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Ryo Horii
Tohoku University
Yale University

Masako Ikefuji
Osaka University

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Natural Disasters in a Two-Sector Model of Endogenous Growth

Masako Ikefuji, Osaka University[†]

Ryo Horii, Tohoku University[‡] and Yale University[§]

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Abstract

Using an endogenous growth model with physical and human capital accumulation, this paper considers the sustainability of economic growth when the use of a polluting input (e.g., fossil fuels) intensifies the risk of capital destruction through natural disasters. We find that growth is sustainable only if the tax rate on the polluting input increases over time. The long-term rate of economic growth follows an inverted V-shaped curve relative to the growth rate of the environmental tax, and it is maximized by the least aggressive tax policy from among those that asymptotically eliminate the use of polluting inputs. Moreover, welfare is maximized under an even milder environmental tax policy, especially when the pollutants accumulate gradually.

Keywords: human capital, global warming, environmental tax, endogenous depreciation, nonbalanced growth path

JEL Classification Codes: O41, H23, Q54

[†]Institute of Social and Economic Research, Osaka University. 6-1, Mihogaoka, Ibaraki, Osaka 567-0047, Japan.

[‡]Correspondence: Graduate School of Economics and Management, Tohoku University. 27-1 Kawauchi, Aoba-ku, Sendai 980-8576, Japan. E-mail: horii@econ.tohoku.ac.jp. Tel: +81-22-795-6265. FAX: +81-22-795-6270.

[§]Economic Growth Center, Yale University. 27 Hillhouse Avenue, New Haven, CT 06511, USA.

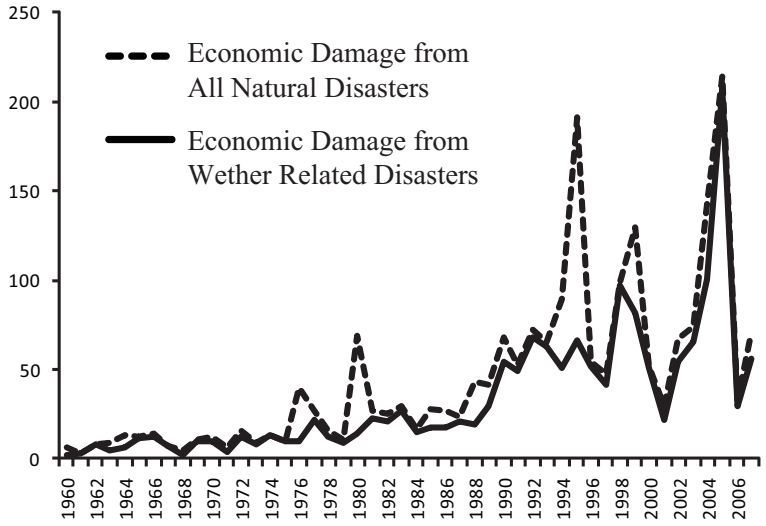

Figure 1: Economic Damage from Natural Disasters Worldwide (in billions of 2005 US dollars). Source: Damage estimates in current US dollars are from EM-DAT, the International Disaster Database, CRED, the Université Catholique de Louvain. Present value estimates in 2005 US dollars calculated using the implicit GDP price deflator from the Bureau of Economic Analysis.

1 Introduction

Natural disasters have a substantial impact on the economy, primarily through the destruction of capital stock. For example, Burton and Hicks (2005) estimated that Hurricane Katrina in August 2005 generated commercial structure damage of $21 billion, commercial equipment damage of $36 billion, and residential structure and content damage of almost $75 billion. These are not negligible values, even relative to the entire U.S. physical capital stock.[1] Figure 1 depicts the time series of the total economic damage caused by natural disasters throughout the world. Although the magnitude of damage caused by Hurricane Katrina may not appear typical, the figure clearly shows a steady and significant upward trend in economic damage arising from natural disasters.

One obvious reason behind this upward trend is the expansion of the world economy. As the world economy expands, it accumulates more capital, which means that it has

[1] In another study of the estimated costs of Hurricane Katrina, King (2005) reported that total economic losses, including insured and uninsured property and flood damage, were expected to exceed $200 billion. See Gaddis *et al.* (2007) for the full cost estimates.

more to lose from a natural disaster of a given physical intensity. However, this simple account cannot fully explain the overall growing trend in damages. To see this, we plot the ratio of the damage from natural disasters to world GDP in Figure 2. As shown, this ratio has been increasing since 1960. On this basis, the figure suggests that each unit of installed capital is facing an increasingly higher risk of damage and loss from natural disasters over time. This observation may then have serious implications for the sustainability of economic growth. Also, observe from Figures 1 and 2 that most economic damage is caused by weather-related disasters.[2] Accordingly, if economic activity is to some extent responsible for climate change, and if climate change affects the intensity and frequency of weather-related disasters,[3] economic growth itself poses a threat to capital accumulation and the sustainability of future growth.

This paper theoretically examines the long-term consequences of the risk of natural disasters on economic growth in a setting where economic activity itself can intensify the risk of natural disasters. We introduce polluting inputs, such as fossil fuels, into a Uzawa–Lucas type endogenous growth model, and assume that the use of polluting inputs raises the probability that capital stocks are destroyed by natural disasters. In the model, we show that as long as the cost of using polluting inputs is constant, economic growth is not sustainable because the risk of natural disasters eventually rises to the point at which agents do not want to invest in capital any further.

Given this result, we introduce a time-varying environmental tax on polluting input, which is shown to have both positive and negative effects on economic growth. On one hand, the faster the environmental tax rate increases, the lower the asymptotic amount of pollution and, therefore, the lower the probability of disasters. This gives households

[2]Specifically, we calculate the sum of damage from storms, droughts, extreme temperatures, floods, mass movements because of climate change, and wildfires.

[3]There is an ongoing scientific debate about the extent to which natural disasters and global warming relate to human activity. The Intergovernmental Panel on Climate Change Fourth Assessment Report (IPCC 2007, p.6) notes, "Anthropogenic warming over the last three decades has likely had a discernible influence at the global scale on observed changes in many physical and biological systems." According to Emanuel (2005) and Webster et al. (2005), increasing sea surface temperatures are suspected of increasing both the frequency and the intensity of hurricanes. We simply assume causality between the emission of greenhouse gases and the frequency of natural disasters. Scientific examination of the validity of this causality is beyond the scope of this paper.

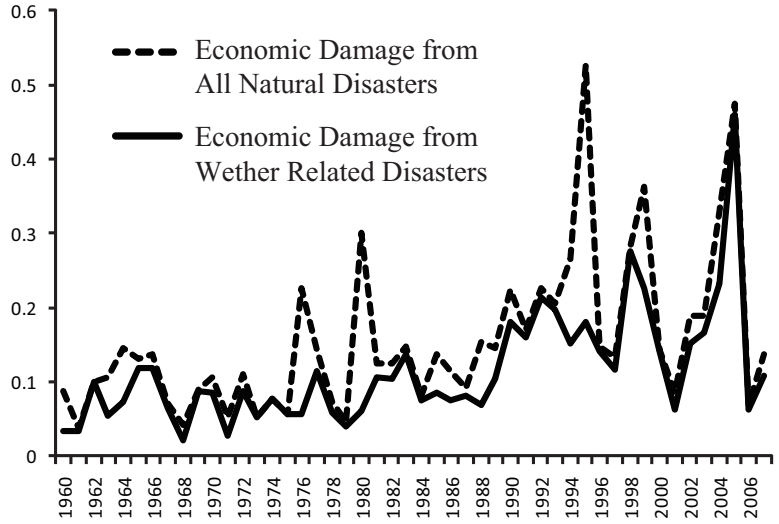

Figure 2: Ratio of Damage from Natural Disasters to World GDP (percent). Data source: World GDP (in current US dollars) is from World Development Indicators, World Bank Data Group.

a greater incentive to save, which promotes growth. On the other hand, the increased cost of using the polluting input by private firms reduces their (effective) productivity at each point in time, and this has a negative effect on growth. This paper shows that these opposing effects give rise to a nonmonotonic relationship between the long-term rate of economic growth and the speed with which the environmental tax increases. We characterize the policy that maximizes the long-term growth rate and examine how it differs from the welfare-maximizing policy. We also examine how the market equilibrium and the optimal policy are affected by the way in which pollutants accumulate.

1.1 Relationship to the literature

The literature on the link between natural disasters and economic growth is relatively new. However, there is an increasing amount of work investigating the theoretical and empirical relation.

On the empirical side, a seminal study by Skidmore and Toya (2002) found using cross-sectional data that the higher frequency of climatic disasters leads to a substitution from physical capital investment toward human capital. Consistent with their finding, our model shows that under appropriate environmental policies, agents accumulate

human capital stock much faster than output and physical capital, enabling sustained growth under limited use of the polluting input. Skidmore and Toya (2002) also found a positive correlation between the frequency of disaster and average growth rates over the period 1960–90, though subsequent studies have shown that this finding may depend on model specification and data. Notably, Raddatz (2007) considered a vector autoregressive (VAR) model for low-income countries with various external shocks, including climatic disasters, and his estimates showed that climatic and humanitarian disasters result in declines in real per capita GDP of 2% and 4%, respectively. Using panel data for 109 countries, Noy (2009) also found that more significant natural disasters (mainly in terms of direct damage to the capital stock) lead to more pronounced slowdowns in production.

The theoretical part of the literature is even more recent.[4] For instance, Soretz (2007) explicitly introduced the risk of disasters into an AK-type one-sector stochastic endogenous growth model and considered optimal pollution taxation. Hallegatte and Dumas (2009) considered a vintage capital model and showed that under plausible parameter ranges, disasters never promote economic growth through the accelerated replacement of old capital. Lastly, using numerical simulations, Narita, Tol, and Anthoff (2009) quantitatively calculated the direct economic impact of tropical cyclones. Our analysis complements these studies by considering both human and physical capital accumulation in addition to the polluting input. This is an important extension, not only because the substitution to human capital accumulation in the presence of disaster risk is empirically supported, but also because theoretically it is the key to sustained and desirable growth.[5] In addition, our methodology can analytically clarify the mutual causality between economic growth and the risk of natural disasters and how this relationship can

[4] Although not directly concerned with disasters, some previous studies analytically examined the effect of environmental quality on economic growth. Bovenberg and Smulders (1995) and Groth and Schou (2007), for example, considered where environmental quality affects productivity. Alternatively, John and Pecchenino (1994), Stokey (1998), and Hartman and Kwon (2005) introduced the disutility of pollution into endogenous growth models.

[5] Using a growth model with pollution and physical capital, Stokey (1998) showed that sustained growth is not desirable even when it is technically feasible. However, Hartman and Kwon (2005) found that Stokey's (1998) result is overturned when human capital is introduced.

be altered by environmental tax policy.[6] Rather than merely considering the optimal tax policy, we consider arbitrary dynamic tax policies and find both welfare-maximizing and growth-maximizing policies.

Finally, our analysis is technically related to Palivos, Wang, and Zhang (1997). In theoretical studies of long-term growth, it is common to focus only on balanced growth paths (BGP). However, it turns out that the risk of capital destruction makes the system of the economy inevitably nonhomothetic, implying that any BGP may not exist. We overcome this problem by extending the method in Palivos *et al.* and consider a broader than usual family of equilibrium paths that asymptote to a BGP only in the long run.

The rest of the paper is organized as follows. Section 2 constructs the model and shows that growth cannot be sustained if the cost of (tax on) the polluting input is constant. We then derive the (asymptotically) balanced growth equilibrium path under a time-varying environmental tax in Section 3. The welfare analysis is in Section 4. Section 5 considers a general version of the model in which pollution accumulates gradually. Section 6 concludes. The Appendix contains the proofs and derivations.

2 The Model

2.1 Production technology and the risk of natural disasters

Consider an Uzawa–Lucas growth model where the economy is populated by a unit mass of infinitely lived homogeneous households owning physical and human capital, and a unit mass of homogenous competitive firms owning a production technology. One difference in our model from Lucas (1988) is that production requires not only physical capital K_t and human capital H_t, but also a polluting input P_t, such as fossil fuels that emit pollutants and greenhouse gases. (For compact notation, we employ subscript $_t$ rather than (t), even though time is continuous.) Specifically, the production function of the

[6] Narita, Tol, and Anthoff (2009) assume that the savings rate is exogenous, while in our model it reacts endogenously to the risk of disasters. In Hallegatte and Dumas (2009), the long-term rate of growth is ultimately determined by the exogenous growth in total factor productivity (TFP), while in our model it is determined by endogenous human and physical accumulation.

representative firm is given by:

$$Y_t = F(K_t, u_t H_t, P_t) = A K_t^{\alpha} (u_t H_t)^{1-\alpha-\beta} P_t^{\beta}, \tag{1}$$

where $u_t \in [0,1]$ is the fraction of time devoted to the production of goods, A is a constant productivity parameter, $\alpha \in (0,1)$ represents the share of physical capital, and $\beta \in (0, 1-\alpha)$ is the share of the polluting input. Note that the production function (1) exhibits constant returns-to-scale with respect to all inputs, including P_t. All output is either consumed or added to the physical capital stock.

The representative firm can use an arbitrary amount of the polluting input P_t;[7] however, the use of the polluting input raises the risk of natural disasters. Specifically, the accumulated stock of pollution determines the frequency of natural disasters as well as the probability distribution of their intensity, and hence affects the proportions of human and physical capital that are lost due to natural disasters. Let us start from a simpler case where the depreciation of the pollution stock is fast enough that we can use the current use of the polluting input P_t interchangeably with the stock of pollution. (We maintain this assumption until we explicitly consider the accumulation process in Section 5). Given P_t, let $Q(P_t)$ denote the frequency of natural disasters per unit time, and $\Phi(r; P_t)$ the distribution function of the proportional damage $r \in [0,1]$ to physical capital, and $\Psi(r; P_t)$ denote that to human capital. We assume that physical and human capital are distributed across infinitely many areas in the economy, and that the damages by natural disasters are uncorrelated across areas.[8] Then, by the law of large numbers, the aggregate losses of physical and human capital stocks per unit time are

$$\Delta K_t = \int_0^1 Q(P_t) r K_t d\Phi(r; P_t) \quad \text{and} \quad \Delta H_t = \int_0^1 Q(P_t) r H_t d\Psi(r; P_t). \tag{2}$$

Observe that these losses are linear functions of K_t and H_t, respectively, multiplied by

[7] We ignore the finiteness of polluting inputs (e.g., fossil fuels), as our focus is on their effect on the risk of natural disasters. Sustainability of economic growth in endogenous growth models with nonrenewable resources has been examined by, for example, Grimaud and Rougé (2003), Tsur and Zemel (2005), and Groth and Schou (2007). Eliasson and Turnovsky (2005) examined the growth dynamics with a resource that recovers only gradually. This paper complements these studies.

[8] This assumption may not be appropriate when a large-scale disaster takes place. In this case, the law of large numbers does not apply and the disaster causes a short-term fluctuation. However, as we focus on the long-term behavior of the economy, the analysis of such fluctuations is left for future research.

functions of P_t. As the (stock of) pollution P_t increases, ΔK_t and ΔH_t increase due to both the increase in the frequency $Q(P_t)$ and the upward shifts of intensity distributions $\Phi(r; P_t)$ and $\Psi(r; P_t)$. Throughout the paper, we focus on the case expressions in (2) can be approximated by linear functions of P_t as follows:

$$\Delta K_t = (\overline{\delta}_K + \phi P_t)K_t \quad \text{and} \quad \Delta H_t = (\overline{\delta}_H + \phi P_t)H_t, \tag{3}$$

where constants $\phi > 0$ and $\psi > 0$ represents the marginal effects of P_t on the expected proportional damages to physical and human capital, respectively.

By incorporating the expressions for damages (3) into Lucas (1988)'s specification for the resource constraints for the physical and human capital stocks, we obtain

$$\dot{K}_t = F(K_t, u_t H_t, P_t) - C_t - (\delta_K + \phi P_t)K_t, \tag{4}$$

$$\dot{H}_t = B(1 - u_t)H_t - (\delta_H + \psi P_t)H_t, \tag{5}$$

where C_t, B, and $1 - u_t$ are aggregate consumption, the constant productivity of human capital accumulation, and the fraction of time devoted to the production of human capital, respectively. Note that constants for depreciation δ_K and δ_H now include both constants for expected damage ($\overline{\delta}_K$ and $\overline{\delta}_H$) and also the depreciation of capital for other reasons as assumed in Lucas (1988). Equations (4) and (5) show that the risk of natural disasters effectively augments the depreciation rates of physical and human capital stocks in proportion to the use of the polluting input.

Observe that, unlike standard endogenous growth models, the right-hand sides of equations (4) and (5) are not homogenous of degree one in terms of quantities (i.e., in K_t, H_t, P_t, and C_t). This implies that a BGP that exhibits the homothetic expansion of all of these variables is not feasible. This has important implications for the possibility of sustained growth, as we discuss below.

2.2 The market economy

We start the analysis with the market economy where the government levies a per unit tax τ_t in terms of final goods on the use of polluting inputs. The tax revenue $T_t = \tau_t P_t$ is equally distributed among households in a lump-sum fashion. At the beginning of the economy, the government announces the tax rate τ_t for all t, and it is assumed that the government can commit to this tax policy.

2.2.1 Behavior of households

Each household is faced with the risk of damage by natural disasters to its physical capital stock, k_t, and its human capital stock, h_t. The insurance market is assumed complete. Under this assumption, it is optimal for the household to take out insurance that covers all of the losses associated with natural disasters. The insurance premium for fully covering the physical and human capital damages, respectively, are equal to their expected losses: $(\overline{\delta}_K + \phi P_t)k_t$ and $(\overline{\delta}_K + \psi P_t)h_t$ from (3). Then, the budget constraint of the household can be written as:[9]

$$\dot{k_t} = r_t k_t + w_t u_t h_t - (\delta_K + \phi P_t)k_t - c_t + T_t, \tag{6}$$

$$\dot{h_t} = B(1-u_t)h_t - (\delta_H + \psi P_t)h_t, \tag{7}$$

where r_t, w_t, and c_t denote the real interest rate, the real wage rate, and the amount of consumption, respectively. Note that in our setting, the costs associated with depreciation and insurance are paid by the owner of the capital.

The utility function of the representative household is given by:

$$\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \tag{8}$$

where $\theta > 1$ is the inverse of the elasticity of intertemporal substitution and ρ is the rate of time preference. We assume $B - \delta_H > \rho$ so that households have sufficient incentive to invest in human capital. Given the time paths of r_t, w_t, P_t, τ_t, and T_t, each household maximizes (8) subject to the constraints (6) and (7). From the first-order condition for the maximization problem, we obtain the Keynes–Ramsey Rule (see the Appendix for the derivation):

$$-\theta \frac{\dot{c}_t}{c_t} = \rho + \phi P_t + \delta_K - r_t. \tag{9}$$

This condition is similar to that obtained in the original Uzawa–Lucas model, except that the depreciation rate is augmented by the risk of natural disasters, ϕP_t.

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[9] Equation (7) implicitly assumes that both the insurance payment and the compensation for human capital damage are in the form of human capital. Obviously, a more realistic setting is that they are in the form of goods (or money). This will not change the equilibrium outcome at the aggregate level as long as the amounts of the insurance payments and the compensation in terms of goods are calculated using the appropriate price of human capital, w_t/B.

In the household's problem, the shadow prices of the physical and human capital stocks are $c_t^{-\theta}$ and $(w_t/B)c_t^{-\theta}$. This means that the value of human capital in terms of physical capital (final goods) is w_t/B, which changes at the rate of $\dot{w}_t/w_t$. We find in the Appendix that the household is indifferent between physical capital investment and human capital investment when the following equation holds:

$$\frac{\dot{w}_t}{w_t} = r_t - (\phi - \psi)P_t - (\delta_K - \delta_H) - B. \tag{10}$$

In (10), the left-hand side (LHS) is the rate of change in the value of human capital in terms of physical capital, while the right-hand side (RHS) represents the difference between the marginal return to investment in physical capital and human capital. In the long run, condition (10) must be satisfied because if it is not, the solution is either $u_t = 0$ or $u_t = 1$ for all agents, and therefore one of the two kinds of aggregate capital stock approaches zero because of depreciation. However, this raises the shadow price of that type of capital stock, which is at odds with the decision of agents not to invest in it. Finally, the respective transversality conditions (TVCs) for the physical capital stock and the human capital stock are:

$$\lim_{t\to\infty} k_t c_t^{-\theta} e^{-\rho t} = 0, \tag{11}$$

$$\lim_{t\to\infty} h_t (w_t/B) c_t^{-\theta} e^{-\rho t} = 0. \tag{12}$$

2.2.2 Behavior of firms

All markets are perfectly competitive. Therefore, the representative firm maximizes profit, taking as given the rental rate r_t and the wage rate w_t, along with the tax rate of the polluting input τ_t. For simplicity, we assume there is no cost associated with the extraction and/or production of the polluting input besides the tax. Then, the firm's problem is written as:

$$\max_{K_t, N_t, P_t} F(K_t, N_t, P_t) - r_t K_t - w_t N_t - \tau_t P_t,$$

where the production function $F(K_t, N_t, P_t)$ is given by (1), and $N_t \equiv u_t H_t$ represents the amount of human capital employed by the firm. The first-order conditions for this problem are:

$$r_t = \alpha \frac{Y_t}{K_t}, \quad w_t = (1 - \alpha - \beta)\frac{Y_t}{N_t}, \tag{13}$$

and $\tau_t = \beta Y_t / P_t$. Note that the final condition means that the profit maximizing amount of the polluting input is $P_t = \beta Y_t / \tau_t$. Substituting this back into the production function (1), output can be written as:

$$Y_t = \left(\widetilde{A} \tau^{-\frac{\beta}{1-\beta}} \right) K_t^{\widehat{\alpha}} N_t^{1-\widehat{\alpha}}, \tag{14}$$

where $\widetilde{A} \equiv \beta^{\beta/(1-\beta)} A^{1/(1-\beta)}$ and $\widehat{\alpha} \equiv \alpha/(1-\beta)$. When written in the form of (14), it becomes clear that the environmental tax lowers the effective TFP, $\widetilde{A}\tau^{-\beta/(1-\beta)}$.

2.3 Equilibrium conditions

Now we can summarize the equilibrium conditions in terms of the motions of five variables: K_t, H_t, u_t, C_t, and P_t. Note that as the population is homogenous and normalized to unity, $K_t = k_t$, $H_t = h_t$, $C_t = c_t$, and $u_t = N_t / H_t$ hold in equilibrium. Substituting factor prices (13) as well as the lump-sum transfer $T_t = \tau_t P_t$ into the budget constraint of households (6) yields the evolution of the aggregate physical capital stock:

$$\frac{\dot{K}_t}{K_t} = \frac{Y_t}{K_t} - \frac{C_t}{K_t} - (\delta_K + \phi P_t). \tag{15}$$

From the production function of human capital (7), the evolution of the aggregate human capital stock is given by:

$$\frac{\dot{H}_t}{H_t} = B(1 - u_t) - (\delta_H + \psi P_t). \tag{16}$$

Substituting factor prices (13) into the arbitrage condition (10), we obtain the evolution of the fraction of time devoted to production u_t:

$$\frac{\dot{Y}_t}{Y_t} - \frac{\dot{u}_t}{u_t} - \frac{\dot{H}_t}{H_t} = \alpha \frac{Y_t}{K_t} + (\psi - \phi) P_t - (\delta_K - \delta_H) - B. \tag{17}$$

The consumption dynamics are given by the Keynes–Ramsey Rule (9) where r_t is replaced by (13):

$$-\theta \frac{\dot{C}_t}{C_t} = \rho - \alpha \frac{Y_t}{K_t} + \delta_K + \phi P_t. \tag{18}$$

Finally, from the firm's first-order condition (see the previous subsection), the amount of polluting input is determined by:

$$P_t = \beta Y_t / \tau_t. \tag{19}$$

The equilibrium dynamics are determined by equations (15)-(19), the TVCs (11) and (12), exogenously given time path of τ_t, and initial levels of K_0 and H_0.

Note that the TVCs can be simply stated using the equilibrium conditions. From (15) and (18), the growth rate of $k_t c_t^{-\theta} e^{-\rho t}$ is $(1-\alpha)(Y_t/K_t) - (C_t/K_t)$. Similarly, from (9), (10), and (16), the growth rate of $h_t(w_t/B)c_t^{-\theta}e^{-\rho t}$ is $-Bu_t$. Therefore, a sufficient condition for the TVC is that these growth rates are negative in the long run:

$$(11), (12) \;\Leftarrow\; \lim_{t\to\infty}((1-\alpha)(Y_t/K_t) - (C_t/K_t)) < 0, \quad \lim_{t\to\infty} u_t > 0. \tag{20}$$

Condition (20) implies that the TVCs are satisfied when more than fraction $1-\alpha$ of output is consumed and the fraction of time used for production converges to a strictly positive value. For use later, we also present the necessary conditions for the TVCs:

$$(11) \;\Rightarrow\; \lim_{t\to\infty}((1-\alpha)(Y_t/K_t) - (C_t/K_t)) \text{ should not be positive,} \tag{21}$$

$$(12) \;\Rightarrow\; \lim_{t\to\infty} \dot{u}_t/u_t \text{ should not be negative.} \tag{22}$$

Condition (22) is slightly weaker than the sufficient condition $\lim_{t\to\infty} u_t > 0$ in that it allows for the possibility that "$\lim_{t\to\infty} u_t = 0$ and $\lim_{t\to\infty} \dot{u}_t/u_t = 0$". In other words, condition (22) states that if the fraction of time used for production converges toward zero, it must do so very slowly.[10]

2.4 Sustainability of growth under a constant tax rate

Let us examine the long-run property of the economy under a simple environmental policy where the government sets a constant per unit tax rate on P_t. Under this policy, and from (19), pollution increases in proportion to output Y_t. Given that the increasing use of the polluting input makes natural disasters increasingly more frequent, it appears that economic growth is not sustainable under such a static environmental policy. The following proposition formally shows that this insight is correct.

[10] If $\lim_{t\to\infty} \dot{u}_t/u_t < 0$, the rate of change in $h_t(w_t/B)c_t^{-\theta}e^{-\rho t}$, which is $-Bu_t$, converges towards zero very rapidly. In that case, $h_t(w_t/B)c_t^{-\theta}e^{-\rho t}$ cannot reach zero in the long run, and therefore the TVC (12) is violated. To show this statement mathematically, define $V^h(t) \equiv \log(h_t(w_t/B)c_t^{-\theta}e^{-\rho t})$. Given the growth rate of $h_t(w_t/B)c_t^{-\theta}e^{-\rho t}$ is $-Bu_t$, it follows that $\dot{V}^h(t) = -Bu_t$. The TVC (12) is equivalent to $\lim_{t\to\infty} V^h(t) = -\infty$. For arbitrary $T > 0$, $\lim_{t\to\infty} V^h(t) = V^h(T) - B\int_T^\infty u_t\,dt$. The first term is finite. In addition, when $\lim_{t\to\infty} \dot{u}_t/u_t < 0$, the integral of the second term is also finite. Therefore, the TVC is violated if $\lim_{t\to\infty} \dot{u}_t/u_t < 0$.

Proposition 1 *If the per unit tax on the polluting input is constant, then economic growth is not sustainable in the sense that aggregate consumption cannot grow in the long run.*

Proof: The proof goes via *reductio ad absurdum*. Suppose that the government sets a constant environmental tax rate (i.e., $\tau_t = \tau_0$ for all t) and consumption grows in the long run (i.e., $\lim_{t\to\infty} \dot{C}_t/C_t > 0$). The Keynes–Ramsey Rule (18) can be rewritten from (19) as:

$$-\theta\frac{\dot{C}_t}{C_t} = \rho + \delta_K - \left(\alpha - \frac{\phi\beta}{\tau_0}K_t\right)\frac{Y_t}{K_t}. \tag{23}$$

For the LHS to be negative, the sign of the value in the parentheses on the RHS must be positive. Hence, physical capital K_t must be bounded above by a constant value $\tau_0\alpha/\phi\beta$ (i.e., $\lim_{t\to\infty} K_t < \tau_0\alpha/\phi\beta$).

Next, let us consider the amount of human capital used for production, $N_t = u_t H_t$. Note that from (14), $\dot{Y}_t/Y_t = \widehat{\alpha}\dot{K}_t/K_t + (1-\widehat{\alpha})\dot{N}_t/N_t$ when τ_t is constant. In addition, $\alpha Y_t/K_t = \theta\dot{C}_t/C_t + \rho + \delta_K + \phi P_t$ from (18). Substituting these into the arbitrage condition (17), we obtain:

$$\widehat{\alpha}\frac{\dot{N}_t}{N_t} = -\psi P_t + \widehat{\alpha}\frac{\dot{K}_t}{K_t} - \theta\frac{\dot{C}_t}{C_t} + B - \delta_H - \rho. \tag{24}$$

For consumption C_t to grow, output Y_t must also grow. This means that $P_t = \beta Y_t/\tau_t \to \infty$ under the constant tax rate. On the RHS of (24), the first term diverges to minus infinity, the second and third terms are zero or lower in the long run, and the remaining terms are constants. Therefore, $\dot{N}_t/N_t$ is negative in the long run, implying that the human capital eventually shrinks.

Given the boundedness of K_t and N_t, (14) means that production cannot grow in the long run. This clearly contradicts the initial assumption that consumption grows in the long run. ■

Intuitively, the proof of the proposition explains that under a constant environmental tax rate, agents eventually lose their incentive to save. As long as firms face a constant tax rate on the polluting input P_t, the risk of disasters rises proportionally with output (see equation 19). Then, the insurance cost rises, $\phi P_t = \phi\beta Y_t/\tau_0$, and the marginal rate of return of holding capital falls to $\rho + \delta_K$, where agents no longer want to save.[11] This

[11] Recall from (13) that the rental price of physical capital is $r_t = \alpha Y_t/K_t$. Thus, the last term on the RHS of (23) represents the marginal rate of return of holding capital net of the insurance cost. As K_t

result suggests that, to sustain economic growth, it is necessary to increase the rate of environmental tax over time to prevent the risk of disasters increasing excessively when output grows. In the remainder of the paper, we consider such a time-varying tax policy.

3 Asymptotically Balanced Growth Paths

In the present model, the economy does not typically have a BGP, primarily because the structure of the model is intrinsically nonhomothetic. This is because of the introduction of the endogenous risk of natural disasters (and therefore the endogenous effective depreciation rate of capital). Nonetheless, it does not rule out the possibility that, under an appropriate tax policy, the growth rates of the variables converge, or *asymptote*, to constant values.

Specifically, we seek to find a tax policy under which the equilibrium path satisfies the following property, originally introduced by Palivos *et al.* (1997).

Definition 1 (NABGP) *An equilibrium path is said to be an asymptotically BGP if the growth rates of output, inputs, and consumption converge to finite constant values; that is, if* $g^* \equiv \lim_{t\to\infty} \dot{Y}_t/Y_t$, $g_K \equiv \lim_{t\to\infty} \dot{K}_t/K_t$, $g_H \equiv \lim_{t\to\infty} \dot{H}_t/H_t$, $g_u \equiv \lim_{t\to\infty} \dot{u}_t/u_t$, $g_P \equiv \lim_{t\to\infty} \dot{P}_t/P_t$, *and* $g_C \equiv \lim_{t\to\infty} \dot{C}_t/C_t$ *are well defined and finite. Furthermore, an asymptotically balanced growth path is said to be nondegenerate if* $g_C \geq 0$.[12]

In the remainder of the paper, we focus on nondegenerate, asymptotically balanced growth path(s), referred to as NABGP(s). Note that the requirements for a NABGP also restricts the asymptotic behavior of the tax rate τ_t because $P_t = \beta Y_t/\tau_t$ (equation 19) must be satisfied in the long run. In particular, for g^* and g_P to be well-defined and finite, the asymptotic growth rate of the tax rate

$$g_\tau \equiv \lim_{t\to\infty} \dot{\tau}_t/\tau_t$$

increases, this term falls to $\rho + \delta_K$ and $\dot{C}_t/C_t$ becomes zero.

[12]Palivos *et al.* (1997) call an asymptotically BGP nondegenerate when every production input grows at a positive rate. Note that our definition of nondegeneration is weaker as we only require aggregate consumption not to fall. Indeed, we show that an important case in the analysis is where the growth rate of one production input (namely, the polluting input P_t) is negative and converges to zero. Even in this case, the growth rates of output and consumption can be positive if the growth rates of the other inputs are positive and more than offset the declining use of a certain type of input.

must also be well defined and finite. This means that in the long run, the per unit tax rate on the polluting input must change at a constant rate. The main task of this section is to examine the dependence of the long-term rate of economic growth g^* on the (long-term) growth rate of the environmental tax g_τ. In the first subsection, we present the conditions that must be satisfied for the NABGP. In the second and third subsections, we examine two different possibilities for long-term growth. The final subsection summarizes.

3.1 Conditions for nondegenerate asymptotically balanced growth paths

We first show that, in the long run, the economy cannot grow faster than the growth rate of the environmental tax.

Lemma 1 *On any NABGP,* $g^* \leq g_\tau$.

Proof: in Appendix.

Intuitively, if production grew faster than the tax rate, the use of the polluting input $P_t = \beta Y_t/\tau_t$ would increase without bound, and natural disasters would be increasingly frequent. In such a situation, however, both physical and human capital deteriorate at an accelerating rate, contradicting with the initial assumption that output can grow. One implication from Lemma 1 is that g_τ must be nonnegative ($g_\tau \geq 0$) to support NABGPs. In particular, sustained growth (with $g^* > 0$) is possible only when $g_\tau > 0$; i.e., only when the per unit tax rate increases at an asymptotically constant rate. This confirms the expectation provided at the end of Section 2.4.

Another implication is that $g^* \leq g_\tau$ leads to $g_P \equiv \lim_{t\to\infty} \dot{P}_t/P_t \leq 0$ from (19). Given that the amount of polluting input P_t is nonnegative, this means that P_t converges to a constant value in the long run. We denote this asymptotic value by $P^* \equiv \lim_{t\to\infty} P_t$. Note that $P^* = 0$ if $\dot{P}_t/P_t < 0$. Even though we limit our attention to nondegenerate growth paths, we should not rule out this possibility. It is true that output Y_t is zero if $P_t = 0$ given the Cobb–Douglas function (1), where polluting inputs, such as fossil fuels, are necessary; that is, a balanced growth path in a conventional sense with $P_t = P^* = 0$ is obviously inconsistent with nondegenerate growth. However, in NABGPs where P_t *asymptotes* to P^*, P_t does not necessarily coincide with $P^* = 0$ at any date. Furthermore, $\lim_{t\to\infty} P_t = 0$ does not necessarily mean $\lim_{t\to\infty} Y_t = 0$ as the other production factors in (1), namely K_t and H_t, may grow unboundedly.

Given the asymptotic constancy of P_t, the first-order and transversality conditions determine the growth rates of u_t, K_t, and C_t as follows.

Lemma 2 *On any NABGP,*

(i) u_t, $z_t \equiv Y_t/K_t$, *and* $\chi_t \equiv C_t/K_t$ *are asymptotically constant.*

(ii) $g_u = 0$ *and* $g_K = g_C = g^*$.

The proof is in the Appendix.

Lemma 2 shows that physical capital and consumption grow in parallel with output.[13] In contrast, in our model, the growth rate of human capital is not the same as output. Differentiating the production function (14) logarithmically with respect to time gives $g^* = -\frac{\beta}{1-\beta} g_\tau + \widehat{\alpha} g_K + (1-\widehat{\alpha})(g_u + g_H)$, where we used $N_t = u_t H_t$. This equation implies that the conditions for the NABGP (i.e., $g_K = g^*$ and $g_u = 0$) are satisfied only when:

$$g_H = g^* + \frac{\beta}{1-\alpha-\beta} g_\tau. \tag{25}$$

Equation (25) says that on a NABGP, human capital must accumulate faster than physical capital and output, and the difference is larger when the growth rate of the environmental tax is higher. To see why agents are willing to accumulate human capital more quickly in equilibrium, observe that as the tax rate on the polluting input increases over time, the effective productivity of private firms $\widetilde{A}\tau^{-\beta/(1-\beta)}$ gradually falls (see equation 14). This means that if human capital accumulated at the same speed as physical capital, output would only be able to grow slower than the speed of physical capital accumulation, and the rate of return from investing in physical capital, $r_t = \alpha Y_t/K_t$, would fall. In this manner, raising the tax rate on the polluting input hinders physical capital investment, and consequently induces agents to choose human capital investment an alternate means of saving, as documented by Skidmore and Toya (2002).[14]

[13] Observe that property (ii) of Lemma 2 is a stronger statement than (i); i.e., property (i) holds whenever $g_u \leq 0$, $g_K \geq g^*$, and $g_C \leq g_K$. In the proof of the lemma in the Appendix, we show that all of $g_u \leq 0$, $g_K \geq g^*$, and $g_C \leq g_K$ must hold with equality as otherwise either the transversality conditions (their necessary condition are given by equations 21 and 22) or sustainability ($g_C > 0$) are eventually violated.

[14] Nonetheless, the interest rate r_t is kept constant on the NABGP. This is because as human capital becomes increasingly abundant relative to physical capital, it raises the marginal productivity of physical capital and eventually compensates for the decline in effective productivity.

Now we are ready to summarize the conditions that must be satisfied by any NABGP. For convenience, let us denote the asymptotic values of the key variables by $u^* \equiv \lim_{t\to\infty} u_t \in [0,1]$, $z^* \equiv \lim_{t\to\infty} Y_t/K_t \geq 0$, and $\chi^* \equiv \lim_{t\to\infty} C_t/K_t \geq 0$. Substituting $g_u = 0$, $g_K = g_C = g^*$ and (25) for (15)-(19), the equilibrium conditions that must hold in the long run can be represented as follows.

$$\text{Evolution of } K_t\text{: } g^* = z^* - \chi^* - (\delta_K + \phi P^*), \tag{26}$$

$$\text{Evolution of } H_t\text{: } g^* + \frac{\beta}{1-\alpha-\beta} g_\tau = B(1-u^*) - (\delta_H + \psi P^*), \tag{27}$$

$$\text{Arbitrage condition: } -\frac{\beta}{1-\alpha-\beta} g_\tau = \alpha z^* - B + (\psi - \phi)P^* - (\delta_K - \delta_H), \tag{28}$$

$$\text{Keynes–Ramsey rule: } -\theta g^* = \rho - \alpha z^* + (\delta_K + \phi P^*), \tag{29}$$

$$\text{Asymptotic pollution: } P^* \begin{cases} \geq 0 \text{ if } g^* = g_\tau \quad \text{(Case 1)}, \\ = 0 \text{ if } g^* < g_\tau \quad \text{(Case 2)}. \end{cases} \tag{30}$$

Given the tax policy $g_\tau \geq 0$, which is set by the government, the five conditions (26)-(30) determine five unknowns (g^*, z^*, χ^*, u^*, P^*) on the NABGP. In the following, we explicitly calculate the values for the unknowns as a function of g_τ. Note that, however, there is a complementary slackness condition (30), and we cannot know whether $g^* = g_\tau$ or $P^* = 0$ holds in advance. Thus, we need to examine two possible cases in turn, and then determine which case actually occurs in equilibrium under a particular tax policy.

3.2 Case 1: $P^* \geq 0$ and $g^* = g_\tau$

Let us first examine the possibility that Case 1 in condition (30) holds. In this case, the equilibrium long-term rate of growth coincides with the growth rate of the environmental tax on the steady-state equilibrium path. Substituting $g^* = g_\tau$ into (28) and (29), we obtain the asymptotic value of polluting input:

$$P^* = \frac{1}{\psi}\Big[B - \delta_H - \rho - \Big(\theta + \frac{\beta}{1-\alpha-\beta}\Big) g_\tau\Big], \tag{31}$$

which is decreasing in g_τ. Recall that, as shown by (30), the asymptotic value must be nonnegative: $P^* \geq 0$. From (31), we can see that this condition is satisfied if g_τ is within the following range:

$$g_\tau \leq (B - \delta_H - \rho)\left(\theta + \frac{\beta}{1-\alpha-\beta}\right)^{-1} \equiv g^{\max}, \tag{32}$$

where $g^{\max}$ is positive from the assumption that $B-\delta_H>\rho$. Hence, Case 1 (i.e., $P^*\geq 0$ and $g^*=g_\tau$) is possible only if $g_\tau\in[0,g^{\max}]$.

Using $g^*=g_\tau$, we obtain the asymptotic values of the other variables from (26)-(29)

$$z^*=\frac{1}{\alpha}\left(\theta g_\tau+\delta_K+\phi P^*+\rho\right), \tag{33}$$

$$\chi^*=\frac{1}{\alpha}\Big((\theta-\alpha)g_\tau+(1-\alpha)(\delta_K+\phi P^*)+\rho\Big), \tag{34}$$

$$u^*=\frac{1}{B}\Big(B-(\delta_H+\psi P^*)-\frac{1-\alpha}{1-\alpha-\beta}g_\tau\Big). \tag{35}$$

Substituting (31) into (33)-(35) and using $g_\tau\in[0,g^{\max}]$, we can confirm that $z^*>0$, $\chi^*>0$, $u^*\in(0,1)$, and $(1-\alpha)z^*-\chi^*<0$. The last two inequalities imply that the sufficient condition for the transversality condition, given by (20), is satisfied. In addition, we show in the Appendix that under a reasonable restriction of the parameter values, the NABGP is saddle stable. The following lemma states the results obtained.

Lemma 3 *A NABGP with $P^*\geq 0$ and $g^*=g_\tau$ exists if and only if $g_\tau\in[0,g^{\max}]$. This is characterized by $g^*=g_\tau$ and (31)-(35), and satisfies the equilibrium conditions (26)-(30) and the transversality conditions. In addition, if:*

$$\psi/\phi<(1-2\alpha)/(1-\alpha-\beta), \tag{36}$$

this equilibrium path is locally saddle stable.
The proof of stability is in the Appendix.

Given that the share of physical capital α is around 0.3 in reality, the RHS of condition (36) is likely to be positive. (When $\alpha=0.3$ and $\beta=0.1$, for example, $(1-2\alpha)/(1-\alpha-\beta)=2/3$.) In addition, we expect that the ratio $\overline{\psi}/\overline{\phi}$ will typically be low because the natural disasters affect more directly physical capital than human capital. Therefore, we reasonably assume that parameters satisfy condition (36) throughout the paper.

3.3 Case 2: $P^*=0$ and $g^*<g_\tau$

Next, we examine the possibility that Case 2 in condition (30) holds. In this case, the amount of polluting input asymptotically converges toward zero. Substituting $P^*=0$ for (28) and (29) yields the asymptotic rate of economic growth:

$$g^*=\frac{1}{\theta}\Big(B-\delta_H-\rho-\frac{\beta}{1-\alpha-\beta}g_\tau\Big). \tag{37}$$

Contrary to Case 1, equation (37) shows that the long-term rate of growth is decreasing in g_τ. In particular, for the condition $g^* < g_\tau$ to be satisfied, the rate of environmental tax must be raised faster than $g^{\max}$, where $g^{\max}$ is defined in (32). However, equation (37) also implies that the rate of economic growth becomes eventually negative when g_τ is too high: $g^* < 0$ if $g_\tau > g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. Therefore, a NABGP with $P^* = 0$ and $(0 \leq) g^* < g_\tau$ exists only if $g_\tau \in (g^{\max}, g^{\lim}]$.

Substituting $P^* = 0$ and (37) into (26)-(29), we obtain the values for the other variables on the NABGP:

$$z^* = \frac{1}{\alpha}\Big(B + \delta_K - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big), \tag{38}$$

$$\chi^* = \Big(\frac{1}{\alpha} - \frac{1}{\theta}\Big)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \frac{1-\alpha}{\alpha}\delta_K + \frac{\rho}{\theta}, \tag{39}$$

$$u^* = \frac{1}{B\theta}\Big[(\theta-1)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \rho\Big]. \tag{40}$$

From $g_\tau \in (g^{\max}, g^{\lim}]$, we can confirm that $z^* > 0$, $\chi^* > 0$, $(1-\alpha)z^* - \chi^* < 0$, and $u^* \in (0,1)$, implying that the transversality condition (20) is satisfied. In addition, we show in the Appendix that this NABGP is saddle stable. The following lemma states the results.

Lemma 4 *A NABGP with $P^* = 0$ and $g^* < g_\tau$ exists if and only if $g_\tau \in (g^{\max}, g^{\lim}]$. It is characterized by $P^* = 0$ and (37)-(40), and satisfies equilibrium conditions (26)-(30) and the transversality conditions. In addition, this equilibrium path is locally saddle stable.*

The proof of the stability is in the Appendix.

3.4 Summary

Lemma 3 and Lemma 4 show that there are two possible patterns of long-term growth. Observe that those two possibilities are mutually exclusive—Lemma 3 applies only when the tax policy satisfies $g_\tau \in [0, g^{\max}]$, whereas Lemma 4 applies only when $g_\tau \in (g^{\max}, g^{\lim}]$. Therefore, the NABGP is always unique. The following proposition states the main result.

Proposition 2 *A NABGP exists if and only if the asymptotic growth rate of the per unit tax on the polluting input, g_τ, is between 0 and $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. When*

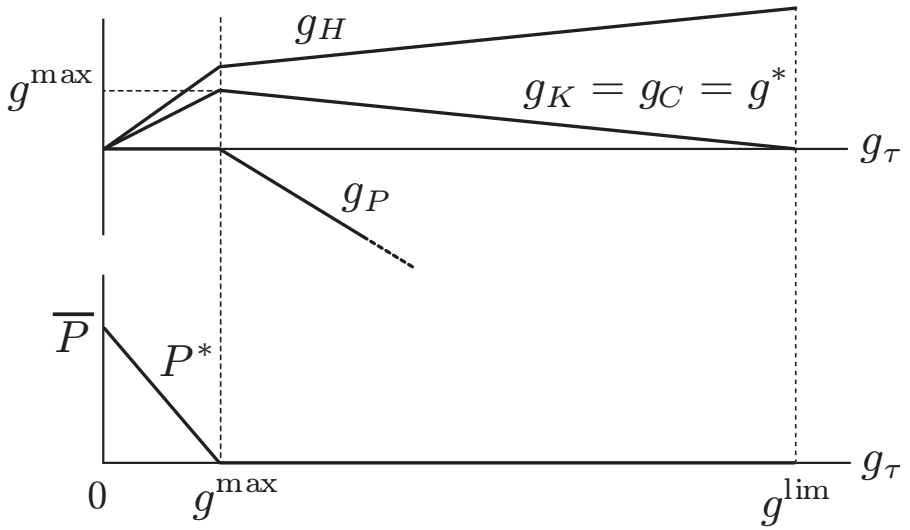

Figure 3: Growth rate of environmental tax and the NABGP. The upper panel shows the relationship between the growth rate of the environmental tax (g_τ) and that of human capital (g_H), physical capital (g_K), output (g^*), and pollution (g_P). The lower panel shows the level to which pollution converges in the long run ($P_t \to P^*$). Parameters: $\alpha = .3$, $\beta = .2$, $\theta = 2$, $\rho = .05$ $B = 1$, $\psi = .005$, $\phi = .01$, $\delta_H = .09$, and $\delta_K = .1$.

it exists, it is unique and locally saddle stable. The long-term rate of economic growth follows an inverted V-shape against $g_\tau \in [0, g^{\lim}]$*, and is maximized at* $g_\tau = g^{\max} \equiv (B - \delta_H - \rho)(\theta + \frac{\beta}{1-\alpha-\beta})^{-1}$.

The asymptotic growth rates of the variables are determined by g_τ in the following way. First, the asymptotic growth rate of output is given by $g^* = g_\tau$ for $g_\tau \in [0, g^{\max}]$ and (37) for $g_\tau \in (g^{\max}, g^{\lim}]$. As shown in Lemma 2, g_C and g_K are the same as g^*, and $g_u = 0$. Next, given g^*, the asymptotic growth rates of human capital and pollution, respectively, are determined by (25) and $g_P \equiv \dot{P}_t/P_t = g^* - g_\tau$ (recall equation 19). Finally, the asymptotic *level* of pollution, P^*, is given by (31) for $g_\tau \in [0, g^{\max}]$ and $P^* = 0$ for $g_\tau \in (g^{\max}, g^{\lim}]$. Figure 3 illustrates these results.

Observe from the figure that when the environmental tax rate is asymptotically constant (i.e., when $g_\tau = 0$), the asymptotic growth rates of all endogenous variables are zero. This means that the economy settles to a no-growth steady state. In this steady state, the amount of pollution converges to $P^* = (B - \delta_H - \rho)/\psi \equiv \overline{P}$, which causes the risk of natural disasters (i.e., the probability of losing physical and human capital) to be

so high that agents lose the incentive to save. Interestingly, the asymptotic level of P_t does not depend on the level of the environmental tax rate, τ_t, as long as the asymptotic growth rate of τ_t is zero. Nonetheless, given $Y_t = \tau_t P_t/\beta$ from (19), a higher tax rate induces the economy to converge to a higher output level. This implies that a higher level of the environmental tax rate promotes growth in the transition, but not in the long run.

When the government raises the per unit tax rate on polluting inputs at an asymptotically constant rate ($g_\tau > 0$), the asymptotic level of P_t can be kept below $\overline{P}$. When g_τ is increased within the range of $[0, g^{\max}]$, the long-run amount of pollution P^* decreases, as does the risk of natural disasters. The reduced risk of natural disasters encourages agents to accumulate capital more quickly. As a result, the growth rate of physical capital g_K increases in parallel with g_τ (i.e., $g_K = g_\tau$). The growth rate of human capital, g_H, also increases with g_τ, and more than proportionately to physical capital. (Recall the discussion in Section 3.1 for why agents are willing to do this.) This makes possible sustained growth without increasing the use of the polluting input.

The long-tern rate of economic growth is maximized at $g_\tau = g^{\max}$, under which the use of polluting inputs P_t converges asymptotically to the zero level ($P_t \to P^* = 0$). However, a further acceleration of the tax rate does not enhance economic growth because it cannot reduce the asymptotic risk of natural disasters (because it is already at the lowest level); rather, it accelerates the fall of the effective productivity of firms, $\widetilde{A}\tau^{-\beta/(1-\beta)}$. As a result, g^* is no longer increasing in parallel with g_τ, but decreasing in g_τ. In particular, if $g_\tau > g^{\lim}$, even though the risk of natural disasters is at its lowest level, the fall of effective productivity is so rapid that it cannot be compensated for by the faster accumulation of human capital. This results in negative growth.

4 Welfare-maximizing Policy

In previous sections, we examined the relationship between the environmental policy and the feasibility of sustained economic growth. Even when production requires polluting inputs and the use of polluting inputs raises the risk of natural disasters, we showed that economic growth can be sustained in the long run if the government gradually increases the tax rate on the polluting inputs. We also found that an environmental policy

maximizes the long-term rate of economic growth. However, this does not necessarily mean that such an environmental policy is desirable in terms of welfare. This section considers the welfare-maximizing policy and examines whether it differs from the growth-maximizing policy.

Let us consider the social planner's problem. The social planner maximizes the representative household's utility (8) subject to resource constraints (4)-(5). From the first-order conditions for optimality, we show in the Appendix that the dynamics of K_t, H_t, u_t and C_t in the welfare-maximizing path are exactly the same as those for the market equilibrium given by equations (15)-(18). The transversality conditions are also the same. The remaining condition for the social planner's problem is that the amount of polluting input should be:

$$P_t = \beta \left(\phi \frac{K_t}{Y_t} + \psi \frac{(1-\alpha-\beta)}{B u_t} \right)^{-1}. \tag{41}$$

Recall that in the market economy, the government sets the tax rate τ_t and firms choose P_t according to $P_t = \beta Y_t / \tau_t$, as shown by equation (19). Therefore, if the tax rate at each point in time satisfies:

$$\tau_t = \phi K_t + \psi H_t \frac{(1-\alpha-\beta) Y_t}{B u_t H_t}, \tag{42}$$

then the firms' decision on P_t in the market equilibrium exactly coincides with the optimality condition (41). Given that the remaining conditions for the social optimum are the same as those for the market equilibrium, this means that the welfare-maximizing allocation can be achieved as a market equilibrium when the government set the environmental tax rate using the following rule (42).[15] This policy rule has an intuitive interpretation: the first term on the RHS of (42) represents the marginal increase in the expected damage to physical capital with respect to P_t, whereas the second term represents that to human capital, both measured in terms of final goods (in particular,

[15]Generally speaking, even when they appear similar, a time-varying policy (a function only of time as considered in the previous section) and a policy rule (a function of state variables such as equation 42) may result in different outcomes if private agents behave strategically. The literature on differential games distinguishes these as the open-loop equilibrium and the Markovian equilibrium. Nonetheless, in the present model, all private agents are price takers and therefore both policies result in the same outcome.

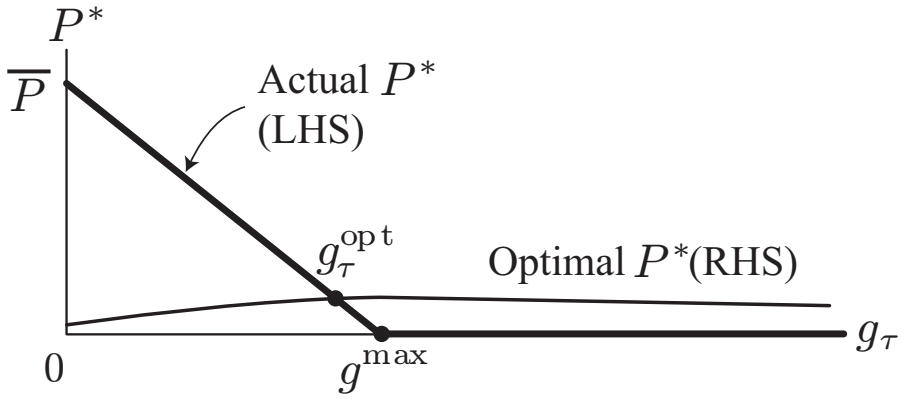

Figure 4: Determination of the optimal growth rate of the environmental tax. This figure plots the RHS and LHS of condition (43) against g_τ. The asymptotic growth rate of the optimal environmental tax is g_τ^{opt}, as given by the intersection, and is lower than the growth-maximizing rate, $g^{\max}$. The parameters are the same as in Figure 3.

$(1-\alpha-\beta)Y_t/(Bu_tH_t)$ is the shadow price of human capital in terms of final goods). Thus, it is optimal to let firms pay the sum of these marginal expected damages on each use of P_t.

Let us characterize the equilibrium path under the optimal tax policy. Similarly to the previous section, we limit our attention to NABGP, where the values of P_t, $z_t \equiv Y_t/K_t$, and u_t in condition (41) are asymptotically constant and converge to P^*, z^*, and u^*. The welfare-maximizing path must satisfy:

$$P^* = \beta\left(\phi\frac{1}{z^*} + \frac{\psi(1-\alpha-\beta)}{Bu^*}\right)^{-1}. \tag{43}$$

Recall that u^* and z^* on the RHS are functions of the asymptotic growth rate of the tax rate, g_τ (see Lemmas 3 and 4). Thus, the RHS of (43) gives the optimal amount of pollution as a function of g_τ. On the other side, the LHS represents the actual amount of pollution in equilibrium, P^*, which is a function of g_τ. The optimal growth rate of the environmental tax, denoted by g_τ^{opt}, must be such that the LHS and the RHS of (43) coincide.

Figure 4 plots the RHS and LHS of equation (43) against g_τ. The actual amount of asymptotic pollution (the LHS) is positive but decreasing in g_τ for $g_\tau \in [0, g^{\max})$, and is zero for $g_\tau \geq g^{\max}$. On the other hand, the optimal amount of asymptotic pollution (the RHS) is positive for all $g_\tau \geq 0$, and at $g_\tau = 0$, is lower than $\overline{P} \equiv (B-\delta_H-\rho)/\psi$ given

that parameters satisfy:[16]

$$(\alpha\phi/(\delta_K + \phi\overline{P} + \rho) + \psi(1-\alpha-\beta)/\rho)\overline{P} > \beta. \tag{44}$$

Therefore, under condition (44), the two curves have an intersecting point $g_\tau^{\text{opt}} \in (0, g^{\max})$, at which point the optimality condition (43) is satisfied. The following proposition formally states this result.

Proposition 3 *Suppose the parameters satisfy condition (44). Then among the NABGP, there exists a path that maximizes the welfare of the representative household (8). This path can be realized by tax policy (42), and the asymptotic growth rate of the optimal per unit tax, g_τ^{opt}, is strictly positive but lower than the growth-maximizing rate, $g^{\max}$.*

Note that condition (44) is satisfied unless both ρ and β are large. Intuitively, it pays to enjoy a high level of consumption, production and, therefore, pollution today at the cost of accepting a higher risk of natural disasters only when the household heavily discounts the future (large ρ) and production substantially relies on polluting inputs (large β). If either the household values the future or the dependence of production on polluting inputs is limited, then sustained economic growth is not only feasible but also desirable. It is also notable, however, that the optimal policy does not coincide with the growth-maximizing policy ($g_\tau^{\text{opt}} < g^{\max}$). Thus, if the government cares about welfare, it should employ a milder policy for protecting the environment than when growth is their only concern. The difference between the growth-maximizing and welfare-maximizing policies is similar to the difference between the golden rule and the modified golden rule. Although an aggressive environmental policy that aims to eliminate the emission of pollutants in the long run (i.e., $P^* = 0$) may maximize the economic growth rate in the very long run, the cost in the form of the reduced effective productivity that must be incurred during the transition can overwhelm the benefit that can be reaped only far in the future.

[16] When $g_\tau = 0$, equations (31), (33), and (35) show that $P^* = (B-\delta_H-\rho)/\psi \equiv \overline{P}$, $z^* = (\delta_K+\phi\overline{P}+\rho)/\alpha$ and $u^* = \rho/B$. Substituting these into both sides of (43) shows that the intercept of the LHS is lower than that of the RHS if (44) holds.

5 General Model with Stock of Pollution

In reality, the risk of natural disasters is often affected not only by how much current firms emit pollution, but also how much they emitted in the past. For example, the use of fossil fuels in the past increases the the stock of greenhouse gases in the atmosphere today, and this affects tropical sea surface temperature, and therefore the risk of disastrous hurricanes. To this point, for simplicity we do not distinguish between the flow of pollution and its stock. This section examines how the long-term properties obtained in previous sections change when pollution stocks affect the risk of natural disasters.

As before, we assume that firms use a polluting input (e.g., fossil fuels), causing them to emit pollution. Let E_t denote the emission of pollution by firms per unit of time. One unit of polluting input yields one unit of emission, and thus E_t also represents the amount of polluting input used by firms. Then, the production function (1) should be modified to:

$$Y_t = F(K_t, u_t H_t, E_t) = A K_t^{\alpha} (u_t H_t)^{1-\alpha-\beta} E_t^{\beta}, \tag{45}$$

where we substituted E_t for P_t. The emission adds to the pollution stock P_t, which is now defined by:

$$P_t \equiv \gamma \int_{-\infty}^{t} E_s e^{-\delta_P (t-s)} ds. \tag{46}$$

There are now two parameters in the accumulation process: γ represents the marginal impact of emissions on the pollution stock, and δ_P denotes the depreciation rate of the pollution stock (e.g., the fraction of greenhouse gases being absorbed by the oceans during a unit of time). If δ_P is smaller, use of a polluting input today has an impact on the environment for a longer period in the future. We assume the risk of natural disasters is affected by the pollution stock P_t, as described by (3). The law of motion for physical capital can then be written as:

$$\dot{K}_t = F(K_t, u_t H_t, E_t) - C_t - (\delta_K + \phi P_t) K_t, \tag{47}$$

whereas that for human capital stock remains the same as (5). Note that P_t in these equations should now be interpreted as the pollution stock at t rather than the amount of polluting input used at t.

5.1 Market economy under stock pollution

In the market economy, the government levies an environmental tax τ_t on each unit of polluting input E_t used by the firm. Similar to the analysis in Section 2.2.2, the first-order conditions for firms are (13) and:

$$E_t = \beta Y_t / \tau_t. \tag{48}$$

The behavior of households is exactly the same as described in Section 2.2.1. In this setting, the equilibrium dynamics of $\{K_t, H_t, u_t, C_t, E_t, P_t\}$ are characterized by conditions (15)-(18), (46), and (48). Let us consider the NABGP, where the growth rates of all inputs, output, and consumption are asymptotically constant in the long run (Recall Definition 1). The following proposition shows that the long-run property of the equilibrium is unaffected by the introduction of accumulated pollution.

Proposition 4 *In an economy where pollution accumulates through (46) and (48), a NABGP exists if and only if the asymptotic growth rate of the per unit tax on polluting input, g_τ, is between 0 and $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. This is characterized by $g^* = g_\tau$ and (31)-(35) if $g_\tau \in (0, g^{\max}]$, and by $P^* = 0$ and (37)-(40) if $g_\tau \in (g^{\max}, g^{\lim}]$. The level of emission is asymptotically constant at $E^* = (\delta_P/\gamma)P^*$.*
The proof is in the Appendix.

The asymptotic growth rate of the economy is again an inverted V-shape against the growth rate of the environmental tax, as illustrated in Figure 3. Note that the long-run amount of pollution stock P^* does not depend on the parameters of pollution accumulation (γ and δ_P). This is interesting because if δ_P is smaller, the effect of emissions on the pollution stock remains for a longer time, and therefore P_t would become higher, provided that the amount of emissions is the same; i.e., independence of P^* from these parameters implies that the amount of emissions must change with the parameters. In fact, from (48) and Proposition 4, we see that the level of output asymptotes to $Y_t = \tau_t E_t/\beta \to \tau_t \delta_P P^*/(\beta\gamma)$, which is lower when the effect of pollution remains for a longer time.[17] This means that the amount of production, and therefore the amount of emissions, is adjusted so that the pollution stock becomes asymptotically P^*, which

[17] When δ_P is lower and the amount of production is the same, the pollution stock becomes higher and disasters occur more frequently. This reduces the capital stocks (K_t and H_t) because the capital stocks are

depends on the growth rate of τ but not on δ_P and γ. As a result, the difference in the accumulation process (δ_P and γ) has level effects on output, but not growth effects.

5.2 Welfare-maximizing policy under stock pollution

Next, let us turn to welfare maximization. The social planner maximizes welfare (8) subject to resource constraints (5), (46), and (47). In the Appendix, we solve the dynamic optimization problem and again find that the dynamics of K_t, H_t, u_t, and C_t in the welfare-maximizing path are exactly the same as those for the market equilibrium (equations 15-18). The optimal amount of emissions is given by:

$$\begin{aligned} &E_t = -\frac{\beta Y_t C_t^{-\theta}}{\gamma \lambda_t}, \\ &\text{where} \quad \lambda_t = -\int_t^{\infty} C_s^{-\theta} \left(\phi K_s + \psi \frac{(1-\alpha-\beta)Y_s}{B u_s} \right) e^{-(\rho+\delta_P)(s-t)} ds \end{aligned} \tag{49}$$

which represents the shadow value of one additional unit of polluting stock, which is, of course, negative. The optimal stock of pollution is obtained by substituting (49) into (46).

Observe that the only difference between the market equilibrium and the welfare-maximizing path is between (48) and (49). In particular, when the government sets the tax rate by:

$$\tau_t = \frac{-\gamma \lambda_t}{C_t^{-\theta}} = \gamma \int_t^{\infty} e^{-\delta_P (s-t)} \left(\phi K_s + \psi \frac{(1-\alpha-\beta)Y_s}{B u_s} \right) \left(\frac{C_s^{-\theta} e^{-\rho(s-t)}}{C_t^{-\theta}} \right) ds, \tag{50}$$

the market economy coincides with the welfare-maximizing path; i.e., (50) gives the optimal policy when pollution accumulates. When a firm emits pollution in year t, it has negative effects on the environment for all years $s \geq t$. The integral on the RHS represents the cumulative negative effects of emissions for year t. More precisely, the first part of the integral, $e^{-\delta_P(s-t)}$, is the portion of emissions remaining by year s. The second part, $\phi K_s + \psi(1-\alpha-\beta)Y_s/(Bu_s)$, is essentially the same as (42), representing the marginal negative effect of the polluting stock in year s. The final part, $C_s^{-\theta} e^{-\rho(s-t)}/C_t^{\theta}$, is the

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destroyed more frequently by disasters and because households lose the incentive to accumulate capital stocks when they face a higher probability of disasters. Lower capital stocks imply lower production and thus lower emissions. The same holds when γ is larger.

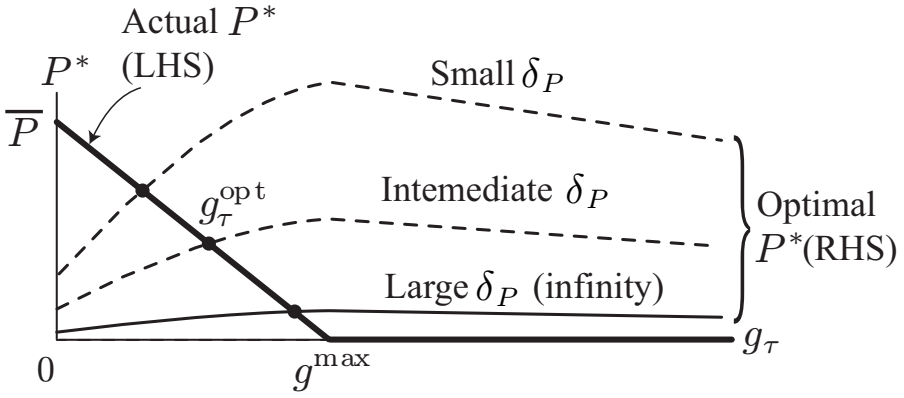

Figure 5: Optimal tax policy when pollution accumulates.

intertemporal marginal rate of substitution between year s and t, and represents how we discount the future.

While equation (50) has a natural interpretation, the implementation of the optimal policy is not obvious because the optimal tax rate in year t depends on the whole time path of the economy in the future, which in turn depends on the whole path of the tax rate in the future. Following Section 4, we solve this problem by focusing on the family of NABGPs. In the NABGPs, $Y_s = Y_t e^{g^*(s-t)}$, $C_s = C_t e^{g^*(s-t)}$, $K_s = K_t e^{g^*(s-t)}$, $u_t = u^*$, $Y_s/K_s = z^*$ hold asymptotically. Substituting these for (50) and calculating the integral, we can see that on a NABGP, the tax rate should be:

$$\tau_t = \frac{\gamma Y_t}{(\theta-1)g^* + \rho + \delta_P}\left(\frac{\phi}{z^*} + \psi\frac{1-\alpha-\beta}{Bu^*}\right). \tag{51}$$

This implies that from equation (48) and Proposition 4, the amount of pollution should asymptotically be:

$$P^* = \frac{\gamma E^*}{\delta_P} = \frac{\gamma\beta Y_t}{\delta_P \tau_t} = \beta\left(1 + \frac{(\theta-1)g^* + \rho}{\delta_P}\right)\left(\frac{\phi}{z^*} + \psi\frac{1-\alpha-\beta}{Bu^*}\right)^{-1}. \tag{52}$$

Recall that g^*, u^*, and z^* on the RHS are functions of the asymptotic growth rate of the tax, g_τ (see Proposition 4). Thus, the RHS of (52) gives the optimal amount of pollution stock as a function of g_τ. On the other side, the LHS (the actual amount of pollution stock in equilibrium, P^*) is also a function of g_τ. The optimal g_τ is such that the LHS and the RHS coincide.

The figure plots the RHS and the LHS of condition (52) against g_τ, for the three different levels of δ_P. Observe that when δ_P is infinitely large, the term $((\theta-1)g^*+\rho)/\delta_P$

vanishes and condition (52) coincides with (43). Intuitively, when the effect of emission depreciates very quickly, only the current use of the polluting input affects the risk of natural disasters, as we considered in previous sections.[18] Thus, the optimal policy is the same as in Section 4.

However, when δ_P is finite (i.e., when the effects of emissions remain for some time), the RHS is higher than in the previous case. Accordingly, the intersecting point in Figure 5 moves toward the upper left. The following proposition summarizes this finding.

Proposition 5 *Suppose that pollution accumulates through (46) and (48), where δ_P is finite. Then, the asymptotic growth rate of the optimal tax rate, g_τ^{opt}, is lower than in Proposition 3. Moreover, as δ_P becomes smaller (i.e., when the effects of emissions remain for a longer time), g_τ^{opt} falls and the asymptotic pollution, P^*, rises. The optimal long-tern rate of economic growth is also lower than in Proposition 3 and falls as δ_P becomes smaller.*

Previously, we have shown in Proposition 3 that in the case where pollution does not accumulate, the welfare-maximizing environmental policy is less strict than the growth-maximizing policy. Proposition 5 shows that, when emissions have a longer-lasting effect, it is optimal to adopt an even less strict environmental tax policy. This implies that the gap between the growth-maximizing policy and the welfare-maximizing policy is even larger when pollution accumulates.

We can again interpret this apparently paradoxical result in terms of time preference. When emissions have a longer effect, the larger part of the social cost of using the polluting input comes long after the benefit of using the polluting input (i.e., larger output) is realized. Thus, as long as the agent discounts the future, there is more social gain in accepting a high level of pollution stock and lower growth in the long run than where pollution does not accumulate. As a result, it is optimal to increase the environmental tax more slowly.[19]

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[18] Formally, when both γ and δ_P are infinite ($\gamma = \delta_P \to \infty$), Proposition 4 shows $E^* = P^*$ holds. This implies that we can consider emissions and the pollution stock interchangeably, as in Section 4.

[19] To understand this argument intuitively, let us consider two extreme cases where the argument in the text does not hold. First, observe that $(\theta - 1)g^* + \rho$ in condition (52) represents the rate of fall in the marginal utility $C_t^{-(\theta-1)}e^{-\rho t}$. If this expression were zero, there would be no benefit from frontloading

6 Conclusion

In this paper, we analyzed the sustainability of economic growth in a two-sector endogenous growth model when taking into account the risk of natural disasters. Here, polluting inputs are necessary for production, though they also intensify the risk of natural disasters. In this setting, we obtained the following results.

First, economic growth can be sustained in the long run only if the per unit tax on the polluting input increases over time. Although economic growth *ceteris paribus* induces private firms to use more of the polluting input, this environmental policy can lead firms to use more human capital (e.g., by investing in alternative technologies), which decreases their reliance on polluting inputs, and thereby prevents the risk of disaster from rising to a critical level. However, it should be noted that we do not consider the cost associated with extracting resources or the finiteness of these inputs. If the cost is significant and changes for some reason, the environmental tax rate must be adjusted to absorb these changes. A next step in our research agenda would be to integrate the analysis of natural disasters with a study of the finiteness of natural resources. This is clearly beyond the scope of this first attempt.

Second, the long-term rate of economic growth follows an inverted V-shaped curve relative to the growth rate of the environmental tax. When the rate of environmental tax is currently slowly growing, its acceleration will reduce the asymptotic level of emissions and the risk of natural disasters. This process enhances the incentive to save and hence promotes economic growth. When the rate of environmental tax is already fast growing, the asymptotic level of pollution is fairly small so that further acceleration of the environmental tax excessively impairs the productivity of private firms. This works against economic growth. Therefore, economic growth can be maximized with the choice of the most gradual increase in the environmental tax rate that minimizes the amount of pollution in the long run.

Third, social welfare is maximized under a milder (i.e., more slowly increasing) envi-

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output. In fact, condition (52) would become the same as (43) if $(\theta - 1)g^* + \rho$ were 0. Second, observe that $\delta_P = \infty$ means that the whole negative effect of emissions is realized immediately; i.e., there is no gap in time between the increased output and the increased risk of disasters. Thus, (52) becomes the same as (43) when $\delta_P = \infty$. Except for these two extremes, (52) always implies a lower g_τ^{opt} than (43).

ronmental tax policy than the growth-maximizing policy. This may appear paradoxical in that welfare considerations justify more pollution than when growth is the foremost policy concern. This is because maximization of the long-term rate of growth requires the minimization of the asymptotic level of pollution, but this can only be achieved only in the long run. As long as people discount the future, aiming for this ultimate goal would be too costly in terms of the efficiency loss that must be incurred in the transition. Thus, a milder environmental policy is more desirable in terms of the discounted sum of expected utility. Moreover, when pollutants accumulate gradually and remain in the air for longer, the transition process takes more time and, therefore, the welfare-maximizing environmental tax policy is even milder.

Appendix

Optimization of the household (Section 2.2.1)

The current value Hamiltonian for the household's maximization problem is:

$$\begin{aligned}\mathcal{H} = \frac{c_t^{1-\theta}-1}{1-\theta} &+ \nu_t \left(r_t k_t - (\delta_K + \phi P_t)k_t + w_t u_t h_t - c_t + T_t\right) \\ &+ \mu_t \left(B(1-u_t)h_t - (\delta_H + \psi P_t)h_t\right),\end{aligned}$$

where ν_t and μ_t are the shadow prices associated with the accumulation of physical and human capital, respectively. The first-order conditions for this problem is given by:

$$\nu_t = c_t^{-\theta}, \tag{53}$$

$$\mu_t = \frac{w_t}{B}\nu_t, \tag{54}$$

$$\frac{\dot{\nu}_t}{\nu_t} = \rho + \phi P_t + \delta_K - r_t, \tag{55}$$

$$\frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{\nu_t}{\mu_t} w_t u_t - B(1-u_t) + \delta_H + \psi P_t. \tag{56}$$

Differentiating the log of (53) with respect to time gives $\dot{\nu}_t/\nu_t = -\theta \dot{c}_t/c_t$. Substituting this into (55) gives the Keynes–Ramsey rule (9) in the text. Differentiating the log of (54) with respect to time gives $\dot{\mu}_t/\mu_t = \dot{w}_t/w_t + \dot{\nu}_t/\nu_t$. Substituting it and (55) into (56) gives the arbitrage condition (10) in the text. Finally, the transversality conditions for this problem are $\lim_{t\to\infty} k_t \nu_t e^{-\rho t} = 0$ and $\lim_{t\to\infty} h_t \mu_t e^{-\rho t} = 0$. Eliminating μ_t and ν_t from these conditions by (53) and (55) gives (11) and (12) in the text.

Proof of Lemma 1 (Section 3.1)

Suppose that $g^* > g_\tau$ (i.e., $\lim_{t\to\infty} \dot{Y}_t/Y_t > \lim_{t\to\infty} \dot{\tau}_t/\tau_t$). Then, $P_t = \beta Y_t/\tau_t \to \infty$. From (16) and $u_t \leq 1$, this means $\dot{H}_t/H_t \leq B - \delta_H - \psi P_t \to -\infty$. This contradicts with the definition of the NABGP, in which $g_H \equiv \lim_{t\to\infty} \dot{H}_t/H_t$ is finite.

Proof of Lemma 2 (Section 3.1)

On the NABGP where $\dot{H}_t/H_t$ and P_t are asymptotically constant, equation (16) implies that u_t must also be asymptotically constant. This means that the growth rate of u_t is zero or negative (i.e., in the case of $u_t \to 0$), but from (22) we know that the TVC for human capital accumulation is satisfied only when the growth rate of u_t is nonnegative. Therefore, $g_u = 0$. Next, as $\dot{C}_t/C_t$ and P_t are asymptotically constant, equation (18) implies that the value of Y_t/K_t must also be constant in the long run. This means that the growth rate of Y_t/K_t is zero or negative. However, if $Y_t/K_t \to 0$, equation (15) states $\dot{K}_t/K_t < 0$, which means that $Y_t = (Y_t/K_t) \cdot K_t \to 0$. This is inconsistent with our definition of a NABGP, where $g_C \geq 0$. Therefore, the growth rate of Y_t/K_t must be zero; i.e., $g_K = g^*$. Finally, given that $\dot{K}_t/K_t$ and Y_t/K_t are asymptotically constant, equation (15) in turn implies that C_t/K_t must also be asymptotically constant. Recall that the TVC for physical capital (21) requires that C_t/K_t must not be smaller than $(1-\alpha)(Y_t/K_t)$, which converges to a strictly positive constant as shown above. Therefore, the growth rate of C_t/K_t must not be negative rather zero; i.e., $g_C = g_K = g^*$.

Proof of Lemmas 3 and 4 (Sections 3.2 and 3.3)

In this subsection, we establish the stability of the equilibrium path stated in Lemma 3 and Lemma 4. The equilibrium path is characterized by a four-dimensional dynamics system of $\{K_t, H_t, u_t, C_t\}$, where the laws of motion for these variables are given by (15)-(18).[20] In this dynamic system, K_t and H_t are predetermined state variables, whereas u_t and C_t are jumpable. Therefore, the system is both stable and determinate when it has a stable manifold of dimension two.

[20]Note that by making use of (14), (19), and $N_t = u_t H_t$, Y_t and P_t appearing in the LHS of (15)-(18) can be expressed in terms of K_t, H_t, u_t and τ_t, where the motion of τ_t is given exogenously by the government.

For convenience, we transform this system into another four-dimensional system in $\{u_t, \chi_t, z_t, P_t\}$, where $\chi_t \equiv C_t/K_t, z \equiv Y_t/K_t$ and $P_t \equiv \beta Y_t/\tau_t$. This transformed system is equivalent to the original system, as $\{K_t, H_t, u_t, C_t\}$ can be represented in terms of $\{u_t, \chi_t, z_t, P_t\}$.[21] Therefore, saddle stability (and determinacy) can be established by confirming that this transformed system has a two-dimensional stable manifold. Using (14) and (15)-(19), we can write the dynamics of the system as:

$$\dot{u_t} = u_t \left(Bu_t - \chi_t + \beta z_t + \Lambda P_t + \frac{1-\alpha-\beta}{\alpha}(B + \delta_K - \delta_H) - \frac{\beta}{\alpha} g_\tau \right), \tag{57}$$

$$\dot{\chi_t} = \chi_t \left(\chi_t - \frac{\theta-\alpha}{\theta} z_t + \frac{\theta-1}{\theta}\phi P_t - \frac{\rho}{\theta} + \frac{\theta-1}{\theta}\delta_K \right), \tag{58}$$

$$\dot{z_t} = z_t \left(-(1-\alpha-\beta) z_t + \Lambda P_t + \frac{1-\alpha-\beta}{\alpha}(B + \delta_K - \delta_H) - \frac{\beta}{\alpha} g_\tau \right), \tag{59}$$

$$\dot{P_t} = P_t \left(-\chi_t + \frac{\alpha + (1-\alpha-\beta)\beta}{1-\beta} z_t + \Omega P_t + \frac{1-\alpha-\beta}{\alpha} B - \frac{\alpha+\beta}{\alpha} g_\tau + \frac{(1-2\alpha-\beta)\delta_K - (1-\alpha-\beta)\delta_H}{\alpha} \right) \tag{60}$$

where Λ and Ω are constants defined by $\Lambda \equiv (1-\alpha-\beta)(\phi-\psi)/\alpha$ and $\Omega \equiv ((1-2\alpha-\beta)\phi - (1-\alpha-\beta)\psi)/\alpha$.

We first examine the stability of the NABGP for the case of $g_\tau \in [0, g^{\max}]$. As examined in Section 3.2, the steady state of the transformed system, denoted by $\{u^*, \chi^*, z^*, P^*\}$, is given by (31) and (35)–(33). Applying a first-order Taylor expansion of equations (57)–(60) around this steady-state yields:

$$\begin{bmatrix} \dot{u_t} \\ \dot{\chi_t} \\ \dot{z_t} \\ \dot{P_t} \end{bmatrix} \simeq \left[\begin{array}{c|ccc} u^*B & -u^* & \beta u^* & \Lambda u^* \\ \hline 0 & & & \\ 0 & & J_1 & \\ 0 & & & \end{array}\right] \begin{bmatrix} u_t - u^* \\ \chi_t - \chi^* \\ z_t - z^* \\ P_t - P^* \end{bmatrix}, \tag{61}$$

where,

$$J_1 \equiv \begin{bmatrix} \chi^* & -\frac{\theta-\alpha}{\theta}\chi^* & \frac{(\theta-1)\phi}{\theta}\chi^* \\ 0 & -(1-\alpha-\beta)z^* & \Lambda z^* \\ -P^* & \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta}P^* & \Omega P^* \end{bmatrix}.$$

We want to show that the Jacobian matrix of (61) has two positive and two negative eigenvalues. From the block-triangular structure of the matrix, one eigenvalue is $u^*B > 0$,

[21] Equivalence is confirmed in that the inverse transformation is well defined. Specifically, $K_t = \tau_t P_t/(\beta z_t)$, $C_t = \tau_t P_t \chi_t/(\beta z_t)$, and $H_t = \left(\tau^{1/(1-\beta)-\widehat{\alpha}}/\widetilde{A}\right)^{1/(1-\widehat{\alpha})} z_t^{\widehat{\alpha}/(1-\widehat{\alpha})} P_t/(\beta u_t)$.

and the other three are given by the eigenvalues of the submatrix J_1. The characteristic equation for J_1 is:

$$-\lambda^3 + \mathrm{tr}(J_1)\lambda^2 - \mathrm{M}(J_1)\lambda + \det(J_1) = 0, \tag{62}$$

where $\mathrm{tr}(J_1)$ is the trace of J_1, $\mathrm{M}(J_1)$ the sum of the principal minors, and $\det(J_1)$ the determinant. These are given by:

$$\begin{aligned}
\mathrm{tr}(J_1) &= \left\{\frac{\theta+\beta-\alpha}{\alpha} - \frac{(1-2\alpha)\phi}{\alpha\psi}\Big(\theta + \frac{\beta}{1-\alpha-\beta}\Big)\right\} g_\tau + \frac{\beta}{\alpha}\delta_K + \frac{\alpha+\beta}{\alpha}\rho \\
&\quad + \left\{\frac{(1-2\alpha)\phi}{\alpha\psi} - \frac{1-\alpha-\beta}{\alpha}\right\}(B-\rho-\delta_H), \\
\mathrm{M}(J_1) &= \begin{vmatrix} \chi^* & -\frac{\theta-\alpha}{\theta}\chi^* \\ 0 & -(1-\alpha-\beta)z^* \end{vmatrix} + \begin{vmatrix} -(1-\alpha-\beta)z^* & \Lambda z^* \\ \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta}P^* & \Omega P^* \end{vmatrix} + \begin{vmatrix} \chi^* & \frac{(\theta-1)\phi}{\theta}\chi^* \\ -P^* & \Omega P^* \end{vmatrix}, \\
&= -\frac{1-\alpha-\beta}{\alpha}\left\{(\phi-\psi)(z^*-\chi^*) + \frac{\alpha\phi\chi^*}{\theta(1-\alpha-\beta)}\right\} \\
&\quad -\frac{1-\alpha-\beta}{\alpha}\cdot\frac{z^*}{P^*}\big\{(\theta-\alpha)g_\tau + (1-\alpha)\delta_K + (1-2\alpha)\phi P^* + \rho\big\}, \\
\det(J_1) &= \frac{\psi(1-\alpha-\beta)}{\theta} z^*\chi^* P^*,
\end{aligned}$$

We determine the sign of the real parts of the roots of (62) based on Theorem 1 of Benhabib and Perli (1994).

Theorem 1 (Benhabib-Perli) *The number of roots of the polynomial in (62) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \qquad \mathrm{tr}(J_1) \qquad -\mathrm{M}(J_1) + \frac{\det(J_1)}{\mathrm{tr}(J_1)} \qquad \det(J_1).$$

Under the assumption that $\psi/\phi < (1-2\alpha)/(1-\alpha-\beta)$, we have $\mathrm{tr}(J_1) > 0$,[22] $\mathrm{M}(J_1) < 0$, and $\det(J_1) > 0$. Thus, the above theorem implies that there is only one eigenvalue with positive real parts in the matrix J_1. Combined with $Bu^* > 0$ obtained before, we have two positive eigenvalues in total. This completes the stability analysis for the case of $g_\tau \in [0, g^{\max}]$ (and therefore the proof of Lemma 3).

Turning to the case of $g_\tau \in (g^{\max}, g^{\lim}]$, the (asymptotic) steady state of the transformed system for this case is given by $P^* = 0$ and (38)–(40) in Section 3.3. The Taylor

[22]This can be confirmed by noting that $\mathrm{tr}(J_1)$ is linear in g_τ and that it is positive at both ends (i.e., $\mathrm{tr}(J_1) > 0$ at $g_\tau = 0,\ g^{\max}$.)

expansion of equations (57)–(60) around this steady state yields essentially the same expression as (61), with the only difference that submatrix J_1 is replaced by:

$$J_2 = \begin{bmatrix} \chi^* & \cdots & \cdots \\ 0 & -z^*(1-\alpha-\beta) & \cdots \\ 0 & 0 & g^* - g_\tau \end{bmatrix},$$

where g^* is the asymptotic growth rate of output, which is defined by (37). As J_2 is a triangular matrix, its eigenvalues are simply given by its diagonal elements. Observe that $g^* - g_\tau$ represent the asymptotic growth rate of $P_t = \beta Y_t/\tau_t$. As discussed in Section 3.3, it is negative in this case (i.e., when $g_\tau \in (g^{\max}, g^{\lim}]$). Therefore, J_2 has one positive eigenvalue (χ^*) and two negative ones ($-z^*(1-\alpha-\beta)$ and $g^* - g_\tau$). This completes the stability analysis for the case of $g_\tau \in (g^{\max}, g^{\lim}]$ and the proof of Lemma 4.

■

Details of welfare maximization (Section 4)

The current value Hamiltonian for the social planner's problem is:

$$\begin{aligned}\mathcal{H} = \frac{C_t^{1-\theta}-1}{1-\theta} &+ \nu_t^o[AK_t^\alpha (u_t H_t)^{1-\alpha-\beta} P_t^\beta - C_t - (\delta_K + \phi P_t)K_t] \\ &+ \mu_t^o[B(1-u_t)H_t - (\delta_H + \psi P_t)H_t],\end{aligned}$$

where ν_t^o and μ_t^o are the planner's shadow prices associated with the accumulation of physical capital and human capital, respectively. The first-order conditions are:

$$\nu_t^o = C_t^{-\theta}, \tag{63}$$

$$\frac{\dot{\nu}_t^o}{\nu_t^o} = \rho + \phi P_t + \delta_K - \alpha \frac{Y_t}{K_t}, \tag{64}$$

$$\mu_t^o = \frac{(1-\alpha-\beta)Y_t}{B u_t H_t}\nu_t^o, \tag{65}$$

$$\frac{\dot{\mu}_t^o}{\mu_t^o} = \rho - \frac{\nu_t^o}{\mu_t^o}(1-\alpha-\beta)\frac{Y_t}{H_t} - B(1-u_t) + \delta_H + \psi_t P_t. \tag{66}$$

$$\frac{\beta Y_t}{P_t} = \phi K_t + \psi_t(\mu_t^o/\nu_t^o)H_t, \tag{67}$$

The resource constraints for the social planner's problem are (15) and (16). Differentiating the log of (65) with respect to time, eliminating $\dot{\nu}_t^o/\nu_t^o$ and $\dot{\mu}_t^o/\mu_t^o$ by (64) and (66), and then eliminating ν_t^o/μ_t^o by (65) gives condition (17). Similarly, differentiating the log

of (63) with respect to time and eliminating $\dot{\nu}_t^o/\nu_t^o$ by (64) gives (18). The transversality conditions for this problem are $\lim_{t\to\infty} K_t\nu_t^o e^{-\rho t} = 0$ and $\lim_{t\to\infty} H_t\mu_t^o e^{-\rho t} = 0$, which are the same as those for the market equilibrium. Finally, eliminating (μ_t^o/ν_t^o) from (67) by (65) yields condition (41).

Proof of Proposition 4 (Section 5.1)

The proof is essentially similar to the discussion in Section 3. Note that equation (48) implies $\dot{\tau}_t/\tau_t = \dot{Y}_t/Y_t - \dot{E}_t/E_t$, the RHS of which is asymptotically constant from the definition of NABGPs. Thus, the growth rate of τ_t is also asymptotically constant and written as $g_\tau = g^* - g_E$, where g_E is the asymptotic growth rate of emission. From this, we can show that the asymptotic growth rate of economy g^* cannot exceed g_τ. Observe that if $g^* > g_\tau$, the previous equation implies $g_E > 0$. This means emission E_t grows without bound, stock P_t also grows without bound from (46), natural disasters occur increasingly frequently, and physical and human capital are destroyed at an ever-increasing rate. As this is obviously incompatible with NABGPs, $g^* \leq g_\tau$ must hold.[23]

Given $g^* \leq g_\tau$, it results that the asymptotic growth rate of emissions is zero or negative ($g_E = g^* - g_\tau \leq 0$). In fact, $E_t > 0$ and $g_E \leq 0$ means that the amount of emissions E_t is asymptotically constant: $E_t \to E^* \geq 0$. Moreover, from (46), the stock of pollution is also asymptotically constant: $P_t \to P^* \equiv (\gamma/\delta_P)E^* \geq 0$. It is easy to see that $P^* = 0$ holds when $g^* < g_\tau$, because $g_E < 0$ and therefore $P^* = (\gamma/\delta_P)E^* = 0$. Thus, we have:

$$P^* \begin{cases} \geq 0 \text{ if } g^* = g_\tau & \text{(Case 1)}, \\ = 0 \text{ if } g^* < g_\tau & \text{(Case 2)}, \end{cases} \tag{30}$$

which is exactly the same as what we attained in Section 3.1. The remaining conditions that characterize the NABGP are also the same (conditions 26 to 29) because they are derived from (15)–(18), which were not changed by the introduction of pollution stocks. Therefore, the discussions in Sections 3.2 and 3.3) are still valid, which yield (31)–(40).

[23] See the proof of Lemma 1 in the Appendix for a formal discussion.

Details of welfare maximization with stock pollution (Section 5.2)

Note that the definition of pollution stock (46) implies that P_t evolves according to $\dot{P}_t = \gamma E_t - \delta_P P_t$. Using this, the current value Hamiltonian for the social planner's problem can be written as:

$$\mathcal{H} = \frac{C_t^{1-\theta} - 1}{1-\theta} + \nu_t^o [A K_t^\alpha (u_t H_t)^{1-\alpha-\beta} E_t^\beta - C_t - (\delta_K + \phi P_t) K_t] \\ + \mu_t^o [B(1-u_t) H_t - (\delta_H + \psi P_t) H_t] + \lambda_t [\gamma E_t - \delta_P P],$$

where λ_t is the shadow price of pollution stock. The first-order conditions are given by (63)–(66) and:

$$\frac{\beta Y_t}{\gamma E_t} = \frac{-\lambda_t}{\nu_t^o} \tag{68}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\nu_t^o}{\lambda_t} \phi K_t + \frac{\mu_t^o}{\lambda_t} \psi H_t + \rho + \delta_P. \tag{69}$$

The TVCs are $\lim_{t\to\infty} K_t \nu_t^o e^{-\rho t} = 0$, $\lim_{t\to\infty} H_t \mu_t^o e^{-\rho t} = 0$, and $\lim_{t\to\infty} P_t \lambda_t e^{-\rho t} = 0$. Similar to the analysis for Section 4 (see above), it can be shown that conditions (63)–(66) and the first two TVCs are the same as the market equilibrium. Note that $P_s \geq P_t e^{-\delta_P (s-t)}$ holds for all $s \geq t$ from $\dot{P}_t = \gamma E_t - \delta_P P_t$ and $E_t \geq 0$. This inequality and the TVC for P_t jointly imply:

$$0 = \lim_{s\to\infty} P_s \lambda_s e^{-\rho s} \geq \lim_{s\to\infty} \lambda_s P_t e^{-\delta_P (s-t)} e^{-\rho s} = P_t e^{-\rho t} \lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} \geq 0$$

which means[24]

$$\lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} = 0. \tag{70}$$

In the following, we derive the value of λ_t from (69) and (70). Substituting s for t in (69) and multiplying both sides by $\lambda_t e^{-(\rho+\delta_P)(s-t)}$ gives:

$$\dot{\lambda}_s e^{-(\rho+\delta_P)(s-t)} - (\rho+\delta_P) \lambda_s e^{-(\rho+\delta_P)(s-t)} = \left(\nu_s^o \phi K_s + \mu_s^o \psi H_s\right) e^{-(\rho+\delta_P)(s-t)}. \tag{71}$$

Observe that the LHS of (71) is the derivative of $\lambda_s e^{-(\rho+\delta_P)(s-t)}$ with respect to s. Thus, we can calculate the definite integral of the LHS from $s = t$ to $s \to \infty$, which becomes:

$$\left[\lambda_s e^{-(\rho+\delta_P)(s-t)}\right]_{s=t}^{\infty} = \lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} - \lambda_t = -\lambda_t,$$

[24]Note that P_t cannot become 0 in finite t, although it may asymptote to 0.

where the second equality follows from (70). As this must coincide with the definite integral of the RHS of (71), we obtain:

$$-\lambda_t = \int_t^\infty \left(\nu_s^o \phi K_s + \mu_s^o \psi H_s\right) e^{-(\rho+\delta_P)(s-t)} ds. \tag{72}$$

Eliminating ν_t^o and μ_t^o from (68) and (72) using (63) and (65) gives (49) in the text.

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