Before deriving the second order conditions, we state some relationships

¹Lower case letters indicate partial differentiation.

that are required for later arguments concerning signs:

$$\begin{aligned} L\alpha_{\ell} &= Y_{\ell\ell} \frac{L^2}{Y} + \alpha(1 - \alpha); \\ K\beta_{\kappa} &= Y_{\kappa\kappa} \frac{K^2}{Y} + \beta(1 - \beta); \text{ and} \\ L\beta_{\ell} &= K\alpha_{\kappa} = Y_{\ell\kappa} \frac{KL}{Y} - \alpha\beta. \end{aligned}$$

The second order conditions for a maximum are $M_{\kappa\kappa} < 0$, $M_{\ell\ell} < 0$, and $D = M_{\kappa\kappa}M_{\ell\ell} - M_{\kappa\ell}^2 > 0$. The second order derivatives of M after substituting $\epsilon = \frac{1}{Y}$ and $R = \frac{\beta PY}{\epsilon K}$ are:

$$M_{\kappa\kappa} = \frac{\beta PY}{\epsilon L K^2} \left[-\beta \epsilon Y \frac{\partial \eta}{\partial Y} + \frac{\beta}{\epsilon} - 1 + \beta_{\kappa} \frac{K}{\beta} \right] < 0.$$

$$M_{\ell\ell} = \frac{\alpha PY}{\epsilon L^3} \left[-\alpha \epsilon Y \frac{\partial \eta}{\partial Y} + \frac{\alpha}{\epsilon} - 1 + \alpha_{\ell} \frac{L}{\alpha} \right] < 0.$$

$$M_{\kappa\ell} = \frac{\alpha\beta}{\epsilon} \frac{PY}{L^2 K} \left[-\epsilon Y \frac{\partial \eta}{\partial Y} + \frac{1}{\epsilon} + \frac{L\beta_{\ell}}{\alpha\beta} \right] \leq 0.$$

The sign of $M_{\kappa\kappa}$ is negative because:

$$-\beta \epsilon \frac{\partial \eta}{\partial Y} = \beta \epsilon \frac{1}{\eta^2} \eta_y < 0; \text{ and because } \frac{\beta}{\epsilon} - 1 + \beta_{\kappa} \frac{K}{\beta} = (Y_{\kappa\kappa} \frac{K^2}{Y}) + (\beta(1-\beta) + \frac{\beta}{\epsilon} - 1)$$

is negative if the last term on the right hand side is negative. This will be true when α and β are between zero and unity as postulated. A similar argument holds for $M_{\ell\ell}$.

After considerable algebra we arrive at

$$\begin{aligned} D &= \frac{\alpha\beta}{\epsilon} \frac{P^2 Y^2}{L^4 K^2} \left[Y \frac{\partial \eta}{\partial Y} \left\{ \alpha_{\kappa} K - \frac{\beta_{\kappa}}{\beta} K\alpha + \beta_{\ell} L - \frac{\alpha_{\ell}}{\alpha} L\beta + \epsilon \right\} - L \epsilon_{\ell} - K \epsilon_{\kappa} \right. \\ &\quad \left. + \frac{1}{\epsilon^2} \left\{ \frac{LK}{\alpha\beta} (\alpha_{\ell} \beta_{\kappa} - \alpha_{\kappa} \beta_{\ell}) \epsilon \right\} \right] > 0. \end{aligned}$$

The term modifying $Y \frac{\partial \eta}{\partial Y}$ can be reduced to $\frac{LK}{Y} (2Y_{\kappa\ell} - Y_{\kappa\kappa} \frac{Y_\ell}{Y_\kappa} - Y_{\ell\ell} \frac{Y_\kappa}{Y_\ell})$

which is positive and so the net affect of $Y \frac{\partial \eta}{\partial Y}$ is positive. The second term on the right hand side, $-L\epsilon_\ell - K\epsilon_\kappa$, is positive by assumption. This leaves only the third term in doubt. At issue is the sign of $\alpha_\ell \beta_\kappa - \alpha_\kappa \beta_\ell$. It must be shown to be positive either by itself or in conjunction with the second term. From our assumption that ϵ_ℓ and ϵ_κ are negative, it is clear that if α_κ and β_ℓ are positive, they must be respectively less than β_κ and α_ℓ . Consequently, for this case $\alpha_\ell \beta_\kappa - \alpha_\kappa \beta_\ell$ will be positive. On the other hand, if α_κ and β_ℓ are negative, consider the extreme case where $Y_{\ell\ell} = Y_{\kappa\kappa} = Y_{\ell\kappa} = 0$. This extreme case makes the two last terms on the RHS algebraically as small as possible so that if they are positive under these conditions, they must be positive for all negative values of α_κ and β_ℓ . Thus, for the case where α_κ and β_ℓ are negative assume the limiting condition $Y_{\ell\ell} = Y_{\kappa\kappa} = Y_{\ell\kappa} = 0$, let Z describe the second and third terms when this condition holds, and prove that

$$Z = -L(\alpha_\ell + \beta_\ell) - K(\alpha_\kappa + \beta_\kappa) + \frac{1}{\epsilon^2} \left\{ \frac{LK}{\alpha\beta} (\alpha_\ell \beta_\kappa - \alpha_\kappa \beta_\ell) \epsilon \right\} > 0.$$

After some substitutions into the above equation we obtain

$$Z = \frac{1}{\epsilon} [LK\epsilon \{(\alpha - \alpha^2) + (\beta - \beta^2)\} + L\alpha(\epsilon - 1) + K\beta(\epsilon - 1)].$$

Inspection shows that Z will be positive whenever ϵ is greater than unity and α and β are between 0 and one. Therefore, $D = M_{\kappa\kappa} M_{\ell\ell} - M_{\ell\kappa}^2$ is positive as required.

The Illyrian Crest is defined as the locus of tangencies between isoquants and isoscale contours. Since the slope of any isoscale contour is

$$\left. \frac{dK}{dL} \right|_{\bar{\epsilon}} = - \frac{\epsilon_l}{\epsilon_k} = - \frac{Y_{ll}L + Y_{kl}K + Y_l(1-\epsilon)}{Y_{kk}K + Y_{kl}L + Y_k(1-\epsilon)},$$

and the slope of any isoquant is $\left. \frac{dK}{dL} \right|_{\bar{Y}} = - \frac{Y_l}{Y_k}$, a tangency occurs if and only if $\left. \frac{dK}{dL} \right|_{\bar{\epsilon}} = \left. \frac{dK}{dL} \right|_{\bar{Y}}$. Consequently all points on the Illyrian Crest satisfy the

condition

$$\frac{Y_{ll}L + Y_{kl}K}{Y_{kk}K + Y_{kl}L} = \frac{Y_l}{Y_k}$$

The comparative static effects of changes in the interest rate on inputs are obtained by using Cramer's Rule to solve

$$M_{kk} \frac{\partial K}{\partial R} + M_{kl} \frac{\partial L}{\partial R} = -M_{kr}$$

$$M_{kl} \frac{\partial K}{\partial R} + M_{ll} \frac{\partial L}{\partial R} = -M_{lr}$$

$$\text{for } \frac{\partial K}{\partial R} \text{ and } \frac{\partial L}{\partial R}.$$

The second order derivatives with respect to R are:

$$M_{kr} = - \frac{1}{L} \text{ and } M_{lr} = \frac{K}{L^2}$$

The solutions for changes in inputs are therefore:

$$D \frac{\partial L}{\partial R} = \frac{\alpha PY}{L^4} \left[-\epsilon Y \frac{\partial \eta}{\partial Y} + \frac{\epsilon_l L}{\epsilon \alpha} \right] < 0$$

$$D \frac{\partial K}{\partial R} = - \frac{\beta PY}{L^3 K} \left[-\epsilon Y \frac{\partial \eta}{\partial} + \frac{\epsilon_k K}{\epsilon \beta} \right] > 0.$$

where D is the denominator for Cramer's Rule which we have already shown to be positive. The signs $\frac{\partial K}{\partial R} < 0$ and $\frac{\partial L}{\partial R} > 0$, are immediate. Note that

$$\frac{\partial K}{\partial R} = - \frac{\alpha}{K \epsilon} \frac{\partial K}{\partial Y} = -(\epsilon \frac{\partial K}{\partial Y}) \frac{\partial K}{\partial P}$$

as stated on page 15.

The effect of changes in R upon Y are derived from the equation

$$\frac{\partial Y}{\partial R} = \frac{\partial Y}{\partial L} \frac{\partial L}{\partial R} + \frac{\partial Y}{\partial K} \frac{\partial K}{\partial R} \quad \text{yielding}$$

$$D \frac{\partial Y}{\partial R} = \frac{\alpha \beta}{\epsilon} \frac{PY^2}{L^4 K} \left[\frac{\epsilon_L}{\alpha} - \frac{\epsilon_K}{\beta} \right]$$

The sign of $\frac{\partial Y}{\partial R}$ is given by the relationship,

$$\frac{\partial Y}{\partial R} \begin{matrix} < \\ > \end{matrix} 0 \text{ as } \frac{\epsilon_L}{\epsilon_K} \begin{matrix} > \\ < \end{matrix} \frac{Y_L}{Y_K}; \quad \text{or } \frac{\partial Y}{\partial R} \begin{matrix} < \\ > \end{matrix} 0 \text{ as } - \frac{dK}{dL} \begin{matrix} > \\ < \end{matrix} - \frac{dK}{dL} \begin{matrix} > \\ < \end{matrix} \frac{1}{Y}$$

The effect of a shift in demand is calculated in the same fashion. We assume that $\frac{\partial P}{\partial \xi} = 1$ and $\frac{\partial \eta}{\partial \xi} \geq 0$. The second order derivatives with respect to the shift parameter ξ are:

$$M_{K\xi} = \frac{\beta Y}{LK} \left(\frac{1}{\epsilon} - P \frac{\partial \eta}{\partial \xi} \right); \quad \text{and } M_{L\xi} = \frac{\alpha Y}{L^2} \left(\frac{1}{\epsilon} - \frac{1}{\alpha} - P \frac{\partial \eta}{\partial \xi} \right).$$

The application of Cramer's Rule gives:

$$D \frac{\partial K}{\partial \xi} = \frac{\alpha \beta}{\epsilon} \frac{PY^2}{L^4 K} \left[\epsilon Y \frac{\partial \eta}{\partial Y} - \frac{\epsilon_L L}{\epsilon \alpha} + \left(\frac{L}{\alpha} \alpha_L - \frac{L \beta_L}{\beta} - 1 \right) P \frac{\partial \eta}{\partial \xi} \right] > 0; \quad \text{and}$$

$$D \frac{\partial L}{\partial \xi} = \frac{\alpha \beta}{\epsilon L^3 K} \left[- \frac{\beta \epsilon}{\alpha} Y \frac{\partial \eta}{\partial Y} + \frac{\epsilon_K K}{\epsilon \beta} + \left(\frac{K}{\beta} \beta_K - \frac{K}{\alpha} \alpha_K - 1 \right) P \frac{\partial \eta}{\partial \xi} \right] \leq 0.$$

The sign of the latter is indeterminate because $P \frac{\partial \eta}{\partial \xi} \leq 0$, and also

$\left(\frac{L}{\alpha} \alpha_L - \frac{L}{\beta} \beta_L - 1 \right) = \frac{L^2}{\beta} Y_{LL} - \frac{LK}{\alpha Y} Y_{LK} < 0$. This means that if $\frac{\partial \eta}{\partial \xi}$ takes on a large enough negative value, $\frac{\partial L}{\partial \xi}$ may be positive. However, for smaller values and

particularly when $\frac{\partial \eta}{\partial \xi} = 0$ we have $\frac{\partial L}{\partial \xi}$ negative.

The effect of shifts in demand on output are found to be,

$$D \frac{\partial Y}{\partial \xi} = \frac{\alpha \beta PY^3}{\epsilon L^4} [K \epsilon_K - L \frac{\beta}{\alpha} \epsilon_L + P \frac{\partial \eta}{\partial \xi} \{ \beta L (\frac{\alpha_L}{\alpha} - \frac{\beta_L}{\beta}) + \alpha K (\frac{\beta_K}{\beta} - \frac{\alpha_K}{\alpha}) - \epsilon \}] \frac{1}{Y} < 0.$$

When $\frac{\partial \eta}{\partial \xi} < 0$ we cannot determine the sign of $\frac{\partial Y}{\partial \xi}$. However, if $\frac{\partial \eta}{\partial \xi} = 0$, then

$$\frac{\partial Y}{\partial \xi} \begin{matrix} \leq \\ > \end{matrix} 0 \text{ as } - \frac{dK}{dL} \begin{matrix} \left| \right. \\ \left| \right. \end{matrix} \begin{matrix} \leq \\ > \end{matrix} - \frac{dK}{dL} \begin{matrix} \left| \right. \\ \left| \right. \end{matrix} \begin{matrix} \frac{1}{\epsilon} \\ \frac{1}{Y} \end{matrix}$$

The possibility of a negative supply response is quite sensitive to the magnitude of $\frac{\partial \eta}{\partial \xi}$. If we use an upward shift in a linear demand function such that the slope is not changed by the shift, we can show that $\frac{\partial Y}{\partial \xi}$ is always positive. This is done by deriving the following two expressions for shifts in linear demand curves:

$$P \frac{\partial \eta}{\partial \xi} = - \frac{1}{\eta} = \frac{1-\epsilon}{\epsilon} ; \text{ and } Y \frac{\partial \eta}{\partial Y} = \frac{1-\epsilon}{\epsilon} (1 - \frac{Y}{P}) ;$$

and then substituting them into the general expression stated above for $\frac{\partial Y}{\partial \xi}$.

This yields,

$$\frac{\partial Y}{\partial \xi} \begin{matrix} \left| \right. \\ \text{linear} \\ \text{demand} \\ \text{shift} \end{matrix} = \frac{\alpha \beta PY^3}{\epsilon L^4 K^2} [- \frac{\beta}{L} (\frac{\alpha_L}{\alpha} - \frac{\beta_L}{\beta}) - 1 + \epsilon] > 0.$$

A calculus derivation for the effects of technological progress is given only for the case of a non-neutral increase in the marginal product of labor relative to the marginal product of capital. We assume that from an original point of equilibrium (L_0, K_0) the production surface is twisted so that the slope of the isoquant through (L_0, K_0) is increased but $Y(L_0, K_0) = Y_0$ and $\epsilon(L_0, K_0) = \epsilon_0$ are left unchanged. Denoting λ as an arbitrary increment to Y_L and μ as

the resulting decrement to Y_K , we have $\epsilon_o Y_o = (1 + \lambda) Y_L + (1 - \mu) Y_K$, so that $\mu = \frac{\lambda \alpha_o}{\beta_o}$. The mathematical derivation proceeds by substituting $\alpha(1 + \lambda)$ for α and $\beta(1 - \frac{\lambda \alpha}{\beta})$ for β into the first order conditions and then treats λ as a production shift parameter. It is necessary to recalculate the second order derivatives M_{LL} , M_{KK} and M_{LK} as well as to calculate the newly required $M_{L\lambda}$ and $M_{K\lambda}$. We spare the reader this and jump immediately to our conclusion:

$$D \frac{\partial K}{\partial \lambda} = \frac{\alpha^2 \beta}{\epsilon^2} \frac{(PY)^2}{L^4 K} \left[-(1 + \frac{\alpha}{\beta}) \epsilon Y \frac{\frac{1}{\partial \eta}}{\partial Y} + \frac{\epsilon L}{\alpha \beta} \right] < 0;$$

$$D \frac{\partial L}{\partial \lambda} = \frac{\alpha \beta}{\epsilon^2} \frac{(PY)^2}{L^3 K^2} \left[\epsilon^2 Y \frac{\frac{1}{\partial \eta}}{\partial Y} - \frac{K}{\beta} (\beta_K + (1 + \lambda) \alpha_K) \right] > 0;$$

and finally,

$$D \frac{\partial Y}{\partial \lambda} = \left(\frac{\alpha \beta PY}{\epsilon L^2 K} \right) \frac{2Y}{\beta} \left[\frac{\epsilon L}{\alpha} - \frac{K}{\beta} (\beta_K + (1 + \lambda) \alpha_K) \right] \leq 0.$$

The sign of $\frac{\partial L}{\partial \lambda}$ is necessarily positive so long as λ is small. Similarly, for infinitesimal changes in λ , the sign of $\frac{\partial Y}{\partial \lambda}$ is the same as the sign of $\frac{\partial Y}{\partial R}$ since the terms within the brackets are identical. Thus,

$$\frac{\partial Y}{\partial \lambda} \begin{matrix} < \\ > \end{matrix} 0 \text{ as } - \frac{dK}{dL} \bigg|_{\epsilon} \begin{matrix} > \\ < \end{matrix} - \frac{dK}{dL} \bigg|_{\bar{Y}}.$$