# Suburbanization and Transportation in the Monocentric Model

Nathaniel Baum-Snow
Brown University[1]

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[1] I thank Nathan Schiff for excellent research assistance. All errors are mine.

## Abstract

This paper presents a version of the monocentric city model that incorporates heterogeneous commuting speeds by introducing radial commuting highways. The paper proves several general results about this model. Simulations of a conservative specification of the model imply that each additional highway ray causes about a 10 percent decline in central city population. Given that observed aggregate central city population declined by 17 percent in the U.S. between 1950 and 1990, this model implies that highways can account for more than the full decline in the population of central cities over this period.

# 1 Introduction

One standard result from the classical monocentric land use theory developed by Alonso (1964), Muth (1969) and Mills (1967) is that an increase in transport speed reduces population density throughout a city. A natural implication of this comparative static is that improvements to the transportation infrastructure may be an important explanation for falling urban population density. Indeed, despite robust population growth in metropolitan areas, urban population density has been declining rapidly. As documented in Baum-Snow (2006), the aggregate population in central cities as defined by their geography in 1950 declined by 17 percent between 1950 and 1990 despite national population growth of 64 percent during this period. This paper examines the extent to which the simple mechanism of the monocentric city model can account for the post World War II decline in aggregate central city population.

I adapt a version of the standard monocentric city model to allow for heterogeneity in commuting speeds through the introduction of radial highways. As in the standard monocentric framework, all workers must commute to the center to work and choose a residential location based on the trade-off between lost wages due to travel time and their preference for space. The endogenously determined land rent function distributes identical individuals over the available land such that in equilibrium, everyone's utility is equal and population density is decreasing in commuting time.

The implications of extending this model to allow for heterogenous travel speeds have been explored in several ways. Some past work (as in Leroy and Sonstelie, 1983) has introduced heterogeneity in commuting speed through different travel modes. While allowing mode choice is clearly a relevant approach for a few big cities, in only 6 MSAs did at least 20 percent of the population commute by public transportation in 1960 and by 1990 this had dropped to just one (New York). Moreover, most previous versions of this model have symmetric or one-dimensional space. The model developed in this paper is similar to that formulated by Anas and Moses (1979). Like Anas & Moses, this model incorporates heterogeneous commuting speeds through radial highways into the monocentric framework. As such, equilibrium land use structure exhibits heterogeneity in residential density at a given radius from the city center. While more stylized than Anas & Moses' model in some ways, this model generates analytical implications that apply quite generally across preference specifications and city structures. This model also generates simulation results that are quantitatively robust to a variety of metropolitan area structures.

This paper shows that even with a conservative specification for the utility function and the commuting technology, the monocentric city model implies that new highways are likely to be an important element needed to explain urban population decentralization. For a metropolitan area with half of its residents living in the central city absent any highways, simulation results indicate that the first few highway rays each cause about a 10 percent decline in central city population, with the marginal effect declining 1 or 2 percentage points for each additional ray. Simulations of the model yields effects of new highways that are quantiatively consistent with the empirical results in Baum-Snow (2006). The range of simulation estimates in this paper imply that construction of new highways can account for as much as one-third of the decline in aggregate central city U.S. population relative to national population growth.

This paper proceeds as follows. Section 2 proposes the model. Section 3 proves a few relevant analytical results using this model. Section 4 presents simulation results using two different utility functions and extends the analysis to incorporate congestion. Finally, Section 5 concludes.

# 2 The Model

Each metropolitan area is treated as its own atomistic general equilibrium system. There is a continuum of $N$ individuals, each of whom commutes to the central work location and earns an exogenously given wage $w$ per unit time. Households have direct preferences over a composite consumption good $z$ and space $s$ given by $U(z, s)$. $U$ is increasing and weakly concave in each of its arguments and both goods are weakly normal. Because all individuals are identical, an equilibrium consists of a situation in which everybody has the same utility $u$ and cannot gain higher utility by moving to an alternate residential location.

Highways are modelled as rays emanating from the city's core along which the travel speed is $\frac{1}{b\gamma}$. Travel at speed $\frac{1}{b}$ is possible along any ray from the origin and along any line perpendicular to a highway. $\gamma$ is interpreted as the speed ratio on surface streets to highways. Rays are distributed evenly such that they serve the maximum number of people possible. $M$ denotes the number of rays in the metropolitan area. I index space in polar coordinates $(r, \phi)$, where $\phi$ is the angle to the nearest highway ray. The individual living at each location $(r, \phi)$ chooses the commuting route to minimize her total travel time. Given this commuting technology, the minimum time it takes to travel from the center $(0, 0)$ to $(r, \phi)$ is:[1]

$$L(r, \phi) = \min[br, br\widetilde{L}(\phi)] \tag{1}$$

where for most of the analysis I take $\widetilde{L}(\phi) = \gamma\cos(\phi) + \sin(\phi)$. As such, individuals use the highway for part of their commutes if $\phi < \widetilde{\phi}$ where $\widetilde{\phi}$ solves $\widetilde{L}(\phi) = 1$.[2] Define $\overline{\phi} = \min[\widetilde{\phi}, \frac{\pi}{M}]$. If $\widetilde{\phi} > \frac{\pi}{M}$, everybody uses the highway for part of their commutes. $\overline{\phi}$ is the angle at which individuals are indifferent between using the highway for part of their commutes and commuting directly downtown on surface streets. If $\gamma = 0.5$ there exist people who do not use a highway for any part of their commutes for $M < 5$. I set the pecuniary cost of commuting to 0.[3]

Individuals' total time endowment is normalized to 1 such that time is spent either working or commuting. The price of land is given by $R(r, \phi)$ and the price of the consumption good is normalized to 1. As such, the following equation represents

[1] Inspection of maps reveals that most city streets travel along straight lines. I use the technology $\widetilde{L}(\phi) = \gamma\cos\phi + \sin\phi$ because it yields the most conservative effects of a new ray in the simulations while retaining travel along straight lines only. Anas and Moses (1979) use the technology $\widetilde{L}(\phi) = \gamma + \phi$ in which travel along any ray through the center or around any circle centered at the CBD is possible at speed $\frac{1}{b}$.

[2] Solving this equation, $\widetilde{\phi} = 2\arctan\left(\frac{1-\gamma}{1+\gamma}\right)$

[3] As long as pecuniary travel cost is an increasing linear function of $L(r, \phi)$, its exclusion does not affect the qualitative urban form implied by this model. Provided the wage is high enough that pecuniary travel cost is a small fraction of income, it will not matter quantitatively either.

each individuals' resource constraint:

$$z + R(r,\phi)s = w[1 - L(r,\phi)] \tag{2}$$

$R(r,\phi)$ is determined endogenously as described in Section 3 below. I denote the compensated demand functions for the composite good and space as $\widetilde{z}(R(r,\phi),u)$ and $\widetilde{s}(R(r,\phi),u)$ respectively. Equilibrium demand for space at $(r,\phi)$ given M rays I denote as $s^M(r,\phi)$. I denote the distance from the central business district to the border between the central city and the suburbs as $r_c$. This variable is exogenously determined and serves purely as a boundary within which to count the change in population as highway rays are added to the metropolitan area.

Each ray influences commuting time in the wedge bounded by angle $\overline{\phi}$ on either side of it. In the remaining land within the angle $2\pi - 2M\overline{\phi}$, commuting time is not affected by the rays. Without loss of generality, I distribute rays to influence commuting time between 0 and $2M\overline{\phi}$ radians only. Metropolitan areas are large enough to accommodate the mass of $N$ individuals with the urban fringe $r_f^M(\phi)$ endogenously determined by the equilibrium rent function and the market rental rate for rural land. Denote $\overline{r}_f^M = r_f^M(\phi)$ for the region $\phi \geq \widetilde{\phi}$, if it exists. Since equalization of utility implies that the travel time from the fringe to the center is the same at every point on the fringe, the fringe is captured by the following function:

$$\begin{aligned} r_f^M(\phi) &= \frac{\overline{r}_f^M}{\widetilde{L}(\phi)} \text{ if } 0 \leq \phi \leq \overline{\phi} \\ &= \overline{r}_f^M \quad \text{if } \phi > \overline{\phi} \end{aligned} \tag{3}$$

The bid-rent function for space determines the market land rent for space at each location. It is defined as the maximum an individual would be willing to pay to reside at location $(r,\phi)$ given utility level $u$:

$$\psi[L(r,\phi),u] = \max_s \left\{ \frac{w[1 - L(r,\phi)] - Z(s,u)}{s} \right\} \tag{4}$$

$Z(s,u)$ is achieved by inverting the utility function.

## 3 Equilibrium Land Use and Highways

This model retains the primary standard results of the classic monocentric city model, except that the uniform linear correspondence between travel time and distance to the center is broken. As seen by differentiating Equation (4), land rent is decreasing and convex in travel time from the city center. Since all individuals are identical, equilibrium land rent at each location equals the bid-rent. The equilibrium land use pattern in the metropolitan area is thus determined jointly by the demand function for space $\widetilde{s}(\psi(br\widetilde{L}(\phi), u^M), u^M)$, a condition stating everyone has a place to live and an equation that equalizes rent at the edge of the populated area to rural land rent:

$$\psi(b\overline{r}_f^M, u^M) = \max_s \left\{ \frac{w[1 - b\overline{r}_f^M] - Z(s, u^M)}{s} \right\} = R_a \tag{5}$$

Solving equation (5), we have $\overline{r}_f^M = q(u^M)$. Substitution of the demand function and the rearranged fringe rent function into the market clearing condition for space yields:

$$\begin{aligned} N &= 2M \int_0^{\overline{\phi}} \int_0^{\frac{q(u^M)}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]} \\ &\quad +(2\pi - 2M\overline{\phi}) \int_0^{q(u^M)} \frac{rdr}{\widetilde{s}[\psi(br, u^M), u^M]} \end{aligned} \tag{6}$$

Equation (6) determines the equilibrium utility level $u^M$ in the city. Fujita's (1989) discussion of existence and uniqueness of the equilibrium applies with the replacement of distance to the center with travel time.

Figure 1 presents a schematic diagram of the spatial structure of the city and what happens to urban form when a new ray is introduced. A new highway ray represents a decline in commuting time for a sector of the metropolitan area. This elicits two effects. First, the price of land decreases because more land is accessible for each given commuting time. Holding the agricultural land rent $R_a$ constant, this means that for $\phi > \overline{\phi}$, the fringe moves in towards the center. The decrease in land rent causes agents to increase land consumption via a price effect, inevitably pushing some people further from the center and lowering population density as a result. Second, average net income rises, causing people to consume more land (assuming land is normal) through a wealth effect, also pushing them away from the center. Since people consume more of both space and the composite consumption good, the new highway causes the equilibrium utility level in the city to rise. In addition, the highway causes the residential land area of the metropolitan area to rise.

Proposition 1 formalizes these results. It states that an additional ray always causes (i) equilibrium utility in the metropolitan area to rise, (ii) the portion of the urban fringe in a region where nobody uses the highway for part of her commute to move inwards and (iii) the portion of the urban fringe on the new highway ray to move outward.

*Proposition 1.* If $M' = M + 1$ then in equilibrium

$$\text{i) } u^{M'} > u^M \quad \text{ii) } \overline{r}_f^{M'} < \overline{r}_f^M \quad \text{iii)} \frac{\overline{r}_f^{M'}}{\gamma} > \overline{r}_f^M$$

**Proof** See Appendix A.

An intuitive way to see the structure of different urban equilibria with highways is through examination of bid-rent functions. Indeed, bid-rent and population density are proportional assuming several standard specifications of the utility function. All utility functions that fulfill the standard assumptions generate 0 population density at the fringe rent of $R_a$. Figure 2 shows how the changing shape of the bid-rent function implies the different city structures in 0, 1 and 2 ray environments. Each line in the figure is a bid-rent function conditional on the angle to the nearest ray and the number of rays in the city. The first argument gives the travel time to the CBD and the second argument gives the equilibrium utility level. The travel time of $rb$ indicates that the bid-rent function is relevant for $\phi > \overline{\phi}$ while the travel time $\gamma rb$ indicates that the bid-rent function is relevant for $\overline{\phi} = 0$. The superscript on the second argument $u$ gives the number of rays in the city.

As is a standard result from monocentric theory, an increase in the utility level in the city is associated with a decline in bid-rent for the same travel speed. This increase in utility associated with extra rays causes the bid-rent function to shift down and intersect $R_a$ at a distance that is closer to the CBD. Given a utility level $u^M$, the bid-rent functions are the same at the center since travel time is equal at 0. As seen in Figure 2, as $\phi$ increases from 0 to $\overline{\phi}$, the bid-rent function gets steeper, representing the increased relative value individuals place on being near the CBD due to the lower average commuting speed. Bid-rent along the ray intersects $R_a$ at a distance exceeding the fringe absent the ray. This reflects the fact that in order to house the full population, some extra space has to be claimed along the highway.

While the average population density in the metropolitan area declines with new highways, one area of the city sees increased population density. Individuals move to live near the new ray and enjoy shorter commuting times. As such, an area of the city near the urban fringe and near the highway sees increased population density as a result of the new highway. To see that population density increases near the highway but decreases elsewhere, it is instructive to compare population density in the 0 and 1 ray equilibria at the coordinates $(\overline{r}_f^0, 0)$ and $(\overline{r}_f^1, \overline{\phi})$ as an example. Note that while

$$\frac{1}{s^1(\overline{r}_f^0, 0)} > \frac{1}{s^0(\overline{r}_f^0, 0)}$$

it is also true that

$$\frac{1}{s^1(\overline{r}_f^1, \overline{\phi})} < \frac{1}{s^0(\overline{r}_f^1, \overline{\phi})}$$

These two conditions are readily seen in Figure 1. In addition, note that

$$\frac{1}{s^1(\varepsilon, 0)} < \frac{1}{s^0(\varepsilon, 0)} \tag{7}$$

for $\varepsilon$ arbitrarily small. Relationship (7) holds because from Proposition 1, $u^1 > u^0$ and $\psi_u < 0$, but commuting cost declines only marginally, implying that $\psi(\varepsilon\gamma b, u^1) < \psi(\varepsilon b, u^0)$. Therefore, demand for space increases at this point through both price and income effects and population density declines as a result. Thus if $r_c$ is sufficiently small, central city population is assured to decline with each new highway ray. This result makes up the main argument in the proof of Proposition 2.

*Proposition 2.* For $M' = M+1$ and $r_c$ sufficiently small, population of the central city assuming $M'$ rays is less than population of the central city assuming $M$ rays.

Proof Central city population is given by:

$$N_c = 2M \int_0^{\overline{\phi}} \int_0^{r_c} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]} + (2\pi - 2M\overline{\phi}) \int_0^{r_c} \frac{r dr}{\widetilde{s}[\psi(br, u^M), u^M]} \tag{8}$$

From Proposition 1, $u$ is increasing in $M$. Using the inequalities $\psi_u < 0$, $\frac{\partial \widetilde{s}}{\partial R} < 0$ and $\frac{\partial \widetilde{s}}{\partial u} > 0$,

$$\frac{1}{\widetilde{s}[\psi(br, u^{M'}), u^{M'}]} < \frac{1}{\widetilde{s}[\psi(br, u^M), u^M]}, r > 0$$

In addition,

$$\frac{1}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^{M'}), u^{M'}]} < \frac{1}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]}, 0 < r < r^*(\phi), 0 < \phi < \overline{\phi}$$

Since at all angles from the center population density falls at some small radius, population density falls in the entire region near the center of the city for an increase in rays.

# 4 Simulations

While the previous section develops the mechanism by which new highway rays in the monocentric model cause declines in urban population density, further restrictions on the model are required to determine the magnitude of these declines. This section addresses this question through simulation. It is important to note that the model specified in this paper is developed in order to facilitate simulation. The model's parsimony means that only a few parameters need be chosen. Furthermore, the choice of commuting technology limits the magnitude by which highways can influence central city population loss. The requirement that commuters must access the highway by using a perpendicular road limits the influence of new highways more than any other technology in which travel only along straight lines is possible. As such, with an appropriate specification of preferences, these simulation results represent a near lower bound on the effect of highways on central city population as implied by this model. After presenting results from simulating the model presented in Section 2, I extend the analysis to incorporate congestion.

## 4.1 Basic Results

I focus primarily on simulation examples using the utility function $U = z + \alpha \ln(s)$. Quasilinear utility is a convenient preference specification for two reasons. First, the quasilinear utility function's income elasticity of demand for space of 0 limits the response of population density to faster transportation infrastructure to be driven only by a price effect. Secondly, quasilinear utility is convenient because results are very stable over a wide range of values for the shape parameter of the utility function, the wage, base travel speed and metropolitan area population. Appendix B derives the mathematical expressions used for the simulations and describes the procedure used.

The simulation exercise follows the evolution of central city population as rays are added to the metropolitan area. Taking central city population in the 0 ray equilibrium as given, Table 1 traces out the evolution of the change in central city population as a function of the number of rays. Each row shows the effect on central city population of each marginal ray, holding the radius of the central city fixed. The central city radius is determined by the fraction of metropolitan area population assumed to reside in the central city listed in the left-most column. In 1950, about 50 percent of metropolitan area populations lived in central cities both in the aggregate and on average in the United States. Each panel displays the log difference in central city population under different assumptions about speeds on surface streets and highways. Changing these two speeds while holding the speed ratio constant does not affect results. Results are nearly identical for reasonable values of metropolitan area population, the wage and the shape parameter of the utility function.

Results in Table 1 show that the first ray causes between a 7 and 18 percent decline in central city population assuming half of the population resides in the central city in the 0 ray equilibrium. Panel A shows that if speed on the highway is double the speed on surface streets, in a 0-ray equilibrium in which half the population lives in the central city, the first ray causes the log central city population to fall by 0.11, with the fall in central city population declining 1-2 percentage points for each additional ray. The percent decline in central city population falls with increases in central city population share in the 0 ray equilibrium and equivalently with increases in the central city radius. Panels B and C show that the magnitudes of these marginal effects are decreasing in the speed ratio on surface streets to highways. Given a speed ratio of $\frac{25}{60}$, the first few rays cause 18, 14 and 11 percent central city population declines respectively. Given a speed ratio of $\frac{35}{60}$, the first few rays cause 7, 6 and 5 percent declines respectively[4] These simulation results are invariant to reasonable values of $w$, $N$ and $\alpha$. Note that because these results are invariant to these deep parameters, structural estimation of the model using quasilinear preferences would yield the results in Table 1.

Table 2 presents analogous results using the Cobb-Douglas utility function. These results show effects within 0.04 greater in absolute value because Cobb-Douglas preferences imply an income elasticity of demand for space of 1 as compared to that of 0 implied by quasilinear preferences. Assuming a speed ratio of 2, metropolitan areas with half of their population in the central city lose about 13 percent of their population for the first ray, 11 percent for the second ray and 9 percent for the third ray. This effect declines by about 1 percentage point for each additional ray there-

[4] Simulation results using the Anas-Moses (1979) commuting technology in which highway users must travel around a circle centered at the CBD to access a highway give effects within 0.02 smaller in magnitude than those reported in Table 1.

after. Panels B and C of Table 2 show that Cobb-Douglas results exhibit similar magnitudes the quasilinear results assuming both higher and lower speed ratios on highways to surface streets. The magnitude of these results are robust to other reasonable choices of $w, N$ and the shape parameter of the utility function. They change by less than 5 percentage points with changes in $N$ and the shape parameter.

## 4.2 Adding Congestion

The model presented in Section 2 assumes that the transport infrastructure is not congestible. In this subsection, I extend the model to handle congestion in a simple way and simulate it. I relegate the consideration of congestion to an extension of the model presented in Sections 2 and 3 for two reasons. First, the existence of congestion means that the simulation results from the previous subsection will no longer be invariant to metropolitan area population. Second, empirically congestion did not start to become a major source of commuting time loss until the 1990s.

A straightforward way of extending the analysis to incorporate congestion, similar to that employed by Anas and Moses (1979), is to make $\gamma$ an increasing function of the population affected by a highway:

$$
\begin{aligned}
\gamma &= f\left(\int_0^{\overline{\phi}} \int_0^{\frac{\overline{r}_f^M}{\widetilde{L}(\phi)}} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^M), u^M]}\right) \\
f' &\geq 0
\end{aligned} \tag{9}
$$

Denote $\gamma_M$ to be the equilibrium value of $\gamma$ given $M$ rays. Congestion represents another force pushing population density up near the center of the city in response to an increase in $N$ because $\frac{dN_c}{d\gamma} > 0$ and $\frac{df}{dN} > 0$. Similarly, this formulation of congestion implies that *ceteris paribus,* more rays lead to a weak decline in the equilibrium value of $\gamma$ because they induce some highway users to move from using other rays, thereby reducing the population around each ray. Thus holding the number of lanes constant, the effect of the first ray on central city population is smaller than it would be without congestion. The effect of the second ray is larger than it would be if $\gamma$ were fixed at $\gamma_1$ because the next ray causes $\gamma$ to fall to $\gamma_2$. The parameterization of the function $f$ and the profiles of the price and wealth effects of demand for space as a function of $\gamma$ determine whether the response of the central city population to the second ray in a world with congestion is less or greater than in a world without congestion.[5]

I evaluate the potential importance of congestion by simulating the model allowing $\gamma$ to be determined by the equilibrium population using the highway. I take the formula for speed on congested highways from the Texas Transportation Institute's (TTI) 2004 Mobility Report, assuming that each new highway is 4 lanes in each direction and that 2 individuals commute together in each vehicle.[6] Given that only about half the U.S. population commutes to work outside the home, the 2 person per car assumption allows the model to better capture commuting patterns for observed population levels. Reduced highway speeds start at an average annual daily traffic of 13,260 vehicles.

Unlike the results presented in the previous subsection, simulation results incorporating congestion are sensitive to the population of the metropolitan area. Table 3 presents simulation results with $\gamma$ endogenized to account for the number of road users according to the formula detailed in Appendix Table 1. Numbers in Table 3 are calculated using the same parameters as are used to simulate Table 1 Panel A. With this formulation of the congestion function and transportation infrastructure, congestion starts to reduce travel speeds when the metropolitan area reaches about 500,000 people. As such, Panel A reports simulation results for a metropolitan area of this size. Panel A shows that congestion reduces the reduction in central city population caused by the first ray by up to 4 percentage points. Five rays is enough infrastructure to eliminate all congestion in this case.

Table 3 Panel B reports results for a metropolitan area of 1 million inhabitants.

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[5] Vickrey (1969) proposes a more complicated formulation which in the context of this model would manifest itself as a "flow congestion" term in the travel time function: $L(r, \phi) = \min[br, br(\gamma \cos\phi + \sin\phi) + \lambda \int_0^{r\cos\phi} \left(\frac{N(v)}{t(v)}\right)^k dv]$ where $N(v)$ is the number of commuters using the highway between $v$ and the edge of the MSA and $t$ is the throughput of the highway.

[6] I alter the TTI's congestion function slightly in order to make it monotonic. Details are in Appendix Table 1.

A metropolitan area of 1 million inhabitants with half its population in the central city in the 0 ray equilibrium sees central city population drop by 3 percent for the first ray, 4 percent for the second, third and fourth rays and 5 percent for the fifth ray. At 10 rays, the spatial distribution of the population looks the same as in the uncongested case because there is enough transportation infrastructure to bring $\gamma$ back to its uncongested value.

Data collected by the Texas Transportation Institute indicates that congestion is not likely to be a major force mitigating highways' influence on changing residential land use patterns since 1950. Among the 139 largest metropolitan areas in 1950, the average ratio of free-flow to congested traffic speeds on limited access highways was 1.16 in 1990. Therefore, it appears that communities built up their highway infrastructure almost sufficiently to fulfill the increased travel demand associated with their rising and decentralizing populations.

# 5 Conclusions

This paper proposes a land use and commuting model that incorporates radial highways. I show that new highways affect urban form by causing the population to spread out along the highways. In addition, holding the population of the metropolitan area constant, the urban fringe in areas not near the highway moves inwards. This simple model implies that highway construction has induced a sea change in urban land use patterns. Results from simulating the model imply that indeed new highways are likely to have had a sizable impact on central city populations. Applying the simulation results in Table 1 Panel A to observed average highway construction of about 2.5 rays per metropolitan area, counterfactual central city population estimates imply that the full decline of 17 percent in aggregate central city population can be explained by highway construction. Baum-Snow (2006) shows that the magnitude of empirical estimates of the effect of highways on central city population are similar to those in the simulation results reported in this paper.

The simple stylized mechanism of the monocentric model generates simulation results with magnitudes that are quite robust to different metropolitan area structures. Similarly, the analytical results require only the standard weak regularity assumptions about the utility function. The cost of using this mechanism is that it is highly stylized. Indeed, casual empirical observation reveals that employment decentralization has occurred apace with residential decentralization, a phenomenon that is ignored in the model presented here. While others have proposed models that endogenize employment location, these models have few general analytical comparative static implications. As such, it is valuable to understand the extent to which a simple tractable model featuring highway construction can explain suburbanization.

# A Proof of Proposition 1

If $M' = M + 1$ then in equilibrium

**i)** $u^{M'} > u^M$: Compare a city with 0 rays and 1 ray. Equation (6) implies that

$$
\begin{aligned}
0 &= 2\int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^1), u^1]} \\
&\quad + (2\pi - 2\overline{\phi}) \int_0^{q(u^1)} \frac{r dr}{\widetilde{s}[\psi(br, u^1), u^1]} - 2\pi \int_0^{q(u^0)} \frac{r dr}{\widetilde{s}[\psi(br, u^0), u^0]}
\end{aligned} \tag{10}
$$

Suppose that $u^0 > u^1$. I prove by contradiction that utility cannot be decreasing in $M$. Using Equation (5), the Implicit Function Theorem and the Envelope Theorem, $\frac{d\overline{r}_f}{du} = -\frac{-\frac{\partial z(s,u)}{\partial u}}{-wb} < 0$ so $q(u^1) > q(u^0)$. Using the fact that space is a normal good, $\widetilde{s}[\psi(L(r,\phi), u^1), u^1] < \widetilde{s}[\psi(L(r,\phi), u^0), u^0]$. These two conditions imply that

$$
2\pi \left[ \int_0^{q(u^1)} \frac{r dr}{\widetilde{s}[\psi(br, u^1), u^1]} - \int_0^{q(u^0)} \frac{r dr}{\widetilde{s}[\psi(br, u^0), u^0]} \right] > 0 \tag{11}
$$

and applying Equation (11) to (10) we have

$$
\int_0^{\overline{\phi}} \int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}} \frac{r dr d\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi), u^1), u^1]} < \overline{\phi} \int_0^{q(u^1)} \frac{r dr}{\widetilde{s}[\psi(br, u^1), u^1]} \tag{12}
$$

But

$$\overline{\phi}\int_0^{q(u^1)}\frac{rdr}{\widetilde{s}[\psi(br,u^1),u^1]} = \int_0^{\overline{\phi}}\int_0^{q(u^1)}\frac{rdrd\phi}{\widetilde{s}[\psi(br,u^1),u^1]}$$
$$< \int_0^{\overline{\phi}}\int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}}\frac{rdrd\phi}{\widetilde{s}[\psi(br,u^1),u^1]}$$
$$< \int_0^{\overline{\phi}}\int_0^{\frac{q(u^1)}{\widetilde{L}(\phi)}}\frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]}$$

which contradicts (12). Thus, it must be that $u^1 > u^0$. An analogous argument follows for all $M > 0$.

**ii)** $\overline{r}_f^{M'} < \overline{r}_f^M$: To understand how equilibrium land use changes with M, we must first understand how the equilibrium land rent function changes with M. We can express land rent in terms of the bid-rent function:

$$\psi(L(r,\phi),u) = \max_s\left\{\frac{w[1-L(r,\phi)]-Z(s,u)}{s}\right\} \tag{13}$$

Using the envelope theorem, $\frac{\partial\psi}{\partial u} < 0$. Thus given result i, areas of the city with no change in travel times see land rents fall with $M$. Therefore, since $R_a$ does not change, fringe distance in these same areas also falls with $M$.

**iii)** $\frac{\overline{r}_f^{M'}}{\gamma} > \overline{r}_f^M$: Once again, consider the case of moving from a regime with 0 rays to a regime with 1 ray. Result i states that utility rises with M and ii) shows that equilibrium land rent falls with M for $\phi > \overline{\phi}$. Thus since space is a normal good, $\widetilde{s}[\psi(L(r,\phi),u^1),u^1] > \widetilde{s}[\psi(L(r,\phi),u^0),u^0]$ in the region $\phi > \overline{\phi}$. Also note from result ii that $\psi(0,u^1) < \psi(0,u^0)$.

To examine the $\phi < \overline{\phi}$ region, it is instructive to think about the shape of the bid-rent function for land in the 0-ray equilibrium compared to that in the 1-ray equilibrium at $\phi \geq \overline{\phi}$ and $\phi = 0$. The derivative of the rent function as a function of

$r$ is:

$$\psi_r = -\frac{w\min[b, b\widetilde{L}(\phi)]}{\widetilde{s}[\psi(L(r,\phi),u),u]} \tag{14}$$

The rent function is thus less steep in the region $\phi < \overline{\phi}$ than in the remainder of the metropolitan area. Further, the rent function at all points is less steep in the 1-ray equilibrium than the 0-ray equilibrium. Define $r^*(\phi)$ to solve $\psi(rb\widetilde{L}(\phi),u^1) = \psi(rb,u^0)$ for the region $\psi(rb,u^0) > R_a$. Given the normality of land and the fact that the fringe distance is the furthest from the center at $\phi = 0$, it must also be true that for $r \leq r^*(\phi)$, $s^1(r,\phi) > s^0(r,\phi)$. Using the market clearing condition for space and the result that $\overline{r}_f^1 < \overline{r}_f^0$ :

$$\begin{aligned} & N - (2\pi - 2\overline{\phi})\int_0^{\overline{r}_f^0} \frac{rdr}{\widetilde{s}[\psi(br,u^0),u^0]} - 2\int_0^{\overline{\phi}}\int_0^{r^*(\phi)} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^0),u^0]} \\ > \; & N - (2\pi - 2\overline{\phi})\int_0^{\overline{r}_f^1} \frac{rdr}{\widetilde{s}[\psi(br,u^1),u^1]} - 2\int_0^{\overline{\phi}}\int_0^{r^*(\phi)} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]} \end{aligned}$$

or

$$2\int_0^{\overline{\phi}}\int_{r^*(\phi)}^{\frac{\overline{r}_f^1}{\widetilde{L}(\phi)}} \frac{rdrd\phi}{\widetilde{s}[\psi(br\widetilde{L}(\phi),u^1),u^1]} > 2\int_0^{\overline{\phi}}\int_{r^*(\phi)}^{\overline{r}_f^0} \frac{rdrd\phi}{\widetilde{s}[\psi(br,u^0),u^0]}$$

That is, there remain more people to be housed in the region $r \in [r^*(\phi),\infty)$ X $\phi \in (0,\overline{\phi})$ in the 1-ray equilibrium than the 0-ray equilibrium. $r^*(\phi)$ must exist for some $\phi$, otherwise not everybody could be housed in the 1-ray equilibrium. By definition of $r^*$ and the fact that $\psi_r(b\gamma r,u^1) > \psi_r(br,u^0)$, rent must be greater in the 1-ray equilibrium than the 0-ray equilibrium in the region $r > r^*(\phi)$. Because $R_a$ is the same in both equilibria, the extent of the city at $\phi = 0$ must be greater in the 1-ray equilibrium than the 0-ray equilibrium. The same argument follows for all $M > 0$. *Q.E.D.*

# B Derivations

This appendix derives the expressions used to simulate the model assuming quasi-linear and Cobb-Douglas utility functions.

## B.1 Quasilinear Utility

This is a variant of the example worked out in Glaeser and Kahn (2003). The utility function is $U = z + \alpha \log(s)$. Using $-\frac{\partial Z}{\partial s} = \psi(L(r, \phi), u)$,

$$\frac{\alpha}{s} = \frac{w[1 - L(r, \phi)] - u + \alpha \log s}{s}$$

Solving for s yields

$$s = e^{\frac{u}{\alpha}+1} e^{-\frac{w}{\alpha}} e^{\frac{w}{\alpha}L(r,\phi)} \tag{15}$$

At the urban fringe, we have $R_a = \psi(b\overline{r}_f^M, u^*)$ giving us

$$e^{\frac{u}{\alpha}+1} = \frac{\alpha}{R_a} e^{\frac{w}{\alpha}(1-b\overline{r}_f^M))} \tag{16}$$

where $\overline{r}_f^M$ is the fringe at $\phi > \overline{\phi}$ assuming $M$ rays. This expression justifies that since everyone must have the same utility, equalizing utility at the fringe implies $r_f^M(\phi) = \frac{\overline{r}_f^M}{\widetilde{L}(\phi)}$.

Combining the expressions for utility and demand for space yields:

$$s^*(r, \phi) = \frac{\alpha}{R_a} e^{-\frac{w}{\alpha}b\overline{r}_f^M} e^{\frac{w}{\alpha}L(r,\phi)} \tag{17}$$

Using the land market clearing condition yields an equation that determines $\overline{r}_f^M$:

$$N = \frac{R_a}{\alpha} e^{\frac{w}{\alpha}b\overline{r}_f^M} (\frac{wb}{\alpha})^{-2} \left[ \begin{array}{c} (\frac{wb}{\alpha})^2 2M \int_0^{\overline{\phi}} \int_0^{\frac{\overline{r}_f^M}{\widetilde{L}(\phi)}} re^{-\frac{w}{\alpha}rb\widetilde{L}(\phi)} dr d\phi \\ +(2\pi - 2M\overline{\phi})[1 - e^{-\overline{r}_f^M wb/\alpha} - \overline{r}_f^M \frac{wb}{\alpha} e^{-\overline{r}_f^M wb/\alpha}] \end{array} \right] \tag{18}$$

In the simulations, the integral with respect to $\phi$ was solved numerically. The expression in the text for $N_c$ is the same as that listed above for $N$, except the upper limit of integration with respect to $r$ is $r_c$.

## B.2 Cobb-Douglas Utility

This is a variant of the example worked out in Chapter 3 of Fujita (1989). The utility function is $U = z^a s^\beta$. Using $-\frac{\partial Z}{\partial s} = \psi(L(r,\phi), u)$,

$$-e^{\frac{u}{\alpha}}(-\frac{\beta}{\alpha})s^{-\frac{\beta}{\alpha}-1} = \frac{w[1-L(r,\phi)] - e^{\frac{u}{\alpha}}s^{\frac{-\beta}{\alpha}}}{s}$$

Solving for s yields

$$s^*(r,\phi,u) = e^{u/\beta}\alpha^{-\alpha/\beta}w^{-\alpha/\beta}(1-L(r,\phi))^{-\alpha/\beta} \tag{19}$$

At the urban fringe, we have $R_a = \psi(\overline{r}_f^M, u^*)$ giving us

$$e^{\frac{u}{\beta}} = \frac{\beta}{R_a}w^{1/\beta}\alpha^{\alpha/\beta}(1-\overline{r}_f^M b)^{1/\beta} \tag{20}$$

where $\overline{r}_f^M$ is the fringe at $\phi > \overline{\phi}$ assuming $M$ rays. This expression justifies that since everyone must have the same utility, equalizing utility at the fringe implies $r_f^M(\phi) = \frac{\overline{r}_f^M}{\widetilde{L}(\phi)}$.

Combining the expressions for utility and demand for space yields:

$$s^*(r,\phi) = \frac{\beta}{R_a}(1-\overline{r_f}^M b)^{1/\beta}w(1-L(r,\phi))^{-\alpha/\beta} \tag{21}$$

Using the land market clearing condition yields an equation that determines $\overline{r}_f^M$:

(22)

$$N = (1-\overline{r}_f^M b)^{-\frac{1}{\beta}}\frac{R_a}{w}\frac{1}{1+\beta}\left[\begin{array}{c} 2M(1+\beta)\int_0^{\overline{\phi}}\int_0^{\frac{\overline{r}_f^M}{\widetilde{L}(\phi)}} r[1-rb\widetilde{L}(\phi)]^{\frac{\alpha}{\beta}}drd\phi \\ +(2\pi - 2M\overline{\phi})[(1-b\overline{r}_f^M)^{\frac{\alpha}{\beta}}((\overline{r}_f^M)^2 - \frac{\alpha\overline{r}_f^M}{b} - \frac{\beta}{b^2}) + \frac{\beta}{b^2}] \end{array}\right]$$

In the simulations, the integral with respect to $\phi$ was solved numerically. The expression for $N_c$ is the same as that listed above for $N$, except the upper limit of integration with respect to r is $r_c$:

$$
\begin{aligned}
N_c &= (1 - L(\overline{r}_f))^{-\frac{1}{\beta}} \frac{R_a}{w} \frac{1}{1+\beta} \\
&* \left[ \begin{array}{c} 2M \int_0^{\overline{\phi}} (1 - br_c \widetilde{L}(\phi))^{\frac{\alpha}{\beta}} (r_c^2 - \frac{\alpha r_c}{b\widetilde{L}(\phi)} \\ - \frac{\beta}{b^2 \widetilde{L}(\phi)^2}) + \frac{\beta}{b^2 \widetilde{L}(\phi)^2} d\phi \\ +(2\pi - 2M\overline{\phi})[(1 - br_c)^{\frac{\alpha}{\beta}} (r_c^2 - \frac{\alpha r_c}{b} - \frac{\beta}{b^2}) + \frac{\beta}{b^2}] \end{array} \right]
\end{aligned} \tag{23}
$$

I impose the constraint $\alpha + \beta = 1$.

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## Figure 1: The Effect on Urban Form of a New Ray

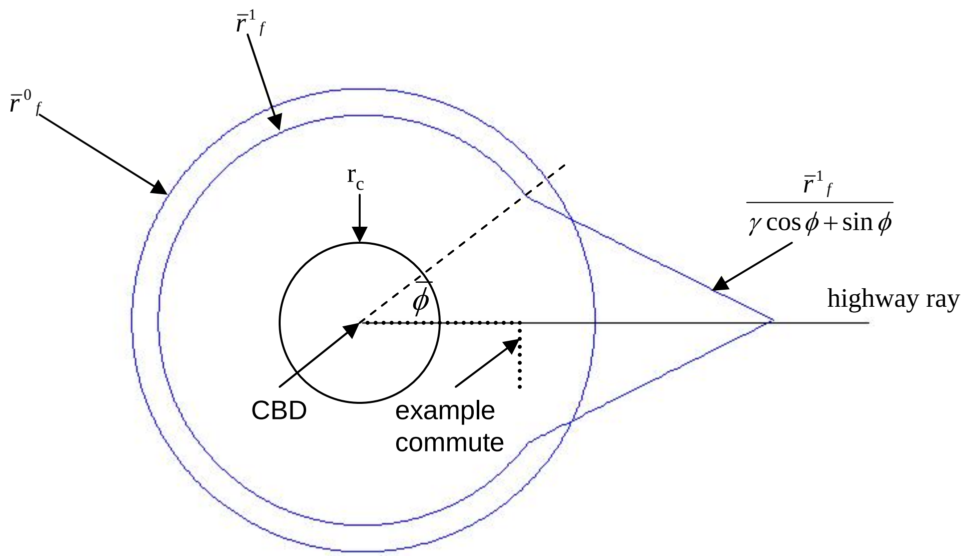
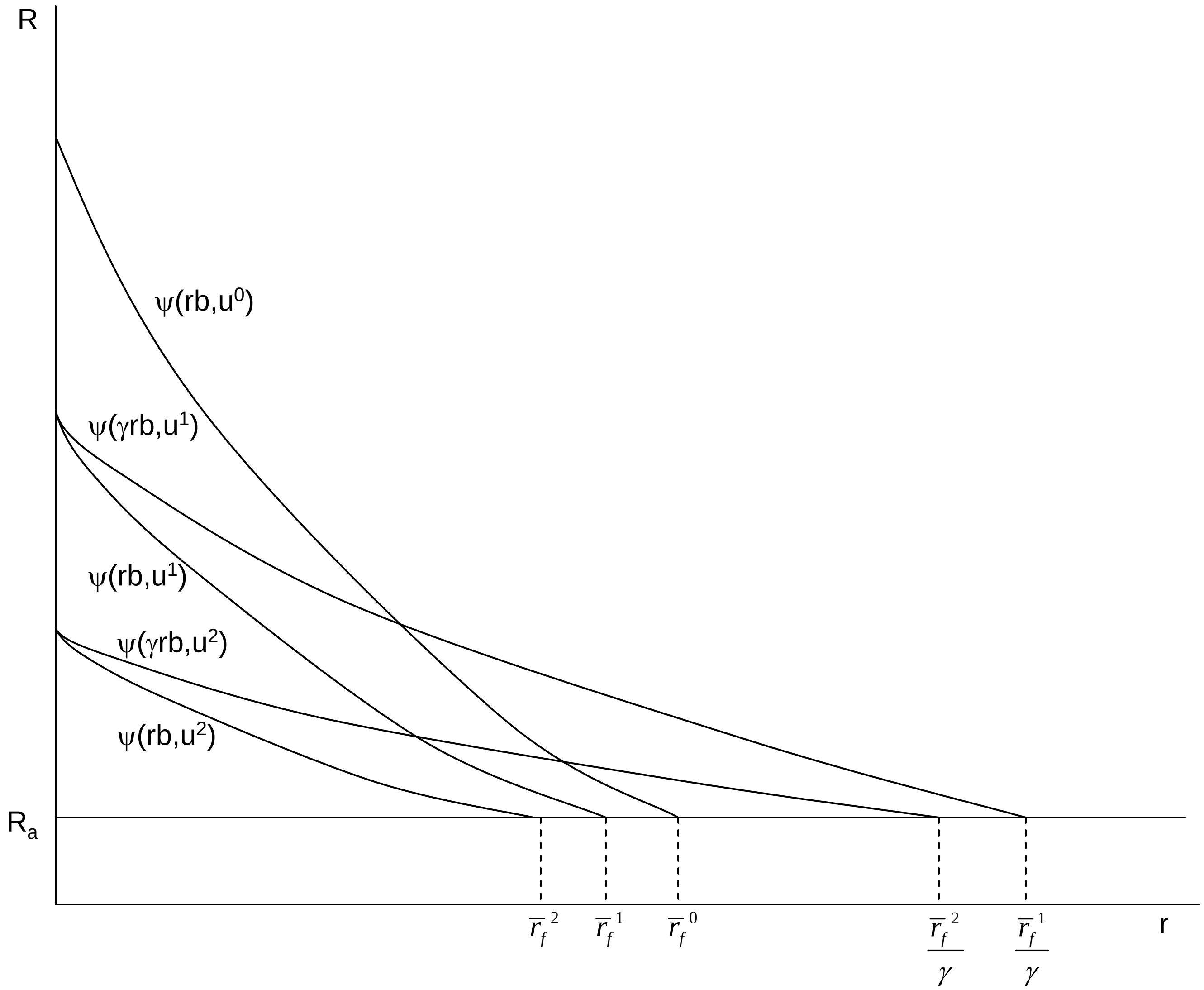


Figure 2

Graphical Depiction of Rent Functions in 0, 1 and 2 Ray Equilibria



## Table 1: Simulations Using Quasilinear Utility
## Δ Log Center City Population for a Marginal Ray

### Panel A: 30 mph on Surface Streets, 60 mph on Highways

| Fraction in CC with 0 Rays | Number of Rays 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.10 | -0.15 | -0.12 | -0.11 | -0.09 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 |
| 0.25 | -0.13 | -0.11 | -0.10 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.50 | -0.11 | -0.09 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 |
| 0.75 | -0.08 | -0.07 | -0.05 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 |

### Panel B: 25 mph on Surface Streets, 60 mph on Highways

| Fraction in CC with 0 Rays | Number of Rays 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.10 | -0.23 | -0.18 | -0.15 | -0.12 | -0.10 | -0.08 | -0.06 | -0.05 | -0.04 | -0.04 |
| 0.25 | -0.21 | -0.16 | -0.13 | -0.11 | -0.09 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 |
| 0.50 | -0.18 | -0.14 | -0.11 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.75 | -0.14 | -0.10 | -0.08 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 | -0.02 |

### Panel C: 35 mph on Surface Streets, 60 mph on Highways

| Fraction in CC with 0 Rays | Number of Rays 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.10 | -0.09 | -0.08 | -0.07 | -0.07 | -0.06 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.25 | -0.08 | -0.07 | -0.06 | -0.06 | -0.05 | -0.05 | -0.04 | -0.04 | -0.03 | -0.03 |
| 0.50 | -0.07 | -0.06 | -0.05 | -0.05 | -0.04 | -0.04 | -0.03 | -0.03 | -0.02 | -0.02 |
| 0.75 | -0.05 | -0.04 | -0.04 | -0.03 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 | -0.01 |

Notes: The utility function used is $U = z + .3\ln(s)$. Holding gamma constant, reported results do not change for all reasonable travel speeds on surface streets. Gamma gives the ratio of surface street travel speed to highway travel speed. Results are invariant to reasonable wage levels and metropolitan area populations. Magnitudes are within .01 for other reasonable coefficients on ln(s).

## Table 2: Simulations Using Cobb-Douglas Utility
## $\Delta$ Log Center City Population for a Marginal Ray

### Panel A: 30 mph on Surface Streets, 60 mph on Highways

| Fraction in CC | Number of Rays | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| with 0 Rays | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
| 0.10 | -0.15 | -0.13 | -0.11 | -0.10 | -0.09 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 |
| 0.25 | -0.14 | -0.12 | -0.10 | -0.09 | -0.08 | -0.07 | -0.05 | -0.04 | -0.04 | -0.03 |
| 0.50 | -0.13 | -0.11 | -0.09 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.75 | -0.11 | -0.09 | -0.07 | -0.06 | -0.05 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 |

### Panel B: 25 mph on Surface Streets, 60 mph on Highways

| Fraction in CC | Number of Rays | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| with 0 Rays | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
| 0.10 | -0.24 | -0.19 | -0.15 | -0.13 | -0.11 | -0.08 | -0.07 | -0.06 | -0.05 | -0.04 |
| 0.25 | -0.23 | -0.18 | -0.14 | -0.12 | -0.10 | -0.08 | -0.06 | -0.05 | -0.04 | -0.04 |
| 0.50 | -0.21 | -0.16 | -0.13 | -0.10 | -0.08 | -0.07 | -0.05 | -0.05 | -0.04 | -0.03 |
| 0.75 | -0.18 | -0.13 | -0.10 | -0.08 | -0.07 | -0.05 | -0.05 | -0.04 | -0.03 | -0.03 |

### Panel C: 35 mph on Surface Streets, 60 mph on Highways

| Fraction in CC | Number of Rays | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| with 0 Rays | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
| 0.10 | -0.09 | -0.08 | -0.08 | -0.07 | -0.06 | -0.06 | -0.05 | -0.04 | -0.04 | -0.03 |
| 0.25 | -0.09 | -0.08 | -0.07 | -0.06 | -0.06 | -0.05 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.50 | -0.08 | -0.07 | -0.06 | -0.05 | -0.05 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 |
| 0.75 | -0.06 | -0.06 | -0.05 | -0.04 | -0.04 | -0.04 | -0.03 | -0.03 | -0.02 | -0.02 |

Notes: The utility function used is $U = .7\ln(z)+.3\ln(s)$. The time endowment is 10 hours per day. The wage is set to 100 per day. Metropolitan area population is set to 1 million. Results are at most 0.02 smaller for a metropolitan area population of 100,000 and within 0.01 for a metropolitan area population of 10 million.

## Table 3: Quasilinear Utility With Congestion
## Δ Log Center City Population for a Marginal Ray

### Panel A: Population of 500,000

| Fraction in CC | Number of Rays | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| with 0 Rays | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
| 0.10 | -0.11 | -0.12 | -0.12 | -0.11 | -0.10 | -0.07 | -0.06 | -0.05 | -0.04 | -0.03 |
| 0.25 | -0.10 | -0.10 | -0.10 | -0.10 | -0.08 | -0.06 | -0.05 | -0.04 | -0.03 | -0.03 |
| 0.50 | -0.09 | -0.09 | -0.08 | -0.08 | -0.07 | -0.05 | -0.04 | -0.03 | -0.03 | -0.02 |
| 0.75 | -0.06 | -0.06 | -0.06 | -0.06 | -0.05 | -0.03 | -0.03 | -0.02 | -0.02 | -0.02 |
| gamma | 0.543 | 0.529 | 0.517 | 0.505 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |

### Panel B: Population of 1 million

| Fraction in CC | Number of Rays | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| with 0 Rays | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 |
| 0.10 | -0.05 | -0.05 | -0.06 | -0.06 | -0.07 | -0.08 | -0.18 | -0.12 | -0.08 | -0.05 |
| 0.25 | -0.04 | -0.05 | -0.05 | -0.06 | -0.06 | -0.07 | -0.16 | -0.10 | -0.07 | -0.04 |
| 0.50 | -0.03 | -0.04 | -0.04 | -0.04 | -0.05 | -0.05 | -0.13 | -0.09 | -0.06 | -0.04 |
| 0.75 | -0.02 | -0.03 | -0.03 | -0.03 | -0.03 | -0.04 | -0.09 | -0.06 | -0.05 | -0.03 |
| gamma | 0.675 | 0.664 | 0.653 | 0.640 | 0.626 | 0.611 | 0.551 | 0.523 | 0.506 | 0.500 |

Notes: The utility function used is U = z+.3ln(s). Each panel shows simulation results assuming that the speed on surface streets is 30 mph. Speed reductions due to congestion only occur on highways according to the function given in Appendix Table 1. The values given for gamma apply to the larger number of rays listed in the column headers. Results are invariant to reasonable wage levels. Magnitudes are within .01 for other reasonable coefficients on ln(s).

## Appendix Table 1: The Congestion Function

| TTI Congestion Function | | Congestion Function Used for Simulations | |
|---|---|---|---|
| Annual Average Daily Traffic *(thousands)* | Travel Speed On Highway *(miles per hour)* | Annual Average Daily Traffic *(thousands)* | Travel Speed On Highway *(miles per hour)* |
| Less than 15 | 0 | Less than 13.26 | 0 |
| 15 - 17.5 | 74.45-1.09*aadt | 13.26 - 17.57 | 74.45-1.09*aadt |
| 17.5 - 20 | 109.76-3.1*aadt | 17.57 - 20.59 | 109.76-3.1*aadt |
| 20 - 25 | 135.08-4.33*aadt | 20.59 - 24.44 | 135.08-4.33*aadt |
| Greater than 25 | 72.03-1.75*aadt | Greater than 25 | 72.03-1.75*aadt |

Notes: The TTI congestion formula is found in Schrank & Lomax (2004). It is altered for the purposes of the simulations because near the kink points it is not monotonic. The only difference between the two functions is the set of points at which the linear functions being evaluated change.