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LEARNING BY DOING AND DYNAMIC COMPARATIVE ADVANTAGE

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LEARNING BY DOING AND DYNAMIC COMPARATIVE ADVANTAGE

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In this paper, learning on the basis of cumulative production experience is integrated into a dynamic model of international trade. The result is a restatement of the infant-industry argument for protection, in terms of learning by doing.

The economic applications of learning by doing have been given considerable attention by Asher [3], Hirsch [7], Arrow [2] and, most recently, by Fellner [5]. In Arrow's model, the competitive market solution is shown to be suboptimal, since private entrepreneurs are not compensated for the learning which occurs as a result of their production. This feature is incorporated into the model below.

Dynamic models of international trade also occupy an important place in the recent literature. McKinnon [9], Baldwin [4], Findlay [6], and Jones [8] have each suggested interesting extensions to the static model of comparative advantage.

In the following sections these two approaches are brought together. Part I presents geometrically a two-period model of international trade. It is demonstrated that in the absence of learning by doing, free trade produces an optimal solution.

With the addition of learning by doing in one industry in Part II, however, free trade is shown to be generally suboptimal. Hence interference with the structure of internal prices by tariff or subsidy is justified.

Part III presents the argument in algebraic form, adding the possibility of learning in both industries. It is indicated that even if rates of learning are equal in each industry, free trade is not in general optimal, since future changes in the international terms of trade must be taken into account.

Part IV discusses possible modifications of the model. In extending the model to three or more periods it is shown that one must consider whether learning in one period has a positive or a negative effect on learning in subsequent periods.

Finally, Part V presents the conclusions.

I

This section will present a two-period model of an open economy which produces two products, machines and cloth, using as inputs machines and labour. By assumption, the machinery industry and the cloth industry will have different factor intensities. It will be assumed further that the relevant production functions exhibit constant returns to scale and that the technology of production remains constant over the two periods. The country will be small enough that its international terms of trade may be considered fixed, but not necessarily at the same rate in each period. Planners and entrepreneurs will be assumed to know at the beginning what these terms of trade will be in the two periods. The total labour force will be constant over the two periods, while the initial capital stock will be augmented for the second period by period-one production and imports of machinery. Finally, capital will be assumed to last for at least two periods, so that there is no depreciation.

The objective of the exercise for the planners will be to maximize the final stock of capital subject to constraints on the consumption of cloth in both periods. At issue is whether the economy should operate under free trade or whether it should protect one or the other of its two industries.

The diagram in figure 1(a) presents the situation in the first period. Here, the northern and western axes measure machinery, while the southern and eastern axes measure cloth. The initial capital stock is represented in quadrant II by the distance OA^1. This stock must be divided between the two industries in some proportion. Accordingly, the initial capital stock in the machinery industry will be measured from 0 to the left, while the initial capital stock of the cloth industry will be measured from A^1 to the right. Thus a point such as E^1 signifies initial capital stocks of OE^1 and A^1E^1 in machinery and cloth respectively. For each amount of capital such as OE^1 in machinery, there will be a corresponding first-period total product E^1F^1. In this way the total product of capital in machinery curve, TPK_m^1, is traced. Similarly, for the cloth industry, A^1E^1 yields E^1G^1 in cloth, and the curve TPK_c^1 is generated. It should be mentioned that the labour force will be assumed to be divided such that for each division of capital the value of the marginal product of labour is equal in the two industries. Note also that the TPK_m^1 curve is convex when viewed from above (as in the TPK_c^1 curve from below), owing to different factor intensities in the two industries.

Every point such as E^1 in quadrant II corresponds in quadrant I to a point such as Q^1 on the country's frontier of production possibilities for cloth and machinery. For example, Q^1 may be found by taking the distance OJ^1 in quadrant I equal to E^1G^1 in quadrant III (using the 45^o angle in quadrant IV), along with the distance J^1Q^1 equal to E^1F^1. Similarly, the other points along D^1B^1 may be found from the corresponding divisions of the initial capital stock.

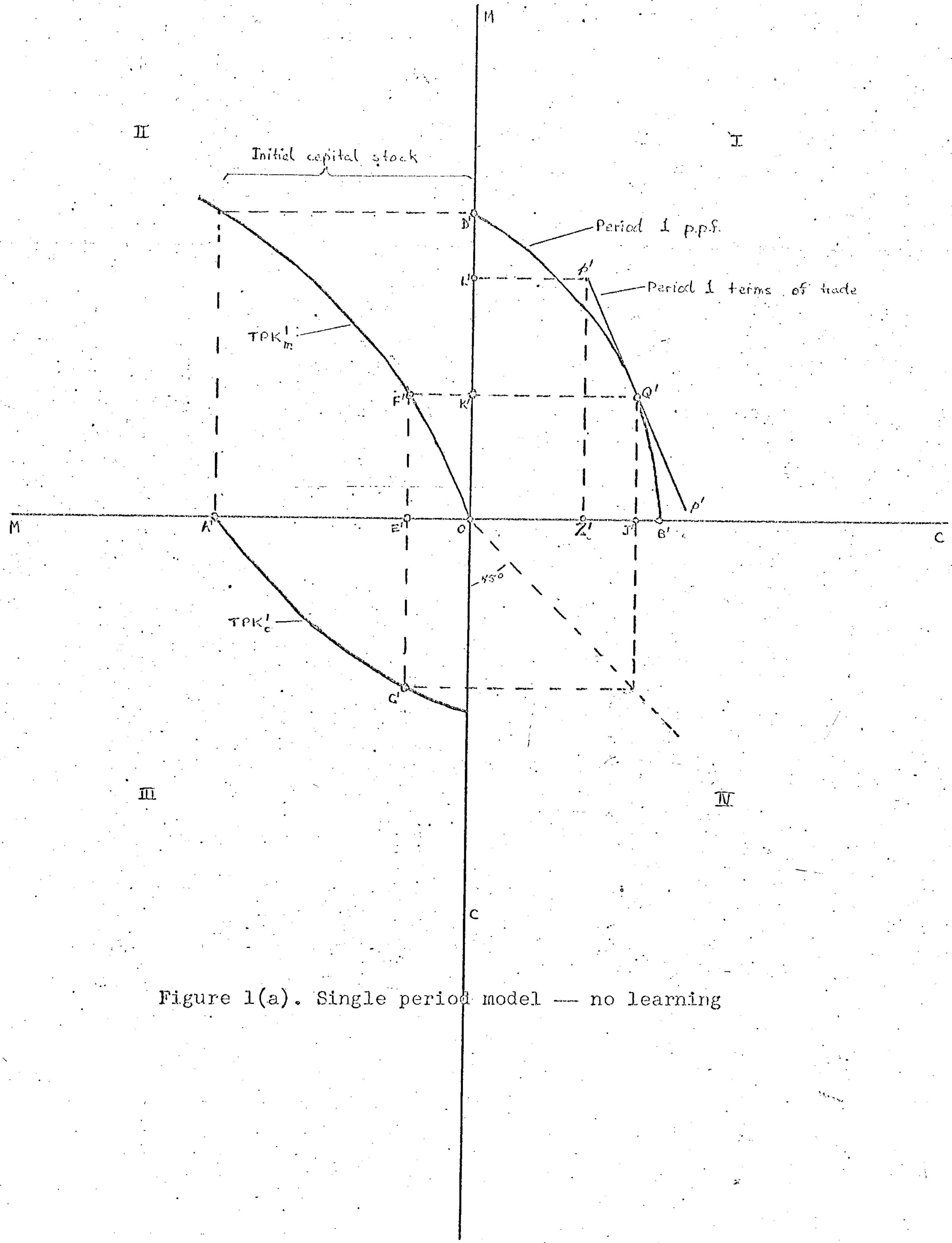

Figure 1(a). Single period model — no learning

The solution which maximizes the capital stock at the end of the first period may now be found. Suppose that first-period consumption is constrained at OZ^1. Then if the slope of p^1p^1 represents the first-period terms of trade, the economy should produce at Q^1, where this line is tangent to the production possibilities frontier. At this point, production of cloth will be OJ^1 and production of machinery OK^1. But the amount Z^1J^1 of cloth will be exported in exchange for the amount K^1L^1 of machinery. This is the solution that will be reached under free trade. In this way the availability of new machinery at the end of period one, OL^1, will be maximized, subject to the constraint on the consumption of cloth and the initial stock of capital.

In figure 1(b), the model is extended for a second period. The period-one supply of machinery, $OL^1 = A^1N^1 = A^1A^2$, is invested, increasing the total capital stock at the beginning of period 2 from OA^1 to OA^2 (the capital being assumed to last forever). Now this stock must be divided as before between the two industries. Since the labour force has been assumed constant, the ratio of labour to capital will have decreased for the economy as a whole. Moreover, with labour distributed between the two industries such that its value of marginal product is the same in each, the equilibrium amount of labour combining with any <u>given</u> amount of capital will fall in each industry. With a smaller ratio of labour to capital , however, the total product of this given amount of capital in either cloth or machinery will be less than in period one. Accordingly, the curves TPK_m^2 and TPK_c^2 will lie closer to the horizontal axis than their counterparts in period one. Note also that the TPK_c^2 curve begins at A^2 rather than A^1, owing to the enlarged stock of capital.

As in the first period, these two curves may be used to trace a production possibilities frontier, D^2B^2, in quadrant I. If the slope of p^2p^2

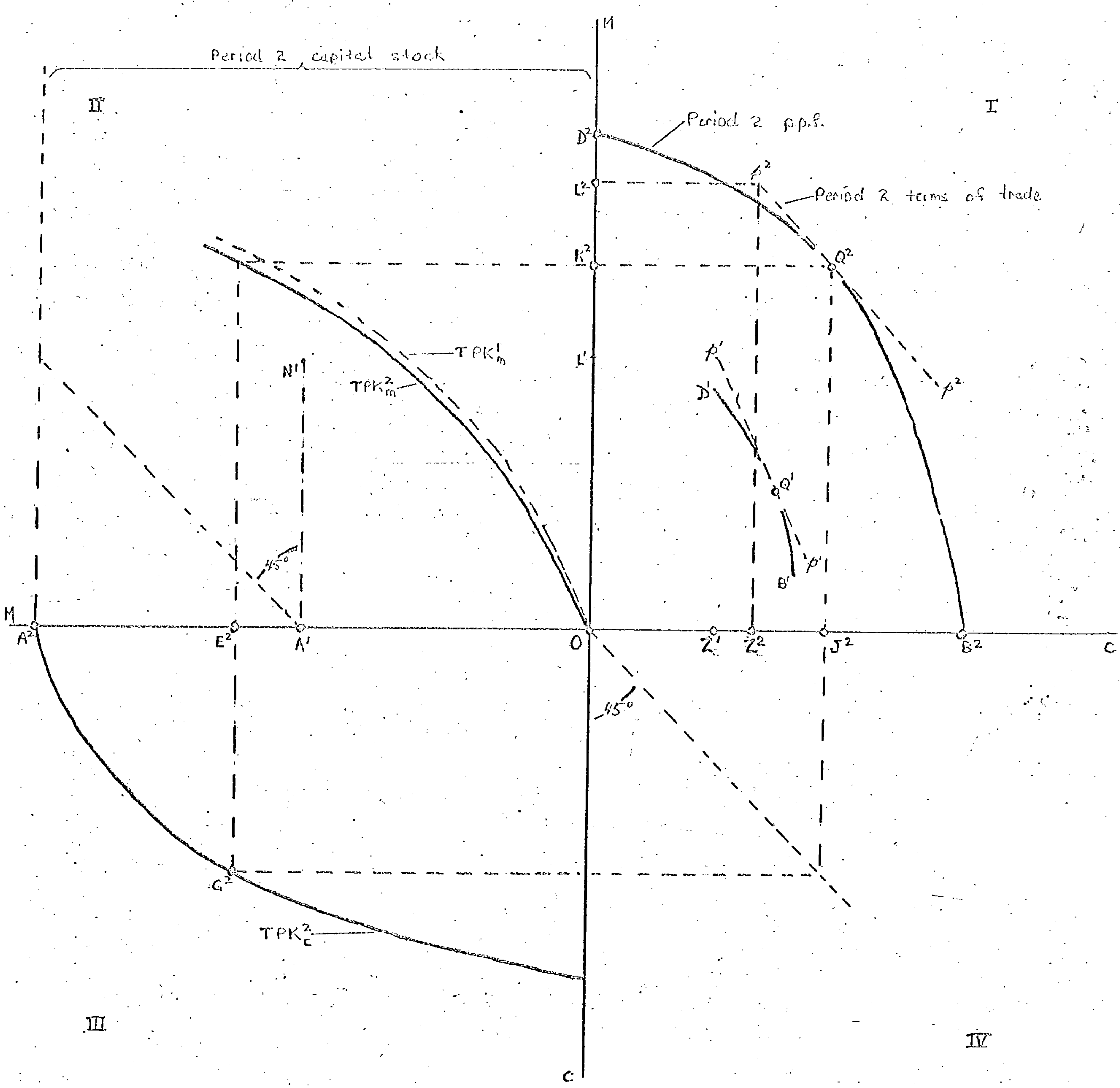

Figure 1(b). Two period model — no learning represents the second-period terms of trade, the optimal production point will be the free-trade point, Q^2, corresponding to OJ^2 of cloth and OK^2 of machinery. With consumption **constrained** at OZ^2, Z^2J^2 of cloth may now be traded for K^2L^2 of machinery. Thus, given OA^2 and OZ^2, the second-period availability of capital, OL^2, has been maximized.

The question which now arises is whether maximizing the availability of capital period by period has maximized the final capital stock, OA^2 plus OL^2. In this model, the only way that first-period production decisions can effect those in the second period is through their effects on the amount of investment in period one. Since first-period investment has been maximized, and since second-period investment has been maximized subject to that first-period investment, it follows that the final capital stock has been maximized. Another way of looking at the situation is to observe that the production possibilities frontier D^2B^2 corresponding to first-period production Q^1 lies outside the production possibilities frontier corresponding to any other point on D^1B^1 (because Q^1 maximized first-period availability of capital). Since Q^2 is the optimal point on this optimal **frontier, it follows** that the final capital stock has indeed been maximized.

Thus in this model with no learning, and with terms of trade determined exogenously, free trade leads to the optimal growth path, where optimal means that the final capital stock has been maximized subject to constraints on consumption.

II

In this section, the model of Part I will be modified by allowing second-period production of one of the goods, machinery, to depend in part on first-period production of that same good. Once again the objective will be to maximize the capital stock at the end of period 2, given an initial supply of capital, and with consumption constrained at OZ^1 and OZ^2 in periods one and two respectively. However, in accord with the learning-by-doing hypothesis, the productivity of the labour force in machinery in period 2 will be assumed to be a positive function of production experience in the machinery industry in period one.

In the diagram of figure 2, learning by doing affects second-period machinery production through the machinery industry's total product of capital curve in quadrant II. Instead of a single second-period curve as in the earlier model, though, there is now a curve for each level of first-period machinery production, with higher curves corresponding to higher levels of first-period output. If first-period production is sufficiently large, it is conceivable that learning by doing may offset the tendency for a lower labour-capital ratio to reduce the marginal product of capital. If so, the period-two curve will be higher than that of period one, as in the diagram.

To every level of period-one machinery production there will also correspond a level of machinery imports and hence, assuming all period-one machinery production and imports are invested, a new stock of capital at the end of period one. Each of these combinations of machinery production and capital stock will then give rise to a second-period production possibilities frontier in quadrant I, in a manner analogous to that of Part I. One of these, for example, will be the frontier corresponding to Q^1, the period-one production point under free trade. If all the benefits of learning in one firm are assumed to accrue to workers or to other firms--that is, if the benefits of learning are external to the firm--the free-trade point will remain the point of tangency between the production possibilities frontier and a line whose slope is equal to the first-period terms of trade.

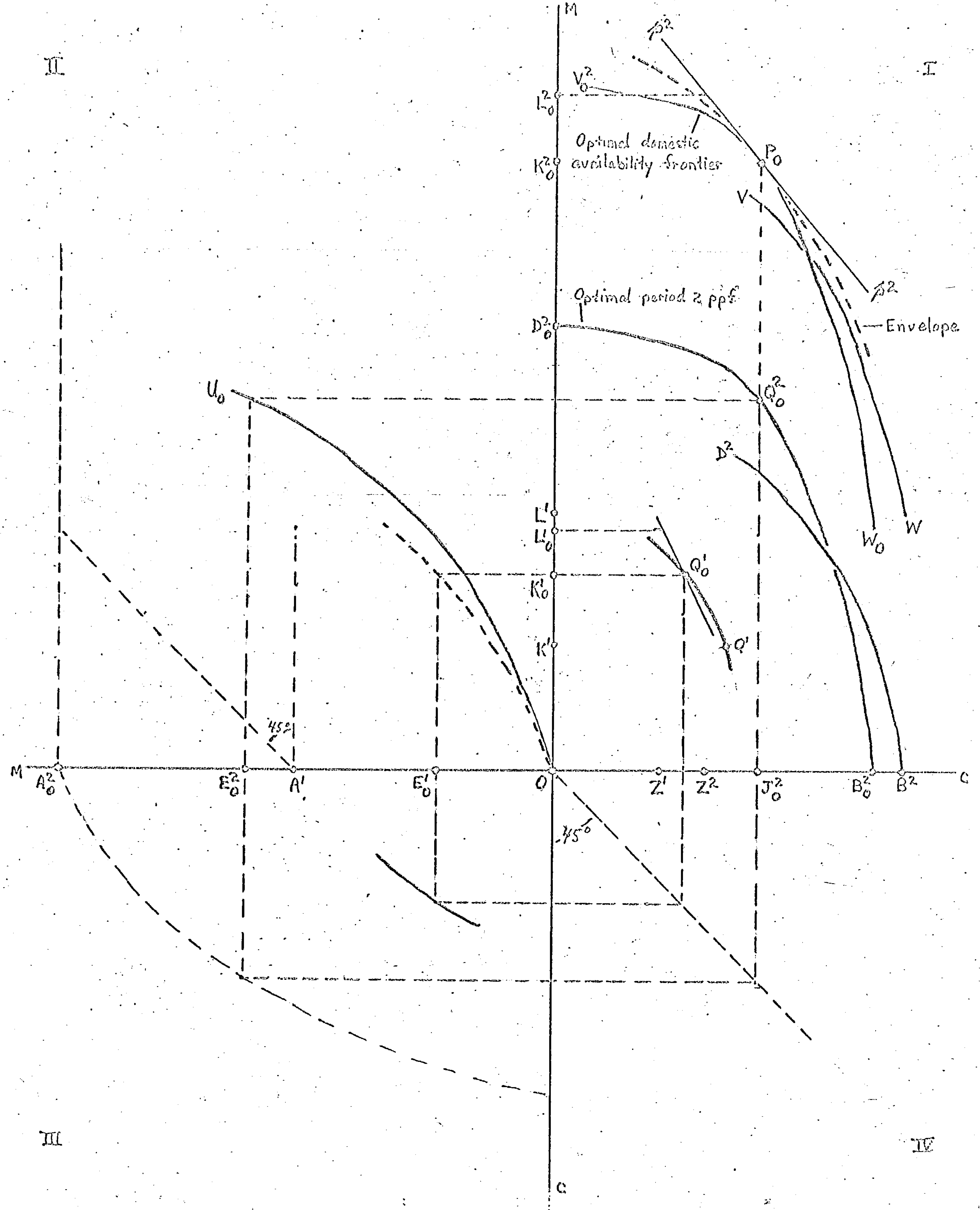

Figure 2. Two-period model — with learning in machinery

It is shown in the last section that the capital stock at the end of period one is maximized under free trade. Now if period-two production is to consist of cloth only, with no machinery, this free-trade solution will still be optimal, for the second-round capital stock is maximized, while the learning that takes place in machinery will never be put to use. Moreover, it may be seen that Q^1 still dominates all points to the right of and below it on the period-one frontier, regardless of what is to be produced in period two. The reason is that Q^1 involves a greater amount of machinery production, and therefore learning, than any of these alternative points, while at the same time it maximizes the availability of capital for the second round.

If a positive amount of machinery is to be produced in period two, however, Q^1 may be dominated by one or more points to the left and above on the first-period frontier. Because of the high cost of domestic machinery production compared to imports to the left of Q^1, the available stock of capital at the end of period one will decline as production points move to the left from Q^1. Offsetting this loss, though, will be second-period gains from learning by doing, as a result of the greater domestic production of machinery in the earlier period. Thus second-period production possibilities frontiers corresponding to first-period production points with increasing amounts of machinery (greater than OK^1) will have increasingly small intercepts with the horizontal axis. But if learning is sufficiently important they will have larger intercepts with the vertical axis, until the loss in efficiency begins to outweigh the gains from learning and the vertical intercepts also grow smaller. Compare, for example, the frontier D^2B^2 corresponding to Q^1 with the frontier $D_0^2B_0^2$ corresponding to the point Q_0^1. In effect, there is a tradeoff between the static gains from comparative advantage and the dynamic gains from learning.

To bring out more clearly the effects of this tradeoff on the final capital stock, one may add to each of the period-two production possibilities frontiers the corresponding period-one additions to the capital stock. Thus the investment that results from first-period production at Q^1, OL^1, may be added vertically to each point on D^2B^2, the second-period frontier corresponding to Q^1. The result, VW, is a period-two domestic availability frontier, showing the combinations of cloth and machinery corresponding to period-one production point Q^1 that are available within the economy before trade at the end of period 2, as a result of both period-one saving and period-two production. Similarly, if the amount OL_0^1 is added vertically to every point on $D_0^2B_0^2$, the result is the second-period domestic availability frontier V_0W_0 corresponding to the first-period production choice Q_0^1. In this way, a family of period-two availability frontiers may be generated in quadrant I, with one frontier corresponding to each point on the first-period frontier of production possibilities. (Note that the slopes of corresponding domestic availability and production possibilities frontiers will be equal at points such as P_0 and Q_0^2 which are equidistant from the vertical axis, since one frontier is simply a vertical projection of the other.)

Now what is the optimal time path for production? To answer this question it is necessary to add to quadrant I the envelope of the second-period domestic availability frontiers[1]. If p^2p^2 again represents the second-period terms of trade, the optimal point of second-period domestic availability will be the point on the envelope with slope p^2p^2. In the

[1]The issue of convexity of the envelope arises here. As mentioned, in moving to the left through the family of domestic availability frontiers, one sacrifices productivity of investment for productivity of learning. If both learning and investment are subject to decreasing returns, it seems probable that the envelope will be convex viewed from above.

diagram, that point is P_0 on the domestic availability frontier V_0W_0. If the amount $Z^2J_0^2$ in cloth is now traded for $K_0^2L_0^2$ in machinery, the final capital stock will be maximized at OA^1, the initial endowment, plus OL_0^2, total investment over the two periods. The corresponding point of second-period production is then Q_0^2 on the production possibilities frontier $D_0^2B_0^2$. Since the slope of the frontier at this point is the same as the slope of V_0W_0 at P_0, Q_0^2 will be reached under free trade, provided that the corresponding point of first-period production, Q_0^1, has been chosen.

The problem is to assure that first-period production occurs at the optimal point, Q_0^1. Since the period-two solution will generally involve positive production of machinery, Q_0^1 will lie to the left of Q^1, the free-trade point. The reason is that free trade leads to a sub-optimal level of production in the industry subject to learning. To reach Q_0^1, therefore, the internal price ratio will have to be altered by taxes or subsidies so that the internal terms of trade are equal to the slope of the production possibilities frontier at Q_0^1.

What investment program will make possible this optimal development path? From the diagram it may be seen that Q_0^1 may be produced only if the initial capital stock is divided in the ratio of $A^1E_0^1$ to E_0^1O between cloth and machinery respectively. When optimal period-one machinery production, OK_0^1 and machinery imports, $K_0^1L_0^1$, are added to initial stock, OA^1, the capital stock at the beginning of the second period is OA_0^2. Then if OU_0 is the period-two total product of capital curve for the machinery industry which corresponds to Q_0^1, Q_0^2 may be produced only if the second-period capital stocks of the cloth and machinery industries are $A_0^2E_0^2$ and E_0^2O respectively. Therefore, the required amount of investment in machinery at the end of period one is $E_0^2E_0^1$, while the required investment in cloth is the difference between this amount and total period one investment $A_0^2E_0^2$.

It may be concluded, therefore, that when there are benefits from learning by doing which are external to the firm, free trade may no longer be relied upon to produce an optimal path of production and investment.

III

The geometric model of dynamic comparative advantage presented in the last section will now be developed algebraicly. To add realism, learning from production experience will be permitted in both industries. However, the production processes will be assumed to be sufficiently different that learning in one industry will have no effect on the potential productivity of workers in the other industry. Thus there is no substitutability between the skills acquired in the two industries.

The notation to be used is as follows:

K = the initial capital stock or investment fund.

I_i^j = period j investment in industry i.

P_i^j = period j production in industry i.

Q_c^j = period j domestic consumption of cloth (exogenously determined).

X_c^j = period j exports of cloth.

p^j = the period j international terms of trade, expressed in units of machinery per unit of cloth (also exogenously determined).

L_i^j = an unspecified function expressing the amount of learning by workers in industry i during period j.

where $i = c$, m (for cloth and machinery respectively) and $j = 0,$ 1, 2. As before it will be assumed that capital lasts forever.

The objective will be to maximize the final capital stock, equal to $K + I_m^1 + I_c^1 + I_m^2 + I_c^2$, subject to the following conditions:

(i) Total initial investment in the two industries is equal to the initial investment fund.

$$I_m^0 + I_c^0 = K$$

(ii) Consumption of cloth in period one equals production in that period less exports, where production is a function of the amount of capital in the industry.

$$Q_c^1 = P_c^1(I_c^0) - X_c^1$$

(iii) Total investment in period one equals machinery imports plus domestic machinery production, where machinery imports are equal to the value of cloth exports, and machinery production is a function of the amount of capital in the industry.

$$I_m^1 + I_c^1 = p^1 X_c^1 + P_m^1(I_m^0)$$

(iv) Consumption of cloth in period two equals production in that period less exports. In this period, production is a function of previous investment in the industry, and of the amount of learning in the industry in period one, where this learning is in turn a function of the amount of consumer goods production in that period.

$$Q_c^2 = P_c^2 \left\{ I_c^1,\ I_c^0,\ L_c^1 \, [P_c^1 \, (I_c^0)] \right\} - X_c^2$$

(v) Total investment in period two equals machinery imports plus domestic production.

$$I_m^2 + I_c^2 = p^2 X_c^2 + P_m^2 \left\{ I_m^1,\ I_m^0,\ L_m^1[P_m^1(I_m^0)] \right\}$$

To solve this maximization problem, form the following Lagrangian expression, substituting for $I_m^2 + I_c^2$ from (v).

$$1. \qquad V = K + I_m^1 + I_c^1 + p^2 X^2{}_c + P_m^2$$
$$- \ \lambda_1 \left\{ I_m^0 + I_c^0 - K \right\}$$

$$- \lambda_2 \left\{Q_c^1 - P_c^1 + X_c^1\right\}$$
$$- \lambda_3 \left\{I_m^1 + I_c^1 - p^1 X_c^1 - P_m^1\right\}$$
$$- \lambda_4 \left\{Q_c^2 - P_c^2 + X_c^2\right\}$$

Partial differentiation yields the following conditions for optimality, assuming second order conditions have been fulfilled (see Appendix for formal proof).

$$\text{A.} \qquad \frac{dP_m^2}{dI_m^1} = p^2 \cdot \frac{dP_c^2}{dI_c^1} \qquad \text{or} \qquad \frac{dP_m^2/dI_m^1}{dP_c^2/dI_c^1} = p^2$$

$$\text{B.} \qquad \frac{dP_m^1/dI_m^0}{dP_c^1/dI_c^0} = \frac{p^1\left(1 + p^2 \cdot \frac{dP_c^2}{dI_c^1}\right) + p^2 \cdot \frac{dP_c^2}{dL_c^1} \cdot \frac{dL_c^1}{dP_c^1}}{1 + \frac{dP_m^2}{dI_m^1} + \frac{dP_m^2}{dL_m^1} \cdot \frac{dL_m^1}{dP_m^1}}$$

Equation A is the requirement pertaining to conditions in period two--the familiar tangency condition. It states that the marginal rate of transformation of machinery into cloth in period two (the slope of the production possibility frontier in absolute terms) should be equal to the international terms of trade. As in the geometric presentation, free trade is optimal in the final period.

Equation B, the first period requirement, is more interesting. It says that the internal marginal rate of transformation of machinery into cloth in period one should equal the first-period terms of trade, adjusted for

(i) the rates of learning in each industry, and

(ii) the subsequent period's international terms of trade.

If the rates of learning are zero in both industries, it may be seen from equations 9 and 10 in the Appendix that equation B becomes the tree-trade condition

$$\frac{dP^1_m/dI^0_m}{dP^1_c/dI^0_c} = \frac{p^1(1 + p^2 \cdot \frac{dP^2_c}{dI^1_c})}{1 + \frac{dP^2_m}{dI^1_m}} = p^1$$

that the internal terms of trade be equal to the international terms of trade.

Contrary to what one might at first suppose, even if rates of learning are equal in the two industries, free trade will not generally be optimal. Suppose, for example, that for an additional unit of period-one production there will be a one-tenth unit increase in period-two production in both sectors:

$$\text{i.e.} \quad \frac{dP^2_c}{dL^1_c} \cdot \frac{dL^1_c}{dP^1_c} = \frac{dP^2_m}{dL^1_m} \cdot \frac{dL^1_m}{dP^1_m} = 0.1$$

From equation B it may be seen that the marginal rate of transformation in period one should equal p^1 (the free-trade condition) only if $p^2 = p^1$--that is, if there is no change in the terms of trade from one period to the next. If $p^2 \neq p^1$, the value of the learning which occurs in one industry will be greater than the value of the learning in the other even though the crude rates of learning are equal. Another way to say this is that any change in the terms of trade enhances the value of the learning-type externalities in one industry relative to those of the other. Unless the former industry is protected in the first period, its period-one production will fall short of the level required to generate the level of learning that will maximize income in period two.

IV

It is worthwhile at this point to consider possible extensions of the two-period model developed in Parts II and III.

To begin with, it might appear that a model of three or more periods is the next logical step. The main advantage of such a model over one with just two periods is that the former permits consideration of the effects--positive or negative--which learning in one period may have on the rate of learning in a subsequent period. If the algebraic model of Part III is extended to three periods, for example, one of the key terms is what may be called an <u>interference effect</u>:

$$\frac{dL_m^2}{dL_m^1} \cdot \frac{dL_m^1}{dP_m^1} \quad \text{and} \quad \frac{dL_c^2}{dL_c^1} \cdot \frac{dL_c^1}{dP_c^1}$$

the effect of learning in period one on learning in period two.

In a learning function of the type

$$X_1 = aX_2^{-b} \qquad a,b > 0$$

where X_1 is the labour input per unit of output and X_2 is cumulative output, an increase in learning now will reduce the productivity boost resulting from a given increase in future production[1]. Presumably an increase in present learning helps to exhaust the amount which remains to be learned, thereby reducing the learning from a given amount of future production, other things being equal. In the above case, then, dL_m^2/dL_m^1 would be negative, lowering the total benefits from first-period learning in machinery.

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[1]One may show this by taking the second derivative of the above function

$$d^2X_1/dX_2^2 = ab(b+1)X_2^{-b-2}$$

and noting that it is positive; i.e., the first derivative increases as X_2 increases. Therefore the first derivative multiplied by -1, $-dX_1/dX_2$, which might be called the rate of learning, decreases with increasing production experience.

Alchian [1] has suggested, however, that the progress elasticity coefficient, b, in the above learning function may itself by an increasing function of past experience in related types of production. Hirsch [7] found some evidence to support this hypothesis. It would appear that in addition to the usual learning function, there is a "learning to learn" function by which workers acquire the techniques for learning the production skills they will acquire in later assignments. When these two effects are combined, it is by no means certain that dL_m^2/dL_m^1 will be strongly negative. Indeed, in the early stages of industrialization it is conceivable that rather than reducing the impact of future production on productivity, present learning may actually reinforce the learning resulting from future production experience.

A second possible modification of the model of Parts II and III would be to allow for depreciation of the capital stock. As long as the capital in each industry depreciates at the same rate, the methodology of Part III may be used unchanged. If the depreciation rates are different, however, equations 12 and 13 of the Appendix may not be simplified quite so easily.

A third change would be to permit productivity in one industry to be a function of both cumulative production in the same industry and production experience in the other industry. While the algebra would become somewhat more complicated, such a change would certainly be feasible.

Finally, it should be noted that this paper has not dealt with technological change in the usual sense of exogenous increases (either disembodied, or embodied in one or both factor inputs) in the output that can be obtained from given factor supplies. One of the problems that has puzzled development economists has been the inability of less-developed countries to take full advantage of the technology available from abroad. A hypothesis which may merit further attention is that the ability of a developing economy to absorb this exogenous technological change may be a function of the previous production experience of its managers and workers.

V

The incorporation of learning theory into a dynamic model of international trade produces a number of insights into the process of economic development.

1. In the first place, it becomes apparent that rather than being solely exogenously determined by geography and factor supplies, a country's comparative advantage at a point in time may be in part a function of its past production experience. Correspondingly, its future comparative advantage will be partly shaped by production decisions taken now.

2. In the second place, if the benefits of learning by doing are external to the firm, competitive markets will not in general yield an optimal development path. To reach such a path, government intervention in the price system will be necessary.

3. Finally, any such intervention will have to take account of a number of factors, including

    (a) relative rates of learning in each industry,

    (b) future changes in international prices and

    (c) the extent to which learning in one period either reinforces or interferes with learning in subsequent periods.

Thus the occurrence of learning by doing may provide valid grounds for the protection of infant industries. It is only when the future does not matter, as in the final period of the model presented in this paper, that an economy subject to the dynamic effects of learning can afford to permit free trade. Along the route to utopia, when present production experience can affect future comparative advantage, the government must interfere with the domestic structure of prices to assure optimality.

Appendix: Derivation of Equations A and B of Part III

Partial differentiation of equation 1 of Part III yields

2. $$\frac{dV}{dI_m^0} = \frac{dP_m^2}{dI_m^0} + \frac{dP_m^2}{dL_m^1} \cdot \frac{dL_m^1}{dP_m^1} \cdot \frac{dP_m^1}{dI_m^0} - \lambda_1 + \lambda_3 \cdot \frac{dP_m^1}{dI_m^0}$$

3. $$\frac{dV}{dI_c^0} = - \lambda_1 + \lambda_2 \cdot \frac{dP_c^1}{dI_c^0} + \lambda_4 \cdot \frac{dP_c^2}{dI_c^0} + \lambda_4 \cdot \frac{dP_c^2}{dL_c^1} \cdot \frac{dL_c^1}{dP_c^1} \cdot \frac{dP_c^1}{dI_c^0}$$

4. $$\frac{dV}{dX_c^1} = - \lambda_2 + \lambda_3 . p^1$$

5. $$\frac{dV}{dI_m^1} = 1 + \frac{dP_m^2}{dI_m^1} - \lambda_3$$

6. $$\frac{dV}{dI_c^1} = 1 - \lambda_3 + \lambda_4 \cdot \frac{dP_c^2}{dI_c^1}$$

7. $$\frac{dV}{dX_c^2} = p^2 - \lambda_4$$

From 7,

8. $$\lambda_4 = p^2$$

From 5,

9. $$\lambda_3 = 1 + \frac{dP_m^2}{dI_m^1}$$

From 6 and 8,

10. $$\lambda_3 = 1 + \lambda_4 \cdot \frac{dP_c^2}{dI_c^1} = 1 + p^2 . \frac{dP_c^2}{dI_c^1}$$

Equations 9 and 10 imply equation A of Part III.

From 4 and 10,

$$11. \quad \lambda_2 = \lambda_3 \cdot P^1 = \left(1 + p^2 \cdot \frac{dP_c^2}{dI_c^1}\right) P^1$$

From 3, 11 and 8,

$$12. \quad \lambda_1 = \left(1 + p^2 \cdot \frac{dP_c^2}{dI_c^1}\right) \cdot \frac{dP_c^1}{dI_c^0} + p^2 \cdot \frac{dP_c^2}{dI_c^0} + p^2 \cdot \frac{dP_c^2}{dL_c^1} \cdot \frac{dL_c^1}{dP_c^1} \cdot \frac{dP_c^1}{dI_c^0}$$

From 2 and 9,

$$13. \quad \lambda_1 = \frac{dP_m^2}{dI_m^0} + \frac{dP_m^2}{dL_m^1} \cdot \frac{dL_m^1}{dP_m^1} \cdot \frac{dP_m^1}{dI_m^0} + \left(1 + \frac{dP_m^2}{dI_m^1}\right) \cdot \frac{dP_m^1}{dI_m^0}$$

If capital lasts forever, and if there is no capital-embodied technological change,

$$\frac{dP_m^2}{dI_m^0} = \frac{dP_m^2}{dI_m^1} \quad \text{and} \quad p^3 \cdot \frac{dP_c^2}{dI_c^0} = p^3 \cdot \frac{dP_c^2}{dI_c^1}$$

Therefore, from A,

$$14. \quad \frac{dP_m^2}{dI_m^0} = p^3 \cdot \frac{dP_c^2}{dI_c^0}$$

Equations 12, 13 and 14 together imply equation B of Part III.

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