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A GROWTH MODEL OF INFLATION,
TAX EVASION AND FINANCIAL REPRESSION

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Abstract

In this paper we study the effects of policies of financial repression on long term growth and try to explain why optimizing governments might want to repress the financial sector. We also explain why inflation may be negatively related to growth, even though it does not affect growth directly. We argue that the main reason why governments repress the financial sector is that this sector is the source of "easy" resources for the public budget. The source of revenue stemming from this intervention is modeled through the inflation tax. Our model has the implication that financial development reduces money demand. Hence, if the government allows for financial development the inflation tax base, and the chance to collect seigniorage, is reduced. To the extent that the financial sector increases the efficiency of the allocation of savings to productive investment, the choice of the degree of financial development will have real effects on the saving and investment rate and on the growth rate of the economy.

We show that in countries where tax evasion is large the government will optimally choose to repress the financial sector in order to increase seigniorage taxation. This policy will then reduce the efficiency of the financial sector, increase the costs of intermediation, reduce the amount of investment and reduce the steady state rate of growth of the economy. Financial repression will therefore be associated with high tax evasion, low growth and high inflation.

KEY WORDS: Growth, Inflation, Tax Evasion, Financial Repression

This paper explores some reasons behind the existence of financial repression and their economic consequences. It is widely recognized that financial markets and financial intermediation are important determinants of the economic performance of a nation.¹ Many governments in history, however, have introduced a whole host of laws, regulations, taxes, restrictions and controls on the behavior of financial intermediaries together with restrictions on the development and introduction of new financial instruments and markets.

Before the 1970s, many economists favored policies of financial repression on several grounds. First, it was argued that the government needed to impose anti-usury laws thereby intervening in the free determination of interest rates. Second, strict control and regulation of the banking system was said to give the monetary authorities a better control over the money supply. Third, it was thought that governments knew better than markets and private banks what the optimal allocation of savings was or what kind of investments were more or less desirable from a social perspective. Fourth, financial repression was identified with interest rates below market rates, which reduced the costs of servicing government debts.

Some of the recent growth literature deals with the theoretical links between financial intermediation and growth along two lines:² first, it analyzes how financial intermediation affects economic growth; second, it studies how economic growth might itself affect the evolution and growth of financial intermediation. Some of the papers exploring the first link study the effects of policies of repression of the financial system (in the form of taxes, restrictions and regulations of various sorts) on the rate

¹For example, the 1989 issue of the World Bank's World Development Report was entirely devoted to the role of financial markets and intermediation for the process of economic growth.

²See Greenwood and Jovanovic (1991), Bencivenga and Smith (1991), Levine (1990a, 1990b), De Gregorio (1991), and Greenwald and Stiglitz (1989).

of economic growth. The main implication of these papers is that policies of repression of the financial sector lead to a reduction in the rate of growth of the economy.

From the empirical side, there is a large body of evidence showing that financial repression leads to low growth rates.³ The question is the following: if both theory and evidence suggest that financial repression policies have adverse effects on growth, why do some governments choose to follow such policies?. This is one of the key issues that will be addressed in this paper.

The second (and related) question we ask in this paper is why inflation seems to be negatively correlated with growth in a cross-section of countries. A number of recent empirical studies use the Summers and Heston (1988) data set to show that countries that have higher inflation rates seem to grow less after a number of other variables are held constant (see, for instance, DeGregorio (1991), Fisher (1991) and Roubini and Sala-i-Martin (1991)). Some economists have interpreted this correlation as evidence that inflation is bad for the economic performance of a country. We argue in this paper that this negative partial correlation is likely to be spurious as both high inflation and low economic growth are caused by policies of financial repression.

It is our view is that the main reason why governments stay in the way of private financial evolution is that the financial sector is the potential source of "easy" resources for the public budget. Governments have the power to follow policies of financial repression. By this we mean that they have the option and capability of not allowing the financial sector to operate at its full potential by introducing all kinds of regulations, laws, and other non-market restrictions to the behavior of banks and other general financial intermediaries. The source of public

³See Fry (1988), World Bank (1988), Gelb (1989), King and Levine (1992), Levine (1991), DeGregorio (1991), and Roubini and Sala-i-Martin (1991, 1992).

income stemming from this intervention will be modeled through inflation tax.⁴ Our model, as most models of money demand, will have the implication that more financial development (which can be interpreted as a reduction in the transaction costs of converting non-liquid to liquid assets) reduces the need for people to carry money. Hence, when governments allow the financial system to develop, the base for the inflation tax is reduced and, as a consequence, so are the chances to collect seigniorage. To the extent that the financial sector increases the efficiency of the allocation of savings to productive investment projects, financial repression will also have real effects, as the amount of physical capital accumulation is smaller for every level of private savings. Thus, financial repression is bad for growth.

We incorporate the possibility of tax evasion by assuming that the effective income tax rate is different from the official rate and that the elasticity of reported income with respect to actual income is less than one. The degree of tax evasion differs across countries and it depends on the availability of tax-evasion technologies (possibly due to differing efficiencies in collecting income taxes) and on prevailing attitudes with respect to the reporting of private income.

Imagine that the government, through regulation and other non-market interventions, can control the degree of financial development (this is what we call financial repression). Given the rate of money growth, the income tax rate and the degree of tax evasion, the choice of financial repression implies two different effects: on the one hand, it reduces income and therefore decreases the income-tax base. On the other hand, it increases real money demand and therefore raises the inflation tax base. We show that in countries where changes in actual income do not lead to

⁴Clearly this is not the only source of income the government gets from repressing the financial sector. Mandatory purchases of government debt and below market interest rates are other important sources of public income. The regulation of the reserve requirement plays an important role but we think of it as a part of the overall inflation tax or seigniorage (see Brock (1989)).

large changes in reported income (ie, where tax evasion is large) the government will choose to repress the financial sector in order to expand money demand and increase the tax rate on money.

Summarizing, in order to increase the revenue from money creation, governments subject to large income-tax evasion choose to increase seigniorage by repressing the financial sector and increasing inflation rates. This policy will tend to reduce the amount of services the financial sector provides to the whole economy. Given the amount of savings in the economy, aggregate investment and therefore the aggregate growth rate will be lower. This introduces a spurious negative correlation between inflation and growth in the sense that a third variable (repression) affects both inflation and growth.

It is important to note that our results are robust to the criticism that only final output should be taxed. Even though we model money as entering in the utility function, we think of money as an intermediate input that makes life easier because it saves people trips to the bank (much like refrigerators save people trips to the supermarket). Kimbrough (1986) and Faig (1986) show that if money is an intermediate input, then the inflation tax should be zero because it is optimal not to tax inputs in an economy where all final outputs can be taxed. The reason why our results are robust to this criticism is that they apply when there is tax evasion. That is, when it is not possible to tax all final output, it will be optimal to tax some of the inputs.

The rest of the paper is organized as follows. In section 2 we present a simple growth model of inflation and growth that is an extension of Sidrauski (1967) to the linear technology of Rebelo (1991). We show that, if variation in technologies and money supply rules are independent across countries, this simple model predicts a negative correlation between inflation and growth, even though inflation does not have direct effects on growth. In section 3, we expand the model and incorporate

tax evasion, inflation and financial repression. We use the model to show the effects of financial repression on growth and argue that optimizing government might find it advantageous to repress the financial sector in order to collect revenue. Some concluding remarks follow in the last section.

1. A simple model of inflation and growth.

In this section we want to analyze the relation between the inflation rate and economic growth. We start with a simple version of the Sidrauski (1967) model where, in order to get positive long run growth rates, the technology is modified along the lines suggested by Rebelo (1991). That is the production function takes the form

$$(1.1) \quad y(t) = A \cdot k(t)$$

where $y(t)$ is per capita output at time t , $k(t)$ is the capital-labor ratio, and A is the constant marginal productivity of capital. We assume that the economy is populated by infinitely lived consumers or dynasties who derive utility from the only consumption good — c — and from the stock of real money per person — m .

Households are therefore assumed to maximize a utility function of the form

$$(1.2) \quad U_0 = \int_0^{\infty} e^{-\rho t} N(t) u(c(t), m(t)) dt$$

where ρ is the personal discount rate, $N(t)$ is the total amount of people alive at time t , which is assumed to grow at an exogenous rate n , $m(t) = M(t)/N(t)P(t)$, and $P(t)$ is the price level. In order to bound utility we also assume $\rho > n$.

The conventional interpretation of money in the utility function is that money "makes life easier" since it allows people to get consumption goods without having to go to the bank and transform bonds into consumption goods all the time. Following Feenstra (1986), we could redefine consumption and introduce money and transaction costs explicitly in the budget constraint rather than in the utility function without changing the basic results. We further assume that the utility function is time separable and logarithmic⁵ in c and m , that is

$$(1.3) \quad u(c,m) = (1-\beta)\ln(c(t)) + \beta\ln(m(t))$$

The budget constraint requires that per capita investment and real money accumulation to be equal to per-capita savings (non consumed resources). Savings, in turn, equal output (Ak) plus transfers (v) minus consumption (c), minus the erosion of real balances via the inflation tax and population growth ($m \cdot (n+m)$) and minus the erosion of per-capita physical capital by population growth:

$$(1.4) \quad \dot{k} + \dot{m} = Ak - c + v - nk - (n+\pi)m$$

where v is the per capita transfer of money from the government and π is the inflation rate. Individuals maximize (1.2) subject to (1.3) and (1.4), given the initial stocks of nominal money and real capital. The set of first order conditions are well

⁵A slightly more general utility function would be $\{[c(t)^{(1-\beta)}m(t)^\beta]^{(1-\sigma)} - 1\}/(1-\sigma)$ where $\sigma \gg 0$.

Our functional form is the particular case of this one when $\sigma=1$. Our simplifying assumption of $\sigma=1$ is of no substantial importance since the effects of changes in σ on savings and growth rates are well known. See for instance Blanchard and Fischer (1989).

known (see Appendix A). The optimal demand for money is

$$(1.5) \quad m^d(t) = \frac{c(t) \beta}{(1-\beta)R(t)}$$

where $R=A+\pi$ is the nominal interest rate. Note that firms' profit maximization implies that the real interest rate, r , is equal to the marginal productivity of capital, A .

From (1.5) we see that real money demand is a positive function of consumption and a negative function of the opportunity cost of holding it, the nominal interest rate. Unlike the Baumol-Tobin "square-root" money demand function, this one does not imply increasing returns to monetary services. In other words, it does not imply that a doubling of consumption needs is associated with less than double the amount of monetary services. The Euler equation implies a growth rate of consumption equal to

$$(1.6) \quad \dot{c}(t)/c(t) \equiv \gamma_c = A - \rho.$$

Since (1.6) is true all the time, consumption always grows at the steady state rate so it displays no transition in real time. We assume a very simple form of government: it prints money at a constant rate μ and transfers it to private agents in a lump-sum fashion, v . Accordingly, seigniorage taxes are the only governments revenue source. Thus, the public budget constraint is:

$$(1.7) \quad m\mu = v$$

It can be shown that the nominal interest rate is constant at all points in time.

The money demand equation implies that consumption and real money balances grow at the same rate all the time. Thus, there is no transition for either m or c . The transversality condition ensures that the following policy functions hold for all t :

$$(1.8a) \quad c(t) = (\rho - n) k(t)$$

$$(1.8b) \quad m(t) = \frac{\rho - n}{(1 - \beta) \cdot (A + \mu - n)} \cdot k(t)$$

(see Appendix A for a derivation of this result). Hence, there is no transition for the capital stock and the level of output either so all the variables of this model grow at the constant steady state growth rate all the time:

$$(1.9) \quad \gamma_m = \gamma_c = \gamma_k = A - \rho > 0 \text{ for all } t.$$

Note from (1.8b) that if the rate of growth of money μ increases, m falls for every level of k . In other words, if we plot $\ln(m(t))$ over time, we get that if there is a discrete increase in the rate of nominal money growth μ , the upward sloping line (with slope $A - \rho$) 'jumps' discretely in the level of $m(t)$, but keeps the same slope. It is in this sense that μ has real effects. The reason is that an increase in μ implies an immediate increase in the rates of inflation and nominal interest; this increase in the opportunity cost of holding money leads to a fall in m (relative to c and k). Utility levels of course fall also since money enters in the utility function.

There are several key implications arising from this model. First, while the rate of growth of money has welfare effects, it does not directly affect the rate of growth of real variables of the economy. Second, from the definition of real money

balances and the equilibrium growth rate of real money, we find that the equilibrium rate of inflation is

$$(1.10) \quad \pi = \mu - n - (A-\rho) \quad \text{for all } t$$

The larger the productivity parameter, A , the larger the growth rate of real money demand and, therefore, the smaller the inflation rate. The intuition is simple: high A implies high consumption growth. Because people demand more real money the larger c , higher rates of consumption growth are associated with higher rates of growth of real money. Given the rate of nominal money growth, in order to get high real money growth, it must be the case that prices do not grow as much. That is, π is low. Thus, cross country variation in A implies a negative correlation between inflation and growth. Cross country variation in rates of money growth, μ , on the other hand, implies no correlation between inflation and growth since μ affects π but not the growth rate. Hence, if there is independent cross-country variation in μ and A , the correlation between inflation and growth will be negative, even though nominal money growth has no direct effect on real growth.

Two final caveats on this model. First, if the government cares about the utility of the representative agent the model predicts that governments will set μ so as to get a negative inflation rate. That is, the Friedman Optimum Quantity of Money rule of zero nominal interest rates applies (so $\pi = -r$). In the real world we see countries with high inflation rates for long periods of time (see Dornbusch and Fischer (1991) for evidence on a number of countries with π around 30% for many years). Our simple model does not explain this phenomenon. This is the objective of the model in the next section.

Second, the negative correlation between inflation and growth arises only if the cross-country variations in γ and μ are independent. This assumption is unrealistic,

especially if A is interpreted in a broad sense so as to include taxes and other forms of government intervention. To the extent that both parameters contribute to public revenue, they will be related through the public budget constraint. In the next section we consider a model with explicit distortionary taxes on income, tax evasion and financial repression.

2. Seigniorage, tax evasion and financial development.

In this section we want to formalize the links between financial repression, tax evasion, inflation and growth. As in the previous section, we assume that the economy is populated by infinitely lived consumers or dynasties who derive utility from the only consumption good and from per capita real money stock (1.2). The marginal utility of money is decreasing in financial development. That is, "the more automatic teller machines (the more financially developed the economy) the lower the marginal benefit of holding money". We think of this assumption as reflecting the negative effect of financial development on the transaction costs (costs of transforming bonds into money). The utility function takes the form

$$(2.1) \quad U = \int_0^{\infty} e^{-(\rho-n)t} [\ln(c(t)) + \beta(F) \cdot \ln(m(t))] dt$$

where F reflects the level of financial sophistication, $\beta'(F) < 0$ and $\beta''(F) > 0$. Notice that the assumed utility has the property that the marginal utility of money is a decreasing function of F.⁶ The government imposes a proportional income tax at a constant rate τ . We want to allow for the possibility of cross-country differences in

⁶This specification also implies that the level of utility of money is decreasing in F. As it will be apparent later on, what matters for our results is that F affects the marginal utility of money.

the degree of tax evasion: countries have different attitudes towards paying taxes due to preference factors (a history of public waste and inefficient provision of public services will lead private agents to be less willing to pay taxes) and technological factors (different governments will have different access to technologies for tax collection that detect tax evaders; and, as argued by Stigler (1970), governments may want to prosecute evaders to different degrees across countries). We therefore assume that reported income, RY , is a positive function of actual income, Y , and a negative function of the tax rate, τ (the larger the tax rates, the larger the incentives to evade taxes):

$$(2.2) \quad RY = h(Y, \tau)$$

with $0 \leq h_Y \leq 1$ and $h_\tau < 0$, and where RY is reported income and Y is actual income. The existence of tax evasion would be represented by $\partial RY / \partial Y$ being less than one (changes in actual income lead to less than proportional changes in reported income) and maybe, but not necessarily, $\partial RY / \partial \tau$ being small (changes in tax rates lead to small changes in reported income). One functional form we will use is

$$(2.2)' \quad RY(t) = aY(t)/\tau(t)^{1-\zeta}, \quad 0 < \tau < 1$$

where ζ and a are parameters between zero and one that relate to the available technology for avoiding taxes. Governments with poor technologies for collecting taxes get low reported income per unit of actual income (reflected in low values of a) and a lower elasticity of reported with respect to actual income (reflected in ζ and a). Large ζ and a correspond to efficient legal systems that impose large penalties on tax evaders (efficient police and tax collection departments, etc.) which leads people to report most of their income (note that if $a=1$ and $\zeta=1$, we have that

all income is reported, $RY=Y$, and all increases in income are reported also, $\partial RY/\partial Y=1$.⁷ It should also be observed that (2.2)' implies that, while the official tax rate is τ , the effective one paid by private agents is $a\tau^{\zeta}$, which we assume is less than τ .⁸

As in section 1, aggregate production takes the following linear form

$$(2.3) \quad Y_t = AK_t$$

Although the assumption of linear technology is crucial to get closed form solutions and non-zero steady state growth, it is not essential to our story. We could have

⁷While we are assuming, for simplicity, that the tax evasion technology is given, it is clear that governments can change such a technology through investments in resources such as higher expenditures on tracking tax evasion and reforms of the taxation system.

⁸While it is assumed that τ should be less than unity, in most countries the highest feasible tax rate is less than one. In fact, suppose that there are two types of agents in an economy. One group cannot tax evade (for example, labor income earners whose taxes are retained at the source as in most countries); the other can evade (for example, self-employed people, earners of financial income and entrepreneurs). Then, even if the second group evades taxes, the government may not be able (and/or willing) to choose very high tax rates because of its inability to distinguish between the two groups of agents. In this case, formal tax rates above unity are infeasible but, more importantly, the actual formal maximum tax rate is likely to be well below unity given the negative labor supply effects of high tax rates on the non-evasion group.

This is also the reason why the government cannot compensate higher degrees of tax evasion (lower a 's) simply by increasing formal tax rates so as to reach its target effective tax rates. In spite of the fact that in this model tax evasion does not represent a real resource cost, in reality, tax evasion does not represent a tax veil that can be undone by formal higher tax rates. In fact, as tax evasion increases, the constraint of the maximum formal tax rate (unity or less) will become binding at some point, constrain the behavior of governments and force them to switch to other forms of taxation (such as financial repression and high inflation). This point will be clear at the end of this section when we turn to the formal optimal taxation analysis.

Alternatively, one could model tax evasion as implying a direct real resource cost so that output available for consumption and investment is reduced because of the resources wasted in tax evasion efforts induced by higher tax rates. In this case, the results of the model will be qualitatively the same but repression of the financial sector will be optimal for any given level of tax evasion. For more on this, see the discussion below on optimal taxation.

decreasing returns to capital, in which case the steady growth rate would be zero, just as in the neoclassical growth model. Since we interpret K in a broad sense, the transition to the steady state would take a long time. Hence, we may want to think of the present model as describing an economy characterized by "long transitions towards steady states".

As in McKinnon (1973) we define as "financial repression" as the set of policies, laws, regulations, taxes, distortions, qualitative and quantitative restrictions and controls imposed by governments which do not allow financial intermediaries to operate at their full technological potential.⁹ We will imagine that the efficiency with which savings are allocated to investment purposes depends on the degree of financial development in the economy: the more developed (or the less repressed) the financial sector the more efficiently savings will be allocated to investment projects. A more developed and less repressed financial sector increases the microeconomic efficiency of the whole macroeconomy for a number of reasons: First, it contributes to the efficient allocation of the capital stock to its best use. Second, it helps to collect and screen information (in a world of imperfect or costly information, individuals may not know who wants to borrow or lend). Third, an inefficient or repressed financial sector will be characterized by high costs of financial intermediation caused by several factors. In repressed financial sectors, competition is limited and oligopolistic conditions will lead to high costs of intermediation; moreover, the more limited variety of financial instruments and markets for financial intermediation (such as the lack of equity and bonds markets and the inefficiency of commercial banks) will also raise intermediation costs. Fourth, if financial

⁹While economic theory suggests that a certain degree of regulation of financial markets might be optimal in the presence of uncertainty, market failures, moral hazard and adverse selection issues, the concept of financial repression, as defined by McKinnon, can be interpreted as those interventionist policies that are not aimed at dealing with the above externalities but rather have the direct and indirect objectives of providing revenue for the government.

intermediation is nonexistent or very costly, private entrepreneurs will be forced to self finance their investment projects. This may lead them to undertake projects that are smaller (and therefore less efficient) than ones they would undertake otherwise undertake. An additional problem is that such agents may also have to self accumulate nominal assets, whose real value gets subsequently eroded by high inflation rates.

From a macroeconomic point of view, all this means that economies more financially developed (or less repressed) are able to transform a given amount of savings into more efficiency units of physical capital. Since, in our model, there is only one type of aggregate physical capital, we will think of a better financial system as generating more units of physical capital for every level of savings: we assume that, while a dollar of savings can be accumulated into a dollar of money balances without any leakage, the fraction of a dollar of savings that will be intermediated into real capital accumulation will be an increasing function of F , the degree of development (and efficiency) of the financial sector.¹⁰ Formally this can be described as:

$$(2.4) \quad \frac{1}{\varphi(F)} \dot{K}/N + \dot{M}/PN = Y - T - C + V$$

where T are total income tax payments per person and where $\varphi(F) < 1$ for $F < F^*$ and $\varphi(F) = 1$ for $F = F^*$ where F^* is the highest level of financial development given by the current available technology (as it is described, for instance, by

¹⁰An alternative way to introduce financial repression in the model (see Roubini and Sala-i-Martin (1992) for an example) is to assume that investment is always equal to savings ($I = S$) but that financial development directly affects the average productivity of capital in the linear production function, i.e. $Y = \varphi(F)$ with $\varphi'(F) > 0$. The growth effects and the normative results of the model are identical under this alternative specification (see the working paper version of this paper for such an alternative specification and results).

Greenwood and Jovanovic (1991). This budget constraint says that the after-tax per capita savings (non consumed resources) are equal to per capita investment net of the resources used in the financial intermediation process, plus money accumulation. Defining lower case variables as the real per capita versions of their upper case counterparts, (2.4) can then be rewritten as:

$$(2.4') \quad \dot{z} = \varphi(F) (y - t - c + v) - n z - \pi \varphi(F)m$$

where $z = k + \varphi(F) m$. The maximization of utility (2.1) subject to the constraint (2.4) yields a money demand function of the form (see Appendix B)

$$(2.5) \quad m^d(t) = \beta(F)c(t)/R(t)$$

where $R(t)$ is the nominal interest rate at time t , $R(t) = (1 - a\tau^s)A\varphi(F) + \pi(t)$ and $(1 - a\tau^s)A\varphi(F)$ is the real post-tax interest rate. That is, real money demand is a positive function of consumption and a negative function of the opportunity cost of holding it, the nominal interest rate. Notice also that money demand is a negative function of the level of financial development of the economy (F) for all levels of nominal interest rates. This reflects the idea of financial development lowers the transaction cost of transforming non-liquid into liquid assets. The interest rate elasticity of money is equal to 1 and, in particular, it is independent of F . This result comes from the assumed constant intertemporal elasticity utility function. The

Euler equations imply that the rate of consumption growth equal to

$$(2.6) \quad \gamma_c = \dot{c}/c = \varphi(F)A(1-a\tau^\zeta) - \rho.$$

There are several aspects of (2.6) that need to be highlighted. First, as long as F and τ are constant, consumption grows always at the same constant rate all the time. Second, consumption growth is independent of nominal variables. This is another form of Sidrauski's result: changes in the rate of growth of money do not affect the steady state growth rate of consumption. Yet this does not mean that money is "superneutral" because changes in the rate of money creation will have an effect on the desired stock of real money. As its name indicates, real money is a real variable that reflects the provision of monetary services which, in our setup, affect utility.¹¹ Third, the growth rate is a positive function of the degree of financial development, F , and a negative function of the tax rate, τ . In Figure one we use a simple graph to illustrate this point. The line called RC (which stands for Return to Consumption) is the Euler equation for private consumers and can be read as the return to savings (r) has to equal the return to consumption ($\rho + \gamma$), which is an upward sloping function in the r - γ space. The return to investment (RI) is the after tax marginal product of capital: $r = \varphi(F^*)A(1-a\tau^\zeta)$. In the absence of financial repression, the growth rate is determined by the crossing point. Policies of financial repression, however, introduce a wedge between the two schedules. The growth rate is consequently lower: an F lower than the technologically feasible F^* leads to a decrease in the real post-tax return to savings ($\varphi(F)A(1-a\tau^\zeta) < \varphi(F^*)A(1-a\tau^\zeta)$), and a consequent decrease in investment and growth rates.

To close the model we need to specify the behavior of the government. In

¹¹In a setup with an explicit shopping technology, money and inflation would affect consumption as well due to the role of transaction costs.

order to focus on the distortions induced by financial repression and non lump-sum taxation, we assume that the government can neither borrow nor lend. This assumption is reasonable if we think of governments that have limited access to international borrowing and enjoy little confidence from private domestic savers, possibly because the existing levels of debt are so extremely high that private agents in the economy believe that there is a large incentive for the government to default on its debt obligations. Under these conditions, the government will have to do without any domestic or international borrowing. The public constraint is

$$(2.7) \quad V = \dot{M}/NP + T$$

Because the economy will end up growing at positive rates, we need to make some assumption on the scale of the government. If, following Sidrauski, we were to set the size of the transfers to some constant, the size of the government would be negligible asymptotically as the economy grows.¹² We therefore assume that the size of the government sector relative to the economy is constant or that transfers are proportional to the stock of capital:¹³

$$(2.8) \quad V(t) = \epsilon K(t)$$

where ϵ will be the policy parameter that tells how large the government sector is relative to the economy. As we mentioned above, the tax revenue is based on

¹²Following Barro (1991) and Barro and Sala-i-Martin (1990) we could allow for productive public spending. Alternatively, we could introduce government spending on goods and services in the utility function. The implications of such extensions are well known and we leave them as an exercise to the reader.

¹³Since $y = A k$, this is the same as transfers being proportional to output or consumption.

income taxes, with constant average and marginal tax rate τ so

$$(2.9) \quad T(t) = a\tau Y(t)$$

Finally, we assume that the government sets the nominal growth rate of money at a constant level μ so

$$(2.10) \quad \dot{M}(t)/M(t) = \mu$$

The resulting budget constraint in per capita terms is the following

$$(2.11) \quad \mu m + a\tau Y = v$$

The implied social budget constraint is

$$(2.12) \quad \dot{k} + n k = \varphi(F) (Ak - c)$$

or

$$(2.12') \quad I = \varphi(F) \cdot S$$

where I represents gross investment and $(Ak - c)$ are savings. In words, a given level of savings transforms into a larger or a smaller level of capital accumulation depending on the level of financial development $\varphi(F)$. Another way to think about (2.12) is that the increase in the capital labor ratio is equal to savings (total output per capita minus private consumption) net of the resources lost in the financial intermediation process. This is what we meant by the financial sector increasing the

efficiency of the allocation of savings into investment.

The transversality condition requires that capital grow at the same rate as consumption at all points in time. The policy function for consumption is

$$(2.14) \quad c(t) = [\rho - n + \varphi(F)Aa\tau^{\zeta}] \cdot k(t)$$

(see Appendix B). An interesting point is that a higher tax rate τ reduces the growth rate γ_c but it increases the level of consumption $c(t)$ given the capital stock. In fact, an increase in τ has an income and a substitution effect. The substitution effect implies that higher tax rates lead to a lower return to savings, to a lower quantity of savings and higher consumption). The income effect leads to a decrease in the rate of growth of the economy and, therefore, a decrease in permanent income and consumption. Given $k(0)$, the substitution effect here dominates so $c(0)$ increases. The adverse effect on growth leads to an ambiguity in the effect of τ on welfare.

Similarly, an increase in the degree of financial development F will increase consumption: first, higher F leads to an increase in the rate of growth of the economy and therefore a permanent increase in income and consumption (income effect); second, an increase in F represents an increase in the return to savings, and should induce a fall in consumption (substitution effect). As long as a is different from zero, the income effect dominates the substitution effect and an increase in F leads to an increase in consumption (in the case when $a=0$, that is, no effective income taxes, the two effects exactly cancel out and consumption remains

unchanged). The equilibrium inflation rate and nominal interest rate are, respectively

$$(2.15a) \quad \pi = \mu - n - [\varphi(F)A(1-a\tau^\zeta) - \rho]$$

$$(2.15b) \quad R(t) = \rho - n + \mu$$

which are constant at all points in time (see Appendix B). The inflation rate is decreasing in F and increasing in τ . The nominal interest rate is independent of both τ and F : On one hand, a higher τ (or a lower F) reduce the real interest rate; on the other hand a higher τ (or a lower F) increases the equilibrium rate of inflation. For the log utility form that we considered, the two effects exactly offset. Using (2.14), the policy function for $m(t)$ is:

$$(2.16) \quad m(t) = \beta(F) \frac{\rho - n + \varphi(F)Aa\tau^\zeta}{\rho - n + \mu} k(t)$$

note that there is a negative relation between μ and $m(t)$ given $k(t)$ through the inflation rate, and a positive relation between F and $m(t)$, and τ and $m(t)$ since both F and τ tend to affect $c(t)$ positively (which leads people to demand more money for transactions). Real money balances always grow at the same rate as consumption and capital. As shown above, this growth rate is always constant. Using (2.16), the government budget constraint can be rewritten as

$$(2.17) \quad \epsilon = \mu \beta(F) \frac{\rho - n + \varphi(F)Aa\tau^\zeta}{\rho - n + \mu} + a\tau^\zeta A.$$

where, recall, ϵ is the size of the government relative to the stock of capital. Public revenue per unit of capital is the sum of seigniorage and income tax collections. It

is assumed that the government can control the rate of money growth μ , the tax rate τ and (through policies of financial repression) the level of development of the financial system, F . The derivatives of revenue with respect to these three arguments are analyzed below:

Revenue and Money Growth.

$$(2.18a) \quad \partial \epsilon / \partial \mu = \beta(F) \left[\frac{(\rho - n)}{(\mu + \rho - n)} \right]^2 [\rho - n + \varphi(F) A a \tau \zeta] > 0$$

Money growth has two offsetting effects on revenue: first, it increases the seigniorage tax rate which increases total seigniorage; second, it increases the equilibrium rate of inflation and the nominal interest rate which reduces the inflation tax base, $m(t)$. Since nominal money growth does not affect the level or the growth rate of per capita income, it does not affect income tax collection. Under our particular specification, the economy is always on the left hand side of the inflation "Laffer curve" so higher nominal money growth goes along higher seigniorage.

Revenue and Financial Repression.

The effect of financial repression on public revenue is given by:

$$(2.18b) \quad \partial \epsilon / \partial F = \frac{\mu}{\rho - n + \mu} \{ A a \tau \zeta [\varphi \beta' + \beta \varphi'] + (\rho - n) \beta' \}.$$

More financial development (high F) affects real money demand and therefore the inflation tax base. It does so through four different channels. First, it lowers the per capita demand for money at given nominal interest rates. This tends to

reduce seigniorage. Second, it increases real interest rates and, consequently, increases the first component of the nominal interest rate which further reduces money demand. Third, it increases the growth rate of the economy which reduces the steady state inflation rate. This tends to reduce the demand for real balances. Unless the utility function is almost linear, the first and second effects clearly dominate the third effect so the per capita stock of real money is a decreasing function of the level of financial development. In our particular case of logarithmic utility, the second and third terms exactly offset so financial development does not affect nominal interest rates. The sum of these first three effects on seigniorage, $\mu \cdot \frac{\rho-n+\varphi(F)Aa\tau^{\zeta}}{\rho-n+\mu} \cdot \beta'(F)$, is unambiguously negative. The fourth effect is that larger F increases desired initial consumption and therefore, money demand. This increases the tax base for seigniorage. This term, $\mu \cdot \frac{\beta(F)}{\mu+\rho-n} \cdot a\tau^{\zeta} \cdot A \cdot \varphi'(F)$ is positive.

The net effect is therefore ambiguous but depends on the degree of tax evasion a . In fact, for $a=0$, a change in F does not affect consumption and the second term is equal to zero. In this case, the above derivative is certainly negative and financial development leads to a net reduction of revenues. This means that governments which face a lot of tax evasion will certainly find that their total revenue rises when they repress the financial sector. As a becomes larger, the second term in (2.18b) becomes more important. In fact, as A is larger the substitution effect of a change in F on consumption is smaller (since any increase in the real return to savings generated by a higher F leads to a lower post-tax return to savings when tax evasion is low, i.e. a is high). Hence, the likelihood of $\partial\epsilon/\partial F$ being positive is an increasing function of a .

In summary, financial development decreases real money demand and therefore the inflation tax base. Thus, the total net effect of an increase in financial development on the government revenues is negative in the presence of high levels of

tax evasion. Conversely, in such an environment financial repression increases both seigniorage and government revenues.

Revenue and Distortionary Taxes.

Finally, the effect of an increase in the tax rate τ on tax revenues is given by

$$(2.18c) \quad \partial \epsilon / \partial \tau = \beta(F) \frac{\mu}{(\mu + \rho - \pi)} \varphi(F) A a \zeta \tau^{\zeta-1} + \zeta a \tau^{\zeta-1} A > 0$$

An increase in the tax rate has two positive effects on total revenue. First, higher income tax rates directly lead to higher total revenue (this is the second term in (2.18c)). Second, it reduces the return to investment so people increase short-run consumption and their demand for money. This increases the base for the inflation tax (this is the first term in (2.18c)).

The above results suggest that countries where it is relatively easy to evade income taxes (low a and ζ) will be countries where the elasticity of public revenue with respect to income tax rates is small and the elasticity of public revenue with respect to the level of financial development is high. Such governments will find it easier to raise revenue by repressing the financial sector (lower F) and printing money (higher μ) than by levying taxes (higher τ).

Optimizing Government and Welfare.

From an optimal taxation point of view, the crucial question is whether optimizing governments will actually choose to distort their economies. To answer this question, we need to specify some objective function for the government. For

instance, governments might not like to print money, but they may have some (exogenous) target rate of growth of money, $\bar{\mu}$. Alternatively, we could assume a loss function for the government such as those used in the tax smoothing and optimal seigniorage literature (Barro (1979), Mankiw (1987)): for example, governments dislike high inflation and high tax rates, but like greater financial development. It is easy to show that if a government has such a preference function and a budget constraint like (2.17), such a government will choose positive rates of inflation, financial repression and tax rates. In other words, the government will find optimal to repress the financial sector even if this leads to lower long-run growth.

The key question is whether a benevolent government which cares about the utility of the representative agent will also find it optimal to financially repress its economy. Formally, if the government maximizes the U_0 in (2.1) subject to the policy functions $c_t = \bar{c}(k_t)$ and $m_t = \bar{m}(k_t)$ and subject to $k_t = k_0 e^{\gamma t}$ and the government budget constraint, is it still possible to make a case for the desirability of financial repression? By substituting the policy functions derived above into (2.1), we obtain the following reduced form utility function:

$$(2.19) \quad U_0 = \frac{1}{\rho-n} \{ (1+\beta) \ln([\rho-n+\varphi(F)Aa\tau^\zeta]) - \beta \ln(\rho-n+\mu) + \beta \ln(\beta) + \\ + (1+\beta) \ln k_0 + (1+\beta) \frac{(1-a\tau^\zeta)\varphi(F)A-\rho}{\rho-n} \}$$

with the following partial derivatives:

$$(2.20a) \quad \partial U_0 / \partial \mu = \frac{-\beta(F)}{(\rho-n)(\rho-n+\mu)} < 0$$

$$(2.20b) \quad \partial U_0 / \partial F = \frac{\beta'(F)}{\rho-n} \{ \ln [k_0 (\rho-n)] + \frac{A\varphi-\rho}{\rho-n} + 1 + \ln[\beta(\rho-n+\mu)] \} + \\ + \frac{1}{\rho-n} \left[(1+\beta) \frac{A\varphi'-\rho}{\rho-n} \right]$$

Note that the second term in (2.20b) is unambiguously positive and the first term is negative. We assume that the parameters are such that the overall effect is positive which means that the overall effect of financial development on people's utility is positive.¹⁴

$$(2.20c) \quad \partial U_0 / \partial \tau = \frac{(1+\beta) a \zeta \tau^{\zeta-1} A \varphi}{(\rho-n) (\rho-n+a\tau^{\zeta} \varphi A)} \\ - \frac{(1+\beta)}{(\rho-n)^2} a \cdot A \cdot \varphi \zeta \tau^{\zeta-1} < 0$$

The first-order conditions for the optimizing government are

$$(2.21) \quad (\partial U_0 / \partial \tau) / (\partial \epsilon / \partial \tau) = (\partial U_0 / \partial F) / (\partial \epsilon / \partial F) = (\partial U_0 / \partial \mu) / (\partial \epsilon / \partial \mu) = \Delta$$

where the partial derivatives are written in (2.18) and (2.20) respectively and where

¹⁴If we this result did not hold, we would have the finding that the government would always financially repress the economy since that increases people's utility and, furthermore increases public revenue. This result, however, would not be too interesting.

Δ is a Lagrange multiplier. Conditions (2.21) are the Ramsey Rule of optimal taxation. These three equations together with the government budget constraint specify optimal values of τ^* , F^* , and μ^* given the parameters a , ζ , ϵ , etc. It should be noted that the government would not just want to maximize the growth rate γ because agents care also about the levels of c_t and m_t : in particular, low tax rates induce higher growth rates but they also lead to an increase in savings, and a fall in both the level of current consumption and in the level of money demand.

This very complicated system of nonlinear equations can be solved numerically. Numerical simulations suggest that if φ'' is sufficiently small there is an interior solution where the government chooses a level of F that is lower than the maximum, i.e. the government chooses to repress the financial sector. The simulations suggest that when tax evasion is large (small a and ζ), optimal F is small and optimal μ is large. More specifically, for high values of a and ζ (low tax evasion) it is optimal not to repress the financial sector (so that F will be chosen by the government as equal to its maximum level). In this case, most of the revenues of the government will be income taxes plus a small amount of seigniorage derived from the fact that the optimal inflation rate is positive even when the financial sector is not repressed.

For values of a and ζ below a certain critical value, even higher statutory tax rates will not deliver enough revenue to pay for government spending. At this critical value of a , the formal tax rate will be set to its maximum feasible level but the total revenue from income taxes will not cover all government spending. At this point, reducing the degree of financial development will start to be optimal since it increases both money demand and seigniorage revenues. For values of a below the critical value, the revenues from income taxes will fall even further (since the statutory rate is already set at its maximum level) so that it will be optimal to further increase the degree of financial repression and, at the same time, choose high levels of the inflation rate in order to increase the seigniorage component of

revenues.^{15, 16}

The model therefore implies that the greater the level of tax evasion, the less the government will derive its revenues from income taxes and the more will it choose to repress the financial sector so as to induce large money balances and, consequently, a large inflation tax base. It is then optimal for the government to tax money at high μ (notice that, as α is smaller the growth losses of low F are offset by lower effective tax rates so the effect on growth is ambiguous). Since the elasticity of revenue with respect to income taxes is small because people evade such taxes, it pays the government to lower F because the loss in actual revenue is relatively small.

As an extreme case, consider a country whose level of tax evasion is so large

¹⁵It is clear that the reason why zero repression of the financial sector is optimal for a range of values of the level of tax evasion is that, in this model, tax evasion does not lead to a real resource cost. If we make the alternative assumption that such a real resource cost occurs (as resources are wasted in tax evasion efforts caused by higher tax rates), we would always get an interior solution where a certain amount of financial repression is optimal for any degree of tax evasion. This alternative specification would require assuming that output net of resources wasted in tax evasion (Y^*) is a negative function of tax rates (since higher tax rates lead to higher tax evasion efforts) or: $Y^* = (1 - \theta(\tau)) Y$ where $\theta'(\tau) > 0$.

¹⁶The result that the optimal inflation rate is positive even when there is zero repression of the financial sector, rather than being equal to Friedman's rule as set out in section 2, depends on two assumptions of the model: first, income taxes are distortionary; second, money is equivalent to a final good since it enters in the utility function. In models where money is an intermediate good, such as Kimbrough (1986), Faig (1986), the optimal inflation rate is zero since it is optimal to tax final goods only. Our result about the optimality of a positive inflation rate would, however, hold even if we had considered a model where money was an intermediate good rather than a money in the utility function model. The reason is that, as tax evasion increases, the formal tax rate reaches its maximum and taxes on final goods (or income) are not enough to cover government spending. In this case, the government will be forced to resort to its only other source of revenue, inflation and seigniorage taxes. In other words, when the maximum formal and effective tax rate is reached, further taxation of final goods or income become impossible and the government will have to tax the intermediate good, i.e. money. Moreover, in the presence of tax evasion and illegal economic activities that are cash-intensive, taxation of money balances is optimal even if money is an intermediate input. In fact, if the tax evading sector is cash intensive, the only (and optimal) way to tax its final output is to tax the monetary balances used in the transactions in this sector.

that the government has lost control over its ability to generate revenue from income taxes. This corresponds to the case where $a=0$. Condition (2.21) can be rewritten as

$$(2.21)' \quad (\partial \epsilon / \partial F) / (\partial \epsilon / \partial \mu) = (\partial U_0 / \partial F) / (\partial U_0 / \partial \mu) = \Delta$$

By using the partial derivatives in (2.18) and (2.20) and setting $a=0$, we note that the first term in (2.21)' represents a negative relation between F and μ . The second term in (2.21)' represents a positive relation between F and μ if φ'' is sufficiently small. The crossing point gives an F^0 and a μ^0 that represents the optimal level of financial development and the optimal rate of money growth¹⁷ (see figure 2). Obviously, as a gets higher, the government start using some income taxes to finance its revenues and for sufficiently large a 's (economies with little tax evasion) we already showed that the government will choose not to repress the financial sector since that yields no positive revenue yet it still entails losses in utility.

The results that we have obtained can be summarized as follows. In order to increase the revenue from money creation, governments of countries with inefficient tax systems (high tax evasion) may optimally choose to increase per capita real money demand by repressing the financial sector. As a side effect, this policy will tend to reduce the amount of services the financial sector provides to the whole economy and a given level of savings will be intermediated in a lower level of investment. This will reduce the steady-state rate of growth of the economy.

In the absence of direct data on tax evasion and the size of the underground economy, our story has the following empirical implications: countries with high tax

¹⁷If φ'' is sufficiently small, the solution will be interior.

evasion are countries with large financial repression, high monetary growth and high inflation rates. As a consequence of the real effects of the distortions in the financial sector, these economies will have lower (pre-tax) real interest rates, higher base money per capita and lower per capita growth. In Roubini and Sala-i-Martin (1991, 1992) we found evidence in favor of the implications of the model. In particular we found that, after controlling for other determinants of economic growth, measures of financial repression (such as negative real interest rates, high required reserves ratios, measures of distortions in capital and financial markets and high inflation rates) negatively affect growth rates in cross-country regressions such as those estimated by Barro (1991). In particular, all these proxies for financial repression entered negatively and significantly affected the growth rates in the estimated equation for growth.

3. Concluding Remarks.

In this paper, we build a model of financial repression, tax evasion, inflation and growth that allows us to examine the effects of policies of financial repression on long-term growth. We are able to explain why optimizing governments might want to repress the financial sector. Our view is that the main reason why governments repress the financial sector is that this sector is the potential source of "easy" resources for the public budget, here modeled as seigniorage revenues.

The main results of the paper are the following: First, because policies of financial repression tend to reduce the amount of services the financial sector provides to the whole economy and, therefore, to reduce aggregate investment for given levels of savings, financial repression will have adverse effects on long-run growth. Second, in order to increase the revenue from money creation, governments of countries with inefficient tax systems (high tax evasion) may optimally choose to increase per capita real money demand by repressing the financial sector and choose high rates of money

growth. Third, as a result, high financial repression will be associated with high monetary growth, high inflation rates, high seigniorage and low economic growth. This will tend to generate a spurious negative correlation between inflation and growth. These implications seem to conform well with the the existing empirical evidence.

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Appendix A: The Non-Transition of the Simple Model

The first order conditions are:

$$\begin{aligned} e^{-(\rho-n)t}(1-\beta)/c &= \lambda \\ e^{-(\rho-n)t}\beta/m &= \lambda(A+\pi) \\ \dot{\lambda} &= \lambda(A-n) \\ \lim_{t \rightarrow \infty} \lambda(t)(m(t)+k(t)) &= 0 \end{aligned}$$

Plugging the government budget constraint (1.7) into the private budget constraint (1.4), we get the social budget constraint

$$(A.1) \quad \dot{k} = Ak - c - nk$$

If we integrate (A.1) between 0 and T, we get

$$(A.2) \quad k_T = [k_0 - c_0/(\rho-n)]e^{(A-n)T} + c_0/(\rho-n) e^{(A-\rho)T}$$

Use the transversality condition to get

$$(A.3) \quad \lim_{T \rightarrow \infty} e^{-(A-n)t} k_t = 0 \quad \text{and} \quad \lim_{T \rightarrow \infty} e^{-(A-n)t} m_t = 0.$$

Using (A.2) in (A.3):

$$(A.4) \quad \lim_{T \rightarrow \infty} [k_0 - c_0/(\rho-n)] + \lim_{T \rightarrow \infty} c_0/(\rho-n) e^{-(\rho-n)T} = 0.$$

For the first term to go to zero, it must be the case that it is zero all the time (it

is independent of T). Hence

$$(A.5) \quad c_0 = (\rho - n)k_0.$$

For the second term to go to zero, we must impose $\rho > n$ (which we assumed in order to get bounded utility). Rewrite k_t as

$$(A.6) \quad k_t = k_0 e^{(A - \rho)t}$$

(A.6) implies that also k_t grows at the steady state rate all the time so that there is no transition to the steady state for k either. Finally, $\lim_{T \rightarrow \infty} e^{-(A - n)t} m_t$ is always zero if $\rho > n$ since the first term falls at $(A - n)$ and the policy functions are

$$(A.7) \quad c_t = (\rho - n)k_t \quad \text{for all } t.$$

$$(A.8) \quad m_t = \frac{\rho - n}{(1 - \beta)(A + \mu - n)} k_t \quad \text{for all } t.$$

Appendix B: The Non-Transition of the Second Model.

The first order conditions are the following

$$e^{-(\rho-n)t} / c(t) = \lambda(t) \varphi(F)$$

$$e^{-(\rho-n)t} \beta(F) / m(t) = \lambda(t) R(t) \varphi(F)$$

$$\lambda(\varphi(F)(1-a\tau^{\zeta})A-n) = -\dot{\lambda}$$

$$\lim_{t \rightarrow \infty} \lambda(t)[k(t)+m(t)] = 0$$

We can divide both sides of the budget social constraint (2.12) by k and integrate between 0 and t to get

$$(B.1) \quad k(t) = \left[k_0 - \frac{c_0}{A\varphi-n-\gamma_c} \right] e^{(\varphi(F) \cdot A-n)t} + \frac{c_0}{A\varphi-n-\gamma_c} e^{\gamma_c t}$$

where γ_c is constant and given by (2.6). If we integrate the third first order

condition we also get that $\lambda(t) = \lambda_0 e^{-[(1-a\tau^{\zeta})\varphi(F)A-n]t}$, where λ is the dynamic multiplier. The transversality condition says $\lim_{t \rightarrow \infty} \lambda(t)[m(t)+k(t)] = 0$. Using (B.1)

and the same argument used in appendix A this implies the following policy function

$$(B.2) \quad c_0 = [\rho-n+\varphi(F)Aa\tau^{\zeta}]k_0$$

and we need to require $\rho > n$ (which we assumed at the outset in order to get bounded utility). Plugging (B.2) in (B.1), we get that k_t grows at the same rate as consumption at all points in time. Thus, we are always in steady state and there is no transitional dynamics. Note that (B.2) is a closed form solution for the policy function. It applies at all times $c_t = [\rho-n+\varphi(F)Aa\tau^{\zeta}]k_t$ given that c and k grow at

the same rate all the time.

We can also derive the equilibrium inflation rate and nominal interest rate.

Starting from the definition of the nominal interest rate we get:

$$(B.3) \quad R(t) = r(t) + \pi(t) = \varphi(F)A(1 - a\tau^{\zeta}) + (\mu - n - \gamma_m) = \\ = \mu + \rho - n + \gamma_R = R$$

which implies:

$$(B.4) \quad \dot{R} = R^2 - (\mu + \rho - n)R$$

This is a differential equation in R that is unstable at $R = \mu + \rho - n$ and stable at $R = 0$.

But $R = 0$ is not feasible with $\mu > 0$. Thus, the only solution to this differential equation is $R = \rho + \mu - n$ at all points in time. Hence, R is constant, $\gamma_R = 0$ and $\gamma_m = \gamma_c$ at all points in time.

FIGURE 1

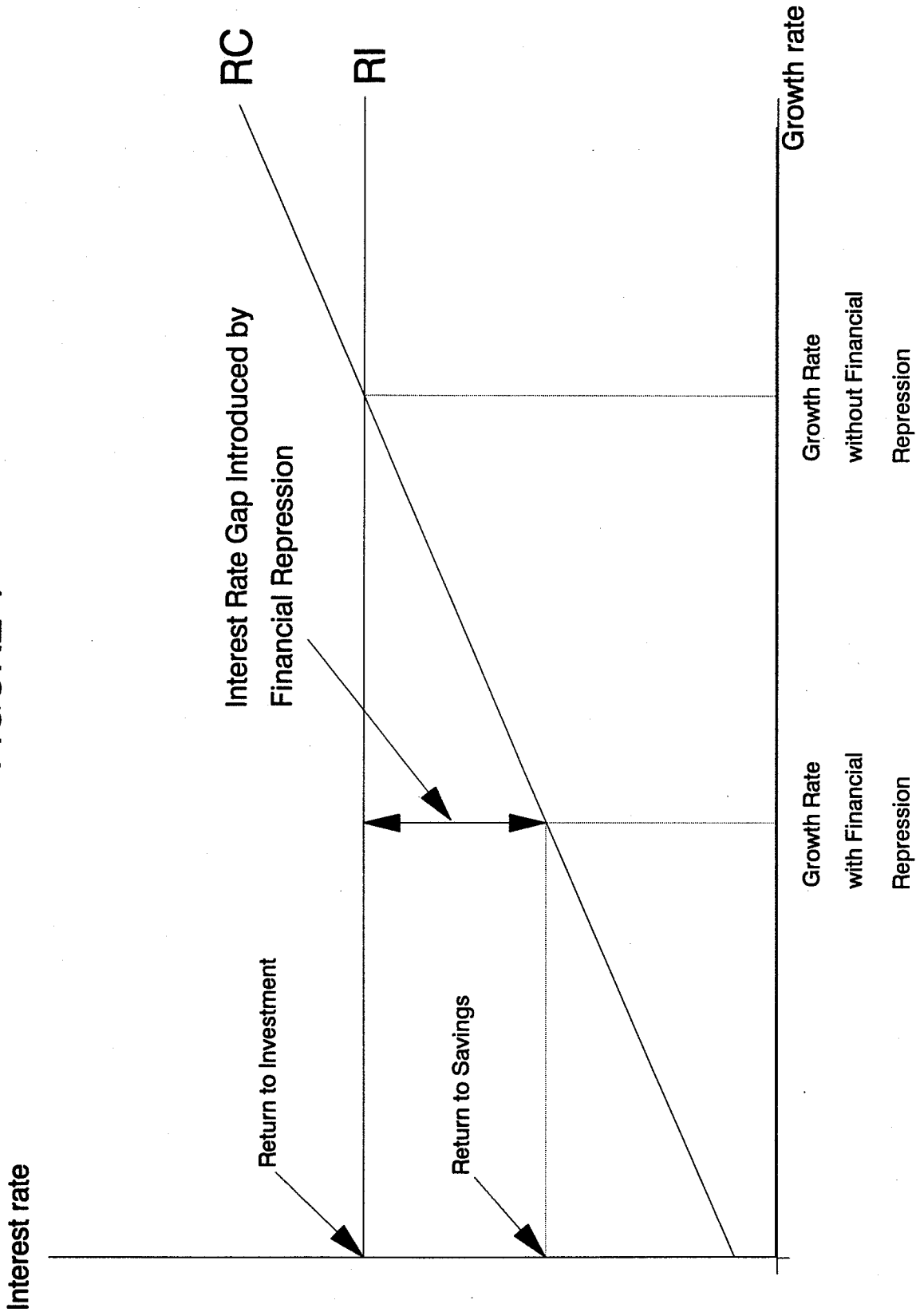


FIGURE 2

