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DEVELOPMENT, STRUCTURAL CHANGES, AND URBANIZATION

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ABSTRACT

This paper constructs an equilibrium model by formalizing a trade-off between the gains from trade based on increasing returns to specialization and transaction costs. The relationship between development, structural changes, and urbanization is investigated. In addition, the function of a free market in searching for the efficient market structure is explored.

Introduction

As far as the problem of production was concerned, Adam Smith [1776] and Allyn Young [1928] emphasized the productivity implications of economic organization (the division of labor). Neoclassical microeconomics cannot explore such implications for the following reason. For production functions with constant returns to scale, an agent's productivity of a good is not greater when he produces only this good than when he produces many goods. On the other hand, production functions with increasing returns to scale cannot be used to characterize the level of specialization within a firm. The concept of economies of scale presupposes a complete separation of pure consumers from pure producers. "Scale" relates to a firm which is a pure producer, but is irrelevant to a pure consumer.

The separation of pure producers from pure consumers is a basis of Debreu's theoretical framework and neoclassical microeconomics. This artificial separation has perhaps misled economic theory. In autarky, there is neither a pure consumer nor a pure producer; each individual is a producer/consumer. The division of labor will increase the portion of a person's production that is not consumed by himself (i.e. the portion sold to other people) and increase the portion of a person's consumption that is not produced by himself (i.e. the portion purchased from other people). We can view this change as an increase in the degree of separation between production and consumption though each person is a producer/consumer even in the division of labor. The degree of such separation depends on the level of division of labor (or inversely on the degree of self-sufficiency). As to endogenizing the level of division of labor and thereby the degree of such separation, Debreu's framework is irrelevant since in his framework, pure consumers are completely separated from pure producers and the degree of the separation of consumption from production cannot be defined.

In order to capture the ideas of Smith and Young, this paper specifies production functions for each producer/consumer such that an individual's productivity increases with the level of specialization and the aggregate transformation curve for the whole economy depends positively on the level of division of labor.

There are several implications of this method of specifying production functions for the theory of equilibrium. First, each individual is a producer/consumer. He must decide how many goods are self-provided, i.e. what is the level of specialization. Hence, the model in this paper can be used to endogenize the level of division of labor. According to conventional microeconomics, pure consumers cannot choose the level of specialization since they must buy all goods from firms.

Second, a Cobb-Douglas utility function is specified for each producer/consumer. Consequently each individual as a consumer prefers diverse consumption and as a producer prefers specialized production. This implies that the division of labor will incur great transaction costs. Therefore, there is a trade-off between economies of specialization and transaction costs. In other words, our method of specifying production functions makes the level of division of labor crucial for productivity; while transaction efficiency is critical for the determination of the level of division of labor. Because of increasing returns to specialization, the production possibility frontier (PPF) is associated with extreme specialization. Extreme specialization will, however, incur prohibitively great transaction costs since people prefer diverse consumption. Hence, the welfare frontier may differ from the PPF.[1] A natural conjecture is that a competitive equilibrium will balance the trade-off between economies of specialization and transaction costs, and that improvement of transaction efficiency will move the equilibrium closer to the PPF, resulting in an increase in the division of labor. The major purpose of this paper is to prove this conjecture. In other words, Smith's conjecture of the "invisible hand" and his insights into increasing returns to specialization will be reconciled in this paper.

A crucial assumption leading to this result is that labor is specific for each person who is able to produce all goods. This assumption combined with the assumption of free entry ensures the first welfare theorem even if there exist increasing returns to specialization. Intuitively, we can see that if increasing returns to specialization are specific for each individual, economies of scale are limited. Since production functions are specified for each producer/consumer, prices are determined by the numbers of individuals selling different goods. This number cannot be manipulated by any individual because of the assumption of free entry and the assumption that each person is able to produce all goods. Hence, nobody is able to manipulate prices. Nevertheless, if labor can be divided in fine detail and the population size is very large, economies of division of labor may be very large. Hence, a competitive market may be compatible with the substantial economies of the division of labor. Therefore, we can use this method to develop the concepts of equilibrium and Pareto optimum in relation not only to the resource allocation for a given level of division of labor, but also to the determination of the level of division of labor and productivity.

The implications of this method for other fields of economic analysis are important. The level of division of labor based on increasing returns to specialization is intimately related to the extent of market, trade dependence, trade pattern, market structure, and economic structure, so these can be made endogenous in our model. Productivity is related to the level of division of labor which depends on transaction efficiency, which is in turn affected by urbanization, government policies, and institutional arrangements. Hence, our model can be used to investigate the impacts of urbanization, government policies, and institutional arrangements on the equilibrium level of division of labor (related to the market structure, trade dependence, and so on) and productivity. This paper shows that the equilibrium trade volume depends positively on the absolute degree of increasing returns to specialization in production and transaction, and negatively on the average distance between a pair of neighbors. The trade pattern is determined by the relative degree of increasing returns to specialization in producing different goods and relative preference for different goods. In addition, it is shown that increases in diversification of the economic structure, concentration of production, integration of the economy, specialization, and the output share of roundabout productive activities are different versions of the evolution of division of labor resulting from improvements of transaction efficiency which are in turn caused by urbanization, liberalization policies, or changes in institutional arrangements.

Many economists have proposed similar ideas. Nevertheless few among them have been successful in formalizing them. On the other hand, the formal models proposed by mainstream economists are often inconsistent with these ideas. This may be due to the difficulty of formalizing the ideas of Smith and Young. For example, many economists (see, e.g. Helpman and Krugman [1985] and Herberg and Tawada [1982]) point to problems which are considered to make an equilibrium model with increasing returns to specialization and transaction costs unmanageable. Such problems include the issue of corner solutions based on increasing returns to specialization, the problem of infinite combinations of individual corner solutions in solving for equilibrium, notorious complications in dealing with indexes of variables in models with transaction costs, and the problem of existence of equilibrium. Formalizing the notion of increasing returns to scale is much easier than formalizing the notion of increasing returns to specialization. This might explain why it is hard to find microeconomic equilibrium models that formalize the theory of production proposed by Smith and Young.

In this paper, we try to formalize the essence of the ideas of Smith and Young as well as to keep an equilibrium model tractable. Our techniques for achieving these two goals are to specify a specific transaction technology and to devise a multiple-step approach to handling the issue of combinations of corner solutions. In order to get around the problem of the existence of equilibrium, we propose a specific model. Using this model, we can prove the existence of equilibrium although it is impossible to reach a general conclusion on this issue for a model with increasing returns to specialization.

Fortunately, these measures are not only effective in keeping the model tractable, but also useful in working out the meaningful comparative statics of equilibrium. Many interesting economic phenomena which are not addressed in the conventional theory can be explained by the kind of model in this paper.

This paper is organized as follows. Section I sets out a model with three goods. Sections II-VI develop a multiple-step approach to handling the model with increasing returns to specialization. Section II solves for the corner solutions for the individual decision problem. Section III solves for all candidates for equilibrium in various market structures. Section IV solves for the restricted Pareto optimum in each market structure and the full Pareto optimum. Section V investigates the relationship between equilibria and the Pareto optima. Section VI solves for an equilibrium and investigates its comparative statics. Some simple conclusions are summarized in the final section.

I A Model with Three Goods

Let us first consider an economy with M consumers/producers and 3 consumer goods. The self-provided amounts of these goods are x, y, and z, respectively. By self-provided we shall mean that quantity of a good produced by an individual for his own consumption. The amounts of these goods sold at the market are x^s, y^s, and z^s, respectively. The amounts of these goods purchased in the market are x^d, y^d, and z^d, respectively. An "iceberg" type of transaction technology is characterized by the coefficient k. Fraction k of a shipment disappears in transportation. Thus, $(1-k)x^d$, $(1-k)y^d$, and $(1-k)z^d$ are the amounts a person receives from the purchases of three goods, respectively.

Furthermore, we assume that (1-k) depends on the quantity of labor used in transaction. (1-k) can be viewed as transaction service. Such services are categorized into self-provided ones and traded ones. Let $1-k = T + T^d$ where T is the self-provided quantity of transaction service and T^d is the quantity purchased of transaction service. The more transaction service $T+T^d$, the greater portion of a purchase is received by its buyer. Here, $T+T^d \leq 1$. Signifying the quantity sold of transaction service by T^s, transaction technology and production functions are thus given by

$$\text{(I-1a)} \qquad x+x^s = L_x^a \qquad y+y^s = L_y^b \qquad z+z^s = L_z^c \qquad T+T^s = L_T^t$$

where $x+x^s$, $y+y^s$, $z+z^s$, and $T+T^s$ are the output levels of four goods and service, respectively. L_s is the amount of labor used in producing good (or service) s where s = x, y, z, T. (I-1) is assumed to be identical for all individuals. In such iso-elasticity production functions parameters a, b, c, and t characterize the returns to specialization. If a, b, c, t > 1, then there are increasing returns to specialization. Adopting the concept of localized technology proposed by Sah and Stiglitz [1986], we assume that the total quantity of labor available for an individual is specific for him. Let this quantity be one; there is an endowment constraint of the specific labor for an individual

$$\text{(I-1b)} \qquad L_x + L_y + L_z + L_T = 1, \qquad 0 \leq L_i \leq 1, \qquad i = x, y, z, T$$

This method of specifying production functions is substantially different from the conventional one. This system of production functions differs from the production functions associated with the U-shaped average cost curve and those with global increasing returns to scale. The production possibility frontier (PPF) of this system of production functions is associated with extreme specialization. Therefore, if there is no transaction cost, equilibrium is the extreme division of labor (each individual has extreme specialization and different individuals specialize in producing different goods).

In our model the reason that people prefer some division of labor is that internal economies of scale are very limited because labor is specific for each individual. This point distinguishes increasing returns to specialization from increasing returns to scale.

Assume, further, that the transaction service T (or T^d) is related only to the quantity traded and there are transaction costs related to the distance between a pair of trade partners; thus there is a location problem. Suppose that all people are evenly located and the geographic distance between a pair of neighbors is a constant. The distance between a pair of trade partners may differ from the distance between a pair of neighbors. If all trade partners of an individual are located in a circle with radius R and with his location as the origin, it can be shown that the average distance between this individual and his trade partners is proportional to R and the number of these trade partners N is proportional to R^2. Hence, the average distance between this individual and his trade partners is proportional to $\sqrt{N}$. If all the trade partners supply different goods to this individual, the number of traded goods for him is $n = N+1$. For simplicity, we assume that the number of trade partners of an individual, $N = n - 1$, where n is the number of traded goods for him.

Assume that the transaction cost coefficient K characterizes the transaction cost related to the average distance between a pair of trade partners; then the K fraction of $(T+T^d)x^d$ or $(T+T^d)y^d$, or $(T+T^d)z^d$ disappears on the way from a seller to a buyer. The relation between K and the number of trade partners of an individual, N, is given by

$$K = s\sqrt{N} \qquad \text{(I-2)}$$

where s is a constant depending on the distance between a pair of neighbors and π. Taking (I-2) into account, the amounts consumed of the three goods are $x+(1-K)(T+T^d)x^d$, $y+(1-K)(T+T^d)y^d$, and $z+(1-K)(T+T^d)z^d$, respectively. The utility function is identical for all individuals:

$$U = [x+(1-K)(T+T^d)x^d]^\alpha \; [y+(1-K)(T+T^d)y^d]^\beta \; [z+(1-K)(T+T^d)z^d]^\gamma \tag{I-3}$$

where $0 < \alpha, \beta, \gamma < 1$ and $\alpha + \beta + \gamma = 1$. The maximal value of $T+T^d$ is 1.

We assume free entry for all individuals into any sector and that M is large. These assumptions imply that individuals treat prices parametrically.

II. The Individual Optimal Decision

This section and the four sections to follow are devoted to devising a multiple-step approach to handling the model with increasing returns to specialization. In our model, the individual decision problem implicitly includes three problems: to choose the optimum level of specialization (in other words, the optimal number of traded goods is a decision variable), to choose the optimal composition of traded goods, and to choose the optimal quantities of consumption, production, and trade. This problem is unmanageable in a step.

If there are increasing returns, some variables will take zero values in the individual optimal decision and in equilibrium. Hence, an individual needs to enumerate all possible combinations of zero and non-zero variables before identifying his optimal decision. Also we need to enumerate all possible combinations of these individual combinations of zero and non-zero variables before identifying the Pareto optimum and equilibrium although we can exclude some of these combinations from the list of candidates for equilibrium by application of the Kuhn-Tucker theorem (as shown in Appendix 1).

There are five steps in solving for equilibrium. This section will solve for individually rational decisions for given prices and for each combination of zero and non-zero variables. Section III solves for the candidate for equilibrium (or corner equilibrium) for each combination of the above individual combinations. Section IV solves for the Pareto optimum candidate for equilibrium. Section V investigates the relationship between equilibrium and the Pareto optimum. Finally, equilibrium and its comparative statics are solved.

The individual decision problem is

$$\text{(II-1)} \qquad \text{Max:} \quad U = [x+(1-K)(T+T^d)x^d]^\alpha \; [y+(1-K)(T+T^d)y^d]^\beta \; [z+(1-K)(T+T^d)z^d]^\gamma$$

$$\text{s.t.} \qquad x+x^s = L_x^a \qquad y+y^s = L_y^b \qquad z+z^s = L_z^c \qquad \text{(production function)}$$

$$T + T^s = L_T^t \qquad \text{(transaction technology)}$$

$$L_x + L_y + L_z + L_T = 1 \qquad \text{(endowment constraint)}$$

$$p_T T^s + p_x x^s + p_y y^s + p_z z^s = p_T T^d + p_x x^d + p_y y^d + p_z z^d \qquad \text{(trade balance)}$$

where i, i^s, i^d, and L_i (i = x, y, z, T) are decision variables, which may take on zero or positive values. p_i is the price of good (or service) i.

If a, b, c, t > 1, the optimal decision is certainly a corner solution. By combination of zero and non-zero values of the variables, there are several possible corner solutions. We shall call such a combination a "structure." An individual needs to enumerate and compare utilities in all structures before choosing a structure. Therefore, an individual must solve for the corner solutions for each structure and his decision making process consists of two stages. In the first stage all structures are enumerated. An individual solves for the efficient allocation (how much should be produced, consumed, and traded of each good) for given prices and for each structure. In the second stage, he decides what should be produced, and what should be sold and purchased, i.e. which structure should be chosen. Section III will discuss this problem. [2]

There are 29 structures of four general types:

(1) Autarky (x,y,z), i.e. an individual self-provides three goods. For this structure

$$x^s = y^s = z^s = x^d = y^d = z^d = T = T^d = T^s = L_T = 0$$

In other words, the amounts sold and purchased of the three goods are zero, as are the amounts sold, purchased, and self-provided of transaction services.

(2) Structure (i/j), i.e. an individual sells good (or service) i and purchases good (or service) j, i,j = x,y,z,T. For such structures

$$k^s = k^d = i^d = j = j^s = L_j = 0 \quad \text{for } k \neq i, j$$

where index k denote the goods other than goods i and j. By 2 permutations of four factors, we obtain 12 structures of this type: (x/y), (y/x), (x/z), (z/x), (y/z), (z/y), (T/x), (x/T), (T/y), (y/T), (T/z), and (z/T).

(3) Structure (i/jk), i.e. an individual sells good (or service) i and purchases goods (or service) j and k. i, j, k = x, y, z, T. For such structures

$$r^s = r^d = i^d = j = k = j^s = k^s = L_j = L_k = 0 \quad \text{for } r \neq i, j, k$$

where index r denote the goods other than goods i, j, and k. There are 12 structures of this type: (x/yz), (x/yT), (x/zT), (y/xz), (y/xT), (y/zT), (z/xy), (z/xT), (z/yT), (T/xy), (T/xz), and (T/yz).

(4) Structure (i/jkr), i.e. an individual sells good (or service) i and purchases goods (or service) j, k, and r, i, j, k, r = x, y, z, T. For such structures

$$i^d = j = k = r = j^s = k^s = r^s = L_j = L_k = L_r = 0 \quad \text{for } j, k, r \neq i$$

There are four structures of this type: (x/yzT), (y/xzT), (z/xyT), (T/xyz). Appendix 1 has proven

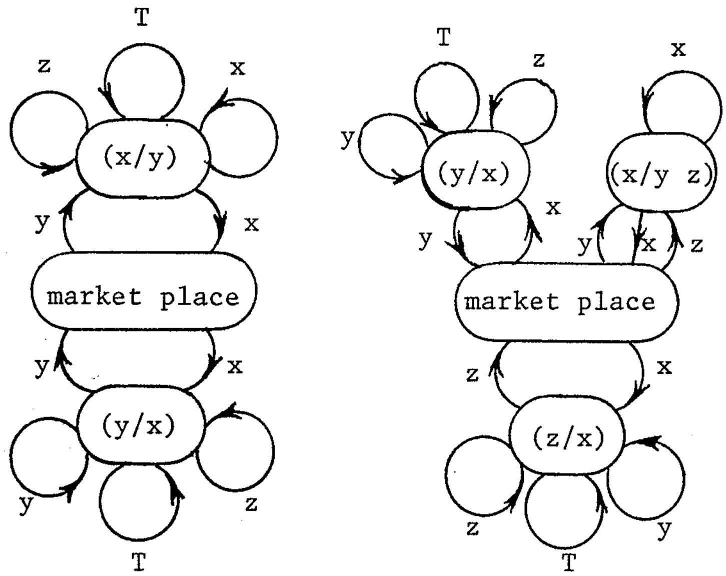
Proposition 1

According to the necessary condition for an optimum decision, an individual does not produce and purchase a good at the same time, and sells only one good (if any).

This proposition implies that the remaining structures, e.g., (ijk/ijk), (ij/jk), (ij/k), etc. do not satisfy the necessary condition for maximizing individual utility. Hence, these structures will not be concerned. Letting the relevant variables in problem (II-1) take on a zero value, an individual can solve for his optimal decisions for each structure. These individual decisions include individual supply, which is a constant depending on a, b, c, α, β, γ, t, and s, and individual demand, depending positively on the relative prices of the goods he sells to the goods he purchases, and his supply. Inserting the individual optimal decision for each structure and for given prices into the utility function gives an indirect utility function for each structure. The indirect utility functions differ from structure to structure although the original utility function is identical for all individuals. In an indirect utility function, the number of relative prices is one less than the number of traded goods. Here there are 29 alternative structures for each individual that might be rational.

III. The Markets and Candidates for Equilibrium (Corner Equilibria)

As in solving for the individual decisions, if there are increasing returns, we need to enumerate all combinations of structures before identifying the



(a) Market P (b) Market B

Figure 1

equilibrium and the Pareto optimum. This section first investigates how structures are combined to constitute markets. We will then enumerate all combinations of structures and solve for the "candidates" for equilibrium (corner equilibrium). The corner equilibrium is an analogue to the corner solution in the optimization problem for an individual. It is a concept used to solve for the equilibrium in a calculation of several steps. Of course, a corner equilibrium would never come into being if it was not a full equilibrium. [3]

Defining a combination of several mutually consistent structures as a market, there are many market configurations, such as those shown in Figure. Figure 1 (a) is a market that combines structure (x/y) (an individual sells x and buys y, and self-provides z and T) and

(Please insert Figure 1 here)

(a) Market P (b) Market B

Figure 1

structure (y/x) (an individual sells y and buys x, and self-provides z and T). We refer to this market as P. Figure 1 (b) is a market that combines structure (x/y z) (an individual sells x and buys y and z, and self-provides T), structure (y/x) (an individual sells y and buys x, and self-provides z and T) and structure (z/x) (an individual sells z and buys x, and self-provides y and T). We refer to this market as B. Note that some structures are not mutually consistent, e.g., structure (x/y) and (x/z) are not mutually consistent, i.e. there is no demand for x and no supply of y and z although there are supply of x and demand for y and z for a combination of these two structures. Therefore, a combination of (x/y) and (x/z) cannot constitute a market.

In the analysis of competitive equilibrium, we will use the concept of a basic market. A basic market is defined as a market for which one cannot obtain another market by dropping any of its structure. In Figure 1, the two markets are basic markets. However, a combination of structures (x/y z), (y/x z), (y/x) and (z/x) is not a basic market because we can obtain market B in Figure 1 (b) by dropping structure (y/x z).

Many structures can be combined with a basic market to obtain a new, non-basic market. For example, by combining market B with any of the other 6 structures which do not involve trade in T and are not in market B we can obtain $\Sigma_{j=0}^{6} C_6^j = 2^6$ markets, where C_6^j is j combination of 6 factors. For a basic market, the number of traded goods equals the number of structures. For a non-basic market, the number of traded goods is smaller than the number of structures.

We will enumerate all possible markets, and solve for all candidate equilibria (corner equilibria), then find the Pareto optimum corner equilibrium and the full equilibrium.

From the individual optimal decisions, the individual supply and demand functions for each structure can be derived. Let M_i be the number of individuals selling good i; market clearing conditions can be specified for each basic market. For example, if the market is P, a combination of structure (x/y) and structure (y/x), shown in Figure 1 (a), there are the market clearing conditions in market P

(III-1a) $$M_x x^s = M_y x^d \qquad M_x y^d = M_y y^s$$

where M_x and M_y are the numbers of the individuals selling x and y, respectively.

The individual budget constraints for the two component structures are

(III-1b) $$p_{xy} x^s = y^d \qquad p_{xy} x^d = y^s$$

where $p_{xy} \equiv p_x/p_y$. One of these two equations implies the other due to Walras' law.

From (III-1a) and (III-1b), it can be derived that

(III-2) $$p_{xy} = y^d/x^s = y^s/rx^s$$

where $r \equiv M_x/M_y$ is the ratio of the number of individuals choosing structure (x/y) to the number of individuals choosing structure (y/x). We need solve only for the relative numbers of individuals choosing different structures because we can derive $M_x = rM/(1+r)$ and $M_y = M/(1+r)$ from $M_x + M_y = M$. Hence, M_x and M_y can be found if r is known.

According to the individual optimal decisions, x^s and y^s are constants depending upon a, b, c, α, β, γ, s, and t. Hence, (III-1b) implies p_{xy} depends inversely on r. Inserting (III-2) into indirect utility functions in structures (x/y) and (y/x), the utilities may be expressed as functions of r. Setting U(x/y) = U(y/x), we have

(III-3) $$U(x/y) \equiv r^{-\beta} G(x/y) = r^{\alpha} G(y/x) \equiv U(y/x)$$

where G's depend on a, b, c, α, β, γ, t, and s. The intersection of U(x/y) and U(y/x) determines a corner equilibrium value of r. Inserting the value of r back into utility function, (III-3) gives a corner equilibrium utility U*, which is real income as well as the real returns to labor in this market because of the assumption that each person has one unit of labor.

For the category of non-basic markets, there is the following proposition:

Proposition 2

There does not exist a corner equilibrium for any non-basic market.

For a non-basic market, there are m structures and n traded goods, and m > n. According to Appendix 1, in any optimal structure an individual sells at most one type of good. This implies that the types of structures selling the same type of good exceed one if m > n. The necessary conditions for a corner equilibrium lead to m-1 conditions of utility equalization. These m-1 equations contain n-1 relative prices. Noting the fact that these m-1 equations are log-linear, non-homogeneous, and independent of one another, this system has no consistent solution since m > n. Therefore, proposition 2 has been established.

Noting the following two points, we can prove the existence of a corner equilibrium for a given basic market.

(A) For a given basic market with n traded goods and n component structures, the individual supplies of all traded goods are constants and the individual demands for all traded goods are functions of relative prices. The indirect utility functions are determined only by relative prices of the traded goods. Letting the n indirect utility functions equal one another, we obtain (n-1) log-linear equations containing (n-1) relative prices. Again, noting that these equations are non-homogeneous and independent of one another, we can solve for a vector of log-relative prices. The relative prices are positive.

(B) Given the relative prices solved in (A), (n-1) independent market clearing conditions can be transformed into a system of equations containing (n-1) relative numbers of individuals selling the different goods. This system looks like a system of linear equations associated with an input-output system. For example, in the market with three traded goods, such a system is

$$\text{(III-4a)} \qquad \begin{pmatrix} 1 & -\beta/(\alpha+b) \\ -\gamma/(\alpha+\gamma) & 1 \end{pmatrix} \begin{pmatrix} M_{yx}\, p_{yx}\, y^s \\ M_{zx}\, p_{zx}\, z^s \end{pmatrix} = \begin{pmatrix} x^s \\ x^s \end{pmatrix}$$

If the number of goods traded by individuals selling good z is less than that traded by the whole market, e.g. structure (z/x) trades two goods and structures (x/y,z) and (y/x,z) trade three goods, then (III-4a) becomes

$$\text{(III-4b)} \qquad \begin{pmatrix} 1 & -\beta/(\alpha+\beta) \\ -1 & 1 \end{pmatrix} \begin{pmatrix} M_{yx}\, p_{yx}\, y^s \\ M_{zx}\, p_{zx}\, z^s \end{pmatrix} = \begin{pmatrix} x^s \\ 0 \end{pmatrix}$$

where $M_{ij} \equiv M_i/M_j$ is the relative number of individuals selling good i to those selling good j. M_{yx} and M_{zx} are unknown. It is easy to see that the solution vector $(M_{yx}, M_{zx})'$ for this kind of system of linear equations is positive.

Considering the positiveness of equilibrium relative prices shown in (A), the positiveness of the solution of (III-4) guarantees the existence of corner equilibrium in a basic market. This leads us to

Proposition 3

There exists a corner equilibrium for any basic market.

There are multiple corner equilibria. All these corner equilibria comprise a set of the candidates for equilibrium. These candidates satisfy the following conditions

(i) All excess demands for goods are zero for uniform positive relative prices of goods and a uniform positive price of labor (the real returns to labor).

(ii) Individuals maximize their utilities for given prices and for a given basic market.

Note that we have not yet imposed full maximization by individuals at the moment since this can be done only when all candidates for equilibrium are enumerated. By enumerating all corner equilibria, we will find the Pareto optimum corner equilibrium and prove that it is a full equilibrium in the next two sections.

IV. The Pareto Optimum Corner Equilibrium

This section first proves that a corner equilibrium is Pareto efficient for a given basic market. Then, we prove that all non-basic markets are neither equilibria nor Pareto optimal. Finally the relationship between the real return to labor in a corner equilibrium and the preference and technology parameters will be investigated.

If the market is P as shown in Figure 1 (a), i.e., some individuals choose structure (x/y) and other individuals choose structure (y/x), then we can derive the necessary conditions for a restricted Pareto efficient allocation from the problem in Appendix 2. By "restricted", we shall mean that the market is given. From the necessary conditions of that problem, it can be shown that the corner equilibrium found in sections II and III is the restricted Pareto optimum for the market.

Following this procedure, we can show that each corner equilibrium is the restricted Pareto optimum for a given basic market. Moreover, Appendix 2 has proven

Proposition 4

Each corner equilibrium is Pareto efficient for a given basic market and all non-basic markets cannot satisfy the necessary conditions for the Pareto optimum.

Combining this with proposition 2, we conclude that neither equilibrium, nor the Pareto optimum is associated with a non-basic market. This leads us to

Proposition 5

Both candidates for an equilibrium and for the Pareto optimum market are associated with some basic market.

By comparing real returns to labor in different basic markets, we can find the Pareto optimum corner equilibrium; it is the corner equilibrium with the maximum real return to labor. This Pareto optimum corner equilibrium depends upon t, s and the differences among (a,α), (b,β), (c,γ).

Given the number of traded goods, it can be shown that (i) the corner equilibrium including as traded goods those with larger preference parameters is Pareto superior to that including as traded goods those with smaller preference parameters; and (ii) the corner equilibrium including as traded goods those with a high return to specialization is Pareto superior to that including as traded goods those with a small return to specialization. (i) and (ii) leads us to

Proposition 6

The corner equilibrium including as traded goods those with a large preference parameter has a greater real return to labor, and the corner equilibrium including as traded goods those with higher returns to specialization has a greater real return to labor.

A proof to this proposition is in Appendix 3.

Assume that $a = b = c = t$ and $\alpha = \beta = \gamma$, then differences in the real returns to labor in various markets depends only upon the number of traded goods and service. The composition of traded goods has no effect on the real returns to labor. With this assumption, there are 5 possible market configurations:

(1) Autarky. We refer to it as market A. This market consists of structure (x,y,z).

(2) Partial division of labor in production. We refer to it as market P. This market consists of structure (x/y) and (y/x).

(3) Complete division of labor in production. We refer to it as market C. This market consists of structure (x/yz), (y/xz) and (z/xy).

(4) Partial division of labor in production and transaction. We refer to it as market PT. This market consists of structure (x/yT), (y/xT), and (T/xy).

(5) Complete division of labor in production and transaction. We refer to it as market CT. This market consists of structure (x/yzT), (y/xzT), (z/xyT), and

(T/xyz).

Having solved the corner equilibria in these five markets and compared the real returns to labor, we obtain

Proposition 7

(1) For s (the average distance between a pair of neighbors) < 0.54, the real returns to labor in markets P and PT cannot be the maximum.

(i) Market A has the maximum real return to labor if a < 0.41-0.25log(1-1.414s).

(ii) Market C has the maximum real return to labor if 0.41-0.25log(1-1.414s) < a < 2.77-1.09log(1-1.732s).

(iii) Market CT has the maximum real return to labor if 2.77-1.09log(1-1.732s) < a.

(2) For 0.54 < s < 0.58, the real return to labor in market P cannot be the maximum.

(i) Market A has the maximum real return to labor if a < 0.41-0.25log(1-1.414s).

(ii) Market C has the maximum real return to labor if 0.41-0.25log(1-1.414s) < a < 4.20-1.04log(1-1.414s).

(iii) Market PT has the maximum real return to labor if 4.20-1.04log(1-1.414s) < a < 2.77-1.09log(1-1.732s).

(iv) Market CT has the maximum real return to labor if 2.77-1.09log-(1-1.732s) < a.

(3) For 0.58 < s < 0.71, the real returns to labor in markets P and CT cannot be the maximum.

(i) Market A has the maximum real return to labor if a < 0.41-0.25log(1-1.414s).

(ii) Market C has the maximum real return to labor if 0.41-0.25log(1-1.414s) < a < 4.20-1.04log(1-1.414s).

(iii) Market PT has the maximum real return to labor if $4.20-1.04\log(1-1.414s) < a$.

(4) For s > 0.71, market A has the maximum real return to labor for any value of "a".

Note, the Pareto optimum corner equilibrium has the maximum real return to labor. The next section will prove that all non-Pareto optimum corner equilibria are not full equilibria and that the Pareto optimum corner equilibrium is a full equilibrium.

V. The Relationship Between the Equilibrium and the Pareto Optimum

This section will establish

Proposition 8

For an increasing returns economy, the Pareto optimum corner equilibrium is an equilibrium and equilibrium is Pareto optimal. Moreover, the Pareto optimum allocation with equal utilities for all individuals is an equilibrium.

Because all non-basic markets are incompatible with equilibrium and the Pareto optimum, only basic markets are concerned in proving this proposition.

To prove this proposition, it suffices to establish that

(1) if an equilibrium exists, it is an element of a set that consists of the corner equilibria in all basic markets;

(2) the Pareto optimum corner equilibrium exists and is an equilibrium;

(3) all non-Pareto optimum corner equilibria are not equilibria.

On the basis of these three statements, it is trivial to show the final part of proposition 8 because the only difference between the Pareto optimum and equilibrium is that the former may have the utilities different from individual to individual, but for the latter all people have equal utilities.

It is easy to show (1) because the set of all markets includes all possible combinations of corner solutions for individual optimal decisions following the definition of a market. The equilibrium is certainly associated with an element of the market set since the definition of equilibrium requires optimization of individual decisions. This, combined with proposition 5, yields that the equilibrium, if it exists, is an element of a set that consists of the corner equilibria in all basic markets.

To justify (2), we need to prove the existence of a Pareto optimum corner equilibrium and prove that this corner equilibrium satisfies the full maximization of individual utilities. Such full maximization will ensure that this corner equilibrium is a full equilibrium because a corner equilibrium satisfies all conditions for equilibrium except ensuring optimal choice of structure by an individual.

Propositions 3 and 5 can be used to justify the existence of a Pareto optimum corner equilibrium.

The definition of a Pareto optimum corner equilibrium combined with proposition 4 implies that this corner equilibrium ensures the full maximization of individual utilities. Therefore, (2) can be proven. That is, there exists a Pareto optimum corner equilibrium and this corner equilibrium is an equilibrium.

(3) is easy to prove. Since the notion of equilibrium in this paper is extremely neoclassical, equilibrium has to ensure that each person chooses the corner solution that maximizes his utility. Non-Pareto optimal corner equilibria are not equilibria because in each of these equilibria every person's utility is not maximized with respect to his corner solutions. Proposition 8 can thus be established.

This section has developed an approach to analyzing a model with increasing returns to specialization. Such an approach has been used to show that the equilibrium achieves a Pareto optimum. By solving for all corner equilibria, we can analyze under what conditions the equilibrium will shift from one corner equilibrium to another. Therefore, this approach is useful for analyzing the development of market structure and equilibrium. The next section applies this approach to investigate the comparative statics of this model.

VI. Equilibrium and Its Comparative Statics

Propositions 5, 7, and 8 lead us to

Corollary 1

For sufficiently small s, increasing economies of specialization will case the equilibrium to evolve from autarky first to complete division of labor in production (market C), then jump to complete division in production and transaction (market CT). For sufficiently large s, equilibrium is autarky for any degree of increasing returns to specialization. For the values of s in between, increasing economies of specialization will make equilibrium gradually evolve from autarky first to market C, then to market PT, finally to market CT. In this evolution, the goods with relatively large parameters of preference and degree of increasing returns to specialization will be traded before other goods are involved in the market.[4]

Here, "s" is the average distance between a pair of neighbors. Moreover, propositions 4, 5, and 6 lead us to

Corollary 2

For sufficiently large economies of specialization, decreasing "s" will involve more goods in the division of labor and the market.

The two corollaries tell us that the number of traded goods is determined by the absolute degree of increasing returns to specialization and the average distance between a pair of neighbors, while the composition of traded goods is determined by the relative degree of increasing returns to specialization and the relative preference for different goods. Also, the corollaries say that a sufficiently large degree of increasing returns to specialization in production as well as in transaction is a necessary but not sufficient condition for the division of labor, while a sufficiently small s is a necessary but not sufficient condition for the division of labor. Sufficient increases in "a" as well as in "1/s" will produce a "take-off" of the division of labor and productivity.

Moreover, these two corollaries mean that there is some substitution between "a" and 1/s. For example, if the degree of increasing returns to specialization is not large, then urbanization can promote the division of labor and productivity by increasing 1/s. If "a" is large, then we may still have the developed division of labor even if population is dispersed (large "s").

If the degree of increasing returns to specialization in production as well as in transaction is a sufficiently large constant, then urbanization can decrease "s," thereby producing a "take-off" of the equilibrium level of division of labor. This take-off will enhance trade dependence (the ratio of trade volume to income), the extent of market (demand for traded goods), and per capita real income (the real returns to labor, or real productivity). We have worked out the formula for these three variables in different markets. They are increasing when the market evolves to the complete division of labor from autarky as s decreases. Moreover, our model can be used to show that the following variables also change with the evolution.

(1) Self-sufficiency decreases as specialization develops.

(2) Diversification of the economy increases as the number of traded goods and professional sectors increase.

(3) Degree of concentration of production increases as the total output share

of a traded good produced by one producer rises.

(4) Degree of integration of the economy develops as the number of trade partners of each individual increases.

(5) Transaction efficiency increases and the ratio of the value of transaction service (or roundabout production, or intermediate production) to the value of consumer goods increases as the division of labor evolves.

All these phenomena, some of them apparently contradictory, are different versions of the evolution of division of labor. This evolution is caused by the increases in 1/s and a.

In order to explore the difference between the theory of structural change proposed here from the conventional one, two examples are discussed. First, we compare our theory of structural change with the theory of Lewis and Fei and Ranis. According to Lewis [1955] and Fei and Ranis [1964], development is a process in which surplus labor in the agricultural sector is transferred to the industrial sector. Sufficient surplus of agricultural output is a necessary condition for starting this process. In order to compare their models with our theory, let us assume that in our model x is food, y is clothing, and z is house. If "a" is very small and s is very great, then our equilibrium is autarky. All individuals self-provide three goods and live scattered throughout the rural area. We can call this a traditional agricultural sector. In this economy each individual produces all goods he needs. A decrease in "s" generated by urbanization or an increase in "a" generated by a technical innovation will shift our equilibrium to the division of labor, e.g. to market CT. In this market the previous "natural agents" are changed to professional farmers, workers specializing in the production of clothing or housing, and professional traders producing T respectively. In this transition, we see that the professional farmers are less than the previous "natural agents" (producing all goods) and population is concentrated in urban areas. This looks like a shift of labor from the agricultural sector to the industrial sector. But this actually is a process of the evolution of division of labor. Each individual switches to professional activity from self-sufficient activity. In autarky, low productivity is not because of "labor surplus", but because of the low level of division of labor. Hence, "labor surplus" as well as "surplus of agricultural products" are not necessary conditions for this shift of economic structure. From our theory proposed in this paper, the necessary conditions for this transition of economic structure are the sufficiently high degree of increasing returns to specialization (a, b, c, t) and sufficiently small "s". Therefore, the key issue for economic development and transition of economic structure is the initiation and speeding up of the evolution of the division of labor rather than the existence of a labor surplus.

If "s" is interpreted as the transaction cost related to government interference with the market system, e.g., a trade tax and restrictions on market exchange, our model will tell us that a liberalization policy will substantially decrease "s" and raise the equilibrium level of division of labor thereby generating economic development. Indeed, such transaction costs imposed by a government in a less developed country are large and the success of liberalization policies in the development of "four small tigers" (Taiwan, Hong Kong, South Korea, and Singapore) is the best support for this theory.

The second example is the theories of Kuznets [1966] and Chenery [1979]. According to their theory of structural changes, the transition of economic structure is based on an increase in per capita income. According to our theory, however, the increase in per capita income and all other phenomena listed above are different versions of the evolution of the division of labor. It does not make sense to explain one version of this evolution by another. This paper explains the evolution by the improvements of transaction efficiency resulting from changes in the level of urbanization, policy, or institutional arrangements and explains structural changes by this evolution. Yang [1988] explores a mechanism behind the evolution of the division of labor in the context of a dynamic equilibrium model without exogenous changes in "a" and "s".

Conclusions

The major accomplishment in this paper is to distinguish increasing returns to specialization from increasing returns to scale and to develop a multiple-step approach to handling the model with increasing returns to specialization. Also, this paper shows that a competitive market can efficiently integrate economies of specialization (which is endogenous to individuals) into economies of division of labor in the whole society (which is exogenous to individuals). A free market endogenously determines an efficient level of division of labor by balancing a trade-off between increasing returns to specialization and transaction costs. In addition, the comparative statics of our model shed new light on urban economics and the issue of structural change.

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Notes

1. According to conventional microeconomics, the welfare frontier coincides with the PPF.

2. This two-stage set up is just for convenience of exposition. In fact all these decisions are determined simultaneously.

3. Many papers on international trade, e.g. Ethier [1986], show that in models with increasing returns to scale, there are multiple equilibria and some of them are unstable. In our model all corner equilibria except the Pareto optimum one are not full equilibria. This point distinguishes our model from the models with increasing returns to scale.

4. Yang [1988] shows that introducing intermediate goods and the market for labor into the model in this paper, we can justify Coase's theory of the firm; while all results in this paper still hold.

Appendix 1: The Choice of Structures

This appendix proves proposition 1 in section II. First, the first part of this proposition is established. Assume $x^d > 0$; x^d can be solved from the budget constraint. Inserting this expression of x^d and the rearranged production function of x into the utility function yields

(1-1) $$U = \{L_x^a - x^s + (1-K)(1-k)[x^s + p_T(T^s - T^d)/p_x + p_y(y^s - y^d)/p_x + p_z(z^s - z^d)/p_x]\}^\alpha [y + (1-K)(1-k)y^d]^\beta [z + (1-K)(1-k)z^d]^\gamma$$

Differentiating (1-1) with respect to x^s yields

(1-2a) $$\partial U/\partial x^s = -A[1-(1-K)(1-k)] < 0$$

where $0 < (1-K)(1-k) < 1$, and A is a positive magnitude independent of x^s. Canceling y^d or z^d, or T^d by using the budget constraint, it can be shown that

(1-2b) $$\partial U/\partial i^s < 0 \text{ if } i^d > 0, \quad i = x, y, z, T.$$

(1-2) implies that the optimum amount sold of a good is zero if an individual buys this good. In other words, an individual will not buy and sell a good at the same time. Assume $x^d > 0$ (this implies $x^s = 0$ due to (1-2)); then the optimum quantity sold of at least another good has to be positive because of the budget constraint. Without loss of generality, we suppose $y^s > 0$. $\partial U/\partial y^s = 0$ gives the necessary condition for the optimum y^s. Inserting this condition into $\partial U/\partial L_x$ and differentiating the resulting first order derivatives with respect to L_x again yields

(1-3a) $$\partial^2 U/\partial L_x^2 > 0 \text{ if } \partial U/\partial L_x = 0$$

This implies that the optimum value of L_x is either zero or one if $x^d > 0$. $L_x = 1$ conflicts the assumption $y^s > 0$, implied by the assumption $x^d > 0$, and requiring $L_y > 0$. Hence, (1-3a) means

(1-3b) The optimum value of L_x is zero if $x^d > 0$

In other words, an individual will not produce and purchase a good at the same time. This is just the first part of the above proposition.

Next, the second part of the proposition is proven. Without loss of generality, we assume y^s, $z^s > 0$. Because i cannot be negative for i = x, y, z, T and because $i^s = 0$ if $i^d > 0$ due to (1-2), it can be shown that $y^d = z^d = 0$ if y^s, $z^s > 0$. $\partial U/\partial y^s = \partial U/\partial z^s = 0$ gives the necessary conditions for the optimum y^s and z^s. Inserting these conditions into $\partial U/\partial L_y$ and differentiating the resulting first order derivatives with respect to L_y again yields

(1-4a) $\partial^2 U/\partial L_y^2 > 0$ if $\partial U/\partial L_y = 0$

This implies that the optimum value of L_y is either zero or one. $L_y = 0$ conflicts the assumption $y^s > 0$, and $L_y = 1$ conflicts the assumption $z^s > 0$. Hence, (1-4a) means

(1-4b) y^s and z^s cannot be positive at the same time

In other words, an individual will not sell two goods at the same time. This is just the second part of the above proposition.

Appendix 2: Corner Equilibrium and the Restricted Pareto Optimum

If market is (P) in Figure 1 (a), i.e., some individuals choose structure (x/y), signified by subscript 1 and other individuals choose structure (y/x), signified by subscript 2, then we can obtain the necessary conditions for the Pareto efficient allocation from the problem below:

(2-1) Max: $U_1 = x_1^\alpha [T(1-K)y_1^d]^\beta z_1^\gamma$

$x_1, y_1^d, z_1, x_1^s, L_x, T$

$y_2, x_2^d, z_2, y_2^s, L_y, L_z$

s.t. $x_1 + x_1^s = L_x^a \qquad z_1 = (1-L_T-L_x)^c$

$y_2 + y_2^s = L_y^b \qquad z_2 = (1-L_T-L_y)^c \qquad T = L_T{}^t$

$M_1\, x_1^s = M_2\, x_2^d \quad$ or $rx_1^s = x_2^d$

$M_2\, y_2^s = M_1\, y_1^d \quad$ or $y_2^s = ry_1^d$

$U_2 = y_2^\beta [T(1-K)x_2^d]^\alpha z_2^\gamma \geq u$

where u is a constant. The first order conditions are

(2-2) $MRS_{yx}^1 = MRS_{yx}^2\, T^2$

(2-3) $MRS_{zx}^1 = MRT_{zx}^1 \qquad MRS_{zy}^2 = MRT_{zy}^2 \qquad MRS_{zx}^2 = MRT_{zx}^2$

$MRS_{zy}^1 = MRT_{zy}^1 \qquad MRS_{yx}^1 = MRT_{yx}^1 \qquad MRS_{yx}^2 = MRT_{yx}^2$

(2-4) $MRT_{yx}^{1} = MRT_{yx}^{2} T^2$

where $MRS_{ij}^{k} = (\partial U_k/\partial x_j)/(\partial U_k/\partial x_i)$ is the marginal rate of substitution between good i and j for individuals choosing structure k; $MRT_{ij}^{k} = (dx_i/dL)/(dx_j/dL)$ is the marginal rate of transformation between good i and j for individuals choosing structure k.

These are just the necessary conditions for corner equilibrium. From the corner equilibrium for market A, solved in subsection I.B and I.C, we can derive that

$$MRS_{yx}^{1}/T = p_y/p_x = MRS_{yx}^{2} T \quad \text{or} \quad MRS_{yx}^{1} = MRS_{yx}^{2} T^2$$

This is just (2-2). From this equilibrium we can obtain (2-3) and (2-4) too.

Actually, the relative number of individuals, r, is also a decision variable in the problem of Pareto efficient allocation. However, from the problem (2-1), we can solve for the unique x_1^s, x_2^d, y_2^s, and y_1^d; they determine a unique r through the market clearing condition since the number of traded goods equals the number of structures in the basic market.

For a non-basic market, e.g., that consists of structures (z/x y), (x/y z), (z/x y), and (x/y), we have a similar problem of Pareto optimal allocation as (2-1) which maximizes U(z/x y) subject to all individual production functions, transaction technologies, and balance between consumption and production giving that other utilities are not smaller than some constants. However, the relative numbers of individuals in structures (x/y z), and (x/y) are flexible. Name these two numbers as M_1 and M_2 respectively. For the relevant Lagrange function associated with this restricted optimization problem, LA, we can show

(2-5a) $\partial LA/\partial M_1|_{M_1=0} \leq 0$ if $A\partial U(z/xy)/\partial x + B\partial U(z/xy)/\partial y \leq 0$

(2-5b) $\partial LA/\partial M_2|_{M_2=0} \leq 0$ if $A\partial U(z/xy)/\partial x + B\partial U(z/xy)/\partial y \geq 0$

where we use the facts that $M = \Sigma M_i$, M is the total number of individuals, and M_i is the number of individuals in structure i. LA is the Lagrange function.

(2-5) implies that for any value of $A\partial U(z/xy)/\partial x + B\partial U(z/xy)/\partial y$, the Pareto optimum requires either $M_1 = 0$ or $M_2 = 0$. In other words, this non-basic market cannot satisfy the necessary condition for the Pareto optimum.

Applying the above procedure to each market, we can show proposition 4 in section IV.

Appendix 3 A Proof to Proposition 6

Assume, for example, that the market is P, as shown in Figure 1 (a); it is not difficult to show (i) the corner equilibrium including as traded goods those with larger preference parameters is Pareto superior to that including as traded goods those with smaller preference parameters; and (ii) the corner equilibrium including as traded goods those with great return to specialization is Pareto superior to that including as traded goods those with small return to specialization.

(i) Assuming $\alpha = \beta$ and a = b = c = t, we have utilities for structure (x/y) and (y/x):

(3-1a) $\log U(x/y) = \alpha(2\log x + \log T - \log r) + \gamma \log z + \alpha \log(1-K)$

(3-1b) $\log U(y/x) = \alpha(2\log x + \log T + \log r) + \gamma \log z + \alpha \log(1-K)$

where $x = [2\alpha/(3\alpha+\gamma)]^a/2$, $z = [\gamma/(3\alpha+\gamma)]^a$ and $T = [\alpha/(3\alpha+\gamma)]^a$. The condition of utility equalization becomes

(3-2) $E = \log U(x/y) - \log U(y/x) = 2\alpha \log r = 0.$

(i) will be established, if it can be shown that

(3-3) $d\log U^*/d\alpha|_{\alpha=\gamma} = \partial \log U^*/d\alpha|_{\alpha=\gamma} + (\partial \log U^*/\partial r^*)(dr^*/d\alpha)|_{\alpha=\gamma} > 0.$

where U* is real income and $r^* = M^*_x/M^*_y$ is relative number of the individuals choosing different structures in the corner equilibrium. From (3-2) it can be derived that

(3-4a) $dr^*/d\alpha|_{\alpha=\gamma} = -(\partial E/\partial a)/(\partial E/\partial r^*)|_{\alpha=\gamma} = 0$

where $\partial E/\partial a|_{\alpha=\gamma} = 2\log r = 0$ because $r = 1$ if $\alpha = \gamma$, and

(3-4b) $\partial \log U^*/\partial \alpha|_{\alpha=\gamma} > 0.$

(3-4) ensures that (3-3) holds. (3-3) implies that if individuals prefer x and y to z, the corner equilibrium with x and y as traded goods will have a greater real return to labor than that with x and z or with y and z as traded goods.

(ii) Assuming $\alpha = \beta = \gamma$ and a = b = t, (3-1) holds if $x = [2a/(3a+c)]^a/2$, $z = [c/(3a+c)]^c$, and $T = [2a/(3a+c)]^a$. Also, there is the condition of utility equalization (3-2).

(ii) will be established, if it can be shown that

(3-5) $\quad d\log U^*/da|_{a=c} = \partial \log U^*/\partial a|_{a=c} + (\partial \log U^*/\partial r^*)(dr^*/da)|_{a=c} > 0.$

From (3-2) with $x = [2a/(3a+c)]^a/2$, $z = [c/(3a+c)]^c$, and $T = [2aL/(3a+c)]^a$, it can be derived that

(3-6) $\quad \partial \log U^*/\partial a|_{a=c} > 0,$ and $dr^*/da|_{a=c} = -(\partial E/\partial a)/(\partial E/\partial r^*)|_{a=c} = 0$

where $\partial E/\partial a|_{a=c} = 0$.

(3-6) ensures (3-5) to hold. (3-5) implies that if the returns to specialization in producing x and y are greater than that in producing z, then the corner equilibrium with x and y as traded goods will have a greater real return to labor than that with x and z or with y and z as traded goods.

Proposition 6 can be established by using (i) and (ii).