

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
27 Hillhouse Avenue
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 655

CAPITAL MOBILITY IN NEOCLASSICAL MODELS OF GROWTH

Robert J. Barro
Harvard University

N. Gregory Mankiw
Harvard University

Xavier Sala-i-Martin
Yale University

March 1992

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments.

Abstract

The main purpose of this paper is to construct a model of economic growth that is consistent with the growing body of evidence on convergence. We want, in particular, to explain gradual convergence in output and income per person while allowing for an international credit market that equates the real interest rates across economies. The key to our model is that capital is only partially mobile: borrowing is possible to finance accumulation of physical capital but not accumulation of human capital. We show that the assumption of partial capital mobility, imbedded in an open-economy version of the neoclassical growth model, can explain the evidence on convergence.

Perhaps the model in this paper is best applied not to countries or even states but to families. The model may, in this context, be useful for explaining the dynamics and distribution of wealth. It would predict that the most patient families would tend to be the most highly educated and they would own most of the economy's physical capital. Physical non-human wealth would be more highly concentrated than human wealth.

Several recent studies have looked for evidence on convergence, defined here as the tendency for poor economies to grow faster than rich economies. The clearest empirical support for convergence comes from economies that, except for initial conditions, appear similar. Dowrick and Nguyen (1989) reported convergence for OECD countries. Barro and Sala-i-Martin (1991b, 1991c) found that convergence occurred for the U.S. states and the regions of Europe and Japan at a rate of about 2% per year. Moreover, for the U.S. states, state product per capita and state personal income per capita converged at roughly the same rate.

The evidence from larger samples of countries is more controversial. Romer (1987) and Rebelo (1991) emphasized the lack of correlation between initial per capita GDP and the subsequent per capita growth rate for a broad sample of about 100 countries. They interpreted this finding as evidence against the convergence implications of the neoclassical growth model. Yet an alternative interpretation is that these economies, unlike the OECD countries and U.S. states, have substantially different steady states. These differences can reflect disparities in levels of technology, government policies that amount to levels of technology, and preferences that affect the saving rate and fertility. Barro (1991), Levine and Renelt (1990), and Mankiw, Romer, and Weil (1992) reported that, after controlling for differences in rates of accumulation in human and physical capital and some other variables, countries converge at a rate of about 2% per year. Hence, if one allows for heterogeneity, all the data sets confirm that convergence is slow but significant.

Standard theories of economic growth cannot easily explain this finding of convergence. Consider, for example, the neoclassical growth model with diminishing returns to capital. As Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil

(1992) pointed out, this model can explain the observed rate of conditional convergence if one assumes that these economies are closed and that the capital share is about 0.8. A capital share this large is reasonable if capital is viewed broadly to include human and physical components. (A more conventional capital share of around 0.3 implies much faster convergence than that observed in the data.) Yet the assumption of a closed economy is more difficult to justify, especially when applied to economies like the U.S. states. Interest rates are about the same in each state, and substantial borrowing and lending seems to flow across state borders. If one assumes, however, that these economies are open, then the neoclassical growth model implies that capital will move quickly to equalize the marginal product of capital, implying instantaneous convergence in output per worker.

Similarly, the usual endogenous growth models for closed economies cannot fully account for the evidence. The one-sector, AK, model without diminishing returns can explain differences in output per person without differences in real interest rates. But this type of model is inconsistent with convergence if each economy has a fixed technology parameter, A . Two-sector endogenous growth models can explain convergence based on imbalances between physical and human capital (see Mulligan and Sala-i-Martin [1991]). But real interest rates would differ across closed economies and the imbalances would vanish instantaneously across open economies.

The main purpose of this paper is to construct a model of economic growth that is consistent with the growing body of evidence on convergence. We want, in particular, to explain gradual convergence in output and income per person while allowing for an international credit market that equates the real interest rates across economies. The key to our model is that capital is only partially mobile: borrowing is possible to finance accumulation of physical capital but not accumulation of human capital. We show that

this assumption of partial capital mobility, imbedded in an open-economy version of the neoclassical growth model, can explain the evidence on convergence.

The paper is organized as follows. Section I presents the elements of the model. Section II considers a closed economy, and Section III deals with an open economy with perfect capital mobility. These two well-known polar cases provide useful benchmarks for comparison. Section IV then considers partial capital mobility, in which borrowing is possible to finance physical capital, but not human capital. Section V discusses the quantitative relation between the theory and the empirical evidence, and Section VI concludes.

I. The Model

Output is produced with three inputs: physical and human capital and a nonreproducible factor, which we view as raw labor. We assume that the production function is Cobb-Douglas:

$$Y = AK^{\alpha}H^{\beta}(Le^{gt})^{1-\alpha-\beta}$$

where $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$, Y is output, K the stock of physical capital, H the stock of human capital, L the quantity of raw labor, and A a fixed technology parameter. Raw labor grows at the constant, exogenous rate n , and g is the constant, exogenous rate of labor-augmenting technological progress. We assume a one-sector production technology in which physical capital, human capital, and consumables are perfectly substitutable uses of output. If we work as is customary in units of effective labor— $y \equiv Y/Le^{gt}$, $k \equiv K/Le^{gt}$, $h \equiv H/Le^{gt}$ —then the production function is given in

intensive form by

$$(1) \quad y = Ak^{\alpha}h^{\beta}$$

Equation (1) implies that the elasticity of substitution between k and h is one.

We begin with this assumption to keep the model as simple as possible. This assumption is not innocuous, however, and we examine later how the analysis differs when we allow for different elasticities of substitution between the two capital stocks.

The households own the three inputs and rent them to the firms at competitive rental prices. Firms pay a proportional tax at rate τ on output. We interpret this tax to include various elements that affect the incentives to accumulate capital; for example, τ includes the risk of expropriation by the government, strong labor unions, or foreign invaders. The after-tax cash flow for the representative firm is given in units of effective labor by

$$(2) \quad \pi = (1-\tau)Ak^{\alpha}h^{\beta} - w - R_k k - R_h h$$

where w is the wage rate, R_k is the rental price of physical capital, and R_h is the rental price of human capital. In the absence of adjustment costs, the maximization of the present value of future cash flows is equivalent to the maximization of profit in each period. The firm therefore equates the marginal products to the rental prices:

$$(3a) \quad R_k = (1-\tau)\alpha Ak^{\alpha-1}h^{\beta} = (1-\tau)\alpha y/k$$

$$(3b) \quad R_h = (1-\tau)\beta Ak^{\alpha}h^{\beta-1} = (1-\tau)\beta y/h$$

$$(3c) \quad w = (1-\tau)Ak^{\alpha}h^{\beta} - R_k k - R_h h$$

Households are represented by the standard, infinitely-lived, Ramsey consumer with preferences

$$(4) \quad U = \int_0^{\infty} \left(\frac{C^{1-\theta}}{1-\theta} - 1 \right) e^{-(\rho-n)t} dt$$

where C is per capita consumption, ρ the subjective rate of time preference, and θ the inverse of the intertemporal elasticity of substitution. Population, e^{nt} , corresponds to the labor force.

The households own the physical and human capital and also have the net stock of debt, d , per unit of effective labor. They receive income from wages and rentals and spend this income on accumulation of physical capital, accumulation of human capital, and consumption. Hence, the budget constraint is

$$(5) \quad \dot{h} + \dot{k} - \dot{d} = w + (R_k - n - g - \delta)k - (r - n - g)d + (R_h - n - g - \delta)h - c$$

where r is the real interest rate, $c \equiv C e^{-gt}$, and a dot over a variable represents its time derivative. This equation assumes that the relative prices of consumables, physical capital, and human capital are always fixed at unity, and that physical and human capital depreciate at the same rate, δ . We also assume that none of the taxes collected are remitted to households, although our results would not change if these revenues showed up as lump-sum transfers or as government services that did not interact with the choices of consumption.

Households can borrow and lend at the real interest rate r on the domestic bond market. In a closed economy, the debt, d , is zero for the representative household and r is determined by the equilibrium of saving and investment at the national level. In an

open economy, r is determined at the world level and d —the foreign debt per effective worker—can be positive or negative.

To simplify the exposition, we integrate the households and firms by substituting the first-order conditions from equation (3) into the budget constraint in equation (5) to get

$$(6) \quad \dot{h} + \dot{k} - \dot{d} = (1-\tau)Ak^{\alpha}h^{\beta} - (\delta+n+g)(k+h) - (r-n-g)d - c$$

Households maximize utility from equation (4) subject to equation (6), given $k(0) > 0$, $h(0) > 0$, and $d(0)$. In the following sections, we consider this maximization problem under environments that involve different degrees of capital mobility. These differences entail various restrictions on the path of debt, d .

II. The Closed Economy

A. Laws of Motion

The first environment is the closed economy, in which households can borrow and lend on the domestic capital market at the rate r , but cannot borrow or lend on international markets. Hence, $d=0$, and the only difference from the standard Ramsey model is that the technology involves two kinds of capital. The rate of return, r , must equal the net returns on the two kinds of capital, $R_k - \delta$ and $R_h - \delta$, where the rental prices are given in equations (3a) and (3b). These equations imply $k/h = \alpha/\beta$ at all points in time. If the initial value of k/h differs from α/β , then households "jump" to the desired ratio. Since we assume no adjustment costs or irreversibility constraints, this jump is feasible, that is, households can convert excess human capital into physical

capital or vice versa.¹ We can then rewrite the budget constraint from equation (6) in terms of a broad capital stock, $z \equiv k+h$:

$$(7) \quad \dot{z} = (1-\tau)\tilde{A}z^{\alpha+\beta} - (\delta+n+g)z - c$$

where $\tilde{A} = A\alpha^{\alpha}\beta^{\beta}(\alpha+\beta)^{-(\alpha+\beta)}$.

The household's problem now corresponds to the standard formulation in the neoclassical growth model (Ramsey [1928], Cass [1965], and Koopmans [1965]), except that the production function is less concave: the capital share is $\alpha+\beta$, which corresponds to physical and human capital, rather than α , which corresponds only to physical capital. Diminishing returns therefore set in more slowly.

The Euler equation characterizing the solution is familiar:

$$(8) \quad \dot{c}/c = (1/\theta) \cdot [(1-\tau)\tilde{A}(\alpha+\beta)z^{\alpha+\beta-1} - (\delta+\rho+\theta g)]$$

where $(1-\tau)\tilde{A}(\alpha+\beta)z^{\alpha+\beta-1}$ is the after-tax marginal product of capital, which also equals $r+\delta$. Equations (7) and (8) and the transversality condition fully describe the transition of the economy toward the steady state.

B. The Steady State and the Feldstein–Horioka Puzzle.

The steady-state growth rate of the variables in units of effective labor is zero. The per capita variables grow accordingly at the rate of productivity growth, g , and the level variables grow at the rate of growth of population plus productivity, $n+g$. The

¹If physical or human capital are irreversible (as is realistic in most situations), then the model involves transitional dynamics of the sort described for two-capital-goods models in Mulligan and Sala-i-Martin (1991).

steady-state stock of broad capital in units of effective labor is given by

$$(9) \quad z^* = [(1-\tau)\tilde{A}(\alpha+\beta)/(\delta+\rho+\theta g)]^{1/(1-\alpha-\beta)}$$

Hence, z^* is a decreasing function of ρ , θ , δ , and τ , and an increasing function of the level of technology, $\tilde{A}$. The breakdown of broad capital into its two components is given in the steady state by

$$(10) \quad k^* = z^* \alpha / (\alpha + \beta) \quad \text{and} \quad h^* = z^* \beta / (\alpha + \beta)$$

This analysis for closed economies provides an interpretation of the Feldstein-Horioka (1980) puzzle about the strong correlation between saving and investment rates across countries. Suppose that all countries have the same technology and preferences and differ only in the level of the tax rate, τ , which should be interpreted broadly to include various disincentives to invest. The steady-state gross investment rate—for physical and human capital—equals the full depreciation rate, $\delta+n+g$, times the ratio of total capital to output, $(z/y)^*$. Equation (8) implies that $(z/y)^*$ equals $(1-\tau)(\alpha+\beta)/(\delta+\rho+\theta g)$. The steady-state investment and saving rates are therefore

$$(11) \quad (i/y)^* = (1-\tau)(\delta+n+g)(\alpha+\beta)/(\delta+\rho+\theta g)$$

Hence, closed economies with high tax rates—that is, high disincentives for investment—have low steady-state investment and saving rates. The same result holds in the standard formulation that treats expenditures on human capital as consumption. The term, $\alpha+\beta$, in equation (11) is then replaced by α , and investment corresponds only to the expenditures on physical capital.

Now consider the incipient capital movements. The steady-state real interest rate is $\rho + \theta g$ and is therefore independent of the tax rate. If economies differ only in their tax rates, then the steady-state real interest rates are the same for all countries. Economies with low steady-state capital intensities have high marginal products of capital, but they also have high tax rates. The exact offset of these two effects implies that the after-tax marginal products of capital and therefore, the real interest rates are independent of the capital intensities. If we opened up international capital markets, then capital would not flow across countries because the after-tax returns are already equalized. Thus, investment and saving rates would be perfectly correlated across countries even with full capital mobility.

Note that this potential resolution of the Feldstein–Horioka puzzle does not rely on the existence of two capital goods, but applies also to the standard Ramsey model and to a variety of other growth models. Heterogeneity in the incentives to accumulate capital—measured here by a broad concept of the tax rate τ —can explain the positive correlation between investment and saving rates without invoking imperfect international capital markets.

C. Convergence during the Transition

The transitional dynamics of the model can be studied graphically using the familiar phase diagram in Figure 1. The $\dot{c}=0$ equation generates the modified golden rule, and the $\dot{z}=0$ condition defines a hump-shaped curve that reaches its maximum at the golden-rule level of broad capital. The model displays saddle-path stability, and the transversality condition ensures that the economy follows the saddle path during the transition.

To quantify the convergence implications of the model, we follow Barro and Sala-i-Martin (1992) and log-linearize the system, equations (7) and (8), around the steady state. The growth rate of y between times 0 and t can then be written as

$$(12) \quad (1/t) \cdot \{\log[y(t)/y(0)]\} = \left(\frac{1-e^{-\lambda t}}{t}\right) \cdot \{\log[y^*/y(0)]\}$$

That is, the growth rate is a negative function of initial income, $y(0)$, after controlling for the steady-state level of income, y^* . The convergence rate, λ , is a complicated function of the parameters of the model:²

$$(13) \quad 2\lambda = \{\varphi^2 + 4\left(\frac{1-\alpha-\beta}{\theta}\right)(\delta+\rho+\theta g)\left[\frac{\delta+\rho+\theta g}{\alpha+\beta} - (\delta+n+g)\right]\}^{1/2} - \varphi$$

where $\varphi = \rho - n - (1-\theta)g > 0$.

Equation (13) implies that the convergence rate, λ , depends inversely on the share of broad capital, $\alpha+\beta$. If $\alpha+\beta=1$ (that is, if all inputs can be accumulated), then $\lambda=0$. This outcome corresponds to the one-sector linear endogenous growth model of Rebelo (1991). Another important property is that λ is independent of the level of technology, $\bar{A}$, and the tax rate, τ .

It is instructive as a numerical example to plug in some figures that seem approximately valid for the U.S. economy: $n=.01$ per year, $g=.02$ per year, $\delta=.05$ per year, $\theta=2$, $\rho=.02$ per year, $\alpha=.3$, and $\beta=.5$. The rate of convergence implied by this parameterization is $\lambda=.014$ per year. This value is at the low end of the range of estimates of convergence coefficients from the empirical literature mentioned in the introduction.

²This formula corresponds to the one derived in Barro and Sala-i-Martin (1992), except that the capital share is now $\alpha+\beta$ rather than α .

The ratio of capital stocks, k/h , remains constant during the transition, but the ratio of each stock to GDP, k/y and h/y , rises as the economy approaches the steady state. The steady increase in k/y conflicts with the view associated with Kaldor (1961) that the capital-output ratio is virtually constant during the process of development. If $\alpha + \beta = .8$, however, then diminishing returns set in slowly and the model predicts only moderate changes in k/y : if k and h double, then k/y rises by 15%. Since k/y is not precisely constant empirically, the prediction of a slowly rising k/y cannot be considered a serious shortcoming of the theory.

The theory also implies that the marginal products of physical and human capital, and hence, the real interest rate, r , would decline over time. If $\alpha + \beta = .8$ and r begins at 8%, for example, then a doubling of k and h implies that r would fall to 6.9%. This slow decline of the real interest rate—a reflection of the slow onset of diminishing returns—does not conflict with empirical evidence.

The expression for λ in equation (13) simplifies if, following Solow (1956), we assume a constant gross saving rate. This assumption amounts to a restriction on the parameters of the model (see Kurz [1968]). The gross saving rate is constant if

$$(14) \quad \theta = (\delta + \rho) / [(\alpha + \beta)(\delta + n) - g(1 - \alpha - \beta)]$$

and the corresponding gross saving rate is then

$$(15) \quad s = (1 - \tau) / \theta$$

In this case, the accumulation constraint in equation (7) can be written as

$$(16) \quad \dot{z}/z = s(1 - \tau)\tilde{A}z^{\alpha + \beta - 1} - (\delta + n + g)$$

and the steady-state capital intensity is

$$z^* = \left[\frac{s\bar{A}(1-\tau)}{\delta+n+g} \right]^{1/(1-\alpha-\beta)}$$

Equation (12) still provides a log-linear approximation to the growth rate, but the convergence coefficient simplifies to

$$(17) \quad \lambda = (1-\alpha-\beta)(\delta+n+g)$$

which corresponds to the expression given in Mankiw, Romer, and Weil (1992). As before, λ is a decreasing function of the capital share, $\alpha+\beta$, and equals zero when $\alpha+\beta=1$. Also, λ is again independent of the level of technology, $\bar{A}$, and the tax rate, τ .

If the parameters, $(\alpha, \beta, \delta, \rho, n, g)$, take on the values assumed before and if $\tau=.3$, then equations (14) and (15) imply that the gross saving rate is constant at the value $s=.44$ if $\theta=1.59$. Note that this high gross saving rate includes saving in human capital.

If we treat the expenditures on human capital as consumption, then the saving rate that corresponds to physical capital is given by $s \cdot \alpha/(\alpha+\beta)$, which equals .17 for the parameter values assumed before. Finally, we have to consider that only a fraction of the expenditure on human capital appears in measured GDP: a significant portion corresponds to foregone wages. If we accept Kendrick's (1976, Tables A-1 and B-2) estimate that about one-half of spending on human capital appears in GDP, then the predicted saving rate that corresponds to physical capital becomes .23. This figure corresponds well to observed ratios of physical investment to GDP. For the United

States, for example, the ratio of real gross domestic investment to real GDP averaged .21 from 1960 to 1990.³

Equation (17) implies that $\lambda = .016$ if the gross saving rate is constant. This value accords with the range of estimates for convergence coefficients found in the empirical studies cited in the introduction.

The major deficiency of this model is the assumption of a closed economy, an assumption that is especially unappealing for the U.S. states and the regions of European countries. Therefore, we now extend the analysis to allow for capital mobility across economies.

III. The Open Economy with Perfect Capital Mobility

Suppose now that households can borrow and lend at the going interest rate on world capital markets. We assume that the country is small relative to the rest of the world and faces a constant world real interest rate, r^w , which pegs the domestic rate, r .⁴ The rate r^w would be constant if the world were in the kind of steady state that we described above for a closed economy. Goods are tradable internationally, but labor cannot migrate.

The interest rate, r , again equals the net returns on the two kinds of capital, $R_k - \delta$ and $R_h - \delta$, where the rental prices are given in equations (3a) and (3b). But since $r = r^w$,

³The ratio was computed from data in *Citibase*, defining gross investment to include public investment for non-military and military purposes. The average ratio for private investment only is .17. We may also want to add consumer-durables purchases, which averaged 8% of GDP from 1960 to 1990. A further small adjustment would modify GDP to include the service flows from government capital (to the extent that these flows were not already reflected in private output) and consumer durables.

⁴As in Barro and Sala-i-Martin (1991b, chapter 2); we assume $r^w > g + n$, a condition that ensures that the present value of future wages is bounded. Strictly speaking, the small-country assumption also requires $r^w \leq \rho + \theta g$; otherwise, the country's assets and consumption will grow faster than the world's, and the country will eventually not be small.

a constant, the implied values of k and h —and hence, y —are also constant. In other words, the model predicts that a small open economy will jump instantaneously to its steady-state levels of output, physical capital, and human capital per effective worker and will remain there forever. The predicted rates of convergence for output and capital are infinite, a result that conflicts sharply with the empirical evidence discussed earlier.

We could eliminate the infinite speeds of convergence by introducing adjustment costs and irreversibility conditions for physical and human capital. Plausible modifications along these lines do not, however, eliminate the counterfactual prediction that convergence rates would be rapid in an open economy with perfect capital mobility.⁵

We therefore now turn to a model that allows for imperfect capital mobility.

IV. The Open Economy with Partial Capital Mobility

A. Laws of Motion.

We now assume that the amount of debt, d , cannot exceed the quantity of physical capital, k . This assumption introduces an asymmetry between the two capital stocks: k can be used as collateral for international borrowing whereas h cannot.⁶ We are assuming implicitly that domestic residents own the physical capital stock but may obtain part or all of the financing for this stock by issuing bonds to foreigners. The results would be the same if we allowed for direct foreign investment, in which case the

⁵The model has other unappealing implications. As first conjectured by Ramsey (1928), if countries differ in their discount rates, ρ , then the most patient country asymptotically owns all the assets in the world, including all claims on human capital and raw labor. For the rest of the countries, c approaches zero, and the debt eventually mortgages all domestic capital and raw labor. The model also predicts, counterfactually, that GDP would typically behave very differently from GNP.

⁶Cohen and Sachs (1984) and Barro and Sala-i-Martin (1991b, chapter 2) examine a model with one capital good in which the borrowing constraint amounts to $d \leq \nu k$, where $0 \leq \nu \leq 1$. In other words, only the fraction ν of capital serves as collateral. The model considered in the text differs in that k and h are imperfect substitutes as inputs to production, and the choices between k and h determine the fraction of broad capital, $k+h$, that constitutes collateral.

foreigners would own part of the physical capital stock rather than bonds. The important assumption is that domestic residents cannot borrow with human capital or raw labor as collateral and that foreigners cannot own domestic human capital or raw labor. We are, in particular, ruling out any international migration of labor.

There are various ways to motivate the borrowing constraint. Physical capital is more easily repossessed than human capital and is therefore more readily financed with debt. Physical capital is also more amenable to direct foreign investment: a person can own a factory but not someone else's stream of labor income. Finally, one can abandon the terms, "physical capital" and "human capital," and recognize that not all investments can be financed through perfect capital markets. The key distinction between k and h is not the physical nature of the capital but whether the cumulated goods serve as collateral for borrowing on world markets.

We still assume that the world interest rate, r^w , is constant at its steady-state value. We assume also that $r^w = \rho + \theta g$, the steady-state interest rate that would apply if the domestic economy were closed. That is, the home economy is neither more nor less impatient than the world as a whole. The initial quantity of assets per effective worker is $k(0) + h(0) - d(0)$, and the key consideration is whether this quantity is greater or less than the steady-state amount of human capital, h^* . If $k(0) + h(0) - d(0) \geq h^*$, then the borrowing constraint is not binding and the economy jumps to the steady-state values of k , h , and y . In contrast, if $k(0) + h(0) - d(0) < h^*$, then the constraint is binding—that is, $d=k$ applies—and we obtain some new results. We therefore focus on this situation.⁷

⁷If $r^w < \rho + \theta g$, then the domestic economy must eventually become constrained on the world credit market. Hence, our analysis of a debt-constrained economy applies at some time in the future even if not at the initial date. If $r^w > \rho + \theta g$, then the assumption of a small economy is violated eventually, and r^w would have to change (see n. 4).

Since physical capital serves as collateral, the net return, $R_k - \delta$, on this capital still equals the world interest rate, r^ω , at all points in time. The formula for R_k from equation (3a) therefore implies

$$(18) \quad k = (1-\tau)\alpha y / (r^\omega + \delta)$$

Equation (18) ensures that the ratio of physical capital to GDP, k/y , will be constant throughout the transition to the steady state. In contrast, k/y rose steadily during the transition for the closed economy that we considered earlier. (The ratio of human capital to GDP, h/y , rose over time for the closed economy and will turn out still to rise for the open economy.) We mentioned before that the rough constancy over time of k/y is one of Kaldor's (1961) stylized facts about economic development. (Maddison (1980) provides some confirmation of this regularity.) The consistency of the credit-constrained open-economy model with this "fact" is therefore notable.⁸

The result for k from equation (18) can be combined with the production function, $y = Ak^\alpha h^\beta$, to express y as a function of h :

$$(19) \quad y = Bh^\epsilon$$

where $B \equiv A^{1/(1-\alpha)}[(1-\tau)\alpha/(r^\omega + \delta)]^{\alpha/(1-\alpha)}$ and $\epsilon \equiv \beta/(1-\alpha)$ are constants. The condition $0 < \alpha + \beta < 1$ implies $0 < \epsilon < \alpha + \beta < 1$. Thus, the reduced-form production

⁸The precise constancy of k/y in our model depends on the fixity of the world interest rate, r^ω , and on the assumption that the production function is Cobb-Douglas. This production function implies that the average product of capital, y/k , is proportional to the marginal product. Since the marginal product of capital, net of depreciation and taxes, equals the fixed world interest rate, r^ω , the average product, y/k , must be constant. In the analysis considered later, which departs from a Cobb-Douglas technology, the ratio y/k is not precisely constant along the transition path.

function in equation (19) expresses y as a function of h with positive and diminishing marginal product. The convergence implications of this model are therefore similar to those of the closed economy—both models involve the accumulation of a capital stock under conditions of diminishing returns.

The budget constraint from equation (6) can be combined with the reduced-form production function from equation (19), the borrowing constraint, $d=k$, and the condition, $(r^w + \delta)k = (1-\tau)\alpha y$ from equation (18), to get the revised budget constraint:

$$(20) \quad \dot{h} = (1-\alpha)(1-\tau)Bh^\epsilon - (\delta+n+g)h - c$$

Note that the term, $-\alpha(1-\tau)Bh^\epsilon = -\alpha(1-\tau)y$, corresponds to the flow of rental payments on physical capital, $(r^w + \delta)k$ (see equation [18]). Since $d=k$, this term equals the net factor payments from abroad and therefore equals the difference between GNP and GDP. GDP exceeds GNP because the country is constrained on the international credit market and therefore has the positive foreign debt, $d=k$.

Households now maximize utility from equation (4) subject to the budget constraint in equation (20) and a given initial stock of human capital, $h(0) > 0$. (The initial value $h(0)$ equals the given amount of initial assets, which was assumed to be less than h^* .) The Euler equation is

$$(21) \quad \dot{c}/c = (1/\theta)[(1-\tau)(1-\alpha)B\epsilon h^{\epsilon-1} - (\delta+\rho+\theta g)]$$

where $(1-\tau)(1-\alpha)B\epsilon h^{\epsilon-1} = (1-\tau)B\beta h^{\epsilon-1}$ is the after-tax marginal product of human capital. Equations (20) and (21) and the transversality condition fully describe the transitional dynamics of this model.

Because we assumed $r^\omega = \rho + \theta g$, the steady state is the same as that for the closed economy. In particular, $h^* = z^* \beta / (\alpha + \beta)$, as in equation (10), where z^* is the steady-state quantity of broad capital for the closed economy, given in equation (9). Hence, the opportunity to borrow on the world credit market does not influence the steady state, but will turn out to affect the speed of convergence.⁹

B. Convergence Along the Transition.

The system described by equations (20) and (21) and the transversality condition has the usual saddle-path characteristics. Compare the debt-constrained open economy with the closed economy: equation (20) corresponds to equation (7) and equation (21) to equation (8). The only differences are that equation (20) contains $(1-\alpha)B$ as a proportional constant in the production function, whereas equation (7) has $\tilde{A}$; the capital-stock variable is h rather than $z \equiv h+k$; and the exponent on the capital stock is $\epsilon \equiv \beta/(1-\alpha)$ rather than $\alpha + \beta$. Since ϵ and $\alpha + \beta$ are positive and less than one—that is, both models feature diminishing returns—the dynamics of the models are essentially the same.

Equation (13) determines the convergence coefficient, λ , for the closed economy. The only difference in the credit-constrained open economy is that $\alpha + \beta$ has to be replaced by $\epsilon \equiv \beta/(1-\alpha)$. (Recall that the level of the production technology does not influence the rate of convergence.) Hence, the convergence coefficient for the log-linearized version of the credit-constrained open economy is given by

$$(22) \quad 2\lambda = \left\{ \varphi^2 + 4 \left(\frac{1-\epsilon}{\theta} \right) (\delta + \rho + \theta g) \left[\frac{\delta + \rho + \theta g}{\epsilon} - (\delta + n + g) \right] \right\}^{1/2} - \varphi$$

⁹If we had assumed $r^\omega < \rho + \theta g$ —so that the home economy is more impatient than the rest of the world (see n. 7)—then the availability of foreign borrowing would also affect the steady-state position. The open economy would have higher steady-state capital intensities, h^* and k^* , than the closed economy.

where φ is again equal to $\rho - n - (1 - \theta)g > 0$. The coefficient determined from equation (22) is the same value that would arise in a closed economy that had the broad capital share ϵ , rather than $\alpha + \beta$. Since $\epsilon = \beta / (1 - \alpha)$, it follows that $\epsilon < \alpha + \beta$ (using the condition, $\alpha + \beta < 1$). *The credit-constrained open economy therefore works like a closed economy with a broad capital share that is less than $\alpha + \beta$.* Recall that the rate of convergence depends inversely on the capital share (because a smaller capital share means that diminishing returns set in more rapidly). The credit-constrained open economy therefore has a higher rate of convergence than the closed economy. Note, however, that $(\alpha + \beta) \rightarrow 1$ implies $\epsilon \rightarrow 1$ and therefore, $\lambda \rightarrow 0$ in equation (22). Thus, if diminishing returns to broad capital do not apply ($\alpha + \beta = 1$), then the model still does not exhibit the convergence property.¹⁰

The credit-constrained open economy converges faster than the closed economy, but the speed of convergence is now finite for the open economy, and the difference from the closed economy is not large for plausible parameter values. If we use the values for $(\alpha, \beta, \delta, \rho, g, n)$ mentioned before along with $\theta = 2$, then the convergence coefficient implied by equation (22) is .022, compared with .014 for the closed economy. In particular, the value .022 accords well with the empirical estimates of convergence coefficients for open economies, such as the U.S. states and the regions of some western European countries and Japan.

Recall that an open economy with perfect capital mobility converges at an infinite rate. Therefore, our finding is that an open economy with partial capital mobility looks much more like a closed economy than a fully open economy. Although we derived this result so far only for a particular set of parameter values, the conclusion is much more

¹⁰If $\alpha = 0$, so that no capital constitutes collateral, then $\epsilon = \beta$ and λ from equation (22) corresponds to the value from equation (13) for a closed economy (with $\alpha = 0$). If $\beta = 0$, so that all capital serves as collateral, then $\epsilon = 0$ and λ from equation (22) becomes infinite, as in the open economy with perfect capital mobility.

general. As we show in the next section, the predicted convergence coefficient for a partially open economy falls within the range of empirical estimates—roughly .015 to .035 per year—for a wide range of "reasonable" parameter values.

The transition to the steady state involves a monotonic increase in human capital per effective worker, h , from its initial value, $h(0)$, to its steady-state value, h^* .

Equation (19) implies that the growth rate of y is ϵ times the growth rate of h , where ϵ is between zero and one. The ratio, h/y , therefore rises steadily during the transition.

Recall, however, that equation (18) implies that the ratio, k/y , is constant throughout.

Therefore, k grows at the same rate as y and the ratio of human to physical capital, h/k , increases during the transition. Note that, although physical capital serves fully as

collateral, k nevertheless rises gradually toward its steady-state value, k^* . The reason

is the constraint of domestic saving on the accumulation of human capital and the

complementarity between h and k in the production function. When h is low the

schedule for the marginal product of physical capital is low; hence, $k < k^*$ follows even

though domestic producers can finance all acquisitions of physical capital with foreign

borrowing. The gradual increase of human capital impacts positively on the marginal

product of physical capital and leads thereby to an expansion of k .

Foreign borrowing occurs only on loans secured by physical capital, and the

interest rate on these loans is pegged at the world rate, r^w . We can also imagine a

domestic credit market, although the setting with a representative domestic agent

always ends up with a zero volume of borrowing on this market. For loans that are

secured by physical capital, the shadow interest rate on the domestic market must also

be r^w . If we assume that human capital and raw labor do not serve domestically as

collateral, then the shadow interest rate on the domestic market with these forms of

security is infinity (or at least high enough to drive desired borrowing to zero), just as it

is on the world market.

We might assume instead that human capital and raw labor serve as collateral for domestic borrowing but not for foreign borrowing. This situation would apply if the legal system enforces loan contracts based on labor income when the creditor is domestic, but not when the creditor is foreign.¹¹ In this case, the shadow interest rate on domestic lending, collateralized by labor income, equals the net marginal product of human capital. This net marginal product begins at a relatively high value (corresponding to the low starting stock, $h[0]$) and then falls gradually toward the steady-state value, r^w . Thus, the transition features a decrease in the spread between this kind of domestic interest rate and the world rate, r^w . An example would be the curb market for informal lending in Korea (see Collins and Park [1989, p. 353]). The spread between curb-market interest rates and world interest rates was 30–40 percentage points in the 1960s and 1970s, but fell by the mid 1980s to about 15 percentage points.

We can again simplify the formula for the rate of convergence if we assume a constant gross saving rate. The required value of θ for a constant gross saving rate is now

$$(23) \quad \theta^* = (\delta + \rho) / [\epsilon(\delta + n) - g(1 - \epsilon)]$$

and the corresponding gross saving rate (expressed relative to GNP) is $s = (1 - \tau) / \theta^*$.

The rate of convergence is then

¹¹If the foreign loans are made directly to the domestic government, then the collateral involves the security put up by the government. Domestic physical capital may then also not serve well as collateral on foreign loans—if the home government does not force itself to pay up—although it may work better on domestic loans (if the government enforces private loan contracts on the domestic market with physical capital as collateral).

$$(24) \quad \lambda = (1-\epsilon)(\delta+n+g)$$

For the parameters used before, $\theta^* = 1.9$, $s = .37$, and $\lambda = .023$.

Another interesting implication of this model is that, despite the existence of international borrowing and lending, the convergence properties of gross national product and gross domestic product are the same. As noted before, the net factor income from abroad is $-(r^w + \delta)k = -(1-\tau)\alpha y$. Therefore,

$$(25) \quad \text{GNP (per unit of effective labor)} = y - (1-\tau)\alpha y = y[1-\alpha(1-\tau)]$$

Since GNP is proportional to GDP, which corresponds to y , the convergence rates for GNP and GDP are the same. This result suggests that data sets that involve GDP are likely to generate similar rates of convergence as those that involve GNP or measures of national income. Some confirmation of this prediction comes from the study of the U.S. states by Barro and Sala-i-Martin (1991a): the rates of convergence are similar for gross state product per capita and state personal income per capita.

The model implies that the gap between GDP and GNP would be large for a credit-constrained open economy: roughly 20% of GDP for the parameter values ($\alpha=.3$, $\tau=.3$) assumed before. The current-account deficit, which equals the change in physical capital, is correspondingly large. It equals 15% of GDP in the steady state for the parameters assumed before.

It is unusual to find developing countries that have values this high for the GDP-GNP gap and the current-account deficit.¹² We can reconcile the theory with this observation by noting first, that many developing countries are insufficiently

¹²One counter-example is Singapore: its current-account deficit was between 10 and 20% of GDP throughout the 1970s (see *International Financial Statistics, Yearbook*, 1991).

productive to be credit constrained, and second, that the collateral for international debt may be substantially narrower than physical capital. If the coefficient α were less than .3, then the predicted ratios for the GDP–GNP gap and the current–account deficit would be correspondingly smaller.

V. Elasticity of Substitution between Capital Stocks

The levels of k and h determine the fraction of capital that serves as collateral—a higher fraction means that the economy is more open and therefore converges more rapidly to its steady-state position. The choice between k and h depends on how these two forms of capital interact as inputs into the production function. We have assumed thus far that the production possibilities are Cobb–Douglas so that the elasticity of substitution between k and h is unity. If the two types of capital are more or less substitutable in the production function, then the chosen values of k and h and therefore the model's convergence properties will differ from those found before. We illustrate this behavior by generalizing to a CES production function.

The production function for output per worker is now

$$(25) \quad y = f(k, h) = A \{ a(bk)^\psi + (1-a)[(1-b)h]^\psi \}^{\eta/\psi}$$

where $0 < \eta < 1$, $0 < a < 1$, $0 < b < 1$, $-\infty < \psi < 1$, and the magnitude of the elasticity of substitution between k and h is $1/(1-\psi)$. The parameter η is the share of broad capital in output.¹³ The parameters, a and b , determine how the steady-state share of physical

¹³The production function for the level of output can be written as

$$Y = AZ^\eta L^{1-\eta}$$

where Z is a broad measure of capital, given by

capital, $\kappa \equiv k^*/(k^*+h^*)$, changes as the substitution parameter, ψ , varies between $-\infty$ and 1.

As $\psi \rightarrow -\infty$, the production function approaches fixed proportions, $y = A \cdot \text{MIN}[bk, (1-b)h]$, and κ approaches $1-b$. For $\psi=0$, the production function becomes

Cobb-Douglas with $\kappa=a$. (The parameters in equation (1) are $\alpha=a\eta$ and $\beta=(1-a)\eta$.)

As $\psi \rightarrow 1$, k and h become perfect substitutes. The economy uses only one of the capital inputs in the steady state, depending on which is more productive. (Recall that k and h substitute one-for-one in the output stream.) If $a+b > 1$, then the economy specializes asymptotically in k and $\kappa \rightarrow \infty$, whereas if $a+b < 1$, then the economy specializes asymptotically in h and $\kappa \rightarrow 0$. (If $a+b=1$, then $\kappa \rightarrow 1/2$.)¹⁴ We can reduce the number of independent parameters by one by rewriting equation (25) as

$$(26) \quad y = f(k, h) = A'[a'k^\psi + (1-a')h^\psi]^{\eta/\psi}$$

where $A' = A[ab^\psi + (1-a)(1-b)^\psi]^{\eta/\psi}$ and $a' = ab^\psi/[ab^\psi + (1-a)(1-b)^\psi]$. In equation (26), however, the parameters A' and a' vary as ψ changes.

The household budget constraint, analogous to equation (6), now generalizes to

$$\dot{h} + \dot{k} - \dot{d} = (1-\tau)f(k, h) - (\delta+n+g)(k+h) - (r-n-g)d - c$$

$$Z = \{a(bk)^\psi(1-a)[(1-b)h]^\psi\}^{1/\psi}$$

The elasticity of substitution between Z and L is still assumed to be one.

¹⁴These results depend on the condition, $r^\omega = \rho + \theta g$. In this case, the home economy is unconstrained on the credit market in the steady state, and hence, the potential to use k as collateral imparts no asymptotic advantage to k over h . As $\psi \rightarrow 1$, the economy therefore specializes in the steady state in the form of capital that is more productive. If $r^\omega < \rho + \theta g$, then k 's usage as collateral imparts an asymptotic advantage. As $\psi \rightarrow 1$, the economy then specializes in k in the steady state even if $a+b$ is somewhat less than 1.

We consider again a credit-constrained economy for which $d=k$ applies and the world interest rate equals the constant r^w . The budget constraint then simplifies to

$$\dot{h} = (1-\tau)f(k,h) - (r^w + \delta)k - (\delta + n + g)h - c$$

Since k serves as collateral on the world credit market, we have

$$(27) \quad (1-\tau)f_k = r^w + \delta$$

where f_k is the marginal product of k , which can be calculated from equation (26). The budget constraint therefore becomes¹⁵

$$(28) \quad \dot{h} = (1-\tau)[f(k,h) - f_k k] - (\delta + n + g)h - c$$

The Euler equation is

$$(29) \quad \dot{c}/c = (1/\theta)[(1-\tau)f_h - (\delta + \rho + \theta g)]$$

where f_h is the marginal product of h , which can be computed from equation (26).

Equation (29) generalizes equation (21).

If we use the partial derivatives computed from equation (26) to substitute out for f_k and f_h , then equations (27)–(29) define a dynamic system of equations in k , h , and c . In the Cobb–Douglas case, it was easy to eliminate k and then deal with the dynamic

¹⁵In the Cobb–Douglas case, f_k is the fraction α of the average product, $f(k,h)/k$, and the term in brackets in equation (28) simplifies to $(1-\alpha) \cdot f(k,h)$ (see equation [20]). This simplification does not work with a CES technology.

system in terms of h and c . In the CES case, it is possible to use equation (27) to express h as a function of k and then eliminate h from the three-equation system. Then we end up with a non-linear system in k and c . We can use a log-linear expansion of this system around the steady state to study convergence.

We continue to use the parameter values: $\eta=.8$, $n=.01$ per year, $g=.02$ per year, $\delta=.05$ per year, $\theta=2$, and $\rho=.02$ per year. We also assume $r^\omega = \rho + \theta g = .06$ per year.¹⁶ The convergence coefficient, λ , then depends on the substitution parameter, ψ , and the capital-share parameter, a' . The results are clearer if we replace a' by the implied steady-state share of physical capital:

$$(30) \quad \kappa \equiv k^*/(k^*+h^*) = \frac{(a')^{1/(1-\psi)}}{(a')^{1/(1-\psi)} + (1-a')^{1/(1-\psi)}}$$

Figure 2 shows the convergence coefficient, λ , as a function of κ . Each curve corresponds to a particular value of ψ , ranging from $-.9$ to $.9$ (elasticity of substitution between $.1$ and 10). The curve associated with $\psi=0$ corresponds to our earlier model with a Cobb-Douglas production function. A shift in ψ , for a given value of κ , implies a compensating change in a' in accordance with equation (30).

For any value of ψ , λ approaches the closed-economy value, $.014$, as $\kappa \rightarrow 0$, and the fully open-economy value, ω , as $\kappa \rightarrow 1$. For the Cobb-Douglas case, $\psi=0$, λ rises from $.014$ at $\kappa=0$ to $.022$ at $\kappa=.375$ (the value that corresponds to the parameters that we used before) and to $.027$ at $\kappa=.5$. The value of λ reaches $.04$ at $\kappa=.68$, $.05$ at $\kappa=.76$, and $.06$ at $\kappa=.81$. Thus, if the share of capital that constitutes collateral is less than $.5$, then λ is confined to the narrow range of $.014$ to $.027$.

¹⁶In the Cobb-Douglas case, the convergence coefficient, λ , in equation (22) is independent of r^ω . That independence does not hold generally in the CES case.

If k and h are highly substitutable, say $\psi=.75$, then λ again equals .014 at $\kappa=0$, but stays below the Cobb–Douglas value as κ increases. If $\kappa<.5$, then λ ranges between .014 and .021.

If k and h exhibit little substitution, say $\psi=-.9$, then λ is above the Cobb–Douglas value for $\kappa>0$.¹⁷ The value of λ varies in this case between .014 and .030 if $\kappa<.5$.

The main message from Figure 2 is that the full range of variation in the substitution parameter ψ leaves the convergence coefficient, λ , confined to a remarkably narrow range—.014 to .030—if $\kappa<.5$ applies. Hence, the theoretical value of λ for a partially open economy conforms to the range of empirical estimates unless the steady-state share of broad capital that serves as collateral is well above .5. Given the importance of human capital in the production process and the difficulties in using human capital as collateral, we are comfortable with the assumption $\kappa<.5$.

The effects on λ of the other technology and preference parameters work as they did in the closed–economy case, discussed by Barro and Sala-i-Martin (1991a). The main results are that λ rises if η (the share of broad capital) falls, if θ falls (so that people are more willing to substitute intertemporally), and if δ rises. If we assume our standard values for the other parameters, then it takes extreme shifts in θ or δ to move λ outside of the range, (.014, .030), mentioned above. Table 1 shows, for example, that if $\kappa=.3$ and $\psi=0$ (Cobb–Douglas production function), then $\lambda=.020$ at the baseline specification, where $\eta=.8$, $\theta=2$, and $\delta=.05$. The value of λ rises to .035 if θ falls to .5 and declines to .010 if θ increases to 10. Similarly, λ rises to .030 if δ increases to .10 and falls to .016 if δ declines to .03.

The coefficient λ is more sensitive to variations in η , the broad capital share. Table 1 shows that λ rises to .047 if η increases to .6 and falls to .009 if η rises to .9.

¹⁷As $\psi \rightarrow -\infty$, the model approaches the framework described in n.6 in which there is only one type of capital, and people can borrow up to the fraction ν of this capital. The share of capital that constitutes collateral, κ , equals ν in this case.

The empirical finding that λ is in the range of roughly .015 to .030 requires η to lie in the narrow range between .70 and .85 if the other parameters are set at their baseline values. If $\theta=.10$ and $\delta=.03$, then a value of η as low as .55 is consistent with the empirical estimates of λ , whereas if $\theta=0.5$ and $\delta=.10$, then η can be as high as .95.

VI. Conclusion

Economists have long known that capital mobility tends to raise the rate at which poor and rich economies converge. The main message of this paper is that the quantitative impact of this effect is likely to be small. If there are some types of capital, such as human capital, that cannot be financed by borrowing on world markets, then open economies will converge only somewhat faster than closed economies. This prediction is consistent with the empirical literature, which finds that samples of open economies, such as the U.S. states, converge only slightly faster than samples of more closed economies, such as the OECD countries.

We have assumed throughout that economies can be modeled by a representative consumer. If capital-market imperfections of the sort considered here are important, however, then this assumption may cause problems: the credit constraints would influence households in different ways. At any time, some households would face binding borrowing constraints, whereas others would not.

Perhaps the model in this paper is best applied not to countries or even to states but to families. The model may, in this context, be useful for explaining the dynamics and distribution of wealth. Suppose, for example, that all families were described by the model in this paper and that they differed only by their rate of time preference. In steady state, the time preference of the most patient families would determine the interest rate; these families would be the most highly educated, and they would own all the economy's physical capital. The less patient families would face binding borrowing

constraints, and they would own no physical capital. Although these families would save to accumulate human capital, they would have a lower level of human capital than the more patient families. Thus, in this economy, we would observe a positive correlation between ownership of human and non-human wealth. We would also see a highly concentrated distribution of non-human wealth and a more diffuse distribution of human wealth. These predictions seem consistent with the facts.

References

- Barro, R.J., "Economic growth in a cross section of countries," *Quarterly Journal of Economics*, 106, May 1991, 407-443.
- Barro, R.J. and X. Sala-i-Martin, "Convergence across States and Regions," *Brookings Papers on Economic Activity*, no. 1, 1991a, 107-158.
- Barro, R.J. and X. Sala-i-Martin, *Economic Growth*, unpublished manuscript, 1991b.
- Barro, R.J. and X. Sala-i-Martin, "Regional Growth and Migration: a Japan-U.S. Comparison," unpublished, December 1991c.
- Barro, R.J. and X. Sala-i-Martin, "Convergence," *Journal of Political Economy*, 100, April 1992.
- Blanchard, O.J., and S. Fischer, *Lectures on Macroeconomics*, MIT Press, Cambridge MA, 1989.
- Cass, D. "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32, July 1965, 233-240.
- Cohen, D. and J. Sachs, "Growth and External Debt under Risk of Debt Repudiation," *European Economic Review*, 78, December 1988, 1138-1154.
- Collins, S.M. and W. Park, "External Debt and Macroeconomic Performance," in J.D. Sachs and S.M. Collins, eds., *Developing Country Debt and Economic Performance*, University of Chicago Press, Chicago, 1989.
- Dowrick, S. and D. Nguyen, "OECD Comparative Economic Growth 1950-85: Catch-Up and Convergence," *American Economic Review*, 79, December 1989, 1010-1030.
- Feldstein, M.S. and C. Horioka, "Domestic Savings and International Capital Flows," *Economic Journal*, 90, 1980.

Kaldor, N., "Capital Accumulation and Economic Growth," in F.A. Lutz and D.C.

Hague, eds., *The Theory of Capital*, St. Martin's Press, New York, 1961.

Kendrick, J.W., *The Formation and Stocks of Total Capital*, Columbia University Press, New York, 1976.

Koopmans, T.C., "On the Concept of Optimal Economic Growth," in *The Econometric Approach to Development Planning*, North Holland, Amsterdam, 1965.

Kurz, M., "The General Instability of a Class of Competitive Growth Processes," *Review of Economic Studies*, March 1968.

Levine, R. and D. Renelt, "A Sensitivity Analysis of Cross-Country Growth Regressions," unpublished, The World Bank, December 1990.

Lucas, R.E., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, July 1988, 3-42.

Mankiw, N.G., D. Romer, and D. Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, 107, 1992.

Maddison, A., *Phases of Capitalist Development*, Oxford University Press, Oxford, 1982.

Mulligan, C.B. and X. Sala-i-Martin, "Transitional Dynamics in Two-Capital Goods Models of Endogenous Growth," National Bureau of Economic Research, working paper, 1991.

Ramsey, F., "A Mathematical Theory of Saving," *Economic Journal*, 38, 1928, 543-559.

Rebelo, S., "Long Run Policy Analysis and Long Run Growth," *Journal of Political Economy*, 99, June 1991.

Romer, P., "Crazy Explanations for the Productivity Slowdown," *NBER Macroeconomics Annual*, 1987.

Solow, R.M., "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70, February 1956, 65-94.

Table 1

Effects of Parameters on Convergence Coefficient

η	θ	δ	λ
.8	2.0	.05	.020
.6	2.0	.05	.047
.9	2.0	.05	.009
.8	0.5	.05	.035
.8	10.0	.05	.010
.8	2.0	.10	.030
.8	2.0	.03	.016
.8	0.5	.10	.056
.8	10.0	.03	.009

Note: The table shows the convergence coefficient, λ , that corresponds to the specified values of η , θ , and δ . The other parameters are $\kappa=.3$, $\psi=0$, $\rho=.02$, $g=.02$, and $n=.01$.

Figure 1: Phase Diagram

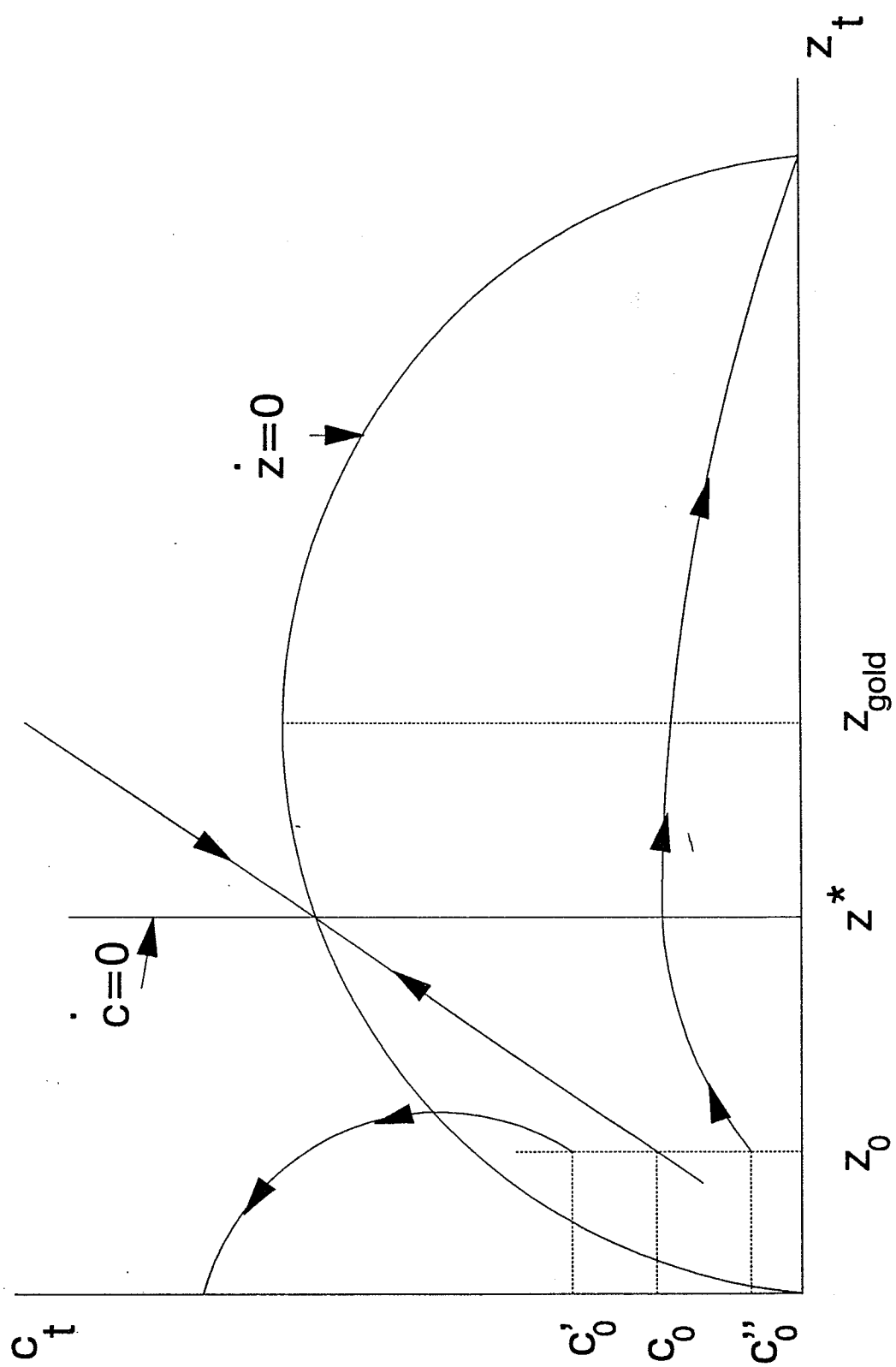


Figure 2 Convergence Coefficient versus Physical Capital Share

