ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
27 Hillhouse Avenue
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 679

TAXATION AND RISK-TAKING ONCE AGAIN (WITH AND WITHOUT TAX REVENUE DISPOSAL)

Giancarlo Corsctti

Terza Universita' degli Studi di Roma

December 1992

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments. Professor Corsetti worked on this project while at Yale University. This is a revised version of the paper originally written December 1991.

ABSTRACT

The effects of taxation on risk-taking crucially depends on what happens to tax revenue. In the literature, this is implicitly disposed of (partial equilibrium), it turns into government consumption or it is redistributed to investors in a lump-sum fashion (general equilibrium). Building upon Merton's intertemporal CAPM, this paper proposes a simple general equilibrium framework of analysis where taxes affect portfolio composition both directly, via tax rates on income from financial assets, and indirectly, via their effect on the stochastic properties of the return on government bonds. In the economy, there exists a set of productive assets characterized by stochastic constant return to scale. The distribution of aggregate output will therefore depend on the portfolio allocation by the private agents. The analysis will discuss the implication of fiscal policy for both intertemporal and intratemporal efficiency, pointing out the difference in results when moving from a partial to a general equilibrium perspective.

KEY WORDS: Taxation, Risk, Stochastic Growth

1. Introduction.

This paper studies the effects of taxation on risk–taking in the framework of a general equilibrium intertemporal model of asset allocation. In the model, tax rates affect portfolio composition both directly, by changing the income flow from taxable financial assets, and indirectly, by determining the stochastic properties of the return on government bonds. On the production side of the economy, there exists a set of alternative production processes, all characterized by stochastic constant return to scale. Thus, aggregate output will depend on the allocation of capital among different productive assets. Distortionary taxes will play two roles. On the one hand, they will determine the distribution of capital among alternative productive assets. On the other hand, they will affect the relative price of consumption at different dates, altering the level of financial wealth as well as the intertemporal allocation.

Since the seminal contribution of Domar and Musgrave [1944], it has been pointed out that, as long as tax laws include loss–offset provisions, taxing risky assets can actually increase their demand. This is because, in the presence of a tax credit when returns are negative, the government shares with private agents part of the investment risk. However, the extent to which risk can be diverted from agents' opportunity set will depend crucially on what happen to tax revenue. In the literature, this is implicitly disposed of, it is fed into government consumption, or it is returned to the private sector in the form of a lump–sum payment. It should be stressed that these schemes refer to very different models. Consider at first a model with a contingent government expenditure where public consumption is proportional to assets' payoff. In the case of a negative realization of these payoffs, public expenditure should be negative, i.e. it should be production rather than consumption. Thus, this specification implicitly assumes that the public sector can resort to some hidden endowment or technology in the economy. As a second example, consider a lump–sum redistribution of the tax proceeds. Even if tax rates shift risk out of the private sector's portfolio of financial assets, tax revenue will reflect the variability of assets. Thus, the level of risk in the opportunity set of the agents will not change at all, even if policies seem to provide some insurance against bad realizations of income from assets.

The main contribution of this paper consists in building a general equilibrium model of taxation and risk–taking without resorting to lump–sum redistribution. Instead, the model focuses on the relationship between fiscal policies and the payoffs of government bonds. The analytical framework presented hereafter, which can be supplemented by simple graphical tools borrowed from mean–variance analysis, mainly draws on Merton intertemporal CAPM, as developed by Eaton [1981] and Corsetti [1992].

This paper is organized as follows. Section 2 will relate our analysis to the literature on taxation, saving and risk–taking. The following three sections develop the model, by describing the main feature of the economy as well as by characterizing the command optimum allocation. Section 6 will focus on a competitive equilibrium with income taxes, with particular reference to the design of optimal policies from the point of view of both intratemporal and intertemporal efficiency. Section 7 will contrast partial and general equilibrium models, while section 8 will illustrate the previous results in the mean–variance space of asset returns.

2. Partial Versus General Equilibrium Views of Taxation and Risk–Taking.

Before Domar and Musgrave [1944], taxation of income from risky assets was commonly seen as discouraging risk–taking. Facing a lowered return, agents would decrease their portfolio holdings of risky investment. However, assume that there are loss–offset provisions, so that negative realizations of asset return can be written off against other taxable income. In this case, as pointed out in the 1944 seminal contribution, the government also shares some of the risk in the investment. In other words, the government shares both losses and gains with the private sector. If agents put more weight on the risk–sharing aspects of taxation, this may actually encourage, rather than discourage, risky investment. In a formalization then widely used in the literature, due to Mossin [1968] and Stiglitz [1969], these effects of taxation are modelled by focusing on a utility–maximizing investor facing a portfolio choice between a risky and a riskless asset. These and later works spelled out conditions under which the original claim by Domar and Musgrave, i.e. that increasing taxation with perfect loss–offset provisions would raise the demand for the risky asset, holds. In particular, conflicting income and substitution effects stemming from tax changes make it clear that, in general, no unambiguous conclusions can be reached. More interestingly, these effects also impede clear–cut conclusions in the case of no loss–offset, when a tax on asset income mainly results in a reduction of its mean return.

The implicit assumption in this analysis is that the portion of risk taken by the government can then be diversified away or shifted to some other sector of the economy. The argument is then essentially one of partial equilibrium. Only a few notable exceptions (Stiglitz [1972], Kanbur [1981] and Kihlstrom and Laffont [1983]) have provided more general frameworks. Stiglitz [1972] is the closest to the present paper, in both spirit and technical features. He develops a one–period model where a single–factor input is used in the production of a single–commodity output. While there are different production processes available, they are all characterized by stochastic constant return to scale. Given initial input endowments, individuals have to choose how much to invest as well as how to allocate their investment among alternative assets.

Among other considerations, Stiglitz [1972] stresses the importance of specifying what is done with the revenue obtained from taxation. If this is disposed of, partial equilibrium results carry over the general equilibrium model just outlined with virtually no change. However, when redistributed as lump–sum payments, a flat–rate tax on assets' return may actually be neutral from the point of view of production efficiency. This is the case in a two asset (one risky, one riskless) world, where the outcomes of the risky investment are perfectly correlated among individuals.

While the intertemporal dimension of the allocation problem is touched upon by Stiglitz [1972], most contributions limit their analysis to portfolio allocation, that is, they focus on the problem of maximizing utility of terminal wealth. However, one should take into account that distortionary taxation also modifies the price of consumption at different dates. As first suggested by Hagen 1970, a simple extension of the one period model to an intertemporal setting can be obtained by assuming a representative individual with instantaneous utility in the form of a iso–elastic function (see also Atkinson and Stiglitz [1980]).

This paper will draw on this literature in an attempt to provide a simple analytical framework illustrating both the intertemporal and portfolio response to changing tax rates, spelling out assumptions regarding public spending and modelling the "redistribution" of tax revenue in excess of spending in the form of both payoffs and new issues of government debt.

3. A Model: Preferences and Technology.

Consider an economy populated by many identical dynastic households characterized by the following preferences

$$(3.1) \qquad E_0 \int_0^\infty \frac{C(t)^{1-R}}{1-R} \exp(-\delta t) \qquad \delta>0,\ R\in(0,\infty)$$

where E_0 is the expectation operator conditional on the information at time t=0, C(t) is the instantaneous consumption rate at time t, R is the elasticity of marginal utility and δ is the rate of time preferences, positive by assumption. Population is assumed constant and normalized to 1. As in the stochastic optimal growth literature, consumption is assumed to be instantaneously deterministic (Merton [1969],[1975]). In a more general framework of analysis, the rate C(t) could be conceived as an index combining both private consumption and public consumption, denoted by C_p and C_g respectively, in the form

$$C(t) = C_p^{\alpha} \, C_g^{(1-\alpha)}. \tag{3.2}$$

where, at an optimum, the parameter α will determine the distribution of C^* between alternative uses.

Production employs a single homogeneous reproducible input, capital, which may be costlessly accumulated and allocated among two different production processes. In other words, there exist two kinds of production workshops characterized by a different level of average productivity. In addition, one is subject to an i.i.d. instantaneous productivity shock, entering the production function in a multiplicative way. Both processes exhibit constant return to capital. Denoting by $m(t)$ the amount of capital employed in the *risky* workshop, $m(t) \equiv \frac{K_1(t)}{K(t)}$, where $K(t)=K_1(t)+K_2(t)$, we can characterize the aggregate instantaneous output flow in the economy as

$$dY(t) = [m(t)\ (\eta\ dt + \sigma\ d\omega) + (1-m(t))\ r\ dt]\ K(t) \tag{3.3}$$

where Y(t) denotes cumulative output net of depreciation, $d\omega$ is a standard Wiener Process with zero mean and unit variance, η and σ are positive constants which denote the instantaneous drift and the instantaneous standard deviation of productivity for the first production processes, while r is the deterministic instantaneous productivity of capital employed in the second workshop. Rewriting output flow as

$$dY(t) = \{[m(t)\ (\eta - r) + r]\ dt + m(t)\ \sigma\ d\omega(t)\ \}\ K(t) \tag{3.4}$$

makes it clear that the output process follows a brownian motion with drift, where the parameters of this process depend on the capital allocation choices by the agents in the economy. As in Cox Ingersoll and Ross [1985], depreciation is stochastic and we cannot rule out the possibility of observing *negative* output flows. In each instant in time, both the mean and variance of the conditional distribution of dY depend on the existing capital stock. Current shocks will have long–lasting effects on the output process to the extent that they affect capital accumulation. However, by linearity of the production functions, the net rate of return to capital will be i.i.d. with mean and variance (η,σ^2) for the first process, while it will be a constant deterministic rate r for the second process.

The government is assumed to absorb a fraction of current output

$$dG(t) = \{g\ [m(t)(\eta{-}r){+}r]dt\ +\ g'\ [m(t)\sigma d\omega(t)]\}\ K(t) \tag{3.5}$$

where G(t) denotes cumulative spending and both g and g' are exogenously given, time–invariant constant parameters. The government could also provide a public good which, to some degree, is a substitute for the private good in consumption. In this case, posing $C_g(t)=\Psi(t)K(t)$, total government spending would be

$$(3.6) \qquad dG(t)+C_g(t)dt = \{[g[m(t)(\eta-r)+r]+\Psi(t)]dt + g'[m(t)\sigma d\omega(t)]\}\ K(t)$$

Note that $dG(t)$ has a simple technological interpretation in terms of a stochastic amount of public goods needed to run the production process in the economy. In this sense, both components of government spending are productive, but in different ways. One will deliver utility directly, while the other one is effectively an input in the production process. As will be discussed below in greater detail, a contingent component in spending (for $g'\neq 0$) will permit us to dispose of some of the production risk taken over by the government through distortionary taxation. However, it should be noted that a non–zero g' raises the possibility of a negative spending, to complement a negative net output.

4. Command Optimum.

Having specified preferences and technology, we can look at the command optimum allocation for our economy. This will provide a benchmark allocation for the remainder of the paper, where we will look at competitive equilibria under different tax and spending regimes. A benevolent social planner, whose objective is to maximize private agents' utility, will face the following problem:

$$(4.1) \qquad \underset{\{C,m\}}{\mathrm{Max}}\ E_0 \int_0^{\infty} \frac{C(t)^{1-R}}{1-R} \exp(-\delta t) \qquad \delta>0,\ R\in(0,\infty)$$

subject to

$$dK(t) = dY(t)-dG(t)-C(t) =$$

$$= \{(1-g)[m(t)(\eta-r)+r] - \frac{C(t)}{K(t)}\}K(t)dt + (1-g')m(t)\sigma K(t)d\omega(t)$$

$$0\leq m(t)\leq 1,\ K(t)\geq 0,\ C(t)\geq 0,\ K(0)=\bar{K}>0$$

The expression for the resource constraint for the economy makes it clear how a contingent component of government spending filters production variability. The risk faced by the consumer will in fact be a fraction (1–g') of the (before–spending) production risk. Following Merton [1969] and assuming that an interior solution for $m(t)$ exists, the optimal consumption rate and capital allocation are

$$C(t) = c\,K(t) \tag{4.2}$$

$$m(t)^{*} = m^{*} = \frac{(\eta-r)(1-g)}{R\sigma^{2}(1-g')^{2}} \tag{4.3}$$

where consumption is linear in capital and the distribution of capital between the two productive assets depends positively on the mean return differential and negatively on both relative risk aversion and the variability of the risky asset. The equilibrium value of c will be

$$c^{*} = \frac{R-1}{R}\left[(1-g)\,(m^{*}(\eta-r)+r) - .5R[m^{*}\sigma(1-g')]^{2}\right] + \frac{\delta}{R} \tag{4.4}$$

whereas, as shown in Merton [1969], a positive value for c will be sufficient to ensure that the transversality condition of the problem is satisfied[1].

5. The Market Economy.

In this section, we will lay out the main assumptions about the behaviour of both institutions and private agents.

5.1 Firms

Firms behave competitively. For simplicity, we will assume that they issue no

[1] See Merton 1969 or a textbook in Financial Economics such as Ingersoll 1987, for a derivation of the solution in the text. Brock and Malliaris 1982 is also a standard reference.

bonds as well as that their number in the market is given. Given the form of the production function, the latter assumption has no relevant implications for our problem. On the other hand, if we allowed for bond financing in the presence of differential tax treatment of debt and equity (for example, interest-deducibility provisions), this would provide an additional mechanism through which distortionary taxation could affect production (see for example Stiglitz [1972]).

By the condition of a zero profit, we equate the before-tax return to capital to its marginal product in each of the two alternative production processes. We will have $\eta dt+\sigma d\omega$ and rdt, for the risky and the riskless productive asset respectively.

5.2 The Government

The government faces a menu of tax instruments including taxes on asset income as well as taxes on both consumption and wealth. At each moment in time government spending can be financed both by collecting tax revenues and by issuing government debt. Assume that the government only issues consols paying an instantaneously riskless coupon at the rate of u unit of output. Denoting by $B(t)$ the number of consols outstanding at time t, with unit price $q_B(t)$, and denoting by $dT(t)$ the instantaneous tax revenue, the government budget identity can then be written as

$$(5.2.1) \qquad d[q_B(t)B(t)] = dG(t)-dT(t)+q_B(t)B(t)\left[\frac{u dt}{q_B(t)} + \frac{dq_B(t)}{q_{B(t)}}\right]$$

where $\frac{dq_B(t)}{q_{B(t)}}$ are capital gains on bond holding.

Assume then that the government can tax firms with different risk characteristics in a selective way. If we allow for different tax rates on expected output and output innovations, we obtain the following general form for the tax revenue function:

$$\text{(5.2.2)} \quad dT(t) = \{[\tau_1(m\eta)+\tau_0(1-m)r]dt + \tau_1'(m\sigma)d\omega(t)\}\ K(t) + [\Gamma W(t) + \mathbb{C}C(t)]dt$$

where T(t) is cumulative tax revenue; τ_1 and τ_1' are time–invariant tax rate on the expected output and output innovation of the risky productive asset; τ_0 is the tax rate on output from the riskless production process; W(t) is private wealth in real terms and Γ is the rate at which this is taxed; $\mathbb{C}$ is the tax rate on consumption.

Throughout the paper, we also assume that the government is able to precommit itself to a given policy, announced and immediately effective at t=0, so that we will not address policy–related, time–consistency issues. As usual, though, it will be possible to verify that in some cases the policies which solve the optimality problem for a benevolent government will indeed be self–enforcing.

5.3 Private Wealth

In a market economy, private agents will hold both capital and government bonds (in the context of a representative agent model, we can abstract from financial assets in zero net supply). Private financial wealth in real terms W(t) can then be written as

$$\text{(5.3.1)} \qquad W(t) = K(t) + q_B(t)B(t)$$

The evolution of W(t) can be characterized as follows. Given the stochastic structure of our economy, each rate of return, including both income and capital gains, can be broken down into an anticipated and an unanticipated component. Since by assumption technological shocks are the only source of uncertainty in the economy, it is reasonable to conjecture that all risky assets in the economy will depend on the same stochastic variables, which means, they will all be perfectly correlated. Thus, in the case of government consols we can pose

$$\text{(5.3.2)} \qquad \frac{u dt}{q_B(t)} + \frac{dq_B(t)}{q_{B(t)}} = r_B(t)dt + \sigma_B(t)d\omega(t)$$

where r_B (t) and $\sigma_B(t)$ will be endogenously determined in general equilibrium. Denote now by $n(t)$ the share of capital in wealth, so that $n(t)W(t) \equiv K(t)$. If we assume that agents consume at the non-stochastic rate C(t), the process of wealth accumulation can be described as

$$\text{(5.3.3)} \quad dW(t) = n(t)W(t)\Big[m(t)[(1-\tau_1)\eta-(1-\tau_0)r] - (1-\tau_0)r - \frac{C(t)}{K(t)} \Big]dt +$$

$$+ \; n(t)W(t)[(1-\tau_1^{\prime}) \; m(t)\sigma d\omega(t)] \; +$$

$$+ \; [1-n(t)]W(t) \; [r_B(t)dt+\sigma_B(t)d\omega(t)] - [\Gamma W(t) + \mathbb{C}C(t)]dt$$

where, by using the definition of the weights $n(t)$ and $m(t)$, we denote the portion of capital employed in the first (risky) and the second (riskless) workshop with $n(t)m(t) = \frac{K}{W}\frac{K1}{K}$ and $n(t)[1-m(t)] = \frac{K}{W}\frac{K2}{K}$, respectively. As opposed to the centralized allocation mechanism underlying the results discussed in Section 4, the decentralized economy is characterized by two competitive markets: the capital market and the government bond market. In the first one, firms rent capital from consumers, in the second one the government and private agents trade consols. In the next sections, we will characterize the competitive equilibrium conditional on a given tax structure for given policy parameters (not necessarily the optimal ones). We will then proceed by focusing on the design of an optimal policy, having as welfare–related benchmark the command optimum allocation described above.

6. Competitive Equilibrium with Net Income Taxes.

The set of equilibria we consider in this section is characterized by a tax structure which includes only net income taxes, which is, τ_1, τ_0 and τ_1' in (5.2.2). We will therefore abstract from both wealth and consumption taxes (i.e. $\Gamma=\mathfrak{C}=0$). Moreover, for the sake of simplicity, we will disregard the possibility that the public good deliver direct utility to private agents. We will first solve for the competitive equilibrium conditional on a given set of policy parameters. Then, we will address the issue of an optimal policy. Last, we will compare the portfolio effects of policy reform with the results of an analysis in partial equilibrium, which is, under the hypothesis that tax revenue is simply disposed of.

The consumer problem in this economy is that of finding both an optimal saving rule and an optimal portfolio rule as to maximize

$$\text{(6.1)} \qquad \underset{\{C,\, m,\, n\}}{\text{Max}} \; E_0 \int_0^{\infty} \frac{C(t)^{1-R}}{1-R} \exp(-\delta t) \qquad \delta>0,\ R\in(0,\infty)$$

subject to

$$dW(t) = n(t)W(t)\Big[\, m(t)[(1-\tau_1)\eta-(1-\tau_0)r] - (1-\tau_0)r - \frac{C(t)}{K(t)}\Big]dt +$$

$$+\; n(t)W(t)[(1-\tau_1')\; m(t)\sigma d\omega(t)] + [1-n(t)]W(t)\; [r_B(t)dt+\sigma_B(t)d\omega(t)]$$

$$K(t)\geq 0,\ C(t)\geq 0,\ K(0)=\overline{K}>0,\ W(t)\geq 0$$

Once again drawing on Merton's construction, the first order conditions for this problem yield

$$\text{(6.2)} \qquad C(t) = \chi(t)\; W(t)$$

(6.3)
$$\eta(1-\tau_1)-r_B(t) - R\left[m(t)n(t)(\sigma(1-\tau_1^{'})-\sigma_B(t)) + [1-n(t)(1-m(t))]\sigma_B(t)\right][\sigma(1-\tau_1^{'})-\sigma_B(t)] = 0$$

(6.5)
$$r(1-\tau_0)-r_B(t) - R\left[m(t)n(t)(\sigma(1-\tau_1^{'})-\sigma_B(t)) + [1-n(t)(1-m(t))]\sigma_B(t)\right][-\sigma_B(t)] = 0$$

where $\chi(t)$ must be non negative in order to satisfy the transversality condition. The first condition expresses consumption as linear in wealth. The following two expressions are standard first order conditions for the portfolio allocation when all risky assets are perfectly correlated.

Closed form solutions for $\chi(t)$, $n(t)$ and $m(t)$ require that the parameters underlying the stochastic return on bond $r_B(t)$ and $\sigma_B(t)$ be known. A possible strategy of solution is as follows. As preferences exhibit constant relative risk aversion while both policy and technology-related parameters are assumed to be time-invariant, we can conjecture that the equilibrium portfolio shares are independent of wealth and therefore constant in steady state. Consider the government budget identity. Under our conjecture, differentiating the definition of the market value of government debt in terms of wealth $q_B(t)B(t)=n(t)W(t)$ yields $d[q_B(t)B(t)]=n(t)dW(t)$. By dividing the latter by the former expression, it follows that both private wealth and the market value of debt grow at the same instantaneous stochastic rate, $\frac{dW(t)}{W(t)} = \frac{d[qB]}{qB}$. Therefore, by equating these two expressions and solving for the parameters of the return on bonds, we obtain

(6.5)
$$r_B(t) = \{(1-g)[m(t)(\eta-r)+r]- \frac{C}{K}\} + + \frac{n(t)}{1-n(t)}\{\tau_1 m(t)\eta+\tau_0(1-m(t))r-g[m(t)(\eta-r)+r]\}$$

$$\sigma_B(t) = \{(1-g')m(t)\sigma\} + (\tau_1'-g')m(t)\sigma \frac{n(t)}{1-n(t)} \tag{6.6}$$

The equilibrium expected return on bonds, $r_B(t)$, is the sum of two components. The first one (included in the first curly brackets) is the expected rate of capital accumulation. The second component is the expected primary surplus (tax revenue in excess of government spending), measured in bond units. *Mutatis mutandis*, an analogous structure characterizes the stochastic term of the bond return. This will be the sum of the standard deviations of both the growth rate and the primary surplus, also measured in bond units.

An intuitive interpretation of expressions (6.5) and (6.6) is the following. The first element in curly brackets reflects demand–induced capital gains and losses occurring when the investors try to adjust their portfolio share of bonds to the current level of capital. The second element is simply the flow of asset income. Note that, even if we do not consider taxes on bond income explicitly (they would not change the terms of the problem substantially), it may help to think of $r_B(t)$ and $\sigma_B(t)$ as net of taxes.

By using these expressions together with the first order conditions for the consumer problem, we are able to characterize both the portfolio and the saving decisions of the representative agent and check whether our conjecture about the time–invariance of equilibrium portfolio shares is verified in steady state. Let us first focus on the portfolio of productive assets. The share of capital in the first (risky) production process is

$$m(t) = m = \frac{\eta(1-\tau_1) - r(1-\tau_0)}{R\sigma^2(1-g')(1-\tau_1')} \tag{6.7}$$

As conjectured, m is independent of wealth and time. A simple comparison of this expression with the corresponding one for the command optimum $m(t)^* = m^* = \frac{(\eta-r)(1-g)}{R\sigma^2(1-g')^2}$ makes it clear that an optimal policy from the point of view of production efficiency requires not only different tax rate across assets but also the ability to tax the expected and the unexpected components of return *selectively.* In particular, it must be the case that

$$g=\tau_1\eta=\tau_0 r,\; g'=\tau_1' \tag{6.8}$$

Since an interior solution for the portfolio of productive assets requires $r<\eta$, then τ_0 must be higher than τ_1. The riskless asset should be taxed at an higher rate as far as production efficiency is concerned. Moreover, as long as $g'\neq g/\eta$, also $\tau_1'\neq\tau_1$ so that the government should be able to discriminate between different components of assets' return. Note that the optimal policy implies a balanced budget rule for the government. So, the next question is whether this policy rule is optimal also from the point of view of the intertemporal consumption/saving decision. Again, a possible way to explore this issue consists in comparing the intertemporal allocation in a competitive economy where the proposed optimal taxation policy is in place with the command optimum allocation. If the two are identical, it follows that our candidate tax rule is indeed optimal.

Define an indicator of fiscal policy, ϕ , as

$$\phi \equiv \tau_1 m\eta + \tau_0(1-m)r - g[m(\eta-r)+r] - (\tau_1'-g')R(1-g')m^2\sigma^2 \tag{6.9}$$

Note that ϕ will be zero if the government pursues the policy described in (6.8). By using the definition of ϕ, closed-form equilibrium expressions for the optimal consumption rate out of wealth *and* the optimal portfolio composition (which solves

(6.2)–(6.4)) can be written as follows:

$$(6.10) \qquad \chi(t) = \chi = \frac{R-1}{R}\left[(1-g)[m(\eta-r)+r] - \phi - .5R[(1-g')m\sigma]^2\right] + \frac{\delta}{R}$$

$$(6.11) \qquad n(t) = n = \frac{\chi}{\chi+\phi}$$

Since $C(t) = \chi\, W(t) = \chi\, \frac{K(t)}{n}$, the rate of consumption out of capital can be obtained by substituting the expression for the capital share in wealth (6.11) in the optimal consumption rule, which is,

$$(6.12) \qquad \frac{\chi}{n} = \chi + \phi = \frac{R-1}{R}\left[(1-g)[m(\eta-r)+r] - .5R[(1-g')m\sigma]^2\right] + \frac{\delta}{R} + \frac{\phi}{R}$$

If the policy (6.8) holds, ϕ is zero and this rate coincides with c^* in (4.4) characterizing the command optimum allocation. The balanced budget rule corresponding to a production efficient tax structure is indeed also optimal from the point of view of intertemporal allocation. This is a rather striking conclusion. One may wonder how it is possible that we are able to attain a first–best allocation while being constrained to use "distortionary" tax rates. The reason is quite simple and lies in the form of the spending function. Since government consumption is a given (stochastic) linear function of the capital stock, public spending is not independent of investment decisions by private agents, because one additional unit of output invested today will lead to a higher (expected) expenditure in the future. In a decentralized economy with lump–sum taxes, the private sector may fail to consider this link (see Corsetti [1991]). Non–lump–sum taxes are then the appropriate tool to make agents face the full intertemporal implications of their saving behavior. Therefore, in our simple framework with a representative agent and no market imperfection (such as intra–temporal externalities), public debt does not perform any economic role. A non–zero debt outstanding only increases the amount of distortions implied by the corresponding *level* of taxation needed to service it.

To sum up, our first example shows a case where both production and intertemporal efficiency entail a unique, identical policy rule, which consists in an instantaneously balanced budget with selective tax rates both across returns and across the unexpected and the expected component of the risky asset. Now, it could be interesting to consider a slightly different specification where production and intertemporal efficiency may be supported by different sets of policies. Consider for example a spending function where g'=0 and the government consumes some fraction ζdt of the capital stock. In this case, the command optimum allocation requires

$$m = \frac{\eta - r}{R\sigma^2} \tag{6.13}$$

This allocation can bc supported by two sets of policies. The first one consists in choosing tax rates which make tax parameters cancel out from (6.7) under the constraint that the government budget is instantaneously balanced. In other words, we pose $\tau_1'=0$ and solve the following system of linear equation

$$\begin{aligned} -\eta\tau_1+r\tau_0 &= 0 \\ m\eta\tau_1+(1-\mu)r\tau_0 &= \xi \end{aligned} \tag{6.14}$$

The solution is $\tau_1=\xi/\eta$ and $\tau_0=\xi/r$. Once again, the riskless asset is taxed at a higher rate. Because of the balanced budget rule, by the same argument made in the case of (6.8), this policy is intertemporally efficient. Nonetheless, there now exists a second policy consistent with production efficiency. It follows immediately that taxing assets' income at a flat rate $\tau_1=\tau_1'=\tau_0$ will lead to the desired distribution of capital among productive assets:

$$m = \frac{(\eta - r)(1-\tau)}{R\sigma^2(1-\tau)} = \frac{\eta - r}{R\sigma^2} \tag{6.15}$$

Unfortunately, this particular result holds for any tax rate. It is then apparent that the corresponding policy may not be optimal from the point of view of intertemporal efficiency. One the one hand, the tax rate may be too high (or too low, if we allow for negative public debt). On the other hand, even if the government tried to keep the magnitude of tax–distortions low, the presence of a contingent component in the tax function would make it impossible to balance the budget at each instant in time. Feasibility of the policy would require a non–zero supply of government debt in equilibrium, the service of which, in turn, will impose a sub–optimal level of taxation.

7. Income Taxes: Partial vs. General Equilibrium.

By partial equilibrium analysis we mean a situation where tax revenue is disposed of. The share of output variability absorbed by the government is not to appear anywhere in the private agents' budget constraints. To this respect, it is important to stress that in our construction, as in reality, net tax revenue can be negative. The disposal of tax revenue thus implies some hidden public endowment which supplies resources whenever needed.

In terms of the model developed in the previous sections, a partial equilibrium analysis would correspond to a particular balanced budget policy rule according to which public spending instantaneously adjusts to tax revenue. In other words, g and g' would now be endogenous variables, balancing the government budget in the back of the maximization problem. In this case, the optimal portfolio rule for the representative investor will be

$$m(t) = m = \frac{\eta(1-\tau_1) - r(1-\tau_0)}{R\sigma^2(1-\tau_1')^2} \tag{7.1}$$

Now, consider a flat–rate tax on assets income. It is evident that, since (7.1) becomes $m = \frac{\eta - r}{R\sigma^2(1-\tau)}$, an increase in τ will indeed raise risk–taking in the economy, by augmenting the amount of existing capital employed in the risky production process as opposed to the riskless one.

Let us now compare this conclusion with our results in the previous section. By (6.7), with a flat–rate income tax, the private optimal allocation of capital will be $m = \frac{\eta - r}{R\sigma^2(1-g')}$. In other words, the portfolio of productive assets will not be affected by the tax reform. Reforms will mainly change the intertemporal allocation, which is the rate of capital accumulation, but not the distribution of the existing stock between different uses.

This conclusion should not come as a surprise, as partial and general equilibrium models of taxation and risk–taking indeed refer to very different model specifications. In particular, tax revenue (or risk) disposal entails that the importance of the insurance role of government policy grows with the tax rate. On the other hand, in a general equilibrium model where revenue collection is kept separated from spending, it is the latter which provides capital income insurance (in terms of a filter between production and private risk). The level of a flat–rate income tax only affects the intertemporal choice.

8. A Graphical Analysis

The results discussed so far can be illustrated by using the standard graphical tools of mean–variance analysis. Consider the space (E,σ), which is the space of the expected value and standard deviation of returns. Given both the assumption of constant returns to scale in production and the assumption of stationarity underlying our specification, each return will identify a point in the (E,σ) space. Consider Figure 1, where our economy is analyzed from a purely technological vantage point. In this figure, we can find a point with coordinates $(r(1-g),0)$ for the riskless asset as well as a point with coordinate $(\eta(1-g),\ \sigma(1-g'))$ for the risky asset. These points are denoted with the letter A and B. In Section 3 we have shown that the aggregate production function will result from the combination of these productive assets in the portfolio of private agents. Thus, once m is known, expected output and output variability per unit of capital are also completely determined. They will have coordinates $(1-g)[m(\eta-r)+r]$ and $(1-g')m\sigma$, respectively, which correspond to the point T along the segment connecting A to B. Obviously, the share of the risky assets in production will be proportional to m. Thus, the level of social risk, given by the coordinate of the return to the aggregate production process on the y–axis, is not exogenous, but depends on the capital–allocation choice by the private sector. By the same token, the parameter m will identify the expected return to capital for the economy as a whole.

Since returns are distributed as a brownian motion, there is no loss of generality in assuming that preferences are locally quadratic in the (E,σ) space. We can therefore draw a map of upward–sloping indifference curves, as in Figure 2. It is apparent that maximizing expected utility will lead the representative investor to choose the combination of productive assets at the point of tangency between the segment AB and the indifference curve IC. This figure depicts the allocation corresponding to the command optimum in Section 4.

If the government budget is instantaneously balanced, as in the case of a competitive equilibrium with an optimizing benevolent government, points like E in

Figure 2 will also indicate the return to the market portfolio. However, suppose that, starting at that point, the government undertakes a series of tax reforms which keep m unchanged, i.e. it alters τ_0, τ_1 and τ_1' in such a way that, given (6.7), the initial value of m is not affected. For an unchanged capital–allocation among productive assets, we will now have a non–zero debt outstanding, because an instantaneously balanced budget rule will no longer be feasible. It is clear that the position of the return to productive assets in the space will change according to the new tax rates. It is less apparent, however, what happens to the return on the *market* portfolio. We can characterize this rate by combining the return on bonds and on capital with the weights n and $(1-n)$ respectively. We obtain

$$\text{(8.1)} \qquad r_m dt+\sigma_m d\omega(t) = \Big[(1-g)[m(\eta-r)+r]-\phi\Big]dt + (1-g')m\sigma d\omega(t)$$

This rate corresponds to points like E, S and F in Figure 3. As a first observation, note that, since we keep m constant by construction, the variability of the market rate (8.1) can only vary with the spending parameter g'. Yet, from a partial equilibrium vantage point, one could perceive that the government increases or decreases productive investment risk by changing tax parameters. However, once net of spending, production risk will be fully reflected by the market portfolio. Secondly, note that the expected return will vary with what with have called an indicator of fiscal policy, ϕ. An increase in ϕ will move the market to the left (point S), a decrease in ϕ to the right (point F). By (6.12), these points correspond to different growth paths for the economy: higher consumption rate and slower growth in S, lower consumption rate and faster growth in F. Our discussion throughout the paper makes it clear that only point E corresponds to the simultaneous achievement of both intertemporal and intratemporal efficiency.

Finally, the slope of the portfolio lines connecting different assets returns will depend on the set of tax rates. These lines need not be parallel across different equilibria. We have drawn three possible portfolio *loci* in Figure 3. Recall that all risky assets are perfectly correlated in our specification. In this case, once we know two out of the three relevant returns (on riskless and risky productive assets as well as on bonds), by a no–arbitrage condition the third return must lie on the portfolio line connecting the other two.

References

Atkinson Anthony B. and Joseph E. Stiglitz [*1980*], *Lectures on Public Economics*, McGraw Hill

Brock W.A. and A.G.Malliaris [1982], *Stochastic Methods in Economics and Finance*, North Holland

Corsetti, Giancarlo [1991], "A Mean–Variance Approach to Endogenous Growth: Fiscal and Financial Policies in the Long Run", mimeo, Yale University

Cox John C., Jonathan E. Ingersoll and Stephen A. Ross [1985], "An Intertemporal General Equilibrium Model of Asset Prices", **Econometrica** 53 363–384

Domar Evsey D. and Richard A. Musgrave [1944], "Proportional Income Taxation and Risk Taking", **Quarterly Journal of Economics** 58, 388–422

Hagen K.P. [1970], "Taxation and Investment Behaviour under Uncertainty: A Multiperiod Portfolio Analysis", Theory and Decisions 1, 269–294

Ingersoll J. [1987], *Theory of Financial Decision Making*, Rowman & Littlefield

Kanbur, S.M. [1981], "Risk Taking and Taxation: An Alternative Perspective", **Journal of Public Economics** 15, 163–184

Kihlstrom, Richard E. and Jean Jacques Laffont [1983], "Taxation and Risk Taking in General Equilibrium Models with Free Entry", **Journal of Public Economics** 21, 159–181

Merton Robert C. [1969], "Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case", in **Review of Economics and Statistics**, 247–257, reprinted in R.C. Merton [1990], *Continuous Time Finance*, Basil Backwell

Merton Robert C. [1975], "An Asymptotic Theory of Growth under Uncertainty", **Review of Economic Studies**, 375–394

Sadmo, Agnar [1985], "The effects of Taxation on Savings and Risk Taking", in A.J.Auerbach and M.Feldstein, eds., *Handbook of Public Economics*, Elsevier Science Publisher B.V. [North Holland]

Stiglitz, Joseph E. [1969], "The Effects of Income, Wealth and Capital Gains Taxation on Risk–Taking", **Quarterly Journal of Economics** 83, 262–283

Stiglitz, Joseph E. [1972], "Taxation, Risk Taking, and the Allocation of Investment in a Competitive Economy" in M.C. Jensen, ed., *Studies in the Theory of Capital Markets*, Praeger, New York

Figure 1

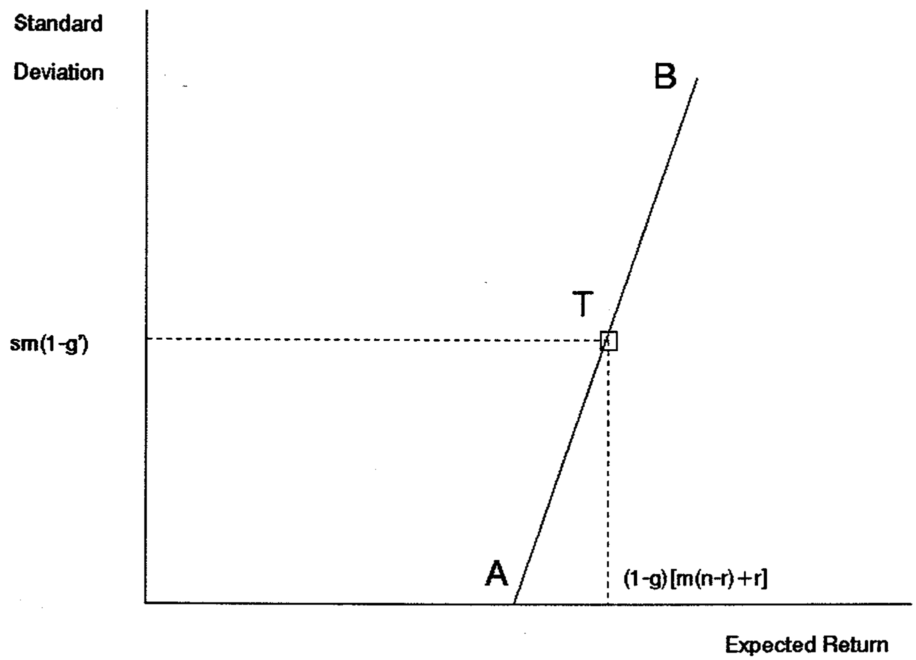


Figure 2

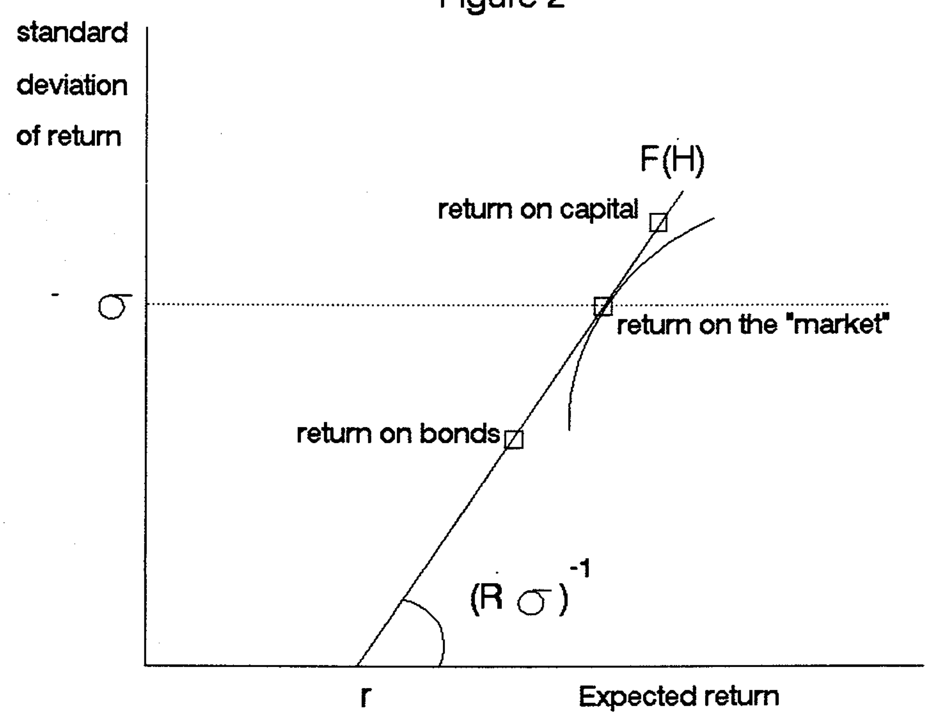
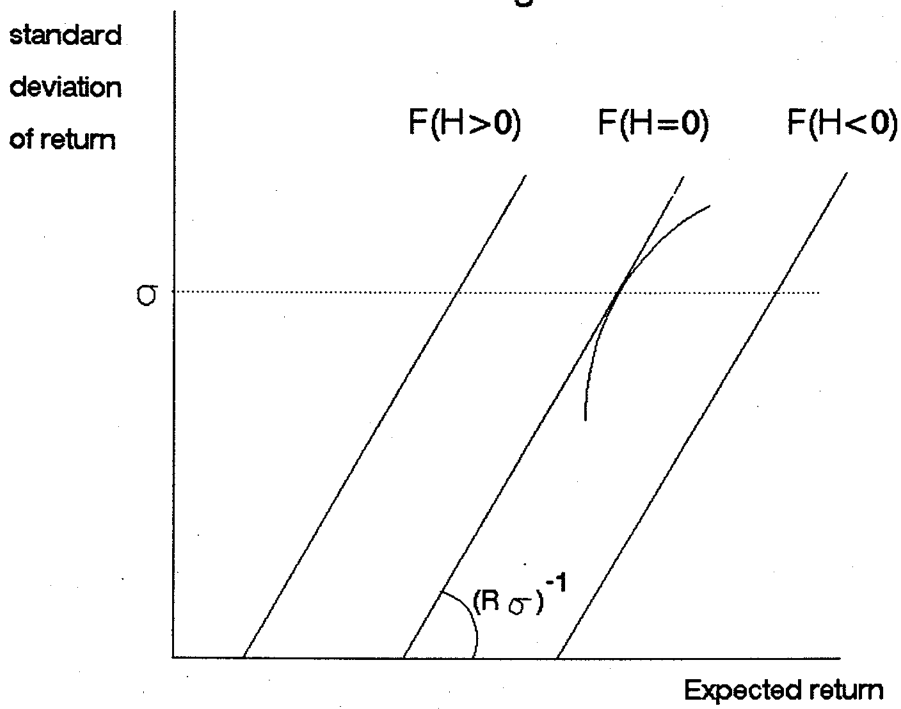


Figure 3


standard
deviation
of return
F(H>0)
F(H=0)
F(H<0)
σ
(R σ)^-1
Expected return