An Empirical Model of Mainframe Computer Investment*

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Abstract

This paper formulates a stochastic optimal stopping model for the investment of mainframe computer systems in the telecommunications industry. It describes the investment behavior by focusing on unique features of computer systems, which are associated with technological development. The optimal investment rule is the solution of a stochastic dynamic programming model that specifies the system administrator's objective to maximize profits through three main choices: 'keep', 'upgrade', or 'replace'. If replace, there are various capacity choices. The model depends on unknown parameters which govern both the profit structures of the task level of the company and the system administrator's expectation of the future values of the state variables.

Using a detailed data set on computer holdings by one of the world's largest telecommunication companies, I investigate the key explanatory facts of computer replacement and estimate the model with the nonlinear-nested fixed point algorithm (NLS-NFXP). The estimation requires two procedures: (i) a parametric approximation procedure which converts the contraction fixed-point problem into a nonlinear least squares problem; (ii) maximum likelihood estimation method to estimate the unknown parameters. I also show the effectiveness of the parametric approximation method in comparison with the discretization method.

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The estimation supports the observed explanatory facts of the data in general, allowing for better understanding of the replacement behavior in an era of rapidly evolving computer technology. Simulations of the estimated model predicts the data well enough to assure that the firm follows an optimal investment strategy to replace and upgrade its computer system by keeping track of the rapid development of computer technology and demand for its services. Several policy experiments are accomplished to show the versatility of the model.

Keywords: Mainframe computers, Technological progress, Optimal replacement, Optimal upgrade, DP model, Parametric approximation, Nonlinear-Nested Fixed Point Algorithm (NLS-NFXP)

JEL Classification: C3, C4, C6, L1, L6, Q3.

1 Introduction

Despite the importance of computers in the "information economy", comparatively little is known about the factors affecting investment decisions, including timing of upgrade and replacement choices. In the face of rapid technological progress and steadily declining costs, consumers and firms must decide whether to upgrade or replace existing computer systems now, or wait to purchase a faster/cheaper system in the future.

Regarding systems replacement in general, there has been previous research[1] on the replacement of bus engines (Rust, 1987) and aircraft engines (Kennet, 1994). However, computers differ from the engines in the following respects. First, while bus and aircraft engines are replaced due to physical depreciation, such as natural wear-out and mechanical failure, replacement of computer systems are usually caused by technological depreciation. The main reason to replace engines is to prevent a future failure and capacity improvement is a secondary reason in research on the replacement of engines. As a result, state variables are the hours of operation and the history of engine shutdowns in case of engines replacement, which represent various measurements of physical depreciation. In contrast, though the prevention of future failure can be a reason to replace or upgrade computer systems, the main reason is to improve performance and meet demand for service. Thus, the aforementioned variables may not be appropriate in a model for computer systems replacement. In the case of computer systems, the replacement caused by physical depreciation accounts for a relatively small fraction of the entire set of replacements. Thus, one of the major features of replacements of computer systems is technical depreciation. One of supporting examples is as follows: According to Moore's law each new CPU (Central Processing Unit) contains roughly twice as much capacity as its predecessor in every 18 months. In the storage industry, density has been doubling every 12 months, which is faster than the speed of CPU development.

For example, Figure 1 illustrates that how Moore's law explains developing trend of computer technology in terms of Intel CPUs. The time frame of my data starts from 1989 and ends on 1999, where 1M transistors per CPU (486 DX Processor) has changed to over 24M transistor CPU (Pentium III Processor) according to Figure 1. In that period, computer technology had been developed tremendously and the technological obsolescence is accelerating. Table 1 also presents how Moore's law acts in development of various components of computer system. We may note that there is a tremendous improvement in computer technology between 1984 and 1997.

[1] There are other related research regarding cement kilns (Das and Rust) and nuclear power plants (Rothwell and Rust, 1995).

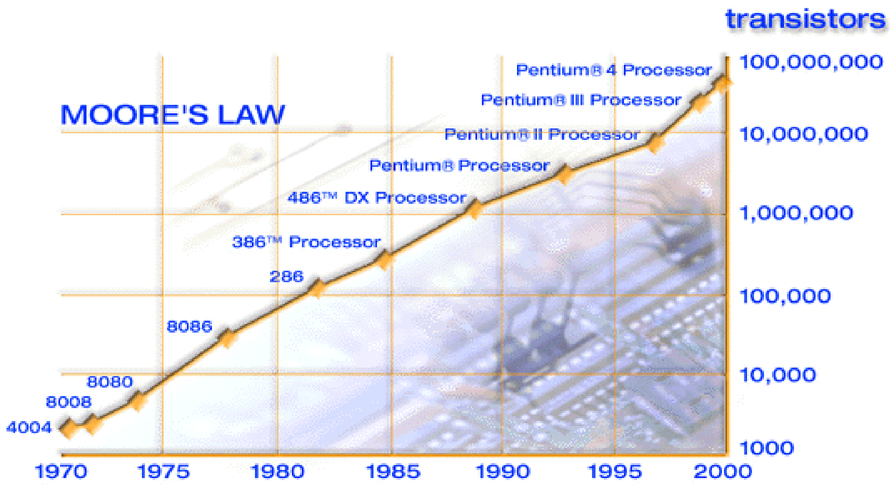

Figure 1: Source - Intel

Furthermore, we expect much faster technological development between 1997 and 2009. Thus, possible candidates of state variables should reflect this developing trend. Possible candidates are the following: (i) an introduction of new operating system (a new operating system may require a more advanced system to work properly); (ii) the difference of CPU speed between the current system and the fastest system available. The continuous introduction of new CPUs in the market makes the relative speed of old CPUs decrease and thus the relative operating costs become higher than having systems with new CPUs, i.e., technically, the CPUs of the current systems continuously depreciate.

Table 1[2]

Moore's Law in Action

Year	**1979**	**1984**	**1997**	**2009**
RAM	16K	128K	12mb	3251mb
Hard Drive	128K	400K	750mb	203,187mb
Speed	2	10	150	40,637
Cost	$5,000	$3,900	$1,400	$10

Second, in computer systems, upgrade is an alternative to replacement when attempting to

[2] Source: Intel

improve performance. In case of bus or aircraft engines, there is no upgrade choice[3]. In fact, for computer systems, upgrading is sometimes the first choice over replacement. Therefore, replacement of computer systems requires us to deal with a more complicated decision process than that of engine replacements. Here, we first have to decide whether to replace, upgrade, buy an additional system, or keep the current system. These decisions are considered as the main choices. Contingent on these main choices, we are confronted with a set of sub-choices. For instance, a replacement decision requires other subsequent choices, such as the capacity and the brand of new computer systems, which require several aspects of the multiple discrete choice model.

Third, an introduction of new software, such as a new operating system (OS), is one of main reasons to replace computers, since a new operating system may require a more advanced system or a larger capacity to work properly. For example, each new OS has a minimum requirement of computers' specification and this minimum requirement tends to increase over time with newer operating systems.

Fourth, unlike cases of engine replacements, vendors' service support may play an important role in the replacement decision of computers. Since vendors usually do not support old systems without an extra service contract, maintaining old computer systems may be more costly than replacing them with new computers[4].

This paper presents a dynamic programming model of a firm's decision of whether to keep, upgrade, or replace an existing computer subject to uncertainty in the demand for services and over the timing and magnitude of future cost reductions of new computer systems. I estimate this model using a detailed data set on computer holdings of one of the world's largest telecommunications companies. An initial analysis of these data leads to the following conclusions. First, the durations between successive upgrades or replacements have become shorter during the last two decades, possibly reflecting the increased rate of technological progress in computing equipment during this time period. Second, computer replacements occurred roughly at a 6-year cycle at the beginning of the sample period, decreasing to 5-year cycle at the end of the period. Third, I show that when

[3] Even though engine maintenances for better performance can be considered to be an upgrade choice, I assume the engine maintenance as a behavior of "do nothing", since it is difficult to have better performance without replacing it in case of engines due to the nature of engines.

[4] Other unique feature is as follows: We may examine to what extent the replacement behaviors are done individually or on a "fleet replacement" basis due to costs of training administrative staff. In many cases, block purchases of computer can give the firm a quantity discount. These features can be considered in the future extension upon availability of richer data.

increases on demand for the services of the computer begin to exceed its processing capacity, the firm is more likely to expand its capacity via an upgrade of the existing computer rather than a purchase of a new computer if the existing computer is relatively new, but more likely to replace the computer as its age approaches the length of the replacement cycle. These facts support that the presence of rapid technological progress affects the firm's replacement and upgrade policy along with the economic development.

The formal analysis begins in section 4. I develop a stochastic dynamic programming model to see whether these stylized facts of replacement and upgrade behavior can be rationalized as an optimal investment strategy for this firm. In the model, the firm has three possible actions at each time period: keep, upgrade, or replace. If replace, there is an array of capacity choices for a new computer system. The state variables include the processing capacity of the current system, the level of demand for this processing capacity, the age of the current system, and the current market price of a standardized unit of processing capacity. The technological depreciation and the relative performance of each computer system are measured by composite measures of all four state variables in the model. The model depends on unknown primitive parameters that specify the firm's profit function and its expectation of future values of the state variables, with its expectation of future reductions in the price of computing capacity playing a critical role in the model's predictions of the optimal length of the replacement cycle.

In section 5, I investigate of a parametric approximation, which greatly reduces the computational burden involved in solving the infinite-horizon version of the model. The parametric approximation procedure converts the contraction fixed-point problem into a nonlinear least squares problem. I show that this latter problem can be solved much more rapidly than standard methods based on discretization of state space. I also show the effectiveness of the parametric approximation method in comparison with a sample result from discretization.

In section 6, I estimate the model using a nonlinear nested fixed point algorithm (NLS-NFXP) incorporating a parametric approximation method to solve the DP problem. The nonlinear nested fixed point algorithm is a maximum likelihood estimation, in which outside of maximum likelihood estimation, the above nonlinear least square estimation (NLS) is performed to calculate fixed points and inside of maximum likelihood estimation, based on the NLS, to estimate unknown parameters. The estimation results support the observed stylized facts in general, allowing for a better understanding of replacement behavior of firms in the era of rapidly growing computer technology. Based on the estimation results, I conduct several simulations to illustrate the estimation results and to show how the proposed model predict the data. Section 7 investigates some policy implications of the model by deriving the aggregate demand functions for investment of mainframe computer systems. Section 8 finally provides some concluding comments and directions for future research.

2 Summary of related literature

Rust (1987)'s seminal work on systems replacement provides a general template for approaching this topic. In this paper, he formulates a regenerative optimal stopping model for bus engine replacement to describe the behavior of the superintendent of maintenance at the Madison Metropolitan Bus Company.

In particular, Rust presents that the superintendent's decision-making behavior on bus engine replacement can be implemented as an optimal stopping rule. It is a strategy for deciding when to replace current bus engines, and is given as a function of observed and unobserved state variables. The optimal stopping rule is formulated as the solution to a stochastic dynamic programming problem that formalizes the trade-off between the conflicting objectives of minimizing maintenance costs and minimizing unexpected failures of bus engines.

This paper is important in at least two aspects. First, it provides a general framework that can be used to analyze replacement behavior in various fields. It is the first research that uses a "bottom up approach" for modeling replacement investment. Second, the paper develops a nested fixed-point algorithm for estimating dynamic programming models of discrete choices. The algorithm is very useful in solving problems that arise typically in investigating replacement behavior. The results in the paper have been widely applied since its publication, and have been extended by many authors in numerous directions[5].

Despite its significant role in replacement research, Rust's model was not intended for computer systems. In contrast, there has been several articles related to the investment of computer systems, namely, Hendel (1999), Ito (1997), and Greenstein and Wade (1997). Hendel presents a multiple-discrete choice model for the analysis of differentiated products that are durable goods in a continuous process of technological change. Hendel develops a model of PC purchasing behavior to deal with the main feature of PC demand, which is multiple-discreteness. That is, Firms spread their purchases over various brands of computers with characterizing in block-purchase.

[5] aircraft engine mainetnance: Kennet (1994), cement Kilns: Das and Rust, and nuclear power plants: Rothwell and Rust (1997)

The proposed model, along with a new data set on PC holding, permits demand estimation at the micro-level. His model is very useful in explaining an optimal replacement behavior of PCs, since one of the most important features of PC replacement is also the block-purchase.

Ito (1997) presents an empirical investigation of the source of investment adjustment costs. Since mainframe computers are often the central pieces of hardware in business information systems, the author examines the dynamics of micro-level investment behavior in order to infer the size of implicit adjustment. She identifies the lumpiness of adjustment costs and concludes that the variation in adjustment costs arises due to the different degree of organizational friction in the investment processes of mainframe computers. She also finds that adjustment costs did not increase with the level of engineering adjustment activities, such as development of new software for new computer systems. Though Ito rightly points to the importance of adjustment cost in investment behavior, she pays little attention to the role of technology in the adjustment cost.

Greenstein and Wade (1997) investigate the product life cycle in the commercial mainframe market. In particular, they examine the entry and exit behavior of mainframe computers in the market using the hazard and Poisson models. The hazard model helps to estimate the probability of product exit and the Poisson model helps to estimate the probability of introduction. Additionally, this paper indicates many important market structures which may cause entry and exit of products, such as cannibalization, vintage and degree of competitiveness.

Also, there are several articles by Bresnahan, and Bresnahan and Greenstein (1997) which investigate the structural changes of mainframe computer market regarding to technological changes.

Unfortunately, previous research regarding systems replacement (Rust (1987) and Kennet (1994)) do not focus on the effect of technological progress on replacement decisions in general nor on its effect on the replacement of computer systems. Moreover, the literature regarding the investment of computers also does not deal with replacement of computers. Greenstein and Wade, and Bresnahan and Greenstein focus on supply side of computers, even if the replacement of computer systems focuses on choices of demand side. Also, even though Hendel focuses on choices of demand side, his model is different from what I want to show in this research. First, my model is associated with replacement behavior. Second, in case of the mainframe computers data, the firm tends to keep the same brand of mainframe computers in favor of easier services, when it decides to replace a current system. Thus, the brand choice is disregarded[6]. However, even though our model is different from Hendel's model, we still form the model implicitly as

[6] However, this assumption can be released in future extension.

a multiple discrete choice model in a sense that first, an actual replacement decision is based on current stocks of computer systems. That is, there are various tasks for the firm and each tasks has a fixed number of mainframe computer systems. Thus, aggregation of tasks of the firm and replacement choices over aggregated tasks can be viewed as choice for number of computers. Second, simultaneous choice for replacement timing should be accompanied along with the choice over the aggregated tasks.

3 The Data

3.1 Summary of the Data

I obtained data from one of the biggest telecommunication companies in the world. It handles over 60 percent of the entire phone services in the market at which it operates. It also offers several other telecommunication services, such as cellular PCS (Personal Communications Service), internet, cable, and satellite communication services. The company has 864 hosts (including workstations) and about 39,000 PCs as of 1998. These hosts and PCs are spread out in 400 regional headquarters and regional offices. All regional headquarters operate independently and own their computer systems, even though there is difference in terms of capacity. Therefore, in most cases, each regional headquarter decides maintenance and investment of its mainframe computers independently.

The computer systems in the company can be divided into two parts according to use: (i) research use, and (ii) service and management use. Since computers for research use are purchased and replaced on project basis, their maintenance activities do not reflect technological depreciation[7]. Thus, I only consider computer systems for only service and management use in the data for this research. I also do not include the replacement of PCs in the company, since in PC replacement there is no upgrade activity and there only are block purchases and replacements. There are several tasks within service and management use. Table I presents important tasks in service and management use.

The time frame of the data set starts from 1989 and ends on 1999. The data prior to 1989 are incomplete, though some computer systems have a history starting from earlier dates, such as 1977, 1979, 1983, and 1985. Within this time frame (1989-99), I have a full history of upgrade and

[7] Mainframe computers in research use have only finite horizon bases of research project, which is different from an assumption of the model, an infinite horizon case.

replacement for 123 computer systems in the company. The data consists of dates of introduction, purchase prices, specifications, dates of upgrades and replacements, prices of upgrades and replacements, details of replacements and upgrades, such as system specifications. The numbers of customers for services provided by the firm also are available as a form of monthly data.

Price data for CPU, hard drive, memory, and other hardware were obtained from several computer databooks[8], online computer resources[9], and manufacturers' web sites[10].

3.2 Explanatory Investigation of the Data

I divided all computer systems in the sample into two categories in terms of the two different standards of CPU benchmarks, which are MIPS (Million Instructions per Second) and TPC (Transaction Processing performance Council). Currently, the MIPS standard is in the process of being merged into the TPC standard, which includes the tpm (transactions per minute) and tps (transactions per second). However, since my data set consists of various computer systems and dates, it is very difficult to convert the MIPS standard into the TPC standard. Within each standard, I divide computer systems into different task groups. Once a certain system brand is designated to serve a given task, the later replacement is from the same or at least similar system brand. Table 2-(a) illustrates the different groups of major tasks and number of systems in terms of the two CPU standards. All mainframe computer systems are associated with specific tasks.

Table 2-(b) illustrates the average, minimum, and maximum costs of three activities, namely, new purchase, upgrade and replacement in terms of the two CPU standards.

As I expected, for both standards, the costs of upgrade are less than the costs of new purchase or replacement. According the computer industry databooks, the cost per unit capacity decreases over time. For example, with a base year of 1982 as 100, the cost in 1998 is measured as 1. Based on this information, the firm has increased the capacities of computer systems tremendously, since the average price of replacement is the same or higher than the average costs of new purchases. This phenomenon can also be confirmed in the several databooks of the computer industry. That is, costs of high-end computer systems, such as mainframe computers in the market, have not decreased and have at time slightly increased over time.

Figure 2 illustrates upgrade and replacement schedules associated with several important

[8] SIA annual data books

[9] CNET.com, ZDnet.com, PC world, and etc

[10] Intel, AMD, MIPS, TPC, SUN, Motorola, Honeywell, Fujitsu, Unisys, Tandem, Samsung, Micron, Seagate, IBM, and etc

tasks in Table 2-(a). Table 3 illustrates the intervals between upgrades and the intervals between replacements.

Table 2-(a)

Computers included in the sample in terms of CPU standards

CPU standard	MIPS	TPC
Number*	48	57
Tasks	Billing-Development	Business Info-Management
	Billing-Management	Customer Development
	General Management	Total Document
	New Customer Info-system	Pre-Billing
	Super High Speed Printer	Line-Management
		Material information

*: number of computers in the sample

Table 2-(b)[11]

Costs for three activities in the sample

Activity	Cost	MIPS	TPC
New purchase	Average	$572,919.9	$968,191.1
	Min	$41,917.7	$20,440.1
	Max	$4,893,545.6	$4,633,600.4
Replacement	Average	$1,082,499.4	$899,340.8
	Min	$16,752.8	$14,854.3
	Max	$7,160,791.3	$3,377,322.2
Upgrade	Average	$263,123.9	$435,181.4
	Min	$2,645.12	$3,251.5
	Max	$3,176,710.1	$2,283,130.1

The first notable fact in Table 3 is that the intervals between replacements are generally much longer than those of upgrades. Second, the maximum intervals between replacement are 61 months and 53 months for MIPS and TPC standards respectively. This is because one of the major reasons for replacing a computer system is the age of the computer, which has an average

[11] The reason of big differences between minimum costs and maximum costs in the various activities is as follows: Since the data consist of various computer systesms, such as workstation, server, and mainframe computers. These varieties make the gaps between two costs much wided.

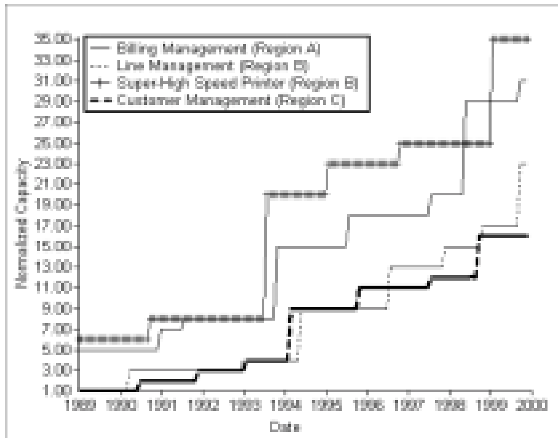

Figure 2: Upgrade and Replacement schedule associated with several important tasks.

5 year life span for the company. In other words, internally regulated policy restricts the life-span of mainframe computers to a 5 year cycle. Figure 3 shows the replacement frequency of computer systems in the firm.

Table 3

Intervals between upgrades and intervals between replacements

		MIPS	TPC
Interval between Replacement	Average	48.3	43.2
	Min	22	19
	Max	61	53
Interval between Upgrade	Average	22.1	16.7
	Min	7	10
	Max	39	32

All numbers are months

This reflects the fact that the computer systems become technologically obsolete after 5 years of use, even though not obsolete physically. Furthermore, this policy has been changed from 6 years to 5 years in recent years, which corresponds to the more rapid speed of technological progress.

Due to the development of the computer industry in the 80's and 90's and the increases in demand for services, the intervals between the two subsequent actions[12] becomes shorter and

[12] Obviously, there are four combinations of actions: (i) upgrade-replacement; (ii) replacement-upgrade; (iii)

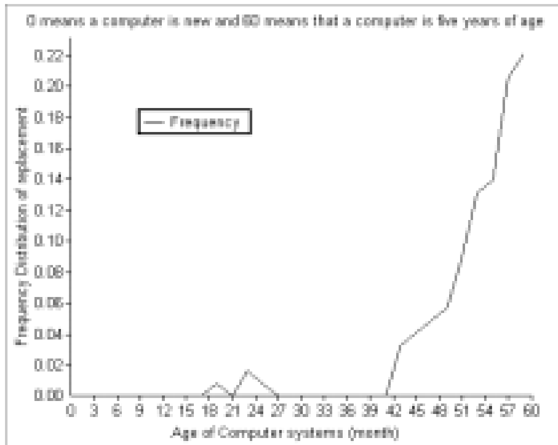

Figure 3: Frequency Distribution of replacements in terms of computers' age.

shorter. Tables 4-(a), (b), and (c) illustrate several examples of the shortening of intervals in certain computer systems assigned to major tasks. One reason for shorter intervals is that the pace of development in the computer industry has become significantly faster and thus the current system becomes obsolete much more quickly.

Table 4-(a)

Examples of Activities and brands of computers in various tasks

Task 1			
Brand of system*	A	B	B
Region	1	2	3
New purchase→first action**	38 months	37 months	24 months
Interval 1st⟶2nd action	23 months	24 months	19 months
Interval 2nd⟶3rd action	20 months	11 months	22 months
Interval 3rd⟶4th action	17 months	8 months	13 months
Interval 4th⟶5th action	15 months	17 months	11 months
Interval 5th⟶6th action	12 months	11 months	12 months

*: A-Unisys system, B-Honeywell and Unisys system (MIPS standard)

**: Actions includes upgrade and replacement

upgrade-upgrade; (iv) replacement-replacement.

Table 4-(b)

Task 2				
Brand of system*	C	C	C	C
Region	1	2	3	4
New purchase → first action	27 months	18 months	18 months	22 months
Interval 1st ⟶ 2nd action	25 months	15 months	16 months	18 months
Interval 2nd ⟶ 3rd action	12 months	12 months	12 months	15 months
Interval 3rd ⟶ 4th action	18 months	15 months	11 months	12 months
Interval 4th ⟶ 5th action	12 months	13 months	12 months	10 months
Interval 5th ⟶ 6th action	11 months	1 0months	.	.

*: C-Tandem system (TPC standard)

Table 4-(c)

Task 3				
Brand of system*	D	D	D	D
Region	1	2	3	4
New purchase → first action	59 months	36 months	42 months	51months
Interval 1st ⟶ 2nd action	23 months	35 months	38 months	34 months
Interval 2nd ⟶ 3rd action	20 months	15 months	13 months	10 months
Interval 3rd ⟶ 4th action	13 months	13 months	9 months	.

*: D-Toray and Fujitsu system (MIPS standard)

Figure 3.1[13] shows that cost per capacity has decreased rapidly from 1994 to the current period. Second, the demand for the services provided by the company is growing tremendously. More frequent upgrades and replacements emerged in 1995, 1996 and 1997, when demand for services increased by greater amounts. However, from mid 1998, there was very little upgrade/replacement observed, since demand decreased significantly due to the economic depression. Figure 3.2[14] shows trend of total demand. The trends of average capacities are illustrated in Figure 3.3 and 3.4[15].[16]

[13] Source: SIA Annual databook

[14] The detailed explanation of the unit of demand is in section 5.1.

[15] In the figures 3.3 and 3.4, Y-axes represents a weighted average of capacity of mainframe computers. Three most important components of computers are CPU, Memory, and Hard Disk. Among these three components, the weights are given such as CPU - 0.5, Memory - 0.25, and Hard Disk - 0.25. Then, these weighted average capacities were discretized to simplify. More details are in the later section.

These weights were confirmed by several system administrators in the company.

[16] They show average capacities in terms of MIPS and TPC standards

Both Figures 3.3 and 3.4 show that capacities increased rapidly from 1994 to 1998. when cost per capacity and demand changed rapidly. However, noticeably since the reduced amounts of cost per capacity is much larger than increased amounts of demand, the effect of cost per capacity on capacity increases seems to be much larger than that of demand.

Average frequency of upgrades for an individual computer system is 2.5 times. The maximum frequency of upgrades in turn is four times. This is because each computer has limited slots for upgrade. Once the upgrade slots are full, the system needs to be replaced to increase its capacity or to meet a growing demand. The average frequency of replacements at each task level is approximately two, though some tasks undergo three or more replacements. Also, there are some tasks which do not undergo any replacement.

Figure 3.1: "Real Price" of Semiconductors (All values are normalized)

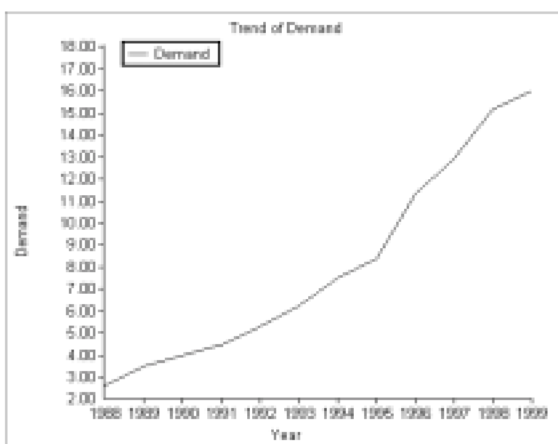

Figure 3.2: Trend of Total Demand (All values are normalized)

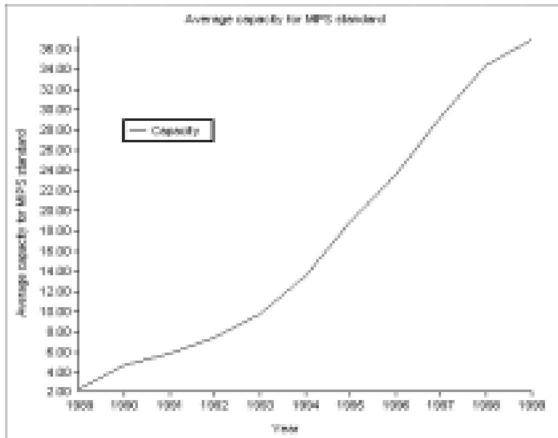

Figure 3.3: Trend of Capacity (MIPS Standard)- All values are normalized

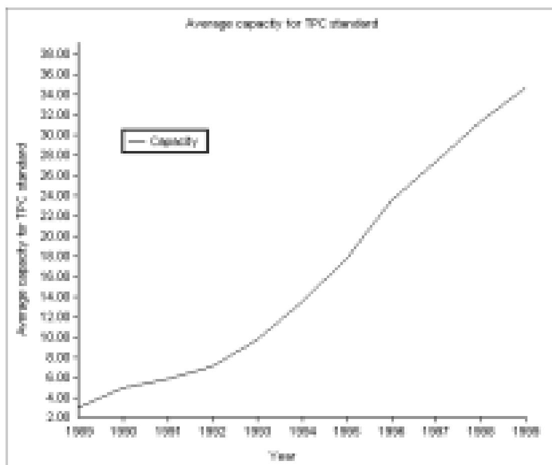

Figure 3.4: Trend of Capacity (TPC Standard)- All values are normalized

4 The Model

This section develops a stochastic dynamic programming model to order to explain the observed pattern of replacement and upgrade observed in the data and to determine whether it can be rationalized as an optimal strategy for the firm. My final objective is to explain the data by deriving a stochastic process $\{a_t, X_t\}$ with an associated likelihood function l $(a_1,..., X_1,...., \theta)$ formed from the solution to a particular optimal stopping problem.

The stochastic DP model consists of a vector of state variables X_t, control variables a_t, a profit function $\pi(X, a)$, a discount factor β, and a Markov transition density $p(X' | X, a)$, representing the stochastic law of motion for the states of computer systems. I assume that the state variable X_t can be partitioned into two components, $X_t = (x_t, \varepsilon_t)$, where x_t is an observed state vector and ε_t is an unobserved state vector. System administrators observe both components of X_t, but the econometrician observes only x_t. The system administrators weigh the consequences of various operating decisions given the states of various computer systems and attempts to perform the best actions. I assume that the result of this decision process can be summarized by a vector of current net benefits (or costs, if negative) corresponding to each operating decision.

4.1 Choice variables

Suppose that, at every month of the year, a system administrator investigates the status of each computer system and decides whether to upgrade, replace, or keep. Thus, the choice set is $A_t = \{0, u, 1\}$, where $(a_t = 0)$ is to keep the system unchanged, $(a_t = u)$ is an upgrade, and $(a_t = 1)$ represents a replacement of system. When the choice is to replace, the system administrator needs to choose the capacity of the new system, i.e., there are n sub-choices of capacities, $K_1, ..., K_n$. Each K_r is a capacity choice for replacement.

The final choice set is as follows: $a : A = \{0, 1, K_1, ..., K_n\}$, i.e., keep $= 0$, upgrade $= u$, and replace $= (K_1, ..., K_n)$.

4.2 State variables

I assume that two of the state variables are discrete, which are the capacity and the age of a current computer system. Two additional variables are continuous, being the demand for services and the cost per capacity in the computer market.

The state set in the model is $x : x_t = \{d_t,\ k_t,\ g_t,\ c_t\}$, where $d_t =$ demand for services, $k_t =$ current capacity of the computer system, $g_t =$ age of each computer system, $c_t =$ real cost per capacity, which can be seen as a market price of capacity. The two state variables g_t and k_t explain internal states of computers and the remaining variables d_t and c_t represent external states of computer systems.

An aggregate demand D_t consists of the sum of the individual demands, d_t for services provided by each task. That is, aggregate demand at time t, $D_t = \sum_j d_{t,j}$, where $d_{t,j}$ is a demand for a task j at time t, and $d_{t,j} = \xi_j D_t$ with $0 < \xi_j < 1$.[17] The aggregate demand for services is assumed to follow an $AR(1)$ process, i.e., $\ln(D_t) = a + \rho \ln(D_{t-1}) + \mu_t$ with $\mu \tilde{} IID\ N(0.\varpi^2)$ and $|\rho| < 1$

[17] In order to calculate a fraction ξ_j for a demand d_t which a specific task serves, I sum up all capacities of computer systems and assume that a proportion for capacity of a system corresponds to a fraction of demand for a system.

for stationarity. Therefore, $\ln(D_t)$ is distributed as normal with mean $\frac{\alpha}{1-\rho}$ and variance $\frac{\varpi^2}{1-\rho^2}$.

The real cost per capacity, c_t is bounded by zero.

The c_t evolves as follows:

$$c_{t+1} = \left\{ \begin{array}{lll} \delta_t c_t & \text{with} & 1- \ b \\ c_t & \text{with} & b \end{array} \right\} \tag{1}$$

δ_t has a truncated normal distribution with mean μ and ν^2 with a range of $0 < \delta_t < 1$.

Therefore, we have the following probability: $p(c_{t+1} \leq z|c_t) = (1-b) \times p\{\delta_t c_t \leq z\} + b \times I(c_t \leq z)$.

The age variable, g_t represents the age of each computer system. Since the firm has the predetermined rule of replacement according to the age of each system, I intend to keep track of the age of each system.

4.3 Profit function

I assume that each mainframe computer system is specifically associated with a certain task. That is, there is only one computer system per a task. Also, the purchase of additional computers as an alternative to replacement is prohibited. The profit function for a task is as follows:

$$\pi(d_t, k_t, g_t, a_t, \theta_1) = R(g(k_t, g_t), d_t, \theta_1, a_t) - C(f_t(k, a_t), g_t, d_t, \theta_1, \varepsilon) \tag{2}$$

where

$$f_t(k, a_t) = \left\{ \begin{array}{ll} k_{t-1} & a_t = 0 \\ k_{t-1} + h & a_t = U \\ K_r & a_t = K_r \end{array} \right\} \tag{3}$$

where $f_t(k, a_t)$ is a rule of capacity evolution associated with choice, a_t and h is a capacity increase by upgrade with $h = 1, 2$ .and $r = 1, .., n$.

θ_1 is a set of unknown parameters for profit function. Profit function consists of two components, revenue function, $R(g(k_t, g_t), d_t, \theta_1, a_t)$ and cost function, $C(f_t(k, a_t), g_t, d_t, \theta_1, \varepsilon)$. $q(f_t(k, a_t), g_t)$ is an adjusted capacity. $q(f_t(k, a_t)\ , g_t)$ illustrates how capacity contributes to the revenue function. For example, the contribution of capacity will decline as a computer gets old.

Forms of Revenue function can be presented as follows:

$$R(g(k_t, g_t), d_t, \theta_1, a_t) = \left\{ \begin{array}{c} \text{Flexible functional form} \\ \text{Restrictive functional form} \end{array} \right\} \tag{4}$$

Flexible functional forms can be linear, square root, quadratic, cubic, and mixed forms. Restrictive form can be "minimum" function, G, such as

$$R(d_t, k_t, g_t, \theta_1 | a_t) = \mathsf{P} \times G(min(q(k_t, g_t), d_t), g_t, d_t, \theta_1, a_t)$$

where P is a shadow price, such as a rate of use of a certain computer.

Cost function has a following structure:

$$C(k_t, g_t, d_t, a_t, \theta_1, \varepsilon)$$
$$= \left\{ \begin{array}{ll} m(d_t, f_t(k, a_t), g_t, \theta_1) + \varepsilon(0) & a = 0 \\ m(d_t, (f_t(k, a_t) - h), g_t, \theta_1) + UC((f_t(k, a_t) - k_{t-1}), c_t, \theta_1) + \varepsilon(U) & a = U \\ F(f_t(k, a_t), \theta_1) + r(f_t(k, a_t), c_t, \theta_1) - s(k_{t-1}, c_t, \theta_1) + \varepsilon(K_r) & a = K_r \end{array} \right\} \tag{5}$$

In the cost function, $m(d_t, f_t(k, a_t), g_t, \theta_1)$ represents a maintenance cost for "keep" and "upgrade" decisions, since each mainframe computer system should receive a regular maintenance to perform its task uninterruptedly. $UC((f_t(k, a_t) - k_{t-1}), c_t, \theta_1)$ in cost for upgrade decision illustrates an upgrade cost for a new capacity. In case of replacement cost function, $F(f_t(k, a_t), \theta_1)$ is a fixed cost of replacement. $r(f_t(k, a_t), c_t, \theta_1)$ is a variable replacement cost. $s(k_{t-1}, c_t, \theta_1)$ is a value from a scrapped computer. The firm considers any scrapped computer systems to have no resale value. This is in fact not the case that these systems maintain a small resale value on the open resale market. Since $s(k_{t-1}, c_t, \theta_1)$ belongs to cost function, it is expected to have a negative sign. I assume that there is no maintenance cost for replacement.

I incorporate unobserved state variables $\varepsilon(a)$ by assuming that unobserved costs $\{\varepsilon(0)$, $\varepsilon(U)$, $\varepsilon(K_r)\}$ follow a specific stochastic process, which will be described. $\varepsilon(0)$ is an unobserved cost from keeping, such as managerial cost to prevent systems failures, cost for service contracts and some other tolerance costs from not replacing nor upgrading. A positive value for $\varepsilon(0)$ could be interpreted as unobserved system-overloads which inform that a corresponding computer systems should be upgraded or replaced. Also, it could be the expiration of a service contract or an unobserved component failure that requires the corresponding computer to be repaired. A negative value of $\varepsilon(0)$ could be interpreted as a report from a system administrator that a computer system has enough capacities to cover the current demand and is working smoothly. $\varepsilon(U)$ is an unobserved cost associated with upgrading computer systems. A negative value of $\varepsilon(U)$ could indicate that an upgraded computer system has a plenty of upgrade slots and there are enough computing components to upgrade, whereas a positive value could be interpreted that the corresponding computer system has limited upgradeble slots. $\varepsilon(K_r)$ is also interpreted as an unobserved cost when the action of replacement occurs. A positive value of $\varepsilon(K_r)$ could be interpreted as an increasing price of a backup system during replacement period, whereas a negative value could be interpreted as a decreasing price of a backup system. In order to identify these unobserved costs, we need more information. I also have implicitly assumed that the stochastic processes $\{\mathsf{x}_t^m, \varepsilon_t^m\}$ are independently distributed across different computer systems, m except the two state variables, demand for services, d_t and cost per unit capacity, c_t.

4.4 Dynamic Programming model

The optimal value function V_θ for each task is defined by

$$V_\theta(x,\varepsilon) = \max_{a\in A}[\pi(x_t, a, \theta_1) + \varepsilon_t(a) + \beta EV_\theta(x_t, \varepsilon_t, a)] \tag{6}$$

where $EV_\theta = \int\limits_y \int\limits_\eta V_\theta(y,\eta)p(dy, d\eta|x_t, \varepsilon_t, a, \theta_0)$

Then, as an optimal policy rule, a stationary decision rule is defined as

$$\mathsf{a}_t = Z(x_t, \varepsilon_t, \theta) \tag{7}$$

where

$$z(x_t, \varepsilon_t, \theta) := \operatorname*{argmax}_{a\in A(x_t)} [\pi(x_{t,}\mathsf{a}_t, \theta) + \varepsilon_t(a) + \beta E\mathsf{V}_\theta(x_t, a_t, \varepsilon_t)] \tag{8}$$

and z $(x_t$ $, \varepsilon_t,$ $\theta)$ is the optimal control.

4.4.1 Markov transition probability

I follow Rust(1987) in making the standard simple assumption that the transition probability η can be factored as

$$\varphi(x_{t+1},\ \varepsilon_{t+1}\mid x_t,\varepsilon_t,\ a_t,\theta_0) = p(x_{t+1}|\ x_t,\ a_t,\ \theta_0)q(\varepsilon_{t+1}|\ x_{t+1}), \tag{9}$$

where θ_0 is a vector of unknown parameters characterizing the transition probability for the observable part of the state variables. From the setup of choice variables, θ_0 is defined as follows. $\theta_0 = \{a, \rho, \mu, \nu, b\}$.

Rust(1987) refers to the above equation as the "Conditional Independence Assumption (CI)", since the density of x_t is independent on ε_t, and ε_{t+1} is independent upon ε_t conditional on (x_t, a_t) as well.

In order to reach $p(x_{t+1}|x_t, a_t)$, I assume that all state variables are independent on one another. Therefore,

$p(x_{t+1}|x_t, a_t) = p(x^1_{t+1}|x^1_t, a_t)\times\ p(x^2_{t+1}|x^2_t, a_t)\ \times p(x^3_{t+1}|x^3_t)\ \times p(x^4_{t+1}|x^4_t)$.

where $x^1_t = k_t$, $x^2_t = g_t$, $x^3_t = d_t$ and $x^4_t = c_t$.

However, because of the assumption that deterministic evolutions of capacity and age variables depend on the choices, I can focus only on $p(d_{t+1}|d_t)$ and $p(c_{t+1}|c_t)$.

4.4.2 Policies of the actions

ε is assumed to have i.i.d multivariate extreme distribution, i.e.,

$$q(\varepsilon|X) = \Pi_{j\in A(X)} \exp\{-\varepsilon(j)\}\exp\{-\exp\{-\varepsilon(j)\}\}. \tag{10}$$

With this assumption of ε, we can rewrite V_θ in the equation 6 as follows:

$$V_\theta(x,a) = \{\pi(x,a,\theta) + \beta\int_y \sigma\log[\sum_{a'\in\{0,U,K_1\ldots K_n\}} \exp[(\pi(y,a',\theta_1) + \beta EV_\theta(y,a'))/\sigma]]p(dy|x,a,\theta_0)\} \tag{11}$$

where σ is a standard deviation of ε_t.

Then, conditional choice probabilities $P(a_t|\ x,\ \theta)$ are given by

$$P(a=0,\ \text{keep}\ |x,\ \theta) = \frac{\exp\{\pi(x,\theta_1, a=0)\ + \beta EV_\theta(x,\ a=0)\}}{\sum_{a'\in\{0,U,K_1,\ldots,K_n\}}\exp[(\pi(x,a',\theta_1) + \beta EV_\theta(x,a'))/\sigma]} \tag{12}$$

$$\mathsf{P}(\mathsf{a}=U,\ \text{upgrade}\ |\mathsf{x},\ \theta)=\frac{\exp\{\pi(x,\theta_1,\mathsf{a}=U)\quad+\beta EV_\theta(x,\ a=U)\}}{\sum_{a'\in\{0,U,K_1,\ldots,K_n\}}\exp[(\pi(x,a',\theta_1)+\beta EV_\theta(x,a'))/\sigma]} \tag{13}$$

$$\mathsf{P}(\mathsf{a}=K_r,\ \text{replace}\ |\mathsf{x},\ \theta)=\frac{\exp\{\pi(x,\theta_1,\mathsf{a}=K_r)+\beta EV_\theta(x,\ \mathsf{a}=K_r)\}}{\sum_{a'\in\{0,U,K_1,\ldots,K_n\}}\exp[(\pi(x,a',\theta_1)+\beta EV_\theta(x,a'))/\sigma]} \tag{14}$$

4.4.3 Log Likelihood Function

Then, following Rust(1987), we have the two partial log likelihood function at time t as follows:

$$l_t^1=\ln(P(a_t|x_t,\theta)) \tag{15}$$

and

$$l_t^2=\ln(p(x_t|x_{t-1},\theta_0)) \tag{16}$$

where l_t^1 is a log likelihood function of the conditional choice probability and l_t^2 is a log likelihood function of the transition probability. And thus, we have the total log likelihood function in the following:

$$l(x_1,\ldots x_T,a_1,\ldots a_T|x_0,a_0,\theta)=\sum_{t=1}^{T}\ln(P(a_t|x_t,\theta))+\sum_{t=1}^{T}\ln(p(x_t|x_{t-1},\theta_0)) \tag{17}$$

5 Parametric Approximation

The general method to solve the fixed point problem is a discretization of observed state variables. When the observed state variable is continuous, the required fixed point is in fact an infinite dimensional object. Therefore, in order to solve the fixed point problem, it is necessary to discretize the state space so that the state variable takes on only finitely many values. But there are limits regarding this method: (i) "curse of dimensionality"; (ii) the limits it places on our ability to solve high-dimensional DP problems. Despite these limits, this method have been used in many literature.

The discretization method may not be applicable to computer replacement research to solve the fixed point problem, because of the aforementioned problems. The details are in the following:

5.1 An attempt of discretization of the state variables

The most conservative dimension of a possible combination of state variables resulting from discretization in the proposed model is 540,000. Discrete variables, capacity and age are discretized as follows. First, I discretize the age variable, g_t, into bimonthly cycle, even though I have monthly data. Thus, age 1 represents a new computer,[18] and an absorbing state 30 means 5 years of age.[19]

Second, regarding the capacity level, the current data set of the capacity consists of the three elements of CPU, hard drive and memory size. In order to concretize and transform the capacities into actual numbers which can represent the capacity of each computer system, I take a weighted average of these three elements. Since CPU is the most important factor in the capacity of computer systems, I give it a weight of 0.5. On the other hand, I give equal weights to Hard Drive and Memory size, namely 0.25. At this time, I do not separate the capacity into the two standards of CPU benchmark, TPC and MIPS. Even though the weights were confirmed with the system administrators in the firm, their appropriateness will be verified in further research. With transformed capacities of computer systems, I discretize the capacity from 1 to 40. The last state 40 is the absorbing state. Difference between each step is 30. Therefore, 1 represents (1,...,30), and 2 represents (31,...,60), and 40 represents the range, (1171,...,$+\infty$). These two discrete variables should be discretized regardless of the parametric approximation.

The continuous variables, demand and cost per capacity, can be discretized as follows. First, I discretize demand from 1 to 30. Like the actual capacity, the last state 30 is the absorbing state. Demand 1 represents 100,000 to 105,000 users and the absorbing state 30 is from 245,001 to ∞ users.

Second, I discretize the cost per capacity into 15 possible costs such as $\{15, 14, ..., 1\}$. Difference between subsequent prices is a 20% price drop. I restrict maximum price drops in one period to just 2 steps. These assumptions are based on several research data, computer industry databooks, and Moore's Law [20].

The transition probability matrices, $p(d_{t+1}|d_t)$ and $p(c_{t+1}|c_t)$ are in the appendix.

Therefore, the resulting dimension from the discretization is $540,000 = 30 \times 40 \times 30 \times 15$.

[18] literally 2 months old.

[19] For estimation purpose, I discretize the age variables into months instead of bi-monthly cycle.

[20] The index was created by the informations from SIA (Semiconductor Industry Association)'s annual databooks, the 8th Annual Computer Industry Almanac, ZDnet.com, and Cnet.com.

5.2 Computational Burden

First, solving the fixed point problem requires calculation of the expected value function. That is, $EV_\theta = \int\limits_y \int\limits_\eta V_\theta(y,\eta)p(dy, d\eta|x_t, \varepsilon_t, a, \theta_0)$. Even though the Markov transition probability from discretization is a sparse matrix, it still requires extensive time to calculate expectation of value function. Second, the polyalgorithm method by Rust (1987) takes advantage of the complimentary behavior of the two iterations, which are a combination of contraction iteration and policy iteration[21] This algorithm enjoys a substantial reduction in time calculating the fixed point. However, it is not applicable to solving a dynamic programing model. The reason is as follows: One must have a Frechet derivative $(I - T'_\theta)$ in order to use policy iteration method[22]. But, the dimensionality problem makes it impossible to get the derivatives of T_θ. Thus, the algorithm for the DP problem consists solely of a backward induction, which is simple but takes more time to solve. Therefore, the extended time caused by the two aforementioned reasons seriously affects the calculation time of a nested fixed point algorithm, because the nested fixed point algorithm uses the fixed point algorithm outside of the maximum likelihood estimation.

5.3 Parametric Approximation

5.3.1 Method

To begin with, one needs functional forms for the three value functions, keep, upgrade, and replacement. To find the parametric forms of value functions, I use the simple linear OLS estimations, such as

$$\begin{aligned} V(a &= 0, x) = H(x, \lambda_0) + \psi_0 \\ V(a &= U, x) = H(x, \lambda_U) + \psi_U \\ V(a &= K_r, x) = H(x, \lambda_{K_r}) + \psi_{K_r} \end{aligned} \tag{18}$$

[21] Newton Kantorovich method.

[22] The idea of the policy iteration method, i.e., the Newton Kantorovich iteration is to find a zero solution of the nonlinear operator $F = (I - T_\theta)$ instead of finding a fixed point $EV_\theta = T_\theta(EV_\theta)$. With invertibility of $(I - T_\theta)$ and existence of a Frechet derivative $(I - T^{\theta}_\theta)$, one can do a following Taylor expansion:

$$0 = [I - T_\theta](EV_l)\tilde{}[I - T_\theta](EV_{l-1}) + [I - T'_\theta](EV_l - EV_{l-1}).$$

$$\Longrightarrow EV_l = EV_{l-1} - [I - T'_\theta]^{-1}(I - T_\theta)(EV_{l-1}).$$

where $H(x,\lambda_0)$, $H(x,\lambda_U)$ and $H(x,\lambda_{K_r})$ are flexible functions and linear in λ. ψ_0, ψ_U, and ψ_{K_r} are assumed to be distributed as $N(0,1)$

First, I choose the best functional forms for each value function according to the criteria, $\overline{R}^2$. After extended search for the appropriate functional forms of the three value functions, I have the following results.$V(a \doteq 0, x)$ has 12 parameters ($=\lambda_0$) with 0.983 of $\overline{R}^2$, $V(a \doteq U, x)$ has 15 parameters ($=\lambda_U$) with 0.962 of $\overline{R}^2$ and $V(a=(K_1...K_n),x)$ has 18 parameters ($=\lambda_{K_r}$) with 0.962 of $\overline{R}^2$. Therefore, we have $H(x,\lambda_0) \approx \sum_{i=1}^{12}\lambda_0^i\vartheta_{0,i}(x)$, $H(x,\lambda_U) \approx \sum_{i=1}^{15}\lambda_U^i\vartheta_{U,i}(x)$, and $H(x,\lambda_{K_r}) \approx \sum_{i=1}^{18}\lambda_{Kr}^i\vartheta_{K_r,i}(x(K_r))$.

Second, with the approximated functional forms of the three value functions, I estimate all 45 parameters (λ_0,λ_U, λ_{K_r}) with nonlinear least square estimation, such as

$$\min_{\lambda_0,\lambda_U,\lambda_{K_r}} \sum_j \sum_a [V_a(x_j) - U_a]^2 \tag{19}$$

where

$U_1 = [(\{u(x_t, a_t = 0, \theta_1)$
$+\beta\int_y \sigma \log\left(\sum_{a'\in A(y)} \exp[V_{\alpha'}(y)/\sigma]\right) p(dy|x_t, a_t = 0, \theta_0)\})]$
and
$U_2 = [(\{u(x_t, a = U, \theta_1)$
$+\beta\int_y \sigma \log\left(\sum_{a'\in A(y)} \exp[V_{\alpha'}(y)/\sigma]\right) p(dy|x_t, a_t = U, \theta_0)\})]$
and
$U_3 = [(\{u(x_t, a = K_r))$
$+\beta\int_y \sigma \log\left(\sum_{a'\in A(y)} \exp[V_{\alpha'}(y)/\sigma]\right) p(dy|x_t, a_t = K_r, \theta_0)\})]$.[23]

Solving the above minimization problem enables us to estimate all parameters $\hat{\lambda}_0, \hat{\lambda}_U$, and $\hat{\lambda}_{K_r}$.In fact, a parametric approximation procedure converts the contraction fixed-point problem into a nonlinear least squares problem.

5.3.2 Parametric approximation and discretization: A Comparison

Based on the parameters in Tables 8, 9, 10-(a), and 10-(b), I calculate a fixed point by a discretization method. Comparisons between two value functions from discretization and parametric approximation are illustrated in Figures 4, 5, and 6, which represent cases of keep, upgrade, and

[23] The above three expectations are calculated by a quadrature method

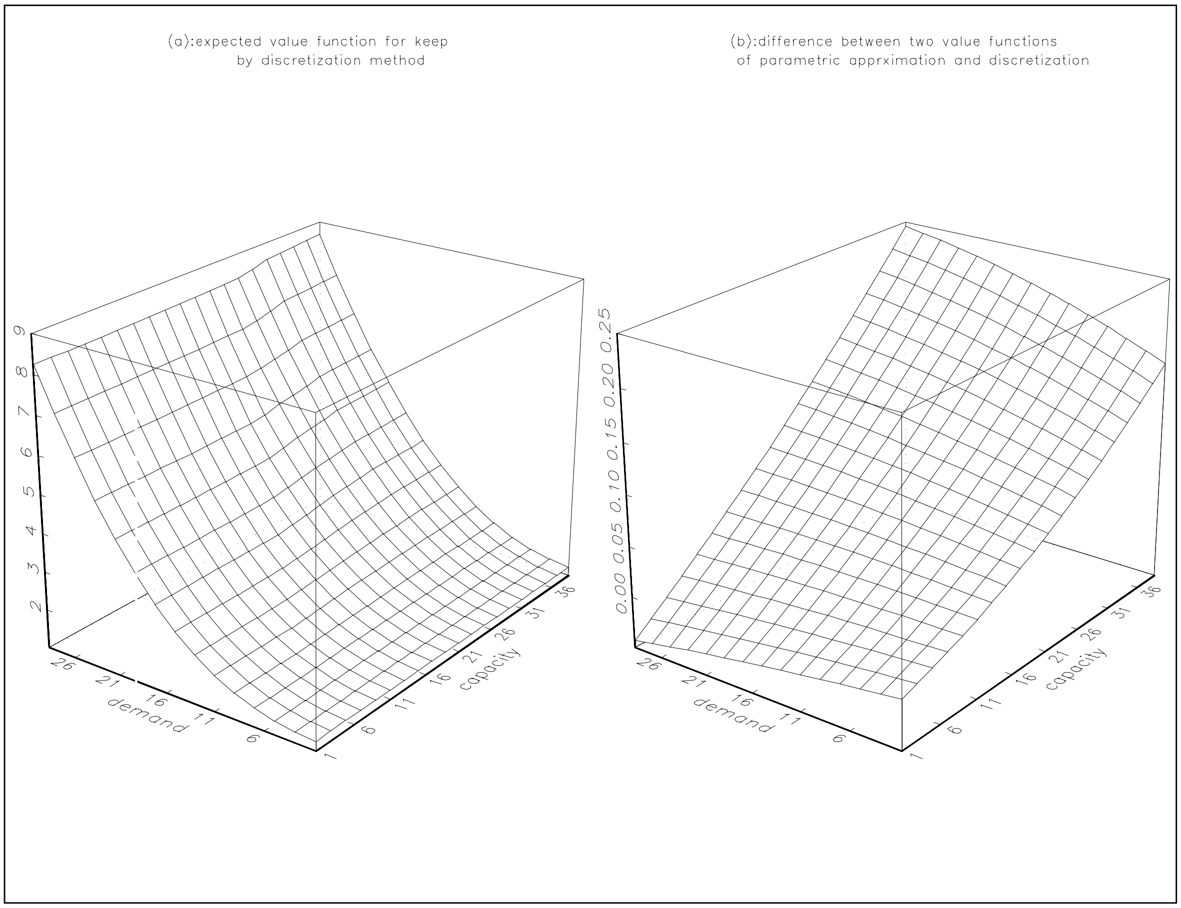

Figure 4: Differences between parametric approximation and discretization

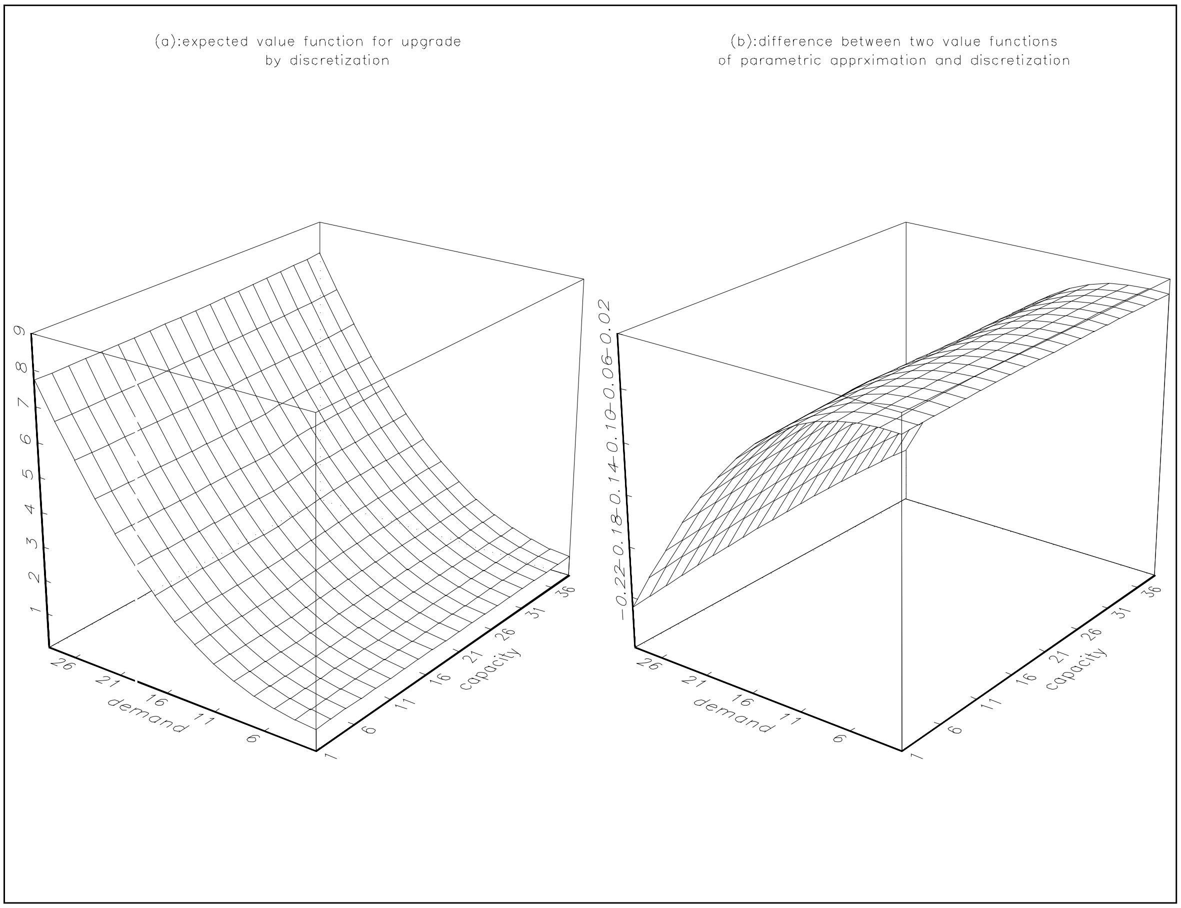

Figure 5: Differences between parametric approximation and discretization

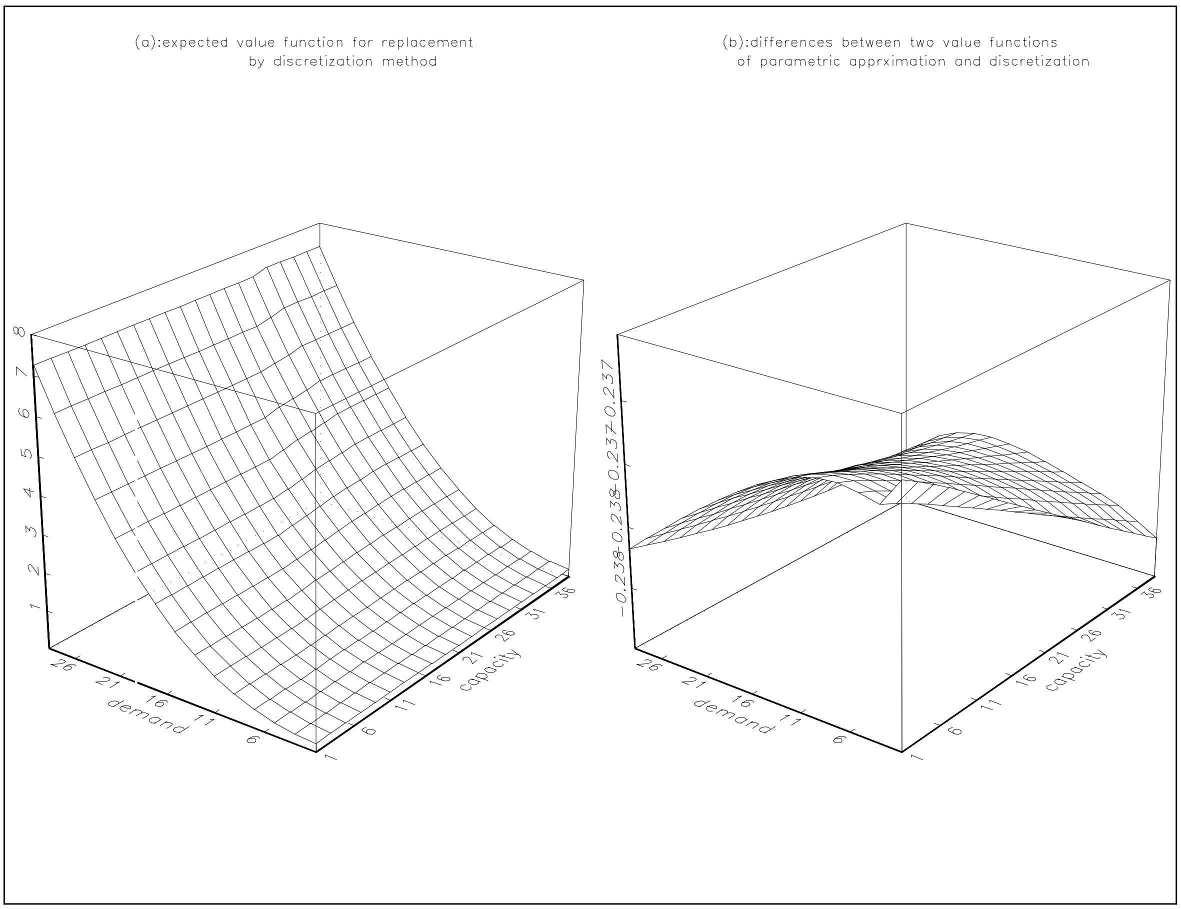

Figure 6: Differences between parametric approximation and discretization

replacement, respectively. In each Figure, graph (a) presents the expected value function[24] by discretization, and graph (b) shows the differences between two value functions from parametric approximation and discretization. Though there is a slight discrepancy in comparisons between two methods, the differences seem to be negligible. Therefore, the parametric approximation solves the proposed DP model as accurately as and more efficiently by speeding up the solution time than a discretization method does.

The forgoing empirical results lead to two main conclusions: (i) along with the nested fixed point algorithm, the parametric approximation method can be a practical, efficient and numerically stable method for estimating certain structural model lacking closed-form solutions with high dimensional state space. (ii) the data are by and large consistent with the prediction of the proposed optimal stopping model of mainframe computers replacement and upgrade. In the following section, interesting behavioral implications of the model will be explored for the purpose of wide application of the model.

6 Estimation

Incorporating the parametric approximation, the estimation requires the nested fixed point algorithm, which is intended to find parameters that maximize the likelihood functions, subject to the constraint that function EV_θ is the unique fixed point. This estimation procedure can be called Nonlinear-Nested Fixed Point Estimation (NLS-NFXP).

One of the benefits of a parametric approximation is that discretization of continuous state variables is no longer required. The two discrete state variables are still discretized in the manner suggested in section 5. Additionally, the age variable is discretized in much finer dimension. It is discretized into months instead of bimonthly cycle. That is, the age variable ranges from 1 to 60, where the absorbing state, 60 represents that computer system is five years of age.

Both restrictive and flexible functional forms have been tried for revenue and cost equations. In detail, one restrictive functional form and several flexible functional forms, such as linear, square root, quadratic, cubic, and mixed forms have been estimated. Among these functional forms, the cubic form presents the best estimation results. In fact, all additional terms from linear to quadratic and from quadratic to cubic forms show significance at 95% level.

The parameters for state variables, θ_0 and parameters for revenue and cost functions θ_1 are

[24] In order to graph value functions in terms of the demand and the capacity variables, I fix the cost per capacity and the age of computer at certain values, such as relatively high cost per capacity and a fairly new computer.

estimated separately. First, the parameters for state variables, θ_0 are estimated. Second, based on the estimates, θ_1 are estimated.

6.1 Nonlinear - Nested Fixed Point Estimation (NLS-NFXP)

The estimation procedure by NLS-NFXP is that, i) outside of the system, θ_0, parameters for state variables are estimated and simulated separately from the structural parameters. ii) inside of the system, θ_1, the structural parameters are estimated by the nested fixed point algorithm. That is, outside of maximum likelihood estimation, the above nonlinear least square estimation (NLS) is performed and fixed points EV_θ is calculated. Based on the fixed points, the maximum likelihood estimation is performed.[25]

The partial log likelihood function in this model is as follows:

$$l_f^1(x_1,...x_T,a_1,...a_T|x_0,a_0,\theta) = \sum_{t=1}^{T} \ln(P(a_t|x_t,\theta)) \tag{20}$$

Where

$$P(a_t|x_t,\theta) = \frac{\exp\{V_\theta(x_t,\ a_t,\lambda_{a_t})/\sigma\}}{\sum_{a'\in\{0,U,K_1,...,K_n\}} \exp[V_\theta(x_t,a',\lambda_{a'})/\sigma]} \tag{21}$$

6.1.1 Functional Forms for Revenue and Cost functions

Flexible form The following Table 5 is cubic functional forms of profit and cost functions of the firms associated with keep, upgrade, replacement, and scrap behaviors.

$q(f_t(k,a_t),g_t,\theta)$ illustrates how capacity contributes to the revenue functions, such as an adjusted capacity. For example, the contributions of capacity will decline as the computer gets old. However, in the revenue function for replacement, the replacement capacity K_r will fully contribute to the revenue function for replacement. $q(f_t(k,a_t),g_t)$ is assumed to be a simple function which is increasing in $f_t(k,a_t)$, decreasing in g_t, such as $q(f_t(k,a_t),g_t,\theta) = (\theta_{35} \times f_t(k,a_t))/\sqrt{g_t}$.

γ is components of a set of unknown parameter, θ_1, which is a measure of unit value for telecommunication services the firm provides. Also, it can be interpreted as an average value of unit demand for aggregated services. l, as a component of θ_1, is a unit labor charge per capacity in order to compensate the shortage of the current adjusted capacity, $(d_t - q(f_t(k,a_t)))$. When demand exceeds the current capacity, the firm usually hires more labor to make up the shortage

[25] The Berndt, Hall, Hausman, and Hall (BHHH) alogorithm is used, along with numerical derivatives.

of the amount of $[l \times (d_t - q(f_t(k, a_t))]$. Table 6 explains the detail of a set of parameters, θ_1 associated with Table 5.

Table 5

Cubic functional forms used in the model as a flexible form

Revenue	Specifications[26]
Keep	$\theta_{11} + \theta_{12}(q(f_t(k, a_t), g_t, \theta_{35}) \times d_t \times \gamma) + \theta_{13}(q(f_t(k, a_t), g_t, \theta_{35}) \times d_t \times \gamma)^2$
	$+\theta_{14}(q(ff_t(k, a_t), g_t, \theta_{35}) \times d_t \times \gamma)^3$
Upgrade	$\theta_{21} + \theta_{22}(h \times \gamma \times d_t) + \theta_{23}(h \times \gamma \times d_t)^2 + \theta_{24}(h \times \gamma \times d_t)^3$
	$\theta_{25}(q((f_t(k, a_t) - h), g_t, \theta_{35}) \times \gamma \times d_t) + \theta_{26}(q((f_t(k, a_t) - h), g_t, \theta_{35}), g_t) \times \gamma \times d_{t,j})^2$
	$+\theta_{27}(q((f_t(k, a_t) - h), g_t, \theta_{35}) \times \gamma \times d_t)^3$
Replacement	$\theta_{31} + \alpha_{32}(f_t(k, a_t) \times \gamma \times d_t) + \theta_{33}(f_t(k, a_t) \times \gamma \times d_t)^2 + \theta_{34}(f_t(k, a_t) \times \gamma \times d_t)^3$
Cost	
Keep	$I(f_t(k, a_t) \geq d_t)\{\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2 + \theta_{54}(f_t(k, a_t) \times m_t)^3\}$
	$+I(f_t(k, a_t) < d_t)\{\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2$
	$+\theta_{54}(f_t(k, a_t) \times m_t)^3 + \theta_{55}(l \times (d_t - q(f_t(k, a_t), g_t, \theta_{35}))$
	$+\theta_{56}(l \times (d_{t,j} - q(f_t(k, a_t), g_t, \theta_{35}))^2 + \theta_{57}(l \times (d_t - q(f_t(k, a_t), g_t, \theta_{35}))^3\}$
Upgrade	$\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2 + \theta_{54}(f_t(k, a_t) \times m_t)^3$
	$+\theta_{61}(c_t \times f_t(k, a_t) \times cp^*) + \theta_{62}((c_t \times f_t(k, a_t) \times cp)^2 + \theta_{63}((c_t \times f_t(k, a_t) \times cp)^3$
Replacement	$\theta_{71} + \theta_{72}(cp \times f_t(k, a_t)) + \theta_{73}(cp \times f_t(k, a_t))^2 + \theta_{74}((c_t \times cp) \times f_t(k, a_t))$
	$+\theta_{75}((c_t \times cp) \times f_t(k, a_t))^2 + \theta_{76}((c_t \times cp) \times f_t(k, a_t))^3$
Scrap	$\theta_{41} + \theta_{42}((k_t \times \gamma)) + \theta_{43}((k_t \times \gamma))^2 + \theta_{44}((k_t \times \gamma))^3$

*: cp is a scale parameter for cost functions from calibration ($cp = 2.013$)

m_t is a unit maintenance cost which is increasing in g_t, such as $m_t = m(g_t, \theta_{81}) = \theta_{81} \times \sqrt{g_t}$

Restrictive form As mentioned earlier, "minimum" function is a revenue function as a restrictive form. A functional form for cost revenue functions associated with the restrictive revenue function is a quadratic function. Table 7 shows the detail of the functional form associated with restrictive form.

[26] The assumptions imposed on these functional forms, $f_1(k_t, g_t, rm)$ and um_t are due to the large number of unknown parameters. These assumptions can be released in further research.

6.1.2 Results of Estimation

Parameters for demand and cost per capacity For simplicity, I estimate the parameters $\theta_0 = \{a, \rho, \mu, \nu, b\}$ which govern the transition probabilities for demand and cost per capacity separately from the parameters of profits function. First, as I mentioned, an aggregated demand D_t equals the sum of an individual demand for a task j, such as $D_t = \sum_j d_{j,t}$ and $d_{t,j} = \xi_j D_t$. In order to calculate a fraction ξ_j for a demand d_t which a specific task serves, I sum up all capacities of computer systems[27] and assume that a proportion for capacity of a system corresponds to a fraction of demand for a system.

Table 6

Explanation of a set of parameters, θ_1 in Table 5

Parameters θ_1	Function	Verification of Parameters
	Revenue	
$\theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}$	Keep	revenue from old components
$\theta_{21}, \theta_{23}, \theta_{23}, \theta_{24}$	Upgrade	revenue from upgraded components
$\theta_{25}, \theta_{28}, \theta_{27}$	Upgrade	revenue from old components
$\theta_{31}, \theta_{32}, \theta_{33}, \theta_{34}$	Replacement	revenue from new components
θ_{35}	Keep, Upgrade	adjusted capacity, $q(f(k,a), g_t, \theta)$
γ	All	an average value of unit demand
	Cost	
$\theta_{41}, \theta_{42}, \theta_{43}, \theta_{44}$	Scrap	scrapped value of computer
$\theta_{51}, \theta_{52}, \theta_{53}, \theta_{54}$	Keep, Upgrade	maintenance cost for keep or upgrade
$\theta_{55}, \theta_{56}, \theta_{57}$	Keep	make up cost for shortage
$\theta_{61}, \theta_{62}, \theta_{63}$	Upgrade	true upgrade cost
$\theta_{71}, \theta_{72}, \theta_{73}$	Replacement	fixed cost*
$\theta_{74}, \theta_{75}, \theta_{76}$	Replacement	variable cost**
θ_{81}	Keep, Upgrade	unit maintenance cost for keep and upgrade, $m(g_t, \theta_{81})$
l	Keep	unit labor charge per capacity

*: Fixed cost for replacement is invariable with respect to cost per unit capacity

**: Variable cost for replacement is variable with respect to cost per unit capacity

Table 8 shows the parametric estimates of state, d_t.

[27] Note that a task uses only one computer system.

Table 7

Minimum functions used in the model as a restrictive form

Revenue	Specifications[28]
Keep	$\theta_{11} + \theta_{12}[\min(q(f_t(k, a_t), g_t, \theta^*_{35}), d_t) \times \gamma \times d_t]$
	$+\theta_{13}[\min(q(f_t(k, a_t), g_t, \theta_{35}), d_t) \times \gamma \times d_t]^2$
Upgrade	$\theta_{21} + \theta_{22}[\min(q(f_t(k, a_t), g_t, \theta_{35}), d_t) \times \gamma \times d_t]$
	$+\theta_{23}[\min(q(f_t(k, a_t), g_t, \theta_{35}), d_t) \times \gamma \times d_t]^2$
Replacement	$\theta_{31} + \theta_{32}[\min(f_t(k, a_t), g_t, \theta_{35}), d_t) \times \gamma \times d_t]$
	$+\theta_{33}[\min(f_t(k, a_t), g_t, \theta_{35}), d_t) \times \gamma \times d_t]^2$
Cost	
Keep	$I(f_t(k, a_t) \geq d_t)\{\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2\}$
	$+I(f_t(k, a_t) < d_t)\{\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2$
	$+\theta_{55}(l \times (d_t - q(f_t(k, a_t), g_t, \theta_{35})) + \theta_{56}(l \times (d_{t,j} - q(f_t(k, a_t), g_t, \theta_{35}))^2\}$
Upgrade	$\theta_{51} + \theta_{52}(f_t(k, a_t) \times m_t) + \theta_{53}(f_t(k, a_t) \times m_t)^2$
	$+\theta_{61}(c_t \times f_t(k, a_t) \times cp^*) + \theta_{62}((c_t \times f_t(k, a_t) \times cp)^2$
Replacement	$\theta_{71} + \theta_{72}(cp \times f_t(k, a_t)) + \theta_{73}(cp \times f_t(k, a_t))^2 + \theta_{74}((c_t \times cp) \times f_t(k, a_t))$
	$+\theta_{75}((c_t \times cp) \times f_t(k, a_t))^2$
Scrap	$\theta_{41} + \theta((k_t \times \gamma)) + \theta_{43}((k_t \times \gamma))^2$

See Table 6 for reference of all unknown parameters except parameters for upgrade revenue, θ_{21} θ_{22}, and θ_{23},which are just parameters for upgrade revenue.[29]

*: θ_{35} was not estimated and used from the result of cubic estimation

Table 8

Parameter estimates for state d_t

d_t	
Parameters	Estimate
α	1.0237 (0.108)
ρ	0.9403 (0.065)
$\overline{R}^2$	0.9021

(standard errors in parentheses)

[28] The assumptions imposed on these functional forms, $f_1(k_t, g_t, rm)$ and um_t are due to the large number of unknown parameters. These assumptions can be released in further research.

[29] I do not separate between old and upgrade components at this time.

The parameters of cost per capacity, c_t are obtained by maximum likelihood estimation method. The log-likelihood function of c_t is as follows.

$$l_f^2(c_1, ...c_T|\theta_0) = \sum_{t=1}^{T} \ln(P(c_t|c_{t-1}, \theta_0)) \tag{22}$$

Table 9 presents the estimation result for c_t.

Table 9

Parameter estimates for state c_t

c_t	
Parameters	Estimate
b	0.759 (0.108)
μ	9.127 (0.065)
ν	8.794 (0.017)
Likelihood	-51.237
Obs. Size	62

(standard errors in parentheses)

Structural estimates of revenue and cost functions Tables 10-(a) and 10-(b) are the results of structural estimation in terms of cubic functional forms in Table 5. The Tables report the structural parameter estimates computed by maximizing the likelihood function l_f^1 in equation (19) using the nested fixed point algorithm. I present structural estimates for the unknown parameters for the cubic specifications suggested in Table 5.[30] The estimation results for $\beta = 0.999$ corresponds to a dynamic model in which the present value of current and future profit streams is maximized by the investment decisions of the firm.

Most parameters of revenue and cost functions are precise and have the expected sign. In Table 10-(b), parameters θ_{42}, θ_{43}, and θ_{44} (except the constant term) of the revenue function for scrapped computers are insignificant at the 95% level, even though I tried several functional forms. This may be caused by two possible reasons. First, the proposed functional form is misspecified. Second, any scrapped computer has a lump sum value regardless of its remaining capacities. According to several interviews with system administrators of the firm, the second assumption seems to be more reasonable, because they do not care about values of scrapped computer systems and they donate in favor of charity, once they replaced old computer systems.

[30] When I tried the estimation additionally with $\beta = 0.99$ and $\beta = 0.95$, there was no distinguishable difference.

Table 10-(a)

Structural parameters (θ_1) estimates for flexible form (cubic)

Parameters	MIPS		TPC	
β	0.999			
Revenue	**Estimate**	**Std.Err**	**Estimate**	**Std.Err**
θ_{11}	13.209	(2.125)	12.031	(1.045)
θ_{12}	1.446	(0.028)	1.746	(0.294)
θ_{13}	1.813*	(1.147)	1.659	(0.314)
θ_{14}	1.124	(0.235)	1.056	(0.125)
θ_{21}	13.774	(1.238)	12.256	(1.001)
θ_{22}	1.114	(0.136)	1.573	(0.354)
θ_{23}	1.901	(0.243)	1.817	(0.347)
θ_{24}	1.298	(0.021)	1.169	(0.185)
θ_{25}	1.741*	(1.045)	2.001	(0.019)
θ_{26}	0.589	(0.002)	1.035	(0.147)
θ_{27}	2.184	(0.029)	3.206*	(3.267)
θ_{31}	12.203	(1.562)	12.322	(1.511)
θ_{32}	2.301	(0.037)	2.540	(0.124)
θ_{33}	1.037*	(0.772)	1.163*	(1.217)
θ_{34}	3.321	(0.056)	3.776	(0.194)
θ_{35}	1.024	(0.014)	1.109	(0.037)
γ	3.524	(0.002)	3.167	(0.351)

Continued in Table 10-(b).

*Not significant at 95% level.

Tables 11-(a) and 11-(b) report the result of structural estimation for "minimum" function as a restrictive functional form. All parameter estimates have the expected sign and fairly reasonable values. Like the estimation results from flexible functional forms, only constant terms for scrapped value of replaced computers are meaningful. Therefore, the aforementioned assumption regarding the value of scrapped computers should be correct one. We note that estimated values of Ps, rate of use of a computer are significant at 95% level. TPC standard computers' rate of use is higher than that of MIPS, which means that computers of TPC standard are being operated more efficiently than those of MIPS standard.

Table 10-(b)

Structural parameters (θ_1) estimates for flexible form (cubic)

Parameters	**MIPS**		**TPC**	
β	0.999			
Cost	**Estimate**	**Std.Err**	**Estimate**	**Std.Err**
θ_{41}	16.269	(1.649)	17.185	(1.197)
θ_{42}	1.532*	(2.487)	1.683*	(1.248)
θ_{43}	0.191*	(1.549)	0.252*	(2.301)
θ_{44}	1.245*	(2.432)	2.421*	(4.579)
θ_{51}	5.102	(1.032)	5.514	(1.154)
θ_{52}	1.338	(0.026)	1.514*	(2.042)
θ_{53}	0.254	(0.032)	0.212	(0.061)
θ_{54}	0.248	(0.003)	0.401	(0.105)
θ_{55}	0.265	(0.063)	0.315	(0.021)
θ_{56}	0.951	(0.106)	0.759	(0.015)
θ_{57}	0.417	(0.053)	0.699	(0.113)
θ_{61}	0.591	(0.017)	1.254	(0.008)
θ_{62}	0.831*	(0.719)	0.954	(0.124)
θ_{63}	1.127	(0.018)	1.551	(0.187)
θ_{71}	4.518	(1.056)	4.341	(0.608)
θ_{72}	2.231*	(1.143)	2.145*	(1.449)
θ_{73}	0.767	(0.014)	0.697	(0.177)
θ_{74}	2.732	(0.516)	2.198	(0.397)
θ_{75}	1.815	(0.059)	1.254	(0.005)
θ_{76}	2.218	(0.218)	1.758	(0.122)
θ_{81}	0.998	(0.157)	0.972	(0.201)
l	1.551	(0.059)	2.485	(0.038)
Likelihood	-5995.35		-6452.05	
Obs. Size	5760		6840	

*Not significant at 95% level

Table 11-(a)

Structural revenue parameters (θ_1) estimates for restrictive form

Parameters	MIPS		TPC	
β	0.999			
Revenue	**Estimate**	**Std.Err**	**Estimate**	**Std.Err**
θ_{11}	19.123	(3.147)	17.448	(2.146)
θ_{12}	2.472	(0.956)	2.875	(0.421)
θ_{13}	2.788*	(1.549)	2.557*	(2.565)
θ_{21}	17.015	(1.025)	16.713	(1.054)
θ_{22}	3.244	(0.556)	3.783*	(2.016)
θ_{23}	2.455*	(1.944)	2.347	(0.301)
θ_{31}	17.576	(2.254)	17.147	(1.512)
θ_{32}	4.174*	(3.145)	4.556*	(4.002)
θ_{33}	3.342	(1.014)	3.794	(0.748)
γ	4.012	(0.845)	4.001	(0.025)
P	0.887	(0.031)	0.965	(0.019)

*Not significant at 95% level.

Continued in Table 11-(b).

We also note that l, prices of unit labor in case of MIPS is lower than those of TPC for both flexible and restrictive functional forms. This may become a explanation for the fact that there are generally more frequent upgrade and replacement activities in case of TPC standard computers that MIPS standards according to Table 3. As I mentioned in section 6.1, if there is a shortage of the current capacity, labor should be employed to compensate it when keeping current computers. Thus, If a labor charge becomes expensive, the firm tends to upgrade or replace current computers instead of keeping it.

[31]Figures 6.1, 6.2, 6.3, and 6.4 show the three policies (keep, upgrade, and replace) and their expected value functions, plotted against various cost per capacity, in the case when demand is lower than the current capacity with age fixed. In the Figures 6.1 and 6.2, the value functions of upgrade fall slightly, as cost per capacity increases due to the amount of upgrade. However, since replacement requires change of the current system as a whole, the cost of replacement will increase tremendously, as cost per capacity increases. Thus, the likelihood of replacement falls

[31] All figures are based on estimated parameters for cubic functional forms.

and reaches zero eventually, as cost per capacity increases. The best choice for keeping up with the current demand becomes the choice of upgrade, when cost per capacity is high enough. Figures 6.3 and 6.4 show where the condition is identical to Figures 6.1 and 6.2 with exception of the age variable. As the computer gets older, replacement becomes more preferable to upgrade. However, as cost per capacity increases, the probability of replacement falls and the probability of upgrade becomes the best choice.

Table 11-(b)

Structural cost parameters (θ_1) estimates for restrictive form

Parameters	**MIPS**		**TPC**	
β	0.999			
Cost	**Estimate**	**Std.Err**	**Estimate**	**Std.Err**
θ_{41}	16.597	(1.085)	17.658	(1.125)
θ_{42}	1.588*	(3.894)	2.014*	(2.497)
θ_{43}	0.544*	(1.255)	0.754*	(2.057)
θ_{51}	5.257	(1.267)	5.953	(1.043)
θ_{52}	1.373	(0.021)	1.044*	(3.089)
θ_{53}	0.324	(0.003)	0.269	(0.002)
θ_{55}	0.299	(0.094)	0.584	(0.164)
θ_{56}	1.194	(0.024)	1.008	(0.021)
θ_{61}	0.601	(0.035)	1.394	(0.018)
θ_{62}	0.927*	(1.719)	1.145	(0.214)
θ_{71}	5.235	(1.311)	5.045	(1.213)
θ_{72}	3.134*	(2.432)	2.954*	(3.112)
θ_{73}	1.112	(0.009)	1.017	(0.052)
θ_{74}	3.811	(0.218)	3.187	(0.122)
θ_{75}	2.014	(0.221)	1.945	(0.045)
θ_{81}	0.998	(0.157)	0.972	(0.201)
l	1.456	(0.024)	2.397	(0.239)
Likelihood	-4845.19		-5067.58	
Obs. Size	5760		6840	

*Not significant at 95% level.

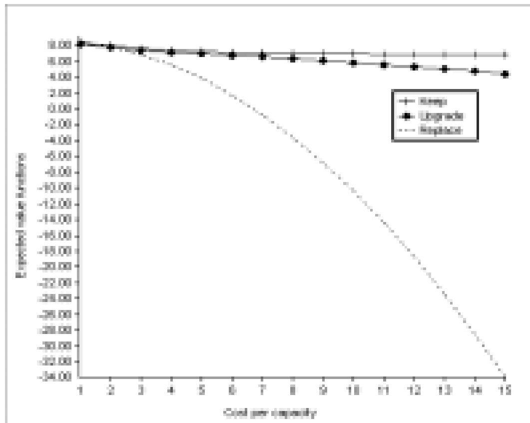

Figure 6.1:Expected value functions of keep, upgrade, and replacement

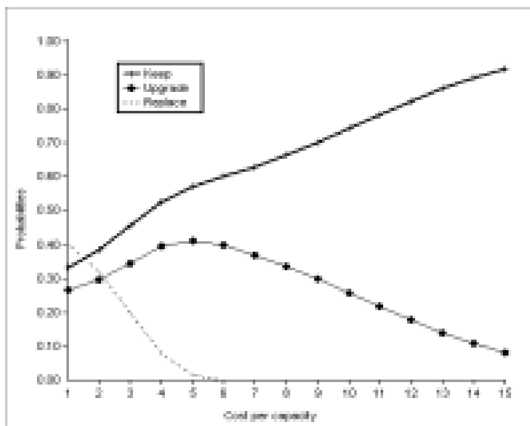

Figure 6.2: Three policy rules with various cost per capacity

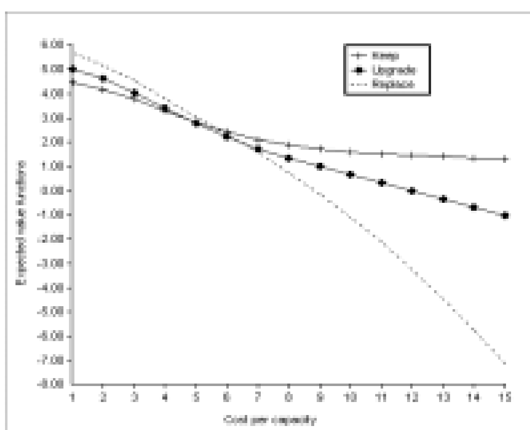

Figure 6.3: Expected value functions of keep, upgrade, and replacement:

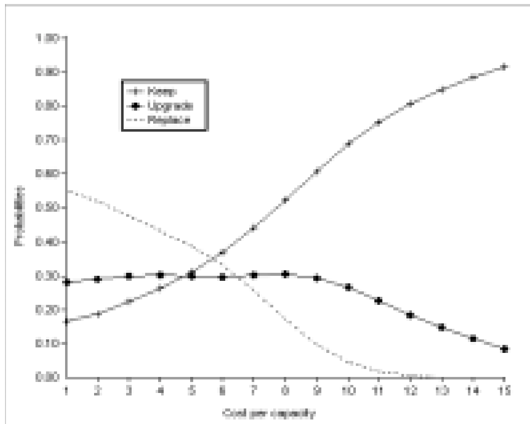

Figure 6.4: Three policy rules with various cost per capacity

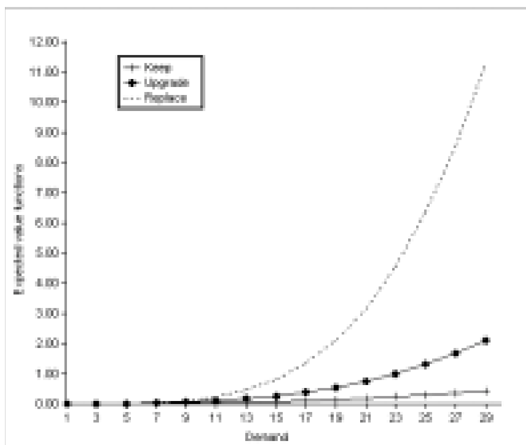

Figure 6.5: Expected value functions of keep, upgrade, and replacement:

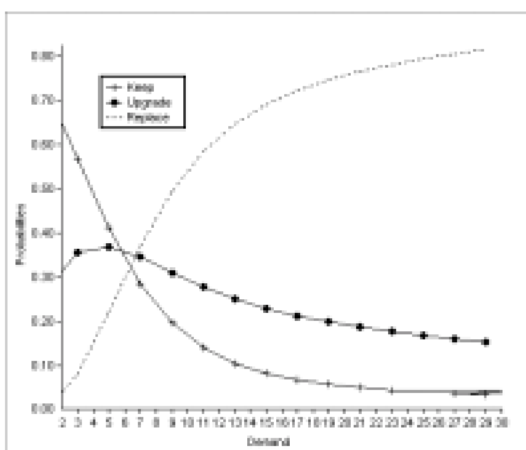

Figure 6.6: Three policy rules with various demand

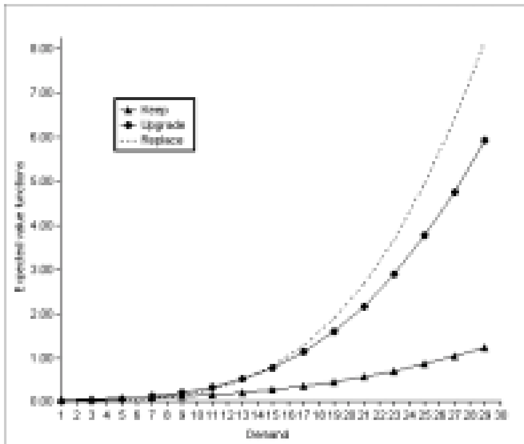

Figure 6.7: Expected value functions of keep, upgrade, and replacement:

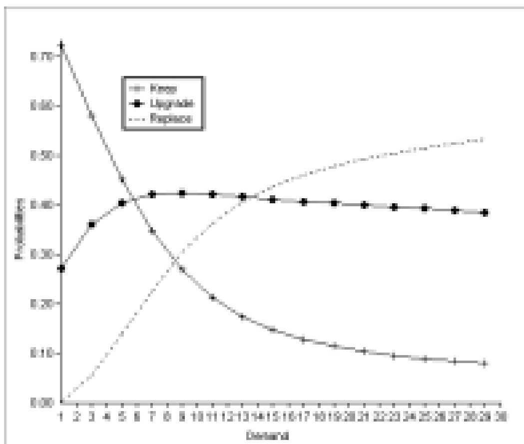

Figure 6.8: Three policy rules with various demand

Figures 6.5, 6.6, 6.7, and 6.8 show how the three polices (keep, upgrade, and replace) and three value functions of old computer system depend on various demands, when capacity and cost per capacity are fixed. In Figures 6.5 and 6.6, as demand increases, the value functions for keep, upgrade and replacement are expressed in smooth increasing curves. However, each policy shows different behavior. Until the points where the capacity is slightly over the current demand, choice of keep is more likely to occur with decreasing tendency. But, beyond the point of the demand, if the system is relatively new, upgrade should be more likely to occur for maximizing profits. However, since the system is relatively old in this case, the choice of replacement outperforms the choice of upgrade and thus, replacement is more likely to occur.

Comparison with Figures 6.7 and 6.8 shows how cost per capacity affects the above situation. When cost per capacity becomes higher (Figures 6.7 and 6.8), upgrade becomes a more reasonable choice than replacement. However, this situation will change, when the demand is much bigger than the current capacity.

Policy for replacement capacity. Figure 6.9, 6.10, 6.11, and 6.12 illustrate how replacement capacity should be chosen depending on future cost and demand, when replacement is considered as an optimal strategy.

Figures 6.9 and 6.10 illustrate effects of capacity choices on expected value functions of replacement according to two cases of demand, high and low. Two Figures are plotted against cost per capacity variable. Figure 6.9 is based on the situation of high demand and Figure 6.10 illustrates a low demand situation. When demand is small enough, there is a little change among expected value functions of replacement with various capacity choices (Figure 6.9). However, when demand is large, the situation changes. Differences among value functions become larger. Value function with large capacity choice falls rapidly (Figure 6.10). This situation suggests that when replacement capacity is decided, future expected demand should be considered. The intuition is that, when demand is expected to be large in the future, the firm should increase its capacity choice of replacement.

Figures 6.11 and 6.12 shows how capacity choices affect expected value functions of replacement subject to two cases of cost per capacity, low and high. Two Figures are plotted against demand variable. Figure 6.11 shows the case of low cost per capacity and Figure 6.11 shows the opposite case.

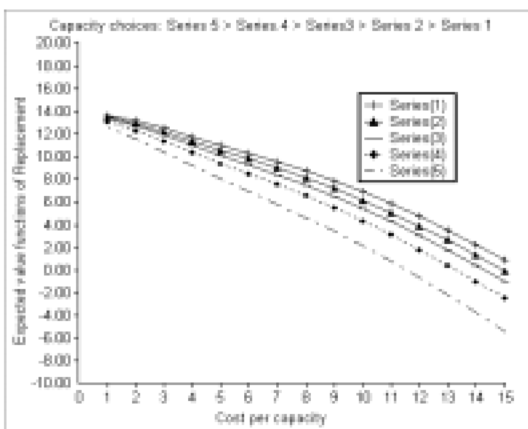

Figure 6.9: Expected value functions with various cost per capacity

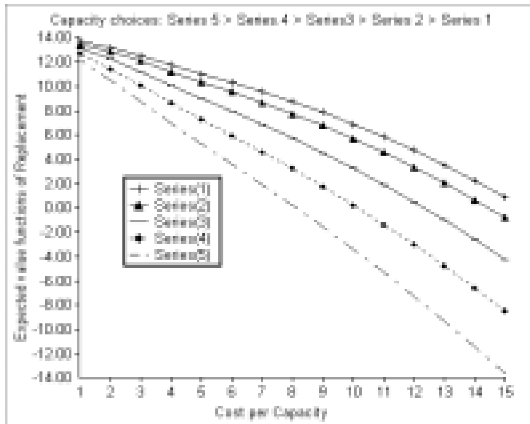

Figure 6.10: Expected value functions with various cost per capacity

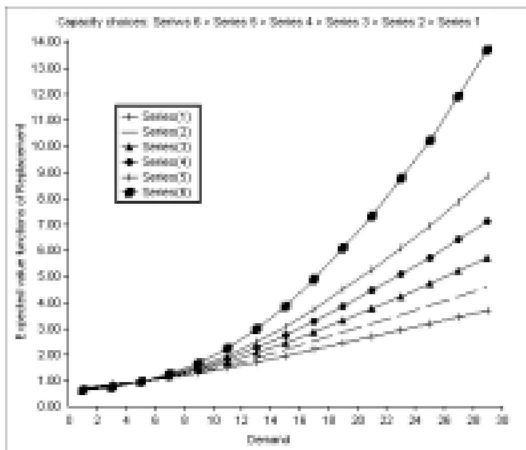

Figure 6.11: Expected value fucntions of replacement depending capacity choices

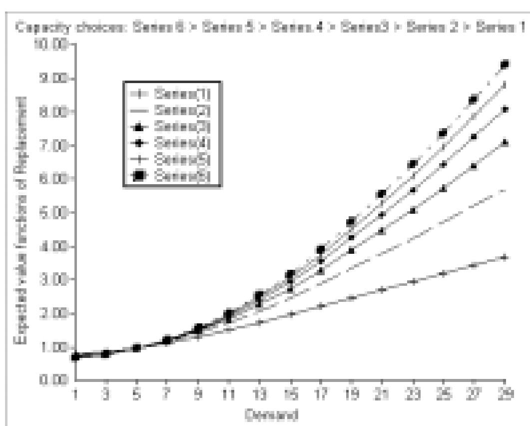

Figure 6.12: Expected value fucntions of replacement depending on capacity choices

Comparison between Figures 6.11 and 6.12 reveals the fact that when cost per capacity is relatively high, increases of capacity choice raise expected values with decreasing rate, because high cost per capacity increases replacement costs more than low cost per capacity does. In contrast, in case of low cost per capacity, increasing capacity choice raises expected values with increasing rate. This fact suggests that when cost per capacity is expected to be high, relatively low capacity is more preferable to high capacity choice.

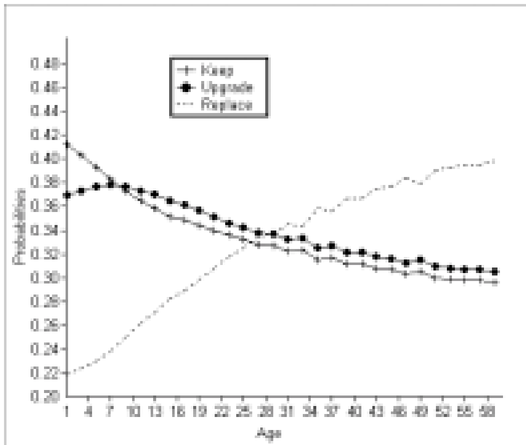

Figure 6.13: Simulated policy rules of keep, upgrade, and replacement

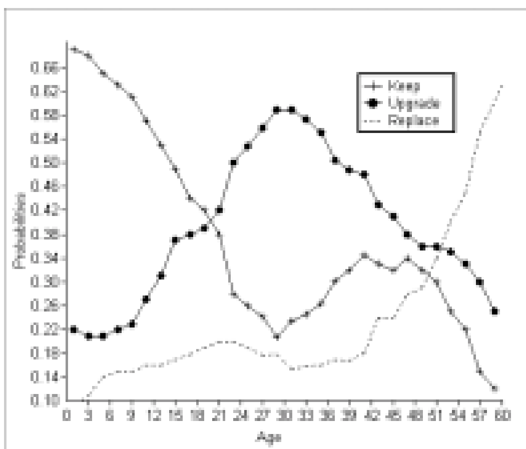

Figure 6.14: Simulated policy rules of keep, upgrade, and replacement

6.1.3 Simulation based on Estimation results

Based on estimated parameters, several simulations are performed to generate simulated data to be compared with real data. Instead of simulating the life of a task, I simulate the life of a computer system in order to investigate policy of upgrade and replacement. Also, In order to investigate frequencies of replacement and upgrade, all computer systems of the firm are simulated instead of only one computer. But, a whole life of a certain task is simulated for the investigation of capacity evolution. The actual decision process is assumed to have randomness, i.e.,. the actual decision varies, even though there is a most probable choice among the three options at each period.

Policy of Upgrade and Replacement Figures 6.13 and 6.14 present three simulated policies in two different situations. Figure 6.13 illustrates the situation in which the cost per capacity decreases rapidly with relatively small starting capacity and demand. Figure 6.14 shows the situation in which the cost per capacity decreases relatively slowly with a large demand for capacity.

The differences between two Figures 6.13 and 6.14 are as follows: The former Figure shows relatively higher likelihoods of keep and replacement than those of Figure 6.14. This is because small capacity requires relatively small maintenance costs. Also, as the computer gets older, replacement will be more profitable than upgrade, due to relatively small cost per capacity. In contrast to the Figure 6.13, the situation is different in Figure 6.14. In the initial phase, keeping is the proper choice. But, the likelihood of keeping is higher than that of Figure 6.13, because large capacity means there is no need for upgrade and replacement. Moreover, upgrade is more profitable than replacement over time in the latter Figure, due to the relatively high cost per capacity.

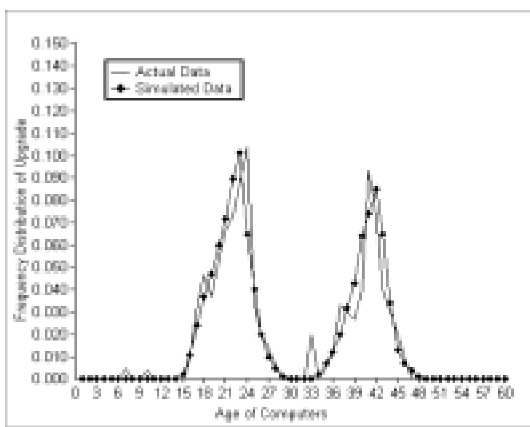

Figure 6.15: Comparison between the simulated data and the actual data

Frequency of Upgrade Figures 6.15 shows the comparison between the simulated data and the actual data in terms of frequency of upgrade with various age of computers. Generally, the shape and tendency of upgrade frequency are similar to each other. Most upgrade activities occur approximately at between one and half or two years of age, and at three and half years of age. The reasons are as follows. First, according to Moore's law, computer capacity becomes doubled every 18 months. Therefore, by upgrading its computer systems, the firm makes continuous efforts to keep track of the current technological progress for the purpose of lowering its operating costs. Second, the firm expands its services in order to meet rapid developing demands by more frequent upgrade activities. In comparison with the actual data, the frequency of the simulated data is slightly higher. But, the difference is minimal and acceptable.

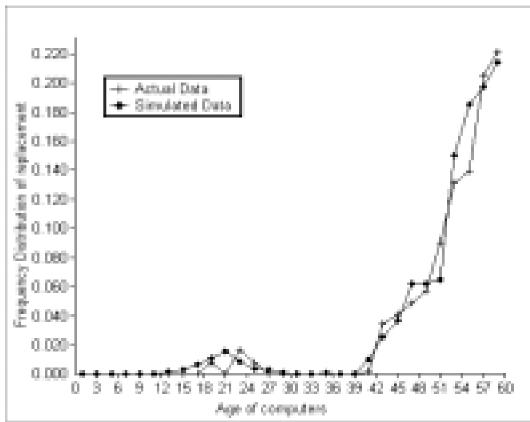

Figure 6.16: Comparison between the simulated data and the actual data

Frequency of Replacement Figure 6.16 illustrates the comparison between the simulated data and the actual data in terms of frequency of replacement with various age of computers. In general, the shape and tendency of replacement are similar to each other. Most replacement activities occur in the range of four and five years of age. Even though frequency of the simulated data is slightly larger than that of the actual data, the difference is minimal and acceptable. One noticeable fact is that several replacement activities occur in the period between one and one and half years of computer age in both data.

This situation can be explained with two compelling reasons. First, as we can notice in the Figure 6.15, the frequency of upgrade, computers tend to be upgraded approximately at two years of their age for the first time. However, replacement is more beneficial to the firm than upgrade during the period in case of some computers, because of costs, efficiency, or unexpected increases of demand the computers serve. Therefore, such computers are replaced as an alternative to upgrade in order to increase their capacities. Second, the firm tends to replace computer systems which fail to accomplish their given tasks after an initial testing period[32] where the period is

[32] In general, a lemon experiences any kinds of troubles within one or one and half year.

normally given by one or one and half year in the firm. In general, the simulated data from the estimated parameters show more frequent tendency to replace computer systems than the actual data do.

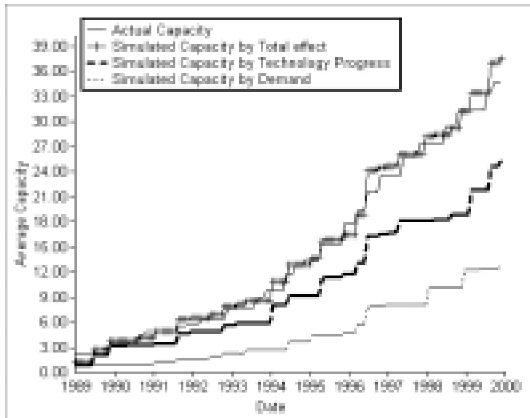

Figure 6.17: Comparison between actual capacity and simulated capacity (TPC case)

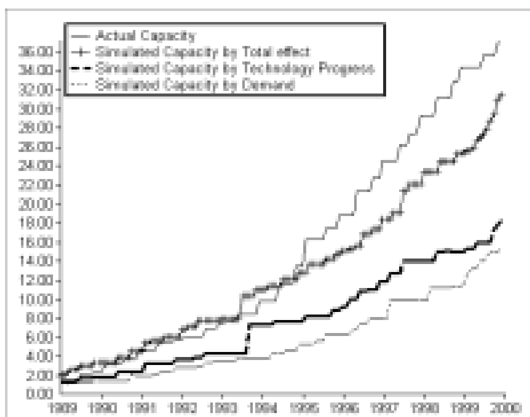

Figure 6.18: Comparison between actual capacity and simulated capacity (MIPS case)

Capacity Evolution of computer systems Figures 6.17 and 6.18 illustrate comparisons of evolution of capacities between the actual data and the simulated data in terms of TPC and MIPS standards respectively. For both Figures, the first graphs, “actual capacity”, present actual capacities from the data in case of TPC and MIPS standards. The second graphs, “simulated capacity by total effect”, show simulated capacities with the same condition as the actual data. The third, “simulated capacity by technology progress”, graphs illustrate simulated data without the effect of demand increase, where only technological progress affects the firm’s decision to keep, upgrade and replace current mainframe computers. The fourth graphs, “simulated capacity by demand”, shows simulated capacities when only demand affects the firm’s decision. The last two graphs at each Figure explain how technological progress and demand increase affect capacity of computer systems separately. First, in Figure 6.17, where computers' capacity is represented by TPC standard, the actual capacity and the simulated total capacity evolve in a very similar manner. The simulated data track the actual data very accurately in case of TPC standard. Second, in case of MIPS standard computer systems (Figure 6.18), actual capacity and simulated capacity evolve with a similar pace at the beginning of the time period. But, as time goes on, there is a slight discrepancy between two capacities. In fact, the actual capacity lies above the simulated capacity. However, general evolving tendencies between two capacities are similar to each other.

Separation of effects by technological progress demand of services shows clearly that technological progress plays more significant role than demand for services does in case of TPC standard computer systems. This confirms our conjecture that technological progress is the first and main source to cause replacement and upgrade of mainframe computer systems in the company. In case of MIPS case, the difference between two effects by technological progress and demand increase is minimal, even though technological progress improves capacity of computers slightly more than demand for services does.

We also note that for both Figures, 6.17 and 6.18, combined capacities from adding two simulated capacities by technological progress and demand at each point of date exceeds the simulated capacities by total effect, which is a capacity influenced by a combination effect of both technological progress and demand increase. The reason is as follows. Addition of two simulated capacities shows overlapped capacities, which mean that there are idle and wasted capacities from the addition, because each effect should account on increases of capacity individually and simultaneously. However, the simulated capacity by total effect is obtained by the optimization decision by considering two effects, technological progress and demand, as a whole effect. This illustrates a fairly expected and reasonable situation.

According to the aforementioned simulations and comparisons with the actual data, the simulated data show more frequent upgrade and replacement activities than the actual data do. However, once any decision between upgrade and replacement has been made, the actual data show more capacity increases than the simulation data do.

Total Expenditure of the firm Figure 6.19 shows a comparison between actual data and simulate data in terms of total expenditure of the firm. First, generally simulated total expenditure tracks the actual total expenditure well enough to ensure the model's usefulness. Second, we can note that the firm's total expenditure does not increase over time, even though there are tremendous increases in computer capacities as in Figures 6.17 and 6.18. This is because real cost per capacity decreases tremendously over time, even though the firm increases its computer systems' capacities greatly. Therefore, we can confirm the fact that decreasing effect of real cost per capacity surpasses increasing effect of capacity of computer systems.

As a result, the proposed model seems to explain the investment strategy of replacement and upgrade of the firm in a fairly reasonable way. Also, the simulations confirm that the firm follows an optimal investment strategy to replace and upgrade its computer systems by keeping track of the rapid development of computer technology and demand for its services.

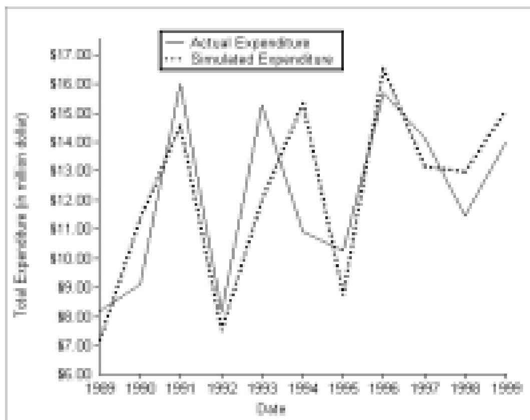

Figure 6.19: Comparison between Actual Total Expenditure and Simulated Total Expenditure

7 Policy Experiments

Even though the model is simple, it leads to a wealth of interesting behavioral implications. In particular, the model can be used to perform a wide variety of "policy experiments" which forecast how changes in various environments of the model, τ_u and τ_r, investment tax credits for upgrade and replacement respectively, or structural parameters, such as γ and β, average unit price of service demand and discount factor respectively, affect the timing and frequency of mainframe computers replacement and upgrade.

7.1 Investment Tax Credits for Replacement and Upgrade

7.1.1 Calculation of the Implied Demand for investment tax credits[33]

In this section, I investigate the effect of tax credits τ_u and τ_r for upgrade and replacement respectively, which were assumed to be zero at the estimation procedure. The upgrade and replacement tax credits, τ_u and τ_r, enter in cost functions as forms of $(1-\tau_u)c_t$ and $(1-\tau_r)c_t$, respectively. In order to see the effects of investment tax credits on the demand system, I first calculate demands for replacement and upgrade investment of mainframe computer systems by demonstrating the bottom-up approach.[34] The upgrade and replacement demand for a computer system is simply the sum of the upgrade and replacement demands generated by individual decision makers. Upgrade and replacement demands for computer systems given a number of month, T, are random functions $\widetilde{d_u}(\tau_u,\tau_r)$ and $\widetilde{d}_r(\tau_u,\tau_r)$. Let $\{a_t^m, x_t^m\}$ be a controlled stochastic process of a computer m. Given an initial distribution, $\Psi_m(a_0^m, x_0^m)$ for the initial states of each computer m, we can compute the probability distribution of the random functions $\widetilde{d_u}(\tau_u,\tau_r)$ and $\widetilde{d}_r(\tau_u,\tau_r)$ using controlled transition density $P(a_t \mid x_t,\theta)p(x_t \mid x_{t-1}, a_{t-1}, \theta_0)$ by integrating out the unnecessary state variables x_t. Then, by varying investment tax credits for computer systems upgrade and replacement, τ_u and τ_r, we can trace out how the entire probability distribution of upgrade and replacement varies as functions of upgrade and replacement tax credits. For simplicity, I compute the expected upgrade and replacement demand functions, $d_u(\tau_u,\tau_r) = E\{\widetilde{d_u}(\tau_u,\tau_r)\}$ and $d_r(\tau_u,\tau_r) = E\{\widetilde{d}_r(\tau_u,\tau_r)\}$ which are expected upward sloping functions of τ_u and τ_r. If we assume that P^* is the equilibrium or long run stationary distribution of the controlled process $\{a_t, x_t\}$.

With an additional assumption that the processes $\{a_t^m, x_t^m\}$ and $\{a_t^k, x_t^k\}$ are independent if $m \neq k$, with exception of the processes for d_t and c_t. I can get the following formula for $d_u(\tau_u,\tau_r)$ and $d_r(\tau_u,\tau_r)$ such as,

$$d_u(\tau_u,\tau_r) = TM\int_0^\infty P_\theta^*(dx, U) \tag{23}$$

$$d_r(\tau_u,\tau_r) = TM\int_0^\infty P_\theta^*(dx, K_r{}^{35}) \tag{24}$$

where T is the number of month and M is the number of computers.

[33] I followed the method used in Rust, Econometrica, Volume 55, Isse 5, 993-1033.

[34] For the bottom-up approach, see Rust, Econometrica, Vol 55, Issue 5 (Sep., 1987), 999-1033).

[35] I fix the capacity choice of replacement as $K_r = 4$.

By varying τ_u and τ_r, it is possible to plot the demand curves with respect to these tax credits.

7.1.2 Behavior of expected demand functions with respect to investment tax credits, τ_u and τ_r.

By parametrically varying investment tax credits τ_u and τ_r, we can trace out the equilibrium distribution P^*_θ as a function of τ_u and τ_r. By the formulae 23 and 24, we can compute expected demand curves for upgrade and replacement investments with respect to two tax credits. Figures 7.1 and 7.2 summarize the resulting demand curves.

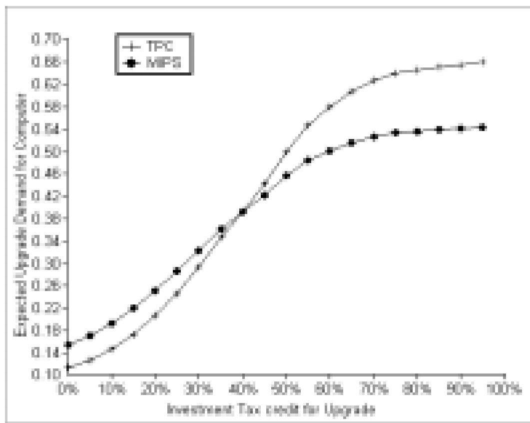

Figure 7.1: Expected Upgrade Demand function for MIPS and TPC

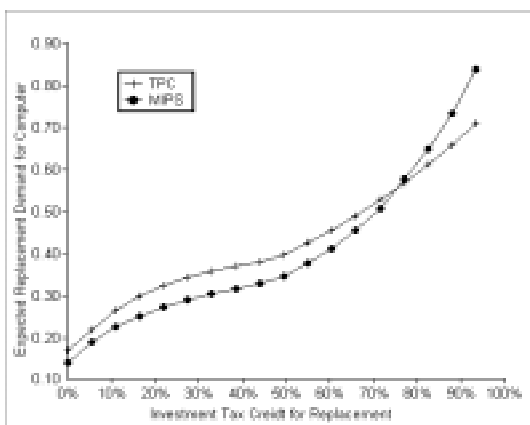

Figure 7.2: Expected Replacement Demand function for MIPS and TPC

Effects of Upgrade Tax credit Figure 7.1 presents the expected upgrade demand functions, $d_u(\tau_u, \overline{\tau_r})$ for both MIPS and TPC standards with respect to τ_u where the investment tax credit for replacement, τ_u is fixed at a certain value[36] for simplicity. As we expect, both demand

[36] which is zero at this time

functions show upward sloping curves. Zero investment tax credit is a natural situation which corresponds to the above estimation result and 100% means that upgrade cost is entirely subsidized by government. At the lower parts of investment tax credit, the changes of tax credit makes rapid changes in upgrade demand. However, at the higher part of the upgrade tax credit, a change of demand show a slow increasing pattern. The Figure suggests that upgrade tax credit seems not to fully induce upgrade of computer systems. The reasons are, first old and new components exist simultaneously in the upgraded computer systems. Therefore, these mixed components can incur the following inferior situations: (i) maintenance costs will be much higher than new system, and (ii) a contribution to an actual capacity which computer actually perform is smaller than that of new system. Second, there is a natural limitation of upgrade behavior, which is that a number of upgrade slots at each mainframe computer is limited to four or five.

In comparison of two graphs, TPC and MIPS standards, the former shows more sensitive behavior than the latter. This suggest that since computers of TPC standard require more upgrade costs[37], and thus the tax credit in case of TPC standard can mitigate the firm's upgrade costs more than that of MIPS standard, the upgrade tax credit is more effective on the TPC systems than those of MIPS standard.

Effects of Replacement Tax credit Figure 7.2 shows the expected replacement demand functions, $d_r(\overline{\tau_u}, \tau_r)$ for both MIPS and TPC standards with respect to τ_r with a certain fixed value $\overline{\tau_u} = 0$. The graphs illustrate an upward behavior like expected upgrade demand functions. But, the upward pattern is very different from the upgrade demands. Since replacement costs are much larger than those of upgrade, there show small increases in lower part of tax credit because of less credits. However, as the tax credit for replacement increases, the demand starts to increase with increasing rate, since larger amount of costs are subsidized by the tax credit than the case of upgrade. Compared with Figure 7.1, replacement demand reaches over 80% at very high level of replacement tax credit, even though upgrade demand reaches just at 60%. This is because new computer after replacement does not require extra maintenance costs and have new service contracts, such as warranty in near future. Thus, investment tax credit is more effective on the case of replacement than that of upgrade.

In comparison of two graphs in the Figure 7.2, TPC and MIPS standards, although the former starts at higher level of replacement demand than the latter, TPC standard shows relatively slower

[37] An average since upgrade cost of TPC's is almost twice as much as MIPS's, according the data,

demand growth than the latter. This is because computers with MIPS standard incur higher cost for replacement than TPC equipped computers, and thus large tax credit can mitigate relatively larger cost-burden in case of MIPS standard than in TPC standard. Therefore, the replacement tax credit is more effective on the MIPS standard computers than those of TPC standard.

As a result, the tax credit practices suggest that i) the investment tax credits should be imposed on the relatively higher costs of computer systems in order to be more effective. ii) the investment tax credit is more effective as a form of the replacement tax credit than that of the upgrade.

7.2 Average Unit Price of Service Demand

In order to find out the behavior of upgrade and replacement demand with respect to an average unit price of service the firm provides, γ, again, first I should compute the expected upgrade and replacement demand functions, $d_u(\gamma) = E\{\widetilde{d_u}(\gamma)\}$ and $d_r(\gamma) = E\{\widetilde{d_r}(\gamma)\}$ respectively. Let $P_\theta^*(a, x)$ be the long-run equilibrium distribution of the controlled process $\{a_t, x_t\}$. With an additional assumption of independency[38] which I mentioned earlier, I can get the following formula for $d_u(\gamma)$ and $d_r(\gamma)$ such as,

$$d_u(\gamma) = TM \int_0^\infty P_\theta^*(dx, U) \tag{25}$$

$$d_r(\gamma) = TM \int_0^\infty P_\theta^*(dx, K_r) \tag{26}$$

where T is the number of month and M is the number of computers.

By varying γ, we can plot upgrade and replacement demand curves with respect to unit price of service demand. Figure 7.3 and 7.4 summarize the resulting upgrade ($d_u(\gamma)$) and replacement ($d_r(\gamma)$) demand curves for TPC and MIPS standards respectively. As expected, all demand functions show upward sloping curves. For both Figures, expected upgrade demand curves ($d_u(\gamma)$) increase with an increasing rate at a lower part of average unit price and become flat at higher prices. In case of replacement demands ($d_r(\gamma)$), the behavior is different from $d_u(\gamma)$. i.e., demand increases smoothly at a lower part of price and becomes steeper at higher average prices. This is because the price is not enough to induce the replacement behavior at a lower unit prices and as price increases, replacement which requires much higher cost than upgrade does is more preferable

[38] The processes $\{a_t^m, x_t^m\}$ and $\{a_t^k, x_t^k\}$ are independent if $m \neq k$

because higher prices can cover a larger part of replacement costs[39]. Higher price can also induce better quality services of the firm which requires newer and faster computer systems in order for the firm to maintain the current, high priced demand.

Note that the estimated equilibrium price, γ equals to 3.167. Until less than twice of the equilibrium price, upgrade demand lies above the replacement demand. After the price, replacement demand becomes larger than upgrade demand. There shows the same phenomenon in case of MIPS standard computer systems in Figure 7.4, where the equilibrium γ equals to 3.524. Therefore, we can conjecture that an average price should be at least twice as much as the equilibrium price in order to have more likelihoods of replacement than upgrade in this case.

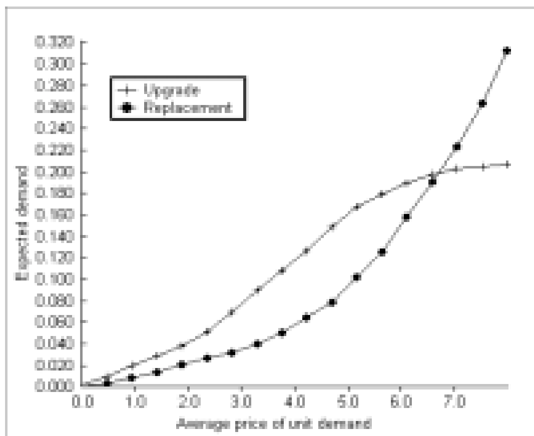

Figure 7.3: Expected demand curves of upgrade and replacement (TPC standard computers)

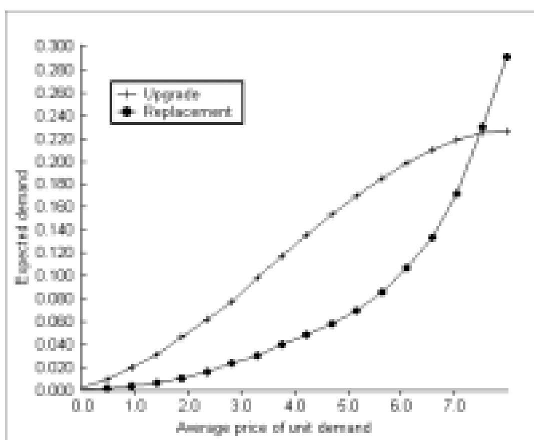

Figure 7.4: Expected demand curves of upgrade and replacement (MIPS standard computers)

[39]Replacement is the better choice than upgrade in order to improve a computer's performance, when the other conditions remain the same.

Comparison between two Figures, 7.3 and 7.4 gives the following interesting points. First, when upgrade demand lies above that of replacement, the gaps between replacement and upgrade curves at each Figure are different from each other. The gap in TPC standard is narrower than that of MIPS standard. This is because the difference between upgrade and replacement costs in MIPS standard computers is larger than that of TPC standard computers. Therefore, an average demand price in case of MIPS standard should be much higher than that of TPC standard in order for replacement demand to exceed upgrade demand[40]. Second, at any given unit prices of service demand, the upgrade demand for MIPS standard computers is larger than the upgrade demand for TPC standard, because the upgrade cost is smaller in case of MIPS standard. Similarly, the replacement demand for MIPS standard computers is smaller than the replacement demand for TPC standard, since the average replacement cost of MIPS standard is larger than that of TPC standard computers. As a result, the above simple policy implications show us that how the firm can modify its upgrade and replacement policies to deal with various environments.

8 Conclusion and Future Research

8.1 Conclusion

The Contribution of the paper can be summarized in the following unique respects.

A). Methodological Contribution

(1). The paper find an optimal investment model including replacement and upgrade which is applied to technologically deteriorating systems. It presents an empirical model to analyze a system administrator's forward looking behavior regarding investment of mainframe computer systems. Taking into account the unique features of computer systems, especially technical depreciation, a stochastic dynamic programing model is developed to see whether the explanatory facts of investment behavior could be rationalized as an optimal investment strategy for the firm.

(2). The paper is the first one to apply a combination of the Nested fixed point algorithm and the Parametric approximation method (NLS-NFXP) to a high dimensional fixed point problem. The paper utilizes a series of estimation techniques, such as a combination of a Parametric estimation and a Nested fixed point algorithm. The parametric approximation is used to circumvent problems, such as curse of dimensionality and computational burden incurred by discretization

[40] As mentioned above, the replacement demand exceeds the upgrade demand at about $\gamma = 7.2$ in case of MIPS standard and at about $\gamma = 6.4$ in case of TPC standard according to figures 7.3 and 7.4.

and substitutes a non-linear least squares estimation method for a fixed point iteration. The speed up in solution time is sufficiently large to make it feasible to estimate the unknown parameters of the model by maximum likelihood.

B). Empirical Contribution

(1). This research find a new and unique data set regarding computer investment from one of the world's biggest telecommunication companies. The explanatory facts from the actual data provide evidence for the observation that the firm's replacement and upgrade behaviors fully reflect the technical depreciation.

(2). Estimation and simulation results show that the dynamic programming model can solve the firm's optimal investment decision and at least, in case of TPC standard computer systems, they confirm that technological progress plays more important roles than demand increase in investment decision. As a result, the firm seems not to use an arbitrary rule of thumb in deciding to upgrade and replace its mainframe computers so rapidly, but rather the firm appears to have a very sophisticated understanding of the impact of technological progress resulting from Moore's Law and is taking advantage of this progress to significantly reduce its operating costs and provides better service to its customers.

8.2 Further Research

First, an introduction of new software, such as a new operating system, is one of main reasons to replace computers, since a new operating system may require a more advanced system to work properly. Even though I am fully aware of importance of role of new software, limits of information regrading software which the company uses prevent me from including this factor as a state variable of the model. In future research, this should be considered as well.

Second, replacement of PCs is another significant decision problem in the current economy. Thus, rationalization of PC replacement behavior is a part of future research. One of the main features of the PC holding is block purchases and replacements. In order to explain PC replacement policy, there should be a different model from the current our research. For example, there will be different choice sets from the replacement of mainframe computers. i) the upgrade choice will be dropped, since the upgrade is not a main choice to improve PCs' performances, because of relatively low cost of replacement of PCs and limited upgrade capacity. ii) brands and number of computers as well as timing issue should be parts of choice variables in case of PC replacement, which require several important aspects of multiple discrete choice random utility model.

Third, issues on the relationship between R&D expenditure and technological progress, technology cycles, and multiple equilibria can be considered as another important extension of the research. The most important factor to induce more replacement demand is faster technological progress in the computer industry. The induced frequent replacement demand will cause more R&D expenditure on the technology sector, which can stimulate faster technological progress. Again, this faster technological progress in the computer industry will decrease the cost per unit capacity of computers or increase the rate of technological progress, which can induce again more replacement demand by decreasing operating cost of each firm. As a result, such technology cycles endogenize the technological progress which was taken as an exogenous variable previously. In such technology cycles, there exist multiple steady states and each one represents different "equilibrium" relationship between R&D and computer replacement demand. When the economy is slow, the aforementioned technology cycles have an equilibrium at a lower point. At this lower equilibrium point, demand for replacement investment decreases, which causes lower R&D expenditure at the technology sector. This lower expenditure makes slower technological progress, which directly affects slower price reduction of computer systems. As a result, firms are not willing to replace their computer systems frequently, because of high replacement costs. However, when economy is booming, the technology cycles have an equilibrium at a higher point. Also, the cycles can be upset by various factors, such as shocks that reduce demand, R&D expenditure, or technological progress.

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9 Appendix A

9.1 Transition matrices of demand and cost per capacity for discretization purpose.

$p(D_{t+1}|D_t)$ is as follows.

	1	2	3	...	29	30
1	h_0^*	$1 - h_0$	0	...	0	0
2	h_1	h_0	$1 - h_0 - h_1$	...	0	0
3	0	h_2	h_0	...	0	0
...	0	0	0	...	...	...
29	0	0	...	0	h_0	$1 - h_0$
30	0	0	...	0	0	1

*: $h_0 = 0.89$.

$p(pc_{t+1}|pc_t)$ is as follows.

	15	14	13	...	2	1
15	g_0^*	g_1^{**}	$1 - g_0 - g_1$	...	0	0
14	0	g_0	g_1	$1 - g_0 - g_1$	0	0
13	0	0	g_0	...	0	0
...	...	...	0	...	0	0
2	0	0	...	....	g_0	$1 - g_0$
1	0	0	...	0	0	1

*: $g_0 = 0.28$, **: $g_1 = 0.7$