Minimum Wage Effects on Labor Market Outcomes under Search with Bargaining*

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Abstract

Building upon a continuous-time model of search with bargaining in a stationary environment, we analyze the effect of changes in minimum wages on labor market outcomes and welfare. While minimum wage increases lead to employment loses, they may be welfare-improving in at least two well-defined senses. A key determinant of the welfare impact of a minimum wage increase is the Nash bargaining power parameter. We discuss identification conditions of this model, which extends the analysis in Flinn and Heckman (1982). Maximum likelihood estimates of the model are obtained using recent Current Population Survey data. Our empirical results indicate that recent minimum wage increases resulted in welfare improvements for young labor market participants. We also present some more speculative evidence concerning the level of the "optimal" minimum wage.

1 Introduction

Determining the equilibrium effects of minimum wage changes on labor market outcomes is a challenging modeling and estimation problem; arriving at policy recommendations is a task even more daunting. Faced with the inherent difficulties of modeling equilibrium labor market events given the limited amount of data to which researchers have access, much recent research has been performed outside of an explicit modeling framework, with researchers pursuing the more limited objective of carefully describing the observed effects of recent minimum wage changes using quasi-experimental methods [see Card and Krueger (1995), hereafter CK, for a summary of these studies and a comprehensive, critical survey of most of the previous research done in this area]. In our view, these recent studies have been particularly useful in indicating that the "textbook" competitive

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model of the labor market, which has been used as an interpretive framework for the bulk of empirical studies using aggregated time series data, may have serious deficiencies in accounting for minimum wage effects on labor market outcomes when confronted with disaggregated data.

While the quasi-experimental results have raised a number of interesting challenges to orthodox theory, few cogent models have been advanced which are consistent with the results which have been found. Some explanations which have been put forth would not seem to be testable given our current data resources. In fact, it appears difficult to operationalize many of the explanations proposed even for the more modest purpose of empirical implementation.

A fully-elaborated, competitive equilibrium model of the effects of minimum wages presumably would require a system of labor demand functions for various observably-differentiated classes of labor and capital in addition to a well-specified system of labor supply functions. Using individual-level data on individuals at a point in time or from a short panel, the requisite information on the demand-side of the market would be missing, so that to estimate an equilibrium model in this case requires one to make a number of stringent assumptions concerning the behavior of firms [see Eckstein and Wolpin (1990), van den Berg and Ridder (1993), and Flinn (1997a) for examples of the estimation of equilibrium labor market models using only individual-level data on workers]. Even after making such assumptions, it may well be the case that important demand-side parameters will be unidentified, thus making it problematic to conduct policy experiments. Therefore this approach has some severe limitations from the point of view of implementation, and additionally may not be able to generate certain key features of the observed wage distribution, such as the probability mass at the minimum wage.

It has long been recognized that for the imposition of minimum wages to have a beneficial effect on the welfare of all labor market participants on the supply side of the market, firms must have some degree of monopsony power [see for example Manning (1995)]. Search frictions and the existence of match-specific capital are capable of providing this. The existence of both seems in accord with common sense and empirical findings [on the existence of match-specific capital see Miller (1984) and Flinn (1986), for example]. Below we formulate a model which includes match-specific capital, job search, and worker-firm bargaining over match-specific rents, that is capable of accounting for most of the empirical observations cited by CK in their survey of minimum wage research based on individual level data (1995, ch. 7).

To illustrate some of the features of wage distributions (and unemployment phenomena) CK point to, consider Figure 1. This figure contains plots of wage distributions and the distributions of on-going spells of unemployed search for individuals in the March Current Population Survey (CPS) who were between the ages of 16 and 24, inclusive. Three years of data are presented, 1996 through 1998. These years are noteworthy because the prevailing (nominal) minimum wage in each year is different: \$4.25 in 1996, \$4.75 in 1997, and \$5.15 in 1998.[1]. In each of the wage distributions we can discern several "spikes," though all except the minimum wage occur at "focal points" for respondents, e.g., integer values. In each of these three years approximately 7 percent of employed respondents have a reported wage rate exactly equal to the minimum wage. If we can assume that the other mass points in the distribution are produced by reporting error, then since the minimum wage does not occur at a natural focal point in any of the years we may be entitled to claim that the wage distribution for young labor market participants can be (roughly) characterized as having a mass point at the minimum wage with wages continuously distributed above it.

[1]In constructing the histograms (and in the subsequent empirical work) hourly wage rates less than the statuatory minimum wage or greater than \$30 per hour have been ignored.

Card and Kruger and others have made much of the potential of a "spillover" effect of minimum wage increases. We shall give a formal definition below (and present a more detailed analysis of this issue in a companion paper (Flinn (2000))), but the basic notion is that changing from a minimum wage of \$4.25 to one of \$4.75, for example, can impact the "characteristics" of the wage distribution above \$4.75. It is not clear from the plots if the conditional distributions of wages above the higher of the two minimum wages in subsequent years, but in Flinn (2000) we present some evidence that indicates that this was the case.

Finally, we wish to record a few observations about unemployment experiences of the sample members. Using standard definitions of the unemployment state, we find that although both the nominal and real minimum wage rate increased in 1997 and 1998, the proportion of labor market participants who were employed actually increased slightly, going from .865 in 1996 to .871 in 1997 and ending at .875 in 1998. Such a finding will not be consistent with the model presented below, at least the simple version of it analyzed here. On a more positive note, the steady state version of our model produces the implication that the distribution of on-going spells of unemployment should belong to the negative exponential family. While there is a large degree of "heaping" in the unemployment histograms presented in Figure 1, the empirical distributions don't seem wildly at odds with this model implication. Thus on most accounts, the model developed and estimated below satisfies the CK desiderata.

The model is in some sense a logical descendent of the econometric model of search of Flinn and Heckman (1982) - hereafter referred to as FH - and the econometric model of minimum wage effects on the distribution of wages and the probability of employment estimated by Meyer and Wise (1983a,b) - hereafter referred to as MW. FH formulated an equilibrium continuous-time search model in which searchers encountered potential employers according to a Poisson process; upon meeting, the potential value of the contact was determined by a draw from a fixed distribution G. In their analysis the match value was arbitrarily assumed to be divided evenly between workers and firms. The setup of the basic model proposed here is similar, with the important difference that rents are divided using an explicit Nash-bargaining criteria. The bargaining power parameter which appears in this formulation is of key importance in determining the welfare consequences [on labor market participants] of imposing, or increasing, a minimum wage.

It is worth elaborating on the role of the bargaining power parameter in this research. As we have noted, this parameter did not appear in the formulation of FH - for one reason because it is a more difficult parameter to interpret than are the others in their model. We think of it as comprising a type of summary statistic of the labor market "position" of a particular group. For example, the match value distribution for low-skilled workers may be stochastically dominated by the match value distribution for high-skilled workers, but in addition low-skilled workers may be at a disadvantage due to their having little bargaining power. Their low bargaining power may derive from there being many substitutes for them in the production process, or to the relative number of low-skilled workers to the number of positions for this type of worker. In this sense, the parameter cannot be really thought of as "primitive" since significant policy changes - such as a doubling of the minimum wage - may result in participation effects or substitution responses by firms which change the labor market "position" of the group, and hence change the bargaining power parameter. Thus comparative statics exercises and policy experiments performed with the estimates obtained from this model will only be valid locally, that is, for small changes in policy variables. This limitation is not too disturbing, since it probably applies to all empirical research which has been conducted in this area.

MW estimated a model of minimum wage effects using individual-level data which allowed them to infer what the wage distribution and employment level would have been in the absence of a minimum wage. Their model was both original and suggestive. While it has been criticized by a number of researchers [e.g., CK (pp.232-236) and Dickens et al. (1997)], primarily for relying on functional form assumptions for identification and for choosing a parameterization which rules out the possibility of employment increases in response to a minimum wage increase, it remains one of the better econometric attempts to identify minimum wage effects using individual-level data in the literature [another example is Heckman and Sedlacek (1981)]. From our perspective, the main weakness of their model is the arbitrary specification of the manner in which a minimum wage "distorts" the preexisting distribution wage distribution. In our model, optimizing behavior by searchers and firms determines the nature of this "distortion," and it is roughly consistent both with the MW specification and with the empirical evidence cited in CK.

In this research we attempt to integrate theory and measurement in analyzing minimum wage effects on the labor market. For example, while there are a number of papers in the literature which consider the possibility of welfare-enhancing minimum wage rates [e.g., Drazen (1986), Lang (1987), Rebitzer and Taylor (1996), Swinnerton (1996)], the frameworks in which the models are set tend to be relatively abstract and the models themselves are typically unestimable. In this research, the same model which is used to define conditions under which minimum wages can be welfare-improving for the supply side of the market is estimated using individual-level data. With the parameter estimates obtained, we are able to check whether actual minimum wage changes have been welfare-improving for searchers and employees, as well as to make some informed speculation as to the optimal level of the minimum wage [once again, considering only searchers and employees] under certain well-defined welfare criteria.

The plan of the paper is as follows. In Section 2 we develop the bargaining model in a continuous-time search environment both in the absence and in the presence of a binding minimum wage. Section 3 contains a discussion of the welfare effects of the imposition of a minimum wage, and presents two summary measures of these effects. We also explore in some detail the effect of changes in minimum wage laws on labor market outcomes such as the wage distribution and employment rate. In Section 4, we begin by developing an econometric framework which is used to estimate the model. Issues of model identification are considered in some depth. In Section 5 we present results from estimating various specifications of the bargaining model using CPS data from the March surveys in 1992 through 1998. We perform some analyses of actual and potential changes in nominal minimum wage rates on population welfare using model estimates. Section 6 concludes.

2 Labor Market Search with Bargaining

In this section we describe the behavioral model of labor market search with matching and bargaining. The model is formulated in continuous time and assumes stationarity of the labor market environment. In the first subsection we derive the decision rules for terminating search and for dividing the match value between worker and firm in the absence of minimum wages. In the following subsection we describe the manner in which minimum wages affect search behavior and the division of the match value.

Throughout we assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels which is given by $G(\theta)$. When a potential employee and a firm meet, which happens at rate λ, the productive value of the match (θ) is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework. The searcher's instantaneous discount rate is given by $\rho > 0$. The rate of (exogenous) termination of employment contracts is $\eta \geq 0$. While unemployed individuals search, their instantaneous utility is given by b, which can assume positive or negative values. For simplicity, we assume that employed individuals do not receive alternative offers of employment, i.e., there is no on-the-job search. It is straightforward to adapt the current framework to that case, however.[2]

2.1 Labor Market Decisions without Minimum Wages

We assume that the only factor of production is labor, and that total output of the firm is simply the sum of the productivity levels of all of its employees. Then if the firm "passes" on the applicant - that is, does not make an employment offer - its "disagreement" outcome is 0 [it earns no revenue but makes no wage payment]. The applicant's disagreement value is the value of continued search, which we denote V_n. For any given value of V_n there exists a corresponding critical "match" value $\theta^* = \rho V_n$ (ρ is the instantaneous discount rate), which has the property that all matches with values at least as great as θ^* will result in employment while all those matches of lower value will not. For any $\theta \geq \theta^*$, the wage offer is given by

$$w(\theta, V_n) = \arg\max_w \, [V_e(w) - V_n]^{\alpha} \left[\frac{\theta - w}{\rho + \eta}\right]^{1-\alpha}, \tag{1}$$

where without loss of generality it has been assumed that the firm shares the employee's effective rate of discount, $\rho + \eta$.

The value of employment at a wage of w is easily determined. Consider an infinitesimally small period of time Δt. Over this "period," either the individual will continue to be employed at wage w or will lose their job, which occurs at rate η. Then

$$V_e(w) = \frac{w\Delta t}{1 + \rho\Delta t} + \frac{1}{1 + \rho\Delta t}\left[\eta\Delta t\, V_n + (1 - \eta\Delta t)\, V_e(w)\right] + \frac{o(\Delta t)}{1 + \rho\Delta t}, \tag{2}$$

where the term $(1 + \rho\Delta t)^{-1}$ is an "infinitesimal" discount factor associated with the small interval Δt, $\eta\Delta t$ is the approximate probability of being terminated from one's current employment by the end of Δt, and $o(\Delta t)$ is a term which has the property that $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$. Note that the first term on the right hand side of [2] is the value of the wage payment over the interval, which is the total payment $w\Delta t$ multiplied by the "instantaneous" discount factor [think of the payment as being received at the end of the interval Δt]. After collecting terms and taking the limit of [2] as $\Delta t \to 0$, we have

$$V_e(w) = \frac{w + \eta V_n}{\rho + \eta}. \tag{3}$$

We now substitute [3] into [1] so as to simplify the problem as follows:

$$\begin{aligned} V_e(w) - V_n &= \frac{w + \eta V_n}{\rho + \eta} - V_n \\ &= \frac{w - \rho V_n}{\rho + \eta}, \end{aligned}$$

[2]Since the CPS data used in the empirical analysis are essentially cross-sectional, it is virtually impossible to construct the kind of event history data required to estimate a model with sophisticated turnover phenomena.

so that we get the well-know expression

$$\begin{aligned} w(\theta, V_n) &= \arg\max_w \, [w - \rho V_n]^\alpha \, [\theta - w]^{1-\alpha} \\ &= \alpha\theta + (1-\alpha)\rho V_n. \end{aligned}$$

We can now move onto computing the value of nonemployment. Using the same setup as above for defining the value of employment, we begin with the $\Delta t-$period formulation which is

$$V_n = \frac{b\Delta t}{1+\rho\Delta t} + \frac{1}{1+\rho\Delta t}\{\lambda\Delta t \int \max[V_n, V_e(w(\theta, V_n))] \, dG(\theta)$$
$$+(1-\lambda\Delta t) \, V_n\} + \frac{o(\Delta t)}{1+\rho\Delta t},$$

where $\lambda\Delta t$ is the approximate probability of encountering one potential employer over the interval. Rearranging and taking limits, we have

$$\rho V_n = b + \lambda \int_{\rho V_n} [V_e(w(\theta, V_n)) - V_n] \, dG(\theta).$$

Since

$$\begin{aligned} V_e(w(\theta, V_n)) &= \frac{\alpha\theta + (1-\alpha)\rho V_n + \eta V_n}{\rho+\eta} \\ &= \frac{\alpha\theta - \alpha\rho V_n}{\rho+\eta} + V_n, \end{aligned}$$

we have

$$V_e(w(\theta, V_n)) - V_n = \frac{\alpha\theta - \alpha\rho V_n}{\rho+\eta}.$$

Then the final (implicit) expression for the value of search is

$$\rho V_n = b + \frac{\alpha\lambda}{\rho+\eta} \int_{\rho V_n} [\theta - \rho V_n] \, dG(\theta). \tag{4}$$

Note that this expression is identical to the expression for the reservation value in a model with no bargaining when θ is the payment to the individual except for the presence of the factor α. This is not unexpected, since when $\alpha = 1$, the entire match value is transferred to the worker, and thus search over θ is the same as search over w.

Now we can summarize the important properties of the model. The critical "match" value θ^* is equal to ρV_n, which is defined by [4]. Since at this match value the wage payment is equal to $w^* \equiv w(\theta^*, V_n) = \alpha\theta^* + (1-\alpha)\theta^* = \theta^*$, the reservation wage is identical to the reservation match value. The probability that a random encounter generates an acceptable match is given by $\tilde{G}(\theta^*)$, where $\tilde{G}$ denotes the survivor function, $1 - G$. The rate of leaving unemployment is $\lambda\tilde{G}(\theta^*)$. As we can see from [4], since θ^* is a decreasing function of α, rates of unemployment are higher when searchers have more bargaining power.

The observed wage density is a simple mapping from the matching density. Since

$$\begin{aligned} w(\theta, V_n) &= \alpha\theta + (1-\alpha)\theta^* \\ \Rightarrow \tilde{\theta}(w, V_n) &= \frac{w - (1-\alpha)\theta^*}{\alpha}, \end{aligned}$$

then the density function of observed wages is given by

$$h(w) = \begin{cases} \frac{\alpha^{-1} g(\tilde{\theta}(w, V_n))}{\tilde{G}(\theta^*)} & w \geq \theta^* \\ 0 & w < \theta^* \end{cases} .$$

There are a few results concerning the relationship between the equilibrium wage distribution and α that will be useful for drawing comparisons between the minimum wage and no minimum wage cases. The first comparative statics result should be intuitively obvious, and we state it without proof.

Proposition 1 *V_n is an increasing function of α.*

The bargaining power parameter translates into the searcher getting a larger share of any given match value, which makes the value of search greater. In this sense, increasing α is analogous to shifting a wage offer distribution from F to F', where F' first order stochastically dominates F, in a nonequilibrium search model without bargaining.

Proposition 2 *In a given labor market with bargaining power parameter α, let the equilibrium wage offer distribution be given by $H(w;\alpha)$. If $\alpha' > \alpha$, then $H(w;\alpha')$ first order stochastically dominates $H(w;\alpha)$.*

Proof: We show that the statement is true for infinitesimal changes in α, which can then be extended to the finite change case. The equilibrium wage c.d.f. can be written as

$$H(w;\alpha) = G(\frac{w - (1-\alpha)\rho V_n(\alpha)}{\alpha}),$$

from which it follows that

$$\begin{aligned} \frac{\partial H(w;\alpha)}{\partial \alpha} &= g(\frac{w - (1-\alpha)\rho V_n(\alpha)}{\alpha}) \\ &\quad \times \{\frac{\rho \tilde{V_n}(\alpha)}{\alpha} - \frac{(1-\alpha)\rho}{\alpha}\frac{\partial V_n(\alpha)}{\partial \alpha} - \frac{w - (1-\alpha)\rho V_n(\alpha)}{\alpha^2}\}. \end{aligned}$$

Since $\partial V_n(\alpha)/\partial\alpha > 0$ and $g(x) \geq 0$ for all x, it follows that this partial derivative is nonpositive for all $w \geq \rho V_n(\alpha)$ since

$$\begin{aligned} \text{sgn}\{\frac{\rho \tilde{V_n}(\alpha)}{\alpha} - \frac{w - (1-\alpha)\rho V_n(\alpha)}{\alpha^2}\} &= \text{sgn}\{\alpha \rho \tilde{V_n}(\alpha) - w + (1-\alpha)\rho V_n(\alpha)\} \\ &= \text{sgn}\{\rho V_n(\alpha) - w\} \\ &\leq 0, \ \forall w \geq \rho V_n(\alpha). \end{aligned}$$

¥

It will be useful to carry an example through the first part of the paper, particularly in order to emphasize the key role the bargaining power parameter α plays in determining the substantive implications of the model. In our example labor market we set the rate of arrival of offers (λ) to the value .5 (so that job contacts occur every 2 "periods" on average), the rate of job dissolutions (η) is set to .02 (so that the average length of a job is 50 periods), ρ is set to .01, and the instantaneous return from search (b) is set to -1. The firm-searcher matching distribution is assumed to be uniform with support $[0, 10]$. We will compute the equilibrium wage distribution for values of α in the set {.25,.50,.75,1.00}.

Figure 2.a plots the uniform p.d.f. which represents $g(\theta)$ in this case. Figure 2.b plots the mapping from draws of θ into wage offers under the four alternative values of α, that is $w_\alpha(\theta, V_n(\alpha)) = \alpha\theta + (1-\alpha)\rho V_n(\alpha)$. Note that α affects the equilibrium mapping both directly through the slope and indirectly through the disagreement point $\rho V_n(\alpha)$. Figures 2.c-f plot the equilibrium wage p.d.f.s for the four α values. Increasing α in the uniform case simply results in increases in the lower and upper bound of the support of the equilibrium wage distribution, which is itself uniform.

2.2 Labor Market Decisions in the Presence of Minimum Wages

Now consider the case in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage, m, is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation the firm can adjust so as to "undo" the minimum wage payment requirement.

We impose the minimum wage in the framework established in the previous section. As should be clear, any $m \leq \theta^*$ has no effect on the behavior of applicants or firms and thus would be a meaningless constraint. Thus we consider only the effects of an imposition of an $m > \theta^*$.

Recall that the expected value of the match from the point of view of the firm is proportional to $(\theta - w)$. Firms cannot earn positive profits on matches which have a value less than m. Since $m > \theta^*$, an immediate implication of the imposition of the minimum wage is that fewer contacts will result in jobs - the standard employment effect.

In terms of wage payments, the minimum wage acts solely as a side constraint on the Nash bargaining problem. Formally, the revised problem is given by

$$w(\theta, V_n) = \arg\max_{w \geq m} [V_e(w) - V_n]^\alpha \left[\frac{\theta - w}{\rho + \eta}\right]^{1-\alpha},$$

where the only difference from [1] is the restriction $w \geq m$. The effect on the solution is relatively intuitive. Under the "constrained" Nash bargaining problem, there will exist a value of search which we denote $V_n(m)$ [Note that this value is *not* equal to V_n - it will be defined below]. If we ignore the minimum wage constraint in determining the wage payment given a match value of θ and the search value $V_n(m)$, we get

$$w(\theta, V_n(m)) = \alpha\theta + (1-\alpha)\rho V_n(m). \tag{5}$$

Under this division of the match value, the worker would receive a wage of m when $\theta = \hat{\theta}$, where

$$\hat{\theta}(m, V_n(m)) = \frac{m - (1-\alpha)\rho V_n(m)}{\alpha}.$$

Then if $\hat{\theta} \leq m$, all "feasible" matches would generate wage offers at least as large as m. When $\hat{\theta} > m$, this is not the case. When θ belongs to the set $[m, \hat{\theta})$, the offer according to [5] is less than m. However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of m for all $\theta \in [m, \hat{\theta})$. Wages for acceptable θ outside of this set are determined according to [5].

We can now consider the individual's search problem given this wage offer function. Using the Δt interval formulation,

$$
\begin{aligned}
V_n(m) \quad = \quad & \frac{b\Delta t}{1+\rho\Delta t} + \frac{\lambda\Delta t}{1+\rho\Delta t}\{\int_m^{\hat{\theta}(m,V_n(m))} \left[\frac{m+\eta V_n(m)}{\rho+\eta}\right] dG(\theta) \\
& + \int_{\hat{\theta}(m,V_n(m))} \left[\frac{\alpha(\theta-\rho V_n(m))}{\rho+\eta}\right] dG(\theta) + V_n(m)G(m)\} \\
& + \frac{(1-\lambda\Delta t)}{1+\rho\Delta t} V_n(m) + \frac{o(\Delta t)}{1+\rho\Delta t}.
\end{aligned}
$$

Taking limits after collecting terms, we have

$$
\begin{aligned}
\rho\tilde{V_n} \quad = \quad & b + \frac{\lambda}{\rho+\eta}\{\int_m^{\hat{\theta}(m,V_n(m))} [m-\rho V_n(m)]dG(\theta) \\
& + \alpha \int_{\hat{\theta}(m,V_n(m))} [\theta-\rho V_n(m)]dG(\theta)\}.
\end{aligned}
\tag{6}
$$

We shall often refer to the value $\rho V_n(m)$ as the "implicit" reservation wage. Unlike the situation in which a binding minimum wage is not present, this value is not the minimal acceptable wage and match value. The acceptable wage/match value is rather the imposed minimum value m. Nonetheless, the value $\rho V_n(m)$ is of critical importance in determining equilibrium wages and the welfare effects of minimum wage changes.

Conditional on the value of a binding minimum wage m, the equilibrium wage distribution is described by

$$
p(w|m) = \begin{cases} \frac{\alpha^{-1} g(\tilde{\theta}(w,V_n(m)))}{\tilde{G}(m)} & w > m \\ \frac{\tilde{G}(m)-\tilde{G}(\hat{\theta}(m,V_n(m))}{\tilde{G}(m)} & w = m \\ 0 & w < m \end{cases}
\tag{7}
$$

The minimum wage side constraint produces an equilibrium wage distribution which has a mass point at m and has wages being continuously distributed on the interval (m, ∞).[3]

Let us reconsider our uniform example after a minimum wage of 7.5 has been imposed; since the distribution now has a mass point, it is more convenient to plot the c.d.f. as opposed to the p.d.f. Figure 3.a plots the c.d.f. of the matching distribution. Figure 3.b contains the equilibrium wage offer mapping from θ to w when $m = 7.5$. Note that for the case of $\alpha = .25$, the equilibrium wage function maps all values of $\theta \geq 7.5$ into a wage offer of $w = 7.5$. At least when the distribution G has bounded support, this demonstrates that the imposition of a minimum wage can result in a degenerate wage offer distribution at the minimum, as we see in Figure 3.c. In the case of $\alpha = .5$, the equilibrium wage distribution has a substantial mass point at 7.5, with a relatively "narrow" range of wages above it. When $\alpha = .75$ or 1 (Figures 3.e and 3.f), the minimum wage does not substantially affect the equilibrium wage distribution, which is not to say that the welfare effects of the imposition of such a minimum wage in these cases are inconsequential.

[3]This statement is predicated on θ being a continuously distributed random variable with unbounded support on R_+.

3 Welfare Effects of Minimum Wages

This section is divided into four parts. In the first we propose two different methods for assessing the welfare effects of the imposition of a minimum wage into a market without a minimum wage.[4] Throughout the analysis we shall confine our attention to the supply side of the market [i.e., unemployed searchers and employees]. While the profit levels of firms will be a function of m, our model does at least imply that firms earn non-negative profits for any m; in this sense, any choice of m is "feasible." We then go on to consider the employment effects of minimum wages in the context of our model, and follow this with a discussion of the manner in which the wage distribution shifts. We conclude with a discussion of some results concerning "optimal" minimum wages under the welfare criteria defined in the first subsection.

3.1 General Welfare Measures

In our stationary model of the labor market, it seems reasonable to define two different criteria for determining whether a homogenous group of labor market participants are "better off" after the imposition of a minimum wage then before it. These definitions are as follows:

Definition 3 (WC1) *A group of labor market participants searching and bargaining in an environment characterized by $\Psi \equiv (\rho\ \lambda\ G\ \alpha\ \eta)'$ have higher welfare after the imposition of minimum wage m if the ex ante value of the labor market career is greater after the imposition of m.*

Since all individuals enter the labor market in the nonemployment state, the value of nonemployment is the *ex ante* value of any group member's labor market career. In this sense this welfare measure is very much a steady state concept. It is clear that under this definition searching for "optimal" minimum wage rates is also a rather simple exercise, conceptually at least, because welfare is characterized by a scalar when the population is homogeneous.

Another conceptual welfare experiment, in some sense more "realistic," examines the effects of the imposition of a minimum wage on the welfare of members of a homogeneous group which has already entered the labor market. For simplicity, consider the case in which there currently does not exist a minimum wage. At the time the minimum wage is imposed, some group members will be nonemployed while others will be employed at match values greater than or equal to ρV_n. How will the imposition of the minimum wage affect the welfare of all members of the group at the point in time when it is imposed? This is the question addressed by our second criterion.

Definition 4 (WC2) *A group of labor market participants currently in the market enjoy a welfare gain from the imposition of a minimum wage m if the value of their labor market state is no less after the imposition of m than it was before for all participants and is strictly greater for some.*

Note that without a minimum wage, at any point in time some proportion of group members will be nonemployed, which as a state has value V_n, while the remainder will be employed at match values greater than ρV_n; the value of the labor market state for these individuals can be written as $V_e(w(\theta, V_n))$. Under WC2, minimum wages will only be said to have an unambiguously beneficial effect on welfare if the value of being in the labor market increases for some individuals and decreases

[4] It is straightforward to amend the analysis to assess welfare changes when a market with a minimum wage changes it to a different level. This topic is implicitly treated when we discuss the selection of "optimal" minimum wages below.

for none *given their labor market status at the time of the imposition of the minimum wage*. In this sense, this criterion essentially insists on Pareto improvement and thus is more stringent than WC1.

We now turn to stating the necessary and sufficient conditions for minimum wages to be welfare improving under our two alternative definitions.

Proposition 5 *A minimum wage m is welfare-improving under WC1 if and only if $V_n < V_n(m)$.*

Proof: Since all group members begin their labor market careers in the nonemployment state, the value of this state is the *ex ante* expected value of the labor market career.¥

This result is basically definitional. As we shall see below, one especially interesting aspect of this model is that minimum wages, for some environments Ψ, can be welfare increasing under WC1. It is also possible that "binding" minimum wages can be welfare improving under WC2 given that the following more stringent condition is met.

Proposition 6 *A minimum wage m is welfare-improving under WC2 if and only if*

$$V_n(m) \geq V_n + \frac{\alpha(m - \rho V_n)}{\rho + \eta}. \tag{8}$$

After the imposition of the minimum wage some individuals employed at acceptable matches will become unemployed. In particular, all individuals who are employed at matches in the set $S_1 \equiv [\rho V_n(\Psi), m)$ will become unemployed under the minimum wage m, while individuals with match values in the set $S_2 \equiv [m, \infty)$ will remain employed. Since m must be binding for the minimum wage to result in a welfare gain, we must have $m > V_n(\Psi)$ so that $V_n'(\Psi, m) > V_n(\Psi)$ under [8]. Thus all individuals who are not employed when the minimum wage is imposed are strictly better off after it is. Consider the individuals with match values in the set S_2. The wage received by these agents after the minimum wage is imposed is

$$\begin{aligned} \max[m, \alpha\theta + (1-\alpha)\rho V_n'(\Psi, m)] \quad &> \quad \alpha\theta + (1-\alpha)\rho V_n(\Psi) \\ &\quad\;\; \theta \in S_2. \end{aligned}$$

Thus all individuals with matches in S_2 are strictly better off after the imposition of the minimum wage. Individuals with matches in the set S_1 lose their jobs as a result of the imposition of the minimum wage. Since the value of pre-minimum wage employment is increasing in θ, the highest employment value is associated with a match of m and is equal to the right hand side of [8]. Thus if the value of nonemployment after the minimum wage is no less than the right hand side of [8] all individuals are at least as well of after the minimum wage as before it, and some are better off. If the inequality in [8] is reversed, at least some individuals are worse off after imposition of the minimum wage.¥

The condition for minimum wages to be welfare-improving under WC1 is a necessary but not sufficient condition for them to be welfare-improving under WC2. If the value of nonemployment increases after the minimum wage, all nonemployed searchers and employed agents with match values in the set S_2 are better off. However, some individuals with job matches in the set S_1, in particular those with matches toward the "high end" of this interval, will suffer a welfare reduction. Condition [8] ensures that no one with match values in this set suffers a welfare loss.

It is of some interest to consider situations under which the minimum wage can be welfare-decreasing. Under WC1 the minimum wage is welfare-decreasing when the value of search decreases.

With “bad” choices of the minimum wage it is not difficult to encounter such a situation, even for labor market environments Ψ where one can demonstrate that there exist some minimum wages which would increase the value of search.

Can a minimum wage make all group members worse off in the sense of WC2? It is clear that for this to be the case the value of search must decrease. In this case, we know that all group members who are unemployed at the time of the imposition of the minimum wage will be worse off, as will be all individuals who lose their jobs [i.e., individuals employed at matches in S_1]. Moreover, all individuals with matches in the set S_2 who have match values greater than $\hat{\theta}(m, V_n'(m, \Psi))$ will earn lower wages when the value of search declines. The individual with the largest potential gain would be the one with a match value exactly equal to m, who will be paid m after the imposition of the wage and who would have had the smallest pre-minimum wage among the set of individuals who remain employed. For this individual to be worse off requires that

$$V_n'(m) < V_n + \frac{1-\alpha}{\eta}(\rho V_n - m).$$

Thus this is the necessary and sufficient condition for all individuals to be worse off under WC2 after the imposition of a binding minimum wage m.

It is interesting to note that certain pairs of (Ψ, m) will be welfare increasing under WC1 or WC2, others will be decreasing under both, and some will be increasing under WC1 but not WC2. Some of these possibilities are illustrated in Figure 4. We consider the impact of the imposition of a minimum wage of \$7.50 in our example labor market for two cases, $\alpha = .25$ and $\alpha = .75$. We assume that all other market parameters are the same, and set the values $\rho = .01$, $b = -1$, $\lambda = .5$, and $\eta = .02$ (recall that the matching distribution is taken to be uniform on the interval [0,10]). Figures 4.a and 4.b contain the equilibrium wage functions before and after the minimum wage of \$7.50 is imposed. As was noted when discussing Figure 3, for the case of $\alpha = .25$ the relatively high minimum wage results in an equilibrium wage function that has all its mass concentrated at the minimum wage. For the case of $\alpha = .75$, about 17 percent of wage observations are at the minimum wage, with the remainder being continuously (and uniformly) distributed on the interval (7.5,9.1].

The proportionate change in the value of each θ state for the two cases is plotted in Figures 4.c and 4.d. In the case in which participants have a low level of bargaining power there is a marked increase in the labor market value of each draw $\theta \in [0, 10]$,with the welfare increase being greater than 20 percent on average. Conversely, when bargaining power is high (.75), each state θ has a lower value under the minimum wage than without it. The proportionate welfare losses in this case are not large in comparison with the gains that were registered for the case of $\alpha = .25$.

We conclude this example by comparing properties of the wage distributions before and after the minimum wage change for the two cases. As we see in Figure 4.e, after the imposition of a minimum wage of \$7.50, the wage distribution for the case of $\alpha = .25$ becomes degenerate at \$7.50. In addition, we note that the new wage offer distribution first order stochastically dominates the original one (with no minimum wage).[5] For the case in which $\alpha = .75$, the new wage distribution does have a “spike” at \$7.50, as was previously noted, but is continuously distributed above it. The wage distribution observed after the minimum wage change does not first order stochastically dominate the original distribution in this case. These observations are in line with the analytic

[5] Distribution F_2 first order stochastically dominates distribution F_1 if $F_1(x) \geq F_2(x)$ for all x and $F_1(x) > F_2(x)$ for some x.

results we obtain in Section 3.3 below.

3.2 Employment Effects

What of standard employment effects in this model? Clearly, if members of a homogeneous group are labor market participants prior to the imposition of the minimum wage, then after the imposition of a binding minimum wage the employment rate will decrease. In particular, the change in the steady state probability of employment will be

$$\frac{\lambda\widetilde{G}(m;\mu)}{\eta+\lambda\widetilde{G}(m;\mu)}-\frac{\lambda\widetilde{G}(\rho V_n;\mu)}{\eta+\lambda\widetilde{G}(\rho V_n;\mu)}<0$$

since $m > \rho V_n$. Thus in this case the model is not consistent with some recent findings of small positive employment effects of minimum wage increases, but is consistent with the large number of studies of minimum wage effects on employment using cross-sectional, panel, and time series data. One of the major points of our analysis is that the imposition of a minimum wage has the potential to be welfare-increasing, even though employment effects are always negative, at least for groups participating in the market both before and after the change.

3.3 Effects on the Wage Distribution

Within this model the effects of imposing a minimum wage on the accepted wage distribution are complex. The minimum observed wage will always increase in response to the imposition of a binding minimum wage or when a binding minimum wage is increased.. If the minimum wage is welfare increasing in the sense of increasing the value of search, then the new observed wage distribution will stochastically dominate the original one.[6] However, if the minimum wage reduces the value of search, no "positive" results can be obtained. We analyze these effects in detail in Flinn (2000); what follows in this subsection is a brief summary of some of the more important results obtained in that paper. The results refer to comparisons between wage offer distributions observed under two different minimum wages, m and m', where $m < m'$. In order to apply them to the situation in which the labor market moves from not having a minimum wage to imposing a binding one, simply set $m = 0$. Proofs of all propositions can be found in Flinn (2000).

Proposition 7 *Let the wage distribution under the minimum wage m' be given by $F_2(w)$ and that under m be given by $F_1(w)$, for $m < m'$. Then F_2 first order stochastically dominates F_1 iff*

$$\frac{\tilde{G}(m)}{\tilde{G}(m')}\geq\frac{\tilde{G}(\frac{z-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(\frac{z-(1-\alpha)\rho V_n(m')}{\alpha})}\ \textit{for all}\ z\geq m'. \tag{9}$$

This result can be used to derive the following sufficient condition for the "new" wage distribution to first order stochastically dominate the original one.

Corollary 8 *If $V_n(m') > V_n(m)$, then F_2 first order stochastically dominates F_1.*

Unfortunately, however, this condition is only sufficient. If are goal is to infer the direction of the welfare change (i.e., $sgn(V_n(m')-V_n(m))$ from the relationship between the two observed wage distributions, the most useful result available to us is the following.

[6] The proof of this claim is not available in this draft.

Corollary 9 *If F_2 does not first order stochastically dominate F_1 then $\tilde{V}_n(m') < \tilde{V}_n(m)$.*

In reality, F_2 may not first order stochastically dominate F_1 due to a variety of features of the two distribution functions. Our model specification places restrictions on the way in which FOSD can fail. In particular, if F_2 does not FOSD F_1, there must exist some x^* such that $F_2(x) \leq F_1(x)$ for all $x \leq x^*$ and $F_2(x) > F_1(x)$ for all $x > x^*$. That is, the c.d.f.s should intersect either never (in which case F_2 first order stochastically dominates F_1) or once and only once (in the case of failure of FOSD).[7] Multiple crossings of the c.d.f.s could be produced only by sampling variability or model misspecification.

We now turn our attention to characterizing and analyzing the phenomenon commonly referred to as "spillover." Consider a wage rate w such that $w > m' > m$. Then under either value of the minimum wage the density of accepted wages at w exists.[8] Consider the ratio of the density at w under m' and m, which is in essence a likelihood ratio. Then we define

$$\begin{aligned} L(w; m, m') &= \frac{\frac{\alpha^{-1} g(\tilde{\theta}(w, V_n(m')))}{\tilde{G}(m')}}{\frac{\alpha^{-1} g(\tilde{\theta}(w, V_n(m)))}{\tilde{G}(m)}} \\ &= \frac{\tilde{G}(m) \times g(\tilde{\theta}(w, V_n(m')))}{\tilde{G}(m') \times g(\tilde{\theta}(w, V_n(m)))}. \end{aligned}$$

The ratio $\tilde{G}(m)/\tilde{G}(m')$ in $L(w; m, m')$ we might refer to as the *truncation effect* of the minimum wage change. Since $\tilde{G}(m) > \tilde{G}(m')$, this effect is always greater than 1 and is independent of the value of w, $w > m'$; we will write it as $T(m, m')$. We view this effect on the ratio of wage densities at w as rather mechanical and uninteresting. Instead, what we will refer to as the *spillover effect* is the term

$$S(w; m, m') = \frac{g(\tilde{\theta}(w, V_n(m')))}{g(\tilde{\theta}(w, V_n(m)))}.$$

In this way we have constructed a decomposition of the likelihood ratio of the wage density at w before and after the wage change, which is

$$L(w; m, m') = T(m, m') S(w; m, m').$$

It will be convenient to work with an additive decomposition of the log likelihood ratio, or

$$\ln L(w; m, m') = \ln T(m, m') + \ln S(w; m, m').$$

Using the logarithmic decomposition, it is clear that the truncation effect shifts $\ln L$ by the uniform amount $\ln T(m, m')$. Furthermore we know that $\ln T(m, m') > 0$ for any two binding minimum wages $m' > m$. Our main interest is in the manner in which the *shape* of the wage density above m' changes with a change in the minimum wage. We will assess this by looking at the manner in which $\ln L(w; m, m')$ varies in w. That is, we are interested in

$$\frac{\partial \ln L(w; m, m')}{\partial w} = \frac{\partial \ln S(w; m, m')}{\partial w}.$$

We work with the logarithm of the likelihood ratio so that the truncation effect can be ignored.

[7] The single-crossing property is not sufficient to produce second order stochastic dominance.

[8] For purposes of this discussion we assume that the matching distribution has unbounded support, which implies that the wage distribution will share this characteristic as well whenever $\alpha > 0$.

Definition 10 *The quantity $\partial \ln S(w; m, m')/\partial w$ is called the* shape perturbation *at w associated with the minimum wage increase from m to m'. We denote this quantity by $SP(w; m, m')$.*

In general, minimum wage changes result in changes in the shape of the density above the new minimum wage. It is interesting to consider when this would not be the case. We begin with one readily checkable sufficient condition for the absence of shape perturbations.

Proposition 11 *Assume that $G(\theta)$ is continuously differentiable on its support $Q \subseteq R_+$, where Q is a connected set. Then there is no spillover when moving from minimum wage m to m' if and only if at least one of the following:*

1. $V_n(m') = V_n(m)$
2. $g(\theta) = \tau^{-1} \exp(\beta\theta)$ *for all $\theta \in Q$, where $\tau = \tau(\beta, Q) = \int_Q \exp(\beta x)\, dx < \infty$.*

The types of distributions which satisfy condition 2 are relatively familiar ones. When $\beta = 0$, then we have $g(x) = \tau^{-1}$ on Q, which implies that Q is a finite interval $[\underline{\theta},\overline{\theta}]$, with $0 \leq \theta < \theta < \infty$, so that $\tau = [\overline{\theta} - \underline{\theta}]$. In this case G corresponds to a uniform distribution. When $\beta < 0$, we have the case of a negative exponential distribution. When $Q = R_+$, then $\tau = |\beta|^{-1}$. When Q is a bounded subset of R_+, then g is a truncated negative exponential density. Finally, when $\beta > 0$, for integrability of the density we require that Q be a bounded subset of R_+. In other analytic respects this case closely resembles that of $\beta < 0$.

A result which follows immediately from condition 2 of the above proposition is the following

Proposition 12 *If condition 2 of Proposition 11 is satisfied, then*

$$f(w; m) = \delta(m, m') f(w; m'), \; \forall w > m' > m,$$

where

$$\delta(m, m') = \frac{\tilde{G}(m')}{\tilde{G}(m)} \frac{g(-\frac{1-\alpha}{\alpha} \rho \tilde{V}_n(m))}{g(-\frac{1-\alpha}{\alpha} \rho \tilde{V}_n(m'))}.$$

In Flinn (2000) these results and others are used to characterize conditions under which changes in certain properties of observed wage distributions can be used to infer the direction of the welfare effects resulting from the minimum wage change. While some researchers have implicitly claimed that the direction of welfare effects can be determined solely by the impact of minimum wage changes on employment rates, it is clear that even within a simple wealth-maximizing search framework changes in wage distributions must be considered as well. This is an especially challenging task since the wage distribution is a function and not a scalar, unlike the employment rate. The simplicity of the bargaining environment that generates the equilibrium wage distribution has allowed us to generate a few useful comparative statics results that can be used for this purpose.

3.4 Optimal Minimum Wages

The model yields several interesting results regarding the welfare effects of minimum wages. When we speak of welfare in this section, we will be referring only to the value of search in the nonemployment state, V_n.

In terms of maximizing the value of search, the "optimal" minimum wage depends especially critically on the value of the bargaining parameter α. Let us fix all parameters in the model with the exception of the α. Now α is contained in the unit interval, and let us consider the implications of imposing a binding minimum wages at the two extreme values α can assume. If $\alpha = 0$, then the firm has all the bargaining power and the worker will earn zero rents in equilibrium. If the opportunity cost of labor market participation is 0, say, all firms will pay individuals with sufficiently high match values a fixed wage of w_0, where w_0 is set so as to equate the value of search to 0. Any match value $\theta \geq w_0$ will result in a job offer of w_0, with all the rents from the match going to the firm. Now if a binding minimum wage is imposed, fewer jobs will be observed in equilibrium [the probability that a contact results in a job offer will decline in proportion to $(\tilde{G}(w_0) - \tilde{G}(m))$]. However, since labor market participation yielded no rents to the worker, the decrease in employment probabilities has no welfare effect. For those individuals who encounter a match at least as large as m, the compensation rate has increased from w_0 to m. Therefore the expected value of labor market participation is greater than 0, and the welfare of labor market participants has increased.

Now consider the case when the employee has all the bargaining power, or $\alpha = 1$. In this case our bargaining model becomes the standard partial-partial equilibrium search model. In this case the matching distribution G and the wage offer distribution are identical. The reservation match value in this case is that value which maximizes the value of the searcher's problem. If a binding minimum wage greater than ρV_n is imposed, this forces the searcher to use a suboptimal rule and thus results in a decrease in the value of search. Thus in this case, the imposition of a binding minimum wage is always welfare reducing for labor market participants.

In the case in which $\alpha = 0$ we know that a minimum wage always improves welfare, no matter what the values of the other parameters that characterize the environment, while in the case in which $\alpha = 1$ the opposite is true. Unfortunately, the properties of the function $V_n(m)$ are not straightforward to derive for all other intermediate cases, especially when working with an unspecified function G. In the empirical work conducted below, we make the assumption that G is log normal. In all of the cases we have investigated using this functional form assumption, the function $V_n(m)$ has either been monotonically decreasing once the minimum wage becomes binding or has exhibited single-peakedness so that a single, optimal minimum wage exits. We present some illustrative evidence below.

When the labor market consists of a number of different types of agents, the restriction that all groups operate under a common minimum wage clearly raises social choice questions. When all value functions for all types of agents are single-peaked (as functions of m), then there is a social relation generated by pairwise majority voting which is complete and transitive [see, e.g., Mas-Colell et al, 1995, Section 21.D]. When all functions are not single-peaked, as we have found in the empirical analysis below, this is not the case. We provide further discussion of this issue at the end of the empirical section.

4 Econometric Model

The model as formulated relies heavily on stationarity assumptions, which enable us to use what are otherwise essentially cross-sectional data sets to estimate dynamic models. The CPS point-in-time sample only contains information on the length of time individuals currently unemployed have been actively searching for a job and the hourly wage rate and/or gross weekly earnings and usual hours worked per week for those reporting that they are currently employed. No information is available on the length of time an employed individual has worked for their current employer. A substantial amount of demographic information is available for each individual as well, information which is largely ignored in the present empirical analysis.

In terms of sample heterogeneity, we will consider there to be a finite number of population types, $k = 1, ..., K$. In terms of the empirical application below, we will restrict $K = 2$. For much of the analysis we will place no restriction on the parameters characterizing the labor market of the different types, so that the model is characterized by sets of parameters Ψ_k, $k = 1, ..., K$. With no restrictions on the parameters across types, each can be viewed as inhabiting their own labor market, and only the data pertaining to individuals of type k will be informative for the parameter vector Ψ_k. In light of this, we begin our discussion of identification conditions with the homogeneous (sub)population case.

Flinn and Heckman (1982) showed that when only accepted wage information is available (in addition to information on the time spent in the various labor market states) consistent estimation of the population matching distribution function G requires that a parametric assumption be made.[9] They also demonstrated empirically that a number of inferences may be quite sensitive to the particular parametric family of distributions that are selected. We shall show that this is even more true when bargaining is explicitly considered.

In most empirical implementations of behavioral search models an allowance is made for measurement error in the wage data (but not in the duration measures). Allowing for measurement error may be indicated when the sample contains very low wage observations, which without measurement error would imply a very low reservation wage. When allowing for on-the-job search, which we do not do here, measurement error is required for the model of expected wealth maximization to be consistent with direct job-to-job transitions in which the destination job offers a lower wage than the job which was left.

In the presence of a binding minimum wage, it is not possible to add "classical" measurement error to the model. If the reported wage distribution is assumed to be given by the convolution of the distribution of actual wages and some continuously distributed i.i.d. random variable, then even though the true wage distribution is of the mixed continuous-discrete type, the observed wage distribution will be absolutely continuous. Since this is inconsistent with even the most casual inspection of the data (see Figure 1, for example), classical measurement error cannot be introduced. While one could consider allowing for "contamination" in the data, i.e., that some unknown proportion τ of wages are measured with classical error while the remainder are not, the estimation of τ would seem not to be possible without further arbitrary assumptions. As a result, we assume throughout that all observations are measured exactly. While we have already noted that a large number of observations are "heaped" at focal points, our hope is that the regular pattern of the heaping process will not bias our estimates to any great degree.

The likelihood function based on the CPS data is constructed as follows (to avoid notational clutter we will drop the subpopulation index k).Each individual observation can be characterized by a the pair of observations (t_i, w_i), $i = 1, ..., N$. The variable t_i is the length of the on-going spell of unemployed search, which is positive if the sample member is unemployed at the time of the survey and is otherwise equal to 0. If the individual is employed, the hourly wage w_i is recorded. If the individual is paid an hourly wage, w_i corresponds to the hourly wage rate they report. If

[9]They considered the case of an exogenously-determined wage offer distribution, but all of the arguments for the necessity of parametric assumptions in that problem carry over to the endogenous wage distribution case considered here.

the individual is not paid on an hourly basis, the hourly rate is imputed by dividing the gross weekly wage by the usual hours of work per week. This imputation procedure is standard, but is particularly problematic in this application because of the likely undercount of individuals paid exactly the minimum wage that will result. On the positive side, the vast majority of our sample members report an hourly wage rate so that the imputation procedure has to be utilized for less than 20 percent of the sample in any given year.

There are essentially three components of the likelihood function, one for the unemployed, one for the employed paid the minimum wage, and one for those paid more than the minimum. Let us first consider the contribution of a sample member who is unemployed with an on-going unemployment of spell of length t. Given being in the unemployed state, the likelihood of finding someone in an on-going spell of length t in the steady state is given by $r_u(t) = \tilde{F}_u(t)/E(t)$, where $\tilde{F}_u$ is the population survivor function of unemployment durations and $E(t)$ is the population mean duration of unemployment spells. When the population unemployment spell duration distribution is of the negative exponential form, it is well-known that $r_u(t)$ is equal to the population density. Thus, given unemployment, the density associated with an on-going spell length of t is

$$f_u(t|u) = \lambda\tilde{G}(m)\exp(-\lambda\tilde{G}(m)t),$$

where the acceptance match value of m presumes the existence of a binding minimum wage (in the absence of one m is replaced by ρV_n in the above expression). Using standard ergodic results, the steady state probability of unemployment is given by

$$p(u) = \frac{\eta}{\eta + \lambda\tilde{G}(m)},$$

so that the joint likelihood of (t, u) is given by

$$f(t, u) = \frac{\eta\lambda\tilde{G}(m)\exp(-\lambda\tilde{G}(m)t)}{\eta + \lambda\tilde{G}(m)}.$$

Next we consider the likelihood contribution of an individual who is paid the minimum wage m. We will assume that whenever two or more population members are paid the minimum wage m that the minimum wage is binding for their population type. Assuming that it is, conditional on employment the likelihood of being paid m is given by

$$p(w = m|e) = \frac{\tilde{G}(m) - \tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(m)}.$$

Therefore the likelihood of observing $(w = m, e)$ is

$$p(w = m, e) = \frac{\lambda[\tilde{G}(m) - \tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})]}{\eta + \lambda\tilde{G}(m)}.$$

The contribution of individuals paid more than the minimum wage is determined as follows. To be paid a wage $w > m$, it must be the case that the match value exceeds $(m - (1-\alpha)\rho V_n(m))/\alpha$. Thus the likelihood of being a paid a wage w given employment and $w > m$ is

$$f(w|w > m, e) = \frac{\frac{1}{\alpha}g(\frac{w-(1-\alpha)\rho V_n(m)}{\alpha})}{\tilde{G}(\frac{m-(1-\alpha)\rho V_n(m)}{\alpha})}.$$

Furthermore, the probability that a sample member is paid a wage greater than m given that she is employed is

$$p(w > m|e) = \frac{\tilde{G}(\frac{m-(1-\alpha)\rho V_{\mathsf{n}}(m)}{\alpha})}{\tilde{G}(m)},$$

so that the likelihood contribution for such an individual is given by

$$\begin{aligned} f(w, w > m, e) &= \frac{\frac{1}{\alpha} g(\frac{w-(1-\alpha)\rho V_{\mathsf{n}}(m)}{\alpha})}{\tilde{G}(\frac{m-(1-\alpha)\rho V_{\mathsf{n}}(m)}{\alpha})} \frac{\tilde{G}(\frac{m-(1-\alpha)\rho V_{\mathsf{n}}(m)}{\alpha})}{\tilde{G}(m)} \frac{\lambda \tilde{G}(m)}{\eta + \lambda \tilde{G}(m)} \\ &= \frac{\frac{\lambda}{\alpha} g(\frac{w-(1-\alpha)\rho V_{\mathsf{n}}(m)}{\alpha})}{\eta + \lambda \tilde{G}(m)}. \end{aligned}$$

The log likelihood for the parameters describing the labor market environment of the (sub) population can be written as

$$\begin{aligned} L = {} & N[\ln(\lambda) - \ln(\eta + \lambda \tilde{G}(m))] + N_u[\ln(\eta) + \tilde{G}(m)] \\ & - \lambda \tilde{G}(m) \sum_i t_i + N_m \ln(\tilde{G}(m) - \tilde{G}(\frac{m - (1-\alpha)x}{\alpha_k})) \\ & - (1 - N_u - N_m) \ln(\alpha) + \sum_{\{i:\ w_{\mathsf{i}} > m\}} \ln(g(\frac{w_i - (1-\alpha)x}{\alpha})), \end{aligned} \tag{10}$$

where N is the number of individuals in the (sub) population, N_u is the number of unemployed, N_m is the number paid the minimum wage, and $x \equiv \rho V_n(m)$ is the "implicit" reservation wage for this group of individuals. Recall that G will be assumed to belong to a parametric family, so that it can be characterized in terms of a finite-dimensional parameter vector ζ. Then the parameters that appear directly (or indirectly) in the log likelihood are λ, η, ζ, α, and x. The value x is not a parameter per se, but rather is a scalar that is determined endogenously and is a function of all the parameters of the model For purposes of estimation and discussion, however, we shall treat it as a parameter of the model.

4.1 Identification Issues

Flinn and Heckman (1982) extensively investigated identification conditions in this model for the special case in which $\alpha = 1$ and with no binding minimum wage. While the data over which the likelihood was defined was slightly different in the case they examined, many of their results carry over directly to the present case. In particular, they found that the parameter λ is identified, while the elements of the parameter vector ζ(that characterizes G) are identified given that the distribution is "recoverable." The two-parameter log normal family we assume below belongs to this class of distributions. They also show that η is identified from information on the length of employment spells. We do not have this type of information in the CPS data, but under steady state assumptions we can identify this parameter from the proportion of the sample who are unemployed.

In terms of identifying the other parameters of the search model (after implicitly assuming that $\alpha = 1$), they noted that the log likelihood depends only on the parameters b and ρ through the reservation wage x. Their strategy for identification and estimation was to first consistently estimate x from the smallest wage observed in the sample (i.e., the first order statistic). Given this (strongly) consistent estimator of x, they concentrated the log likelihood and estimated the parameters λ, η, and ζ. Given these consistent estimates, they noted that the functional relationship $\hat{x} = \rho V_n(\hat{\lambda}, \hat{\eta}.\hat{\zeta}, b, \rho)$ could be used to find a consistent estimator of either ρ (conditional on an assumed value of b) or b (conditional on an assumed value of ρ), but not both simultaneously.

Our estimation problem is considerably more challenging due to our desire to estimate α and is also differentiated from the Flinn-Heckman analysis by our incorporation of the minimum wage constraint. We will begin by discussing issues connected with the estimation of α both in the absence and presence of a binding minimum wage. We focus a great deal of attention on the identification of α both because it is such a crucial parameter in assessing the welfare impacts of minimum wages and also because there exists no detailed consideration of the problem of which we are aware.

In a homogenous (sub) population, we can see from inspection of [10] that all of the sample information regarding the parameter α will come from those individuals who are employed, so we begin by looking at the relationship between the empirical wage distribution and the underlying model parameters. In order to provide a unified treatment of the identification problem, define the value

$$\tilde{w} = a + b\theta,$$

where under the Nash bargaining structure

$$\begin{aligned} a &= (1-\alpha)\rho V_n(m) \\ b &= \alpha. \end{aligned}$$

For notational simplicity let $x \equiv \rho V_n(m)$ as above, and let the reservation value in the absence of a binding minimum wage be captured by $\rho V_n(0)$. We will think of the $\tilde{w}$ as latent variables, and will term the distribution of $\tilde{w}$ the "potential" wage distribution. Let Q be the c.d.f. of potential wages, and let q be its associated p.d.f. Then we have

$$\begin{aligned} Q(w) &= G\left(\frac{w-a}{b}\right) \\ q(w) &= \frac{1}{b} g\left(\frac{w-a}{b}\right). \end{aligned}$$

The linearity of the mapping between θ and $\tilde{w}$ causes difficulties when we attempt to disentangle the bargaining power parameter and the value of search from the parameters that characterize the matching distribution. This problem is essentially insurmountable when the matching distribution belongs to a location-scale family.

Definition 13 *Let P be a family of distributions with cumulative distribution functions of the form $P(\mu + \sigma x)$, $\sigma > 0$. Then assuming that P is everywhere differentiable, its corresponding density is*

$$p(x) = \frac{1}{\sigma} l(y),$$

where $y = (x-\mu)/\sigma$ and l is not a function of μ or σ. Then P is a location-scale invariant family.

Some well-known location-scale invariant families of distributions include the normal, the negative exponential, and the uniform. For example, a negative exponential density can be written as

$$p(x; \mu, \sigma) = \frac{1}{\sigma} \exp(-y) \chi[c \leq x]. \tag{11}$$

Definition 14 *Let r_i be a 1-1 transformation from X to X, $i = 1, 2$. A new transformation, termed the composition of r_1 and r_2, is given by*

$$\tilde{r} = r_2 \circ r_1,$$

and is determined by the applying the transformation r_2 after the transformation r_1 has been applied.

Definition 15 *A class of transformations Υ is closed under composition if $r_1 \in \Upsilon$, $r_2 \in \Upsilon$ implies that $r_2 \circ r_1 \in \Upsilon$.*

Finally consider the following formal definitions of location and scale parameters.

Definition 16 *A parametric distribution G possesses a scale parameter if and only if the distribution of x/ω_i is independent of ω_i for some $i \in \{1, ..., \#(\zeta)\}$.*

Definition 17 *A parametric distribution L possesses a location parameter if and only if the distribution of $x - \omega_i$ is independent of ω_i for some $i \in \{1, ..., \#(\zeta)\}$.*

We now use these formal definitions to consider the identification problem under the two possible scenarios.

4.2 No Binding Minimum Wage

We will first consider identification issues in the situation in which no binding minimum wage is present in the labor market. In such a case, the decision to accept a job is determined by the relationship between a match value, or equivalently, a wage offer, and some fixed critical value x. As has been discussed at length above, the critical value x is a function of all of the parameters that characterize the economic environment.

Under the assumption that there is no measurement error in wages, an estimator of the critical value x can be obtained using the order statistic

$$\hat{x} = \min_{\{i:\ w_i > 0\}} \{w_i\}$$

Using well-known properties of order statistic estimators, Flinn and Heckman show that $\hat{x}$ is a strongly consistent estimator of x with convergence rate equal to the size of the employed sample (N_e in this case).

The accepted wage p.d.f. is given by

$$\begin{aligned} f(\tilde{w}|\tilde{w} > x; a, b, \zeta) &= \frac{f(\tilde{w}; a, b, \zeta)}{\tilde{F}(x; a, b, \zeta)},\ w > x \\ &= \frac{\frac{1}{b} g\left(\frac{w-a}{b}; \zeta\right)}{\tilde{G}\left(\frac{\hat{x}-a}{b}; \zeta\right)},\ w > x. \end{aligned}$$

Using our order statistic estimator of x, $\hat{x}$, we can form the concentrated log likelihood associated with the accepted wage distribution as

$$L_c(a, b, \zeta; \hat{x}) = \ln \prod_{i=1}^{N_e} \frac{\frac{1}{b} g\left(\frac{w_i - a}{b}; \zeta\right)}{\tilde{G}\left(\frac{\hat{x}-a}{b}; \zeta\right)}.$$

Our first result concerns the possibility of identifying $a, b,$and ζ when G is a location-scale family.

Proposition 18 *Assume that G is known up to location and scale parameters γ_1 and γ_2, respectively. Then no parameters are identified solely from L_c.*

Proof: Since G is a location-scale family, the random variable θ can be written as $\theta = \gamma_1 + \gamma_2 y = r_1(y)$, where y has the known p.d.f. $g_0(y)$. The mapping of θ into the potential (and observed in this case) wage w is given by $w = a + b\theta = r_2(\theta)$. The class of linear transformations is closed under composition, so that $r_2 \circ r_1$ is also linear, with

$$w = r_3(y) = \gamma_3 + \gamma_4 y,$$

where

$$\begin{aligned} \gamma_3 &= a + b\gamma_1 \\ \gamma_4 &= b\gamma_2. \end{aligned} \tag{12}$$

The concentrated log likelihood can be written as

$$L_c = -N_e \ln(\gamma_4) + \sum_i \ln g_0(\frac{w_i - \gamma_3}{\gamma_4}) - N_e \ln \tilde{G}_0(\frac{\hat{x} - \gamma_3}{\gamma_4}).$$

Under standard regularity conditions on the c.d.f. G_0, consistent maximum likelihood estimates exist for γ_3 and γ_4. From inspection of [12] we can see that apparently we have to solve for four unknowns with two pieces of information, $\hat{\gamma}_3$ and $\hat{\gamma}_4$. The situation is not quite as bad as this, since we can rewrite [12] as

$$\begin{aligned} \hat{\gamma}_3 &= (1-\alpha)\hat{x} + \alpha\gamma_1 \\ \hat{\gamma}_4 &= \alpha\gamma_2, \end{aligned} \tag{13}$$

after having substituting consistent estimators for unknown parameter values whenever possible. We have reduced our task to determining three unknowns from two pieces of information. This is still insufficient to enable us to define consistent estimators for all elements of $(\alpha, \gamma_1, \gamma_2)$ without further assumptions.¥

There are of many assumptions, or "normalizations," that we might consider imposing to break the identification impasse. One obvious, but perhaps not very palatable, solution is to consider matching distributions that do not have scale parameters, at least not unknown ones. We have seen that the bargaining power parameter α acts as a rescaling of the match value. If the match value distribution does not have an unknown scale parameter itself, then bargaining power can, in general, be identified from the scale of the observed (i.e., truncated) wage distribution. The following proposition shows that this approach is sufficient to identify α.

Proposition 19 *The bargainning power parameter α is identified from L_c if the matching distribution does not possess an unknown scale parameter.*

Proof: If the distribution G does not possess a scale parameter, than there is no element of the parameter vector ζ, ζ_j, for which the distribution of θ/ζ_j is independent of ζ_j. In terms of the accepted wage distribution, the absence of a scale parameter in G implies that there is no ζ_j such that the distribution of $\frac{w-a}{\zeta_j b}$ is independent of ζ_j. Then the scale of the untruncated distribution conveys independent information regarding b ($\equiv \alpha$).

In the case of the accepted (i.e., truncated) wage distribution, the concentrated *likelihood* is

$$\prod_i \frac{\frac{1}{b}g(\frac{w_i-a}{b};\zeta)}{\tilde{G}(\frac{\hat{x}-a}{b};\zeta)}.$$

If the survivor function $\tilde{G}$ is multiplicatively separable in the sense that $\tilde{G}(xy) = \tau Q(x)Q(y)$, then we note that we can write $\tilde{G}(\frac{1}{b}(w_i - a);\zeta) = \tau Q(\frac{1}{b};\zeta)Q(w_i - a;\zeta)$. It follows that $g(\frac{w_i-a}{b};\zeta) = -\tau Q(\frac{1}{b};\zeta)Q'(w_i - a;\zeta)$, and the concentrated likelihood becomes

$$\begin{aligned} & \prod_i \frac{-\tau Q(\frac{1}{b};\zeta)Q'(w_i - a;\zeta)}{\tau Q(\frac{1}{b};\zeta)Q(\hat{x} - a;\zeta)} \\ = & \prod_i \frac{-Q'(w_i - a;\zeta)}{Q(\hat{x} - a;\zeta)}, \end{aligned} \tag{14}$$

which is not a function of b $(= \alpha)$. The only survivor function that possesses the multiplicative separability property is that of the Pareto. The Pareto survivor function can be written as $\tilde{G}(x;c,d) = c^d x^{-d}$, so that $\tilde{G}(xy) = \tau c^d x^{-d} y^{-d}$, with $Q(x) \equiv x^{-d}$. In this case the concentrated likelihood is not a function of either b or the (joint) location-scale parameter of the Pareto, c. However, since [14] yields a consistent estimator of a, we have assumed knowledge of the location-scale parameter c, and since $a = (1-\alpha)x + \alpha c$, consistent estimates of a and x along with knowledge of c allows us to obtain a consistent estimator of α.¥

The results of this section point to the extreme difficulty of identifying the bargaining power parameter, and demonstrate formally the nature of the "scale" problem. The only solution is the unappealing one of utilizing a matching distribution with known scale, an apparently arbitrary device. We will now investigate the extent to which the presence of a binding minimum wage exacerbates or alleviates the problem of identifying α.

4.3 Binding Minimum Wage

From the standpoint of estimation, one of the first things to note is that the presence of a binding minimum wage "masks" the value of the "implicit" reservation wage. By definition, the minimum value of wages observed in the sample is the binding minimum wage m, and by the structure of the model we know that $x < m$. Therefore, concentrating the log likelihood function on a consistent (order statistic) estimator of x is no longer feasible. The parameter x must be simultaneously estimated with all other parameters, if possible.

Aside from this difference, the case of the binding minimum wage is clearly distinguished by the mass point in the observed wage distribution at m. The conditional log likelihood for this case can be written as

$$L_c(a,b,\zeta) = \ln\left[\left(\frac{\tilde{G}(m;\zeta) - \tilde{G}(\frac{m-a}{b};\zeta)}{\tilde{G}(m;\zeta)}\right)^{N_m} \prod_{\{i:\, w_i>m\}} \frac{\frac{1}{b}g\left(\frac{w_i-a}{b};\zeta\right)}{\tilde{G}\left(m;\zeta\right)}\right],$$

where N_m is the number of employed individuals who are paid exactly the minimum wage. In considering identification issues in this section, we will continue to focus on the case in which location and scale parameters are possibly present in the matching distribution G. It will also help in our discussion if we distinguish between the information contained in the wage distribution conditional on being paid more than m and the event of being paid more than m (conditional on employment).

Let γ_1 and γ_2 denote location and scale parameters in the distribution G (if the distribution has such parameters) and let the remaining parameters of ζ be denoted ζ'. Consider the distribution of wages conditional on $w_i > m$. Since the probability of being paid more than m is given by $\tilde{G}(\frac{m-a}{b};\zeta)/\tilde{G}(m;\zeta)$, the log likelihood for these data is of the form

$$\begin{aligned} L_{w>m} &= \ln \prod_{\{i:\, w_i>m\}} \frac{\frac{1}{b} g\left(\frac{w_i-a}{b};\zeta\right)}{\tilde{G}\left(\frac{m-a}{b};\zeta\right)} \\ &= \ln \prod_{\{i:\, w_i>m\}} \frac{\frac{1}{\gamma_4} g\left(\frac{w_i-\gamma_3}{\gamma_4};\zeta'\right)}{\tilde{G}\left(\frac{m-\gamma_3}{\gamma_4};\zeta'\right)}. \end{aligned}$$

Under standard regularity conditions on the distribution G, consistent estimators of γ_3 and γ_4 (as well as the remaining parameters ζ') can be obtained from this log likelihood $L_{w>m}$.

Now consider the "concentrated" log likelihood that can be formed from the event of being paid the minimum wage. This has the form

$$L_m = N_m \ln\left(1 - \frac{\tilde{G}(\frac{m-\hat{\gamma}_3}{\hat{\gamma}_4},\hat{\zeta}')}{\tilde{G}(\frac{m-\gamma_1}{\gamma_2},\hat{\zeta}')}\right) + (N_e - N_m)\ln\left(\frac{\tilde{G}(\frac{m-\hat{\gamma}_3}{\hat{\gamma}_4},\hat{\zeta}')}{\tilde{G}(\frac{m-\gamma_1}{\gamma_2},\hat{\zeta}')}\right).$$

Thus this concentrated log likelihood contains the two unknown parameters, γ_1 and γ_2. Since it is based on essentially one piece of information, N_m/N_e, two parameters cannot be uniquely determined.

In terms of location and scale parameters, we have

$$\begin{aligned} & \gamma_1 \\ & \gamma_2 \\ \gamma_3 &= (1-\alpha)x + \alpha\gamma_1 \\ \gamma_4 &= \alpha\gamma_2 \end{aligned}$$

If the matching distribution contains both location and scale parameters, they cannot be identified. Without knowledge of γ_1 and γ_2, neither α or x can be determined from consistent estimates of γ_3 and γ_4. Thus if the matching distribution contains separate location and scale parameters, α cannot be uniquely determined within a homogenous population.

If the matching distribution does not possess a scale parameter, so that no element of ζ plays the role of γ_2, then the scale of the distribution of wages greater than the minimum identifies α. Information from L_m serves to determine γ_1 uniquely, since there is one piece of information and one unknown parameter to determine. With consistent estimators of γ_3, γ_1, and a available, a consistent estimator of x can be obtained.

Similarly, if the matching distribution contains no location parameter, then no parameter in ζ plays the role of γ_1. Since consistent estimators of γ_3 and γ_4 are available from $L_{w>m}$, L_m can be used to obtain a consistent estimator of γ_2. Using the consistent estimators γ_4 and γ_2, we can obtain a consistent estimator of α. Since the location parameter $\gamma_3 = (1-\alpha)x$ when γ_1 is not present, the consistent estimators of α and γ_3 can be used to determine a consistent estimator of x.

In practice, of course, we will estimate the log likelihood L_c directly, and this log likelihood combines the informative elements contained in both $L_{w>m}$ and L_m.We have merely attempted to demonstrate, in a fairly intuitive fashion, the conditions for identification of α in the presence of a minimum wage (within a homogeneous population). The results are essentially the same as in the case in which no binding minimum wage was present. If the matching distribution has unknown scale and location parameters, the bargaining power parameter cannot be identified in either case. For the case in which a minimum wage is present, the probability mass at m provides an "extra" piece of information to use, but this is counterbalanced by the presence of an additional unknown parameter, x.

We knew from the analysis of Flinn and Heckman that identification of search models requires that parametric assumptions be made regarding the matching distribution G (and that the assumed distribution be "recoverable). The present analysis demonstrates that to identify the bargaining power parameter, additional restrictions must be placed on the parametric family. If we choose a distribution without a scale parameter, such as the χ^2 distribution, then identification of α follows immediately whether or not a binding minimum wage is present. But the assumption of a known scale parameter for G is very strong indeed. If we are to estimate α in a homogeneous (sub) population, a more tenable route would be to consider distributions without a location parameter. The discussion indicates that this restriction is sufficient, in principle, to identify α. In conducting the empirical analysis discussed below we have assumed that θ follows a two-parameter log normal distribution, with

$$G(\theta; \gamma_2, \sigma) = \Phi(\ln(\frac{\theta}{\gamma_2})/\sigma), \tag{15}$$

where γ_2 is the scale parameter of the distribution and σ is the shape parameter. Thus the two-parameter log normal, besides being a popular choice for fitting compensation distributions, has the important characteristic of not having a location parameter. According to our analysis, whether or not a minimum wage is binding, the log normality assumption should be sufficient to enable identification of $(\gamma_2, \sigma, \alpha, x)$ from the observed wage distribution.

While we have demonstrated that identification is possible under this particular distributional assumption, as a practical matter in samples of the size we encounter in practice identification may be tenuous. To get some idea of the importance of this problem, we performed a very small Monte Carlo experiment. We set the minimum wage (m) to the value 3.5, the implicit reservation wage x to 3.0, the bargaining power parameter α to .4, the scale parameter γ_2 to e (the natural number), and σ to 1. In each of 1000 samples, we drew 2000 acceptable draws from G (i.e., draws of θ that exceeded m), generated the implied wage rate, and obtained maximum likelihood estimates of all of the parameters. In all of the estimation problems the algorithm converged, though in about one-fourth of the cases the Hessian could not be inverted and in about five percent of cases the estimate of α was near the boundary of the parameter space at 0. In these "unstable" cases, the estimates of the parameters characterizing the matching distribution assume values that imply that the probability mass of the matching distribution is very much to the right of the distribution used to generate the data. Most noticeably, in these cases estimates of γ_2 are typically three or four times greater than the true value of e.

The sampling distributions of the estimates obtained from the experiment are displayed in Figure 5. Panel 5.a contains the sampling distribution of estimates of the implicit reservation wage. Prior to plotting the values, we have deleted 13 values of $\hat{x}$ that were less than 1.5; these values typically arose in the "unstable" cases discussed above. We see that in general the value of x is precisely determined from the accepted wage data. The same is generally true with respect to the estimation of α, the results of which appear in Figure 5.b. Since the estimates of α are constrained to lie in the unit interval, the plot contains all of the 1000 estimates, even the "unstable" ones. The problem cases correspond to the cluster of estimates of α near 0. Aside from these results, most of the other estimates fall within a relatively small neighborhood of .4, the actual value of α.

Results are less encouraging with respect to estimation of the scale parameter of the matching distribution, γ_2. Recall that this parameter is only distinguishable from α due to the fact that no location parameter is present in the two-parameter log normal distribution. The histogram contains the estimates of γ_2 after 80 values greater than 5 were discarded. These very high values of $\hat{\gamma}_2$ were all associated with runs in which $\hat{\alpha}$ was approximately equal to 0. The histogram plotted in Figure 5.c reveals a systematic underestimation of the true value of γ_2 (that is approximately 2.71) across the "well-behaved" samples.

The distribution of estimates of σ is presented in Figure 5.d. For purposes of graphical presentation, 14 values of $\hat{\sigma}$ that were greater than 1.5 were omitted for purposes of presentation. It is interesting to note that even in the runs in which estimation was "unstable," the estimate of σ was virtually always reasonable. The histogram reveals that estimates of σ are tightly clustered around the true value of 1. This perhaps should not be too surprising since, as a shape parameter, the severe problems associated with disentangling γ_2 and α are largely avoided.

This example has provided some evidence that even when we have access to relatively large samples of wage draws from distributions that theoretically should enable identification of the four parameters we are attempting to identify, in practice precise estimation of these parameters may not be possible. This will especially be true when the real world problems of measurement error and the misspecification of the true underlying matching distribution are added to the mix. In the empirical work below, we will impose additional "exclusion" restrictions to increase the accuracy of model estimates under assumptions linking the labor markets of observationally-distinguishable sample members. Needless to say, such increases in accuracy come at the expense of imposing further assumptions on the model, assumptions which will be untestable from a practical perspective. We briefly summarize the types of restrictions we will impose in the following subsection.

4.4 Additional Identification Devices

In empirical work utilizing a Nash-bargaining structure, it is common to assume symmetric bargaining, i.e., $\alpha = .5$ (see, e.g., Del Boca and Flinn (1994) and Eckstein and Wolpin (1993)). Such a "normalization" is imposed both because the difficulty of identifying the parameter α has been apparent for some time, and also due to modeling considerations. In terms of the second point, recall that in our formulation of the behavioral model we assumed that both workers and firms shared a common effective discount rate. If we were to ignore the fact that the searcher has an "outside option" when bargaining with the firm, i.e., the value of continued search, bargaining over the match surplus could be placed in the framework of a Rubinstein alternating-offers game. It is well known that in the case of equal discount rates, the solution is equivalent to the symmetric Nash bargaining solution.

As the first step in the empirical analysis we simply restrict $\alpha = .5$ for every subpopulation considered. Besides conforming with common practice, this case serves as something of a benchmark. We were also interested in determining whether positive "optimal" minimum wages were ever indicated given this normalization. Roughly speaking, we know that the likelihood that binding minimum wages will improve the welfare of labor market participants is decreasing α. The empirical question we address is whether the restriction that $\alpha = .5$ effectively rules out a welfare-improvement role for minimum wages. Using the samples below, this indeed appears to be the case.

Next, we considered the estimation of the model under some common forms of exclusion restrictions. Throughout the empirical analysis we allow their to be two types of agents, which in our application are whites and nonwhites. We will gain some additional information regarding labor market parameters by making some cross-market restrictions. In particular, we will assume that the matching distributions of whites and nonwhites are identical, while allowing all other (estimable) parameters to differ. At the same time we will impose the restriction that the bargaining power parameter for whites, α_w, is equal to .5. Free parameters will then include all estimable arrival rate parameters, the values of search for whites and nonwhites, two parameters characterizing the common log normal θ distribution, and the bargaining power parameter for nonwhites, α_o. The estimates obtained using these restrictions seem relatively plausible and are used in discussing welfare implications at the end of the next section.

We have also attempted estimation of a subset of parameters by imposing an intertemporal restriction. A different nominal minimum wage was in place in March of 1996, 1997, and 1998. We allow for "approximately" neutral price changes across the years, and assume that changes in nominal minimum wages are unanticipated. Thus we view the wage distributions in each of these three years as being determined by a time invariant "real" match value distribution G, a time invariant bargaining power parameter α, and a year-specific implicit reservation wage, x_t. At the imposition of a new minimum wage, all existing contracts are rebargained instantaneously, so that the current wage distribution reflects the influence of the contemporaneous x_t and the existing "real" minimum wage m_t. We only use wage distribution information when imposing these restrictions, since even if agents continue to use stationary rules the contemporaneous probability of employment and durations of unemployed search will reflect past values of the state variables.

Finally, we estimate all parameters of the model after assuming values for ρ and b. In the discussion of identification to this point, we have treated the implicit reservation wage as a free parameter to be estimated. This is justified if either or both of the primitive parameters ρ and b are unknown. When both are known, the functional equation defining the implicit reservation wage yields another piece of information that can be used in breaking the location-scale identification problem. We illustrate this method of identification of α, and other estimable parameters, for a few different assumptions on (ρ, b). Since we have no firm convictions regarding the appropriate values for these parameters, and since the implied estimates of α from this method are so low, we view these results as mainly illustrative.

5 Empirical Results

The data used contain information on 16 to 24 year olds [inclusive] from the March 1992-1998 person files of the Current Population Survey. We focused attention on this age group since minimum wage workers are disproportionately young; our feeling was that if minimum wage effects on welfare were determined to be small for this group of participants, they are likely to be even more insignificant for older individuals. Most of the specifications have been estimated on each of the seven survey years. Our main intention was to investigate the stability of model estimates over time. General economic conditions were quite favorable over this period and inflation was relatively low. In performing the empirical work all wages (including nominal minimum wages) were deflated using the national CPI-U series, with the price index normalized to 1 for 1992. Unemployment durations are measured in months.

Before beginning with a discussion of the detailed results, let us consider the ability of the model to fit the data at our disposal. Figure 6 presents histograms of the actual wage and unemployment duration for the total sample in March 1996. Using estimated model parameters for the full model for that year, we generated a sample of draws for the unemployment spell length and wage distributions. The actual data appears in the top two panels, and the simulated data appear in the bottom two. Aside from the pronounced heaping in the survey data, there seems to be (to us at least) a striking similarity between the wage and unemployment duration distributions. By the structure of the log likelihood, the probability at the mass point m is fit essentially perfectly, as is the proportion of the sample in the employment state. The quality of the fit is particularly striking given that we have aggregated over observable types. As is well known, even if the the hazard rate out of unemployment is constant for whites and nonwhites, if the hazard rate is different in the two subpopulations the marginal hazard rate will not be constant. Even though our estimates indicate that the unemployment hazards are different for the two groups, the marginal unemployment duration distribution displays no marked divergence from a negative exponential. Since most of our econometric specifications possess similar abilities to fit the data, we conclude that the estimated specifications are all broadly consistent with the data.

Table 1 contains descriptive statistics generated from the data. Note from the column on the far right of the table that CPS sample sizes were systematically reduced over the decade, though even in 1998 the aggregate sample contains 1750 individuals. Individuals were included in our analysis if they satisfied the age criterion and indicated that they were employed at the survey date or were actively looking for work. Since wage observations less than the minimum are not consistent with the model, employed individuals reporting a wage less than the nominal minimum wage for the year were excluded. In addition, individuals who reported a (real) hourly wage greater than 30 were excluded. Few individuals were excluded due to having high hourly wage rates, though about two or three percent of the sample in any year was excluded for reporting wages below the minimum. Reporting a wage less than the minimum was taken as an indication that the individual was not a member of the "regulated" labor market being modeled here.

For the total sample we see that the proportion employed, $P(e)$, was relatively constant over time. The proportion of employees paid the minimum wage, $P(w = m|e)$, consistently falls over the period. From March 1992 through March 1996 the nominal minimum wage was fixed at \$4.25, so even with moderate amounts of inflation the decline in the mass point at m is to be expected. In March 1997 and again in March 1998 the nominal minimum wage was raised by substantially more than the inflation rate. Despite this, the proportion paid the minimum wage did not increase. This might indicate that the primitive parameters, such as the matching distribution G, shifted over this period as well.

The average real wages of those paid above the minimum, $E(w|w > m)$, is decreasing for much of the period (as the real minimum wage is reduced), though this statistic increases in the years 1997 and 1998 when the nominal minimum wage was increased. The standard deviation of the distribution of wages above the minimum exhibits some slight decreases over the period as well. The average duration of unemployment spells, $E(t_u|u)$, is relatively stable over the period with the exception of 1995. The standard deviation of unemployment durations is also stable with the exception of 1994 and 1995. Under the model, the distribution of unemployment durations should be negative exponential, so that the mean duration should be equal to the standard deviation of the distribution. The patterns exhibited in these two columns are roughly consistent with this prediction.

The bottom two panels of Table 1 contain the same descriptive statistics computed for the subsamples of whites and nonwhites.[10] Roughly speaking, in any given year slightly more than two-thirds of the total sample are white. For the white subsample we can observe an increase in the employment probability over the period, while for the nonwhite subsample there is no clear pattern in this proportion. While employed whites had a much lower probability of being paid the minimum wage than did blacks in the early 1990s, no difference was evident by the last few years of the period. The same observation is largely valid regarding the mean of the wages above the minimum. While whites have somewhat greater wages in the first few years, the difference is negligible by the latter years of the sample. The rough correspondence between the wage distributions of whites and nonwhites may reflect the impact of business cycle factors operative in this period, and may also be due to differences in the degree of labor market attachment of these two groups. In the future we intend to expand the scope of our empirical work so that differences in school enrollment rates and hours of work can be included in the analysis.

The most marked difference in the descriptive statistics concerns the unemployment duration distribution. There are large decreases in the average length of an unemployment spell for whites over the period, though there is little discernible pattern in this statistic for nonwhites. In virtually every year, the average spell length for whites is at least 20 percent less than the corresponding nonwhite value. These differences will be reflected in the quite disparate estimates of λ we obtain in the white and nonwhite samples.

We begin our presentation of empirical results with those obtained from the maximization of L. Even though the model is identified (in theory) under the log normal assumption on G, attempts to estimate all parameters $(\lambda, \eta, \mu, \sigma, x, \alpha)$ were unsuccessful (in what follows we characterize the two-parameter log normal in terms of μ and σ, where μ is the mean of $\ln(\theta)$). To circumvent the numerical problems we experienced, we adopted the assumption of symmetric Nash bargaining and set $\alpha = .5$.The point estimates and standard errors (in parentheses) of the maximum likelihood estimates for the full sample are presented in Table 2.a. In terms of interpreting the parameter estimates, recall that durations are measured in months. For example, the point estimate of λ in 1992, .472, indicates that on average an offer is received every 2.12 months (1/.472). Similarly, the point estimate of η of .064 indicates that on average jobs last 15.625 months. Although our search model represents a highly stylized view of the labor market, neither figure seems implausible considering the nature of our sample.

The estimates of the parameters of μ and σ indicate that mean of the matching distribution (in 1992) is about 8.29 with a standard deviation of 4.31. Recall that these values are largely determined by the assumption that $\alpha = .5$. Finally, the implied reservation wage is estimated to be 3.551 in 1992. This “parameter” is estimated under the constraint that it be less than the prevailing minimum wage.

The estimates of the parameters of the model by and large are relatively stable over the seven years, with no clear trends in any of the parameter estimates. It is interesting to note that the

[10] The category of nonwhites predominately includes African-Americans and hispanics. While there is some empirical evidence that hispanics have labor market outcomes more similar to whites than to African-Americans, we have chosen our aggregation scheme primarily in response to sample size considerations.

estimates of the implicit reservation wage tend to increase noticeably in 1997 and 1998, years in which the nominal minimum wage was increased. We will see the same pattern in other estimates from different model specifications.

There is more instability in parameter estimates within the white and nonwhite samples as is to be expected because of the smaller sample sizes. Nonetheless, there are some discernible patterns in the sets of estimates. The estimates from the white sample, presented in Table 2.b, indicate that the rates of receiving offers are much higher in 1997 and 1998 than in the earlier years of the period. There is no clear pattern in estimates of the rate of match dissolution, η. Estimates of μ tend to be smaller in the last two years, though estimates of σ are slightly larger than for most of the other years. The change in the matching distribution parameter estimates is attributable, to some extent, to increases in the nominal minimum wage in these two years. Estimates of the implicit reservation wage provide some evidence that the value of unemployed search increased as a result of the minimum wage change.

Estimates of the full model for the nonwhite sample appear in Table 2.c. There is substantially more intertemporal variability in estimates from this sample. The estimates of rates of arrivals of job offers are substantially smaller than the estimates from the white sample for most years, though the biggest difference between the sets of estimates is with respect to η, the rate of dissolution of matches. These are estimated to be substantially higher for nonwhite labor market participants.[11] Estimates of the matching distribution parameters are not markedly different from what we observed for the white sample. As was the case for the aggregate and white samples, increases in the nominal minimum wage seem to have been associated with some improvement in the value of search.

In Tables 3 and 4 we present model estimates in which identification is "enhanced" by placing some restrictions on parameters in the white and nonwhite labor markets. In these specifications, we continue to normalize the bargaining power of whites at .5, but allow the bargaining power of nonwhites to be a free parameter, α_o. In all of the specifications, we restrict the matching distribution of whites and nonwhites to be the same.

In obtaining the estimates presented in Table 3, we only made use of wage data for the employed sample members at each survey date. There are five free parameters to estimate, the implicit reservation wages of whites and nonwhites, the two common matching distribution parameters, and the bargaining power parameter for nonwhites. We note that there is no strong indication that whites and nonwhites have different degrees of bargaining power, conditional on the restrictions that have been imposed. The lowest estimated bargaining power parameter for nonwhites is approximately .42, and the value .5 is almost always within one standard error of the estimate. There is also no significant difference between the values of the implicit reservation wage for whites and nonwhites. As before, we find that the values of search improved for both groups after the imposition of the new minimum wages in 1997 and 1998.

In Table 4 we report estimates from a specification that utilizes all sample information and allows all parameters in the two samples to be different with the exception of the matching distribution. In terms of the estimates of the bargaining power parameter α_o, the results are largely consistent with what we found in Table 3, as was to be expected. Once again the estimates of α_o vary between .42

[11] The reader should bear in mind that we have no direct information on the length of jobs. The estimate of η is obtained under the assumption that the probability of being unemployed at the time of the sample is given by the steady state probability of unemployment. In this sense, η is obtained solely via functional form assumptions, though the estimated values of η are not inconsistent with the average length of jobs observed in data sets with more complete information on labor market experiences, such as the National Longitudinal Survey of Youth - 1979.

and .50 over time. Though this difference seems small, as we will see at the end of this section the values of .42 and .50 have very different implications regarding the value of an "optimal" minimum wage.

In line with the results presented in Table 2, we find that the rate of arrival of job offers tends to be higher for white sample members, and that the rate of match separations is substantially higher for nonwhite sample members. We continue to find evidence of a substantial increase in the value of unemployed search after 1996. There is no systematic difference between the values of unemployed search for whites and nonwhites over time.

We now attempt to estimate a subset of model parameters by exploiting intertemporal variation in the real (and nominal) minimum wage. We have pooled wage information from the years 1996-1998 after having deflated the nominal minimum wage and the wages of all sample members using the CPI-U. We assume that the underlying match distribution as well as the bargaining power parameter are invariant over the period, in line with the stationarity required by our model. We can then identify all of the parameters determining the wage distribution, including the bargaining power parameter and three separate implicit reservation wages for each year. The model is estimated for the entire sample and then for whites and nonwhites separately.

The results appear in Table 5. In the first line of each panel we present results in which the bargaining power parameter has been fixed at .5. The second line contains estimates of the model when α is estimated as a free parameter. The most striking results in the table are probably the estimates of α, which are .20 or less in each panel. Taking these point estimates at face value, they imply extremely large "optimal" minimum wages to overcome the distinct bargaining disadvantage all workers share. However, the standard errors associated with estimates of α are extremely large in all cases, and in none could the null hypothesis that $\alpha = .5$ be rejected. The large standard errors result from the tenuousness of the identification of α. It appears that in order to increase the precision of estimation of α, additional years with different (real) minimum wage rates would have to be added. The cost of doing so is the improbability that primitive parameters will have remained constant for increasingly long periods of time.

Although the standard errors associated with the estimates of α are very large, the implicit reservation wages for the various years continue to be estimated precisely. We continue to find marked improvements in the value of search associated with the minimum wage changes. This is true in the white and nonwhite samples, and we see no large differences in the search values of these two groups.

We conclude the estimation phase with a consideration of an alternative method for identifying model parameters, most especially α. As was described in the previous section, using this method we begin by assuming values for (ρ, b) and no longer treat the implicit reservation wage x as a parameter. Given assumed values of (ρ, b) and guesses as to the values of $(\lambda, \eta, \mu, \sigma)$, a value of α is implied by an implicit reservation value x through the functional equation $x = x(\rho, b, \lambda, \eta, \mu, \sigma, \alpha)$. We substitute this implicit function into the log likelihood and recover all of the other primitive parameters conditional on our assumption concerning (ρ, b).

The results of some of our attempts to implement this procedure are contained in Table 6. We have arbitrarily selected data from the year 1996 to carry out this exercise. We report the results associated with four different sets of assumptions concerning (ρ, b). We found that in order for the model to be consistent with the data, very high discount rates and/or very low (i.e., negative) utility flows when unemployed were necessary. Given these side constraints on the choice of (ρ, b), virtually all estimates of α are extremely low, on the order of what was found in the previous table. The estimates of other parameters in the model are relatively insensitive to the choice of (ρ, b), which is to be expected since they only impact the implicit reservation wage and that is constrained to be less than the match acceptance value of m. These estimates illustrate that this estimation method is feasible to implement, but without particular values of (ρ, b) upon which to focus attention we would hesitate to use estimates from this method to conduct policy experiments.

While all of the specifications we have considered have their drawbacks, if forced to choose among them perhaps those presented in Table 4 might be preferable. We use the estimates from that specification from the year 1997 to illustrate some issues in the selection of "optimal" minimum wages. In performing this exercise we simply use the point estimates of the parameters that characterize the two labor markets. We must also solve for the value of search, so assumptions must be made concerning values of (ρ, b). In performing this exercise we have simply fixed the value of ρ at a monthly rate of .05/12. Using the estimates of all other parameters, we then solved for the value of b consistent with this assumption.

Figure 7 contains a plot of $V_n^r(m)$ for each group $r \in \{w, o\}$. Recall that in this specification the bargaining power of whites is set to .5. The figure illustrates that it is optimal for group w to have no binding minimum wage under this assumption. The value of search is of course flat until the minimum wage becomes binding. For the whites, at the point of becoming binding, the value of search is monotonically decreasing in m. Thus members of this group prefer a minimum wage of 0, or at least a minimum wage less than $\rho V_n^w(0)$, i.e., one which would not be binding.

The situation is different for nonwhites. Given their estimated parameters, and particularly the estimated $\hat{\alpha}_o = .421$, the value of unemployed search is single-peaked and reaches a maximum at the value \$6.29 an hour. If two separate minimum wages could be imposed, no social choice problem would result. However, given the institutional requirement of one minimum wage a nontrivial choice problem emerges. Whites are the largest group in this sample, and let us say that they are the majority of the voting population. Their preference is to have a nonbinding minimum wage. If we were to use a majority voting rule to determine the optimal minimum wage, whites would vote to have a nonbinding one. This wouldn't necessarily rule out a nonzero minimum wage, since they would be willing to vote for any m that didn't affect them. However, the value at which a minimum wage becomes binding for whites is estimated to be lower than it is for nonwhites, meaning that no binding minimum wages will be implemented for either group in this case. This finding is consistent with the political apathy regarding the minimum wage in the U.S. in recent years. Since most members of the labor market are not impacted by the minimum wage (in terms of their own labor market outcomes), or are adversely impacted by increases in the minimum wage, the argument for dramatic increases in m is left to relatively small groups who have little bargaining power vis-a-vis employers. This lack of bargaining power in the labor market may be associated with having little political influence as well.

We do not want to overemphasize the cost of the selection of a "nonoptimal" minimum wage to the nonwhite population, however. The implicit reservation value associated with no minimum wage for nonwhites is 3.847. At their optimal minimum wage of \$6.29 the value of the implicit reservation wage is 3.904, so that moving from no binding minimum wage to one of \$6.29 only increases the value of unemployed search by about 1.5 percent. While increases in the minimum wage may improve the value of unemployed search for at least some groups in the labor market, the results of our empirical comparative statics exercise indicate that such improvements may be of negligible size.

6 Conclusions

We have formulated a simple equilibrium model of wage determination which carries implications for the effects of a minimum wage on accepted wage distributions and unemployment spell lengths which are broadly in accord with the findings of other researchers who have worked with disaggregated data. The model has the capacity to determine the optimal level of the minimum wage, even across heterogenous labor market environments, under well-defined welfare criteria. Our initial experiences with the model indicate that it performs well empirically in terms of fitting marginal distributions of unemployment durations and accepted wage distributions. The most vexing problem is that policy implications hinge critically on assumptions made regarding the underlying matching distribution, and that key parameters are only tenuously identified given the types of data to which we typically have access.

While we are not yet in a position to seriously evaluate alternative minimum wage policies, the analysis performed here delivers an important cautionary message. The fact that minimum wage increases may result in reductions in employment levels does not imply that they decrease welfare for *any* population members. Conversely, the fact that minimum wage increases may result in new wage distributions that stochastically dominate the old ones does not imply that the welfare of all agents (or even any agent) has increased. Only by jointly considering the effect of minimum wages on wage distributions and employment rates can their welfare effects be determined.

Table 1
Descriptive Statistics by Race and Year

Year	$P(e)$	$P(w=m\|e)$	$E(w\|w>m)$	$SD(w\|w>m)$	$E(t_u\|u)$	$SD(t_u\|u)$	N
1992	.850	.103	7.060	2.941	2.790	3.274	2572
1993	.848	.111	6.885	2.884	3.034	3.867	2507
1994	.853	.091	6.597	2.696	3.167	4.070	2258
1995	.871	.056	6.437	2.588	3.634	4.682	2176
1996	.865	.066	6.534	2.676	3.004	3.585	1933
1997	.871	.067	6.610	2.662	2.958	3.275	1826
1998	.875	.067	6.807	2.596	2.728	3.381	1750
				Whites			
1992	.869	.093	7.163	2.929	2.740	3.266	1839
1993	.884	.097	6.998	3.007	2.903	3.876	1774
1994	.870	.081	6.594	2.635	2.903	4.010	1699
1995	.910	.051	6.440	2.593	2.715	3.178	1620
1996	.898	.062	6.520	2.554	2.677	3.531	1419
1997	.914	.067	6.730	2.810	2.464	2.805	1285
1998	.910	.068	6.807	2.604	2.416	3.547	1233
				Nonwhites			
1992	.800	.131	6.768	2.999	2.872	3.299	733
1993	.763	.150	6.564	2.454	3.190	3.863	733
1994	.801	.123	6.609	2.901	3.691	4.155	559
1995	.755	.076	6.427	2.572	4.614	5.728	556
1996	.776	.080	6.581	3.043	3.416	3.625	514
1997	.771	.067	6.274	2.162	3.400	3.599	541
1998	.791	.064	6.808	2.578	3.047	3.185	517

Table 2.a
Estimates of Stationary Search Model
$\alpha = .5$
All Labor Market Participants, 16-24

Year	*Parameters*					N	*ln L*
	$\rho V_n(m)$	λ	η	μ	σ		
1992	3.551	.472	.064	1.904	.650	2572	-6560.721
	(.065)	(.030)	(.005)	(.089)	(.038)		
1993	3.432	.454	.059	1.826	.682	2507	-6421.787
	(.065)	(.033)	(.004)	(.104)	(.042)		
1994	3.482	.432	.054	1.798	.660	2258	-5647.837
	(.053)	(.031)	(.004)	(.091)	(.040)		
1995	3.521	.336	.041	1.897	.588	2176	-5320.681
	(.046)	(.022)	(.004)	(.064)	(.034)		
1996	3.200	.373	.052	2.017	.552	1933	-4822.823
	(.075)	(.024)	(.005)	(.061)	(.005)		
1997	3.785	.517	.050	1.694	.692	1826	-4409.593
	(.040)	(.047)	(.005)	(.111)	(.046)		
1998	4.068	.540	.052	1.781	.631	1750	-4135.353
	(.042)	(.048)	(.005)	(.100)	(.045)		

Table 2.b
Estimates of Stationary Search Model
$\alpha = .5$
White Labor Market Participants, 16-24

Year	*Parameters*					N	*ln L*
	$\rho V_n(m)$	λ	η	μ	σ		
1992	3.529	.450	.055	1.994	.619	1839	-4690.081
	(.089)	(.033)	(.005)	(.095)	(.042)		
1993	3.517	.481	.045	1.819	.705	1774	-4474.583
	(.064)	(.041)	(.005)	(.113)	(.045)		
1994	3.515	.454	.051	1.839	.637	1699	-4167.410
	(.058)	(.037)	(.005)	(.094)	(.043)		
1995	3.563	.453	.036	1.891	.592	1620	-3761.271
	(.047)	(.041)	(.004)	(.071)	(.038)		
1996	3.194	.412	.043	2.041	.533	1419	-3419.079
	(.094)	(.035)	(.005)	(.068)	(.038)		
1997	3.779	.629	.038	1.687	.718	1285	-3015.397
	(.047)	(.075)	(.005)	(.134)	(.053)		
1998	4.074	.632	.041	1.745	.648	1233	-2809.226
	(.048)	(.078)	(.006)	(.128)	(.055)		

Table 2.c
Estimates of Stationary Search Model
$\boldsymbol{\alpha = .5}$
Nonwhite Labor Market Participants, 16-24

Year			*Parameters*			N	*ln L*
	$\rho V_n(m)$	λ	η	μ	σ		
1992	3.618	.625	.087	1.555	.755	733	-1869.641
	(.096)	(.114)	(.011)	(.256)	(.092)		
1993	3.160	.400	.098	1.880	.592	733	-1908.3551
	(.113)	(.040)	(.011)	(.073)	(.066)		
1994	3.398	.444	.067	1.602	.753	559	-1465.601
	(.118)	(.080)	(.010)	(.263)	(.097)		
1995	3.366	.260	.070	1.920	.574	556	-1508.149
	(.125)	(.023)	(.009)	(.140)	(.074)		
1996	3.216	.353	.084	1.924	.619	514	-1377.189
	(.134)	(.039)	(.012)	(.154)	(.076)		
1997	3.784	.410	.087	1.759	.592	541	-1354.360
	(.080)	(.054)	(.012)	(.167)	(.084)		
1998	4.051	.443	.087	1.864	.586	517	-1302.291
	(.087)	(.057)	(.013)	(.165)	(.081)		

Table 3
Estimates of Stationary Search Model
α_w= .5
All Employees, 16-24

Parameter	*1992*	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*
α_o	.422	.425	.495	.497	.499	.421	.500
	(.070)	(.073)	(.097)	(.074)	(.085)	(.081)	(.125)
$\rho V_n^o(m)$	3.553	3.390	3.291	3.399	3.070	3.867	4.088
	(.160)	(.162)	(.206)	(.157)	(.241)	(.087)	(.128)
$\rho V_n^w(m)$	3.588	3.480	3.540	3.558	3.240	3.762	4.060
	(.104)	(.101)	(.081)	(.071)	(.119)	(.074)	(.074)
μ	1.939	1.865	1.801	1.897	2.018	1.741	1.781
	(.068)	(.079)	(.077)	(.052)	(.047)	(.095)	(.091)
σ	.652	.681	.659	.588	.552	.690	.631
	(.037)	(.040)	(.040)	(.030)	(.029)	(.047)	(.044)
N	2185	2127	1927	1895	1673	1591	1531
ln L	-2040.604	-1973.870	-1733.092	-1666.867	-1260.349	-1396.290	-1318.837

Table 4
Estimates of Stationary Search Model
α_w= .5
All Labor Market Participants, 16-24

	1992	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*
Parameter							
α_o	.422	.425	.500*	.497	.500*	.421	.500*
	(.056)	(.053)		(.011)		(.052)	
$\rho V_n^o(m)$	3.553	3.390	3.277	3.400	3.067	3.867	4.088
	(.126)	(.119)	(.098)	(.090)	(.140)	(.057)	(.068)
$\rho V_n^w(m)$	3.589	3.481	3.541	3.558	3.240	3.762	4.060
	(.073)	(.066)	(.052)	(.047)	(.079)	(.048)	(.047)
λ_o	.449	.421	.370	.265	.328	.433	.483
	(.041)	(.037)	(.039)	(.024)	(.031)	(.047)	(.053)
λ_w	.471	.463	.470	.450	.419	.598	.609
	(.035)	(.038)	(.038)	(.039)	(.036)	(.068)	(.065)
η_o	.087	.098	.067	.070	.084	.087	.087
	(.011)	(.011)	(.010)	(.009)	(.012)	(.012)	(.012)
η_w	.055	.045	.051	.036	.043	.038	.041
	(.005)	(.005)	(.005)	(.004)	(.005)	(.005)	(.006)
μ	1.939	1.865	1.800	1.897	2.017	1.741	1.781
	(.088)	(.097)	(.091)	(.064)	(.062)	(.110)	(.099)
σ	.652	.681	.659	.588	.552	.690	.631
	(.038)	(.039)	(.040)	(.034)	(.034)	(.045)	(.044)
N	2572	2507	2258	2176	1933	1826	1750
ln L	-6562.792	-6385.179		-5269.489		-4371.061	

Table 5
Model Estimates with Merged Samples
Employed Individuals, 1996-1998

				Parameter				
Sample	α	μ	σ	$\rho V_n(m_{96})$	$\rho V_n(m_{97})$	$\rho V_n(m_{98})$	L	N
All	.500	1.986	.607	3.770	4.128	4.484	-4458.559	4793
	-	(.037)	(.021)	(.073)	(.077)	(.072)		
All	.159	2.847	.732	3.869	4.183	4.543	-4456.086	4793
	(.276)	(.543)	(.052)	(.075)	(.104)	(.099)		
Whites	.500	1.994	.610	3.795	4.120	4.465	-3342.850	3569
	-	(.043)	(.024)	(.082)	(.091)	(.087)		
Whites	.202	2.661	.717	3.900	4.202	4.548	-3341.783	3569
	(.348)	(.565)	(.069)	(.066)	(.091)	(.087)		
Nonwhites	.500	1.966	.593	3.689	4.148	4.531	-1114.511	1224
	-	(.075)	(.041)	(.160)	(.146)	(.128)		
Nonwhites	.072	3.541	.750	3.691	4.042	4.442	-1112.817	1224
	(.893)	(3.099)	(.071)	(.622)	(.707)	(.737)		

Table 6
Full Model Estimates Conditional on b and ρ
All Sample Members, 1996

Parameter	*Specification* **1**	**2**	**3**	**4**
b	-2	0	-10	-10
ρ	.1	.2	.01	.05
λ	.340	.344	.336	.380
	(.020)	(.019)	(.021)	(.021)
η	.052	.053	.051	.050
	(.005)	(.005)	(.005)	(.004)
μ	3.082	3.027	3.090	2.617
	(.142)	(.156)	(.106)	(.175)
σ	.655	.648	.657	.566
	(.022)	(.019)	(.020)	(.018)
$\rho V_n(m)$	3.551	3.545	3.557	3.358
	(.139)	(.130)	(.101)	(.138)
α	.149	.160	.147	.289
L	-4985.942	-4986.347	-4985.884	-4999.337

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Figure 1.a
Hourly Wages
Employed Individuals, 1996

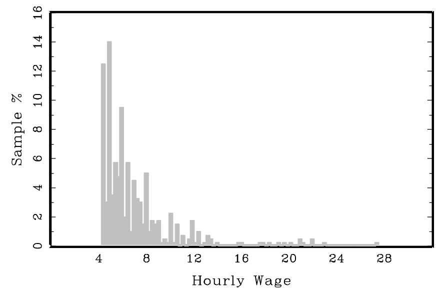

Figure 1.b
On-Going Search Duration
Unemployed Individuals, 1996

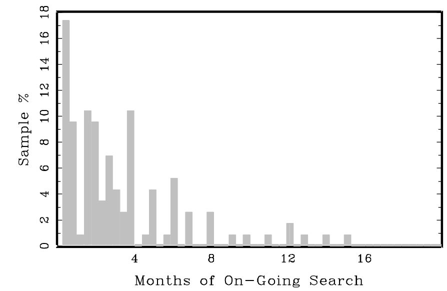

Figure 1.c
Hourly Wages
Employed Individuals, 1997

Figure 1.d
On-Going Search Duration
Unemployed Individuals, 1997

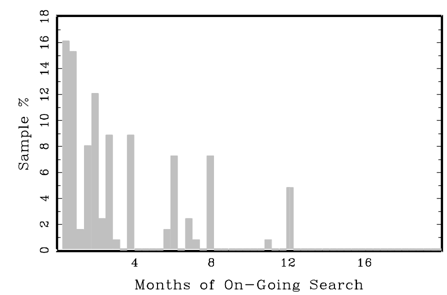

Figure 1.e
Hourly Wages
Employed Individuals, 1998

Figure 1.f
On-Going Search Duration
Unemployed Individuals, 1998

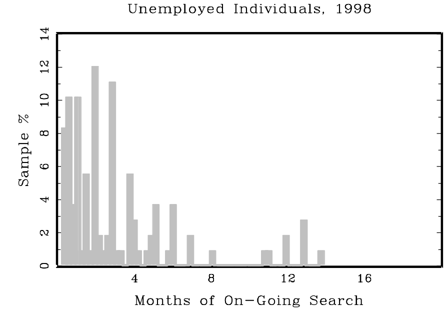

Figure 2.a
Match Value Density

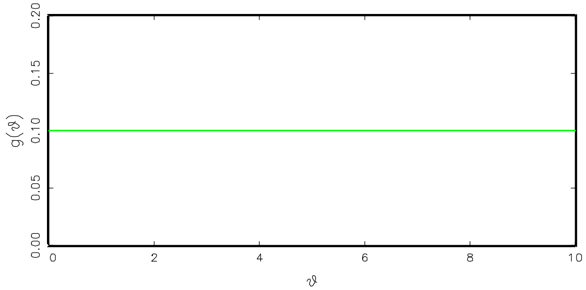

Figure 2.b
Equilibrium Wage Function

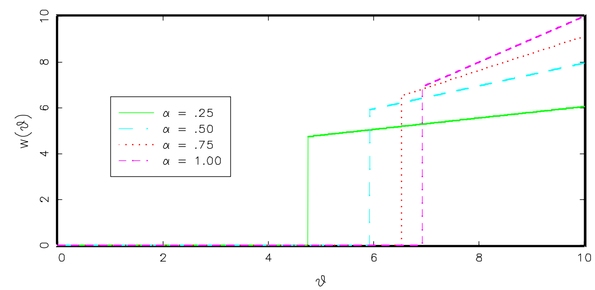

Figure 2.c
Wage Density
α=.25

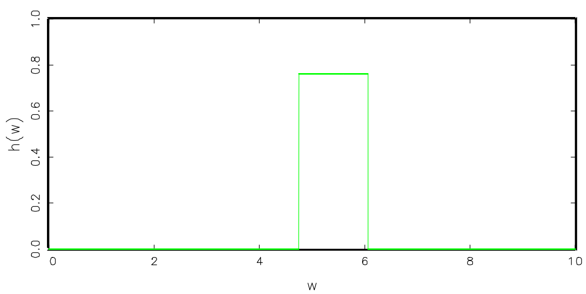

Figure 2.d
Wage Density
α=.5

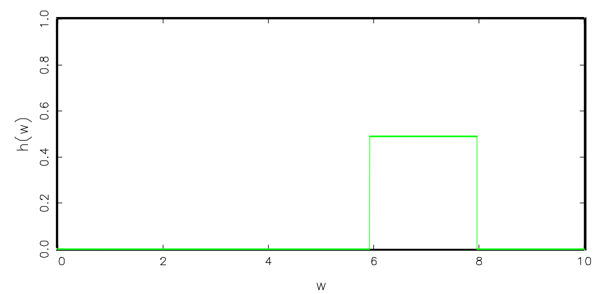

Figure 2.e
Wage Density
α=.75

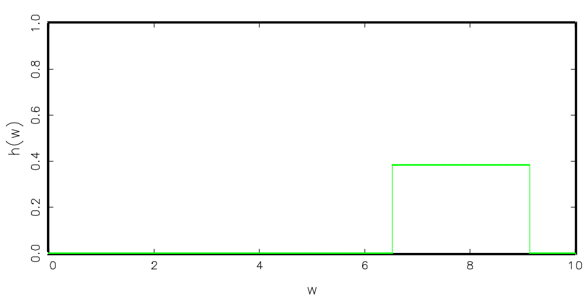

Figure 2.f
Wage Density
α=1

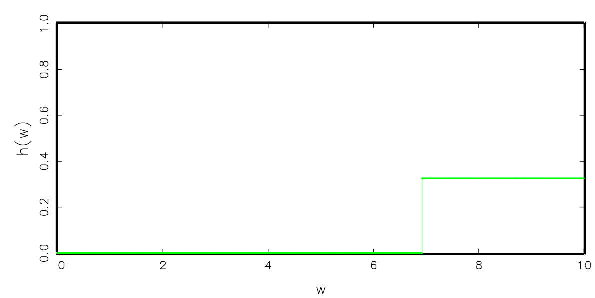

Figure 3.a
Match Value C.D.F.

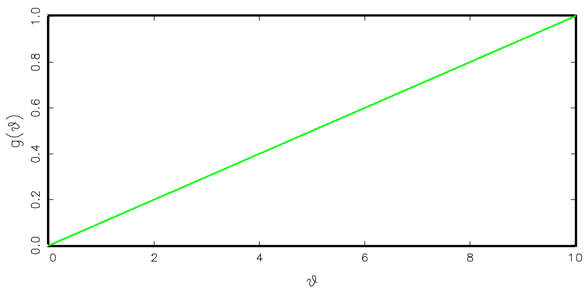

Figure 3.b
Equilibrium Wage Functions

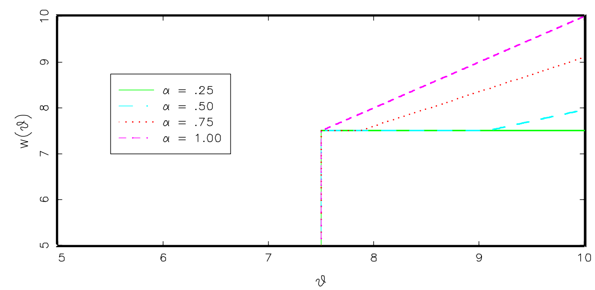

Figure 3.c
Wage C.D.F.
α=.25

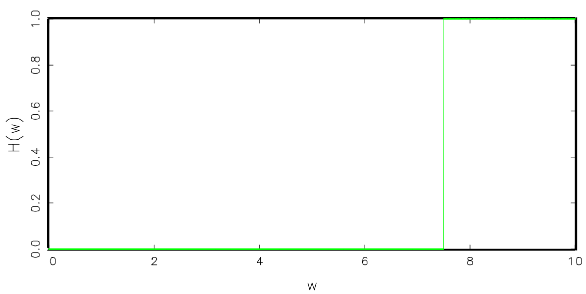

Figure 3.d
Wage C.D.F.
α=.5

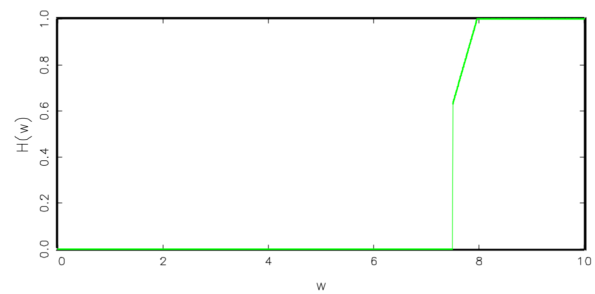

Figure 3.e
Wage C.D.F.
α=.75

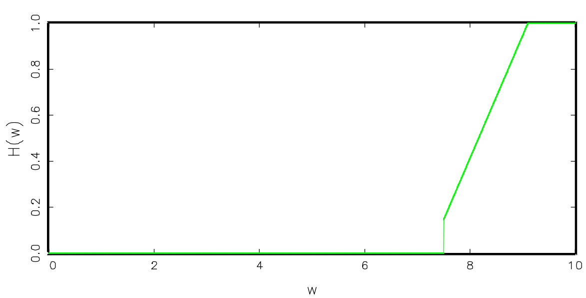

Figure 3.f
Wage C.D.F.
α=1

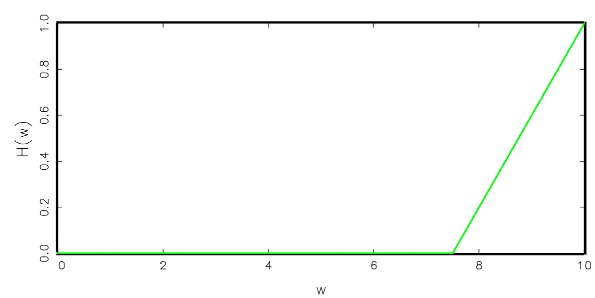

Figure 4.a
Wage Function
α=.25, m=7.50

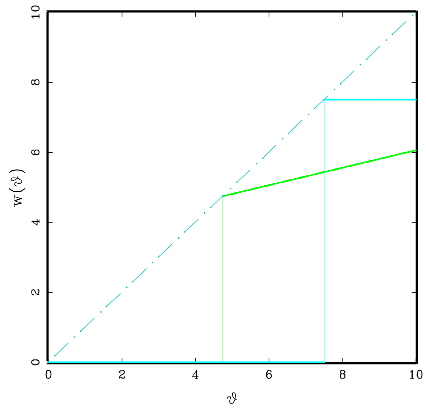

Figure 4.b
Wage Function
α=.75, m=7.50

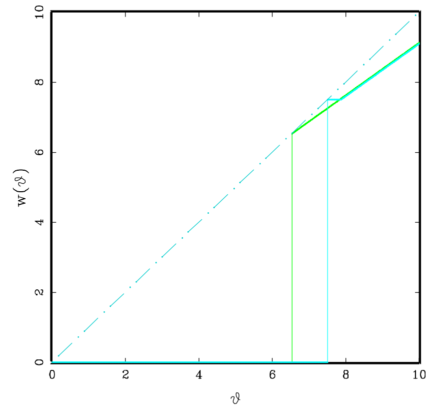

Figure 4.c
Proportionate Change in Labor Market Values
α=.25, m=7.50

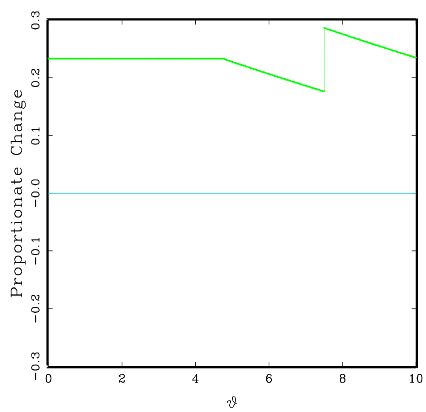

Figure 4.d
Proportionate Change in Labor Market Values
α=.75, m=7.50

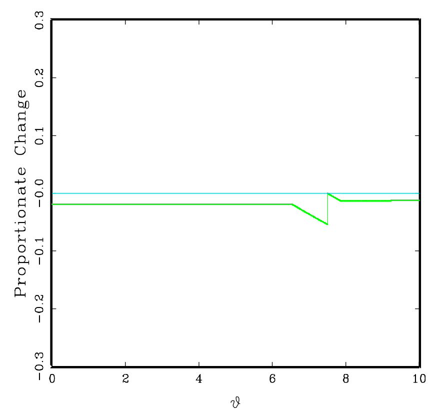

Figure 4.e
Observed Wage C.D.F.
α=.25, m=7.50

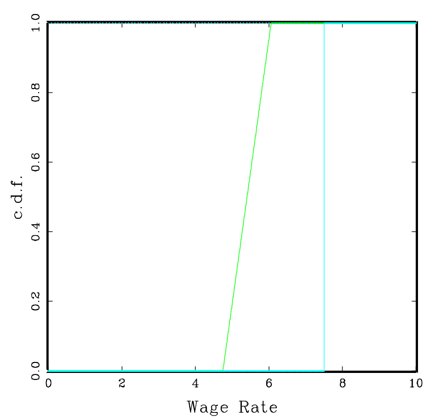

Figure 4.f
Observed Wage C.D.F.
α=.75, m=7.50

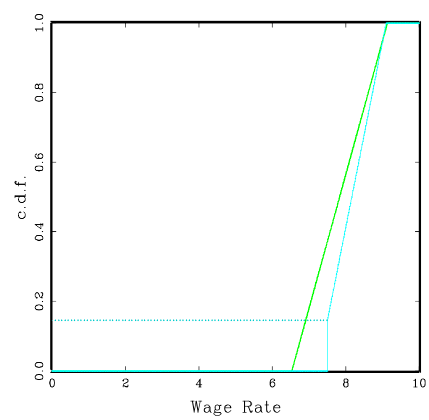

Figure 5.a
Estimates of x
x = 3

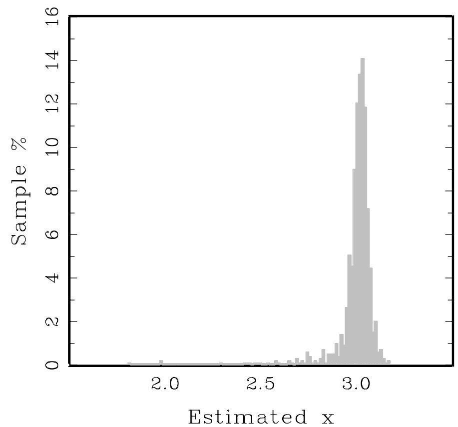

Figure 5.b
Estimates of α
α = .4

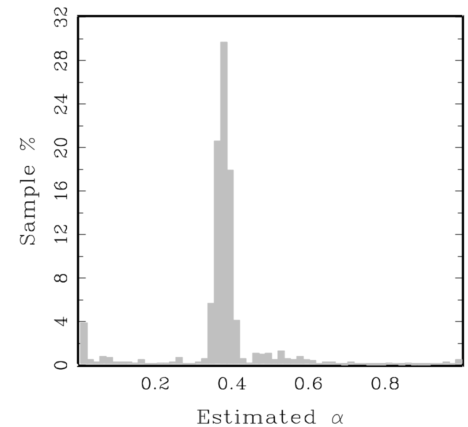

Figure 5.c
Estimated Matching Scale
Median = exp(1)

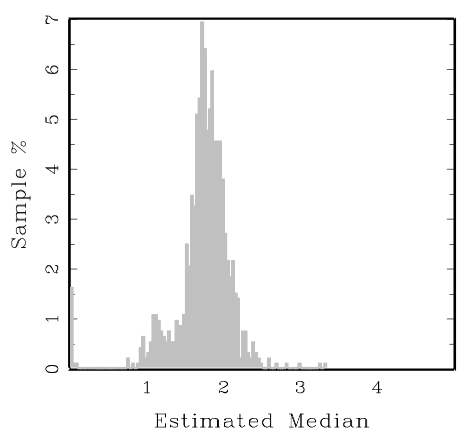

Figure 5.d
Estimates of σ
σ = 1

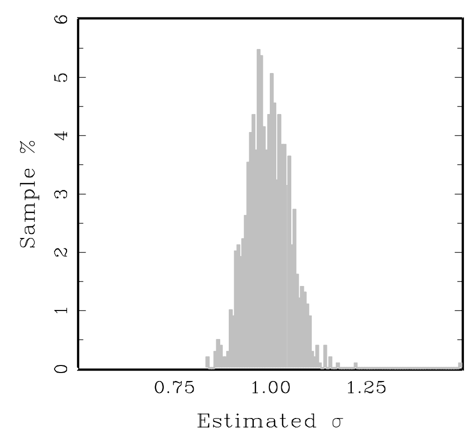

Figure 6.a
Hourly Wages
Employed Individuals, 1996

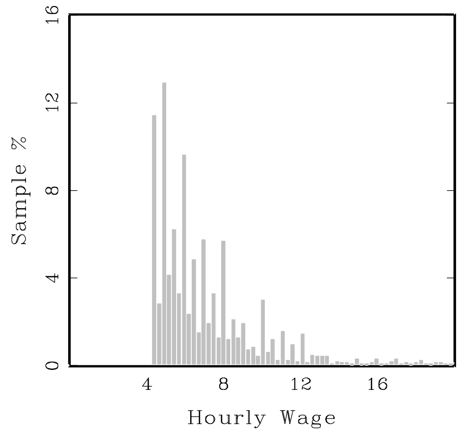

Figure 6.b
On−Going Search Duration
Unemployed Individuals, 1996

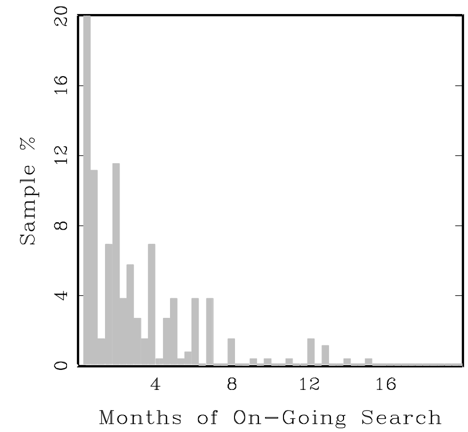

Figure 6.c
Hourly Wages
Simulated Distribution

Figure 6.d
On−Going Search Duration
Simulated Distribution

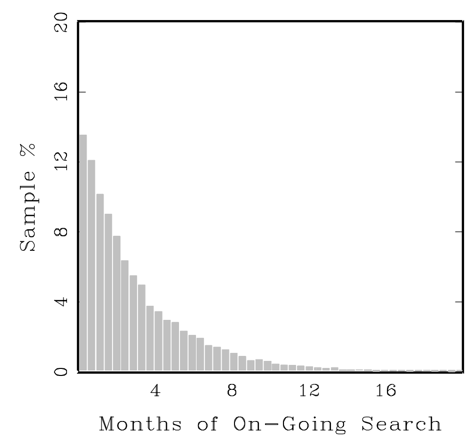

Figure 7
Value of Unemployed Search
Using 1997 Estimates