

The Human Capital Stock: A Generalized Approach

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September 29, 2011

Abstract

This paper presents a new framework for human capital measurement. The generalized framework can (i) substantially amplify the role of human capital in accounting for cross-country income differences and (ii) reconcile the existing conflict between regression and accounting evidence in assessing the wealth and poverty of nations. One natural interpretation emphasizes differences across economies in the acquisition of advanced knowledge by skilled workers.

Keywords: human capital, cross-country income differences, ideas, institutions, TFP, division of labor

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1 Introduction

This paper considers the measurement of human capital. A generalized framework for human capital accounting is developed. Under this framework, human capital variation can play a much bigger role in explaining cross-country income differences than traditional accounting exercises suggest. Moreover, in assessing the wealth and poverty of nations, the existing conflict between regression and accounting evidence can be resolved.

To situate this paper, first consider the literature's standard methods and results, which rely on assumptions about (1) the aggregate production function, mapping capital inputs into output, and (2) the measurement of capital inputs. The traditional production function is Cobb-Douglas. In a seminal paper, Mankiw et al. (1992) used average schooling duration to measure human capital and showed its strong correlation with per-capita output (see Figure 1). Overall, Mankiw et al.'s regression analysis found that physical and human capital variation predicted 80% of the income variation across countries.

The interpretation of these regressions is not obvious however, given endogeneity concerns (Klenow and Rodriguez-Clare 1997). To avoid regression's inference challenges, more recent research has emphasized accounting approaches, decomposing output directly into its constituent inputs (see, e.g., the review by Caselli 2005). A key innovation also came in measuring human capital stocks, where an economy's workers were translated into "unskilled worker equivalents", summing up the country's labor supply with workers weighted by their wages relative to the unskilled (Hall and Jones 1999, Klenow and Rodriguez-Clare 1997). This method harnesses the standard competitive market assumption where wages represent marginal products and uses wage returns to inform the productivity gains from human capital investments. With this approach, the variation in human capital across countries appears modest, so that physical and human capital now predict only 30% of the income variation across countries (see, e.g., Caselli 2005) – a quite different conclusion than regression suggested.

This paper reconsiders human capital measurement while maintaining neoclassical assumptions. The analysis continues to use neoclassical mappings between inputs and outputs and continues to assume that inputs are paid their marginal products. The main difference comes through generalizing the human capital aggregator.

The primary results and their intuition can be introduced briefly as follows. Write a general human capital aggregator as $H = G(H_1, H_2, \dots, H_N)$, where the arguments are the human capital services provided by various subgroups of workers. Denote the standard human capital calculation of *unskilled worker equivalents* as \tilde{H} . The first result of the paper shows that any human capital aggregator that meets basic neoclassical assumptions can be written in a general manner as (Lemma 1)

$$H = G_1(H_1, H_2, \dots, H_N)\tilde{H}$$

where G_1 is the marginal increase in total (i.e. collective) human capital services from an additional unit of unskilled human capital services. This result is simple, general, and intuitive. It says that, once we have used relative wages in an economy to convert workers into equivalent units of unskilled labor (\tilde{H}), we must still consider how the productivity of an unskilled worker depends on the skills of other workers, an effect encapsulated by the term G_1 .

This result clarifies the potential limitations of standard human capital accounting, which focuses on variation in \tilde{H} across countries. Because the variation in \tilde{H} is modest in practice, human capital appears to explain very little.¹ In revisiting that conclusion, one possibility is that G_1 varies substantially across countries. Traditional human capital accounting assumes that G_1 is constant, so that unskilled workers' output is a perfect substitute for other workers' outputs. However, this assumption rules out two kinds of effects. First, it rules out the possibility that the marginal product of unskilled workers might be higher when they are scarce ($G_{11} < 0$). Second, it rules out that possibility that the marginal product of unskilled workers might be higher when they work with skilled workers ($G_{1j \neq 1} > 0$).² In practice, because rich countries are relatively abundant in skilled labor, G_1 will tend to be higher in rich than poor countries, amplifying human capital differences. This reasoning establishes natural conditions under which traditional human capital accounting is downward biased, providing only a lower bound on actual human capital differences across countries.

¹For example, comparing the 90th and 10th percentile countries by per-capita income, the ratio of per-capita income is 20 while the ratio of unskilled worker equivalents is only 2 (see, e.g., the review of Caselli 2005).

²For example, hospital orderlies might have higher real wages when scarce and when working with doctors. Farmhands may have higher real wages when scarce and when directed by experts on fertilization, crop rotation, seed choice, irrigation, and market timing. Such scarcity and complementarity effects are natural features of neoclassical production theory. They are also found empirically in analyses of the wage structure within countries (see, e.g., the review by Katz and Autor 1999).

To estimate human capital stocks while incorporating these effects, this paper further introduces a “Generalized Division of Labor” (GDL) human capital aggregator, which features a constant-returns-to-scale aggregation of skilled labor types

$$Z(H_2, H_3, \dots, H_N)$$

that combines with unskilled labor services with constant elasticity of substitution, ε . This approach has several useful properties. First, the GDL human capital stock can be calculated without specifying $Z(\cdot)$, so that the human capital stock calculation is robust to a wide variety of sub-aggregations of skilled workers. Second, GDL aggregation encompasses traditional human capital accounting as a special case. Third, the human capital stock calculation becomes log-linear in unskilled labor services and unskilled labor equivalents, making it also amenable to linear regression approaches.

Using this aggregator, accounting estimates show that physical and human capital variation can fully explain the wealth and poverty of nations when $\varepsilon \approx 1.6$. Meanwhile, regression estimates suggest values for ε in a similar range. This approach can thus resolve the conflict between regression and accounting evidence in the existing literature. This consistency appears both in the capacity to estimate central roles for human capital and in specific estimates of ε in the generalized aggregator. Moreover, while these calculations are made across countries, existing micro-estimates within countries for related sub-classes of human capital aggregators appear broadly consistent with values of ε in this range (e.g., Katz and Autor 1999, Ciccone and Peri 2005, Caselli and Coleman 2006).

Having established these results about human capital stocks, sources of human capital variation are further investigated by unpacking the skilled aggregator, $Z(\cdot)$. First, skill differences across countries are examined along quantity and quality dimensions. To perform this analysis, it is shown that differences in the quality of skilled workers across economies will tend to appear through differences in labor supply (quantities), not wages (factor prices), providing a further caveat against relying on wage returns alone to make human capital inferences. Second, it is shown - always maintaining neoclassical assumptions - that quality differences between skilled workers in rich and poor countries can be naturally amplified by labor division. Moving beyond the traditional treatment of skilled workers as perfect substitutes, this analysis instead acknowledges that technicians, engineers, medical

professionals, et cetera come in many varieties with highly differentiated task specializations. Such task differentiation may be inevitable in advanced economies where the set of advanced knowledge used in production is too large for any one person to know (Jones 2009). This analysis supports the human capital stock calculations by showing in greater detail where human capital differences across countries may come from.

Lastly, the paper discusses the broader meaning of a “human capital” explanation for the wealth and poverty of nations. A human capital explanation acts to eliminate total factor productivity residuals in explaining economic prosperity, which can be construed as a central goal of macroeconomic research. At the same time, because residual productivity differences are often interpreted as variation in "ideas" or "institutions", a human capital explanation might be interpreted as limiting these other stories. I will argue, to the contrary, that the embodiment of ideas (facts, theories, methods) into people is a good description of what human capital actually is. Further, this process of human capital investment can be critically influenced by institutions. In this interpretation, the contribution of this paper is not in reducing the roles of ideas or institutions, but in showing how the role of human capital can be substantially amplified, making it a central vessel for understanding productivity differences.

Section 2 of this paper develops the generalized framework for calculating human capital stocks. Section 3 considers empirical estimates using both accounting and regression approaches. Section 4 shows how human capital stocks can be unpacked along quality and quantity dimensions and through the division of labor. Section 5 summarizes the results and provides further interpretation.

Related Literature In addition to the literature discussed above, this paper is most closely related to Caselli and Coleman (2006) and Jones (2010). Caselli and Coleman separately estimate residual productivities for high and low skilled workers across countries when allowing for imperfect substitutability between two worker classes. Their estimates continue to use perfect-substitute based reasoning in interpreting a small role for human capital. Jones (2010) provides a model to understand endogenous differences across countries in the quality and quantity of skilled workers and shows that human capital differences expand. These papers will be further discussed below.

2 A Generalized Human Capital Stock

Standard neoclassical accounting couples assumptions about aggregation with the assumption that factors are paid their marginal products. The general framework builds from the following assumptions, which will be maintained throughout the paper.

Assumption 1 (*Aggregation*) *Let there be an aggregate production function*

$$Y = F(K, H, A) \tag{1}$$

where Y is value-added output, $H = G(H_1, H_2, \dots, H_N)$ is aggregate human capital, $K = Q(K_1, K_2, \dots, K_M)$ is aggregate physical capital and A is a scalar. Human capital services of type $i \in \{1, \dots, N\}$ are $H_i = h_i L_i$, where workers of mass L_i provide service flow h_i . Let all aggregators be constant returns to scale in their capital inputs and twice-differentiable, increasing, and concave in each input.

Assumption 2 (*Marginal Products*) *Let factors be paid their marginal products. The marginal product of a capital input X_j is*

$$\frac{\partial Y}{\partial X_j} = p_j$$

where p_j is the price of capital input X_j and the aggregate price index is taken as numeraire.

The objective of accounting is to compare two economies and assess the relative roles of variation in K , H , and A in explaining variation in Y .

2.1 Human Capital Measurement: Challenges

The basic challenge in accounting for human capital is as follows. From a production point of view, we would like to measure a type of human capital as an amount of labor, L_i (e.g., the quantity of college-educated workers), weighted by the flow of services, h_i , such labor provides, so that $H_i = h_i L_i$. The basic challenge of human capital accounting is that, while we may observe the quantity of each labor type, $\{L_1, L_2, \dots, L_N\}$, we do not easily observe their service flows, $\{h_1, h_2, \dots, h_N\}$.

The value of the marginal products assumption, Assumption 2, is that we might infer these qualities from something else we observe - namely, the wage vector, $\{w_1, w_2, \dots, w_N\}$.

The marginal products assumption implies

$$w_i = \frac{\partial F}{\partial H} G_i h_i \quad (2)$$

where w_i is the wage of labor type i .³ It is apparent that the wage alone does not tell us the labor quality, h_i , but rather also depends on $(\partial F/\partial H) G_i$, which is the price of H_i .

To proceed, one may write the wage ratio

$$\frac{w_i}{w_j} = \frac{G_i h_i}{G_j h_j} \quad (3)$$

which, together with the constant-returns-to-scale property (Assumption 1), allows us to write the human capital aggregate as

$$H = h_1 G \left(L_1, \frac{w_2 G_1}{w_1 G_2} L_2, \dots, \frac{w_N G_1}{w_1 G_N} L_N \right) \quad (4)$$

Thus, if wages and labor allocations are observed, one could infer the human capital inputs save for two challenges. First, we do not observe the ratios of marginal products, $\{G_1/G_2, \dots, G_1/G_N\}$. Second, we do not know h_1 . To make further progress, additional assumptions are needed. The following analysis first considers the particular assumptions that development accounting makes (often implicitly) to solve these measurement challenges. The analysis will then show how to relax those additional assumptions, providing a generalized approach to human capital accounting that leads to different conclusions.

2.2 Traditional Development Accounting

In development accounting, the goal is to compare different countries at a point in time and decompose the sources of income differences into physical capital, human capital, and any residual, total factor productivity. The literature (e.g., see the reviews of Caselli 2005 and Hsieh and Klenow 2010) focuses on Cobb-Douglas aggregation, $Y = K^\alpha (AH)^{1-\alpha}$, where α is the physical capital share of income, K is a scalar aggregate capital stock, and $H = G(H_1, H_2, \dots, H_N)$ is a scalar human capital aggregate.

In practice, the labor types $i = 1, \dots, N$ are grouped according to educational duration in development accounting, with possible additional classifications based on work experience

³Recall that the wage is the marginal product of labor, not of human capital; i.e. $w_i = \frac{\partial Y}{\partial L_i}$. This calculation assumes that we have defined the workers of type i to provide identical labor services, h_i . More generally, the same expression will follow if we consider workers of type i to encompass various subclasses of workers with different capacities. In that case, the interpretation is that w_i is the mean wage of these workers and h_i is the mean flow of services (H_i/L_i) from these workers.

or other worker characteristics. Human capital is then traditionally calculated based on *unskilled labor equivalents*.

Definition 1 Define *unskilled labor equivalents* as $\tilde{L}_1 = \sum_{i=1}^N \frac{w_i}{w_1} L_i$, where labor class $i = 1$ represents the uneducated.

This calculation translates each worker type into an equivalent mass of unskilled workers, weighting each type by their relative wages. This construct is often referred to as an "efficiency units" or "macro-Mincer" measure, the latter acknowledging that relative wage structures within countries empirically follow a Mincerian log-linear relationship.

Calculations of human capital stocks based exclusively on unskilled labor equivalents can be justified as follows.

Assumption 3 Let the human capital aggregator be $\tilde{H} = \sum_{i=1}^N h_i L_i$.

Note that this aggregator assumes an infinite elasticity of substitution between human capital types. This perfect substitutes assumption implies that $G_i = G_j$ for any two types of human capital. It then follows directly that the human capital aggregate can be written

$$\tilde{H} = h_1 \tilde{L}_1$$

Thus, as a matter of measurement, the perfect substitutes assumption solves the problem that we do not observe the marginal product ratios $\{G_1/G_2, \dots, G_1/G_N\}$ in the generic aggregator (4) by assuming each ratio is 1.

To solve the additional problem that we do not know h_1 , one must then make some assumption about how the quality of such uneducated workers varies across countries. Let the two countries we wish to compare be denoted by the superscripts R (for "rich") and P (for "poor"). One common way to proceed is as follows.

Assumption 4 Let $h_1^R = h_1^P$.

This assumption may seem plausible to the extent that the unskilled, who have no education, have the same innate skill in all countries. Under Assumptions 3 and 4, we have

$$\frac{\tilde{H}^R}{\tilde{H}^P} = \frac{\tilde{L}_1^R}{\tilde{L}_1^P}$$

providing one solution to the human capital measurement challenge and allowing comparisons of human capital across countries based on observable wage and labor allocation vectors.

2.3 Relaxing the Perfect Substitutes Assumption

To see the implications of Assumption 3 for the conclusions of development accounting, we now return to a generic human capital aggregator $H = G(H_1, H_2, \dots, H_N)$.

Lemma 1 *Under Assumptions 1 and 2, any human capital aggregator can be written $H = G_1(H_1, H_2, \dots, H_N)\tilde{H}$.*

All proofs are presented in the appendix.

This result gives us a general, simple statement about the relationship between a broad class of possible human capital aggregators and the "efficiency units" aggregator typically used in the literature. By writing this result as

$$H = G_1 \times h_1 \times \sum_{i=1}^N \frac{w_i}{w_1} L_i$$

we see that human capital can be assessed through three essential objects. First, there is an aggregation across labor types weighted by their relative wages, $\sum_{i=1}^N \frac{w_i}{w_1} L_i$, which translates different types of labor into a common type - equivalent units of unskilled labor. Second, there is the quality of the unskilled labor itself, h_1 . Third, there is the marginal product of unskilled labor services, G_1 . The last object, G_1 , may be thought of generically as capturing effects related to the division of labor, where different worker classes produces different services. It incorporates the scarcity of unskilled labor services and complementarities between unskilled and skilled labor services, effects that are eliminated by assumption in the perfect substitutes framework. Therefore, the traditional human capital aggregator \tilde{H} is not in general equivalent to the human capital stock H , and the importance of this discrepancy will depend on the extent to which G_1 varies across economies.

Definition 2 *Define $\Lambda = \left(\frac{H^R/H^P}{\tilde{H}^R/\tilde{H}^P} \right)$ as the ratio of true human capital differences to the traditional calculation of human capital differences.⁴*

⁴Note that, for any production function $Y = F(K, AH)$, the term AH is constant given Y and K . Therefore we equivalently have $\Lambda = (\tilde{A}^R/\tilde{A}^P)/(A^R/A^P)$, which is the extent total factor productivity differences are overstated across countries.

It follows immediately from Lemma 1 (i.e. only on the basis of Assumptions 1 and 2) that

$$\Lambda = G_1^R/G_1^P$$

indicating the bias induced by the efficiency units approach.

This bias may be substantial. Moreover, there is reason to think that $\Lambda \geq 1$; i.e., that the perfect-substitutes assumption will lead to a systematic understatement of true human capital differences. To see this, note that G_1 is likely to be substantially larger in a rich country than a poor country, for two reasons. First, rich countries have substantially fewer unskilled workers, a scarcity that will tend to drive up the marginal product of unskilled human capital ($G_{11} < 0$). Second, rich countries have substantially more highly educated workers, which will tend to increase the productivity of the unskilled workers to the extent that highly skilled workers have some complementarity with low skilled workers ($G_{1j \neq 1} > 0$). It will follow under fairly mild conditions that $\Lambda \geq 1$. One set of conditions is as follows.

Lemma 2 *Consider the class of human capital aggregators $H = G(H_1, Z(H_2, \dots, H_N))$ with finite and strictly positive labor services, H_1 and Z . Under Assumptions 1 and 2, $\Lambda \geq 1$ iff $Z^R/H_1^R \geq Z^P/H_1^P$.*

Thus, under fairly broad conditions, traditional human capital estimation provides only a lower bound on human capital differences across economies.

2.3.1 A Generalized Estimation Strategy

In practice, the extent to which human capital differences may be understated depends on the human capital aggregator employed as an alternative to the efficiency units specification. Here we develop an alternative that (i) can be easily estimated and (ii) nests many approaches, as follows.

Lemma 3 *Consider the class of human capital aggregators $H = G(H_1, Z(H_2, \dots, H_N))$. If such an aggregator can be inverted to write $Z(H_2, \dots, H_N) = P(H, H_1)$, then the human capital stock can be estimated solely from information about H_1 , \tilde{H} , and production function parameters.*

This result suggests that there may be a broad class of aggregators that are relatively easy to estimate, with the property that the aggregation of skilled labor, $Z(H_2, H_3, \dots, H_N)$,

need not be measured directly. Moreover, any aggregator that meets the conditions of this Lemma also meets the conditions of Lemma 2. Therefore, in comparison to traditional human capital accounting, any such aggregator allows only greater human capital variation across countries.

A flexible aggregator that satisfies the above conditions is as follows.

Definition 3 Define the "Generalized Division of Labor" (GDL) aggregator as

$$H = \left[H_1^{\frac{\varepsilon-1}{\varepsilon}} + Z(H_2, H_3, \dots, H_N)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (5)$$

where $\varepsilon \in [0, \infty)$ is the elasticity of substitution between unskilled human capital, H_1 , and an aggregation of all other human capital types, $Z(H_2, H_3, \dots, H_N)$.

This aggregator encompasses, as special cases: (i) the traditional efficiency-units aggregator $\tilde{H} = \sum_{i=1}^N H_i$, (ii) CES specifications, $H_\varepsilon = \left(\sum_{i=1}^N H_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$, and (iii) the Jones (2010) and Caselli and Coleman (2006) specifications, which assume an efficiency units aggregation for higher skill classes, $Z = \sum_{i=2}^N H_i$. More generally, the GDL aggregator encompasses any constant-returns-to-scale aggregation $Z(H_2, H_3, \dots, H_N)$. It incorporates conceptually many possible types of labor division and interactions among skilled workers.

By Lemma 3, the aggregator (5) has the remarkably useful property that human capital stocks can be estimated - identically - without specifying the form of $Z(H_2, H_3, \dots, H_N)$.

Corollary 1 Under Assumptions 1 and 2, any human capital aggregator of the form (5) is equivalently $H = H_1^{\frac{1}{1-\varepsilon}} \tilde{H}^{\frac{\varepsilon}{\varepsilon-1}}$.

Therefore, the calculated human capital stock will be the same regardless of the underlying structure of $Z(H_2, H_3, \dots, H_N)$ - we do not need to know the potentially very complicated and difficult to estimate form that this skilled aggregator may take. Related, the understatement of human capital differences across countries is

$$\Lambda_{GDL} = \left(\frac{\tilde{L}_1^R / \tilde{L}_1^P}{L_1^R / L_1^P} \right)^{\frac{1}{\varepsilon-1}} \quad (6)$$

which can be estimated regardless of $Z(H_2, H_3, \dots, H_N)$.⁵ The findings of the traditional perfect substitutes approach are equivalent to the special case where $\varepsilon \rightarrow \infty$.⁶ This generalized division of labor approach will be examined empirically in Section 3. We will see that, under reasonable parameterizations, it allows human capital to replace total factor productivity residuals in explaining cross-country income variation.

2.4 Relaxing the Identical Unskilled Assumption

In comparing human capital across economies, analyses must also specify the relationship between h_1^R and h_1^P . The often implicit assumption is that $h_1^R = h_1^P$ (Assumption 4), i.e. that the unskilled have the same innate skill in different economies. Alternatively, one might imagine that children in a rich country have initial advantages (including better nutrition and/or other investments prior to starting school) that make $h_1^R > h_1^P$.⁷ On the other hand, one might be concerned about selection. Those with little or no schooling are a tiny part of the population in rich countries and a large part in poor countries. Especially in the presence of compulsory schooling programs, the uneducated in a rich country may select on substantial physical or cognitive difficulties, in which case we might imagine $h_1^R < h_1^P$. Because human capital differences across countries track linearly in h_1^R/h_1^P (just as they track linearly in G_1^R/G_1^P) getting this ratio right may be important. This section considers how to relax Assumption 4 and let data determine h_1^R/h_1^P .

2.4.1 Identifying h_1^R/h_1^P

The basic challenge that motivated Assumption 4 is that we do not directly observe h_1^R or h_1^P . However, one can make potential headway by noting that immigration allows us to observe unskilled workers from both a rich and poor country in the same economy. Examining immigrants and native-born workers in the rich economy, one may observe the

⁵From the corollary and the definition of Λ , we have $\Lambda_{GDL} = \left(\frac{\tilde{H}^R/H_1^R}{\tilde{H}^P/H_1^P} \right)^{\frac{1}{\varepsilon-1}}$. The term $\left(\frac{\tilde{H}^R/H_1^R}{\tilde{H}^P/H_1^P} \right)$ is equivalent to the easily measured $\left(\frac{\tilde{L}_1^R/\tilde{L}_1^P}{L_1^R/L_1^P} \right)$, because the h_1^R and h_1^P terms cancel. Therefore, Λ_{GDL} is invariant to assumptions made regarding h_1^R and h_1^P , and, in particular, Assumption 4 is not relevant to this calculation.

⁶Equation (6) also implies that Assumption 3 is a strong version of the traditional accounting framework, which is more generally equivalent to any aggregator written as $\tilde{H} = H_1 + Z(H_2, H_3, \dots, H_N)$. In other words, the traditional calculation is correct should unskilled worker services be perfect substitutes for all other worker services.

⁷For example, Manuelli and Sheshadri (2005) makes such arguments.

wage ratio

$$\frac{w_1^R}{w_1^{R|P}} = \frac{(\partial F^R / \partial H^R) G_1^R h_1^R}{(\partial F^R / \partial H^R) G_1^R h_1^{R|P}} = \frac{h_1^R}{h_1^{R|P}} \quad (7)$$

where $w_1^{R|P}$ and $h_1^{R|P}$ are the wage and skill of uneducated immigrants working in the rich country. In other words, immigration allows us to observe workers from different places in the same economy, thus allowing us to eliminate considerations of variation in $(\partial F / \partial H) G_1$ across countries in trying to infer variation in h_1 .

If we proceed under the assumption that the unskilled immigrants are a random sample of the unskilled in the poor country, then $h_1^{R|P} = h_1^P$. Therefore $h_1^R / h_1^P = w_1^R / w_1^{R|P}$, and we can calculate the corrected human capital ratio as

$$\frac{H^R}{H^P} = \frac{G_1^R}{G_1^P} \frac{w_1^R}{w_1^{R|P}} \frac{\tilde{L}_1^R}{\tilde{L}_1^P}$$

Of course, one might imagine that unskilled immigrants have higher or lower ability on average than the unskilled who stay behind in the poor country. If immigration selects on higher ability among the unskilled, then $h_1^{R|P} > h_1^P$ and the correction $w_1^R / w_1^{R|P}$ would provide a lower bound on the unskilled skill ratio, understating human capital difference across countries.

3 Empirical Estimation

Given these theoretical results, we reconsider human capital's role in explaining cross-country income variation. We first consider direct accounting and then consider regression evidence. The analysis uses the flexible, generalized division of labor aggregator (5) and emphasizes comparison with the traditional special case.

3.1 Data and Basic Measures

To facilitate comparison with the existing literature, we use the same data sets and accounting methods in the review of Caselli (2005). Therefore any differences between the following analysis and the traditional conclusions are driven only by human capital aggregation. Data on income per worker and investment are taken from the Penn World Tables v6.1 (Heston et al. 2002) and data on educational attainment is taken from Barro-Lee (2001). The physical capital stock is calculated using the perpetual inventory method following Caselli (2005),

and unskilled labor equivalents are calculated using data on the wage return to schooling. These data methods are further described in the appendix.

Again following the standard literature, we will use Cobb-Douglas aggregation, $Y = K^\alpha (AH)^{1-\alpha}$ and take the capital share as $\alpha = 1/3$. Writing $Y_{KH} = K^\alpha H^{1-\alpha}$ to account for the component of income explained by measurable factor inputs, Caselli (2005) defines the success of a factors-only explanation as

$$success = \frac{Y_{KH}^R / Y_{KH}^P}{Y^R / Y^P}$$

where R is a "rich" country and P is a "poor" country. We will denote the success measure for traditional accounting, based on \tilde{H} , as $success_T$.

3.2 Accounting Evidence

Table 1 summarizes some basic data. Comparing the richest and poorest countries in the data (the USA and Congo-Kinshasa), the observed ratio of income per-worker is 91. The capital ratio is larger, at 185, but the ratio of unskilled labor equivalents is much more modest, at 1.7. Comparing the 85th to 15th percentile (Israel and Kenya) or the 75th to 25th percentile (S. Korea and India), we again see that the ratio of income and physical capital stocks is much greater than the ratio of unskilled labor equivalents.

Using unskilled labor equivalents to measure human capital stock variation, it follows that $success_T = 45\%$ comparing Korea and India, $success_T = 25\%$ comparing Israel and Kenya, and $success_T = 9\%$ when comparing the USA to the Congo. These calculations suggest that large residual productivity variation is needed to explain the wealth and poverty of nations. These findings rely on unskilled labor equivalents, $\tilde{L}_1^R / \tilde{L}_1^P$, to measure human capital stock variation. Because unskilled labor equivalents vary little, human capital appears to add little to explaining productivity differences.⁸

3.2.1 Relaxing the Perfect-Substitutes Assumption

The relationship between the traditional success measure and the success measure for a general human capital aggregator is

$$success = \Lambda^{1-\alpha} \times success_T$$

⁸Figure A1 shows $success_T$ when comparing all income percentiles from 70/30 (Malaysia/Honduras) to 99/1 (USA-Congo). The average measure of $success_T$ is 31% over this sample.

which follows from Lemma 1 and the definition of Λ .

One can implement a generalized accounting using the GDL aggregator. From (6) and the data in Table 1, it is clear that Λ_{GDL} can be large. While the variation in unskilled labor equivalents, $\tilde{L}_1^R/\tilde{L}_1^P$, is modest, the human capital variation that corrects for labor division expands according to two objects. One is the relative scarcity of unskilled labor services, L_1^R/L_1^P . The second is the degree of complementarity between skilled and unskilled labor services, as defined by ε .

The literature on the elasticity of substitution between skilled and unskilled labor within countries suggests that $\varepsilon \in [1, 2]$.⁹ Table 2 (Panel A) reports the results of development accounting over this range of ε , focusing on the Israel-Kenya example. The first row presents the human capital differences, H^R/H^P , the second row presents the ratio of these differences to the traditional calculation, Λ_{GDL} , and the third row presents the resulting *success* measure for capital inputs in explaining cross-country income differences. As shown in the table, factor-based explanations for income differences are substantially amplified by allowing for labor division. As complementarities across workers increase, the need for TFP residuals decline.¹⁰ For the Israel-Kenya example, the need for residual TFP differences is eliminated at $\varepsilon = 1.54$, where human capital differences are $H^R/H^P = 10.5$.

One can also consider a broader set of rich-poor comparisons; for example, all country comparisons from the 70/30 income percentile (Malaysia/Honduras) up to the 99/1 percentile (USA/Congo).¹¹ Calculating the elasticity of substitution, ε , at which capital inputs fully explain income differences, shows that the mean value is $\varepsilon = 1.55$ in this sample with a standard deviation of 0.34. Notably, the estimated values of ε typically fall in the same $[1, 2]$ interval as the micro-literature suggests.

⁹See, e.g. reviews in Katz and Autor (1999) and Ciccone and Peri (2005). Most estimates come from regression analyses that may be substantially biased due to the endogeneity of labor supply. Ciccone and Peri (2005) use compulsory schooling laws as a source of plausibly exogenous variation in schooling across U.S. states and find that ε is in a range between 1.3 and 2 depending on the specification. All these estimates consider the elasticity of substitution between high-school and college-educated workers, and they may not extend to primary vs non-primary educated workers. The regression analysis below, however, also suggests ε in this range.

¹⁰The intuition for this result will be discussed in Section 4.

¹¹The Malaysia/Honduras income ratio is 3.8. As income ratios (and capital measures) converge towards 1, estimates of ε become noisier.

3.2.2 Relaxing the Identical Unskilled Assumption

Table 2 (Panel B) further relaxes Assumption 4, allowing h_1^R/h_1^P to be determined from immigration data. Examining immigrants to the U.S. using the year 2000 U.S. Census, the mean wages of employed unskilled workers varies modestly based on the source country (see Figure A2). While the data are noisy, one may infer that mean wages are about 17% lower for uneducated workers born in the U.S. compared to immigrants from the very poorest countries, which suggests $h_1^R/h_1^P \approx .83$ (see Appendix for details).¹²

Such an adjustment lowers the explanatory power of human capital in explaining cross-country variation. The adjustment seems large - it cuts human capital differences by 17%. However, in practice, relaxing Assumption 4 has modest effects compared to relaxing Assumption 3, as seen in Table 2. Now residual TFP differences are eliminated when $\varepsilon = 1.50$ for the Israel-Kenya comparison. Across the broader set of rich-poor examples, additionally relaxing Assumption 4 leads to a mean value of $\varepsilon = 1.58$, with a standard deviation of 0.37.

The human capital stock estimations are summarized for the broader sample in Figure 2. The ratio of human stocks for each pair of countries from the 70/30 income percentile to the 99/1 income percentile is presented. Panel A considers the generalized framework, with $\varepsilon = 1.6$. Panel B considers traditional human capital accounting. We see that, as reflected in Table 1, traditional human capital accounting admits very little human capital variation. It appears orders of magnitude less than the variation in physical capital or income. With the generalized framework, human capital differences substantially expand, admitting variation similar in scale to the variation in income and physical capital.

3.3 Regression Evidence

Regression analysis in the development accounting context is controversial. While Mankiw, Romer, and Weil (1992) find a large R^2 and theoretically credible relationships between capital aggregates and output, Klenow and Rodriguez-Clare (1997) point out the omitted

¹²This micro-data finding stands in contrast to the cross-country analysis of Manuelli and Seshadri (2005), which relies on $h_1^R \gg h_1^P$. Manuelli and Seshadri (2005) can be understood as relaxing Assumption 4 but not relaxing Assumption 3, which means that one will require $h_1^R \gg h_1^P$ to increase the explanatory power of human capital in a cross-country setting. The immigrant wage data is inconsistent with $h_1^R \gg h_1^P$ unless one assumes that uneducated immigrants to the U.S. select on extremely high ability compared to the non-immigrating population. More generally, Table 2 suggests that much more action comes from relaxing Assumption 3.

variable hazards in interpreting such regressions. In practice, because average schooling is highly correlated with income per-capita, regressions of income on schooling variables will tend to show highly significant positive relationships and large R^2 . While this correlation might be causative, it may well not be, and caution is needed.¹³

Given these concerns, the more telling aspect of regression analysis may come less from high R^2 and more from the implied production function parameters. In particular, it is informative whether the estimated production function parameters are consistent with the production functions being estimated in direct accounting exercises. This connection explicitly fails in the traditional analyses: using a (quasi¹⁴) perfect substitutes aggregation of human capital, Mankiw, Romer, Weil (1992) suggests that human capital plays a primary role in cross-country income differences, but explicit accounting using a perfect substitutes aggregation suggests the opposite conclusion.

In this section, we show that the generalized aggregator may avoid this conflict. Continuing with the standard Cobb-Douglas production function, $Y = K^\alpha(AH)^{1-\alpha}$, define income net of physical capital's contribution as $\log \hat{Y} = \log Y - \alpha \log K$. The GDL aggregator implies $\log \hat{Y} = (1 - \alpha)[\log A + \frac{1}{1-\varepsilon} \log H_1 + \frac{\varepsilon}{\varepsilon-1} \log \tilde{H}]$. A regression can then estimate

$$\log \hat{Y}^c = \beta_0 + \beta_1 \log H_1^c + \beta_2 \log \tilde{H}^c + u^c \quad (8)$$

where c indexes countries. The values of ε are then implied by the coefficient estimates.¹⁵

3.3.1 Relaxing the Efficiency Units Assumption

Table 3 presents the regression results. Columns (1)-(3) examine (8) while maintaining Assumption 4. Column (1) shows that the explanatory power of $\log \tilde{H}$ is substantial. The coefficient implies $\varepsilon = 1.34$, with a 95% confidence interval of [1.28, 1.34]. Column (2) considers the explanatory power of $\log H_1$, which is also substantial. The coefficient implies

¹³An additional challenge for the original Mankiw, Romer, and Weil (1992) approach is the linear use of schooling duration for the human capital aggregator, i.e. $H = \sum_{i=1}^N s_i L_i$, where s_i is the number of years of school. This approach is an efficiency units aggregator (Assumption 3) combined with an additional assumption equating skill to years in school, i.e. $h_i = s_i$. However, this combination is inconsistent with the wage evidence. Under Assumption 3, the relative skill h_i/h_j is linear in the relative wage, w_i/w_j , which are *log-linear* in schooling duration, not linear. Thus the assumption that $h_i = s_i$ does not appear supportable with a neoclassical aggregator.

¹⁴See prior footnote.

¹⁵Regression estimates that attempt to account for both physical and human capital simultaneously are much noisier, presumably due to the high correlation between these capital stocks and consequent multicollinearity.

$\varepsilon = 1.70$, with a 95% confidence interval of $[1.50, 2.17]$. Considering $\log \tilde{H}$ and $\log H_1$ together, in column (3), the estimates of ε rise somewhat and become noisier.

3.3.2 Relaxing the Identical Unskilled Assumption

Table 3 columns (4)-(6) further examine (8) while additionally relaxing Assumption 4. Immigrant wage outcomes are used to estimate variation in h_1^c from the U.S. 2000 census, and the measures for $\log \tilde{H}^c$ and $\log H_1^c$ are then adjusted accordingly. (The methodology is further detailed in the Appendix.) In column (4) the coefficient on $\log \tilde{H}^c$ now implies $\varepsilon = 1.64$, with a 95% confidence interval of $[1.50, 1.87]$. In column (5) the coefficient on $\log H_1$ implies $\varepsilon = 1.88$, with a 95% confidence interval of $[1.64, 2.38]$. Joint estimation again raises the ε estimates somewhat and expands the confidence intervals. Many robustness checks have also been considered, varying how \tilde{H} and H_1 are calculated, as further discussed in the Appendix. In general, the regressions tend to point to estimates of ε in similar ranges.

Overall, we see broad consistency between (1) the range of ε that eliminates TFP differences in explicit accounting, (2) regression estimates of ε , and (3) within-country micro-evidence on the substitutability between skilled and unskilled labor. These observations suggest that the GDL aggregator may provide a reasonable theoretical approach, resolving the tension between regression and accounting methodologies while implying that human capital variation can now play a substantial role in explaining income variation across countries. These findings - both Table 2 and Table 3 - are robust to any constant-returns-to-scale specification of the aggregator $Z(H_2, H_3, \dots, H_N)$.

4 Inside Human Capital Stocks

A value of the human capital stock calculations above is that they do not require detailed specification of the aggregator. At the same time, it would be useful to look "underneath the hood" and gain a better understanding of where the variation in stocks may come from. One basic question is whether the increased human capital services in rich countries follow from the quantity and/or quality of skilled labor. It is clear (Figure 1), that the quantity of skilled laborers is much greater in rich countries. It will be shown below that the quality of skilled laborers also appears much greater in rich countries. An ensuing question is then how

quality advantages in rich countries may emerge, and the greater acquisition of knowledge by skilled workers will be offered as a potential explanation. While empirical estimation becomes increasingly challenging, given the lack of consensus (or knowledge) about production functions at this level of detail, the theory provides a road-map for estimation, and illustrative calibrations are offered.

4.1 The Quality and Quantity of Labor Services

We begin with theoretical considerations for inferring labor quality. In general under Assumptions 1 and 2, the relative "quality" of two groups of laborers in an economy is, from (3)

$$\frac{h_i}{h_j} = \frac{w_i G_j}{w_j G_i} \quad (9)$$

where $h_i = H_i/L_i$ is the mean flow of services from the workers in group i . Thus the relative qualities (h_i/h_j) can in general be inferred from relative wages (w_i/w_j), which are factor prices, and the relative marginal products of the human capital intermediates (G_j/G_i), which are the relative prices of the human capital services.

Now consider a type of partial equilibrium experiment, where we hold the relative prices (G_j/G_i) fixed. In that case, increasing the skill ratio by a factor of x would increase the wage ratio by the same factor. This provides standard intuition, for example, for the common practice in economics of using wage returns to schooling variation to make claims about variation in skill returns. Were we to extend such reasoning to comparisons between rich and poor countries, we would infer the relative quality of labor services as

$$\left(\frac{h_i^R/h_j^R}{h_i^P/h_j^P} \right)_{PE} = \frac{w_i^R/w_j^R}{w_i^P/w_j^P} \quad (10)$$

where the subscript "PE" indicates that we are using a type of partial equilibrium reasoning.

Of course, comparing economies with substantially different factor allocations suggests that partial equilibrium analysis could be problematic. In general equilibrium, we need to introduce the possibility that relative prices (G_j/G_i) vary.¹⁶ This caution becomes more precise by allowing for the labor allocation to shift in response to variations in skill returns.

¹⁶Ignoring such price variation in general equilibrium requires Assumption 3 – that different skill classes are perfect substitutes – so that G_j/G_i is a constant. However, as discussed above, perfect substitutability across educational groups appears inconsistent with the empirical evidence (e.g., Katz and Autor 1999, Ciccone and Peri 2005).

To see this, consider a simple stylized theory with endogenous labor supply.¹⁷

Assumption 5 *Let individual income, y , as a function of educational duration, s_i , be $y(s_i) = \int_{s_i}^{\infty} w(s_i)e^{-rt}dt$ where the discount rate r is fixed. Let individuals be identical ex-ante and maximize income with respect to educational duration.*

Workers train for some period of years and then work, with choices over training duration determining the supply of various worker classes. Defining an elasticity of substitution $\sigma_{ij} = -\frac{\partial \ln(H_i/H_j)}{\partial \ln(G_i/G_j)}$, we establish the following result.

Lemma 4 *Under Assumptions 1, 2, and 5, (a) $\frac{\partial \ln(w_i/w_j)}{\partial \ln(h_i/h_j)} = 0$ and (b) $\frac{\partial \ln(L_i/L_j)}{\partial \ln(h_i/h_j)} = \sigma_{ij} - 1$.*

This result provides exactly the opposite intuition from the partial equilibrium reasoning. Namely, the lemma says that quality variation will appear through quantities, not through wages. If labor supply is endogenous as in Assumption 5, then labor allocations shift to neutralize the wage variation. Under the lemma, and comparing two countries with a common discount rate, we have the general equilibrium counterpoint to (10)

$$\left(\frac{h_i^R/h_j^R}{h_i^P/h_j^P} \right)_{lemma} = \frac{G_j^R/G_i^R}{G_j^P/G_i^P} \quad (11)$$

With endogenous labor supply, quality variation becomes divorced from wage variation and appears instead through the allocative shifts that the partial equilibrium reasoning ignored.

To further establish this general equilibrium intuition, and its empirical relevance, consider the following observation (and see Figure 3 below). Skilled-unskilled wage ratios tend to be fairly similar across economies, while skilled-unskilled labor supply ratios tend to be extremely different. That wage returns to education are fairly similar across economies follows naturally from result (a) of the lemma; when labor supply is endogenous, equilibrium investment levels in education drive the rate of return to the local discount rate. How then can a country like the United States sustain high wage returns to education despite a massive increase in the quantity of highly-educated workers? Quality advantages provide an answer. Expanded labor supply lowers the prices for skilled labor services. To maintain such labor supply in equilibrium, one needs quality advantages.

¹⁷The educational choice decision follows Mincer's original theoretical justification for the log-linear wage-schooling relationship (Mincer 1958). The assumption that r is fixed could be relaxed. See also Jones (2010).

The lemma also makes this quantity-quality linkage explicit, via result (b). When the elasticity of substitution between skilled and unskilled labor is greater than one, economies with higher skilled quality will see greater skilled labor supply in response. Moreover, for a given difference in labor quantity, the implied quality differences become increasingly large as the elasticity of substitution falls toward one (because skilled output prices fall increasingly quickly in response to skilled quantity increases, so that larger quality differences are required to maintain the wage ratio). This theoretical reasoning explains why, in Table 2, human capital’s capacity to explain cross-country income differences is increasing as the elasticity of substitution falls.

4.1.1 Empirical Estimates of Quality Differences

Figure 3 presents the variation in wage returns and skilled labor supply when comparing rich and poor countries. Defining L_Z as the mass of skilled workers and w_Z as their mean wage, the variation in wage returns is seen to be exceptionally modest, while the variation in labor supply is enormous. Taking the Israel-Kenya example, the relative skilled labor allocation (L_Z/L_1) is 2300% greater in Israel, while the wage returns (w_z/w_1) are only 20% lower. Taking the USA-Congo example, the relative skilled labor allocation is 17500% greater in the USA, while the wage returns are only 15% lower. Consistent with Lemma 4, massive increases in skilled labor supply can be reconciled with little if any drop in wage returns through variation in the quality of skilled labor services.

To estimate the variation in the quality of skilled services, return first to the human capital stock calculations of Section 3. Using the GDL aggregator in tandem with (9), one can infer the skilled-unskilled ratio of mean service flows as

$$\frac{h_z^R/h_1^R}{h_z^P/h_1^P} = \left(\frac{w_z^R/w_1^R}{w_z^P/w_1^P} \right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{L_z^R/L_1^R}{L_z^P/L_1^P} \right)^{\frac{1}{\varepsilon-1}} \quad (12)$$

where $h_z = Z(H_2, H_3, \dots, H_N)/L_Z$ is the mean flow of services from skilled workers.

Table 4 (Panel A) reports the implied variation in h_z/h_1 , continuing with the rich-poor example in Table 2. Recall that human capital stock variation eliminates residual total factor productivity variation when $\varepsilon \approx 1.6$. At this value of ε , the relative service flows of skilled workers in the rich country appear 98.6 times larger than in the poor country. This empirical finding is consistent with Caselli and Coleman (2006), but now explicitly extended to the general class of skilled labor aggregators, $Z(H_2, H_3, \dots, H_N)$. Thus similar

wage returns are consistent with massive differences in labor allocation when skilled service flows are substantially higher in rich countries.

Skilled service flows can be further articulated by specifying particular skilled aggregators, Z . For example, consider a sub-aggregator of skilled types

$$Z = \left[\sum_{i=2}^N H_i^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (13)$$

where η is the elasticity of substitution among these types. Table 4 (Panel B) presents the implied service flows from these different groups of skilled workers.¹⁸ Taking a range of $\eta \in [1.2, 2]$, the implied skill return advantages for skilled but less than tertiary-educated workers in the rich country are in the interval [69, 103], while the skill returns among the tertiary-educated are in the interval [60, 284].

In sum, labor allocations and wage returns are reconciled when service flows from higher educated workers in rich countries are far higher (as a group) than their service flows in poor countries. The next section considers how the acquisition of knowledge may explain this phenomenon.

4.2 Labor Division and the Acquisition of Knowledge

The analysis above, in dropping the perfect substitutes assumption, equivalently imagines that individuals work on differentiated, interdependent tasks, so that the productivity of unskilled workers depends on the broader human capital context in which they work. For example, the output of dishwashers can now depend on the chef, the output of hospital orderlies on the doctors, and the output of factory janitors on the engineers who design and run the plant.

The GDL estimations also implicitly incorporate variation in the division of labor through the skilled aggregator $Z(\cdot)$. In advanced economies, and especially among the highly educated, skills can be highly differentiated. Not only do skills differ between medical doctors, chemical engineers, computer scientists, molecular biologists, lawyers, and architects, but

¹⁸The skill returns for the sub-groups of workers are calculated using the general result (9) in tandem with (5) and (13). Skill flows are then calculated as

$$\frac{h_{i \neq 1}}{h_1} = \left(\frac{w_i}{w_1} \right)^{\frac{\eta}{\eta-1}} \left(\frac{L_i}{L_1} \right)^{\frac{1}{\eta-1}} \left(\frac{h_Z}{h_1} \frac{L_Z}{L_1} \right)^{\frac{\eta-\varepsilon}{\varepsilon(\eta-1)}}$$

The calculations in Panel B of Table 4 assume $\varepsilon = 1.6$ in the GDL aggregator; i.e. the value of ε where capital variation fully explains the income variation.

skills within professions can be highly differentiated themselves. For example, there are 145 accredited medical specialties in the United States, and MIT offers 119 courses across 8 sub-specialties within aeronautical engineering alone.¹⁹ By comparison, Uganda has 10 accredited medical specialties, and the engineering faculty of Makerere University, often rated as the top university in Sub-Saharan Africa outside South Africa, does not offer any aeronautics courses within its engineering curriculum.

This section considers greater task specialization among skilled workers as a possible explanation for the greater skilled service flows in rich countries. The approach incorporates the classic idea that the division of labor may be a primary source of economic prosperity (e.g., Smith 1776) and builds on ideas in a related paper (Jones 2010), which considers micro-mechanisms that can obstruct collective specialization among skilled workers.

The core observation is that focused training and experience can provide extremely large skill gains at specific tasks. For example, the willingness to pay a thoracic surgeon to perform heart surgery is likely orders of magnitude larger than the willingness to pay a dermatologist (or a Ph.D. economist!) to perform that task. Similarly, when building a microprocessor fabrication plant, the service flows from appropriate, specialized engineers are likely orders of magnitude greater than could be achieved otherwise. Put another way, if no individual can be an expert at everything, then embodying the stock of productive knowledge in the workforce requires a division of labor. Possible limits to task specialization include: (i) the extent of the market (e.g. Smith 1776); (ii) coordination costs across workers (e.g. Becker and Murphy 1992); (iii) the extent of existing advanced knowledge (Jones 2009); and (iv) local access to advanced knowledge (e.g. Jones 2010). Here we consider a simple production theory to explore how variation in task specialization may explain variation in skilled services across economies.²⁰

4.2.1 Production with Specialized Skills

Consider skilled production as the performance of a wide range of tasks, indexed over a unit interval. Production can draw on a group of n individuals. With n individuals, each member of the group can focus on learning an interval $1/n$ of the tasks. This specialization

¹⁹Accredited medical specialties are listed by the *American Board of Medical Specialties*, <http://www.abms.org>. The MIT subject offerings are found in the *MIT Bulletin*, <http://web.mit.edu/catalog/>.

²⁰The following set-up is closest theoretically to Becker and Murphy (1992) and Jones (2010). It differs in part by providing a path toward calibration.

allows the individual to focus her training on a smaller set of tasks, increasing her mastery at this set of tasks. If an individual devotes a total of s units of time to learning, then the time spent learning each task is ns .

Let the skill at each task be defined by a function $f(ns)$ where $f'(ns) > 0$. Meanwhile, let there be a coordination penalty $c(n)$ for working in a team. Let task services aggregate with a constant returns to scale production function that is symmetric in its inputs, so that the per-capita output of a team of skilled workers with breadth $1/n$ will be $h(n, s) = c(n)f(ns)$. We assume that $c'(n) < 0$, so that bigger teams face larger coordination costs, acting to limit the desired degree of specialization.²¹

Next consider the choice of s and n that maximizes the discounted value of skilled services per-capita.²² This maximization problem is

$$\max_{s,n} \int_s^\infty h(n, s) e^{-rt} dt$$

4.2.2 Example

Let $c(n) = e^{-\theta n}$, where θ captures the degree of coordination costs that ensue with greater labor division. Let $f(ns) = \alpha(ns)^\beta$, where α and β are educational technology parameters. It follows from the above maximization problem that²³

$$s^* = \beta/r \tag{14}$$

$$n^* = \beta/\theta \tag{15}$$

and skilled services per-capita are $e^{-\beta} \alpha \left(\frac{\beta^2}{r\theta}\right)^\beta$. Expertise at tasks declines with higher discount rates (r), which reduce the length of education, and with greater coordination costs (θ), which limit specialization.

As a simple benchmark, assume common β around the world. Then the ratio of skilled labor services between a rich and poor country will be

$$\frac{h^R}{h^P} = \frac{\alpha^R}{\alpha^P} \left(\frac{r^P \theta^P}{r^R \theta^R} \right)^\beta \tag{16}$$

²¹For analytical convenience, we will let team size, n , be a continuous variable.

²²Decentralized actors may not necessarily achieve this symmetric, output maximizing outcome. In fact, given the presence of complementarities across workers, multiple equilibria are possible (see Jones 2010). Here we consider the output maximizing case as a useful benchmark.

²³The following stationary points are unique, and it is straightforward to show that they satisfy the conditions for a maximum.

This model thus suggests a complementarity of mechanisms. Differences in the quality of education (α), discount rates (r), and coordination penalties (θ) have multiplicative effects. These interacting channels provide compounding means by which skilled labor services may differ substantially across economies.

4.2.3 Calibration Illustration

We focus on the division of labor. Note from (15) that with common β the equilibrium difference in the division of labor (that is, the team size ratio) is equivalent to the inverse coordination cost ratio, θ^P/θ^R . To calibrate the model, let $\beta = 2.2$, which follows if the duration of schooling among the highly educated is 22 years and the discount rate is 0.1. Further let $\alpha^R/\alpha^P = 1$ and take the Mincerian coefficients as those used to calculate each country's human capital stocks throughout the paper, as described in the Appendix. Figure 4 then plots the implied variation in the division of labor, n^R/n^P , that reconciles (16) with the quality variation h_z^R/h_z^P implied by (12), under the assumption that rich countries have no advantage in education technology.

We find that a 4.3-fold difference in the division of labor can explain the productivity difference between Israel and Kenya (the 85-15 percentile country comparison), and a 2.4-fold difference explains the productivity difference between Korea and India (the 75-25 percentile country comparison). The extreme case of the USA and the former Zaire is explained with a 22-fold difference. These differences would fall to the extent that the education technology (α, β) is superior in richer countries.

Are large division of labor differences reasonable? Systematic measures are not readily available in the micro-literature and await further research, but the anecdotes above about medical and engineering specialization do suggest very large differences, and specialized training clearly raises skills at particular tasks by very large multiples. In any case, the primary observation here is that considerations of task specialization face little theoretical constraint in providing an interpretation for both (1) the large differences in skilled labor allocations across countries and (2) the large differences in human capital stocks estimated in this paper.

5 Discussion

5.1 Summary

This paper introduces a generalized framework for human capital accounting. Traditional development accounting is nested as a special case and, under mild conditions, is shown to provide only a lower bound on human capital variation across economies. A "generalized division of labor" aggregator is introduced, which allows human capital stocks to be calculated with relatively little aggregation structure. This framework can reconcile the conflict between regression analyses (e.g. Mankiw, Romer, and Weil 1992) and traditional accounting approaches (e.g. Klenow and Rodriguez-Clare 1997, Hall and Jones 1999, Caselli 2005). Human capital stocks can now play a central role in explaining the wealth and poverty of nations.

Having established these results about human capital stocks, the paper further considers possible underlying sources of human capital variation. First, the generalized framework is extended to account for quality differences in workers' service flows. Wage variation is found to be a poor guide to quality differences, because labor supply adjustments tend to neutralize the wage effects. In consequence, quality differences tend to appear as quantity differences rather than wage differences. Because skilled labor quantity differences are so large across countries, one infers correspondingly large differences in skilled labor qualities. Second, variation in knowledge acquisition is incorporated into the accounting framework. Increased specialization – allowing greater collective acquisition of knowledge – is seen to amplify skilled service flows, providing a candidate, underlying production theory for the relatively large quality of skilled worker services in advanced economies.

5.2 Interpretations, Implications, and Extensions

The paper's estimates of human capital stocks suggest that cross-country output variation can now be accounted for without relying on residual, total factor productivity (TFP) variation. Because TFP is often interpreted as (i) "ideas" and/or (ii) "institutions", this paper might therefore seem to diminish these explanations for economic development. Such an implication, however, need not follow if (1) ideas are embodied in the capital inputs and (2) the embodiment process is influenced by institutions. In this interpretation, the contribution of this paper is not in reducing the roles of ideas or institutions, but in emphasizing human

capital's potential as a central feature of understanding productivity differences. This paper closes by considering this perspective.

Consider ideas first. Macroeconomic arguments aside, studies of actual production processes through history suggest that the creation and diffusion of ideas are central to understanding productivity.²⁴ Yet one may also claim, by studying any particular production process, that ideas enter production only when they are known and implemented; that is, only when actuated through tangible inputs - the people and their physical inputs that actually make things. In fact, the physical instantiation of an idea may be a good description of what physical capital is (e.g. a microprocessor is a set of ideas etched on silicon) and learning ideas may be a good description of education (e.g. skilled workers are vessels of facts, theories, and techniques). In this view, one doesn't need TFP for ideas to be a centerpiece of economic development.

Putting ideas into production through capital inputs, rather than as a residual, also provides explicit processes in which institutions matter. As further emphasized by the division of labor model, individuals may collectively fail to embody advanced ideas when faced, e.g., with high interest rates, high coordination costs, and poor educational institutions. Institutional features like credit constraints, weak property rights and contracting environments, and poor public good provision would then naturally underpin these investment failures.

Lastly, claiming that ideas enter production through people and their machines leads quickly to an emphasis on the division of labor. If embodiment is needed for ideas to become useful in production, then this next logical step follows to the extent that the set of existing ideas is too large for any one person to know. Differentiated knowledge across workers is then *necessary* to bring the collective set of advanced ideas into production (Jones 2009). Thus, while the division of labor calibration in Section 5 provides one approach, the broader point is that successfully mapping an advanced economy's ideas into productive inputs naturally depends on specialized workers, which suggests that the division of labor is a central aspect of understanding human capital and economic development. Jones (2010) further develops this idea and suggests that, beyond cross-country income differences, division of labor variation can explain a variety of stylized facts about the world economy.

²⁴See, e.g., Mokyr's *The Lever of Riches* (1992) for many historical examples. Nordhaus (1997) provides a powerful example by studying the price of light through time. Conley and Udry (2010) is one of many studies demonstrating that ideas can fail to diffuse in poor countries.

In summary, the analysis in this paper suggests a substantially amplified role of human capital. The findings offer a reconciliation of regression and accounting approaches to human capital measurement. More broadly, the findings are fully consistent with a framework in which investment, ideas, and institutions play substantive roles – but where human capital is drawn to the heart of economic development.

While the framework is applied here to cross-country income differences, the same framework has other natural applications at the level of countries, regions, cities, or firms. Growth accounting provides one direction for future work. The urban-rural economy literature is another direction, where productivity differences from specialization are often suggested as critical but cannot be captured using traditional human capital measures.

6 Appendix

Proof of Lemma 1

Proof. $H = G(H_1, H_2, \dots, H_N)$ is constant returns in its inputs (Assumption 1). Therefore, by Euler's theorem for homogeneous functions, the true human capital aggregate can generically be written $H = \sum_{i=1}^N G_i H_i$. Rewrite this expression as $H = G_1 h_1 \sum_{i=1}^N \frac{G_i h_i}{G_1 h_1} L_i$. Recalling that $w_i = \frac{\partial F}{\partial H} G_i h_i$ (Assumption 2), so that $\frac{w_i}{w_1} = \frac{G_i h_i}{G_1 h_1}$, we can therefore write $H = G_1 h_1 \tilde{L}_1$, where $\tilde{L}_1 = \sum_{i=1}^N \frac{w_i}{w_1} L_i$. ■

Proof of Lemma 2

Proof. If $H = G(H_1, Z)$ is constant returns to scale, then G_1 is homogeneous of degree zero by Euler's theorem. Therefore $G_1(H_1, Z) = G_1(H_1/Z, 1)$. Noting that $G_{11} \leq 0$, it follows that $\Lambda = G_1^R/G_1^P \geq 1$ iff $Z^R/H_1^R \geq Z^P/H_1^P$. ■

Proof of Lemma 3

Proof. By Lemma 1, $H = G_1 \tilde{H}$, providing an independent expression for H based on its first derivative. If the human capital aggregator can be manipulated into the form $H = Q(H_1, Z(H_2, \dots, H_N)) = Q(H_1, P(H, H_1))$, then we have from Lemma 1 $H = Q_1(H_1, P(H, H_1)) \tilde{H}$. This provides an implicit function determining H solely as a function of H_1 and \tilde{H} ; that is, without reference to $Z(H_2, \dots, H_N)$. ■

Proof of Corollary 1

Proof. By Lemma 1, $H = G_1 \tilde{H}$. For the GDL aggregator, $G_1 = (H/H_1)^{\frac{1}{\varepsilon}}$. Thus $H = H_1^{\frac{1}{1-\varepsilon}} \tilde{H}^{\frac{\varepsilon}{\varepsilon-1}}$. ■

Proof of Lemma 4

Proof. Consider two skill categories that require s_i and s_j years of training respectively. Holding these durations fixed, imagine that the skill levels h_i and h_j associated with each may vary. Assumptions 1 and 2 imply (3). Taking logs and differentiating, it follows that $1 = \frac{\partial \ln(w_i/w_j)}{\partial \ln(h_i/h_j)} + \frac{1}{\sigma_{ij}} \left(1 + \frac{\partial \ln(L_i/L_j)}{\partial \ln(h_i/h_j)} \right)$. Under Assumption 5, arbitrage in career choices ($y(s_i) = y(s_j)$) implies $w_i/w_j = e^{r(s_i-s_j)}$. Thus, if labor supply is endogenous, then the

wage returns are pegged to the duration of training, not the skill associated with it. It then follows that $\frac{\partial \ln(w_i/w_j)}{\partial \ln(h_i/h_j)} = 0$. Hence the result. ■

Data Appendix

Capital Stocks

To minimize sources of difference with standard assessments, this paper uses the same data in Caselli's (2005) review of cross-country income accounting. Income per worker is taken from the Penn World Tables v6.1 (Heston et al. 2002) and uses the 1996 benchmark year. Capital per worker is calculated using the perpetual inventory method, $K_t = I_t + (1 - \delta)K_{t-1}$, where the depreciation rate is set to $\delta = 0.06$ and the initial capital stock is estimated as $K_0 = I_0/(g + \delta)$. Further details are given in Section 2.1 of Caselli (2005). As a robustness check, I have also considered calculating capital stocks as the equilibrium value under Assumptions 1 and 2 with a Cobb-Douglas aggregator; i.e., $K = (\alpha/r)Y$, where $\alpha = 1/3$ is the capital share and $r = 0.1$. This alternative method provides similar results as in the main paper.

To calculate human capital stocks, I use Barro and Lee (2001) for the labor supply quantities for those at least 25 years of age, which are provided in five groups: no schooling, some primary, completed primary, some secondary, completed secondary, some tertiary, and completed tertiary. Schooling duration for primary and secondary workers are taken from Caselli and Coleman (2006) and schooling duration for completed tertiary is assumed to be 4 years. Schooling duration for "some" education in a category is assumed to be half the duration for complete education in that category.

For wage returns to schooling, I use Mincerian coefficients from Psacharopoulos (1994) as interpreted by Caselli (2005). Let s be the years of schooling and let relative wages be $w(s) = w(0)e^{\phi s}$. Psacharopoulos (1994) finds that wage returns per year of schooling are higher in poorer countries, and Caselli summarizes these findings with the following rule. Let $\phi = 0.13$ for countries with $\bar{s} \leq 4$, where $\bar{s} = (1/L) \sum_{i=1}^N s_i L_i$ is the country's average years schooling. Meanwhile, let $\phi = 0.10$ for countries with $4 < \bar{s} \leq 8$, and let $\phi = 0.07$ for the most educated countries with $\bar{s} > 8$. Unskilled labor equivalents are then calculated as $\tilde{L}_1 = \sum_{i=1}^N e^{\phi s_i} L_i$ in each country.

As a robustness check, I have considered calculating human capital stocks under a variety of other assumptions. The results using the GDL aggregators are broadly robust to reasonable alternative human capital input measures. The sample mean value of ε at which capital stocks fully explain income variation typically falls in the the [1.5, 2] interval across human capital accounting methods. For example, if we set $\phi = .10$ (the global average) for all countries, then the gap between unskilled labor equivalents widens slightly, since the returns to education in poor countries now appear lower and the returns in rich countries appear higher. The resulting increase in human capital ratio means that capital inputs can explain income differences at somewhat higher values of ε , but still with $\varepsilon < 2$.

Variation in the Quality of Unskilled Labor

Following the analysis in Section 2.4, we estimate the difference in unskilled qualities as $h_1^R/h_1^P = w_1^R/w_1^{R|P}$, where wages are for unskilled workers in the U.S. The term w_1^R is the mean wage for unskilled workers born in the US and $w_1^{R|P}$ is the mean wage for unskilled workers born in a poor country and working in the US.

Wages are calculated from the 5% microsample of the 2000 U.S. Census (available from www.ipums.org). Unskilled workers are defined as employed individuals with 4 or less years of primary education (individuals with educ=1 in the pums data set) who are between the ages of 20 and 65. To facilitate comparisons, mean wages are calculated for individuals who speak English well (individuals with speakeng=3, 4, or 5 in the pums data set).

Figure A2 presents the data, with the mean wage ratio, $w_1^{R|P}/w_1^R$, plotted against log per-capita income of the source country. (National income data is taken from the Penn World Tables v6.1.) There is one observation per source country, but the size of the marker is scaled to the number of observed workers from that source country. The figure plots the results net of fixed effects for gender and each integer age in the sample.

For accounting and regression analysis when relaxing Assumption 4, the (weighted) mean of $w_1^R/w_1^{R|P}$ is calculated for five groups of immigrants based on quintiles of average schooling duration in the source country. The age and gender controlled data is used, although using the raw wage means produces similar findings. The corrections for h_1^R/h_1^P are then applied to the human capital stock in each country. These corrected data are used (only) when Assumption 4 is relaxed – in Panel B of Table 2, Figure 2B, and columns (4)-(6)

of Table 3. One can use other reasonable methods to calculate w_1^R/w_1^{RP} and apply it to the human capital measures, but in general the primary findings of the paper are robust to such variations, because the implications of relaxing Assumption 3 tend to be much greater.

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Table 1: Basic Data

Measure	Rich v Poor		
	99 th / 1 st Percentile (USA/Zaire)	85 th / 15 th Percentile (Israel/Kenya)	75 th / 25 th Percentile (S Korea/India)
Y^R / Y^P (Income)	90.9	16.9	6.3
K^R / K^P (Capital stock)	185.3	43.9	17.4
$\tilde{L}_1^R / \tilde{L}_1^P$ (Unskilled worker equivalents)	1.70	1.33	1.15
L_1^R / L_1^P (Unskilled workers)	.09	.44	.52

Income and capital stock measures are per worker. Data sources and methods are further described in the text and appendix.

Table 2: Human Capital and Income: Accounting Approach

	Elasticity of Substitution Between Unskilled Labor, H_U , and Skilled Aggregate, $Z(H_2, \dots, H_N)$						
	1	1.2	1.4	1.6	1.8	2.0	∞
<i>Panel A: Relaxing Assumption 3</i>							
H^R/H^P	∞	358	21.9	8.6	5.4	4.1	1.3
Λ	∞	269	16.4	6.5	4.0	3.1	1
Success	∞	1050%	163%	88%	64%	54%	25%
<hr style="border-top: 1px dashed black;"/>							
<i>Panel B: Relaxing Assumptions 3 and 4</i>							
H^R/H^P	∞	306	18.6	7.3	4.6	3.5	1.1
Λ	∞	269	16.4	6.5	4.0	3.1	1
Success	∞	950%	147%	79%	58%	48%	23%

This table compares Israel and Kenya, which represent the 85th and 15th percentile countries respectively ranked by income per worker. H^R/H^P is the ratio of human capital stocks. Λ is the ratio of H^R/H^P at the indicated elasticity of substitution to H^R/H^P for the infinite elasticity of substitution case. Success is the consequent percentage of the income variation that is explained by variation in capital inputs. Figure 2 summarizes accounting for a broader set of rich and poor countries and shows that Israel and Kenya provide a useful benchmark, as discussed in the text.

Table 3: Human Capital and Income: Regression Approach

	Dependent variable is $\log \hat{Y}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(\tilde{H})$	2.606** (0.212)		2.049** (0.293)	1.713** (0.143)		1.467** (0.141)
$\log(H_1)$		-0.957** (0.195)	-0.344* (0.158)		-0.762** (0.141)	-0.508** (0.137)
Observations	92	92	92	92	92	92
R-squared	0.508	0.371	0.533	0.445	0.246	0.545

Implied ε						
(from \tilde{H})	1.34 [1.28-1.44]	--	1.48 [1.34,1.83]	1.64 [1.50-1.87]	--	1.83 [1.83-2.28]
(from H_1)	--	1.70 [1.50,2.17]	2.94 [2.01,23.3]	--	1.88 [1.64,2.38]	2.31 [1.85,3.82]

Specifications (1) – (3) maintain Assumption 4. Specifications (4) – (6) relax Assumption 4.

Robust standard errors in parentheses. Robust 95% confidence intervals in square brackets.

* indicates $p < 0.05$, ** indicates $p < 0.01$

Table 4: Human Capital Services by Educational Groups

Panel A: Human capital services, grouping secondary and tertiary educated workers

	Elasticity of Substitution Between Unskilled Labor, H_1 , and Skilled Aggregate, $Z(H_2, \dots, H_N)$					
	1	1.4	1.6	1.8	2	∞
$\frac{(h_z / h_1)^R}{(h_z / h_1)^P}$	∞	1101	98.6	29.5	14.3	0.79

Panel B: Human capital services, secondary and tertiary educated workers treated separately

	Elasticity of Substitution Between Secondary, H_2 , and Tertiary, H_3 , Human Capital Services					
	1.2	1.4	1.6	1.8	2	∞
$\frac{(h_2 / h_1)^R}{(h_2 / h_1)^P}$	68.5	83.9	89.8	92.9	94.8	103
$\frac{(h_3 / h_1)^R}{(h_3 / h_1)^P}$	284	130	100	88.0	81.4	59.6

This table compares Israel and Kenya, which represent the 85th and 15th percentile countries respectively ranked by income per worker. Panel A of this table corresponds to Panel A of Table 2. Panel B considers the implied human capital services for secondary and tertiary educated workers, depending on the elasticity of substitution between their services. In Panel B, the elasticity of substitution in the GDL aggregator is taken to be 1.6 – the mean value in Figure 2 – where capital variation across countries fully explains income differences.

Figure 1: Income per Worker and Mean Schooling Duration

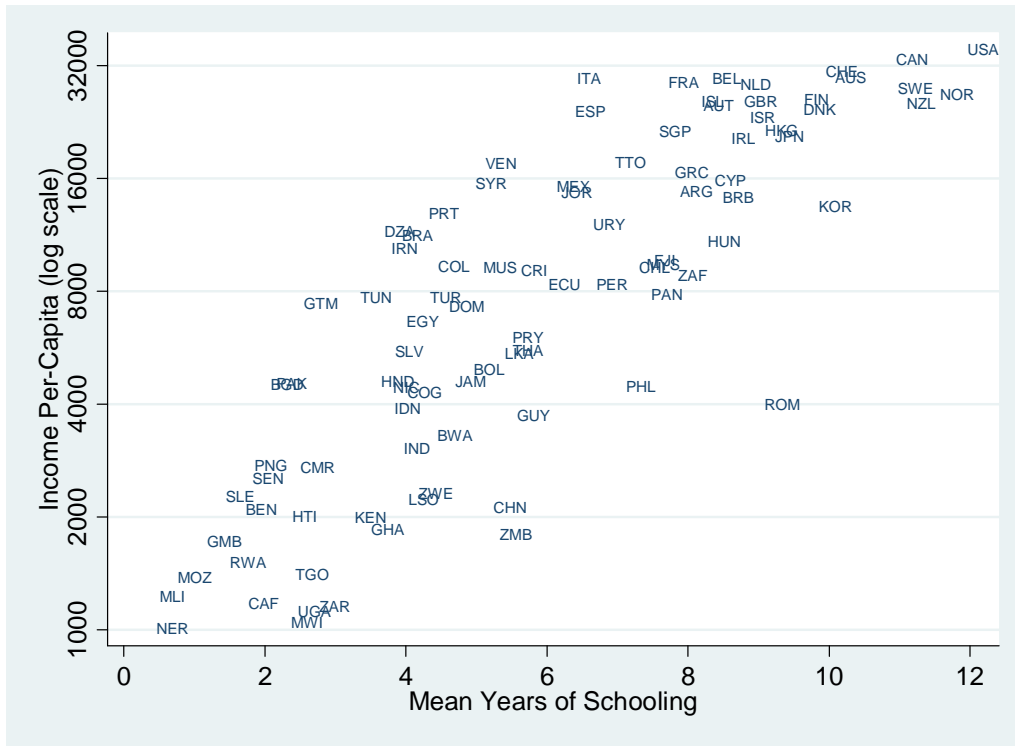
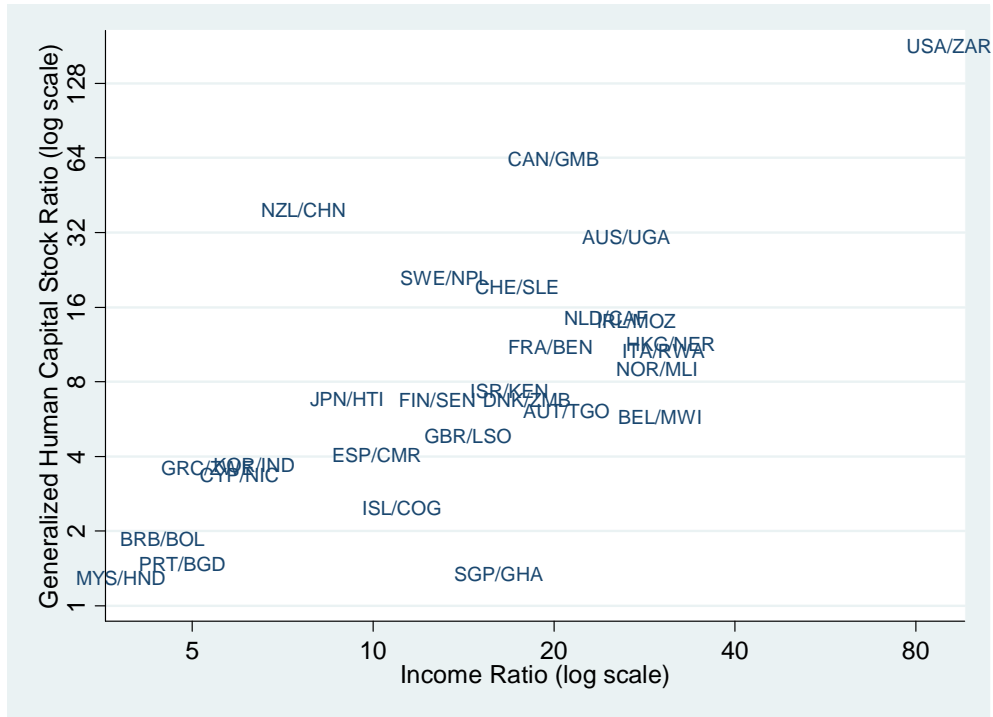


Figure 2: Human Capital Stock Variation

Panel A: Generalized Human Capital Stock



Panel B: Traditional Human Capital Stock

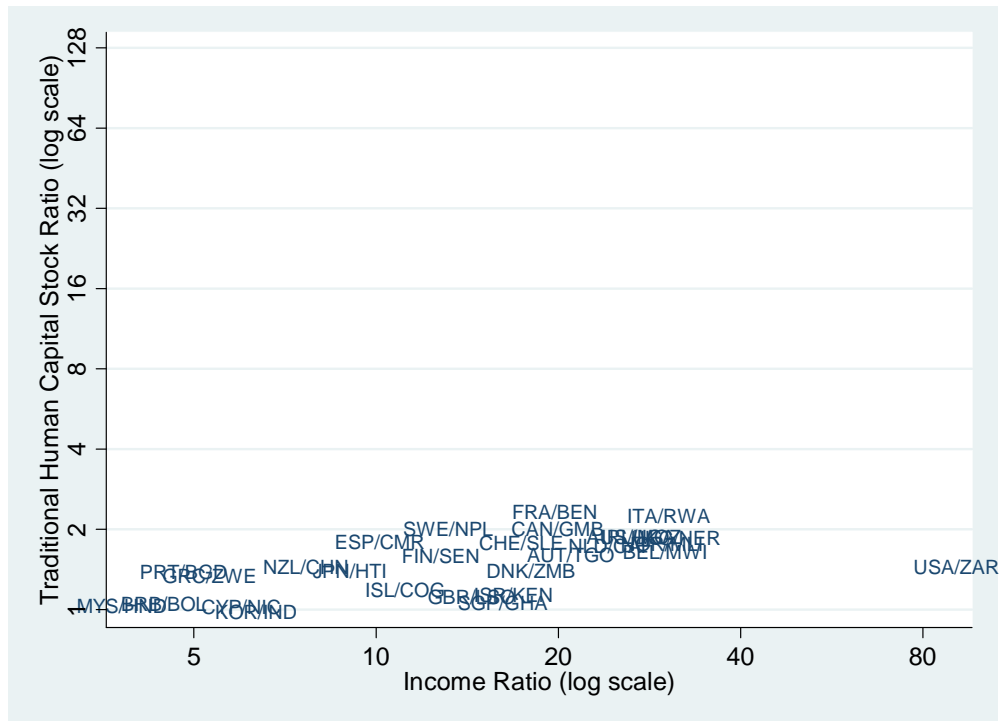


Figure 3: Sources of Human Capital Variation: Labor Supply versus Wages

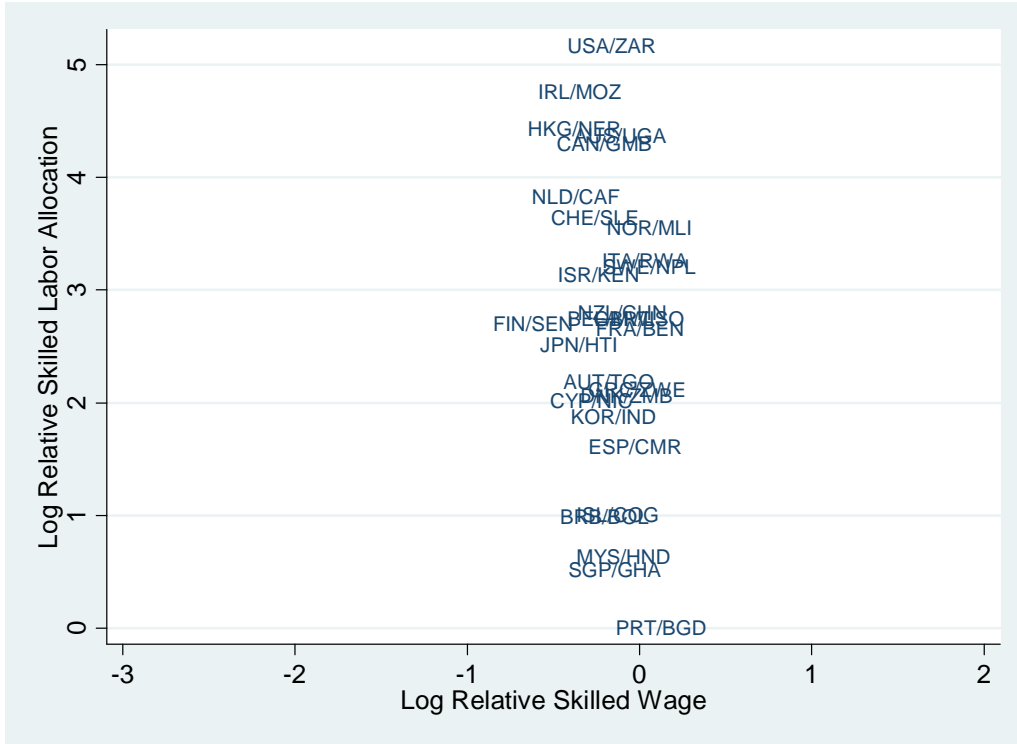


Figure 4: Calibrated Difference in Specialization across Countries

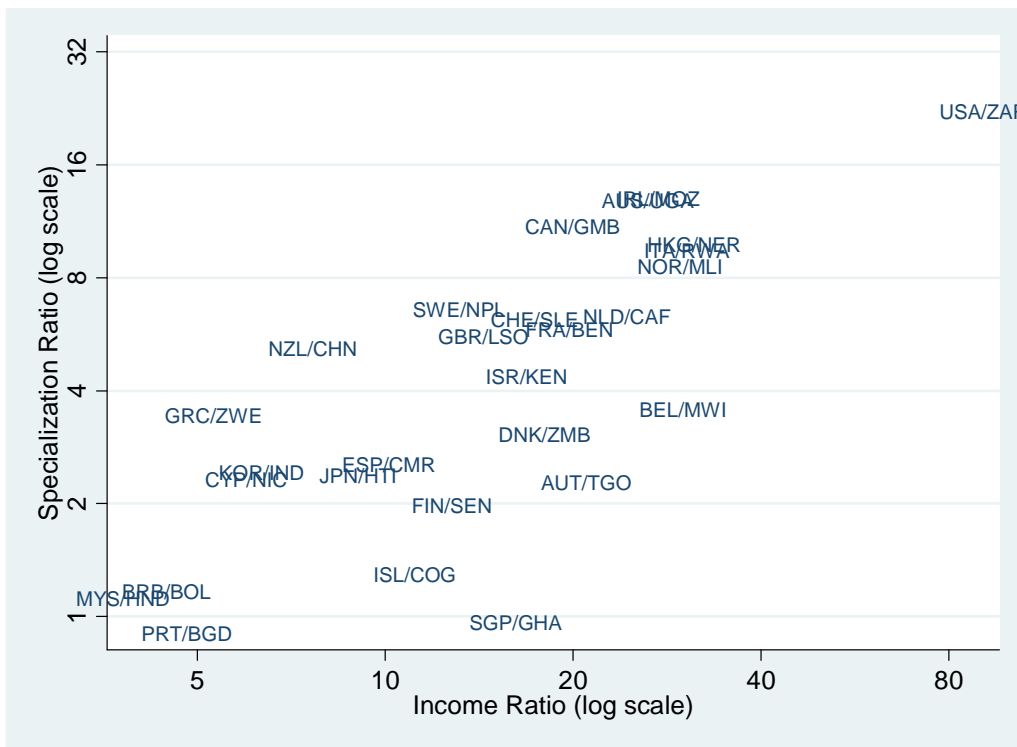
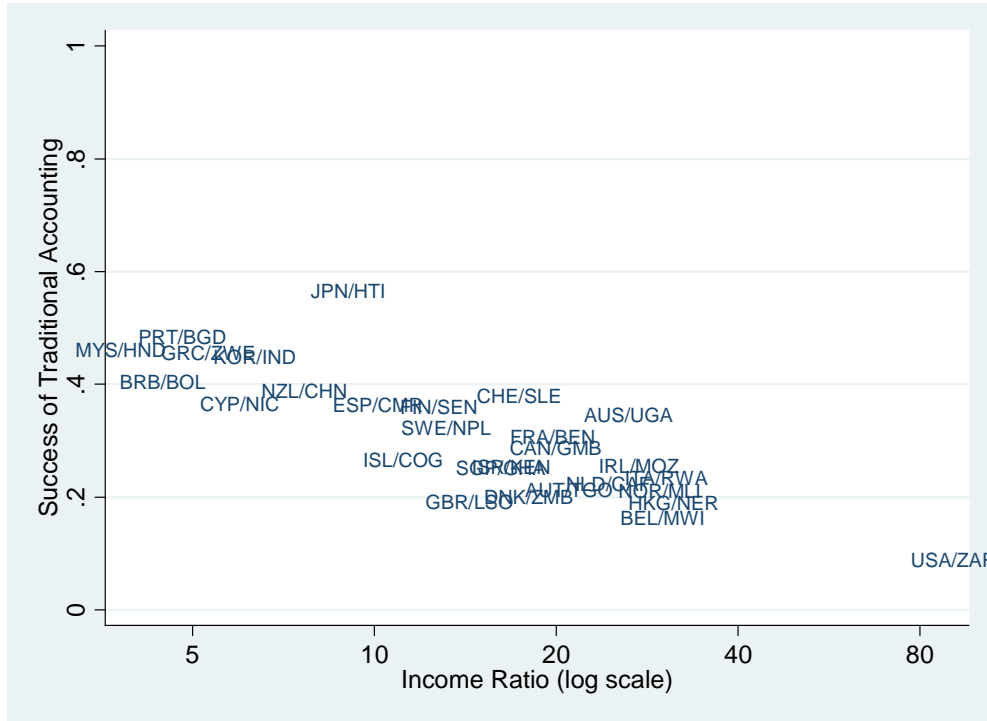


Figure A1: Fraction of Income Difference Explained Using Traditional Human Capital Accounting



**Figure A2: Wage Ratios from 2000 US Census among Employed Unskilled Workers
Conditional on Age and Gender**

